A Jaynes-Cummings (JCM) model of a black hole: upholding both semiclassical spacetime and unitarity

Minseong Kim*

(Dated: May 27, 2025)

Abstract

A Jaynes-Cummings (JCM) model of a Schwarzschild black hole is constructed. The model features two-energy atoms in JCM as Hawking radiation qubits sequentially interacting with an N-energy optical cavity as a black hole, one by one. A qubit that completed interaction flies away as an outgoing radiation, never interacting with the black hole. This setup is essentially the common qubit model deployed in black hole physics, such as in the small corrections theorem. The resulting simulations suggest unitarity after complete black hole evaporation without violating semiclassical spacetime, given by the total energy emitted. The empty black hole quantum branch of superpositions cannot generate lasting entanglement even semiclassically and effective field theory (EFT)-wise, with the effects of the empty black hole branch building up to force the black hole entanglement entropy down. Meanwhile, the rest of the branches feature the familiar EFT black hole horizon entanglement structure. EFT, semiclassical spacetime and unitarity are all safe.

Keywords: Jaynes-Cummings model, black hole information problem, semiclassical physics, qubit model

* mkimacad@gmail.com; ORCiD:0000-0003-2115-081X

CONTENTS

I. INTRODUCTION

The main issue with the semiclassical analysis of black hole physics that combines quantum field theory machinery with largely classical spacetime is the difficulty in reconciling semiclassical spacetime with unitarity. One is often obtained at the expense of the other, and upholding semiclassical spacetime results in the black hole information problem [1, 2].

This paper demonstrates that both semiclassical spacetime and unitarity can be upheld in black hole physics using the Jaynes-Cummings (JCM) model of a black hole without invoking soft hairs [3] - the no-hair semiclassical picture is maintained. Conventionally, semiclassical spacetime is understood to imply that the 'empty black hole' branch (outcome) of superpositions can be ignored. This is not the case, and there is no issue with effective field theory (EFT) itself.

EFT-wise, the empty black hole branch and other non-empty black hole branches imply different horizon vacua - the former does not form a lasting qubit pair entanglement at radial infinity [4], while the latter does (Hawking radiations) [1]. If the former can be ignored, this is not a problem, but we show that this is not the case. This implies that late-time infalling observers do not see vacuum on the black hole interior horizon, but they do for each classical black hole outcome. This resolves the AMPS firewall paradox [5], since it relies heavily on the assumption that the interior horizon is a vacuum state. Furthermore, the small corrections theorem [6] is overturned, since the late-time qubit pair entanglement structure must change even within EFT.

A. Historical background: Wen's JCM model of a black hole

A use of JCM for modeling a black hole is not completely new. [7] uses the JCM model with a reflective cavity such that reflected photons can re-interact with a black hole atom. There are some differences with respect to this paper: [7] uses atoms as black holes instead of a cavity as a black hole. A black hole atom is then embedded in a reflective cavity. This is in contrast to a typical black hole setup where Hawking radiations are outgoing radiations (non-zero transmission ratio) that can be seen at future radial infinity. [7] replicates the Parikh-Wilczek (PW) non-thermal spectrum [8] and provides its quantum toy model description. As a result of reflections, one obtains periodic black hole entanglement entropy curves, understood to provide a potential mechanism of information transfer. The periodic curves of incomplete entropy decreases imply that they are not models of black hole evaporation.

B. Historical background: Small corrections theorem

We now briefly describe the toy qubit model logic of the small corrections theorem [6]. Consider effective bulk field theory. At each step with interval Δt, we consider a single qubit pair to be produced around the black hole horizon, with one qubit falling into the black hole and the other radiated away to the exterior. At step n, counted from step 1, this can be described as:

$$|\psi^{eff}_{pair,n}\rangle = \cos(\phi)|0^{eff}_{R,n}\rangle|0^{eff}_{BH,b}\rangle + \sin(\phi)|1^{eff}_{R,n}\rangle|1^{eff}_{BH,n}\rangle + O(\epsilon), \quad \epsilon \ll 1 \tag{1}$$

