Genetic Algorithm for Optimizing Supply Locations around Tower Crane

By C. M. Tam,[1] Thomas K. L. Tong,[2] and Wilson K. W. Chan[3]

Abstract: Site layout planning is a complicated issue because of the vast number of trades and interrelated planning constraints. To unfold its complexity, this paper aims to confine the study to a particularly defined area of construction: the structural concrete-frame construction stage of public housing projects. In this study, optimization of the tower crane and supply locations is targeted, as they are the major site facilities for high-rise building construction. A site layout genetic algorithm model is developed and a practical example is presented. The optimization results of the example are very promising and demonstrate the application value of the model.

INTRODUCTION

Facilities layout is believed to be the heart of efficient production. Construction site facilities layout planning (FLP), which defines the types, quantities, and positioning of the mechanical plant, storage areas, and fabrication yards, has significant impact on productivity, costs, and duration of construction. Although FLP is such a critical process in construction planning, a systematic analysis of construction site layout is always difficult because of the existence of the vast number of trades and interrelated planning constraints. Practitioners of the building industry lack a well-defined approach in construction site layout planning, especially for high-rise building construction.

Site conditions such as the topographical layout, building tower layout, and adjacent environment are unique for each site. Consequently, they result in a great variation in site layout strategies and approaches. For high-rise building construction, the allocation of temporary facilities keeps changing and is interrelated with the progress of construction work, which further complicates the planning process. Optimization of FLP (which is a nonlinear and discrete system) using the scientific approach is difficult, if not impossible, to achieve. Hence, FLP of construction sites in Hong Kong has been carried out mainly through human judgment. Because of human involvement, there are no conditions that lead consistently to the same result. Therefore, site layout planning is usually an art rather than a science.

To overcome the above, this paper aims to solve the complexities of site layout planning by analyzing a particularly defined stage of construction, structural concrete frame construction of public housing in Hong Kong, in which the use of a tower crane dominates among all the site planning activities.

Lessons can be learned from previous research. However most recorded research works have their limitations. Choi and Harris (1991) proposed a mathematical model for determining the most suitable single tower crane location. The model aims to optimize the position of a tower crane that yields the least transportation time. Rodriguez-Ramos and Francis (1983) proposed a model in locating the parking position of the crane's hook between movements. Gray and Little (1985) developed a computer package using decision flowcharts to optimize crane location for irregular-shaped buildings. Wijesundera and Harris (1989) and Zhang et al. (1999) used the Monte Carlo-simulation approach to optimize crane location. Farrell and Hover (1989) developed a database with a graphical interface to assist in crane selection and location. Most of these research works singled out the tower crane, the most critical facility in high-rise building construction, as the target of optimization. However, most of these studies have neglected the interrelated effect between locations of the tower crane and supply points and the space competition among various supply points. Li and Love (1998) and Philip et al. (1997) applied genetic algorithms (GAs) to optimize a set of predetermined facilities. However, the approach has been much simplified, shapes of facilities were considered as rectangular, and size constraint and space competition between facilities were not taken into account.

The objectives of this paper are to investigate and analyze the relationship between the key storage areas and the tower crane and to develop a GA model to optimize the above facilities, taking into account the complexity of the relationship between these facilities.

GA MODELS

GAs are directed randomized search procedures. They derive their power from the mechanics of natural selection and the survival-of-the-fittest principles (Goldberg 1989). GAs have been popular in many areas (e.g., constrained or unconstrained optimization, scheduling and sequencing, transportation, reliability optimization, and artificial intelligence) (Leu and Yang 1999). In the broadest sense, a GA creates a set of solutions that reproduce based on their fitness in a given environment. The process follows the following pattern:

1. An initial population of random solutions is created.
2. Each member of the population is assigned a fitness value based on its evaluation against the current problem.
3. Solutions with a higher fitness value are most likely to parent new solutions during reproduction.
4. The new solution set replaces the old, a new generation is complete, and the process continues by returning to Step 2.

This sequence implements, in a most simplistic way, the concept of survival of the fittest. The reproductive success of a solution is directly tied to the fitness value it is given during evaluation. In this stochastic process, the least-fit solution has a small chance at reproduction whereas the most-fit solution may not reproduce at all. The outcome of a GA is based on probabilities, just as biological success is grounded in chance.

