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Multi-Objective Dynamic Construction Site Layout Planning in Fuzzy Random Environment

Jiuping Xu [a,b,*], Zongmin Li [b]

[a] State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu 610064, PR China
[b] Uncertainty Decision-Making Laboratory, Sichuan University, Chengdu, 610064, PR China

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ABSTRACT

Construction site layout planning is a fundamental task for any project undertaking. In practice, most of construction site layout planning problems are dynamic, multi-objective and uncertain in nature. To enhance the general practice of dynamic construction site layout planning, this paper proposes a fuzzy random multi-objective decision making model. In this model, two objectives are considered: (1) minimizing the total cost of site layout; and (2) maximizing the distance between the 'high-risk' facilities and the 'high-protection' facilities to reduce the possibility of safety or environmental accidents. After stating the problem and presenting the mathematical formulation, this paper deals with fuzzy random variables to propose an equivalent crisp model. According to the characteristics of the proposed model, a multi-objective particle swarm optimization algorithm (MOPSO) with permutation-based representation is proposed to solve the problem. The approach is applied to the Longtan hydropower construction project to illustrate the effectiveness of the proposed model and algorithm.

1. Introduction

Efficient layout planning of a construction site is fundamental to any successful project undertaking and has a significant impact on finances, safety, and other aspects, particularly for larger construction projects [1]. Since Yeh [2] and Zouein and Tommelein [3] have identified the construction site layout problem, much research has been conducted in this field. A review of this problem can be found in Russell and Gau [4].

The site layout problem can be formulated as a quadratic assignment problem (QAP) aiming at the optimal assignment of L facilities to L predetermined locations [5]. Previous research has mainly involved only a single objective function, e.g. [6,7]. In fact, a good layout demands the fulfillment of several competing and yet often conflicting design objectives, such as operational efficiency maximization, maintaining good employee morale, minimizing travel time and distance for the movement of resources to decrease material handling time. Besides these concerns, legal obligations may also impose safety and permit constraints [8]. As stated by Turskis et al. [9], the complex nature of decision-making in construction design and management requires practitioners to make decisions based on a wider variety of policy considerations in addition to cost benefit analysis and pure technical considerations. Moreover, previous construction site layout planning research has tended to concentrate on static problems, e.g. [2,10]. In static construction site layout problems, the facilities serviced in the different construction phases in accordance with the requirements of the construction work during the whole progress of a construction project are assumed to be the same [11] and ignore the possibility of site space reuse to accommodate different resources at different times, the relocation of resources, and the varying resource space needs over time [12]. An increasing number of studies focus on solving dynamic construction site layout planning problems. For instance, Baykasoglu et al. [13] made the first attempt to show how the Ant Colony Optimization Algorithm can be applied to dynamic construction site layout planning problems with the budget constraints. Ning et al. [12] used a continuous dynamic searching scheme to guide the max–min ant system algorithm to solve the dynamic construction site layout planning problem, and they further developed a computationally decision-making system to solve the dynamic, multi-objective and unequal-area construction site layout planning problem [14].

In practice, there are two types of uncertainties in layout planning problems [15]: the first is from internal, such as decision makers' perception and dissension. This kind of uncertainty is subjective. The second uncertainty is caused by external forces, such as uncertainties in the level of demand, product prices, product mix, equipment breakdowns, variable task times and queuing delays. This kind of uncertainty is objective. In these two kinds of uncertainties, objective information can be dealt using randomness and subjective information can be dealt using fuzziness. Current uncertainty research in construction site layout planning problem concentrates on fuzzy uncertainty, e.g. [16,17]. In reality, because both objective and subjective information simultaneously exist, both fuzziness and randomness should be considered.

* Corresponding author at: State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu 610064, PR China. Tel.: +86 28 85418522; fax: +86 28 85415143.
E-mail address: xujiuping@scu.edu.cn (J. Xu).

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However, so far, there has been no study comprehensively considering multi-objective, dynamic and fuzzy random problems in construction site layout planning. From a realistic aspect, construction site layout planning tends to be dynamic and uncertain in nature and needs to fulfill some conflicting objectives. Therefore this paper proposes a fuzzy random multi-objective decision making model for the dynamic construction site layout planning problems.

As pointed out by Mawdesley and Al-Jibouri [18], site layout planning research tends to concentrate on four main areas in the future:

(1) Developing more suitable models;
(2) Extending existing models to include a time element (dynamic layout);
(3) Adding uncertainty;
(4) Adding multiple criteria for evaluation and study into special cases for specific types of problems.

This study is a beneficial exploration in these four areas as it works on the multi-objective dynamic construction site layout planning under fuzzy random environment and the case study.

As for the solution methods, there are two types of methods employed in solving construction site layout problems: the exact methods and heuristics. Exact methods are only applicable for small-scale problems because of the combinatorial nature of the problem. For large scale construction projects, heuristic methods have distinct advantages [18]. Research shows that artificial intelligence, evolutionary algorithms or evolutionary programming, swarm intelligence, and computer-aided design have been developed to solve construction site layout planning problems. Among these models which use evolutionary algorithms, genetic algorithm (GA) is mostly adopted [20]. Another evolutionary algorithm, particle swarm optimization (PSO) was first proposed by Kennedy and Eberhart in 1995 [21], and has become one of the most important swarm intelligence paradigms. PSO has a superior search performance for many hard optimization problems with faster and more stable convergence rates compared with other population-based stochastic optimization methods. Zhang and Wang [22] successfully used PSO to solve the single objective static construction site layout planning problem. In this paper, the multi-objective particle swarm optimization algorithm (MOPSO) is firstly applied to solve the multi-objective dynamic construction site layout planning problem.

The remainder of this paper is as follows. Section 2 focuses on the problem statement. Assumptions, notations, objective functions, constraints and a method for dealing with fuzzy random variables are presented in Section 3. In Section 4, MOPSO algorithm with permutation-based representation is presented for solving the problem. In Section 5, the proposed model and algorithm are applied in a practical case. Model comparison and algorithm analysis are also presented in this section. Concluding remarks are made in Section 6.

2. Problem statement

Construction site layout deals with the assignment of appropriate site locations for temporary facilities such as warehouses, site offices, workshops, and batch plants. The site layout problem involves multiple sources, requiring the scheduling of activities and space consideration. Project managers, superintendents and subcontractors may jointly agree upon the location of facilities using past experience, trial and error, insight, preference, common sense and intuition [23]. Construction site layout can be delimited as the design problem of arranging a set of predetermined facilities on the site, while satisfying a set of constraints and optimized layout objectives. The effective placement of facilities within the site is significantly influenced by the movement of resources or the interactions between the facilities [12].

In dynamic construction site layout planning problems, the facilities serviced are different in different construction phases according to the construction work requirements. For example, in Fig. 1, the construction project lasts for n years, and according to the facility requirements, the complete projects can be divided into t phases. In phase 1, six facilities are setup, located in six of eight locations; in phase 2, facilities 1, 2, and 6 are still open, facility 4 is closed, while facility 8 changes. In the last phase, only four facilities remain open.

Four quantitative factors: material flows (MF), information flows (IF), personnel flows (PF) and equipment flows (EF) and two qualitative factors: safety/environment concerns (SE) and users' preference (UP) are usually considered in a construction site layout planning problem. According to [12], these six factors are defined as:

- MF: the flow of parts, raw materials, works-in-process and finished products between departments. The MF can be measured as unit per time unit.
- IF: the communication (oral or reports) between facilities. IF can be measured using an involved personnel survey and it can be expressed as the number of communications per time unit.
- PF: the number of employees from one or both facilities that perform tasks from one facility to another.
- EF: the number of material handling equipment (trucks, mixers, etc.) used to transfer materials between facilities.
- SE: the level of safety and environmental hazards, measured by the safety concerns, which may arise when two facilities are close to each other, and may affect site workers by an increase in the likelihood of accidents, excessive noise, uncomfortable temperature or pollution.
- UP: the project manager's desire to have the facilities close to or apart from each other.

This paper considers MF, IF, PF, EF and SE in the mathematical model, and considers the UP (users' preference) in the algorithm design.

In practice, there is often a hybrid uncertain environment in construction projects, especially in large scale construction projects. The imprecision and complexity of large scale construction projects cannot be dealt with simple fuzzy variables or random variables. In fact, fuzzy random environment has been studied in many problems, such as system reliability analysis [24], scheduling problems [25], vehicle routing optimization [26], and inventory problems [27]. These studies show the necessity of considering fuzzy random environment in practical problems. Compared with fuzziness and randomness, fuzzy randomness simultaneously describes both objective and subjective information as a fuzzy set of possible probabilistic models over a range of imprecision. For instance, in one phase, the operating cost of the labor residence is "between 160.8 thousand CNY and 189.3 thousand CNY", and it is "possibly around 180 thousand CNY". Because of scanty analytical data and a variety of other complex factors such as infrastructure destruction and the loss of goods and materials, the "possibly around 180 thousand CNY" is considered as an expected value of a random variable which follows a normal distribution $\mathcal{N}(180, \sigma^2)$. In this case, a triangular fuzzy random variable $(160.8, \varphi(\omega), 189.3)$ in which $\varphi(\omega)$ follows a normal distribution $\mathcal{N}(180, \sigma^2)$ can be used to deal with this kind of uncertain parameters of combining randomness and fuzziness. The situation is similar when considering the interaction costs of different facilities.

3. Modeling

Construction site layout problem is a type of QAP. QAP requires an equal number of facilities and locations. If the number m of facilities is less than the number n of locations, n–m dummy facilities shall be created and zero set-up and transportation cost are assigned to them. If there are fewer locations than facilities, the problem is infeasible. The goals of solving the dynamic construction site layout problems are to minimize the cost for each single facility, the interaction

