Automation in Construction 20 (2011) 459–473

Contents lists available at ScienceDirect

Automation in Construction

journal homepage: www.elsevier.com/locate/autcon

A decision-making system for construction site layout planning

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ARTICLE INFO

Article history:
Accepted 4 November 2010
Available online 9 December 2010

Keywords:
Decision-making system
Construction site layout planning
Max–min ant system
Pareto-based ant colony optimization algorithm

ABSTRACT

A decision-making system, which consists of input, design, evaluation and selection, and output stages, is proposed to solve dynamic, multi-objective and unequal-area construction site layout planning (CSLP) problem. In the input stage, the multiple objectives, schedule planning and site condition are determined. In the design stage, two mathematical optimization models max–min ant system (MMAS) and modified Pareto-based ant colony optimization (ACO) algorithm are employed to solve single objective optimization (SOO) and multi-objective optimization (MOO) problem respectively. In the evaluation and selection stage, the intuitionistic fuzzy TOPSIS method is used to evaluate and select the best layout plan among the generated layout alternatives from the design stage. The performance of the proposed decision-making system, which was verified by a residential building project, shall assist the practitioners in the construction industry to deliver construction projects in a more efficient and effective manner, and thus construction costs could be reduced significantly.

1. Introduction

Construction site layout planning (CSLP) has been recognized as a critical step in construction planning by practitioners and researchers. CSLP is a decision-making process, which involves identifying problems and opportunities, developing solutions, choosing the best alternative and implementing it. Therefore, designing a good site layout plan is a decision-making process that involves first the determination of the CSLP objectives, which are usually multiple, and the layout constraints. Second, identification of the site facilities and the available site space (sometimes it is unequal-area site), which is dynamic during the construction progress. Third, generation of the construction site layout alternatives, which fulfill the layout objectives and constraints, and finally evaluating and selecting the best site layout plan for implementation.

In our previous research [1], a new method using continuous dynamic searching scheme to guide the max–min ant system (MMAS) algorithm, which is one of the ant colony optimization (ACO) algorithms, to solve the dynamic CSLP problem under the two congruent objective functions of minimizing safety concerns and reducing construction cost is proposed. This paper aims to develop a computationally intelligent CSLP decision-making system which consists of four stages: the input stage, the design stage, the evaluation and selection stage, and the output stage to help project managers and planners to solve dynamic and unequal-area CSLP problems under multiple objectives.

2. Construction site layout planning

After extensive literature reviews of previous CSLP research works and discussions with project managers and practitioners of construction operations, some CSLP problems are identified as follows:

First, if the requirements of the construction work were changed during the progress of a construction project, the site layout should be altered accordingly. Under this situation the CSLP problems should be regarded as dynamic problems. However, literature reviews showed that most of the previous CSLP research works concentrated on the static CSLP problems [2–4].

Second, reviews of previous CSLP research works [5–7] showed that the site facilities had been shaped as rectangular blocks, by which they could be assigned to any location. Academically it is termed as *equal-area CSLP problem*. However, this theoretically convenient approach did not consider whether those locations could actually accommodate the assigned facilities in reality. This hypothetical assumption is far away from the real-life situation in CSLP. Moreover, limited research works had been carried out to solve unequal-area CSLP problems [8]. Therefore, there is a research gap in solving unequal-area CSLP problems.

Third, most of the CSLP research works concentrated on the improvement of the computational capability of various optimization algorithms, by which a number of site layout alternatives could be generated for evaluation and selection. Nevertheless little research

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0926-5805/$ – see front matter © 2010 Published by Elsevier B.V.
doi:10.1016/j.autcon.2010.11.014

effort has been placed on how to evaluate and select the final, or the best, construction site layout from the generated site layout alternatives. As a matter of fact that CSLP is basically a trade-off problem, in which construction managers and site planners need to make a decision to select the best construction site layout among the site layout alternatives generated from various optimization algorithms. This approach requires construction managers and planners to consider and balance all the important attributes, such as minimizing costs, safe working environment and efficient material transportation, without sacrificing the required quality. Moreover, some of these important attributes, such as security and noise control are not easy to handle and it is extremely difficult for decision makers to justify which construction site layout should be selected.

Finally, many researchers treated CSLP as a single-objective optimization (SOO) problems, for instance, minimizing the frequency of trips made by construction personnel, minimizing the total transportation costs of resources between facilities, and minimizing the cost of construction facilities or the interactive costs between facilities. Until now, only few researchers have treated CSLP as a multi-objective optimization (MOO) problem, which involves more than a single objective function [9].

In order to solve the aforementioned problems, this study aims to develop a computationally intelligent CSLP decision-making system which consists of four stages: the input stage, the design stage, the evaluation and selection stage, and the output stage (see Fig. 1) to help project managers and planners to solve dynamic, multiple objective and unequal-area CSLP problems.

In this study, the multiple objectives defined in the input stage are minimizing the likelihood of accidents happened to improve the safety level (f_1) and minimizing the total handling cost of interaction flows between the facilities associated with the construction site layout (f_2), which is based on interaction relationship between the facilities.

In the design stage, two optimization models, one is SOO model and the other is MOO model, are employed to solve multiple objective CSLP problems, which could be treated as either SOO problem or MOO problem. Max–min ant system (MMAS) and Pareto-based ACO algorithm are deployed into the SOO model and MOO model to solve the multiple objective CSLP problems respectively. Moreover, the unequal-area grids recognition strategy and continuous dynamic searching scheme [1] are adopted to guide the two ACO algorithms to solve dynamic, multiple objective and unequal-area CSLP problems.

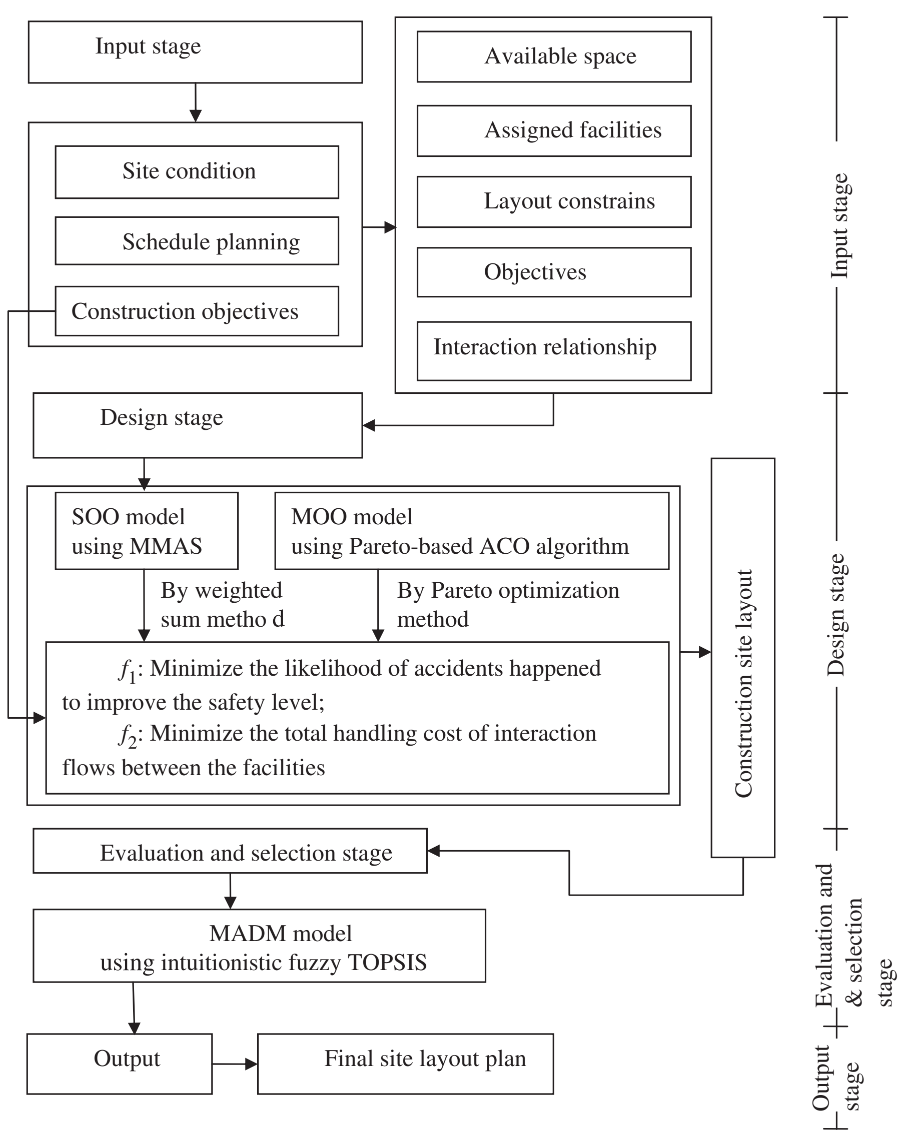


Fig. 1. The flowchart of the proposed CSLP decision-making system.

Table 1
The pseudo-code of MMAS.

Step	Function
1	Select assignment sequence for the facilities
2	Initialize pheromone information τ and define heuristic information η
3	Determine the maximum iteration number N
4	Determine the total ant number m
5	Initialize the iteration number $n=1$
6	Initialize pheromone information τ and define heuristic information η
7	For each ant k
8	Set a parameter q_0 between [0, 1].
9	Generate a random variable q, which is uniformly distributed over [0,1]
10	If $q \le q_0$, an ant k assigns the next unassigned facility i to a free location j in terms of arg max$_{l \in N_i^k}\{\tau_{ij}^{\alpha}[\eta_{ij}]^{\beta}\}$; otherwise, the probability rule $p_{ij}^k(t) = \frac{[\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k}[\tau_{il}(t)]^{\alpha} \cdot [\eta_{il}]^{\beta}}$ is adopted. α and $\beta$are the parameters that determine the relative influence of the pheromone information and the heuristic information respectively, N_i^k is the feasible neighborhood of node i
11	Update the pheromone informationτ, If after the pheromone update we have $\tau \rangle \tau_{\max}$, we set $\tau = \tau_{\max}$; if $\tau \langle \tau_{\min}$, we set $\tau = \tau_{\min}$.
12	Update the iteration number $n = n+1$
13	**If** $i<N$, go to step 7
14	Output the solution (site layout alternatives)

When solving multiple objective CSLP problems, a number of construction site layout alternatives are generated in the design stage of the proposed decision-making system. In the evaluation and selection stage, a multiple attribute decision making (MADM) model using intuitionsitic fuzzy TOPSIS is employed to evaluate and select the best construction site layout plan from the site layout alternatives generated in the design stage. The evaluation attributes such as security, easy supervision and control, ease of expansion, which are neglected in the previous CSLP studies and difficult to quantify, are considered in this study. The rest of this paper is organized in the following manner: first, the structure of the proposed CSLP decision-making system is introduced. Second, the functions and the algorithms used in each stage will be discussed one by one in the following sections. Third, a case study of the residential building project is used to verify the proposed CSLP decision-making system. Finally, a conclusion is drawn.