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In site facilities layout optimization, there exist many problems to be solved (for example, the nonlinearity of the site facilities layout planning system, discreteness of the number, and positions of facilities). Among these problems, one of the important issues is the optimal placement of facilities in sites, on the condition that all facilities are considered simultaneously. GAs are heuristic random search techniques based on the concept of natural selection and natural genetics of a population (Holland 1975; Goldberg 1989). GAs presume that the potential solution of any problem is unique and can be represented by a set of parameters. These parameters are regarded as the genes of a chromosome and can be structured by a string of values in binary form. A positive value, generally known as a fitness value, is used to reflect the degree of ''goodness'' of the chromosome for the problem that would be highly related to its objective value. Because of the distinctive features such as domain independence, nonlinearity, robustness, and parallel nature, GA has been proven to be a versatile and effective approach for solving optimization problems. The successful application of GAs to both combinatorial and discrete optimization problems (Koumousis and Georgiou 1994) motivated the application of GAs for solving mixed-discrete nonlinear optimization problems (Jenkins 1997).

MODEL DESCRIPTION

Three steps are involved in optimizing the locations of the tower crane and supply points. First, the permissible locations of the supply points are determined from the site map with the consideration of the length of the tower crane jib and its capacity radius, required size of the supply points, and other site constraints. The demand points are fixed by the geometric shape of the permanent building. Then the possible locations of the tower crane are plotted, which are dependent on the structural design layout, space provisions of the permanent structure, convenience to other site activities, etc. Finally, a GA model is applied to optimize two outputs: the tower crane and the supply point locations for various trades.

Assumptions

A single-tower-crane optimization model is applied to search for the optimal location in terms of minimal hook transportation time. Existing models have their limitations. First, they tend to oversimplify the site space allocation and positions of tower cranes and neglect the interdependent and space competition relationships between site facilities.

The following assumptions were applied to model development:

- Geometric layout of all demand points is predetermined and fixed.
- For each supply and demand pair, demand levels for transportation are known (e.g., total number of lifts, number of lifts for each batch, and maximum load).
- The material transported between a supply-demand pair is handled by one crane only.
- The horizontal simultaneous movement of crane operations in lifting objects for experienced crane operators is assumed to be 76% of the total duration of the cycle (Kogan 1976). Hence the coefficient α that represents the degree of coordination of hook movement in radial and tangential directions in the horizontal plane is assumed to be 0.25 (Zhang et al. 1999).
- The vertical simultaneous movement of crane operations is assumed to be small for high-rise building construction where the object needs to be lifted to a level that is clear of the building before radial movements can be activated. Hence, the coefficient β that represents the degree of coordination of hook movement in vertical and horizontal planes is assumed to be 1; i.e., the hook moves consecutively in two planes (Zhang et al. 1999).
- The feasible locations for the tower crane and supplied points are limited by the site conditions and shape of the permanent building.
- The area of each supplied point is large enough to accommodate the storage requirements.

Determination of Supply Points

The reach of a crane tower is determined by the length of its jib and its lifting capacity is decided by a radius-load curve where the greater the load, the smaller the crane's operating radius. Hence the locations of both the supply and demand points must fall within the permissible weight-radius circle of the tower crane. Because the demand points are fixed, attention is focused on the permissible locations of supply points.

Model for Calculation of Hook Travel Time (Zhang et al. 1999)

$$T = \max(T_h, T_v) + \beta \cdot \min(T_h, T_v) \tag{1}$$

$$T_v = |ZSi - ZDi|/V_h \tag{2}$$

$$T_h = \max(T_a, T_w) + \alpha \cdot \min(T_a, T_w) \tag{3}$$

$$\rho(D_i) = \sqrt{(XD_i - XCr_i)^2 + (YD_i - YCr_i)^2} \tag{4}$$

$$\rho(S_i) = \sqrt{(XS_i - XCr_i)^2 + (YS_i - YCr_i)^2} \tag{5}$$

$$l_i = \sqrt{(XD_i - XS_i)^2 + (YD_i - YS_i)^2} \tag{6}$$

where l_i = distance between supply and demand points.

