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Mathematical programming models for construction site layout problems

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ABSTRACT

We address the construction site layout problem that determines the locations of temporary facilities. Mathematical programming models for the site layout problem are proposed, which can be solved by state-of-the-art solvers to optimality. A number of safety, health and environmental concerns, such as falling objects, dusts, and noise, are incorporated in the extensions of the mathematical models. We demonstrate, using numerical experiments, the superiority of our proposed mathematical programming model over existing heuristics in terms of solution optimality and the wide applicability in terms of handling practical considerations. Based on the results of conducted experiments, the proposed method achieved a 3–19% improvement on optimality over those of the existing heuristics methods. The contribution of this research work includes the advanced development of a mathematical programming model incorporating extended concerns and solving site layout problems within reasonable time.

1. Introduction

Construction site layout is a planning activity that determines the locations of temporary facilities (TFs) to improve construction efficiency. TFs include warehouses, fabrication shops, maintenance shops, batch plants, administration offices, tool trailers, staging areas, maintenance areas, labor residence facilities, and tower cranes. The types of TFs required vary for projects of different sizes, locations, and other characteristics. The layout of TFs affects the construction time (e.g., for construction sites of limited space), costs (e.g., for large sites with a large amount of material transported between different TFs), safety (e.g., there must be some clearance around a tower crane), and environment (e.g., it is better not to position a noisy workshop close to an office for health and safety reasons). A main objective of the TF layout is to minimize travel times, to remove unnecessary movement of resources, and to reduce the frequency of handling of materials. A well-designed site layout saves considerable nonproductive time that is caused by inefficient coordination of the resources employed, especially for large projects where traveling between facilities can be considerably time consuming. Deriving the best construction site layout is a difficult problem because there are many possible alternatives and it is impractical to evaluation all of the alternatives. In the conventional approach, site layout planning is done based on the experience of planners. The level of accuracy that can be achieved varies among different projects and lacks of stabilities. Site managers and related contractors still need to make further adjustments based on the actual information acquired from sites. The planning activities and qualities of plans seriously rely on experience and accuracy of site information. To improve the current circumstance, the development of computational approaches to guarantee the objective optimality and reliability is necessary.

1.1. Literature review

Because of the practical importance and the computational complexity of TF layout problems, a large number of attempts for them have been made by researchers. The TF layout problem can be classified in different dimensions: (i) discrete vs. continuous construction site layout, (ii) static vs. dynamic site layout, and (iii) layout construction vs. layout improvement. (i) Discrete layout (or called 'facility to location assignment' by Osman et al. [15]) means there are a finite number of candidate locations in a construction site and each TF needs to be assigned to one of the candidate locations. Continuous layout (or called 'facility to site assignment' by Osman et al. [15]) refers to the situation where TFs can be placed at any empty space of a construction site. The continuous empty space is sometimes discretized in solution algorithms for continuous layout problems. (ii) Static layout problems assume that all the TFs are set up and removed at the same time; as a result, a TF cannot overlap spatially with permanent facilities or other TFs. Dynamic layout problems account for the time dimension of the

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construction of the permanent facilities, and hence, after completing their tasks, some TFs will be removed to make room for new TFs to be set up. (iii) Layout construction is to design the layout of TFs from scratch, whereas layout improvement deals with conducting some but not fundamental changes to an initial layout, which is likely to be designed by site managers based on experience.

A large variety of heuristic and meta-heuristic algorithms have been developed to generate solutions for construction site layout problems. We present the stream of studies on discrete site layout in this paragraph and the stream of works on continuous site layout in the next paragraph. Yeh [22] studied a static discrete site layout problem with the objective of minimizing the sum of construction costs of TFs at candidate locations and interactive costs between TFs. An annealed neural network method was developed and tested over two case studies with 12 TFs and 12 candidate locations. Li and Love [7] formulated a static discrete site layout model that minimizes total traveling distance of site personnel between facilities. They proposed a genetic algorithm to solve an example with 11 TFs and 11 candidate locations. Li and Love [8] extended the above study to an unequal-area facility layout problem, where some candidate locations are too small to accommodate large TFs. Unlike the above studies which require deterministic cost parameters in the models, Lam et al. [6] applied fuzzy reasoning to calculate the closeness relation of TFs in the objective function to account for the uncertainty of the parameters, and proposed an ant colony optimization (ACO) algorithm to solve the problem. This research was extended in Lam et al. [5], which proposed a max-min ant system algorithm. Unlike the above static site layout problems, Xu and Li [20] proposed a multi-objective particle swarm optimization algorithm for a dynamic site layout problem with two objectives: one is minimizing the total transportation costs and the other is maximizing the distances between high-risk facilities.

In the stream of literature on continuous construction site layout, Mawdesley et al. [10] applied a genetic algorithm to a problem setting in which the inter-facility distance (minimum and maximum distance between two TFs) is account for. Zouein et al. [24] further took into account the orientations of TFs. In addition to factors considered in the above two studies, Osman et al. [15] developed a computer-aided design (CAD) platform to visually observe the layout; another characteristic of Osman et al. [15] is that it minimizes the “relative proximity weight” instead of transportation costs as the values of transportation costs are difficult to obtain. Elbeltagi et al. [3] and Sanad et al. [17] have also investigated continuous site layout problems using genetic algorithm. In the two studies, Elbeltagi et al. [3] addressed the dynamic site layout problem and Sanad et al. [17] took into account many safety and environment considerations, such as safety zones around construction areas, noisy workshops and facilities that emit gases, and effect of harmful TFs on other facilities. Zhou et al. [23] integrated genetic algorithm and simulation to address continuous construction site layout for tunnels. Ning et al. [13] proposed a max-min ant system algorithm to address the continuous construction site layout problem with two objectives: safety/environment concerns and total transportation cost of flows between the facilities. The models and algorithms were integrated into a decision-making system in Ning et al. [14]. To find Pareto-optimal solutions for site layout, Ning and Lam [12] presented a modified Pareto-based ant colony optimization algorithm and Yahya and Saka [21] developed a multi-objective artificial bee colony algorithm. As opposed to the above studies which have taken into account solely quantitative factors, Ning et al. [11] applied the intuitionistic fuzzy set theory to account for qualitative factors in construction site layout.