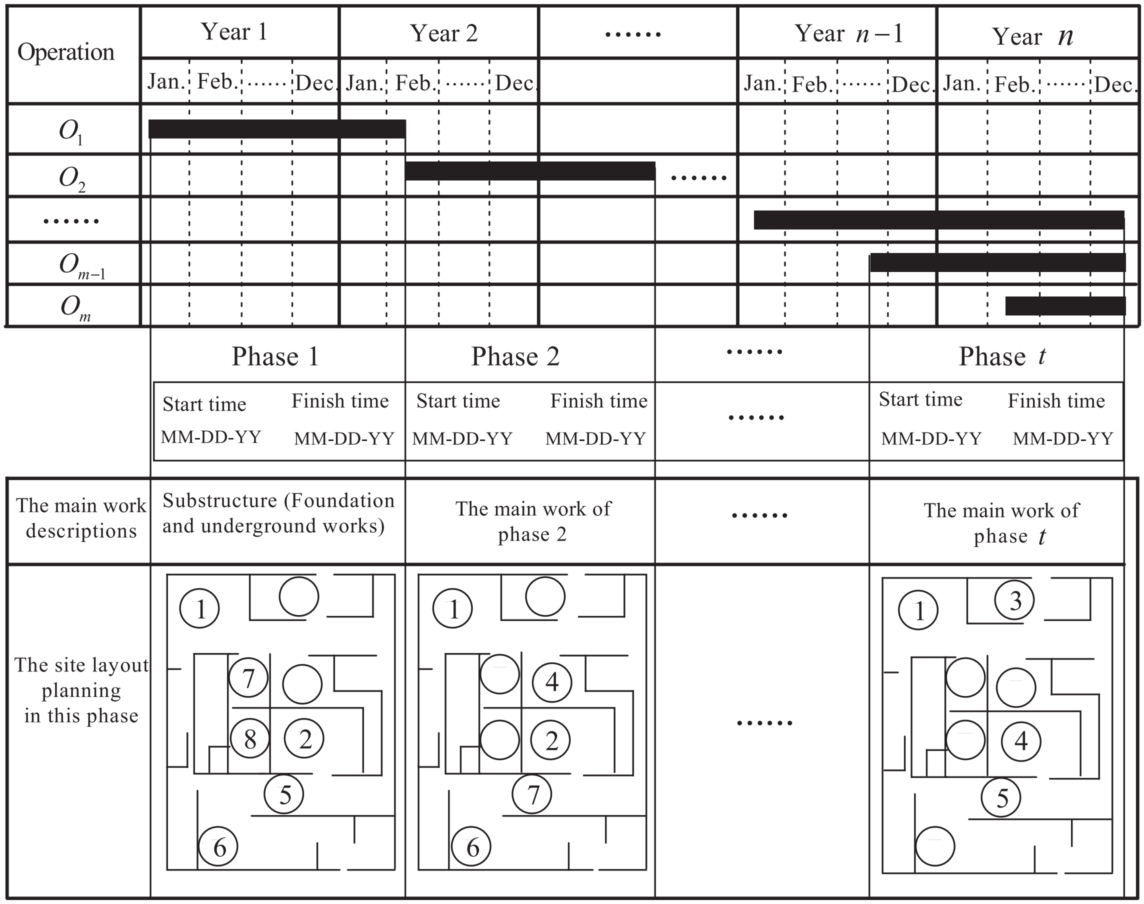

Fig. 1. An example of dynamic construction site layout planning problem.

cost between different facilities, as well as to ensure that the distance between the facilities is optimized to reduce the possibility of safety or environmental accidents. Facilities and locations are interrelated by two kinds of constraints, i.e., area constraints and logical constraints. For example, different facilities cannot be in the same location at the same time, and each facility can be located in only one location in one period. After presenting the mathematical formulation, this paper deals with fuzzy random variables to propose an equivalent crisp model.

3.1. Assumptions

To model the dynamic construction site layout planning problems under fuzzy random environment, the following assumptions are made:

(1) All the possible locations for different facilities can be identified.
(2) F facilities are to be positioned on a site, L locations are available for each facility to position, $L \geq F$.
(3) The interaction cost and the operating cost of facilities are regarded as fuzzy random variables.
(4) All the resource demands at each facility during every period can be satisfied.
(5) For each facility assignment to a candidate location, there are different set-up and removal costs.
(6) The possibility of safety or environmental accidents and the possible loss are proportional to the distance between 'high-risk' facilities and 'high-protection' facilities.

3.2. Notations

To formulate the model, the indices, variables, certain parameters, uncertain variables, and decision variables are as follows:

Indices:

x,y different types of site-level facility index, $x,y \in \{1,2,\cdots,F\}$. Among those, 'high-risk' facilities which may cause safety or environmental accidents, are marked as x^k, 'high-protection' facilities which are vulnerable and are likely to suffer great losses, are marked as y^r,
k 'high-risk' facility index, $k \in \{1,2,\cdots,K\}$,
r 'high-protection' facility index, $r \in \{1,2,\cdots,R\}$,
i,j location index, $i,j \in \{1,2,\cdots,L\}$,
t periods of the problem index, $t \in \{1,2,\cdots,T\}$.

Variables:

η_{xit} denotes the opening of facility x at location i during period t,
ω_{xit} denotes the closure of facility x at location i during period t.

Certain parameters:

α_t an appropriate discount rate,
A_{ij} the distance from location i to location j,
C^s_{xit} the startup cost of facility x at location i during time period t,
C^c_{xit} the closure cost of facility x at location i during time period t,

S_{xt} the area required of facilities x during time period t,
D_i the area of the location i,
w_{kr} the risk weight of 'high-risk' facility x^k near 'high-protection' facility y^r.

Uncertain variables:

$\tilde{\bar{C}}^z_{xit}$ the operating cost at location i during time period t for facility x,
$\tilde{\bar{C}}_{xyt}$ the interaction cost per unit distance of facility x and facility y for unit-distance during time period t.

Decision variables:

δ_{xit} denotes the existence of facility x at location i during period t, with the initial condition of $\delta_{xi0}=0$, $\delta_{xj0}=0$,
δ_{yit} denotes the existence of facility y at location i during period t, with the initial condition of $\delta_{yi0}=0$, $\delta_{yj0}=0$.

Based on the assumptions and notations above, this paper proposes a fuzzy random multi-objective decision making model for dynamic construction site layout planning problems in the following.

3.3. Objective functions

Various layout plans are evaluated to determine the optimum. From the six construction site layout planning factors, two objective functions are derived.

3.3.1. The total cost of site layout planning

The cost for each single facility is composed by three parts, the setup cost, the closure cost and the operating cost. Since η_{xit} is the opening of facility x at location i during phase t, C^s_{xit} is the startup cost. $\eta_{xit}=1$ denotes the facility x is setup at location i during phase t, and C^s_{xit} is a certain variable forecasted by the construction site layout managers, so that $C^s_{xit}\eta_{xit}$ denotes the opening cost. Similarly $C^c_{xit}\omega_{xit}$ denotes the closure cost, and $\tilde{\bar{C}}^z_{xit}\delta_{xit}$ denotes the operating cost of facilities.

It is not enough to just minimize the cost for each single facility. There are imperative MF, IF, PF, and EF between the different facilities. An optimum dynamic site layout plan minimizes the interaction cost between these facilities. In phase t, if facility x is located in location i, facility y is located in location j and there are interaction activities between x and y. Frequently, the future condition is difficult to predict. Thus, as per the assumptions in Section 3.1, the interaction cost of x and y for unit-distance is a fuzzy random variable, marked as $\tilde{\bar{C}}_{xyt}$. Since A_{ij} is the distance from i to j, the interaction cost between x and y during phase t is $\delta_{xit}\delta_{yjt}A_{ij}\tilde{\bar{C}}_{xyt}$.

In addition, for a large scale construction project, the time duration can be lengthy. Some construction projects last several years or even more than ten years, so it is necessary to consider fund time value. a_t denotes an appropriate discount rate to determine the net present value.

Considering the fund time value, the total cost objective C can be expressed as minimizing the following:

$$\sum_{t=1}^{T}\sum_{x,y=1}^{F}\sum_{i,j=1}^{L}\alpha_t\left(C^s_{xit}\eta_{xit}+C^c_{xit}\omega_{xit}+\tilde{\bar{C}}^z_{xit}\delta_{xit}+\delta_{xit}\delta_{yjt}A_{ij}\tilde{\bar{C}}_{xyt}\right). \tag{1}$$

3.3.2. Safety and environmental objective

In the construction industry the risk of a fatality is five times more than that in a manufacturing based industry, and the risk of a major injury is two and a half times higher [28]. Safety and environmental issues are therefore extremely important when designing construction site layout. A well-planned and well-run project should be both safe and efficient to save lives and money, and reduce injury and ill-health.

The 'high-risk' facilities which may cause safety or environmental accidents are marked as x^k, for example, oil depots, explosives storage and dangerous chemicals storage. The 'high-protection' facilities are vulnerable facilities which could suffer great losses if these accidents happen. From previous studies it is known that the nearer of these two kinds of facilities are, the greater the chance of serious accidents will be. Thus, it is better that these 'high-risk' facilities are far from 'high-protection' facilities. δ_{x^kit} denotes facility x^k is located in location i, and δ_{y^rjt} denotes facility y^r is located in location j. So the distance between 'high-risk' facilities and other facilities is denotes as: $\delta_{x^kit}\delta_{y^rjt}A_{ij}$. As w_{kr} is the risk weight of 'high-risk' facility x_k near 'high-protection' facility y_r, and in accordance with the sixth assumption above, the possibility of safety or environmental accidents and possible loss are proportional to the distance between 'high-risk' facilities and 'high-protection' facilities, the safety and environmental objective D can be expressed as minimizing the following:

$$\sum_{t=1}^{T}\sum_{i,j=1}^{L}\sum_{k=1}^{K}\sum_{r=1}^{R}\left(w_{kr}\delta_{x^kit}\delta_{y^rjt}A_{ij}\right)^{-1}. \tag{2}$$

3.4. Constraints

3.4.1. Area constraints

The location area has to meet the facility requirement, therefore,

$$\delta_{xit}S_{xt}<D_i,\forall x\in\{1,2,\cdots,F\},t\in\{1,2,\cdots,T\},i\in\{1,2,\cdots,L\}. \tag{3}$$

Note that the use ratio of the area cannot reach 100%, so that $S_{xt}<D_i$ instead of $S_{xt}\le D_i$.

3.4.2. Logical constraints

To obtain the feasible solution, there are some logical constraints. At most one facility is located in one location, namely, $\delta_{xit}+\delta_{yit}\le1,\delta_{xjt}+\delta_{yjt}\le1$, yet a certain type of facility can be located in more than one location, namely $\delta_{xit}+\delta_{xjt}\ge0,\delta_{yit}+\delta_{yjt}\ge0$. $\delta_{xit},\delta_{xjt},\delta_{yit},\delta_{yjt}$ denotes the existence of the facility, with 1 denoting positive and 0 denoting negative.

η_{xit} is the opening of facility x at location i in phase t, and ω_{xit} is the closure of facility x at location i or j during phase t. Practically, if $\delta_{xi,t-1}=0$, and $\delta_{xit}=1$, namely $\delta_{xit}-\delta_{xi,t-1}=1$, then facility x is opened in location i in phase t, so $\eta_{xit}=1$. $\delta_{xit}-\delta_{xi,t-1}=0$ denotes facility x is neither open nor closed in phase t, and $\eta_{xit}=0,\omega_{xit}=0$. $\delta_{xit}-\delta_{xi,t-1}=-1$ denotes facility x is closed at location i during period t, so $\omega_{xit}=1$, namely:

$$\eta_{xit}=\begin{cases}1 & \text{if } \delta_{xit}-\delta_{xi,t-1}=1\\ 0 & \text{otherwise}\end{cases}\quad\text{and}\quad\omega_{xit}=\begin{cases}1 & \text{if } \delta_{xit}-\delta_{xi,t-1}=-1\\ 0 & \text{otherwise}\end{cases} \tag{4}$$

where $x,y\in\{1,2,\cdots,F\},i,j\in\{1,2,\cdots,L\},t\in\{1,2,\cdots,T\}$.

By the integration of Eqs. (1)–(4), a mathematical model of the dynamic construction site layout planning problem can be obtained. However, as is generally known, it is difficult to achieve optimal programming results when there are the dual uncertainties of randomness and fuzziness. To transform the fuzzy random variables to deterministic variables, this paper uses the following method.