where R represents the black hole exterior radiation(s) and BH represents the black hole interior, with eff representing effective degrees of freedom. von Neumann entropy-wise, we have:

$$\begin{gathered} S(R^{eff}_{n+1} + BH^{eff}_{n+1}) = 0 \\ S(BH^{eff}_{n+1}) = -[\cos^2(\phi)\ln(\cos^2(\phi)) + \sin^2(\phi)\ln(\sin^2(\phi))] \\ S(R_{n+1} + BH_{n+1}) = \epsilon_1 \\ S(BH_{n+1}) > -[\cos^2(\phi)\ln(\cos^2(\phi)) + \sin^2(\phi)\ln(\sin^2(\phi))] - \epsilon_2, \quad \epsilon_1, \epsilon_2 \ll 1 \end{gathered} \tag{2}$$

where R_{n+1} represents the radiation qubit in the exact theory at step $n+1$ and so forth. Now we invoke the strong subadditivity relation (for subspaces with independent degrees of freedom) to prove the small corrections theorem:

$$S(A+B) + S(B+C) \geq S(A) + S(C) \tag{3}$$

Let $S(R_{\leq n})$ refer to the entropy of all radiations to step n in the exact theory. Then:

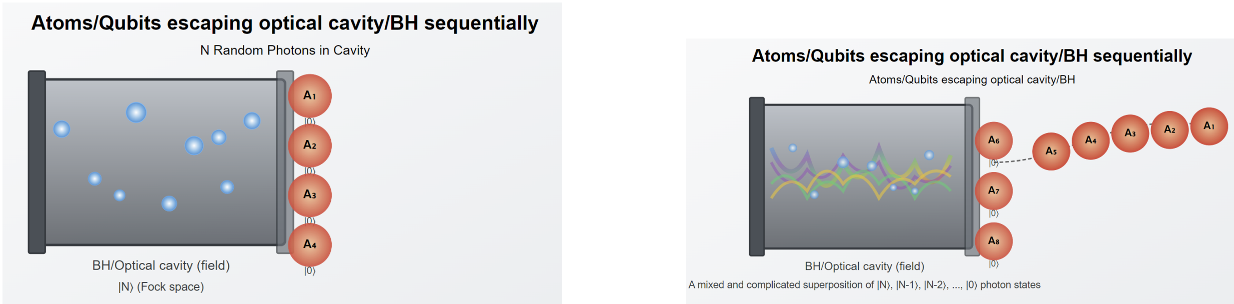
$$\begin{gathered} S(R_{\leq n} + R_{n+1}) + S(R_{n+1} + BH_{n+1}) \geq S(R_{\leq n}) + S(BH_{n+1}) \\ \Rightarrow S(R_{\leq n} + R_{n+1}) \geq S(R_{\leq n}) + S(BH_{n+1}) - S(R_{n+1} + BH_{n+1}) \\ \Rightarrow S(R_{\leq n} + R_{n+1}) \geq S(R_{\leq n}) - [\cos^2(\phi)\ln(\cos^2(\phi)) + \sin^2(\phi)\ln(\sin^2(\phi))] - (\epsilon_1 + \epsilon_2) \end{gathered} \tag{4}$$

Therefore, we obtain the small corrections theorem: the entropy of all radiations must keep increasing monotonically, as long as $-[\cos^2(\phi)\ln(\cos^2(\phi)) + \sin^2(\phi)\ln(\sin^2(\phi))] > \epsilon_1 + \epsilon_2$. The original small corrections theorem takes $\phi = \pi/4$, but this is made just for analytic simplicity [6].

With the JCM model, we show that without violating EFT, the below can be violated at late time evaporation, expressed as an inequality:

$$S(BH^{eff}_{n+1}) \neq -[\cos^2(\phi)\ln(\cos^2(\phi)) + \sin^2(\phi)\ln(\sin^2(\phi))] \tag{5}$$

ϵ_1, ϵ_2 are irrelevant for rest of the paper - we set $\epsilon_1, \epsilon_2 = 0$.