There are few studies that have applied mathematical optimization to construction site layout problems. Easa and Hossain [2] developed a mathematical optimization model for a static continuous site layout problem. The model was solved using LINGO software for an instance of four TFs. By contrast, we take advantage of state-of-the-art mixed-integer programming techniques and solve larger problems much more efficiently. Said and El-Rayes [16] designed an approximate dynamic programming method for a dynamic continuous site layout problem. Approximate dynamic programming, by nature, is a heuristic and cannot guarantee optimal solutions, in contrast to our exact mixed-integer programming approach. Wong et al. [19] used integer programming to solve site layout problems to minimize transport costs. Our model distinguishes from theirs in three aspects: first, we consider many more realistic factors; second, they applied a big-M method to linearize constraints, whereas our linearization approach does not require the big-M; third, we consider two objectives and propose methods to obtain the exact Pareto-frontier for the two objectives. Hammad et al. [4] applied mathematical programming technique for site layout to minimize noise pollution and transport costs. Their model is inherently nonconvex and solvers cannot guarantee global optimality. Furthermore, they proposed an epsilon-constraint method to identify an approximate Pareto-frontier for the two objectives, while our study takes advantage of the discrete nature of the decision variables and proposes cutting planes to obtain the exact Pareto-frontier.

1.2. Objectives and contributions

The objective of this paper is to develop mathematical models for TF layout problems that can be solved by state-of-the-art solvers to optimality. The contribution of the paper is two-fold. First, for a long time, the construction management community has considered it unrealistic to use mathematical programming to address layout problems due to their large set of feasible solutions. However, in the last 30 years, the spectacular advances in computational power and in integer optimization methods have made it possible to address problems that were once thought to be intractable in practical settings [1]. Hence, in contrast to most of the existing literature, which tries to develop heuristic or meta-heuristic approaches, we take advantage of the latest development of mathematical programming techniques in developing mathematical models to solve the problems to optimality within reasonable time. Second, we demonstrate how mathematical models can be applied to handle many practical considerations, especially those that can hardly be dealt with by heuristic methods.

The remainder of this paper is organized as follows. Section 2 presents a mathematical programming model for a basic static discrete construction site layout problem. Section 3 demonstrates how mathematical programming can be applied to handle a large variety of practical considerations. Section 4 reports the results of numerical experiments that are used to validate the effectiveness of the proposed models. Conclusions are presented in Section 5. The symbols used in the paper are listed in Table 1.

2. Mathematical programming model for construction site layout

In this section, we present a mathematical programming model for a basic static discrete construction site layout problem. Then we investigate the computational complexity of the problem and finally we propose techniques to transform the mathematical programming model into an integer linear programming formulation.

2.1. A nonlinear optimization model

A static discrete construction site-level layout problem is concerned with assigning a number of predetermined TFs to a number of predetermined locations. The number of predetermined places is equal to or greater than the number of predetermined TFs. We assume that each of the predetermined places is capable of accommodating any of the TFs, i.e., we consider an equal-area facility layout problem. To minimize the transportation costs between TFs, the problem can be formulated as a nonlinear optimization model:

Table 1
Symbols.

Sets	
Ω_k	Set of candidate locations suitable for TF $k = 1, 2, \ldots, K$ due to closeness requirement to fixed facilities, $\Omega_k \subseteq \{1, 2, \ldots, I\}$
Indices	
$i, j = 1, 2, \ldots, I$	A candidate location for temporary facilities
$k, l = 1, 2, \ldots, K$	A temporary facility
Parameters	
δ_{ik}	Binary parameter that equals 1 if candidate location i is large enough to accommodate TF k and 0 otherwise.
δ_{ik}'	Binary parameter that equals 0 if TF k cannot be set up at candidate location i due to safety considerations and 1 otherwise.
δ_{ik}''	Binary parameter that equals 0 if TF k cannot be set up at candidate location i due to health/environmental considerations and 1 otherwise.
δ_{ijkl}''	Binary parameter that equals 0 if simultaneously setting up TF k at candidate location i and TF l at candidate location j, $k < l$, is not allowed due to health/environmental considerations, and 1 otherwise.
c_{ik}	Safety, health and environmental damages posed to the neighbors by TF k if it is set up at candidate location i.
c_{ijkl}	Safety, health and environmental damages posed if TFs k and l, $k < l$, are simultaneously set up at candidate locations i and j, respectively.
D_{ij}	Distance between candidate locations i and j
I	Total number of candidate locations for temporary facilities
K	Total number of temporary facilities
K'	Total number of fixed facilities
$\sigma(k)$	The location where fixed facilitate k must be set up
q_{kl}	Amount of flow from TF k to TF l, $q_{kk=0}$
Functions of the decision variables	
$C(\mathbf{x})$	Transportation cost for layout plan $\mathbf{x}$
$C(\mathbf{z})$	Transportation cost for layout plan $\mathbf{z}$
$D(\mathbf{x}, \mathbf{z})$	Safety, health and environmental damages caused by layout plan $\mathbf{x}$ (or represented as $\mathbf{z}$)
Decision variables	
x_{ijkl}	Binary variable that equals 1 if TF k is set up at candidate location i and TF l is set up at candidate location j, $k < l$, and 0 otherwise.
$\mathbf{x}$	A layout plan: $\mathbf{x} = (x_{ijkl}, i, j = 1, 2, \ldots, I, k = 1, \ldots, K-1, l = k+1, \ldots, K)$.
z_{ik}	Binary variable that equals 1 if TF k is set up at candidate location i, and 0 otherwise.
$\mathbf{z}$	A layout plan: $\mathbf{z} = (z_{ik}, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K)$.

