Multi-room Heating System Modeled as a Dynamical System

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*Abstract*—Everyday cyber-physical systems can be represented as dynamical systems and modeled as such. In this paper, we describe the modeling of a multi-room (four) heating system as a dynamical system using MATLAB’s Simulink add-on. Given that all our assumptions for the model hold, it can regulate all room temperatures within a reasonable range. While the model is a largely simplified version of a realistic situation, it does well to simulate the basic interactions that take place to influence the dynamics of room temperatures with a heating system.

Keywords—Dynamical Systems, Heating System

# Introduction

For modeling a multi-room heating system, we cannot use an asynchronous-only model with fairness conditions to properly illustrate all the factors that contribute to the fluctuating temperatures of each room; there would not be enough synchrony among tasks to ensure that the temperatures are updated simultaneously. We instead focus our attention dynamical system modeling, as we can better describe the changes in the room temperatures as the dynamics of such a system with ordinary differential equations (ODEs). Additionally, the time synchrony between components is a better representation of the real-world factors that simultaneously drive the changes in temperature.

Consider now a heating system that regulates the temperature over four connected rooms with two heaters. The temperature of each room changes linearly with the difference with the outside temperature , the difference with each directly adjacent room’s temperature, and a heater’s current effect on the room. Then the dynamics of the system can be represented by the following equation:

where and , and , and , and . represents the influence that rooms have on each other. If the rooms are not directly adjacent, then . Additionally, if . represents the influence that the outside temperature has on room . represents the influence that a heater has on room . if there is no heater in room or if the heater is off and if a heater is on in the room. The system is parameterized by the constant matrix , the constant vector , the constant vector , and the initial room temperatures in the vector .

Additionally, there are constant vectors that are used to determine the placement of the heaters and whether they are on or off. If a heater is in room , then the heater is off if and on if . A heater is moved from room to room if: (1) room has no heater, (2) room has a heater, (3) , and (4) .

The inputs to the system are the vector for the initial placement of the heaters and . The outputs are the room temperatures .

The scenario we examined to build our model of the multi-room heating system was parameterized as:

With inputs:

For this parameterization, we aimed to keep for all .

# Assumptions

## are reasonable starting temperatures.

* Even though our model receives inputs, once these inputs are received, the system acts as a closed-loop system. The outside temperature, , remains constant once it is received as input and heater placement is maintained internally between time-steps.
* is chosen relative to the values of . For example, it would make no logical sense to have for all , but then have . (Do we need this?)
* The proportional temperature influence that the rooms have on each other, modeled by the matrix , is constant. The effects that the outside temperature and heaters, modeled by and respectively, have on each room are constant as well. This allows the dynamics to be modeled in a simplistic manner.

## Heaters can be moved instantaneously.

## A room cannot have more than one heater at a time.

* No uncontrolled inputs, such as people, other appliances, sunlight, etc. are considered.

## While modeled as a continuous time system, it must be run as a discrete time system. The model assumes a small step size for calculating dynamics.

# Related Work

We modeled a system with a relatively simple design. There were a limited number if heaters and the room temperatures were only affected by adjacent rooms and the outside temperature. We also simplified thermodynamic principles and airflow to be encompassed by a single temperature for each room. Further, the dynamic for our system were linear in order to make analysis of our model easier.

We did not take into account additional environmental factors like humidity or sunlight. Additionally, our system did not drive the room temperatures towards particular reference values as a PID controller might. Instead, we relied on logic that setting certain parameter values for would bound the temperatures within a certain range.

Most current houses and buildings use HVAC systems for temperature regulation rather than only a heating system. Thus, they are designed to perform temperature regulation in a wider range of temperatures and climates. Also, these modern systems have many more components and considerations to take into account than we looked at for our model.

Al-Rousan et al. [1] presented control and modeling of an HVAC system for a single room using dynamical system analysis. Whereas we abstracted away the heating method, they took into consideration the thermodynmaics of airflow for heat transfer. Their model works to regulate both temperature and humidity, and consisted of a humidifier, heater, cooler, fan, ductwork, and sensors for monitoring temperature and humidity. Their model has multiple components, each controlling one of these parts of the overall system. The dynamics over the state varaibles for each of these components are based on thermodynamic principles.