(a) Initial evaporation configuration

(b) Late-time evaporation configuration

FIG. 1: JCM black hole model configurations at initial and late times

C. Is the Page curve necessary?

In the recent years, the idea that quantum gravity observables are inherently non-local [9] and are different from non-gravitational observables such that information on the black hole interior can be retrieved on the exterior has become somewhat popular - holography of information [10–14]. This is sometimes elevated to the view that the Page curve [15] is not needed in quantum gravity, resolving the information problem in a straightforward way.

Nevertheless, there are merits to an analysis by local non-gravitational observables around the black hole horizon. It is feasible that a quantum gravity theory satisfying holography of information nevertheless has problems with local descriptions around the black hole horizon. It is in this sense that we analyze the Page curve.

Throughout the paper, we assume $\hbar = 1$ for convenience.

II. A JCM MODEL OF A BLACK HOLE

A. Setup

Each kth qubit (atom in original JCM), initially in $|0\rangle$ interacts sequentially with the black hole (optical cavity in original JCM) for duration of Δt_k ("interaction time") from $t = \sum_{j=1}^{k-1} \Delta t_j$ to $t = \sum_{j=1}^{k} \Delta t_j$. Then the qubit flies away, never interacting with the black hole again. The initial state of the black hole is assumed to be $|N\rangle$ at $t = 0$, with N representing the number of particles in the black hole. There are M qubits.

The free Hamiltonian H_{rad} of each qubit is given as, with qubit states satisfying $\langle 0|1\rangle = 0$:

$$H_{rad} = \omega_a \sigma_+ \sigma_- = \omega_a |1\rangle\langle 1| \tag{6}$$

This is simply about only the excited state of the qubit having non-zero energy, with $\sigma_+ = |1\rangle\langle 0|$ and $\sigma_- = |0\rangle\langle 1|$.

The free Hamiltonian H_{BH} of the black hole is given as:

$$H_{BH} = \omega_a a^\dagger a \tag{7}$$

with commutation relations $[a, a^\dagger] = 1$ and $[a, a] = [a^\dagger, a^\dagger] = 0$. This gives us $|n\rangle$ with particle number operator $\hat{n} = a^\dagger a$ such that $\hat{n}|n\rangle = n|n\rangle$ for $n \in \mathbb{N}$. The initial state then satisfies $\hat{n}|N\rangle = N|N\rangle$.

Now most importantly, JCM interaction Hamiltonian H_I that couples a single qubit and the black hole (when under interaction) goes as:

$$H_I = g(\sigma_+ a + \sigma_- a^\dagger) \tag{8}$$

Since we have same ω_a in the free Hamiltonians, this JCM interaction is all about trading non-interaction energy: if the black hole loses energy ω_a, then the qubit obtains ω_a and vice versa. The total number of 'particles' and non-interaction energy are conserved in the model. The value of ω_a is irrelevant for rest of the discussions.

B. Variable interaction time (Δt_k) case

The final outgoing state of the kth qubit is determined with unitary U_k

$$U_k|0, n\rangle = \cos(g\sqrt{n}\Delta t_k)|0, n\rangle - i\sin(g\sqrt{n}\Delta t_k)|1, n-1\rangle \tag{9}$$

where $|q_1, q_2\rangle$ refers to $q_1 \in \{0, 1\}$ (kth qubit), $q_2 \in \{0, 1, .., N\}$ (black hole). For variable interaction time Δt_k, we choose

$$\Delta t_k = \frac{\pi}{4g\sqrt{\langle n_k\rangle}} \tag{10}$$

where $\langle n_k\rangle$ is average excitation numbers in the black hole at the start of the kth interaction. This matches the semiclassical requirement of the small corrections theorem that is reproduced up to phase for $\langle n_k\rangle \in \mathbb{N}$ (with $\phi = \pi/4$):

$$U_k|0, \langle n_k\rangle\rangle \approx \frac{1}{\sqrt{2}}\left(|0, \langle n_k\rangle\rangle + |1, \langle n_k\rangle - 1\rangle\right) \tag{11}$$