3. The structure of CSLP decision-making system

The system structure of the proposed CSLP decision-making system is summarized as below:

The components and methods employed at each stage of the proposed CSLP decision-making system would be discussed below.

3.1. Input stage

There are three components: site conditions, schedule planning and construction objectives in the input stage. From the site conditions and the schedule planning of the construction project, the space availability, the site facilities, the dynamic site layout time interval and the layout

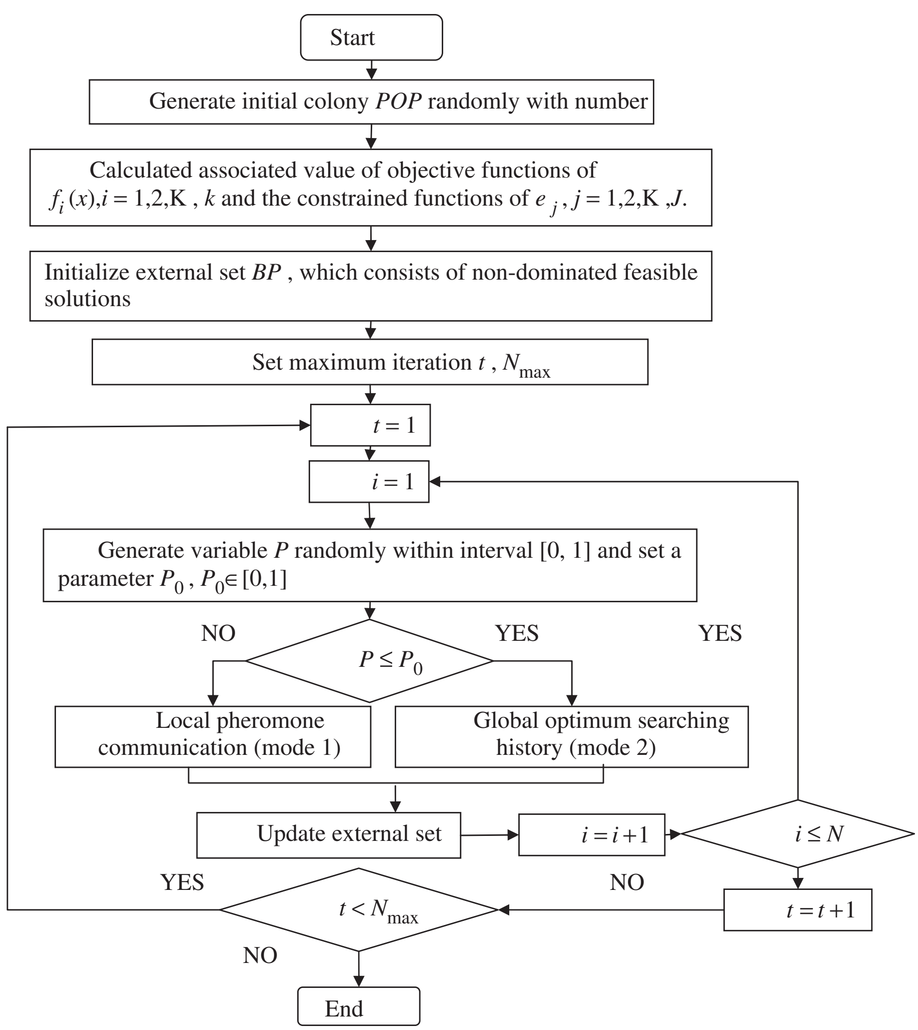
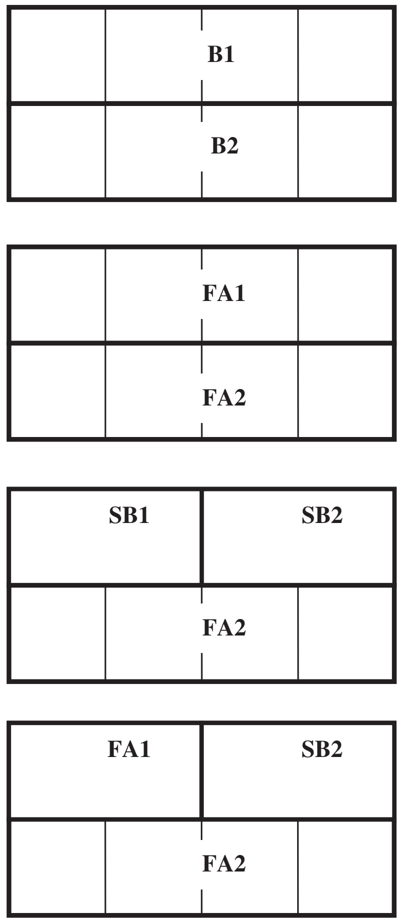


Fig. 2. The workflow of the modified Pareto-based ACO algorithm.



Step 1: Divide the whole construction site into two blocks (B1 and B2) by the maximum facilities' sizes. In this case, the maximum size of the two facilities is 100 ㎡ .Thus the whole construction site is divided into two blocks (B1 and B2) with an area of 100㎡.

Step 2: Assign the facilities of FA1 with sizes of 50㎡ and FA2 with sizes of 100㎡ into two blocks with size of 100㎡ randomly without considering the constraints of locations' sizes.

Step 3. Divide the block of B1 occupied by the FA1 into smaller blocks (SB1 and SB2) with area of 100㎡ equal to area of FA1.

Step 4. Assign the facility of FA1 to the blocks of SB1 and SB2 randomly. The smaller blocks SB1 and SB2 have areas of 50㎡. The facility FA1with an area of 50㎡ can be assigned to any small blocks randomly.

Fig. 3. Unequal-area grid recognition strategy employed in the SOO model and MOO model.

constraints are firstly derived. From the component of construction objectives, the multiple objectives are defined and the interaction relationships are then calculated. In the case study of the proposed decision-making system, the multi-objective functions defined were minimizing the likelihood of accidents happened and minimizing the total handling cost of interaction flows between the facilities. The factors considered in the interaction relationship were material flows, information flows, personnel flows, equipment flows, safety/environment concerns and users' preference) [10,11]. Having the data of the qualitative factors involved in the interaction relationships derived via a questionnaire, the fuzzy rule-based system [12,13] was employed to calculate the interaction relationships between the facilities.

3.2. Design stage

The main function of the design stage of the proposed CSLP decision-making system is the generation of construction site layout alternatives, in terms of the objective functions, for the evaluation and selection stage. In order to have more quality candidate solutions or alternatives for evaluation and selection, the two ACO algorithms were applied to optimize objective functions in the dynamic site layout time interval. In the SOO model, MMAS, which is one of the extensions of ACO algorithms [14] conjoins the weighted sum method, is used to solve the multiple objective CSLP problems, by which the multi-objective functions are transformed into single objective function, and thus MOO problem was mitigated as SOO problem. In the MOO model, the Pareto-based ACO algorithm conjoins the Pareto optimization method [9], which is used to solve continuous combinatorial optimization problem, is being modified to solve discrete CSLP problem in this study. Moreover, the unequal-area grid recognition strategy and continuous searching scheme are employed to guide the two ACO algorithms to solve unequal-area and dynamic CSLP problems.

3.2.1. SOO model using MMAS

In the SOO model MMAS conjoins the weighted sum method, which is a method that scalarizes a set of objectives into a single objective by pre-multiplying each objective with a user-supplied weight, is employed to solve the dynamic, multiple objective CSLP problems. By the weighted sum method, the two objectives (f_1 and f_2) are transformed into single objective function $f$as follows:

$$f = w_1 \cdot f_1 + w_2 \cdot f_2 \quad (1)$$

Where w_1 and w_2 are the weights of objective functions. The task of MMAS is to find the optimal solution for the single objective function $f$in accordance with the principle of ants' searching food. The steps to realize MMAS [15,16] is introduced in Table 1.

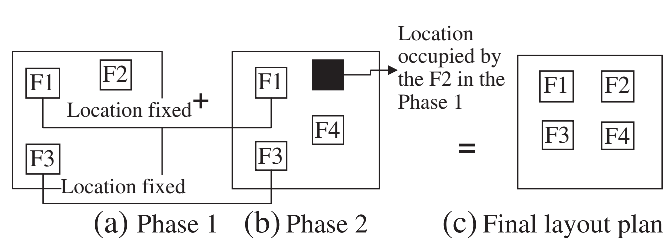


Fig. 4. The continuous dynamic searching scheme employed in the SOO model and MOO model.

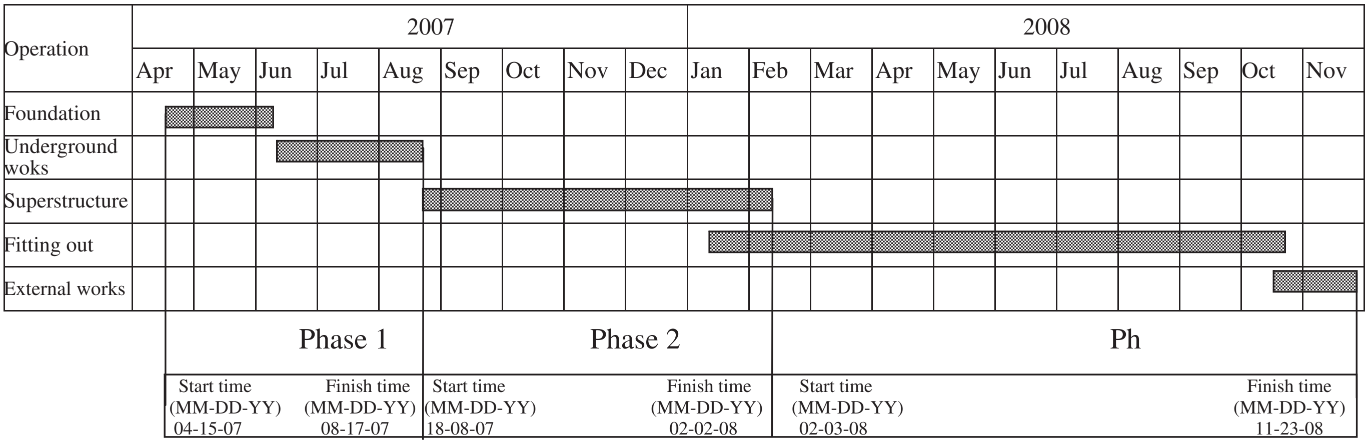


Fig. 5. The key date in each construction phase of residential building.