$$[\text{P1}] \quad \min_{\mathbf{z}} C(\mathbf{z}) = \sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{k=1}^{K} \sum_{l=1}^{K} D_{ij} q_{kl} z_{ik} z_{jl} \tag{1}$$

subject to:

$$\sum_{i=1}^{I} z_{ik} = 1, \quad k = 1, 2, \ldots, K \tag{2}$$

$$\sum_{k=1}^{K} z_{ik} \leq 1, \quad i = 1, 2, \ldots, I \tag{3}$$

$$z_{ik} \in \{0, 1\}, \quad i = 1, 2, \ldots, I, k = 1, 2, \ldots, K \tag{4}$$

The objective function (1) minimizes the total transportation costs between TFs. Constraint (2) imposes that each TF is assigned to exactly one location. Constraint (3) enforces that each location can accommodate at most one TF. Constraint (4) defines the z_{ik}'s to be binary decision variables. As model [P1] has multiplication terms of decision variables, $z_{ik}z_{jl}$, it is inherently nonlinear and cannot be directly solved by off-the-shelf mixed-integer (or integer) linear programming solvers.

Some studies in the literature assume that the number of predetermined locations is equal to the number of predetermined TFs, which simply means $I = K$ in model [P1]. Note that when the number of predetermined locations is greater than the number of predetermined TFs, as opposed to studies in the literature, the mathematical model [P1] does not need to have extra 'dummy' facilities to make both numbers equal.

2.2. Hardness of the problem

It has been stated in the literature [5,6,21] that the TF layout problem is strongly NP-hard. However, the NP-hardness is not demonstrated. Below, we provide a rigorous proof. The key idea is to demonstrate SLP is at least as difficult as an NP-hard problem, the traveling salesman problem (TSP).

The decision version of the TF layout problem of [P1] is in NP; that is, given an assignment of TFs to the candidate locations, it can be determined in polynomial time whether the total transportation cost is larger than a given constant Θ. We then show the NP-completeness of the problem by reducing a well-known strongly NP-hard problem, the traveling salesman problem, into a TF layout problem. The decision version of the TSP is defined as follows: Given a set of nodes $\{1, 2, \ldots, I\}$, distances D_{ij} from node i to node j, and constant Θ, determine whether there is a circle that starts from node 1, visits each of the other nodes once, and returns to node 1, such that the length of the circle is not greater than Θ.

Theorem 1. The decision version of the TF layout problem is strongly NP-hard.

Proof. Suppose that the number of TFs is equal to the number of candidate locations, $K = I$; $q_{1,2} = q_{2,3} = \ldots = q_{I,1} = 1$ and the other q_{kl}'s are 0. The objective function (1) in model [P1] becomes $\Sigma_{i=1}^{I} \Sigma_{j=1}^{I} \Sigma_{k=1}^{I} D_{ij} x_{ijk,k+1}$, where $x_{ijI,I+1}$ is defined to be $x_{ijI,1}$. It follows easily that a solution to the above TF layout problem will have a transportation cost not greater than Θ if and only if the TSP has a circle whose length is not greater than Θ. Thus, the TSP can be solved by solving a TF layout problem. □.

2.3. Linearization

The nonlinear model [P1] cannot be directly solved by off-the-shelf mixed-integer (or integer) linear programming solvers. In order to take advantage of the state-of-the-art progress in the computation of mixed-integer linear programs, we introduce binary decision variables x_{ijkl} that equals 1 if TF k is set up at candidate location i and TF l is set up at candidate location j, $k < l$, and 0 otherwise. Then model [P1] can be transformed to the following integer linear programming formulation:

$$[\text{P2}] \quad \min_{\mathbf{x}} C(\mathbf{x}) = \sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{k=1}^{K-1} \sum_{l=k+1}^{K} (D_{ij} q_{kl} + D_{ji} q_{lk}) x_{ijkl} \tag{5}$$

subject to:

$$x_{ijkl} \geq z_{ik} + z_{jl} - 1, \quad i = 1, 2, \ldots, I, j = 1, 2, \ldots, I, k = 1, 2, \ldots, K-1, l = k+1, k+2, \ldots, K \tag{6}$$

$$x_{ijkl} \leq z_{ik}, \quad i = 1, 2, \ldots, I, j = 1, 2, \ldots, I, k = 1, 2, \ldots, K-1, l = k+1, k+2, \ldots, K \tag{7}$$

$$x_{ijkl} \leq z_{jl}, \quad i = 1, 2, \ldots, I, j = 1, 2, \ldots, I, k = 1, 2, \ldots, K-1, l = k+1, k+2, \ldots, K \tag{8}$$

$$x_{ijkl} \in \{0, 1\}, \quad i = 1, 2, \ldots, I, j = 1, 2, \ldots, I, k = 1, 2, \ldots, K-1, l = k+1, k+2, \ldots, K \tag{9}$$

and Constraints (2)–(4). Constraint (6) requires that $x_{ijkl} \geq 1$ if both z_{ik} and z_{jl} are 1. Since Constraint (9) enforces the condition that x_{ijkl} is either 0 or 1, x_{ijkl} must be 1 if both z_{ik} and z_{jl} are 1. Constraints (7) and (8) impose that x_{ijkl} must be 0 if either z_{ik} or z_{jl} is 1. Altogether, Constraints (6)–(9) formulate the requirement that x_{ijkl} is 1 if and only if both z_{ik} and z_{jl} are 1.