Time for trolley radial movement:

$$T_a = \frac{|\rho(D_i) - \rho(S_i)|}{V_a} \tag{7}$$

Time for trolley tangent movement (Fig. 1):

$$T_w = \frac{1}{V_w} \cdot \arccos\left(\frac{l_i^2 - \rho(D_i)^2 - \rho(S_i)^2}{2 \cdot \rho(D_i) \cdot \rho(S_i)}\right) \quad (0 \le \arccos(\theta) \le \pi) \tag{8}$$

Total Cost

$$TC = \sum_j^n \sum_k^n T \cdot Q_{jk} \cdot C_{jk} \tag{9}$$

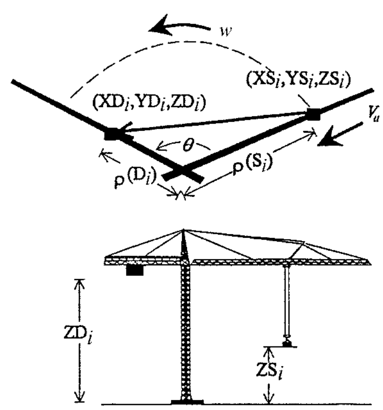

FIG. 1. Hook Travel Time

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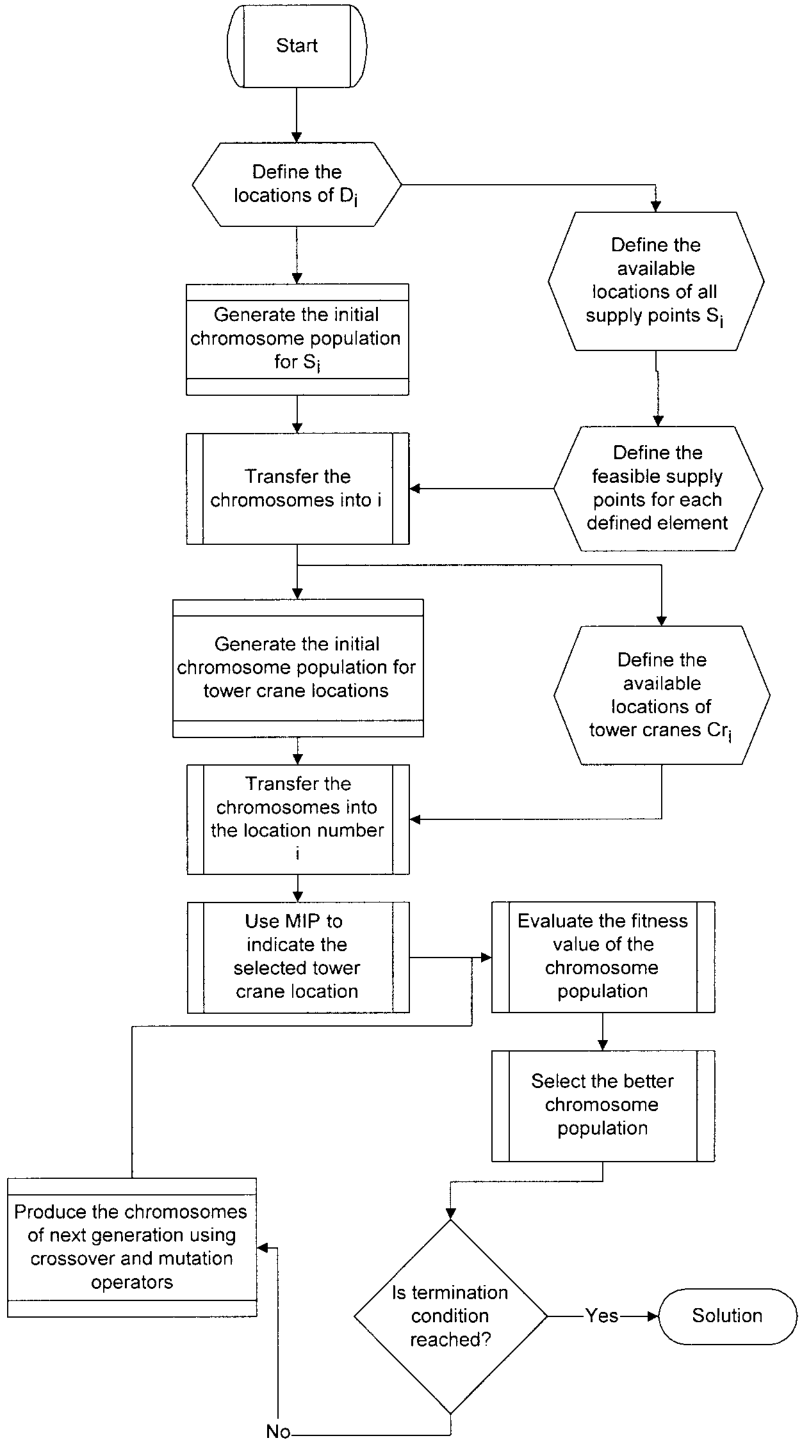