3.5. Dealing with fuzzy random variables

To describe the phenomenon that contains random factors and fuzzy factors synchronously, Kwakernaak proposed the concept of fuzzy random variables for the first time in 1978 [29]. Subsequently, many scholars have defined fuzzy random variables from different perspectives [30,31]. This paper considers the fuzzy random variable defined in a real number set, in which the above definitions are equivalent. This study utilizes the definition proposed by Puri [30]. In this definition, the fuzzy random variable is a measurable function from a probability space to a collection of fuzzy variables. Fuzzy randomness simultaneously describes objective and subjective information as a fuzzy set of possible probabilistic models with a certain range of imprecision.

Without loss of generality, fuzzy random variables are denoted in this problem as $\tilde{\tilde{\xi}} = ([\xi]_L, \varphi(\omega), [\xi]_R)$, where, $\varphi(\omega)$ is a random variable with probability density function $p_\varphi(x)$. Since it has been supposed above that $\varphi(\omega)$ follows a normal distribution $\mathcal{N}(\mu_0, \sigma_0^2)$. Let σ be any given probability level of the random variable, r be any given possibility level of the fuzzy variable. The parameters satisfy:

$$\sigma \in \left[0,\ \sup p_\varphi(x)\right] \ \text{and} \ r \in \left[\frac{[\xi]_R - [\xi]_L}{[\xi]_R - [\xi]_L + \varphi_\sigma^R - \varphi_\sigma^L}, 1\right].$$

σ and r reflect the decision-maker's degree of optimism, and are called respectively the probability level and the possibility level.

The method of dealing with the fuzzy random variable is as follows:

(1) Use statistical methods to estimate the parameters $[\xi]_L, [\xi]_R, \mu_0$ and σ_0 by previous data and professional experience.
(2) Obtain the decision-maker's degree of optimism.
(3) Let φ_σ be the σ-level sets (or σ-cut) of the random variable $\varphi(\omega)$, i.e. $\varphi_\sigma = [\varphi_\sigma^L, \varphi_\sigma^R] = \{x \in R | p_\varphi(x) \geq \sigma\}$, where,

$$\varphi_\sigma^L = \inf\left\{x \in R \middle| p_\varphi(x) \geq \sigma\right\} = \inf p_\varphi^{-1}(\sigma) = \mu_0 - \sqrt{-2\sigma_0^2 \in \left(\sqrt{2\pi}\sigma_0\sigma\right)},$$
$$\varphi_\sigma^R = \inf\left\{x \in R \middle| p_\varphi(x) \geq \sigma\right\} = \sup p_\varphi^{-1}(\sigma) = \mu_0 + \sqrt{-2\sigma_0^2 \in \left(\sqrt{2\pi}\sigma_0\sigma\right)}.$$

The probability density function of the random variable φ with φ_σ^L and φ_σ^R is shown in Fig. 2.

(4) Transform the fuzzy random variable $\tilde{\tilde{\xi}} = ([\xi]_L, \varphi(\omega), [\xi]_R)$ into the (r,σ)-level trapezoidal fuzzy variable by $\tilde{\tilde{\xi}} \to \tilde{\xi}_{(r,\sigma)} = \left([\xi]_L,\ \underline{\xi}, \overline{\xi}, [\xi]_R\right)$, where

$$\underline{\xi} = [\xi]_R - r\left([\xi]_R - \varphi_\sigma^L\right) = [\xi]_R - r\left([\xi]_R - \mu_0 + \sqrt{-2\sigma_0^2 ln\left(\sqrt{2\pi}\sigma_0\sigma\right)}\right),$$
$$\overline{\xi} = [\xi]_L + r\left(\varphi_\sigma^R - [\xi]_L\right) = [\xi]_L + r\left(\mu_0 + \sqrt{-2\sigma_0^2 ln\left(\sqrt{2\pi}\sigma_0\sigma\right)} - [\xi]_L\right).$$

The membership function of $\tilde{\xi}_{(r,\sigma)}$ is as below:

$$\mu_{\tilde{\xi}_{(r,\sigma)}}(x) = \begin{cases} 0 & \text{if} \quad x < [\xi]_L, x > [\xi]_R, \\ \dfrac{x - [\xi]_L}{\underline{\xi} - [\xi]_L} & \text{if} \quad -[\xi]_L \leq x < \underline{\xi}, \\ 1 & \text{if} \quad \underline{\xi} \leq x \leq \overline{\xi}, \\ \dfrac{[\xi]_R - x}{[\xi]_R - \overline{\xi}} & \text{if} \quad \overline{\xi} < x \leq [\xi]_R. \end{cases}$$

Therefore, the fuzzy random variable $\tilde{\tilde{\xi}}$ is transformed into the (r,σ)-level trapezoidal fuzzy variable $\tilde{\xi}_{(r,\sigma)}$. The membership function of $\tilde{\xi}_{(r,\sigma)}$ is shown in Fig. 3.

Based on the method above, the fuzzy random variables in this paper are transformed into (r,σ)-level trapezoidal fuzzy variables as follows:

$$\tilde{\tilde{C}}_{xyt} \to \tilde{C}_{xyt} = \left(\left[C_{xyt}\right]_L,\ \underline{C}_{xyt}, \overline{C}_{xyt}, \left[C_{xyt}\right]_R\right).\mu_{\tilde{C}_{xyt}}$$
$$= \begin{cases} 0 & \text{if} \quad x < \left[C_{xyt}\right]_L, x > \left[C_{xyt}\right]_R, \\ \dfrac{x - \left[C_{xyt}\right]_L}{\underline{C}_{xyt} - \left[C_{xyt}\right]_L} & \text{if} \quad -\left[C_{xyt}\right]_L \leq x < \underline{C}_{xyt}, \\ 1 & \text{if} \quad \underline{C}_{xyt} \leq x \leq \overline{C}_{xyt}, \\ \dfrac{\left[C_{xyt}\right]_R - x}{\left[C_{xyt}\right]_R - \overline{C}_{xyt}} & \text{if} \quad \overline{C}_{xyt} < x \leq \left[C_{xyt}\right]_R. \end{cases}$$

$$\tilde{\tilde{C}}_{xit}^z \to \tilde{C}_{xit}^z = \left(\left[C_{xit}^z\right]_L,\ \underline{C}_{xit}^z, \overline{C}_{xit}^z, \left[C_{xit}^z\right]_R\right).$$

$$\mu_{\tilde{C}_{xit}^z} = \begin{cases} 0 & \text{if} \quad x < \left[C_{xit}^z\right]_L, x > \left[C_{xit}^z\right]_R, \\ \dfrac{x - \left[C_{xit}^z\right]_L}{\underline{C}_{xit}^z - \left[C_{xit}^z\right]_L} & \text{if} \quad -\left[C_{xit}^z\right]_L \leq x < \underline{C}_{xit}^z, \\ 1 & \text{if} \quad \underline{C}_{xit}^z \leq x \leq \overline{C}_{xit}^z, \\ \dfrac{\left[C_{xit}^z\right]_R - x}{\left[C_{xit}^z\right]_R - \overline{C}_{xit}^z} & \text{if} \quad \overline{C}_{xit}^z < x \leq \left[C_{xit}^z\right]_R. \end{cases}$$

(5) Calculate the expected value of the (r,σ)-level trapezoidal fuzzy variables. This paper uses the pessimistic–optimistic adjustment index whose definition is given by Xu and Zhou [32] to calculate the expected value. The index indicates that the expected value of (r,σ)-level trapezoidal fuzzy variable $\tilde{\xi}_{(r,\sigma)} = \left([\xi]_L,\ \underline{\xi}, \overline{\xi}, [\xi]_R\right)$ ($[\xi]_L > 0$) is:

$$E^{Me}\left[\tilde{\xi}_{r,\sigma}\right] = \frac{1-\lambda}{2}\left([\xi]_L + \ \underline{\xi}\right) + \frac{\lambda}{2}\left(\overline{\xi} + [\xi]_R\right), \tag{5}$$

where λ is the optimistic index to determine the combined attitude of decision makers. Note that the fuzzy measure *Me* is to evaluate a confidence degree that a fuzzy variable takes values in an interval. $\lambda = 1$ denotes the decision maker is optimistic and the decision maker thinks that the best case of that event

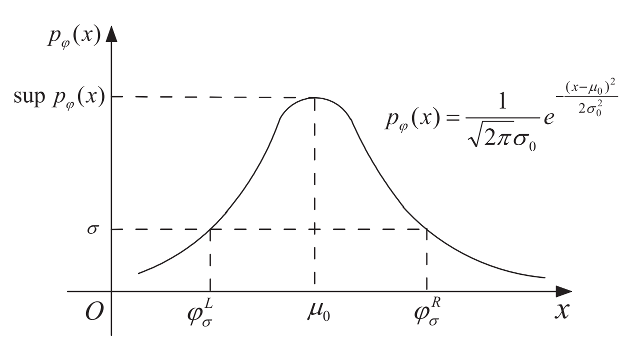

Fig. 2. The probability density function of random variable φ.

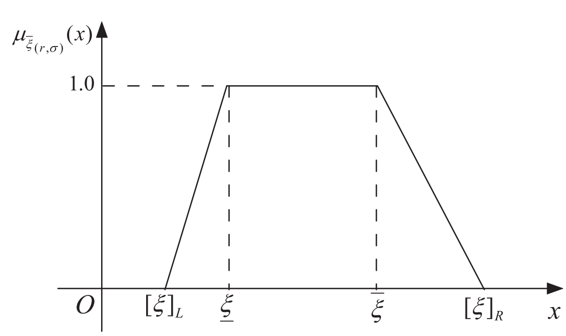

Fig. 3. The (r,σ)-level trapezoidal fuzzy variable $\tilde{\xi}_{(r,\sigma)}$.

has the maximal chance to happen, while $\lambda = 0$ denotes the opposite.