Three issues regarding the integer linear programming model [P2] are noteworthy. First, in the definition of x_{ijkl}, we require $k < l$ to eliminate symmetry. As a result, in objective function (5) the coefficient of x_{ijkl} is $D_{ij}q_{kl} + D_{ji}q_{lk}$, which consists of the transportation cost from TF k to TF l and the transportation cost from l to k. Second, the relation between decision variables x_{ijkl}, z_{ik} and z_{jl} is $x_{ijkl} = z_{ik}z_{jl}$, i.e., x_{ijkl} is equal to 1 if and only if both z_{ik} and z_{jl} are 1. However, since we minimize x_{ijkl} in objective function (5), x_{ijkl} will automatically take the smallest possible value in the optimal solution. As a result, Constraints (7) and (8) can be removed without losing optimality. Third, as model [P2] is an integer linear program, it can be solved by state-of-the-art solvers such as CPLEX and Gurobi.

3. Practical extensions

Models [P1] and [P2] are basic and their purpose is to show the NP-hardness of the problem, the mathematical formulation, and how the mathematical formulation can be linearized and solved by solvers. In this section, we demonstrate how mathematical programming can be applied to handle a large variety of practical considerations.

3.1. Unequal area TF layout problem

Model [P2] assumes that all of the candidate locations can accommodate all of the TFs. If some of the predetermined locations are too small to accommodate some large TFs, then the problem becomes an unequal-area TF layout problem. The unequal-area facility layout problem can be easily handled by setting some of the z_{ik} decision variables at 0. Define binary parameter δ_{ik} that equals 1 if candidate location i is large enough to accommodate TF k and 0 otherwise. Then the unequal-area TF layout problem can be formulated as:

[P3] the same objective function as [P2]

subject to:

$$z_{ik} \le \delta_{ik}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{10}$$

and Constraints (2)–(4), (6) and (9). Constraint (10) ensures that if candidate location i cannot accommodate TF k, i.e., $\delta_{ik} = 0$, then z_{ik} has to be 0, meaning that TF k will not be set up at location i in the optimal solution.

3.2. Safety considerations

Safety is also a factor that should be taken into account in construction facility layout. For instance, proper safety zones around construction areas can protect workers and entities from falling objects. Some clearance from buildings or structures must be kept clear. This consideration can be handled as follows: define binary parameter $\delta_{ik}{}'$ that equals 0 if TF k cannot be set up at candidate location i due to safety considerations and 1 otherwise. Then adding the following constraints to model [P3] will account for the safety considerations:

$$z_{ik} \le \delta'_{ik}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{11}$$

The safety concerns can be considered in practice according to the determinations and judgements by occupational health and safety supervisors, preliminary risk analysis of construction site, or historical hazard and accident records in similar projects. During the construction stage, related safety concerns can also be identified through inspections or sensor readings for further layout improvement processes.

3.3. Health/environmental considerations

In a construction site, some TFs are of high harmful effects, for example, noisy workshops and those emit harmful substances such as gases, dusts, and radiation. These TFs will present health hazards to neighbors, especially if they are positioned adjacent to hospital, schools, or residential buildings. To exclude layout plans that may lead to such health hazards to the neighboring fixed facilities, we define binary parameter $\delta_{ik}{}''$ that equals 0 if TF k cannot be set up at candidate location i due to environmental considerations and 1 otherwise. Then adding the following constraints to model [P3] will account for the health/environmental impact on the neighboring fixed facilities:

$$z_{ik} \le \delta''_{ik}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{12}$$

Another health/environmental consideration is the dangerous interaction between two TFs. For instance, TFs such as noisy workshops and those emit harmful substances should be kept a far distance away from sensitive TFs such as temporary administration offices and labor residence facilities. To take into account this type of environmental consideration, we define binary parameter $\delta_{ijkl}{}''$ that equals 0 if simultaneously setting up TF k at candidate location i and TF l at candidate location j, $k < l$, is not allowed due to health/environmental considerations, and 1 otherwise. Then adding the following constraints to model [P3] will account for the health/environmental considerations of two TFs:

$$x_{ijkl} \le \delta''_{ijkl}, \quad i = 1, 2, ..., I, j = 1, 2, ..., I, k = 1, 2, ..., K-1, l = k+1, k+2, ..., K \tag{13}$$

3.4. Inter-facility distance for convenience of construction

Some facilities may need to be positioned near each other. For example, a crane must be placed close to the building it serves. To reflect this constraint, we define the set of candidate locations suitable for TF $k = 1, 2, \ldots, K$, due to closeness requirement to fixed facilities, as $\Omega_k \subseteq \{1, 2, \cdots, I\}$. The following constraints can be added to model [P3] to allow only some specific locations for each TF:

$$z_{ik} = 0, \quad i \notin \Omega_k, k = 1, 2, ..., K \tag{14}$$

3.5. Fixed facilities

In the above models as well as in many models in the literature, the transportation costs between TFs have been taken into account, whereas the transportation costs between TFs and fixed facilities in the construction site are not incorporated, or at least not incorporated quantitatively (Constraint (13) has accounted for the transportation costs between TFs and fixed facilities in a qualitative manner). We can slightly modify the models to include the transportation costs between TFs and fixed facilities in objective functions. Specifically, we will treat a fixed facility as a temporary one except that its location is pre-determined. Define K' as the number of fixed facilities. Redefine K as the total number of temporary facilities and fixed facilities, where $1, 2, \ldots, K'$ represent fixed facilities and $K'+1, K'+2, \ldots, K$ represent TFs. Redefine I as the number of candidate locations for temporary facilities plus the number of fixed facilities. Define $\sigma(k)$ as the location where fixed facilitate k must be set up, $\sigma(k) = 1, 2, \ldots, I$. Then the TF layout problem considering fixed facilities can be formulated as:

[P4] the same objective function as [P3]

subject to:

$$z_{\sigma(k),k} = 1, \quad k = 1, 2, ..., K' \tag{15}$$

and Constraints (2)–(4), (6), and (9)–(14). Constraint (15) ensures that a fixed facility $k = 1, 2, \ldots, K'$ must be set up at location $\sigma(k)$.

3.6. Layout improvement

The construction site manager may already have a layout plan and he does not intend to have the plan fundamentally changed. Let

$\hat{\mathbf{z}} = (\hat{z}_{ik}, i = 1, 2, ..., I, k = 1, 2, ..., K)$ represent the manager's plan and suppose that the manager allows a change of at most U TFs' locations (e.g., $U = 3$). Then the following constraint can be added to model [P3] to obtain a layout improvement plan:

$$\sum_{i=1}^{I}\sum_{k=1}^{K}\hat{z}_{ik}z_{ik} \geq K - U \tag{16}$$

Constraint (16) guarantees that there are at least $K - U$ TFs are set up at the same locations as they are in plan $\hat{\mathbf{z}}$.

3.7. Multi-objective optimization

The transportation costs are not the solely concern in construction site layout. As mentioned above, the safety, health and environmental concerns are also important factors to be taken into account. Sections 3.2 and 3.3 have formulated the safety, health and environmental concerns in a 'yes/no' manner, i.e., a TF can either be set up at a location without causing any safety, health and environmental problem, or is not allowed to be set up at the location at all. This 'yes/no' may be too restrictive in practical applications.

A more reasonable way to model the safety, health and environmental concerns is to adopt a multi-objective approach: one objective is the transportation cost, and the other objective is the safety, health and environmental damage. In this setting, even if a TF k can be set up at candidate location i ($\delta_{ik}' = 1$ and $\delta_{ik}'' = 1$), it may still cause some safety, health and environmental damages.

To formulate such a bi-objective model, we need to define some new parameters: c_{ik} is the safety, health and environmental damages posed to the neighbors by TF k if it is set up at candidate location i; c_{ijkl} is the safety, health and environmental damages posed if TFs k and l, $k < l$, are simultaneously set up at candidate locations i and j, respectively. Then the following bi-objective model can be formulated:

$$[\text{P5}] \quad \min_{\mathbf{x},\mathbf{z}} \begin{pmatrix} C(\mathbf{x}) \\ D(\mathbf{x},\mathbf{z}) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}(D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \\ \sum_{i=1}^{I}\sum_{k=1}^{K}c_{ik}z_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}c_{ijkl}x_{ijkl} \end{pmatrix} \tag{17}$$

subject to Constraints (2)–(4), (6), and (9)–(15).

It is unlikely for model [P5] to have a solution $(\mathbf{x}, \mathbf{z})$ that minimizes both objectives simultaneously. The purpose of formulating the bi-objective model [P5] is for us to develop algorithms to identify Pareto optimal solutions and the Pareto frontier of the two objective functions. The Pareto frontier can be shown to construction managers to decide which solution to adopt. Because of the discrete nature of the decision variables, we propose the following algorithms to identify all the Pareto optimal solutions:

3.7.1. Algorithm: identifying all the Pareto optimal solutions

Step 0. Define Ψ as the set that saves the Pareto optimal solutions to be generated; $\Psi \leftarrow \emptyset$. Solve [P4] and obtain an optimal solution denoted by $\mathbf{z}^0$ (note that we only mention the $\mathbf{z}$-component of the solution because the $\mathbf{x}$-component is implied by the $\mathbf{z}$-component). Then $\mathbf{z}^0$ has the smallest transportation costs.

Step 1. Solve the following model and its optimal solution is denoted by $\mathbf{z}^*$.

$$[\text{P6}] \quad \min_{\mathbf{x},\mathbf{z}} D(\mathbf{x},\mathbf{z}) = \sum_{i=1}^{I}\sum_{k=1}^{K}c_{ik}z_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}c_{ijkl}x_{ijkl} \tag{18}$$

subject to:

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}(D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \leq \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}(D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl}^{0} \tag{19}$$

where ${x_{ijkl}}^0$ is element in the solution $\mathbf{z}^0$, and subject to the other relevant constraints. Note that because of Constraint (19), $\mathbf{z}^*$ must have the smallest transportation costs; because we minimize the safety, health and environmental damages in objective function (18), $\mathbf{z}^*$ has the smallest damage among all of the solutions that minimize the transportation costs. Therefore $\mathbf{z}^*$ is a Pareto-optimal solution. Set $\Psi \leftarrow \Psi \cup \{\mathbf{z}^*\}$.