FIG. 2. Flowchart of GA Model

GA Model

As illustrated in Fig. 2, the GA modeling procedure is, first, to define the demand and supply point locations by specifying their 3D coordinates. Then, a series of chromosomes are generated, which are mapped to the various location points (including supply and demand points). The fitness value of the first attempt is calculated from the fitness function defined. Mutation and crossover are then applied to change the chromosomes. The fitness value of the new generation is compared with the previous one. The process is repeated until the termination condition has been reached (for example, no change or no improvement of the fitness value in 200 rounds of attempts):

1. Define coordinates of D_i

$$\begin{gathered} D_1(XD_1,\ YD_1,\ ZD_1) \\ D_2(XD_2,\ YD_2,\ ZD_2) \\ \vdots \\ D_i(XD_i,\ YD_i,\ ZD_i) \end{gathered}$$

2. Define the coordinates of all available supply point locations

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define $w = \{S_i | \text{coordinates of all available supply point locations}, \forall_i \in (1, 2, \ldots, n)\}$

so that one has

$$\begin{gathered} S_1(XS_1, YS_1, ZS_1) \\ S_2(XS_2, YS_2, ZS_2) \\ \vdots \\ S_n(XS_n, YS_n, ZS_n) \end{gathered}$$

3. Define feasible supply points for the location of element j

$$\text{define } A_j = \{S_i | S_i \text{ feasible for element } j\}$$

4. Generate chromosomes for the first generation of supply points

$$\text{random numbers } \delta_1, \ldots, \delta_n \in (0, 1)$$

5. Transfer values of chromosomes $\{\delta_i | \text{random numbers}, \forall_i \in (1, 2, \ldots, n)\}$ into location number i for each element A_j

$$S_{Aj} = \{S_i | S_i \subset A_j\}$$

6. Define the available positions for the tower crane location

$$\begin{gathered} Cr_1(XCr_1, YCr_1, ZCr_1) \\ Cr_2(XCr_2, YCr_2, ZCr_2) \\ \vdots \\ Cr_i(XCr_i, YCr_i, ZCr_i) \end{gathered}$$

subject to the constraints that the distance between the supply points S_i and tower crane Cr_i and between demand points D_i to tower crane Cr_i, plus a margin that allows for the size of the storage area, should be within the jib length l_{cr} or the lifting capacity radius.

7. Generate chromosomes for the tower crane positions

$$\text{random number } \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_i \in (0, 1)$$

8. Map values of ε to the location number Cr_i for the tower crane positions.
9. Use the mixed integer program (MIP) to denote the selected tower crane location. Define N to be the integer to indicate the state of selection

$$[N | Cr_i] = 1, \quad \text{selected to be tower crane position}$$

$$[N | Cr_i] = 0, \quad \text{otherwise}$$

where $\forall_i \in (1, 2, \ldots, n)$.

10. Define fitness function

$$\min TC = \min \left\{ N \cdot \left[\sum_{j=1}^{n} \sum_{k=1}^{n} T_{jk} Q_{jk} C_{jk} \right] \right\} \tag{10}$$

11. Crossover and mutation takes place.
12. Repeat the process until the termination condition has been reached.
13. Optimal solution is found.

APPLICATION EXAMPLE

Over the past decades, public housing has occupied >50% of the overall housing market in Hong Kong. Owing to the constraint of land supply, high-rise residential buildings have become a norm. To speed up the construction process, the government encourages standardization in design and mechanization in construction. As a result, all housing projects adapt similar methods but struggle in their attempts to develop efficient temporary facilities. It is noted that there are significant differences in site layouts and temporary facilities used. These directly or indirectly determine the duration of floor construction cycles of public housing projects, which vary from 4–6 days/floor. This variation of 2 days for each floor cycle could lead to a reduction of 3 months for the construction of a typical 40-story housing block and could have a great impact on productivity. Further, the standardized designs and standardized space settings of public housing construction sites offer an ideal prototype for studying site facility layout.