The capabilities of MMAS outperformed the other two main ACO algorithms (ant colony system and ant system) as MMAS first enhances the optimization searching capability by exploiting the best solution found during iteration or during the run of the algorithm. After each iteration, only one best single ant (iteration-best ant or global-best ant) adds pheromone. Second, MMAS limits the range of possible pheromone trails on each solution component to an interval $[\tau_{\min},\tau_{\max}]$ to avoid stagnation of the searching process [15,16].

3.2.2. MOO model using the Pareto-based ACO algorithm

Zhang and Huang [17] proposed the Pareto-based ACO algorithm, in which the principle of Pareto optimization was applied into ACO algorithm, to solve continuous combinatorial optimization problem. In order to modify the original Pareto-based ACO algorithm to solve discrete CSLP problem in this study, two modifications are needed. First, the searching space in CSLP problem is the discrete location in the construction site. In the modified Pareto-based ACO algorithm, the artificial ants shall search from one location to another location directly within the discrete searching space. Second, the parameter of random disturbance factor defied in the original Pareto-based ACO algorithm to solve continuous optimization problem is not applicable in this modified model as the searching location is limited and discrete in the construction site.

Table 2
Site facilities and their dimensions of residential building.

Temporary and fixed facilities (facility number)	Facility type	Number of facilities	Dimension of facilities (m^2)	Serviced phase
Sample room (F1)	non-fixed	1	25	3
Equipment maintenance plant (F2)		1	25	1,2,3
Electrician hut (F3)		1	25	1,2,3
Tool shed (F4)		1	50	1,2,3
Rebar bending yard (F5)		1	100	1,2
Carpentry workshop (F6)		1	100	1,2,3
#1 Material (laydown) area (F7)		1	100	1,2,3
#2 Material (laydown) area (F8)		1	100	1,2,3
#1 Tower crane (F9)	fixed	1	50	1,2
#2 Tower crane (F10)		1	50	1,2
#1 Material hoist (F11)		1	25	2,3
#2 Material hoist (F12)		1	25	2,3
#3 Material hoist (F13)		1	25	2,3
#4 Material hoist (F14)		1	25	2,3
Site office (F15)		1	50	1,2,3
Security hut at site entrance (F16)		1	9	1,2,3
Security hut at site exit (F17)		1	9	1,2,3

The modified Pareto-based ACO algorithm consists of two searching modes: searching mode based on the local pheromone communication (mode 1) and searching mode based on the global optimum searching history (mode 2). The workflow of the modified Pareto-based ACO algorithm is shown in Fig. 2.

For the mode 1 (see the flowchart of Fig. 2), the choice of the ant i is related to the pheromone density of the ant j released and the distance between the ant i and the ant j. The higher the pheromone density, the closer the distance and the greater the probability of ant j is selected. The probability P_j of choosing ant j is

$$P_j = \frac{\theta_j \delta_{ij}}{\sum_{j=1}^{m} \theta_j \delta_{ij}}, j \neq i, j = 1, 2, \cdots, N. \tag{2}$$

Where $\delta_{ij} = 1/d_{ij}$, where d_{ij} is the distance between x_i and x_j. θ_j is pheromone density released in terms of

$$\theta_j = \begin{cases} \lambda_1, & \text{if } x_j \text{ is non} - \text{feasible solution;} \\ \lambda_2, & \text{if } x_j \text{ is feasible solution and } x_i \prec x_j; \\ \lambda_3, & \text{if } x_j \text{ is feasible solution and} \\ & x_i, x_j \text{ is non} - \text{constrianed dominated relation;} \\ \lambda_4, & \text{if } x_j \text{ is feasible solution and } x_j \prec x_i. \end{cases} \tag{3}$$

Where x_i and x_j is the associated solution of the ant $i, j = 1, 2, \cdots, N$, $(i \neq j, N$ is the ant number); $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the four parameters for

Table 3a
The optimal results via SOO model in phase 1.

Construction phasing	Parameters setting		Convergence status within 100 steps	Optimal results	Best results of fixed value ρ
1	$\rho = 0.5$	$\alpha = 2$ $\beta = 1$	Y	9939	9750.2
		$\alpha = 2$ $\beta = 2$	Y	9750.2	
		$\alpha = 1$ $\beta = 2$	N	12952	
1	$\rho = 0.7$	$\alpha = 2$ $\beta = 1$	Y	9941	9941
		$\alpha = 2$ $\beta = 2$	Y	9983	
		$\alpha = 1$ $\beta = 2$	N	10971	
1	$\rho = 0.9$	$\alpha = 2$ $\beta = 1$	Y	9855.8	9714.4
		$\alpha = 2$ $\beta = 2$	Y	9714.4	
		$\alpha = 1$ $\beta = 2$	N	10648	

Table 3b
The optimal results via SOO model in phase 2.

Construction phasing	Parameters setting		Convergence status within 100 steps	Optimal results	Best results of fixed value ρ
2	$\rho=0.5$	$\alpha=2$ $\beta=1$	Y	33725	33536
		$\alpha=2$ $\beta=2$	Y	33536	
		$\alpha=1$ $\beta=2$	N	32499	
2	$\rho=0.7$	$\alpha=2$ $\beta=1$	Y	32661	32661
		$\alpha=2$ $\beta=2$	Y	32768	
		$\alpha=1$ $\beta=2$	N	35137	
2	$\rho=0.9$	$\alpha=2$ $\beta=1$	Y	32415	32415
		$\alpha=2$ $\beta=2$	Y	33642	
		$\alpha=1$ $\beta=2$	N	33746	

the pheromone density due to relationship between x_i and x_j and $\lambda_4>\lambda_3>\lambda_2>\lambda_1$; $x_i \prec x_j$ means solution x_i dominates solution x_j; $x_j \prec x_i$ means solution x_j dominates solution x_i. The amount of pheromone density θ_j released is determined by comparing the qualities of these solutions in terms of Pareto dominance relationship between them.

For the searching mode 2, it is assumed that there are p non-dominated solutions $x=(x_1,x_2,\cdots,x_p)$ in the set BP, which is an external set to keep all non-dominated solutions found during the run of the algorithm. The distance fd_{ij} between the objective function of x_i to the other ants can be calculated using the following equation:

$$fd_{ij} = \sqrt{\sum_{t=1}^{K}\left(f_t(x_i)-f_t\left(x_j\right)\right)^2} \tag{4}$$

where $i=1,2,\cdots,p, j=1,2,\cdots,p, i\neq j$.

And then calculate the shared function value $S(fd_{ij})$

$$S\left(fd_{ij}\right) = \begin{cases} 1-fd_{ij}/\sigma_{\text{share}},\ fd_{ij}<\sigma_{\text{share}}; \\ 0, fd_{ij} \geq \sigma_{\text{share}}. \end{cases} \tag{5}$$

Table 3c
The optimal results via SOO model in phase 3.

Construction phasing	Parameters setting		Convergence status within 100 steps	Optimal results	Best results of fixed value ρ
3	$\rho=0.5$	$\alpha=2$ $\beta=1$	Y	25214	25206
		$\alpha=2$ $\beta=2$	Y	25206	
		$\alpha=1$ $\beta=2$	N	29160	
3	$\rho=0.7$	$\alpha=2$ $\beta=1$	Y	25902	25596
		$\alpha=2$ $\beta=2$	Y	25596	
		$\alpha=1$ $\beta=2$	N	27918	
3	$\rho=0.9$	$\alpha=2$ $\beta=1$	Y	25876	25872
		$\alpha=2$ $\beta=2$	Y	25872	
		$\alpha=1$ $\beta=2$	N	25411	

Where σ_{share} is niching radius. The niche of non-dominated solution is given

$$\text{niche}(i) = \sum_{j=1}^{p} S\left(fd_{ij}\right), i = 1, 2, \cdots, p, i\neq j \tag{6}$$

The smallest niche(i)of the non-dominated solution $i$determines the searching direction of the ants.

3.2.3. Unequal-area grid recognition strategy used in the SOO model and MOO model

In solving unequal-area CSLP problem, the dimensions or the sizes of the facilities and the locations should be considered during the assignment of facilities in the construction site. Using unequal-area grid recognition strategy, an unequal-area CSLP problem can be reduced into an equal-area CSLP problem by assigning the biggest facility area first in the construction site and thus the searching scope is then reduced step by step. Using the sequence of the descending ranking order of facilities' dimensions, the locations for all facilities shall be found.

A brief description of the unequal-area grid recognition strategy employed in the SOO model and MOO model is given in Fig. 3. Suppose there is a construction site with area of 200 m^2, which is divided by grid units of each 25 m^2 area, the two facilities of FA1 with area of 50 m^2 and FA2 with area of 100 m^2 thus occupied two grids and four grids respectively (see step 2 of Fig. 3). Now, the two facilities will be assigned to construction site step by step (see step 3 and step 4 of Fig. 3) accordingly.

3.2.4. The continuous dynamic searching scheme employed in the SOO model and MOO model

In order to guide the SOO model and MOO model to solve dynamic CSLP problem, the continuous dynamic searching scheme was adopted (see the Fig. 4 for details).