C. Fixed interaction time $\Delta t_k = \Delta t$ case

We can instead fix interaction time as constant $\Delta t_k = \Delta t$ ($U_k = U$) such that:

$$\Delta t = \frac{\pi}{4g\sqrt{N}} \tag{12}$$

reproducing only for $n = N$ up to phase,

$$U|0, N\rangle \approx \frac{1}{\sqrt{2}} \left(|0, N\rangle + |1, N-1\rangle\right) \tag{13}$$

D. Simplified model

We can modify interaction unitary $U_k = U$ (fixed interaction time case) to be slightly different from the JCM case:

$$\begin{aligned} U|0, n\rangle &= \frac{1}{\sqrt{2}} \left(|0, n\rangle + |1, n-1\rangle\right) \quad (0 < n) \\ U|0, 0\rangle &= |0, 0\rangle \end{aligned} \tag{14}$$

This simplified model continues to respect conservation of non-interaction energy. The model is analytically simpler and can be beneficial for those insisting on the same interaction behavior regardless of n. We arrive at the same conclusions qualitatively regardless of the models used.

E. Reduced interaction time case: semiclassical spacetime upheld

Interaction time Δt_k or coupling constant g can be reduced by factor k_r such that more M is allowed for the given time. As M becomes larger, the total energy emitted becomes more semiclassical due to the central limit theorem (CLT). This is simplest to explore in the simplified model, though qualitatively conclusions remain the same. U now instead generates in the simplified model of Section II D:

$$\begin{aligned} U|0, n\rangle &= \cos(\pi/(4k_r))|0, n\rangle + \sin(\pi/(4k_r))|1, n-1\rangle \\ U|0, 0\rangle &= |0, 0\rangle \end{aligned} \tag{15}$$

If we can ignore the empty black hole case, variance in energy emitted per each qubit $Var[E_{k,rad}]$ goes as:

$$Var[E_{k,rad}] = \langle (E_{k,rad})^2 \rangle - (\langle E_{k,rad} \rangle)^2 = \sin^2\left(\frac{\pi}{4k_r}\right) - \sin^4\left(\frac{\pi}{4k_r}\right) \tag{16}$$

CLT says that for K such identical qubits, with $\sigma^2 = Var[E_{k,rad}]$:

$$E_{rad,total,k} - K \langle E_{k,rad} \rangle \xrightarrow{d} \mathcal{N}\left(0, K\sigma^2\right) \tag{17}$$

It can be noted that variance is lowered approximately by factor of $(1/k_r)^2$ as k_r is varied, while interaction time is reduced by $1/k_r$, achieving the goal of enlarging the number of interacting qubits within the same time duration and lowering total variance. The point is that with large k_r, we see practically zero variance in total energy emitted from the semiclassical prediction.

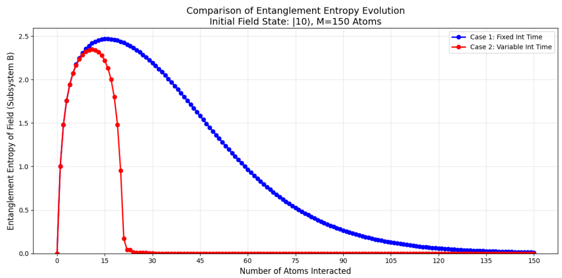
In the models, σ eventually changes over time. However, we can slice time into different small sub-intervals of sufficiently large qubits being emitted and apply CLT to each sub-interval. The CLT conclusion therefore remains.