Over the last 20 years, designs and construction technologies used in public housing projects in Hong Kong have greatly advanced. The design of public housing adopted in the 1980s was the trident type (Fig. 3). Since 1990 the mainstream design has been the harmony type (Fig. 4). The various floors are repetitive in layout, encompassing cycles of similar operations for both types of projects.

As the site layout for high-rise building construction keeps changing at different stages of construction, this study has defined a particular construction stage: the use of a tower crane in concrete frame construction is studied. The major materials transported by tower cranes are (1) large panel formwork; (2) precast concrete façade units; and (3) reinforcing bars (concrete is normally transported by concrete hoists or concrete pumps). Hence, in this model, positioning of the bending yard, façade storage yard, and assembling area of a large panel formwork is considered.

Suppose the three storage areas including the large panel formwork, precast façade units, and bending yard are denoted as A_1, A_2, and A_3. For A_1, there are six possible locations (S_1, S_2, S_3, S_4, S_5, and S_6) after considering its size, shape, and other constraints. For A_2, there are four possible locations (S_1, S_2, S_3, and S_4). For A_3, there are five possible locations (S_5, S_6, S_7, S_8,

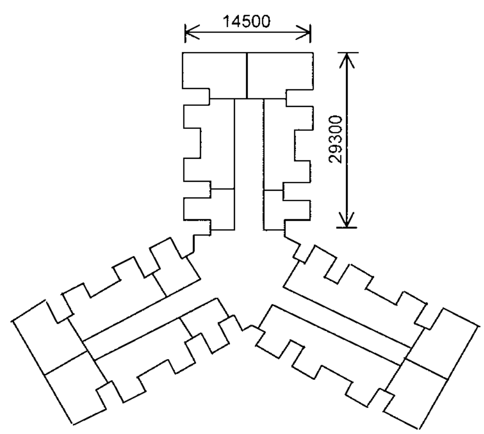

FIG. 3. Typical Floor Plan for Trident Blocks

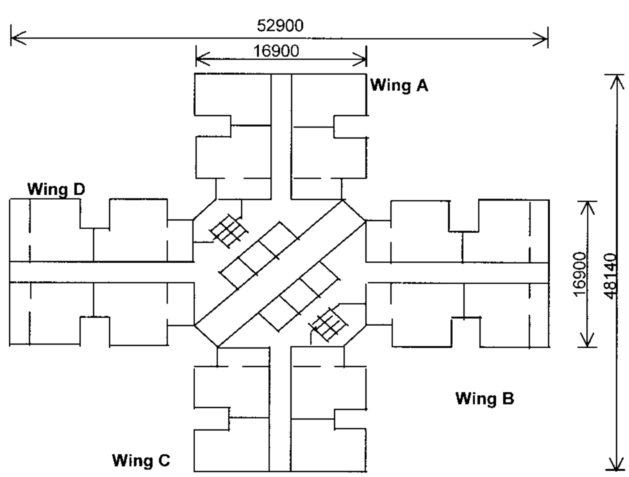

FIG. 4. Typical Floor Plan of Harmony 1 Blocks

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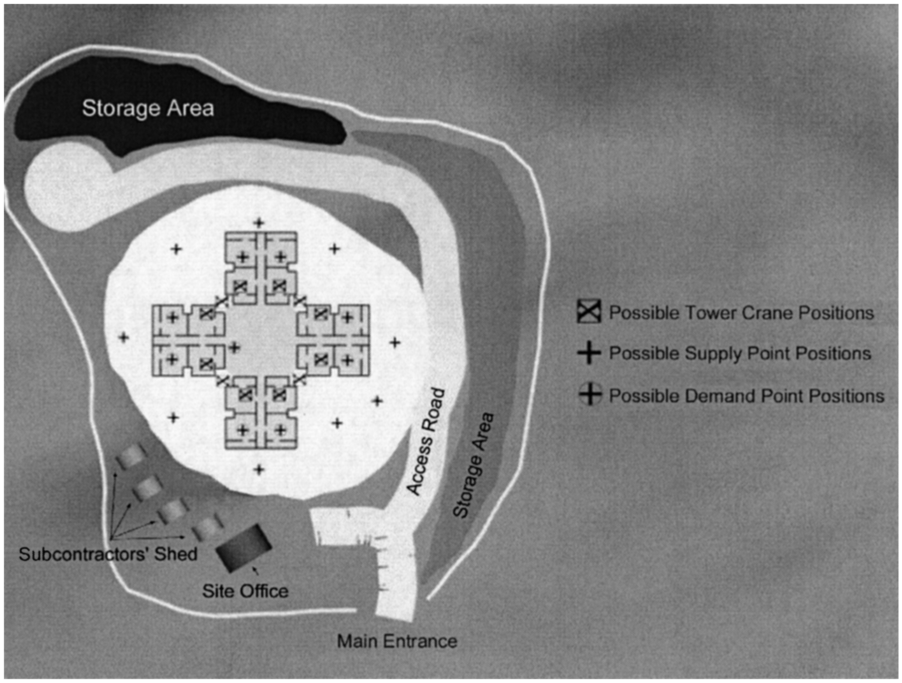