Suppose there were four facilities needed to be assigned on a construction site (see Fig. 4), the facilities of F1, F2 and F3 were involved in the construction operation in Phase 1 and the facilities of F1, F3 and F4 were needed in the construction phase 2. The locations for F1, F2 and F3 were assigned to corresponding locations via the SOO model and MOO model in phase 1. The locations for facilities of F1 and F3 were fixed in the phase 2, the task for the SOO model and MOO model was to find the location for facility of F4 and the location for facility F2 was removed from the assignment process in this phase. With the locations fixing for facilities of F1 and F3 during the phase 2, no double handling cost occurred. Finally, the two layout plans in phase 1 and phase 2 were merged into one layout plan [1]. Just like Tommelein [18] said that there was a single-site layout drawing used throughout the project to include such information regarding facilities on the site at successive time intervals during project construction.

3.3. Evaluation and selection stage

At this stage the MADM model using intuitionistic fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method was employed to evaluate and select the most satisfying, or the best, construction site layout in accordance with the pre-determined attributes, which can be determined by the survey conducted among the management team of a specific contractor. Modified from Li [19] and Tan and Zhang [20], the intuitionistic fuzzy TOPSIS method adopted in this research is explained below.

3.3.1. Intuitionistic fuzzy TOPSIS

Intuitionistic fuzzy sets (IFS), which are the extension of traditional fuzzy sets [21], constitute a generalization of the concept of fuzzy sets. In traditional fuzzy set [22], a membership function is assigned to each element of the universe of discourse a number, which is from an unit interval, to indicate the degree of belongingness to the set under

Table 4
The construction site layout alternatives generated by the SOO model.

Construction site layout alternatives	Adopted parameters setting	Value of f in different phases			Total value of f	Total value of $f$in original layout	Reduction rate (%)		
		Phase 1	Phase 2	Phase 3			f	f_1	f_2
Alternative 1 (L1)	$\rho=0.9$ $\alpha=2$ $\beta=2$	9714.4	32418	26279	68411.4	86851	21.2	17.9	25.0
Alternative 2 (L2)	$\rho=0.9$ $\alpha=2$ $\beta=1$	9815.6	32415	25301	67531.6	86851	22.2	18.1	27.0
Alternative 3 (L3)	$\rho=0.5$ $\alpha=2$ $\beta=2$	9720.3	32362	25206	67288.3	86851	22.5	18.4	27.2

Note: scalarized single objective function f is the weighted representative score for likelihood of accidents that happened and total handling cost between the facilities in the construction site. f_1 is the representative score for likelihood of accidents happened associated with the site layout. f_2 is the representative score for total handling cost between the facilities associated with the construction site layout.

consideration. However, in real-life problems, sometimes people are hard to define imprecise and uncertain situations by membership functions. It is because, in some cases, people can clearly describe their negative feelings rather than the positive feelings. Moreover, one can easily figure out which alternatives one likes or does not like or one does not know how much one really likes it or not. Since IFS give both a degree of membership and a degree of non-membership, therefore, IFS are helpful to model many real-life problems.

Based on the principle of choosing the best alternative which has the shortest distance from the positive ideal solution (PIS) and the longest distance from the negative-ideal solution (NIS), TOPSIS was proposed by Hwang and Yoon [23] to solve MADM problems. Inspired by the work of Li [19], who use linear program to calculate optimal weights between the attributes instead of personnel preference to avoid evident preference on some alternatives and the research works of IFS and TOPSIS done by Tan and Zhang [20], the proposed intuitionistic fuzzy TOPSIS method adopted in this research can be realized by the following successive steps:

Step 1 Calculate intuitionistic fuzzy set decision-making matrix D.

$$\tilde{D} = \begin{array}{c} \\ x_1 \\ x_2 \\ \cdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & \cdots & a_m \end{array} \\ \begin{bmatrix} \tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1m} \\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{r}_{n1} & \tilde{r}_{n2} & \cdots & \tilde{r}_{nm} \end{bmatrix} \end{array} = \begin{array}{c} \\ x_1 \\ x_2 \\ \cdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & \cdots & a_m \end{array} \\ \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \cdots & (\mu_{1m}, \nu_{1m}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \cdots & (\mu_{2m}, \nu_{2m}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{n1}, \nu_{n1}) & (\mu_{n2}, \nu_{n2}) & \cdots & (\mu_{nm}, \nu_{nm}) \end{bmatrix} \end{array} \quad (7)$$

$$\tilde{W} = [\ \tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_m\ ] \quad (8)$$

Where an alternative set $X=[x_1,x_2,\ldots,x_n]$ consists of n alternative candidates in which the most preferred alternative need to be selected, a attribute set Acan be expressed as $A=[a_1,a_2,\ldots,a_m]$, which is based on $m$attributes, each alternative candidate's performance is evaluated; $\tilde{r}_{ij}, (i=1,2,\ldots,n, j=1,2,\ldots,m)$ is the rating of alternative candidate x_i with respect to the attribute a_j; μ_{ij}, $(0\le\mu_{ij}\le1)$ and ν_{ij} $(0\le\nu_{ij}\le1)$ are the degree of membership and the degree of non-membership of the alternative candidate x_i with respect to the attribute a_j under the concept of intuitionistic fuzzy set respectively, where $0\le\mu_{ij}+\nu_{ij}\le1.\tilde{w}_j(j=1,2,\ldots,m)$ is the weight of the attribute a_j. $\tilde{D}$ and $\tilde{W}$ are the intuitionistic fuzzy decision matrix and weight vector respectively.

Step 2 Calculate the attribute weight.
The evaluation of alternatives in term of attribute is an IFS, which can be denoted as $X=\{<x_i,\mu_{ij},\nu_{ij},\pi_{ij}>\}$, where intuitionistic index $\pi_{ij}=1-\mu_{ij}-\nu_{ij}$ is the uncertainty membership of alternative candidate x_i with respect to the attribute a_j, which will bias decision maker's choice. The best result of μ_{ij} will increase to $\mu_{ij}+\pi_{ij}$ if the decision maker considers the situation in an optimistic way. So the decision maker's 'positive' evaluation will fall into the interval $[\mu_{ij},\mu_{ij}+\pi_{ij}]$. Similarly, the decision maker's 'negative' evaluation will fall into the interval $[\nu_{ij},\nu_{ij}+\pi_{ij}]$.
As for the attribute weight, let α_j, β_j and τ_j to be the degree of membership, the degree of non-membership and intuitionistic index of the fuzzy concept of 'importance' of the attribute a_j respectively, where $\tau_j=1-\alpha_j-\beta_j$. The weight $\tilde{w}_j(0\le\tilde{w}_j\le 1, \sum_{j=1}^{m}\tilde{w}_j=1)$ lies in the closed interval $[\alpha_j,\alpha_j+\tau_j]$ if the decision maker takes into consideration of τ_j.
In order to get PIS, the alternative should fulfill the following equations:

$$max\left\{\sum_{j=1}^{m}\left(\mu_{ij}+\pi_{ij}\right)\cdot\tilde{w}_j\right\} \quad (9)$$
$$s.t\begin{cases}\alpha_j\le\tilde{w}_j\le\alpha_j+\tau_j, j=1,2,\ldots,m\\ \sum_{j=1}^{m}\tilde{w}_j=1\end{cases}$$

Table 5
The Pareto optimal results via MOO model.

Construction site layout alternatives	Value of MOF in different phases						Total value of MOF		Total value of MOF in original layout		Reduction rate (%)	
	Phase 1		Phase 2		Phase 3							
	f_1	f_2	f_1	f_2	f_1	f_2	f_1	f_2	f_1	f_2	f_1	f_2
Alternative 4 (L4)	10976	13118	42496	29640	32991	22558	86463	65316	96585	77867	10.5	16.1
Alternative 5 (L5)	10965	12490	42298	28959	32636	22626	85899	64075	96585	77867	11.1	17.7
Alternative 6 (L6)	10831	11969	41890	27538	34321	23967	87042	63474	96585	77867	9.9	18.5

Note: MOF stands for multiple objective functions, f_1 (multiple objective function 1) is the representative score for likelihood of accidents happened associated with the site layout, f_2 (multiple objective function 2) is the representative score for the total handling cost between the facilities associated with the construction site layout.

