Polynomial-Time Solution to the Hamiltonian Path Problem via Fermionic Adiabatic Quantum Computing

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Abstract

We propose a Fermionic Adiabatic Quantum Computing (FAQC) architecture that solves the k-simple path problem—and by extension, the NP-complete Hamiltonian Path problem—in polynomial time. Unlike standard qubit-based adiabatic approaches, which rely on penalty terms to enforce constraints (soft constraints) leading to frustrated glassy landscapes and exponentially closing spectral gaps, FAQC encodes the simple path constraint directly into the antisymmetry of the fermionic wavefunction (hard constraints). We demonstrate that by utilizing the Pauli exclusion principle, the search for a self-avoiding path is transformed from a probabilistic search into a physical filling process. We provide analytical arguments suggesting that the spectral gap of the system scales polynomially as $\mathcal{O}(k^{-1})$ rather than exponentially, implying a total runtime scaling of $\mathcal{O}(k^2)$.

Keywords: fermionic quantum computation, adiabatic quantum computation, Pauli exclusion principle, frustrated glass, exterior calculus

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I. INTRODUCTION

Adiabatic Quantum Computing (AQC) was originally proposed as a paradigm to solve NP-hard optimization problems by slowly evolving a system from the ground state of a trivial Hamiltonian H_i to a problem Hamiltonian H_f [1]. The adiabatic theorem guarantees that if the evolution time T is large enough—specifically $T \gg \Delta_{\min}^{-2}$, where $\Delta_{\min}$ is the minimum spectral gap—the system will remain in the ground state.

Initial hopes were that AQC could evade the quadratic speedup limitation of Grover's search algorithm [2] and potentially solve NP-complete problems in polynomial time. However, this hope has largely faded due to the behavior of the spectral gap Δ. For many combinatorial problems mapped to qubits (e.g., 3-SAT, Traveling Salesman), the problem Hamiltonian corresponds to a *frustrated spin glass* [3].

A frustrated glass refers to a system where competing interactions prevent the simultaneous satisfaction of all local constraints. In the context of qubit-based optimization, this manifests as an energy landscape rugged with numerous local minima. These minima are separated by high energy barriers, meaning the system, much like a disordered glass solidifying, gets stuck in a suboptimal configuration rather than reaching the true crystalline ground state. As the system size N increases, the minimum gap typically closes exponentially ($\Delta \sim e^{-\alpha N}$), necessitating an exponentially long runtime T.

One might naively argue that this vanishing gap could be overcome simply by rescaling the system's energy—multiplying the Hamiltonian by a large factor λ ($H \to \lambda H$). While this amplifies the gap ($\Delta \to \lambda \Delta$), it simultaneously increases the rate of change of the Hamiltonian ($\dot{H} \to \lambda \dot{H}$) if the total runtime T is kept constant. The adiabatic condition effectively scales as $1/\lambda$. Consequently, to counteract an exponentially closing gap ($\Delta \sim e^{-N}$) and maintain the adiabatic condition without exponentially increasing the runtime T, the energy scale λ would need to grow exponentially with problem size ($\lambda \sim e^N$). Such energy scales are physically unrealizable, as they would rapidly exceed the breakdown voltage of circuits, the available laser power in atomic systems, or lead to leakage out of the computational subspace. Thus, the solution must come from changing the scaling behavior of Δ itself, not merely its pre-factor.

In this paper, we propose Fermionic Adiabatic Quantum Computing (FAQC) as a resolution to the energy gap problem for path-finding algorithms. By using physical fermions

instead of qubits, we utilize the Pauli Exclusion Principle to enforce the no-loop constraint of the simple path problem fundamentally. As highlighted by Vourdas [4], the mathematical framework of fermions is naturally described by exterior calculus, where the wedge product $x \wedge x = 0$ provides a rigorous algebraic basis for the self-avoiding property required for simple paths. We show that by exploiting this structure, the system avoids the frustrated glass phase and instead behaves as a Fermi liquid flowing into a potential well.

II.   FORMALISM

A.   Problem Definition

We consider a graph $G = (V, E)$ with N vertices. The objective is to determine the existence of a simple path (a path with no repeated vertices) of length k starting from a source node $s \in V$.

B.   Fermionic Hamiltonian

We assume the implementation strategy for fermionic processing proposed by [5], using neutral atoms in optical tweezers, though alternative implementations are certainly possible and the contents of this paper do not depend on [5].

Let $c_i^\dagger$ and c_i be the creation and annihilation operators for a fermion at site i, satisfying $\{c_i, c_j^\dagger\} = \delta_{ij}$.

The adiabatic evolution is governed by $H(t) = (1 - s(t))H_{\text{source}} + s(t)H_{\text{graph}}$. We imagine fermions initially in a reservoir ('reservoir sites') that flow into a non-reservoir ('graph') region ('graph sites').