FIG. 5. Top View of Possible Supply and Tower Crane Positions, Demand Points

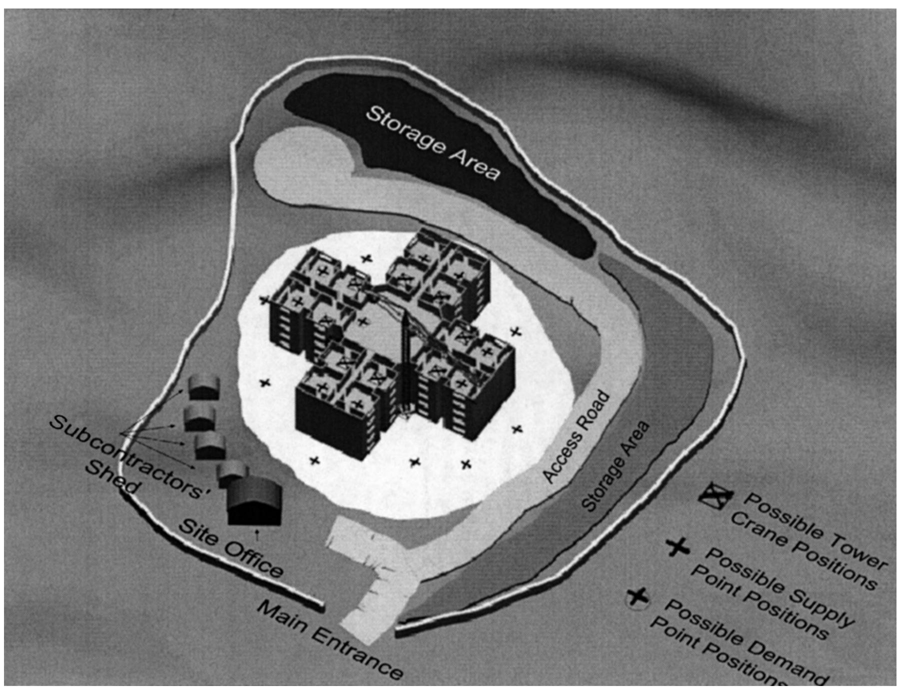

FIG. 6. Side View of Possible Supply and Tower Crane Positions, Demand Points

TABLE 1. Coordinates of Demand Points

Number	x	y	z
D_1	34	41	15
D_2	34	51	15
D_3	51	65	15
D_4	60	65	15
D_5	76	51	15
D_6	76	41	15
D_7	60	26	15
D_8	51	25	15
D_9	43	44	15

TABLE 2. Coordinates of Supply Points

Number	x	y	z
S_1	73	26	2
S_2	83	31	2
S_3	87	45	1.5
S_4	73	67	1.5
S_5	55	73	1.5
S_6	35	67	0
S_7	22	46	0
S_8	36	27	1
S_9	55	15	1

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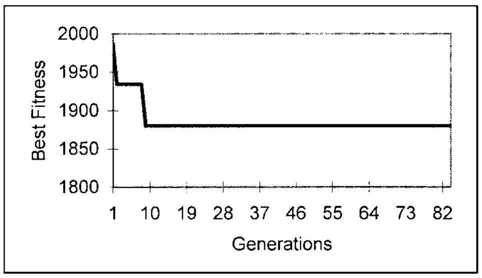
TABLE 3. Coordinates of Tower Crane Positions