Step 2. Solve the following model:

$$[\text{P7}] \quad \min_{\mathbf{x},\mathbf{z}} C(\mathbf{x}) = \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}(D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \tag{20}$$

subject to:

$$\sum_{i=1}^{I}\sum_{k=1}^{K}c_{ik}z_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}c_{ijkl}x_{ijkl} \leq \sum_{i=1}^{I}\sum_{k=1}^{K}c_{ik}\hat{z}_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K}c_{ijkl}\hat{x}_{ijkl}, \quad \hat{\mathbf{z}} \in \Psi \tag{21}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}(1 - \hat{z}_{ik})z_{ik} \geq 2, \quad \hat{\mathbf{z}} \in \Psi \tag{22}$$

and the other relevant constraints. If model [P7] has no feasible solution, go to Step 3; otherwise, its optimal solution is denoted by $\mathbf{z}^*$. Note that because of Constraint (21), the safety, health and environmental damages of $\mathbf{z}^*$ is not greater than any solution in Ψ; Constraint (22) makes sure that $\mathbf{z}^*$ is different from any solution in Ψ; in other words, solutions in Ψ will not be generated again. The objective function (20) minimizes the transportation costs. It should be highlighted that $\mathbf{z}^*$ is not necessarily a Pareto-optimal solution because it is possible that a solution in Ψ has the same damage as $\mathbf{z}^*$ but a lower transportation cost. However, we still let $\Psi \leftarrow \Psi \cup \{\mathbf{z}^*\}$ for the moment and repeat Step 2.

Step 3. Compare each pair of solutions in Ψ and remove those solutions that are not Pareto optimal from Ψ. Then Ψ is the complete set of Pareto optimal solutions. □.

4. Numerical experiments

In this section, we report the results of numerical experiments that are used to validate the effectiveness of the proposed models. The experiments are run on a PC equipped with 3.60 GHz of Intel Core i7 CPU and 16 GB of RAM. The integer programming models are all solved by CPLEX 12.6.3.

4.1. Comparison of solution quality with the literature

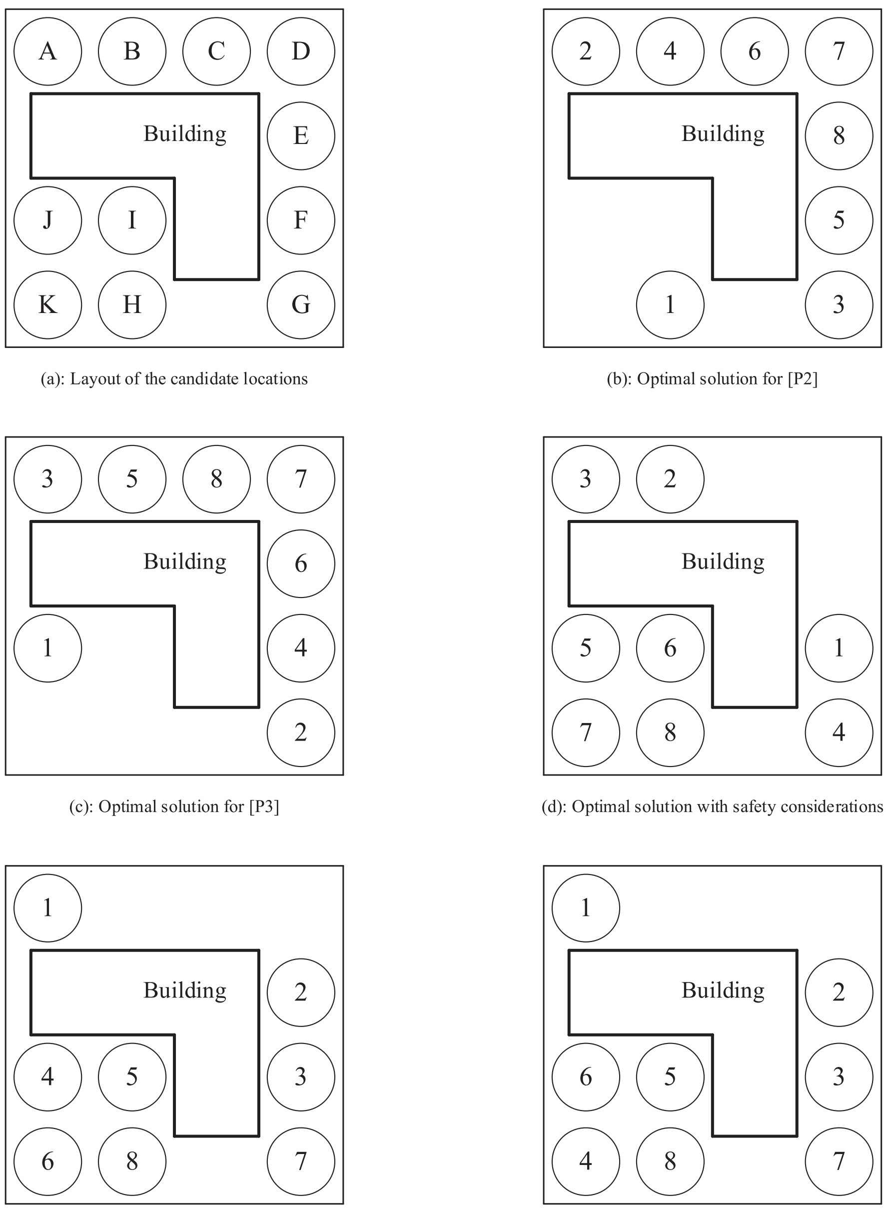
To test the superiority of our proposed mathematical programming models over existing heuristics in terms of solution optimality, we solve four case studies in three papers in the literature and compare our solutions with theirs. The results are shown in Table 2.