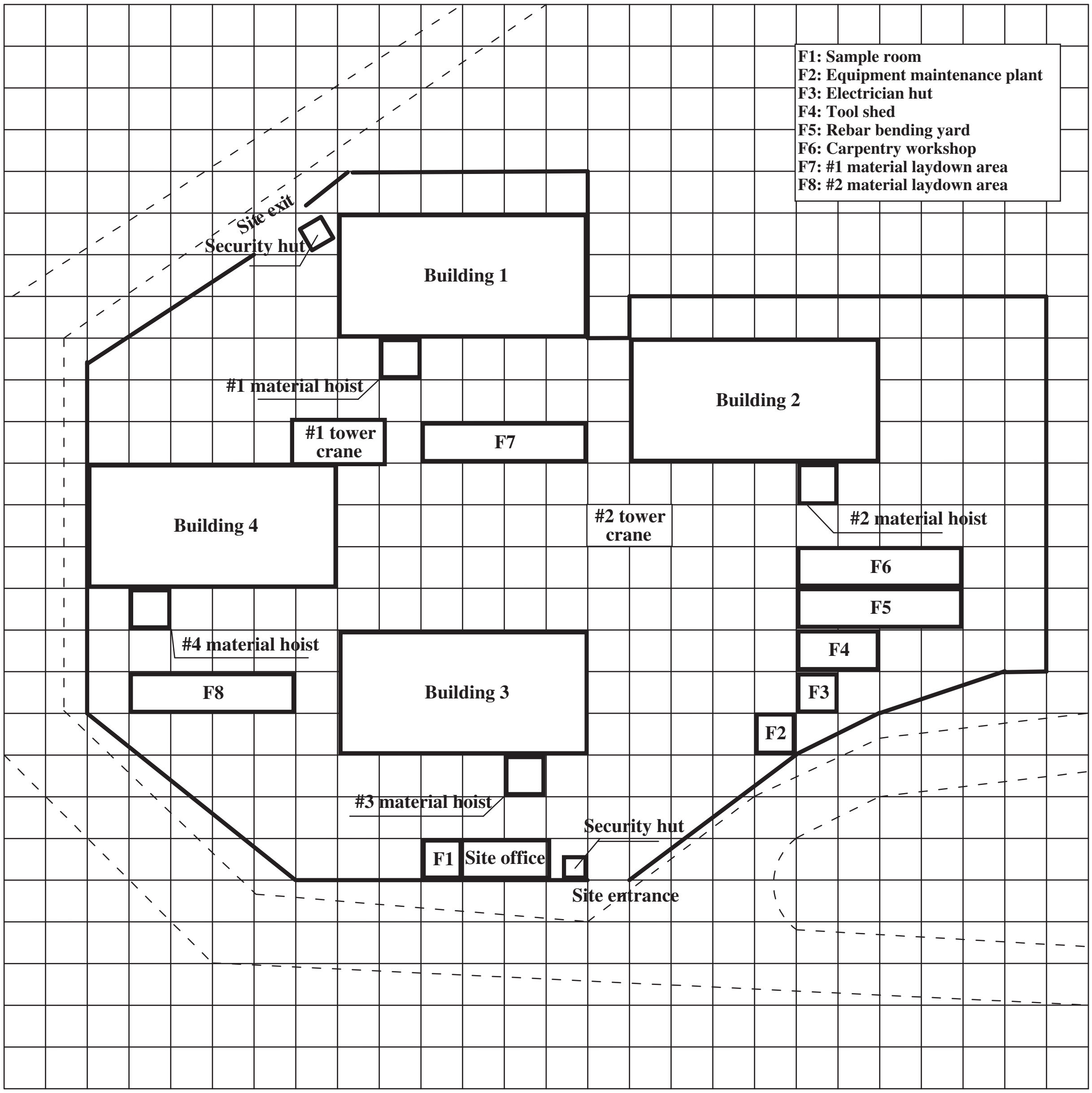


Fig. 6. The original construction site layout.

$$min\left\{\sum_{j=1}^{m}\nu_{ij}\cdot\tilde{w}_j\right\} \tag{10}$$

$$s.t\begin{cases}\alpha_j\le\tilde{w}_j\le\alpha_j+\tau_j, j=1,2,\dots,m\\ \sum_{j=1}^{m}\tilde{w}_j=1\end{cases}$$

Where $\pi_{ij}=1-\mu_{ij}-\nu_{ij}$, is the degree of hesitation of the alternative candidate x_i with respect to the attribute a_j under the concept of IFS.

Integrate Eqs. (9) with (10), we have the following equation

$$max\left\{\sum_{j=1}^{m}\left(\mu_{ij}+\pi_{ij}-\nu_{ij}\right)\cdot\tilde{w}_j\right\} \tag{11}$$

$$s.t\begin{cases}\alpha_j\le\tilde{w}_j\le\alpha_j+\tau_j, j=1,2,\dots,m\\ \sum_{j=1}^{m}\tilde{w}_j=1\end{cases}$$

Based on the Eq. (11), the attribute weight problem can be solved by transforming into a linear programming.

Step 3 Calculate weighted IFS decision-making matrix $\tilde{D}$ (see Eqs. (7) and (8))

$$\overline{r}_{ij}=r_{ij}\cdot\tilde{w}_j, i=1,2,\dots,n; j=1,2,\dots,m. \tag{12}$$

Step 4 Identify the positive-ideal intuitionistic fuzzy solutions (A^+) and negative-ideal intuitionistic fuzzy solutions (A^-)

$$A^+=\left\{\left(\max_j\mu_{ij},\right),\left(\min_j\nu_{ij}\right), j=1,2,\dots,m|i=1,2,\dots,n\right\} \tag{13}$$

$$A^-=\left\{\left(\min_j\mu_{ij},\right),\left(\max_j\nu_{ij}\right), j=1,2,\dots,m|i=1,2,\dots,n\right\} \tag{14}$$

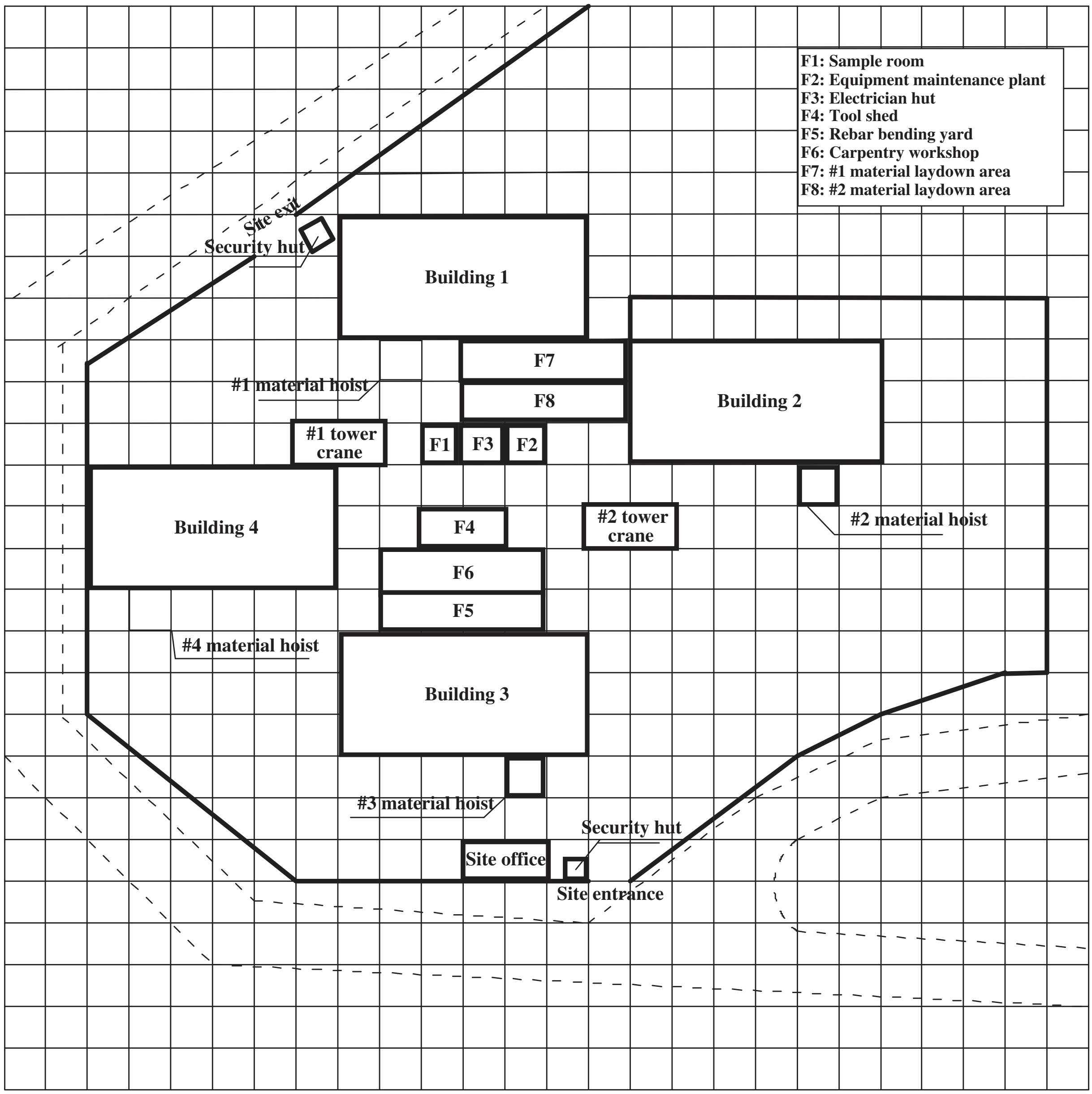


Fig. 7. Construction site layout alternative-L1.

Step 5 Calculate the distance of each alternative to positive-ideal and negative-ideal intuitionistic fuzzy solutions.
In this research, the distances between alternative candidates and positive-ideal and negative-ideal intuitionistic fuzzy solutions are measured by Euclidean distance. The distances from positive-ideal and negative-ideal intuitionistic fuzzy solutions are denoted as E^+ and E^- respectively.

Step 6 Calculate the closeness coefficient of each alternative.
The closeness coefficient $C_i$of each alternative can be calculated by

$$C_i = E_i^- \Big/ \left(E_i^+ + E_i^-\right), i = 1, 2, \ldots, n. \tag{15}$$

Step 7 Rank the preference order of alternatives.
A set of alternatives shall be ranked in accordance to the descending order of C_i. The best alternative is the one with the highest relative C_i.

3.4. Output stage

The output stage is the final step of the proposed CSLP decision-making system. The best construction site layout selected should fulfill both quantitative requirements of multiple objective functions and qualitative requirement of attributes such as security, noise control.

4. Application of the proposed CSLP decision-making system

The proposed CSLP decision-making system was tested and verified by the case study of residential building. The project is located in the city of Beijing, China. There are four residential buildings (Building 1 and Building 4 are sixty stories high; Building 2 and Building 3 are eighty stories high), site offices and construction operational facilities located within the construction site. Other temporary living facilities are located outside the construction site. The task of the proposed model is to find the corresponding locations for the temporary construction operational facilities in the construction site.