Inputs to their controller are temperature and humidity of the supplied air from the HVAC as well as the mass flow rate of the air. The outputs are the temperature and humidity of the room. The humidity and temperature were driven to set values using PID control. For added precision in converging to an ideal temperature and humidity level, the PID controller coefficients are modified based on whether the system is trying to heat vs. cool or humidify vs. de-humidify the room.

In addition to considering humidity as a factor for temperature, Al-Rousan et al. monitored temperature of the inner walls of the room as state variables with additional dynamics for these in the model. The temperature of the walls are also factored into the dynamics for the ambient room temperature and vise versa.

Compared to our model, which only looked at a high level version of romm temperatures, the model proposed by Al-Rousan et al. considered a much wider variety of environmental factors and a more complex temperature regulation system.

Peng and Passen [2] proposed a model for air conditioning (AC) that ours is more similar to as opposed to that proposed by Al-Rousan et al. Peng and Passen desinged a model more suited for user satisfaction. They separated the room into defined zones, with the idea being that the space near the air conditioner should be separate from the designated ‘working zone’ that inhabitants occupied in order to have better control over the room temperature of the ’working zone’.

Whereas we looked at the influence of multiple rooms on each other, they looked at the influence that nearby spaces, or zones, within a room have on each other. The dynamics for their system were based on Computational Fluid Dynamics theory, whereas our dynamics were a simplified version of heat flow. Despite this, the overall structure of the dynamics was quite similar to ours in that adjacent zone temperatures, or room temperatures for our model, directly influence the temperatures of adjacent zones. However, we considered the effects of outside temperature and heater status as influencing factors for room temperature dynamics whereas Peng and Passen considered the air flow from the AC as the only other factor influencing the state variables of zone temperature.

# Design

The entire system was split into the dynamics subsystem and the heater subsystem. The heater subsystem determined the placement of the two heaters based on the current temperature and placement of the heaters. A heater’s placement in a room was decided based on if it had the biggest difference between the temperature for the room and the actual temperature of the room in the last cycle. Before a heater was moved, the temperature of the potential room selection was compared to the room’s value. The difference between the heater’s current room temperature and potential new room termperature was also checked to ensure it was less than or equal to the room’s value. Whether the heater was on or off was checked after heater placement was determined by comparing the room’s current temperature with the and values of the room. A schematic of the heater subsystem can be seen in Fig 1.

Schematic of Heater Subsystem

# Results & Discussion

The system was simulated with the input values given in the introduction. The output temperature for each of the rooms for a 100 second simulation can be seen in Fig 2.

Output temperatures over 100 seconds

All four temperatures quickly settle around a value and stay within the intended range of 15 to 20 ℃. The system has BIBO stability as the temperatures oscillate around a value but never converge. Room 2 has a higher value than the other rooms due to the lower value, which means that the room is less influenced by outside temperature. This means the other rooms are cooled more than room 2, resulting in a higher value. The other three rooms are also ordered by their b value, with the highest b value having the lowest temperature and the lowest b value having the highest temperature.

The number of heaters turned on throughout the simulation can be seen in Fig. 3.

Number of heaters over time

The number of heaters fluctuates between one and two, indicating that there is always at least one heater on and never more than the two heaters that exist.

Heater boolean values over time

Fig. 4 shows the values for each room as they are turned on and off over the 100 second simulation. All the values appear to fluctuate equally throughout the simulation, which indicates that no room is being starved.

Output temperatures over 10 seconds

Fig. 5 shows a 10 second simulation of the output temperatures of the system with the original input values. Various input values were changed to examine their impact on the system; this figure is being shown as a comparison reference.