Based on the objective function and constraint formulas as well as the above method, the mathematical model for the dynamic construction site layout planning problem can be transformed into the following multi-objective expected value model:

$$\begin{cases} \min C = \sum_{t=1}^{T} \sum_{x,y=1}^{F} \sum_{i,j=1}^{L} \alpha_t \Big(C^s_{xit} \eta_{xit} + C^c_{xit} \omega_{xit} \\ \qquad + E^{Me}\left[C^z_{xit}\right] \delta_{xit} + \delta_{xit}\delta_{yjt} A_{ij} E^{Me}\left[C_{xyt}\right] \Big) \\ \max D = \sum_{t=1}^{T} \sum_{i,j=1}^{L} \sum_{k=1}^{K} \sum_{r=1}^{R} \left(w_{kr} \delta_{x^k it} \delta_{y^r jt} A_{ij} \right)^{-1} \\ s.t. \begin{cases} \delta_{xit} S_{xt} < D_i \\ \delta_{xit} + \delta_{yit} \le 1 \\ \delta_{xjt} + \delta_{yjt} \le 1 \\ \delta_{xit} + \delta_{xjt} \ge 0 \\ \delta_{yit} + \delta_{yjt} \ge 0 \\ S_{xt} \ge 0 \\ D_i > 0 \\ \delta_{xit}, \delta_{xjt}, \delta_{yit}, \delta_{yjt} = 0 \ or \ 1 \\ \eta_{xit} \begin{cases} 1 & \text{if } \delta_{xit} - \delta_{xi,t-1} = 1 \\ 0 & \text{otherwise} \end{cases} \\ \omega_{xit} \begin{cases} 1 & \text{if } \delta_{xit} - \delta_{xi,t-1} = -1 \\ 0 & \text{otherwise} \end{cases} \\ \alpha_t = \dfrac{1}{(1+\alpha)^{(t-1)}} A_{ij} \ge 0 x \ne y, i \ne j. \end{cases} \end{cases} \tag{6}$$

4. MOPSO with permutation-based representation

PSO is a population-based self-adaptive search optimization technique. It simulates social behaviors such as birds flocking to a promising position for certain objectives in a multi-dimensional space [21]. PSO is based on a set of potential solutions defined in a given space (called the search space) and conducts searches using a fixed population number (called a swarm) of individuals (called particles) that are updated from iteration to iteration. An n-dimensional position of a particle (called the solution), initialized at a random position in a multidimensional search space, represents the solution to the problem and resembles a chromosome of a genetic algorithm [33]. The particles characterized by their position and velocity, fly through the problem space by following the current optimum particles [21]. Unlike other population-based algorithms, the velocity and position of each particle are dynamically adjusted according to its own flying experience or discoveries as well as those of its companions. In 1995, Kennedy and Eberhart [21] proposed the following to update the position and velocity of each particle:

$$v_{ld}(\tau+1) = w(\tau) v_{ld}(\tau) + c_p r_1 \left[p^{\text{best}}_{ld}(\tau) - p_{ld}(\tau) \right] + c_g r_2 \left[p^{\text{best}(\tau)}_{gd} - p_{ld}(\tau) \right], \tag{7}$$

$$p_{ld}(\tau+1) = p_{ld}(\tau) + v_{ld}(\tau+1), \tag{8}$$

where $v_{ld}(\tau+1)$ is the velocity of lth particle at the dth dimension in the τth iteration, w is an inertia weight, $p_{ld}(\tau)$ is the position of lth particle at the dth dimension, r_1 and r_2 are random numbers in the range $[0,1]$, c_p and c_g are personal and global best position acceleration constants respectively, while, p^{best}_{ld} and p^{best}_{gd} are the personal and global best positions of lth particle at the dth dimension.

As PSO can be implemented easily and effectively, it has been widely applied in solving real-world optimization problems in recent years [34–36,19]. Researchers are also seeing PSO as a very strong competitor to other algorithms in solving multi-objective optimal problems [37] and find it especially suitable for multi-objective optimization [38]. Therefore, a number of proposals have been suggested to extend PSO to solve multi-objective problems [39,34]. The MOPSO approach uses the concept of Pareto dominance to determine the flight direction of a particle and it maintains previously found non-dominated vectors in a global repository that is later used by other particles to guide their own flight [37].

This paper is going to apply MOPSO to solve the model proposed in the last section. The following notations are used:

t	iteration index, $t = 1, \cdots, T$,
i	particle index, $i = 1, \cdots, L$,
d	dimension index, $d = 1, \cdots, N \times \tau$,
j	phase index, $j = 1, \cdots, \tau$,
f	facility index, $f = 1, \cdots, F$,
l	location index, $l = 1, \cdots, N$,
$P_i^{max}(d)$	maximum position value of particle i at the dth dimension,
$P_i^{min}(d)$	minimum position value of particle i at the dth dimension,
r_1, r_2	uniform distributed random number within $[0,1]$,
$w(t)$	inertia weight in the tth iteration,
$v^j_{id}(t)$	velocity of the jth phase of the ith particle at the dth dimension in the tth iteration,
$x^j_{id}(t)$	position of the jth phase of the ith particle at the dth dimension in the tth iteration,
$x^{\text{pbest}}_{id}(t)$	personal best position of the jth phase of the ith particle at the dth dimension in the tth iteration,
$x^{\text{gbest}}_{id}(t)$	global best position of the jth phase of the ith particle at the dth dimension in the tth iteration,
c_p	personal best position acceleration constant,
c_g	global best position acceleration constant,
ARC	the positions of the particles that represent non-dominated vectors in the repository.

4.1. Permutation-based representation and hybrid particle-updating mechanism

There have been various types of solution representations. For example Zhang and Wang [22] used priority-based representation for a static construction site layout planning problem. In this study, the permutation-based representation is used in applying MOPSO method where the candidate solution is represented by the multidimensional particle with ordinal numbers.

All dimensions of a multidimensional particle in the traditional PSO are independent of each other; thus, the updating of the velocity and particle based on Eqs. (7) and (8) is performed independently for each element. As a result, more than one element in an updated particle may have the same value, such as $\{3,5,7,3,\cdots\}$. Particles with the same value at multiple elements represent an infeasible permutation, because two facilities cannot appear at the same locations. Therefore, the MOPSO updating mechanism needs to be modified to eliminate conflicts.

Various genetic crossover operators have been developed for permutation infeasibility. For example, the partially mapped crossover (PMX) [41], the cycle cross-over (CX) [42], and the order crossover (OX) [43]. For the PMX, at each of the arbitrary elements, the two values (i.e., location index) of the two parents are exchanged. Then, the repeated value of another element in one parent is replaced by the mapped value of the specified arbitrary element in the second parent, and vice versa. In other words, the PMX performs swaps for a series of arbitrary elements by exchanging each of them with an element where there is a mapped value of the arbitrary element in the other parent. Fig. 4 illustrates the PMX, where one arbitrary element is specified. Zhang et al. [44] successfully used permutation-based PSO with PMX for a resource-constrained project scheduling problem.

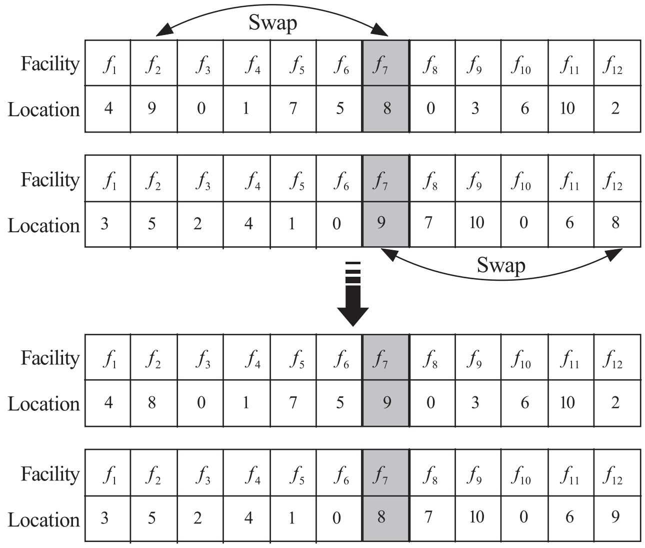

Fig. 4. PMX for the permutation-based representation.

Originally, the velocity computed by Eq. (7) is a distance measure to decide on a new position for the updated particle. A larger velocity means a more distant position or area being explored. When used for the permutation-based particle representation, the N-dimension velocity, i.e., $V_i(t)=\{v_{i1}(t),v_{i2}(t),\cdots,v_{iN}(t)\}$ computed by Eq. (7) corresponds to the distance or gap between the current particle-represented sequence (i.e., the order of locations) and its own local best and the global best sequences (i.e., experiences) found so far. A larger gap indicates that such a particle-represented sequence is more likely to be updated. Therefore, the absolute value of the velocity is used herein to represent the probability that the particle-represented sequence will change. Based on the PMX concept and the N-dimension probability, every dimension of a particle is randomly determined to see if the facility at such a location will be moved to another or if the element will be swapped with another. The "another element" has the value (i.e., facility-index number) the former element maps in the reference particle which is randomly selected from the current particle's local best (experience or sequence) and the swarm's global best. Here the particle to be updated and the reference particle respectively, resemble Parents 1 and 2 in the PMX.

Considering permutation feasibility, the concept of PMX is used in this paper to develop the hybrid updating mechanism for the particle-represented sequence.

4.2. Multi-objective method

The particle-represented solution is checked and adjusted for infeasibility due to the violation of resource constraints before transforming to layout planning. The taxonomy that Reyes-Sierra and Coello [45] proposed to classify the current MOPSOs is aggregating approaches, lexicographic ordering, sub-population approaches, Pareto-based approaches, combined approaches and other approaches. Previous multi-objective optimization of construction site layout planning studies tended to use the aggregating approach, such as [12,46]. However, the weights can be arbitrary.

In the subsequent iterations, the Pareto Archived Evolution Strategy (PAES) [47] is used in this paper. PAES is one of Pareto-based approaches used to update the best position. This approach uses leader selection techniques based on Pareto dominance. The basic idea is to select the particles that are non-dominated in the swarm as leaders. The PAES procedure is outlined below, X_i^{pbest} is the best position vector of particle i, and is initially set equal to the initial position of particle i. X_i^{gbest} and X_i^{lbest} are similarly derived.

Procedure The updating of the best position in the subsequent iterations

generate initial random solution $x_{id}^{\text{pbest}}(t)$ and add it to the archive;

update $x_{id}^{\text{pbest}}(t)$ to produce $x_{id}(t+1)$ and evaluate $x_{id}(t+1)$;

if ($x_{id}^{\text{pbest}}(t)$ dominates $x_{id}(t+1)$) **then** discard $x_{id}(t+1)$;

else if ($x_{id}(t+1)$ dominates $x_{id}^{\text{pbest}}(t)$) **then** replace $x_{id}^{\text{pbest}}(t)$ with $x_{id}(t+1)$, and add $x_{id}(t+1)$ to the archive;

$x_{id}^{\text{pbest}}(t+1)=x_{id}(t+1)$;

else if ($x_{id}(t+1)$ is dominated by any member of the archive) **then** discard $x_{id}(t+1)$;

else apply test ($x_{id}^{\text{pbest}}(t)$,$x_{id}(t+1)$, archive) to determine which becomes the new current solution and

whether to add $x_{id}(t+1)$ to the archive;

until a termination criterion has been reached, return to line 2.

Procedure Test

if the archive is not full;

add $x_{id}(t+1)$ to the archive;

if ($x_{id}(t+1)$ is in a less crowded region of the archive than $x_{id}^{\text{pbest}}(t)$);

accept $x_{id}^{\text{pbest}}(t+1)=x_{id}(t+1)$;

else maintain $x_{id}^{\text{pbest}}(t+1)=x_{id}^{\text{pbest}}(t)$;

else

if ($x_{id}(t+1)$ is in a less crowded region of the archive than $x_{id}^{\text{pbest}}(t)$ for some member $x_{id}^{\text{pbest}}(t)$ on the archive);

add $x_{id}(t+1)$ to the archive, and remove a member of the archive from the most crowded region;

if ($x_{id}(t+1)$ is in a less crowded region of the archive than $x_{id}^{\text{pbest}}(t)$);

accept $x_{id}^{\text{pbest}}(t+1)=x_{id}(t+1)$;

else maintain $x_{id}^{\text{pbest}}(t+1)=x_{id}^{\text{pbest}}(t)$;

else

if ($x_{id}(t+1)$ is in a less crowded region of the archive than $x_{id}^{\text{best}}(t)$);

accept $x_{id}^{best}(t+1)=x_{id}(t+1)$;

else maintain $x_{id}^{\text{pbest}}(t+1)=x_{id}^{\text{pbest}}(t)$.