The **Initial Hamiltonian** (H_{source}) prepares a reservoir of k fermions localized in a source (non-graph) region:

$$H_{\text{source}} = -\mu_{\text{res}} \sum_{j \in \text{Res}} n_j \tag{1}$$

where $n_j = c_j^\dagger c_j$.

The **Problem Hamiltonian** (H_{graph}) describes the physics of the fermions on the graph G:

$$H_{\text{graph}} = H_{\text{hop}} + H_{\text{int}} + H_{\text{pot}} \tag{2}$$

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1. **Hopping (Kinetic Energy):** Allows exploration of the graph topology.

$$H_{\text{hop}} = -J \sum_{(u,v) \in E} (c_u^\dagger c_v + c_v^\dagger c_u) \tag{3}$$

2. **Nearest-Neighbor Attraction:** Encourages the k fermions to form a connected chain.

$$H_{\text{int}} = -V \sum_{(u,v) \in E} n_u n_v \tag{4}$$

3. **Chemical Potential:** A global potential that makes occupying the graph energetically favorable.

$$H_{\text{pot}} = -\mu_G \sum_{v \in V} n_v \tag{5}$$

C. The Pauli Constraint

The crux of the method is the operator identity $(c_i^\dagger)^2 = 0$. In the language of exterior calculus [4], if we associate a 1-form with each site, a path is a wedge product of these forms. If a vertex repeats, the term vanishes. Thus, a non-simple path is not a high-energy state; it is a null state.

III. DETAILED ANALYSIS

A. Ground State Derivation

To verify that the ground state of H_{graph} corresponds to a simple path, we analyze the energy in the strong coupling limit $V \gg J$. In this limit, the kinetic term H_{hop} acts as a perturbation, and the eigenstates are Fock states (definite occupancy).

Case 1: A Simple Path Exists. Let $\mathcal{P} = (v_1, v_2, \ldots, v_k)$ be a simple path of length k. The state $|\psi_{\mathcal{P}}\rangle = c_{v_1}^\dagger \ldots c_{v_k}^\dagger |0\rangle$ has the following energy contributions:

- **Potential:** All k sites are in the graph, so $E_{\text{pot}} = -k\mu_G$.

- **Interaction:** There are $k-1$ nearest-neighbor bonds along the path. $E_{\text{int}} = -(k-1)V$.

The total energy is:

$$E_{\text{path}} \approx -k\mu_G - (k-1)V \tag{6}$$

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Case 2: No Simple Path Exists. If the fermions cannot form a single chain of length k, they must fragment into at least two disjoint clusters (e.g., of size k_a and k_b where $k_a + k_b = k$). The number of internal bonds is at most $(k_a - 1) + (k_b - 1) = k - 2$.

$$E_{\text{frag}} \approx -k\mu_G - (k - 2)V \tag{7}$$

The energy penalty for failing to find a path is:

$$\Delta_{\text{classical}} = E_{\text{frag}} - E_{\text{path}} \approx V \tag{8}$$

Since we set $V > 0$, the simple path state is energetically separated from the no-path states by a macroscopic gap V, ensuring the ground state encodes the correct solution.

B. The Adiabatic Theorem and Error Rates

The rigorous condition for the Adiabatic Theorem states that the probability of transitioning out of the ground state $|0(t)\rangle$ to the first excited state $|1(t)\rangle$ is bounded. Near an avoided crossing, the transition probability is described by the Landau-Zener formula [6]:

$$P_{\text{LZ}} \sim \exp\left(-\frac{\pi\Delta_{\text{min}}^2}{4v}\right) \tag{9}$$

where $v = \left|\left\langle 1|\dot{H}|0\big|1|\dot{H}|0\right\rangle\right|$ is the velocity of the Hamiltonian change at the crossing.

For the full evolution, rigorous bounds on the adiabatic approximation [7, 8] establish that to suppress the diabatic error probability to $P_{\text{err}} < \epsilon$, the runtime T must scale according to the minimum spectral gap Δ_{min}:

$$T \geq \frac{\mathcal{C}}{\epsilon\Delta_{\text{min}}^2} \max_{s\in[0,1]} \left|\left\langle 1(s)|\frac{dH}{ds}|0(s)\big|1(s)|\frac{dH}{ds}|0(s)\right\rangle\right| \tag{10}$$

Thus, the efficiency of the algorithm is entirely determined by the scaling of Δ_{min}. If the gap closes polynomially (as in our fermionic case) rather than exponentially (as in frustrated spin glasses), the required runtime remains polynomial.

C. Exact Analysis of Energy Gap Scaling

In the frustrated glass models typical of qubit encodings, Δ_{min} closes exponentially because the transition requires tunneling through a macroscopic energy barrier (First Order Phase Transition).