F. Simulation results and analysis

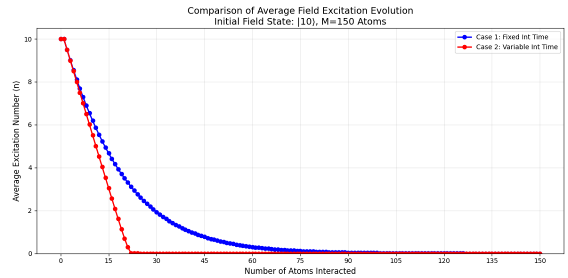
Model simulation results are provided in Figure 2, confirming unitarity even after complete black hole evaporation, regardless of the full JCM or the simplified model, reduction in interaction time or not. Since the interaction time reduction does not change unitarity, semiclassical spacetime is maintained by the discussion in Section II E.

1. On the eventual decrease of entanglement entropy

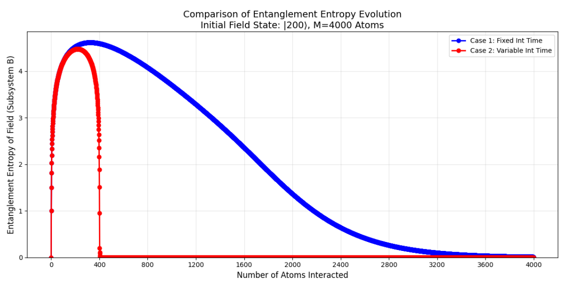
Black hole entanglement entropy has to decrease due to the zero energy lower bound of the black hole - JCM interaction forces energy transfer whenever an interacting qubit is in its ground state, but this is not possible when the black hole is also in its ground state. If not for the lower bound, entropy would have continuously increased. Furthermore, the result has nothing to do with not distinguishing different quantum states of each black hole $|n\rangle$ (those states considered the same $|n\rangle$ within JCM). Even when these states are distinguished, entropy must eventually decrease due to the black hole hitting its ground state thereby completely evaporating. So down to complete evaporation, there is nothing wrong with effective field theory (EFT) in black hole physics. The semiclassical black hole horizon entanglement structure is well-justified, but it should take into account that it does not apply to the complete evaporation (empty black hole) quantum branch/world of superpositions - this branch should have a different vacuum according to EFT.



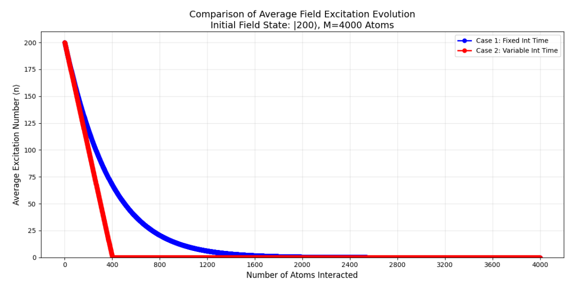
(a) Black hole entanglement entropy evolution. ($N = 10$, $M = 150$, $g = 1$, $k_r = 1$, full JCM)



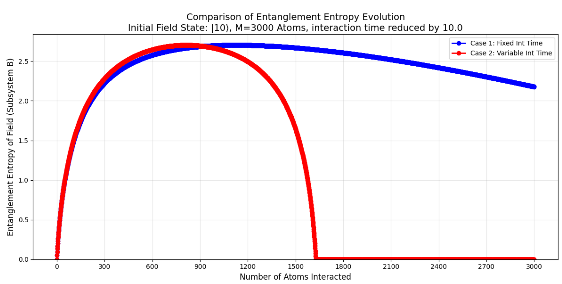
(b) Average black hole excitation number $\langle n \rangle$ plot. ($N = 10$, $M = 150$, $g = 1$, $k_r = 1$, full JCM)



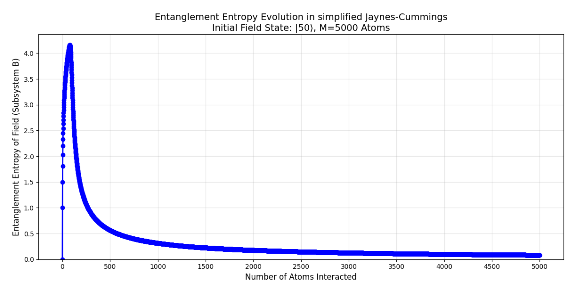
(c) Black hole entanglement entropy evolution. ($N = 200$, $M = 4000$, $g = 1$, $k_r = 1$, full JCM)