4.3. Framework of MOPSO with permutation-based representation

The framework of MOPSO with permutation-based representation to solve the dynamic construction layout planning problems is presented in this section.

Step 1 Initialize L particles as a swarm: for $i=1,\cdots,L$, $j=1,\cdots,\tau$ generate the position of the ith particle with integer random position:

$$X_{if}^j(t)=\left[x_{i1}^1(t),x_{i2}^1(t),\cdots,x_{iN}^1(t);x_{i1}^2(t),x_{i2}^2(t),\cdots,x_{iN}^2(t);\cdots;x_{i1}^\tau(t),x_{i2}^\tau(t),\cdots,x_{iN}^\tau(t)\right]$$

the value is $\{0,1,2,\cdots,N\}$, $P_i^{max}(d)=N$, $P_i^{min}(d)=0$. There can be more than one '0', and other values are unequal in one phase, otherwise, a different facility will be located in the same location which is unacceptable.

As shown in Fig. 5, there are F facilities and N locations, N–F dummy facilities.

Step 2 Decode particles into solutions: For ith particle, X_f^j, if $x_f^j=0$ denotes that facility f does not exist during phase j, while $x_f^j\neq 0$ denotes that facility f is located in location x_f^j during phase j.

Step 3 Check the feasibility of solutions: For $i=1,\cdots,L$, if the feasibility criterion is met by all particles, i.e. $\delta_{xit}S_{xt}<D_i$, then continue. Otherwise, return to step 1.

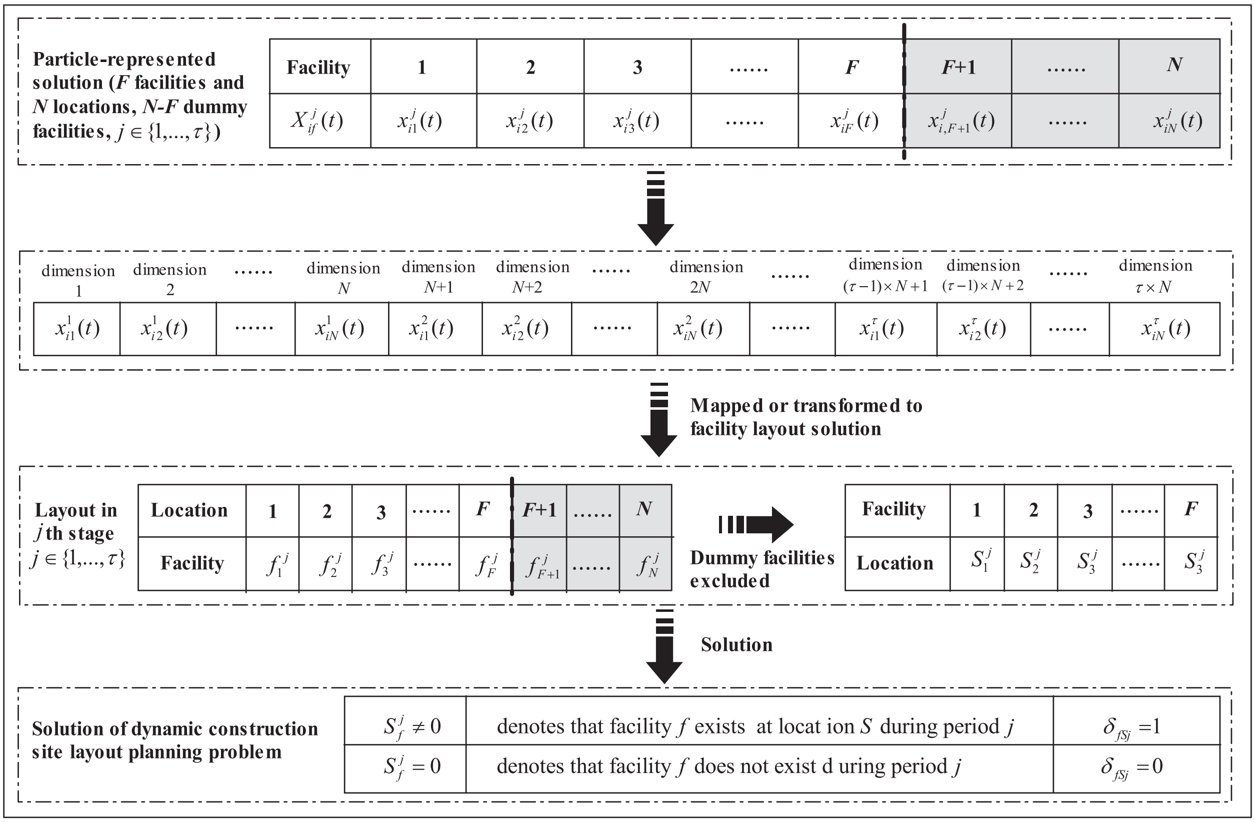

Fig. 5. Decoding method and mapping between MOPSO particles and solutions to the problem.

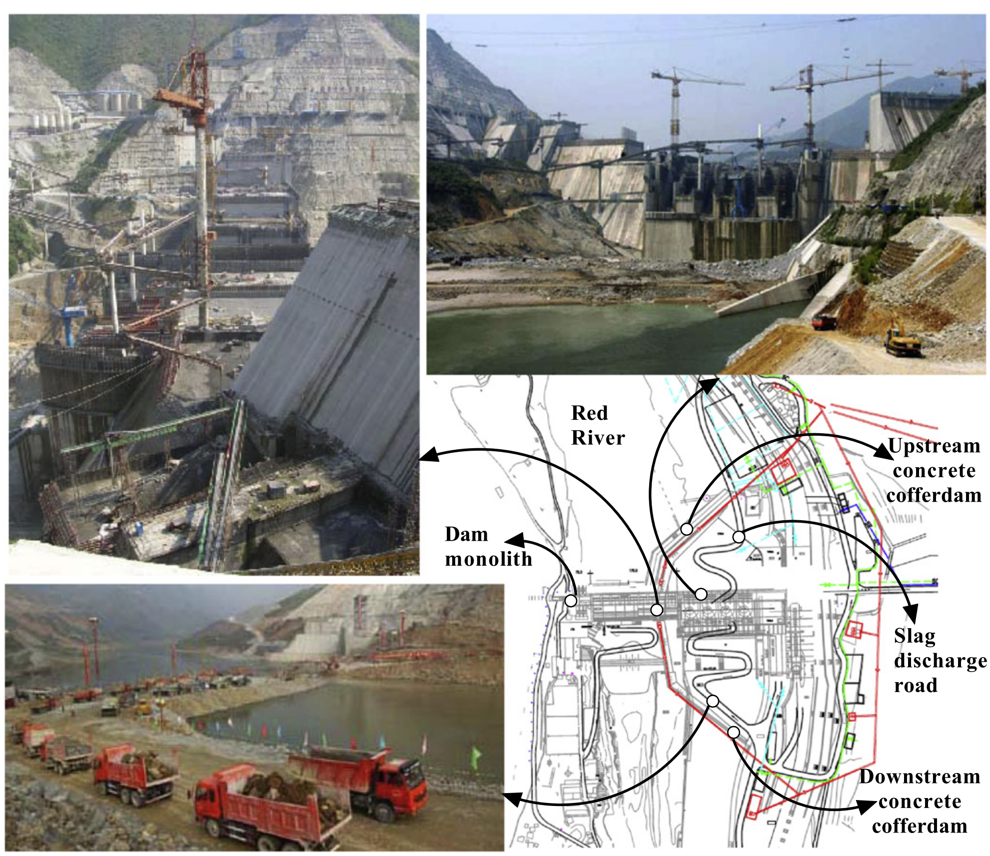

Fig. 6. Longtan hydropower station.

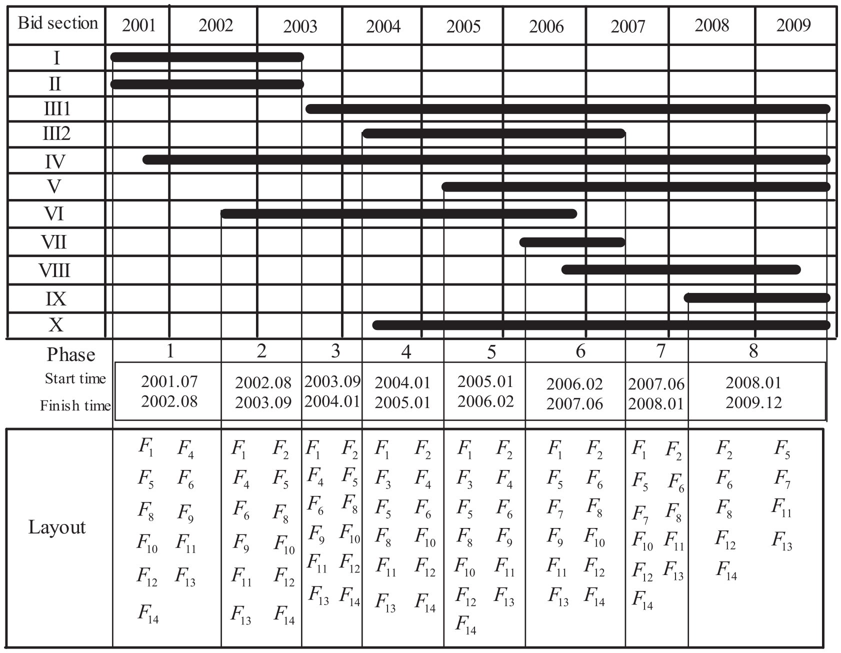

Fig. 7. The construction schedule as well as the phase and facilities required.

Step 4 Initialize the speed of each particle and personal best position: Let velocity $V_{if}(1)=[v_{i1}(1),v_{i2}(1),\cdots,v_{iN}(1)]$, and the value is between 0 and 1. The initial x_{id}^{pbest} is current position.

Step 5 Evaluate each of the particles: For $i=1,\cdots,L$, compute the performance measurement under every objective, and set the objective value as the fitness value of $X_i^j(t)$, represented by $Fitness_i(X_i^j(t))$. For each of the particles, $Fitness_i(X_i^j(t))$ is a 1×2 matrix.

Step 6 Store the positions of the particles that represent non-dominated vectors in the archive denoted as *ARC*.

Table 1
Facilities to be distributed and their required area during each phase.