Number	x	y	z
Cr_1	45	36	30
Cr_2	65	36	30
Cr_3	65	57	30
Cr_4	45	57	30
Cr_5	51	33	30
Cr_6	60	33	30
Cr_7	70	41	30
Cr_8	70	52	30
Cr_9	60	58	30
Cr_{10}	51	58	30
Cr_{11}	42	52	30
Cr_{12}	42	41	30



Initial Total Cost: 1987.33 (US$ 254)
Final Total Cost: 1880.018 (US$ 241)

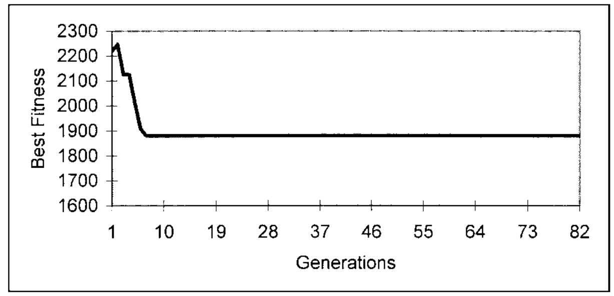
Supply Points				
No.	X	y	z	Choice
A_1	87	45	1.5	S_3
A_2	83	31	2	S_2
A_3	55	15	1	S_9

Location of tower crane				
No.	Choice	x	y	z
Cr_1	0	45	36	30
Cr_2	1 (selected)	65	36	30
Cr_3	0	65	57	30
Cr_4	0	45	57	30
Cr_5	0	51	33	30
Cr_6	0	60	33	30
Cr_7	0	70	41	30
Cr_8	0	70	52	30
Cr_9	0	60	58	30
Cr_{10}	0	51	58	30
Cr_{11}	0	42	52	30
Cr_{12}	0	42	41	30

FIG. 7. Test 1

and S_9). For tower crane positions, site layout planners prefer locating a climbing crane within the structure where the crane must not obstruct other site activities. Sometimes, a static tower crane may be located at the corner of two building wings, depending on the preference of site planners. Hence, it gives 12 possible locations of the tower crane (Cr_1, Cr_2, . . . , Cr_{12}). The top and side views of the site conditions, available supply points, feasible tower crane positions, and demand points are illustrated in Figs. 5 and 6.

Generating a set of four chromosomes {ε_1, ε_2, ε_3, and ε_4} were $\varepsilon_i \in (0, 1)$, the first three for the supply points and the last for tower crane positions, an initial population of random solutions is created. Each chromosome is mapped to a unique supply location S_i for supply points A_1, A_2, and A_3 that avoids repetition of the location for more than one supply point (for example, if S_1 is selected for A_1, S_1 will not be available for A_2). Similarly, the last chromosome is mapped to the tower crane position Cr_i.



Initial Total Cost: 2217.63 (US$ 284)
Final Total Cost: 1880.018 (US$ 241)

Supply Points				
No.	x	y	Z	Choice
A_1	87	45	1.5	S_3
A_2	83	31	2	S_2
A_3	55	15	1	S_9

Location of tower crane				
No.	Choice	x	y	z
Cr_1	0	45	36	30
Cr_2	1 (selected)	65	36	30
Cr_3	0	65	57	30
Cr_4	0	45	57	30
Cr_5	0	51	33	30
Cr_6	0	60	33	30
Cr_7	0	70	41	30
Cr_8	0	70	52	30
Cr_9	0	60	58	30
Cr_{10}	0	51	58	30
Cr_{11}	0	42	52	30
Cr_{12}	0	42	41	30

FIG. 8. Test 2

The coordinates of the supply points, demand points, and tower crane positions are defined in Tables 1–3.

The crane traveling speeds were obtained by site measurements from two public housing sites in Hong Kong and the averages are recorded as follows:

$$V_h \text{ (hoisting velocity of hook)} = 60 \text{ m/min}$$

$$V_a \text{ (radial velocity)} = 53.3 \text{ m/min}$$

$$V_w \text{ (slewing velocity of jib)} = 7.57 \text{ rad/min}$$

The costs of crane time C_{jk} is assumed to be \$1.92 (HK\$15) per minute, and the quantities of material flow for each element per concrete floor cycle Q_{jk} is 10 for A_1, 20 for A_2, and 30 for A_3. The β-value (the degree of coordination of hook movement in vertical and horizontal planes) is assumed to be 0.25, and the α-value (the degree of coordination of hook movement in radial and tangential directions in the horizontal plane) is 1.