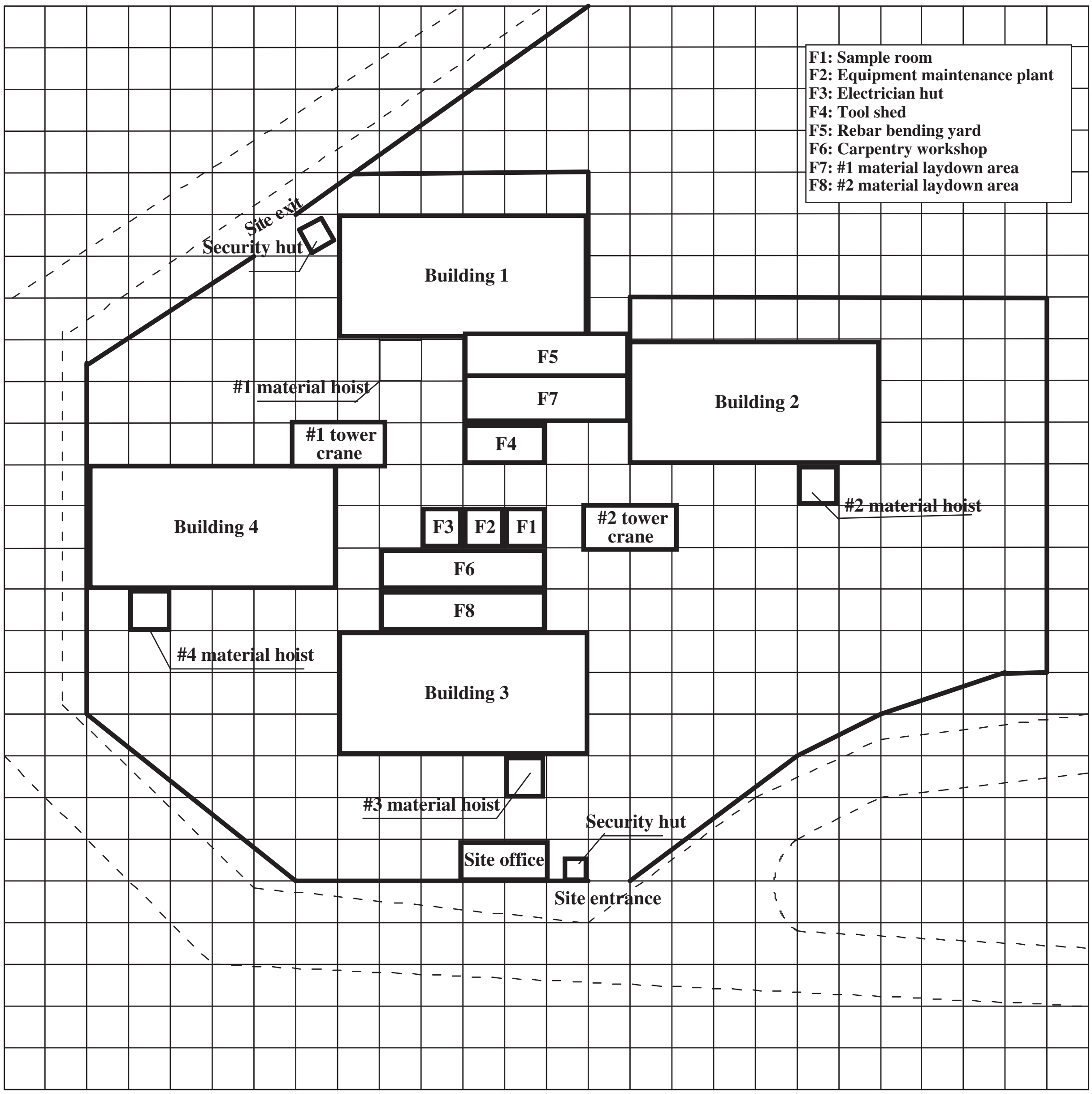


Fig. 8. Construction site layout alternative-L2.

4.1. Input stage

The construction project started in 15 April 2007 and is scheduled to finish by 28 November 2008. The whole construction process is divided into three construction phases for this case study. The start time and finish time of each construction phase are shown in Fig. 5.

In Fig. 5, the site layout plan used in the original contractor's plan document consisted of five construction phases, namely, foundation construction phase, underground section construction phase, superstructure construction phase, finishing works phase and external works phase. In the case study, the three construction phases are used as the dynamic schedule intervals to lay out the site in terms of the time duration and the amount of the facilities serviced in each original construction phase. The main works done in each construction phase are foundation and underground works (Phase 1), superstructure (Phase 2), finishing works and external works (Phase 3). In the original plan, the foundation phase lasted for almost 2 months and there were few facilities serviced in this phase, so there was no need to treat it as a separate phase. In this case study, the foundation phase and the underground section construction phase are merged into one dynamic schedule interval and termed as Construction Phase 1. Superstructure works were finished when the work on the waterproof roof was completed; the period of the superstructure work was used as the dynamic schedule interval and termed Construction Phase 2. The finishing work (masonry, installation of door and window, interior decoration, outer-eave decoration, etc.) began when the waterproof roof was completed; therefore, the finishing works and the external works were merged into Construction Phase 3.

The temporary facilities (fixed and non-fixed facilities) and their corresponding dimensions, serviced phase are shown in Table 2.

In Table 2, tower cranes, material hoists, site office and security huts are fixed. It is because tower cranes are always fixed during the building construction for the high installation fees. The material hoists are used for the transportation of materials to the superstructure and their location are dependent on the structural elements to which they are tied, thus planners always freeze these facilities in certain

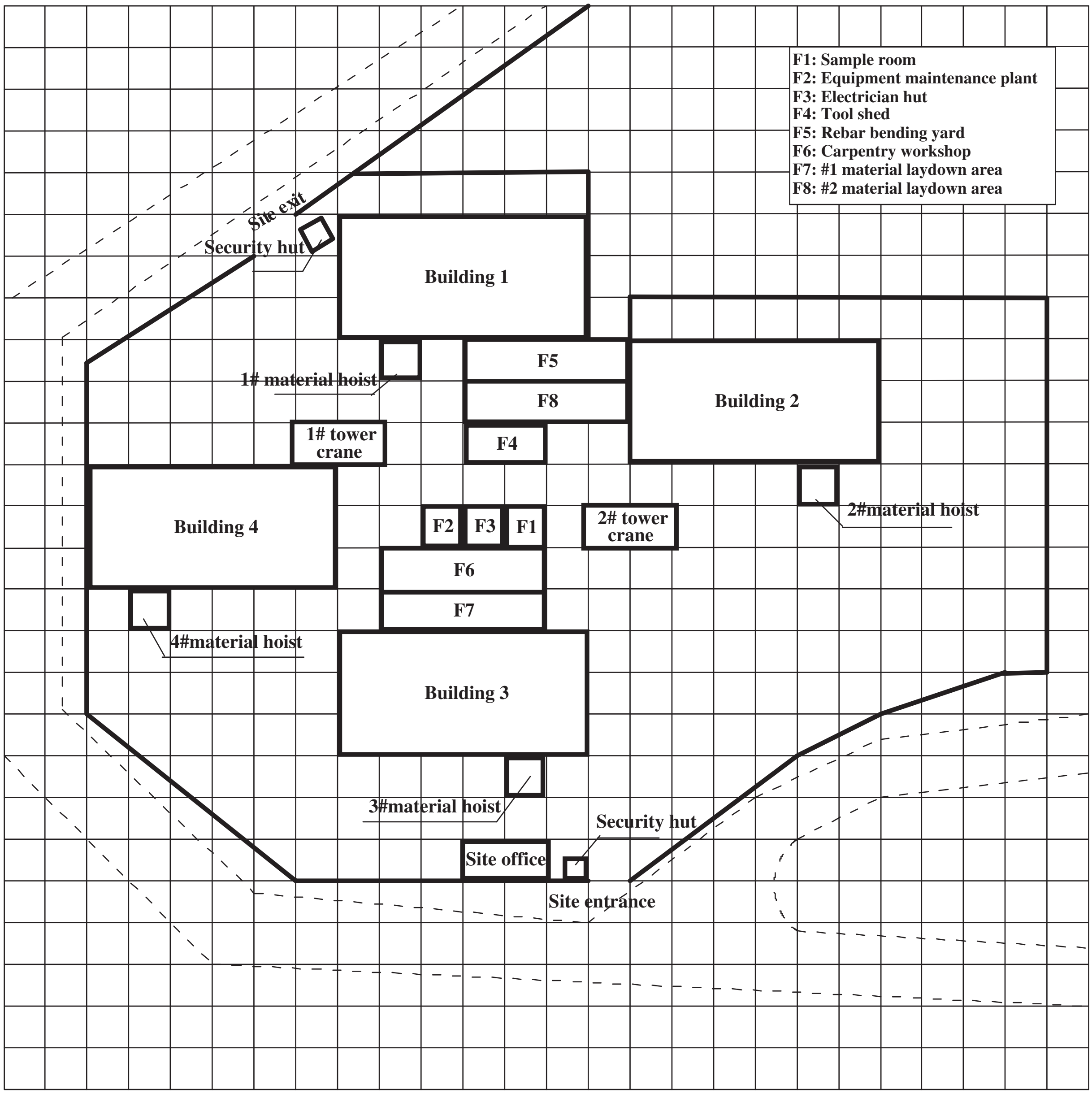


Fig. 9. Construction site layout alternative-L3.

locations first. The location of the main gate is usually pre-determined with reference to the external transportation system. Moreover, security huts are commonly assigned to the site entrance and site exit for ease of supervision and to avoid theft. The site office is usually near the main gate, also for the sake of safety.

4.2. Design stage

In the proposed CSLP decision-making system, there are two models (the SOO model and the MOO model) employed to solve the MOO problem in the design stage. The CSLP problem of this case study was predominantly a MOO problem, in which the multiple objective functions defined are intended to minimize the likelihood of accidents occurring and to minimize the total handling cost between the facilities. In the SOO model, MMAS is used to solve the MOO problems by the weighted sum method. The MOO model is based on the Pareto-based ACO algorithm to solve the MOO problems by Pareto optimization method. The applications of both models are discussed as follows.