Index	Facility	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5	Phase 6	Phase 7	Phase 8
F_1	Reinforcing steel shop	1800	1800	1800	2000	2000	1800	1000	0
F_2	Carpentry shop	0	2500	2500	3000	3000	2500	2000	2000
F_3	Concrete precast shop	0	0	0	1360	1360	0	0	0
F_4	Drill tools repair shop	800	800	800	800	800	0	0	0
F_5	Equipment repairing workshop	3500	3500	3500	3500	3500	3000	3000	3000
F_6	Truck maintenance shop	4000	4000	4000	4000	4000	3500	3000	3000
F_7	Metal and electrical installing workshop	0	0	0	0	0	3000	3300	3300
F_8	Oil depot	1000	1000	1000	1000	1000	1000	1000	1000
F_9	Explosive storage	750	750	750	0	750	750	0	0
F_{10}	Rebar storage	2000	2000	2500	3510	3510	3000	2500	0
F_{11}	Steel storage	2000	2000	2000	2500	2500	2000	2000	2000
F_{12}	Integrated warehouse	800	1000	1000	1000	1000	800	1000	1000
F_{13}	Office	4500	4500	4500	4500	4500	4500	4500	4500
F_{14}	Labor residence	8000	8000	8000	8000	10,000	8000	8000	8000

Step 7 WHILE the maximum number of cycles has not been reached DO

Step 7.1 Compute the speed of each particle using the following formula:

$$v_{id}^j(t+1)=w(t)v_{id}^j(t)+c_p r_1\left[\text{pBest}_{id}^j - x_{id}^j(t)\right] + c_g r_2\left[ARC_{fd}(t) - x_{id}^j(t)\right], \tag{9}$$

where c_R is an acceleration constant, $w(t)=w(T)+\frac{t-T}{1-T}[w(1)-w(T)]$ [40], $ARC_{fd}(t)$ is a solution randomly selected from the repository in each iteration for facility i, the index d is selected in the following way: hypercubes which contain more than one particle are assigned a fitness equal to the result of dividing any number into the number of contained particles. The assignment aims to decrease the fitness of those hypercubes that contain more particles and it can be seen as a form of fitness sharing [41]. Then, roulette-wheel selection method is applied using these fitness values to select the hypercube from which the corresponding particle is taken. Once the hypercube has been selected, a particle is selected randomly within such hypercube.

Step 7.2 Adding the speed produced from the previous step, compute the new positions of the particles using:

$$x_{id}^j(t+1)=x_{id}^j(t)+v_{id}^j(t+1).$$

Step 7.3 Maintain the particles within the search space to prevent them moving beyond their boundaries.

Step 7.4 Evaluate every particle.

Step 7.5 Update the contents of the *ARC* and the geographical representation of the particles within the hypercubes. Insert all the currently non-dominated locations into the repository and eliminate any dominated locations from the repository.

Step 7.6 Use PAES as well as the test procedure to update the best position.
Otherwise, the position in the memory replaces the one by current one; if neither of them is dominated by the other, select one randomly.

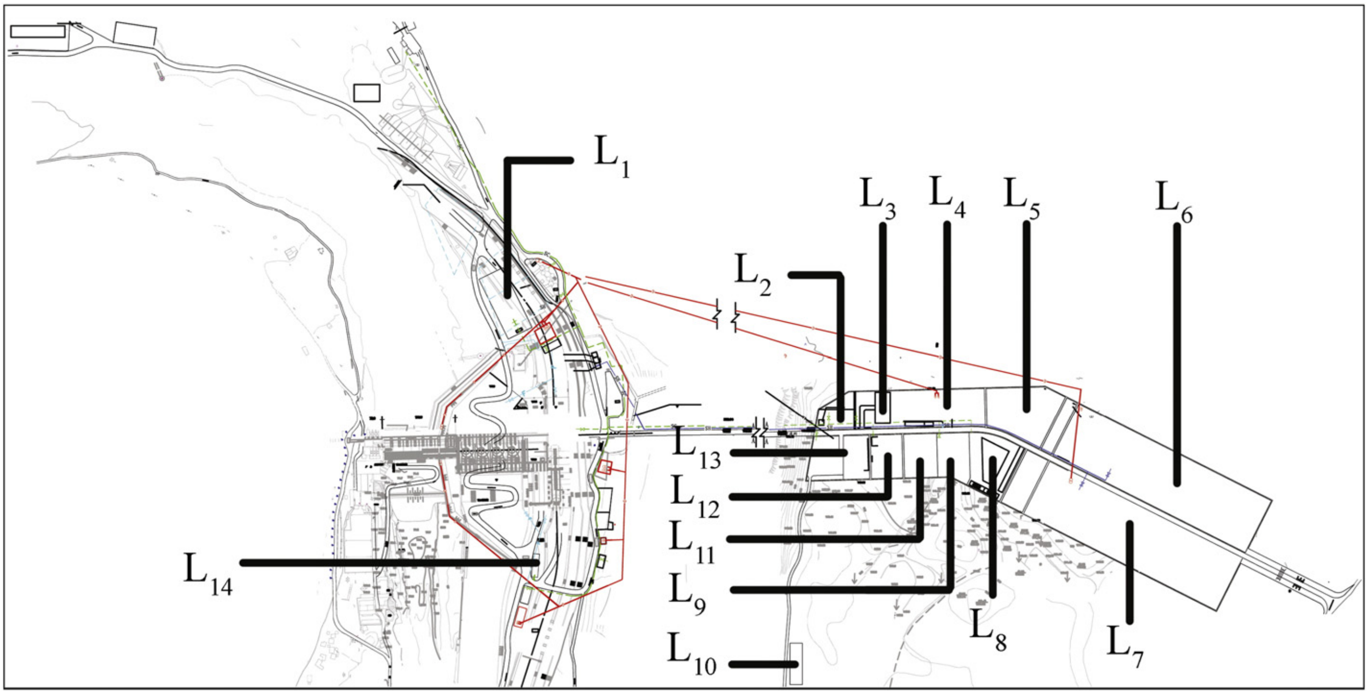

Fig. 8. The distribution of candidate locations.

Step 7.7 Increment the loop counter.
Step 8 END WHILE
Step 9 If the stopping criterion is met, i.e., $t=T$, go to step 10. Otherwise, $t=t+1$ and return to step 7.
Step 10 Decode ARC_{id} as the solution set.

5. Case study

The problem considered in this paper is from the project of Longtan hydropower station. The following is the case representation, the model comparison and the algorithm evaluation.

5.1. Representation of case problem

The Longtan hydropower construction project located in Tian'e county of Guangxi Zhuang Autonomous Region is one of the ten major indicative projects for the national Great Western Development in China and the Power Transmission from West to East strategies. It is a control project for the cascade development of the Red River (Fig. 6). Its layout is as follows: a roller compacted concrete gravity dam, a flood building with seven outlets, two bottom outlets on the river bed, a power stream system capacity with nine installations on the left bank, and navigation structures on the right bank, including a 2-shage vertical ship lift which is used for navigation. The designed impoundment level of Longtan hydropower station is 400 m and the installed plant capacity is 5400 MW. During construction process, tunnel diversion is used to divert the river with the two diversion openings on the left and right banks. The diversion standard is a ten-year flood with a corresponding flow of 14,700 m^3/s.

There are ten bid sections in the principal part of the Longtan large-scale water conservancy and hydropower construction project. Bid i is the excavation treatment of the left bank slope, and the left bank diversion tunnel. Bid ii is the excavation treatment of the right bank slope, the diversion tunnel and the navigation structure. Bid i and bid ii can be done concurrently. Bid iii1 is the river closure, the cofferdam and the excavation treatment of the riverbed. Bid iii2 is the diversion and retaining dam concrete pouring on the left bank. Bid vi is construction of plant, the water diversion and the tailrace system in the underground powerhouse. Bid v is the second excavation treatment of navigation structure and concrete pouring. Bid iv

Table 2
The distances between locations (m).

A_{ij}	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}	L_{12}	L_{13}	L_{14}
L_1	0	1440	1584	1740	1908	2148	2256	1956	1800	2172	1680	1620	1476	1452
L_2	1440	0	144	300	468	708	816	636	516	732	348	264	72	1140
L_3	1584	144	0	156	324	564	672	492	372	1092	204	120	180	1284
L_4	1740	300	156	0	168	408	480	336	216	1248	120	144	228	1440
L_5	1908	468	324	168	0	240	348	108	132	1416	288	264	348	1608
L_6	2148	708	564	408	240	0	108	240	312	1656	420	504	588	1848
L_7	2256	816	672	480	348	108	0	264	360	1680	600	708	828	2028
L_8	1956	636	492	336	108	240	264	0	120	1500	336	444	564	1656
L_9	1800	516	372	216	132	312	360	120	0	1380	168	324	444	1776
L_{10}	2172	732	1092	1248	1416	1656	1680	1500	1380	0	1212	1128	888	1872
L_{11}	1680	348	204	120	288	420	600	336	168	1212	0	84	276	1488
L_{12}	1620	264	120	144	264	504	708	444	324	1128	84	0	192	1404
L_{13}	1476	72	180	228	348	588	828	564	444	888	276	192	0	1212
L_{14}	1452	1140	1284	1440	1608	1848	2028	1656	1776	1872	1488	1404	1212	0

Table 3
The startup costs of facilities at different locations (1000 CNY).

C^s_{xit}														
x	i													
	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}	L_{12}	L_{13}	L_{14}
F_1	367.1	264.5	250.8	264.1	267.5	214.4	223.5	255.5	170.2	159.6	131.0	105.3	174.0	360.9
F_2	157.3	126.4	130.4	84.7	85.5	150.0	138.6	127.7	106.3	180.1	97.3	117.4	98.8	174.9
F_3	126.1	196.4	192.6	267.8	270.6	235.4	223.5	255.5	287.8	160.1	130.4	108.9	218.3	229.6
F_4	105.9	116.1	120.0	127.6	128.3	182.2	95.2	159.6	148.8	138.1	131.4	179.1	163.7	131.2
F_5	126.1	105.2	170.7	95.9	96.5	139.6	127.3	180.9	191.3	159.6	130.0	95.2	109.1	98.4
F_6	126.4	100.6	80.2	74.1	74.6	85.7	63.1	95.8	127.5	148.4	140.7	95.2	131.8	109.3
F_7	84.7	106.0	180.6	169.4	171.1	160.0	85.0	95.8	106.3	116.7	195.6	137.8	142.1	87.4
F_8	96.0	107.6	100.3	105.8	106.4	192.6	191.4	202.8	148.8	138.1	151.1	190.9	87.4	98.4
F_9	105.9	97.0	80.2	95.2	96.5	150.8	212.1	170.4	180.7	169.9	98.3	95.5	109.1	98.3
F_{10}	84.7	95.0	90.3	105.8	106.4	128.6	116.9	106.6	85.0	95.2	98.0	63.0	81.9	109.7
F_{11}	63.0	73.9	103.8	95.9	96.5	117.9	127.3	95.8	97.2	84.9	98.3	63.5	76.2	90.1
F_{12}	63.5	74.9	90.0	63.5	64.6	96.4	106.6	95.8	95.6	74.7	86.9	74.0	98.8	89.1
F_{13}	115.0	126.4	140.7	148.3	149.2	117.9	106.7	95.2	96.8	106.4	97.3	76.7	131.8	164.5
F_{14}	167.3	73.3	60.2	74.0	74.6	107.2	116.9	117.1	85.0	95.6	97.3	63.0	76.5	89.6

is the steel tube processing. Bid iiv, iiiv and xi are the installation of the generating system metal structures, the flood system and the ship lift system respectively. Bid x is electrical installation. Based on the plan for each bid section, the principal part of the Longtan hydropower construction project is divided into 8 phases. The facilities required in each phase are shown in Fig. 7.