In the GA modeling, the mutation rate is 0.05 and the crossover rate used is 0.9. The results of three rounds of modeling with different initial chromosome populations are shown in Figs. 7–9.

CONCLUSIONS

The GA model described helps improve conventional site supply points and tower crane location methods. It gives an objective, quantitative, and scientific way to evaluate the effectiveness of site facility layout.

Experimental results indicate that the model performs satisfactorily. As revealed from the application example, if site planners just randomly allocate the supply points and tower crane locations, the transportation cost will be 5.7% higher

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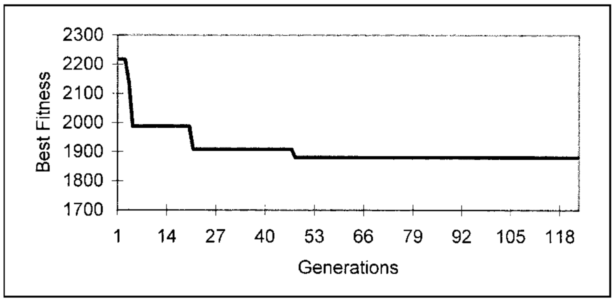

Initial Total Cost: 2216.739 (US$ 284)
Final Total Cost: 1880.018 (US$ 241)

Supply Points				
No.	X	y	Z	Choice
A_1	87	45	1.5	S_3
A_2	83	31	2	S_2
A_3	55	15	1	S_9

Location of tower crane				
No.	Choice	x	y	Z
Cr_1	0	45	36	30
Cr_2	1 (selected)	65	36	30
Cr_3	0	65	57	30
Cr_4	0	45	57	30
Cr_5	0	51	33	30
Cr_6	0	60	33	30
Cr_7	0	70	41	30
Cr_8	0	70	52	30
Cr_9	0	60	58	30
Cr_{10}	0	51	58	30
Cr_{11}	0	42	52	30
Cr_{12}	0	42	41	30

FIG. 9. Test 3

than the optimal solution as shown in Test 1 and 18% in Tests 2 and 3. This fact implies that a systematic approach in site facility planning is important to improve the site production efficiency. An 18% savings in crane traveling time can generate a substantial improvement in site productivity and savings in the time of construction.

The model offers the following superiority over traditional approaches:

- The supply points and possible tower crane positions are obtained realistically according to the site conditions and geometrical layout of the permanent structures. The model can thus generate a more realistic solution.
- Unlike the 2D layout approach of traditional methods, the model developed can handle 3D coordinates of all supply, demand, and tower crane locations.
- A number of supply points are considered simultaneously; thus, it offers a higher variability in the choice of supply points and may result in a more optimal solution.
- Site facility layout is a nondeterministic polynomial problem that is difficult to solve by other polynomial algorithms. GA is an effective tool in handling this kind of nondeterministic polynomial optimization.

Future work will be extended to study the site facility layout at the other stages of high-rise building construction, such as the site mobilization and internal finishing stages.

ACKNOWLEDGMENT

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NOTATION

The following symbols are used in this paper:

A_j = all supply locations available for element j;
C_{jk} = cost of material flow from S_i to D_j per unit quantity and unit time;
$Cr_i(XCr_i, YCr_i, ZCr_i)$ = coordinate of tower crane;
D_i = demand point for element j;
$D_i(XD_i, YD_i, ZD_i)$ = coordinate of demand point i;
i = location number of each position;
j = element number;
k = number of times that transportation of element j needs to repeat;
l_{cr} = jib length of tower crane;
Q_{jk} = quantity of material flow from S_i to D_j;
S_{Aj} = selected supply location number of element j;
S_i = supply point at location number i;
$S_i(XS_i, YS_i, ZS_i)$ = coordinate of supply point i;
T = hook travel time;
TC = total cost;
T_a = time for trolley radial movement;
T_h = hook horizontal travel time;
T_v = hook vertical travel time;
T_w = time for trolley tangent movement;
V_a = radial velocity;
V_h = hoisting velocity of hook (m/min);
V_w = slewing velocity of jib (r/min);
w = all possible and available supply locations;
α = degree of coordination of hook movement in radial and tangential directions in horizontal plane (0 for simultaneous movement and 1 for consecutive movement); and
β = degree of coordination of hook movement in vertical and horizontal planes (0 for simultaneous movement and 1 for consecutive moment).

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