In FAQC, the transition is a transport process. As the potential evolves, the fermions tunnel from the source to the graph. The critical moment occurs when the chemical potentials balance ($\mu_{\text{res}} \approx \mu_G$). At this point, the system resembles a 1D Fermi gas in a box of effective length $L \sim k$.

The excitation spectrum is determined by the kinetic energy operator H_{hop}. For a 1D tight-binding chain of length L, the eigenvalues are:

$$\epsilon_n = -2J \cos\left(\frac{n\pi}{L+1}\right) \tag{11}$$

The relevant gap is the energy cost to create the lowest-energy excitation (e.g., a density fluctuation or particle-hole excitation) near the Fermi level. For finite $L \approx k$:

$$\Delta_{\text{kinetic}} = \epsilon_{L+1} - \epsilon_L \approx \frac{2J\pi}{L} \sin\left(\frac{L\pi}{L+1}\right) \approx \mathcal{O}\left(\frac{J}{k}\right) \tag{12}$$

Even in the presence of interactions V, provided the system remains in the Luttinger liquid phase (gapless in the bulk limit), the finite-size gap scales polynomially as k^{-1}.

D. Phase Transition Analysis: First vs. Second Order

The critical distinction between the failure of qubit-based AQC on NP-complete problems and the success of the proposed FAQC lies in the order of the quantum phase transition encountered during the adiabatic path.

1. First-Order Transitions in Frustrated Systems

In standard qubit annealers, the problem Hamiltonian often encodes a frustrated spin glass. The adiabatic evolution typically encounters a first-order phase transition where the ground state character changes abruptly from a delocalized paramagnetic state to a localized ferromagnetic (or glassy) state.

Consider the transition between two local minima $|\psi_A\rangle$ and $|\psi_B\rangle$ separated by a Hamming distance $d \sim N$. If the driver Hamiltonian is local (e.g., single-spin flips σ_i^x), the matrix element coupling these states vanishes to first order. The states are only connected via an N-th order perturbation process. The gap Δ scales as the effective tunneling amplitude:

$$\Delta_{1st} \approx \frac{\langle\psi_B|(H_{\text{driver}})^N|\psi_A\rangle\langle\psi_B|(H_{\text{driver}})^N|\psi_A\rangle}{(\delta E)^{N-1}} \sim \left(\frac{\Gamma}{\mathcal{E}}\right)^N \sim e^{-\alpha N} \tag{13}$$

where Γ is the transverse field strength and $\mathcal{E}$ is the energy barrier height. This exponential closure implies an exponential runtime $T \sim e^{2\alpha N}$.

2. Second-Order Transition in Fermionic Transport

In the FAQC model, the transition corresponds to the transport of fermions from the Source to the Graph. The order parameter is the particle number density in the graph, $\rho = \langle N_G \rangle / k$, which changes continuously from 0 to 1.

Let us analyze the gap at a critical crossing where it becomes energetically favorable for the m-th fermion to enter the graph. The relevant basis states are $|n = m - 1\rangle$ (Source has $k - m + 1$ particles) and $|n = m\rangle$ (Source has $k - m$). The Hamiltonian term driving this transition is the single-particle hopping operator $H_{\text{hop}} \propto c_v^\dagger c_s$. The matrix element is:

$$\mathcal{M} = \langle n = m | H_{\text{hop}} | n = m - 1 | n = m | H_{\text{hop}} | n = m - 1 \rangle = -J \tag{14}$$

Crucially, this matrix element is **non-zero at first order**. It does not require a simultaneous reconfiguration of N particles, but rather the motion of a single particle at the boundary.

Consequently, the spectral gap at the crossing is determined by the kinetic energy scale J, modified essentially by the finite size level spacing. For a system of size $L \sim k$, the gap behaves as a second-order (continuous) transition:

$$\Delta_{2nd} \sim \frac{J}{k^\beta} \quad \text{(typically } \beta = 1\text{)} \tag{15}$$

Since Δ scales polynomially with k, the adiabatic runtime $T \sim \Delta^{-2}$ remains polynomial. The system avoids the orthogonality catastrophe associated with first-order transitions because the reaction coordinate (particle filling) is coupled linearly to the Hamiltonian parameter.

IV. CONCLUSION

We have outlined a Fermionic Adiabatic Quantum Computing framework. By mapping the hard constraints of the Hamiltonian Path problem to the native antisymmetry of fermions, we bypass the frustrated glass landscape typical of qubit annealers. Our derivation of the ground state confirms that the solution is energetically favored by a gap V, while the

gap analysis shows that the adiabatic speed limit is set by the kinetic gap $\sim k^{-1}$, resulting in polynomial time complexity..

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