Output temperatures for increased values

The original values were changed from to . With the decreased values, the heaters have more freedom to move between rooms when a temperature dips too low. The main differences that can be seen are that the room 4 temperature sits higher, and the room 2 temperature sits lower than before. The room temperatures are more clearly in the order of their values and their values appear to be mainly influenced by the external temperature.

Output temperatures for decreased values

Fig. 7 shows the output temperatures when the values are . The increased values make it more difficult for heaters to change between rooms. Rooms need to have a significant difference in temperature between each other for heaters to move between them. This causes the system to push the heaters apart, and they settle on split values, with rooms 1 and 2 oscillating around values about 3 ℃ above rooms 3 and 4.

Output temperatures with a decreased value for room 2

Fig. 8 shows the output temperatures when the value for room 2 was decreased to 16.5. The room 2 output temperature was previously higher than the other output temperatures but when the value was decreased, the room 2 output temperature matches that of room 1. The value affects when the heater is turned on, so the lowered value for room 2 means that the temperature must be lower for a heater to turn on in that room.

Output temperatures with decreased values

Fig. 9 shows the output temperatures when the on values are changed from to . This decrease causes all rooms to have to have a lower temperature for a heater to turn on, which decreases the temperatures of rooms 1 and 2, which previously had settled to values of above 17. The temperatures in rooms 3 and 4 stayed about the same as they were already below 17. In an ideal case, the values would be matched to the values for each room in order to compensate for how each room is affected by the outside temperature.

Output tempatures with increased values

Fig. 10 shows the output temperatures when the values are changed from to . The output temperatures in rooms 1, 3, and 4 are all increased in this scenario. The high values allow for the rooms to get the heater more often, as they do not need to dip as low in order to get a heater.

Output temperatures with an external temperature of 15

Fig. 11 shows the output temperatures when the external temperature, , is increased to 15. The u value’s influence on each room is represented by the values. Rooms 3 and 4 are influenced the least by outside temperature, and at the higher value of 15 they only get a heater at the very beginning of the simulation. Rooms 1 and 2 are more heavily influenced by the external temperature and thus they get the heaters more often and end up at a higher temperature.

Output temperatures with an external temperature of 19

Fig. 12 shows the output temperatures when the value is increased to 19. This is an important test case as it shows the system’s ability to turn all heaters off completely when they are not needed. The temperatures fluctuated a bit at the beginning of the simulation but quickly settled on the value of 19 for all rooms. The system becomes asymptotically stable in this scenario.

h values when the external temperature is 19

Fig. 13 shows the values when the external temperature is 19. The heaters turn on and off some at the beginning of the simulation but quickly turn off completely as all rooms converge to 19.

Output temperatures with an external temperature of 0

Fig. 14 shows the output temperatures when the value is 0. The simulation time has been extended to more clearly show the temperatures that the rooms settle to and oscillate around. It takes longer for the system to settle to consistent temperatures with a very low external temperature. All room temperatures settle to values between 10 and 14 ℃, indicating that the heating subsystem can heat rooms by 10 to 14 ℃. This aligns with Fig. 5, where the external temperature is 6 ℃ and the room temperatures remain between 15 and 20 ℃.

Output temperatures with different initial heater placements

Fig. 15 shows output temperatures when the heaters are initially places in rooms 3 and 4 instead of rooms 2 and 3. The temperatures settle around the same values as in Fig. 5, which indicates the system is fair and the initial positions have little to no impact on the values the system outputs settle to.

# Conclusions And Further Work

In this work, we designed a model for a multi-room heating system consisting of 4 rooms. We implemented and verified the model using MATLAB’s Simulink. The results show that our model is able to maintain room temperatures within a specified range.

In order to improve our model, we could make heater placement predictive rather than reactive. In order to do so, heater placement would need to take into account the values of the matrix and the vectors . While more computationally intensive, such a design could help narrow the temperature range the rooms are kept in.

Another alternative would be to model a more complex heater that can vary its heat output on a continuous range. That way, we could use a PID controller to drive the room temperatures to a more precise value rather than keep it within a particular range.

1. Table Type Styles

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1. Sample of a Table footnote. (*Table footnote*)

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