In the first case study of Yeh [22], the total "interactive cost" between TFs was solved to be 93 using the annealed neural network method. Our method finds the optimal solution with the total interact cost at 90, a reduction of 3%. For the second case of Yeh [22], our model improves the objective value from − 133 to − 143. In the case of Li and Love [7], the genetic algorithm obtains an objective of 15,090, while our model improves it to 12,284, a reduction of 19%. Similarly, the objective value for the case of Li and Love [8] is also improved by 19%. These comparisons clearly show the superiority of our proposed mathematical programming models over existing heuristics in terms of

Table 2
Comparisons with the literature.

Literature		Our models		CPU time (s)
Cases	Objective value obtained	Optimal objective value	Optimal solution	
Case 1 of Yeh [22]	93	90	[9,12,6,10,8,11,7,4,5,2,1,3][b]	0.04
Case 2 of Yeh [22]	− 133[a]	− 143[a]	[10,12,8,2,7,1,9,11,5,4,3,6]	8.38
Li and Love [7]	15,090	12,284	[11,10,6,3,2,4,7,8,5,1,9]	61.06
Li and Love [8]	15,160	12,320	[11,2,5,8,9,6,4,3,7,10,1]	626.52

[a] The objective values are negative because the case allows the "interactive costs" between TFs to be negative.
[b] This solution means that TF 1 is set up at location 9, TF 2 is set up at location 12, etc.



(a): Layout of the candidate locations

(b): Optimal solution for [P2]

(c): Optimal solution for [P3]

(d): Optimal solution with safety considerations

(e): Optimal solution considering health/ environment impact on the neighours

(f): Optimal solution considering health/environment impact caused by interaction between two TFs

Fig. 1. Site layout in the case study.

solution optimality. Moreover, the last column of Table 2 demonstrates that the optimal solutions are obtained within reasonable time, as the TF layout problem is a tactical-level decision problem and does not require real-time calculation.

4.2. Case study

We then report the result of a case study to show the wide applicability of the proposed models. We consider a site shown in Fig. 1(a). It has 11 candidate locations: A, B, C, D, E, F, G, H, I, J, and K; the

Table 3
Distances between the candidate locations.

	A	B	C	D	E	F	G	H	I	J	K
A	0	2	4	6	8	10	12	8	6	4	6
B	2	0	2	4	6	8	10	10	8	6	8
C	4	2	0	2	4	6	8	12	10	8	10
D	6	4	2	0	2	4	6	10	12	10	12
E	8	6	4	2	0	2	4	8	10	12	10
F	10	8	6	4	2	0	2	6	8	10	8
G	12	10	8	6	4	2	0	4	6	8	6
H	8	10	12	10	8	6	4	0	2	4	2
I	6	8	10	12	10	8	6	2	0	2	4
J	4	6	8	10	12	10	8	4	2	0	2
K	6	8	10	12	10	8	6	2	4	2	0

Table 4
Amount of flow between the temporary facilities.

	1	2	3	4	5	6	7	8
1	0	3	4	5	6	7	8	9
2	3	0	5	6	7	8	9	10
3	4	5	0	7	8	9	10	11
4	5	6	7	0	9	10	11	12
5	6	7	8	9	0	11	12	13
6	7	8	9	10	11	0	13	14
7	8	9	10	11	12	13	0	15
8	9	10	11	12	13	14	15	0

distances (D_{ij}) between the locations are shown in Table 3, which are symmetric, meaning that $D_{ij} = D_{ji}$. Eight TFs need to be set up at these locations; the amount of flow (q_{kl}) between the TFs is shown in Table 4. In practice, the amount of flow between the TFs can be estimated in advance based on material procurement plans, construction schedules, equipment/tools operation protocols, resource allocation dependencies [9], or even historical records of similar projects judged by experienced field experts, depending on different functionalities the TFs served as. Though possible objective logistical effort and cost could be inferred given different resources features (human, equipment, and material) and environmental effects (e.g. weather), this can be served as one of the limitation and is necessary to be investigated in the future.

We first solve model [P2], which addresses the basic construction site layout problem. The optimal objective value (transportation cost) is 2784, and the optimal layout is shown in Fig. 1(b). It can be seen that TFs 6, 7, and 8 are close to each other because the amount of flow between them is very large. It can also be found that TF 1 is far away from the other TFs because only 7 candidate locations (A to G) are close and TF 1 is excluded from these locations because its amount of flow to other TFs is the smallest.

Suppose that location H is too small to accommodate TF 1 and location E is too small for TF 8, i.e., $\delta_{H1} = \delta_{E8} = 0$. We then solve model [P3] and the minimum transportation cost is still 2784. The optimal layout is shown in Fig. 1(c). We find that because the 11 candidate locations are symmetric, the layout in Fig. 1(c) is actually the "mirror" of the layout in Fig. 1(b), and hence their transportation costs are identical.

Suppose further that because of safety considerations, TF 8 cannot be assigned to any of the locations A, B, C, D, E, F, or G, i.e., $\delta_{A8}{}' = \delta_{B8}{}' = \ldots = \delta_{G8}{}' = 0$. The minimum transportation cost is then increased to 2856. The optimal layout is shown in Fig. 1(d). TF 8 has to be set up at a location among H, I, J, and K. Because there is large amount of flow between TF 8 and TFs 5, 6, 7, these three TFs are all assigned to the locations near TF 8. The other four TFs are then assigned to locations near TFs 5, 6, 7, and 8.