4.2.1. Optimization process by the SOO model

The three key parameters in the SOO model are the persistence of the pheromone trails (ρ), the relative influences of the pheromone strengths (α) and the relative influences of the heuristic information (β). In order to investigate the impact to the results caused by the parameter settings, the SOO model was tested under the three sets of parameters in this case study (see Table 3). According to the continuous dynamic searching scheme, when the SOO model was applied to find locations for the seven facilities (F2~F8) in the construction phase 1, the locations for the seven facilities were found. The optimal results via SOO model in phase 1 were documented in Table 3a. In this Table, the layout with the smallest value of f(see Eq. (1)) in which seven facilities were assigned to corresponding locations is selected for further optimization. It is because there were a total of eight temporary facilities (F1~F8) involved in the whole construction process. In order to search the location for the sample room (F1) which is only existed in Phase 3, optimization process was conducted in Phase 3. Thus, the layout alternative to merge the layout plan in construction phase 1 and phase 3, named as Alternative 1 (L1), was generated. If the continuous dynamic searching scheme was started from

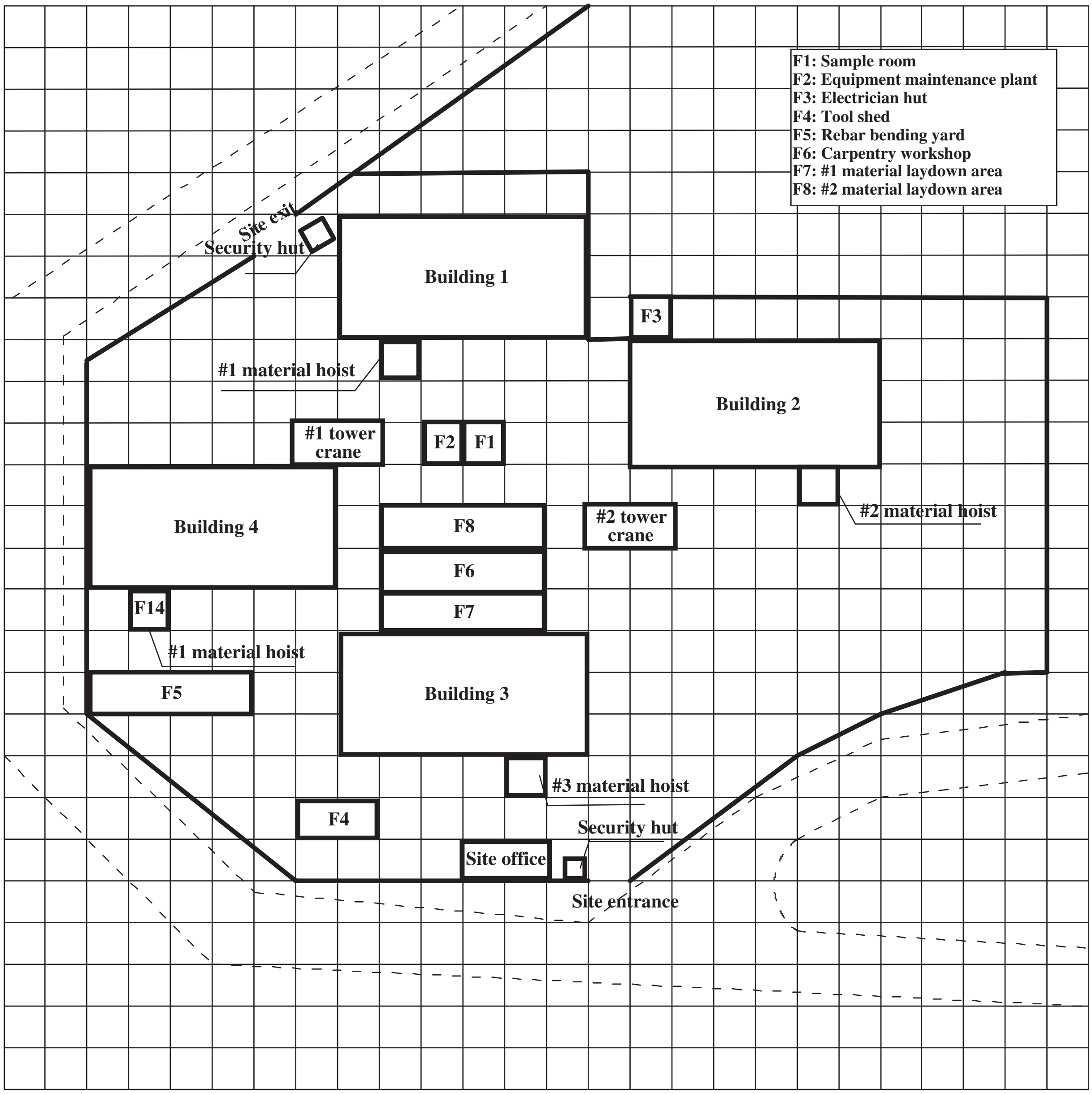


Fig. 10. Construction site layout alternative-L4.

the construction phase 2, in which there are seven facilities (F2 ~ F8) involved, the locations for the seven facilities were determined. The optimal results via SOO model in phase 2 were documented in Table 3b. In order to the location for the sample room (F1) which only existed in Phase 3, optimization process was conducted in Phase 3. In this way, Alternative 2 (L2) was generated. So as the continuous dynamic searching scheme started from construction phase 3. Firstly the locations for the seven temporary facilities (F1 to F4, F6 to F8) were assigned (please see Table 3c for the optimal results via SOO model in phase 2) and then the location for facility rebar bending yard (F5) was identified by optimizing process on Phase 2 and Phase 1, separately. Finally, the third layout alternative-Alternative 3 (L3) was determined. The optimal results (construction site layout alternatives) and their respective representative scores for *f*(see Eq. (1)) are shown in Table 4.

4.2.2. Optimization process by the MOO model

The key parameters in the MOO model are *N*, which is the ant colony population (solution population *N*) and the four parameters λ_1, λ_2, λ_3 and λ_4 for the different levels of pheromone density. The MOO model was tested under the four sets of parameters. When *N* was set to 100, the optimization processes by the MOO model were conducted under $\lambda_1 = 0.01$, $\lambda_2 = 0.1$, $\lambda_3 = 2$, $\lambda_4 = 5$ and $\lambda_1 = 0.01$, $\lambda_2 = 0.2$, $\lambda_3 = 8$, $\lambda_4 = 32$, respectively. When *N* was set to 200, the optimization processes of MOO model were conducted under $\lambda_1 = 0.01$, $\lambda_2 = 0.1$, $\lambda_3 = 2$, $\lambda_4 = 5$ and $\lambda_1 = 0.01$, $\lambda_2 = 0.2$, $\lambda_3 = 8$, $\lambda_4 = 32$, respectively. After setting the parameters, the optimal construction site layouts were processed by the MOO model via the same searching process in SOO model. Therefore, the alternative 4 (L4), alternative 5 (L5), alternative 6 (L6) were generated by the continuous dynamic searching scheme started on the construction phase 1, construction phase 2 and construction phase 3 respectively. The optimal construction site layout alternatives generated by the MOO model, with their corresponding representative scores for *f*are shown in Table 5.

For the comparison and evaluation, the original site layout and six layout alternatives are illustrated in Fig. 6 to Fig. 12.

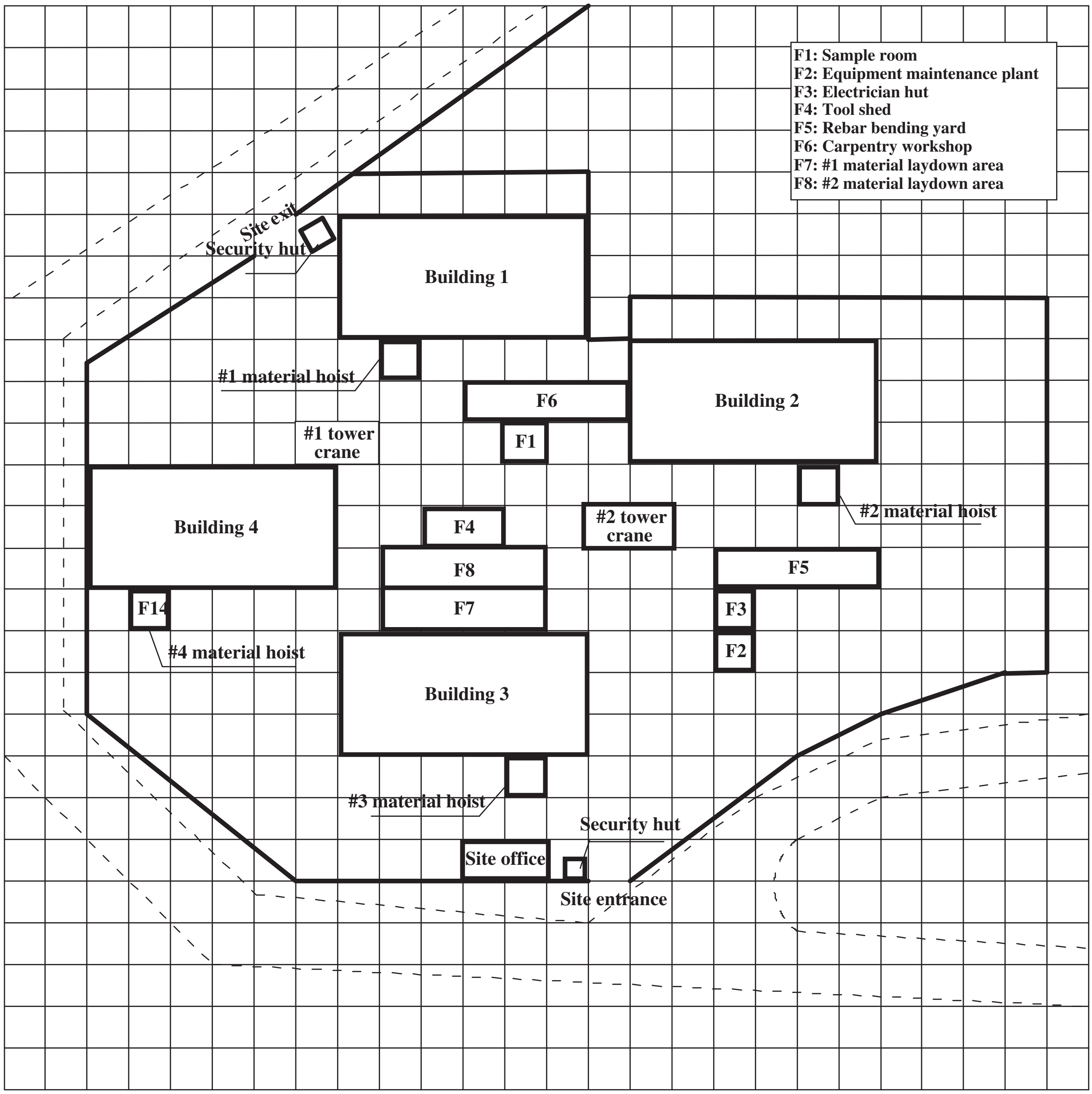


Fig. 11. Construction site layout alternative-L5.