There are 14 main temporary facilities involved in the process of principal part of the Longtan hydropower construction project. These facilities as well as their required area during each phase are listed in Table 1.

As shown in Fig. 8, 14 candidate locations were identified after an investigation of carrying capacity, the slope stability as well as the topographic, geological and traffic conditions. The area of these locations is denoted as D_i, $D_i=$ (6500, 6000, 5750, 7800, 7200, 25,000, 26,000, 6600, 7200, 4600, 7200, 7200, 7400, 4800) (m^2). The distances between these 17 locations are indicated in Table 2. The setup cost of facilities at different locations is listed in Table 3. The closure costs of facilities include dismantling costs, transport costs, material loss costs and function loss costs. In this paper, the closure costs of facilities at different locations are assumed to be 11.3% of startup costs.

Empirically, F_8 (oil depot) and F_9 (explosive storage) are the most likely to cause safety and environmental accidents. Thus, they are identified as 'high-risk' facilities, namely $\{x^1,x^2\}=\{F_8,F_9\}$. F_2 (carpentry shop), F_{13} (office) and F_{14} (labor residence) are identified as 'high-protection' facilities, namely $\{y^1,y^2,y^3\}=\{F_2,F_{13},F_{14}\}$. Decision makers can set the weight by preference. In Longtan's case, the weights are set as: $\{w_{11},w_{12},w_{13}\}=\{w_{21},w_{22},w_{23}\}=\{0.2,0.3,0.5\}$.

The fuzzy random coefficients of the interaction costs per unit distance are listed in Table 4. The fuzzy random operating costs of each facility located in L_2, L_3, L_4, L_9, L_{11}, L_{12} and L_{13} are listed in Table 5. The operating costs of facilities located in L_1 and L_{14} are 5.8% greater because of their distance from the workshop. Similarly, the operating costs of facilities located in L_5, L_6, L_7, L_8 and L_{10} are each 7.5% greater. By the method proposed in Section 3, fuzzy random coefficients can be transformed to determined values.

After running MATLAB program of the proposed MOPSO algorithm, the solutions were obtained and the efficiency of the proposed algorithm was tested.

The algorithm parameters for the case problem were set as follows: swarm size popsize $=500$; iteration max $T=1000$; personal best position acceleration constant $c_p=2$; global best position acceleration constant $c_g=2$; local best position acceleration constant $c_l=1$. The large number of particles and iteration was used to reduce the probability of becoming trapped in a stagnant state [50]. The computer running environment was an intercore 2 Duo 2.00 GHz clock pulse with 2048 MB memory. The case problem was satisfactorily solved using the proposed algorithm within 17 min on average, which is an acceptable time.

The red dots in Fig. 9 are Pareto-optimal solutions, while the blue dots are the best position of the particles in this iteration. Decision makers can choose their preferred plan from these Pareto-optimal solutions. For example, if decision makers determine that the safety and environmental objectives are the most important, they may allow an increased cost to ensure the safest site layout plan. Thus, they would choose far right of the Pareto-optimal solution as shown in Table 6.

Table 4
The fuzzy random coefficients of the interaction costs per unit distance (CNY).

	$\tilde{\bar{C}}_{1,3,t}$	$\tilde{\bar{C}}_{1,10,t}$	$\tilde{\bar{C}}_{2,3,t}$	$\tilde{\bar{C}}_{5,8,t}$	$\tilde{\bar{C}}_{6,8,t}$	$\tilde{\bar{C}}_{7,11,t}$
Phase 1	0	$(3.6,\rho_{1,10,1},7.8)$ $\rho_{1,10,1}\sim\mathcal{N}(5.2,5)$	0	$(8.4,\rho_{5,8,1},16)$ $\rho_{5,8,1}\sim\mathcal{N}(11.5,8)$	$(9,\rho_{6,8,1},18.5)$ $\rho_{6,8,1}\sim\mathcal{N}(12.4,4)$	0
Phase 2	0	$(3.8,\rho_{1,10,2},8)$ $\rho_{1,10,2}\sim\mathcal{N}(5.4,6)$	0	$(8.5,\rho_{5,8,2},15.8)$ $\rho_{5,8,2}\sim\mathcal{N}(11,8)$	$(8.5,\rho_{6,8,2},18.8)$ $\rho_{6,8,2}\sim\mathcal{N}(12.6,8)$	0
Phase 3	0	$(4.1,\rho_{1,10,3},8)$ $\rho_{1,10,3}\sim\mathcal{N}(5.5,4)$	0	$(8.5,\rho_{5,8,3},15.8)$ $\rho_{5,8,3}\sim\mathcal{N}(11,10)$	$(8.5,\rho_{6,8,3},18.8)$ $\rho_{6,8,3}\sim\mathcal{N}(12.6,8)$	0
Phase 4	$(3.6,\rho_{1,3,4},7.8)$ $\rho_{1,3,4}\sim\mathcal{N}(5,5)$	$(4.3,\rho_{1,10,4},8.4)$ $\rho_{1,10,4}\sim\mathcal{N}(5.5,4)$	$(5,\rho_{2,3,4},12.5)$ $\rho_{2,3,4}\sim\mathcal{N}(10,4)$	$(8.5,\rho_{5,8,4},15.8)$ $\rho_{5,8,4}\sim\mathcal{N}(11,4)$	$(8.5,\rho_{6,8,4},19)$ $\rho_{6,8,4}\sim\mathcal{N}(12.8,8)$	0
Phase 5	$(4,\rho_{1,3,5},8.5)$ $\rho_{1,3,5}\sim\mathcal{N}(5.5,6)$	$(4.5,\rho_{1,10,5},9.5)$ $\rho_{1,10,5}\sim\mathcal{N}(5.5,5)$	$(6.7,\rho_{2,3,5},14)$ $\rho_{2,3,4}\sim\mathcal{N}(10.5,5)$	$(10,\rho_{5,8,5},16.8)$ $\rho_{5,8,5}\sim\mathcal{N}(12,6)$	$(10,\rho_{6,8,5},20.8)$ $\rho_{6,8,5}\sim\mathcal{N}(12,8)$	0
Phase 6	0	$(4,\rho_{1,10,6},8.5)$ $\rho_{1,10,6}\sim\mathcal{N}(5,4)$	0	$(8,\rho_{5,8,6},15)$ $\rho_{5,8,6}\sim\mathcal{N}(11,8)$	$(8,\rho_{6,8,6},22)$ $\rho_{6,8,6}\sim\mathcal{N}(12,6)$	$(4,\rho_{7,11,6},6)$ $\rho_{7,11,6}\sim\mathcal{N}(5,2)$
Phase 7	0	$(4,\rho_{1,10,7},7.8)$ $\rho_{1,10,3}\sim\mathcal{N}(5.5,4)$	0	$(8.5,\rho_{5,8,7},15.8)$ $\rho_{5,8,7}\sim\mathcal{N}(11,4)$	$(8.5,\rho_{6,8,7},24)$ $\rho_{6,8,7}\sim\mathcal{N}(12,8)$	$(4,\rho_{7,11,7},7)$ $\rho_{7,11,7}\sim\mathcal{N}(5.4,1)$
Phase 8	0	0	0	$(8.5,\rho_{5,8,8},15.8)$ $\rho_{5,8,8}\sim\mathcal{N}(11,8)$	$(8.5,\rho_{6,8,8},26)$ $\rho_{6,8,8}\sim\mathcal{N}(14,8)$	$(4,\rho_{7,11,8},8)$ $\rho_{7,11,8}\sim\mathcal{N}(6,4)$

On the contrary, if decision makers think the cost objective is the most important, they may choose the minimum total cost plan and sacrifice the safety and environmental objective. The minimum total cost plan is shown in Table 7.

5.2. Model comparison

Here is a comparison between the dynamic construction site layout model considering the fuzzy random factors and other traditional models, including the one without any uncertainty, and the one only considering the fuzziness.

(1) Qualitatively, there are two kinds of uncertainty in construction site layout planning problems, i.e., objective uncertainty which can be dealt by randomness and subjective uncertainty which can be dealt by fuzziness. Only the fuzzy random programming approach explicitly considers the entire range of hybrid uncertain scenarios, which is more reality-oriented.

(2) Quantitatively, for each type, the MOPSO programme runs 10 times with the same parameter values to study the performance. Considering the true determinate data do not exist in practice, and most of them are obtained by ignoring their uncertainty, the determinate data are chosen randomly without any uncertainty considerations. For the fuzzy type, the statement “it is about 5.2 CNY” is understood as “it is 5.2 CNY” to ignore the ambiguous information and the rest are similar. From the results listed in Table 8, it is clear that the fuzzy random type has much better performances than the others, not only in the value of the results, but also in stability.

5.3. Algorithm analysis

In this part, the algorithm used in this paper is analyzed from both qualitative and quantitative aspects.

(1) Qualitatively, this paper compares the GA which is mostly used in the construction site layout problems with the algorithm used in this study. The merit of GA is its strong evolutionary process to find an optimal solution by the operation of crossover, selection and mutation. However, the randomly generated initial generation at the algorithms' beginning affects the solution quality because of the bad gene inherited from the parent generation. Moreover, the searching capability is reduced as GA does not rely on gradient or derivative information [48]. What's more, most GA approaches adopt a sequence-based representation to encode candidate solutions to site layout problems, where each element in the sequence represents a facility, and the number in the element represents the location for the facility. Reproduction of the sequence-represented solutions based on the operators (e.g., crossover and mutation) could lead to infeasible solutions when several elements have the same value of numbers, i.e., overlay of multiple facilities at one location. In the proposed algorithm, the permutation-based representation closely connects the particles of PSO and the solutions to the problem. The hybrid particle-updating mechanism successfully avoids the permutation infeasibility. At the same time, the multi-objective method is introduced to obtain the Pareto optimal solution set. This method provides more effective and non-dominant alternate schemes for the decision makers. Compared with the

Table 5
The fuzzy random coefficients of the operating costs (1×10^3 CNY).