Suppose further that because of health/environment impact on the neighbors, TF 7 cannot be assigned to any of the locations H, I, J, or K, i.e., $\delta_{H7}{}'' = \delta_{I7}{}'' = \delta_{J7}{}'' = \delta_{K7}{}'' = 0$. The minimum transportation cost is then increased to 2904. The optimal layout is shown in Fig. 1(e). Now TF 7 and TF 8 cannot be close to each other. As a result, TFs 4, 5, and 6 are assigned to be close to TF 8 because there is large amount of flow between TF 8 and these three TFs.

Suppose further that TF 6 and TF 8 must be set up at locations whose distance is more than 2 due to health/environmental considerations. This implies that $\delta_{ij,6,8}{}'' = 0$ if $D_{ij} = 2$. The minimum transportation cost is increased to 2920. The optimal layout is shown in Fig. 1(f). We find that the layout in Fig. 1(f) is very similar to that in Fig. 1(e) with a small adjustment of the distance between TF 6 and TF 8.

Next, we examine the application of the bi-objective optimization model [P5]. Suppose that $c_{ik} = 0$ and suppose that c_{ijkl} is as follows:

$$c_{ijkl} = \begin{cases} kl & \text{if } D_{ij} = 2 \\ 0, & \text{if } D_{ij} > 2 \end{cases} \tag{23}$$

That is, c_{ijkl} is not 0 if $D_{ij} = 2$. For example, $c_{KH,4,8} = 4 \times 8 = 32$. All of the constraints in the above are still used. Applying the proposed algorithm, we computed the Pareto frontier, which is plotted in Fig. 2. As expected, the minimum total transportation cost is 2920; at the same

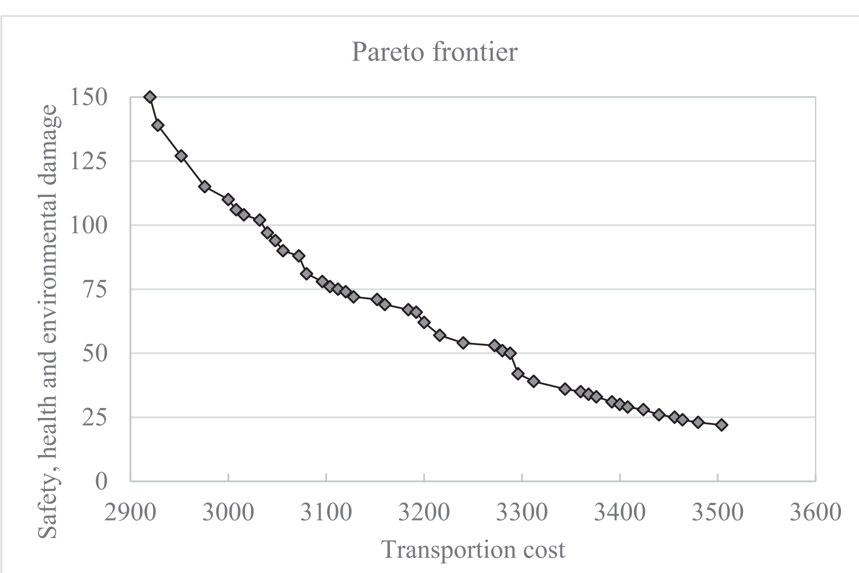


Fig. 2. Pareto frontier of the case study.

time, the total safety, health and environmental damage is 150. If we just minimize the total safety, health and environmental damage, then it will be 22; however, the transportation cost will be increased to 3504. It should be noted that because of the discrete nature of the problem, the Pareto Frontier curve is not convex. In sum, the Pareto frontier in Fig. 2 can be used to assist construction site manager to design a layout plan that balances the tradeoff between the transportation cost and the total safety, health and environmental damage.

4.3. Discussion of the research limitations

According to the results obtained, a superiority on site layout optimality has been identified by using the proposed mathematical programming model and its optimization processes. It tackled the concerns of mathematical programming which is used to be considered unrealistic in site layout planning. It further suggested layout improvement in using the proposed approach is possible given the case study shows the layout problems can be solved within reasonable time.

While the proposed mathematical programming model considering extensive practical factors in solving site layout planning problems, the estimation of safety, health and environmental concerns require objective and predetermined information of the site in order to perform an accurate planning. Related estimation techniques, such as unbiased judgements, historical data in similar projects, and detailed site information, are necessary and they seriously influence the performance of the optimization in practice.

The proposed model targets on solving static and discrete site layout planning problem which can be less practical if the predetermined locations for TPs are not applicable. However, a possible site layout planning procedure is proposed accordingly given the efficient performance the proposed approach can achieve. A preliminary identification for TPs' locations can always be decided judged by experienced site managers. Through the optimization process using our approach, a preliminary result can be offered in advance. Then there are rapid but gradual adjustments, by updating field information to amend the numbers, locations and sizes of potential TFs if needed, for the existing site layout. Moreover, site managers may have several layout plans in mind and are unsure of which plan to use. The proposed model can then consider all locations in the plans as candidate locations and obtain a globally optimal layout based on these candidate plans.

5. Conclusions

This paper has examined the construction site layout problem that determines the locations of temporary facilities. A mathematical model was proposed, which can be solved by state-of-the-art solvers to optimality. This contrasts to the heuristic methods in the literature, which may obtain good but not necessarily optimal solutions. We further demonstrated how mathematical programming can be applied to handle a large variety of practical considerations. Specifically, a number of safety, health and environmental concerns, can be incorporated in the proposed mathematical programming models, either as hard constraints or in a bi-objective formulation. Numerical experiments show that the optimal solutions obtained by the proposed mathematical programming models improve over the ones by existing heuristics for as large as 19%. A case study demonstrates the wide applicability of the proposed models in terms of handling practical considerations. Further efforts will be directed at using mathematical programming to address dynamic construction site layout problems.

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