4.3. Evaluation and selection stage

In the evaluation and selection stage, the six construction site layout alternatives were evaluated and selected using the intuitionistic fuzzy TOPSIS method in terms of the ten attributes, which were identified from 23 common attributes from literature used to evaluate the construction site layout alternatives. In order to identify the essential attributes from the 23 common attributes, the 5-point scale questionnaire (not important is set to 1 and extremely important is set to 5) was designed to survey the key personnel of this project. The aim was to survey to what extent they considered these attributes important in the evaluation of the construction site layouts. For the ten attributes, please see the Table 6.

At this stage of the proposed CSLP decision-making system, the intuitionistic fuzzy TOPSIS method was used to evaluate and select the best construction site layout plan from the six construction site layout alternatives (L1 to L6 generated by the previous stage). The intuitionistic fuzzy set was first determined by the survey to describe the extent of how these six alternatives could fulfill the requirements of a good site layout in terms of the ten essential attributes surveyed and defined. Then, the positive-ideal (best solution) and the negative-ideal (worst solution) intuitionistic fuzzy solutions could be found from the intuitionistic fuzzy sets. The positive-ideal and the negative-ideal intuitionistic fuzzy solutions under the ten essential attributes are recorded in Table 7.

Having the positive-ideal and the negative-ideal intuitionistic fuzzy solutions determined, the task for the following intuitionistic fuzzy TOPSIS method is used to find the best of the best solutions, which has both the nearest Euclidean distance from the positive-ideal solution (the best solution) and the longest Euclidean distance from the negative-ideal solution (the worst solution). The ratio between the longest distance from the negative-ideal solution and the nearest distance to the positive-ideal solution is the closeness coefficient of each alternative. So, the best construction site layout alternative is the one which has the highest closeness coefficient. The alternatives of this case study then could be ranked in terms of their closeness coefficients as shown in Table 8.

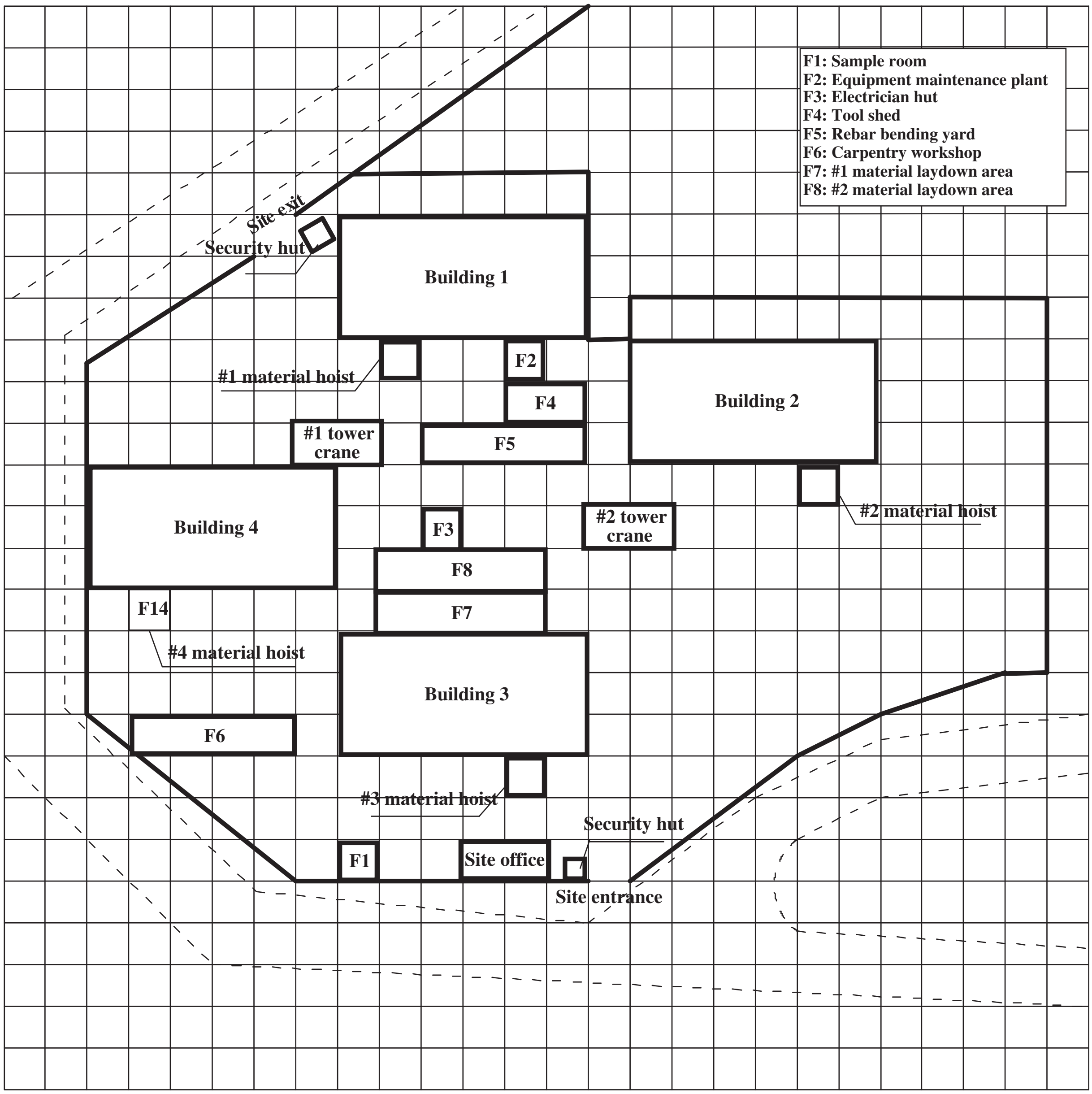


Fig. 12. Construction site layout alternative-L6.

4.4. Output stage

The result of the preference ranking order of the six alternatives generated by the SOO model and the MOO model is in descending order of L2 > L3 > L1 > L6 > L5 > L4. Therefore, Alternative L2, which has the highest closeness coefficient (0.966) among the six alternatives, was selected as the output of the proposed CSLP decision-making system. Alternative L2 is the best site layout which has the shortest distance from the positive ideal site layout and the longest distance from the negative ideal site layout in terms of the ten essential attributes among all the alternatives. In addition, the construction site layout of L3 ($C_i = 0.908$) could also be used as another option by the project manager. The closeness coefficients for L2 and L3 are very close, and there is not a big difference between them. In L2, the #1 and #2 material lay-down areas are located around Building 3 and Building 4, and Building 1 and Building 2, respectively. Thus, the #1 material lay-down area could service Building 3 and Building 4 and the #2 material lay-down area could service Building 1 and Building 2 effectively. The centralized locations for all the facilities in the middle area between the four buildings should boost the construction operation efficiency. The compact arrangement of the facilities should facilitate the construction site expansion on the left-hand side area of the construction site.

Table 6
Ten key attributes to evaluate construction site layout alternatives in residential building.

Attribute no.	Ten attributes
1	Efficient movement of materials
2	Tie-in with external transportation
3	Good space utilization and configuration
4	Ease of expansion
5	Safety
6	Effective movement of personnel
7	Efficient operations
8	Frequency and seriousness of potential breakdowns
9	Security
10	Easy supervision and control

Table 7
Positive-ideal and negative-ideal intuitionistic fuzzy solutions.

Ideal intuitionistic solutions	Project-specific attributes									
	(1)[a]	(2)[a]	(3)[a]	(4)[a]	(5)[a]	(6)[a]	(7)[a]	(8)[a]	(9)[a]	(10)[a]
A^+	(1,0)	(1,0)	(0.9,0)	(0.9,0)	(0.9,0)	(0.9,0)	(0.9,0)	(0.9,0)	(1,0)	(1,0)
A^-	(0.7,0.1)	(0.9,0)	(0.7,0.2)	(0.6,0.2)	(0.8,0.1)	(0.3,0.6)	(0.5,0.4)	(0.6,0.2)	(0.4,0.4)	(0.3,0.6)

[a] Attribute no.

Table 8
Rank the alternatives in terms of closeness coefficient.

Alternative	Distance from positive-ideal intuitionistic fuzzy solution E^+	Distance from negative-ideal intuitionistic fuzzy solution E^-	Closeness coefficient of each alternative C_i	Ranking
L1	0.093	0.249	0.727	3
L2	0.011	0.310	0.966	1
L3	0.030	0.295	0.908	2
L4	0.270	0.053	0.165	6
L5	0.122	0.205	0.628	5
L6	0.109	0.226	0.675	4

5. Conclusion

The CSLP decision-making system is proposed to solve dynamic and unequal-area, multi-objective optimization CSLP problems in this study. The performance of CSLP decision-making system was tested and verified by the case study of a residential building. The representative score of likelihood of accident happened and total handling cost could be reduced from 9.9% to 18.4% and from 16.1% to 27.2% respectively if the proposed CSLP decision-making system recommendations were adopted. The figures showed that an optimal construction site layout planning has a great, positive impact on construction cost. Furthermore, the best construction site layout in the output stage could satisfy both the quantitative aspect of cost reduction and the qualitative aspect of the other attributes, which have been neglected in previous studies.

The grids-recognition strategies used to solve the unequal-area CSLP problem by the SOO model and the MOO model could also be employed using other advanced algorithms and their applications can also be extended e.g. from static to dynamic CSLP problems. At the same time, the strategy would offer an innovative way to solve the unequal-area CSLP problem without increasing the computational complexities.

With the adoption of continuous dynamic search scheme in this decision-making system, the re-handling cost could be avoided, the safety level could be improved and the work productivity could be increased. This continuous dynamic searching scheme could be applied to solve static to dynamic CSLP problems using other advanced algorithms.

Moreover, the MOO problems in the CSLP were solved by the SOO model and MOO model using the weighted sum method and the Pareto-based optimization method respectively. With the applications of the two methods to solve the MOO problems, more quality solutions (L1, L2, L3, L4, L5 and L6) were generated for the evaluation and selection stage using the intuitionistic fuzzy TOPSIS method.

The proposed CSLP decision-making system can be used as a tool to help project managers and planners to design their construction sites under conflicting multiple objectives or congruent objectives, which involve more than one objective functions consisted of quantitative factors, such as handling cost of material. Moreover, the proposed decision-making system will help project managers and planners to design a good construction site layout with consideration of other qualitative factors, such as ease of supervision and control which are neglected in the previous studies.

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