	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5	Phase 6	Phase 7	Phase 8
F_1	$(80.3,\mathcal{N}(85.5,8),90)$	$(76.1,\mathcal{N}(80.5,5),86)$	$(97.1,\mathcal{N}(100.8,6),110)$	$(103,\mathcal{N}(110,10),118.9)$	$(93.3,\mathcal{N}(95.5,4),97)$	$(86.1,\mathcal{N}(88,4),89)$	$(72.1,\mathcal{N}(74.8,6),78)$	
F_2		$(57,\mathcal{N}(62,6),64.8)$	$(55,\mathcal{N}(64,10),70.8)$	$(68,\mathcal{N}(71.5,8),75.8)$	$(76.4,\mathcal{N}(78,6),80)$	$(77,\mathcal{N}(78,2),79.5)$	$(67,\mathcal{N}(71,2),73)$	$(58,\mathcal{N}(60.5,3),63)$
F_3				$(134,\mathcal{N}(138,8),144.5)$	$(135,\mathcal{N}(138,10),146)$			
F_4	$(40.3,\mathcal{N}(42,4),44.3)$	$(40.3,\mathcal{N}(42,4),44.3)$	$(40.3,\mathcal{N}(42,4),44.3)$	$(40.3,\mathcal{N}(42,4),44.3)$	$(39,\mathcal{N}(40.5,2),41.8)$			
F_5	$(53,\mathcal{N}(55.4,5),57)$	$(54,\mathcal{N}(56.4,5),57,8)$	$(54,\mathcal{N}(56.4,5),57,8)$	$(58,\mathcal{N}(59.4,4),62,5)$	$(58,\mathcal{N}(59.4,4),62,5)$	$(55,\mathcal{N}(56.4,3),57)$	$(52,\mathcal{N}(53.4,4),55,3)$	$(50,\mathcal{N}(52.1,4),53,6)$
F_6	$(42,\mathcal{N}(44,2),45)$	$(42,\mathcal{N}(44,2),45)$	$(42,\mathcal{N}(44,2),45)$	$(44,\mathcal{N}(46.4,3),47)$	$(44,\mathcal{N}(46.4,3),47)$	$(42,\mathcal{N}(43.2,3),45)$	$(42,\mathcal{N}(43.2,3),45)$	$(38,\mathcal{N}(40.4,5),43)$
F_7						$(66.4,\mathcal{N}(67.5,4),68)$	$(66.4,\mathcal{N}(67.5,4),68)$	$(66.4,\mathcal{N}(67.5,4),68)$
F_8	$(35.1,\mathcal{N}(36.5,2),38)$	$(35.1,\mathcal{N}(36.5,2),38)$	$(36.3,\mathcal{N}(38.2,2),40)$	$(36.3,\mathcal{N}(38.2,2),40)$	$(36.3,\mathcal{N}(38.2,2),40)$	$(36,\mathcal{N}(37.4,2),38.8)$	$(34.3,\mathcal{N}(35.2,2),36)$	$(34.3,\mathcal{N}(35.2,2),36)$
F_9	$(41,\mathcal{N}(43.6,4),45.6)$	$(41,\mathcal{N}(43.6,4),45.6)$	$(41,\mathcal{N}(43.6,4),45.6)$		$(41,\mathcal{N}(43.6,4),45.6)$	$(41,\mathcal{N}(43.6,4),45.6)$		
F_{10}	$(29.3,\mathcal{N}(31,3),33)$	$(29.3,\mathcal{N}(31,3),33)$	$(29.3,\mathcal{N}(31,3),33)$	$(32.4,\mathcal{N}(34,3),36)$	$(32.4,\mathcal{N}(34,3),36)$	$(32.4,\mathcal{N}(34,3),36)$	$(27.5,\mathcal{N}(29,4),32)$	
F_{11}	$(28.6,\mathcal{N}(30,3),33.2)$	$(28.6,\mathcal{N}(30,3),33.2)$	$(29.6,\mathcal{N}(31.4,4),34)$	$(34.5,\mathcal{N}(36,3),38)$	$(34.5,\mathcal{N}(36,3),38)$	$(33.8,\mathcal{N}(35,3),36.6)$	$(31.2,\mathcal{N}(33,3),34.7)$	$(28.9,\mathcal{N}(30.4,3),32.2)$
F_{12}	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$	$(33,\mathcal{N}(34.5,2),36)$
F_{13}	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$	$(42.3,\mathcal{N}(43,3),44)$
F_{14}	$(52,\mathcal{N}(54.9,4),57)$	$(52,\mathcal{N}(56.7,4),58)$	$(58,\mathcal{N}(62.8,4),65)$	$(59,\mathcal{N}(64.8,4),67)$	$(58,\mathcal{N}(62.8,4),65)$	$(56,\mathcal{N}(58.5,2),63)$	$(53.2,\mathcal{N}(54.1,4),56)$	$(51.2,\mathcal{N}(53.4,4),55.4)$

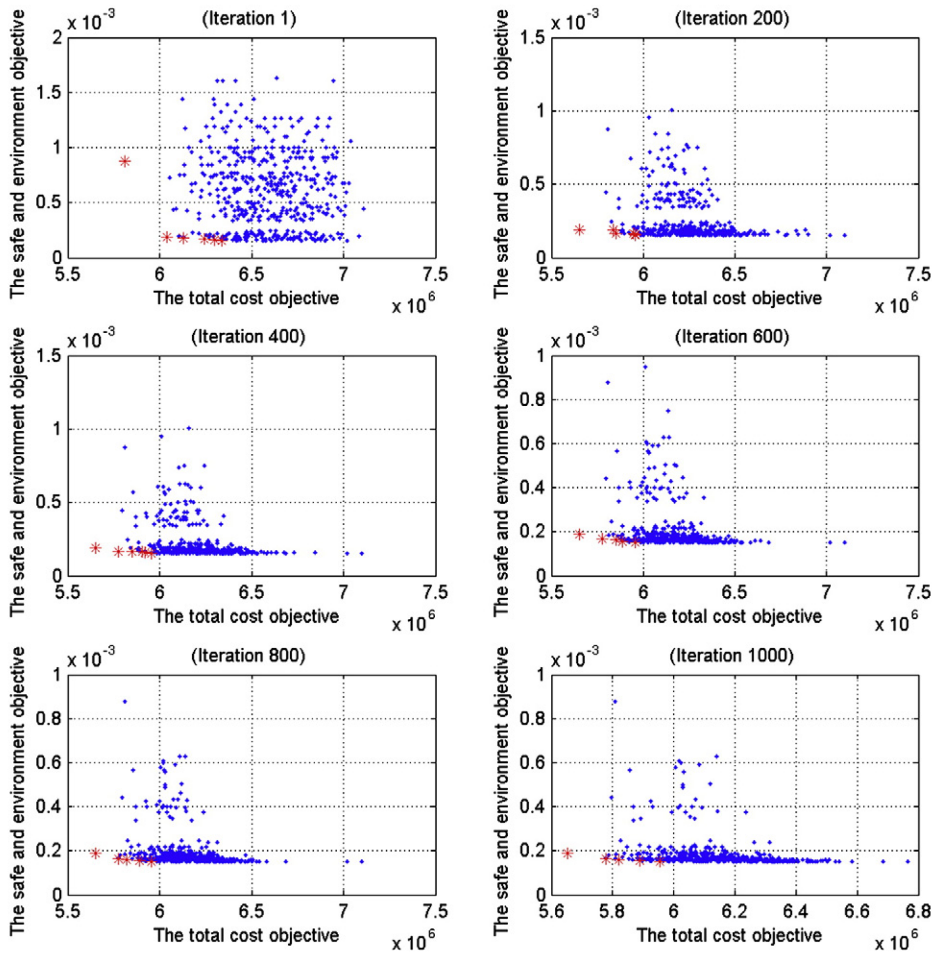

Fig. 9. Pareto optimal solutions of the Longtan case.

weight-sum method used in past research dealing with multi-objective dynamic site layout planning problems, the solutions in this paper have more reference value for the decision makers and reflect the users' preference requirements.

(2) Quantitatively, experimentally comparing the different optimization techniques always involves the notion of performance. For multi-objective optimization, the definition of quality is substantially more complex than single-objective optimization problems. There are many metrics of performance used to measure the distance of the resulting non-dominated set to the Pareto-optimal front, i.e., the distribution of solutions and the extent of the obtained non-dominated front [49].

The distribution of the Pareto optimal solutions is deemed satisfactory as shown in Fig. 9. For further expression of the efficiency of the convergence, three metrics of performance are studied. Table 9 shows the average distance to the Pareto optimal sets, the distribution in combination with the number of non-dominated solutions found and the extent of the Pareto optimal fronts proposed in [49]. The metrics of performance for the Pareto optimal set shown in Table 9 provide a satisfactory result in the efficiency of the convergence. Although there are some fluctuations in the three metrics, the final result has not been affected.

6. Conclusion

This paper proposed a multi-objective decision making model for dynamic construction site layout planning problems under fuzzy random environment and a permutation-based MOPSO algorithm to solve the model. The approach is applied to the Longtan large-scale water conservancy and hydropower construction project to illustrate the effectiveness of the proposed model and algorithm. The main

Table 6
The safest dynamic site layout plan in the Longtan case.

Phase	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
1	3	–	–	1	4	5	–	10	14	9	2	12	6	7
2	3	13	–	1	4	5	–	10	14	9	2	12	6	7
3	3	13	–	1	4	9	–	10	14	11	2	8	6	7
4	12	3	1	13	4	9	–	10	–	11	2	8	6	7
5	12	3	1	13	4	9	–	10	14	11	2	8	6	7
6	12	13	–	–	4	9	5	10	14	11	2	8	6	7
7	12	13	–	–	4	9	5	14	–	11	2	8	6	7
8	–	13	–	–	4	9	5	14	–	–	11	8	6	7

Table 7
The most cost-effective dynamic site layout plan in the Longtan case.

Phase	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
1	8	–	–	1	4	5	–	3	10	11	2	12	6	7
2	8	13	–	1	4	5	–	3	10	11	2	12	6	7
3	12	13	–	14	4	9	–	3	10	11	2	8	6	7
4	12	13	1	14	4	9	–	3	–	11	2	8	6	7
5	12	13	1	14	4	9	–	3	10	11	2	8	6	7
6	12	13	–	–	4	9	5	3	10	11	2	8	6	7
7	12	13	–	–	4	9	5	10	–	11	2	8	6	7
8	–	13	–	–	4	9	5	10	–	–	11	8	6	7

Table 8
Comparisons of three types of model.

Type	The total cost objective (1×10^6)			The safe and environment objective (1×10^{-3})		
	Best result	Worst result	Average result	Best result	Worst result	Average result
Fuzzy random type	5.8404	5.9809	5.9114	0.1521	0.1628	0.1562
Fuzzy type	5.9531	6.2909	6.1736	0.1572	0.1864	0.1711
Determinate type	5.9902	6.4373	6.2889	0.1584	0.3754	0.1816

Table 9
Algorithm evaluation by performance metrics.

Iteration	The average distance metric	The distribution metric	The extent metric
1	0.0936	0.1278	572.3830
200	0.0715	0.3860	864.0474
400	0.0552	0.1369	966.0624
600	0.0425	0.3328	678.6942
800	0.0363	0.5641	752.1259
1000	0.0336	0.6991	994.3038

contributions of this paper are as follows: firstly, this paper focused on the multi-objective dynamic construction site layout problems of large scale construction projects which have great practical significance. Secondly, the fuzzy random uncertainty in the mathematical modeling of construction site layout planning is considered for the first time. Because of the introduction of multi-objective and fuzzy random uncertainty, the proposed model is closer to reality compared with traditional models. Thirdly, to solve the proposed model, a permutation-based MOPSO algorithm was designed, which reduced infeasible solutions effectively and generated Pareto-optimal solutions. Finally, the model and algorithm were successfully applied to a practical case, so that the model and algorithm proposed were proved to be viable and efficient. To develop more suitable models, future research will consider more realistic factors and constraints, such as area utilization, unequal-area departments and multi-floor facility layout.

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