(d) Average black hole excitation number $\langle n \rangle$ plot. ($N = 200$, $M = 4000$, $g = 1$, $k_r = 1$, full JCM)



(e) Black hole entanglement entropy evolution. ($N = 10$, $M = 3000$, $g = 1$, $k_r = 10$, full JCM)



(f) Black hole entanglement entropy evolution. ($N = 50$, $M = 5000$, $g = 1$, $k_r = 1$, simplified model)

FIG. 2: Model simulations. For Figure 2a, 2b, 2c, 2d and 2e, the red and symmetric curves represent the variable interaction time case, while the blue curves represent the fixed interaction time case.

2. On "time" in the model

Since this is a toy qudit model, time is modeled as a function of the number of qubits that interacted with the black hole. This can be re-scaled to fit the Page curve and so the slope of the entropy curve should not be of concern here. Qudits are there as entropy and entanglement generators, not as exact descriptions.

3. On stretched horizon

As seen with the simplified model of Section II D, the idea that the stretched horizon [16] captures all information about the black hole interior is not necessary for the model to make sense. Every $|n\rangle$ of the black hole produces the same entanglement behavior except for $n = 0$, where the black hole ceases to exist.

Nevertheless, it may provide a good interpretation for the no-hair feature of the JCM model (which is actually irrelevant for unitarity) - $|n\rangle$ is $|n\rangle$ regardless of how it is reached, and it does not care about infalling qubits, effectively encoded on the stretched horizon, producing local horizon interactions. It also provides a decent connection to quantum gravity setups like Karch-Randall braneworld, where we have a non-gravitational holographic 'boundary of a boundary' setup and the Page curve may be replicated [17].

III. CONCLUSION

We conclude this paper by looking at the black hole exterior and the black hole interior pictures. In the JCM model, the exterior horizon remains as a horizon vacuum - all qubits are initially in $|0\rangle$. Semiclassical spacetime geometry is also safe for small interaction time. Radiations are the only things that tell exterior observers the non-tranquility of the black hole interior.

For the interior horizon, it is safe to assume any quasi-vacuum (particle) state (each $|n\rangle$, $n \in \mathbb{N}$) for early-time evaporation - this is especially for small interaction time and the simplified model. This is no longer the case for late-time evaporation. We could say this as a partial breakdown of semiclassicality and even possibly 'no drama'. However, for each $|n\rangle$ branch that can be classically measured, nothing different from EFT should be noticeable, so semiclassicality is somewhat preserved. Semiclassical spacetime around the horizon is maintained as well.

According to the JCM model, black hole paradoxes arise because the black hole does not exist for one quantum branch of superpositions, which implies a different entanglement structure. A key contribution of this paper was to show that semiclassical spacetime is not enough to ignore eventual contributions of this empty branch. EFT is safe, semiclassical spacetime is fine, and unitarity is upheld.

DATA AVAILABILITY AND DECLARATION OF INTERESTS

The full JCM model code is available at:

`https://mkimacad.github.io/bh_jaynes_cummings/codes/jaynes_improved.py` and the simplified model code is available at:

`https://mkimacad.github.io/bh_jaynes_cummings/codes/jaynes_simplified.py`.

The author(s) have no funding source to declare. Furthermore, there is no conflict of interests.

[1] Stephen W. Hawking, "Particle creation by black holes," Communications in Mathematical Physics **43**, 199–220 (1975).

[2] Suvrat Raju, "Lessons from the information paradox," Physics Reports **943**, 1–80 (2022).

[3] Stephen W. Hawking, Malcolm J. Perry, and Andrew Strominger, "Soft hair on black holes," Phys. Rev. Lett. **116** (2016), 10.1103/PhysRevLett.116.231301.

[4] W. G. Unruh, "Notes on black-hole evaporation," Phys. Rev. D **14**, 870–892 (1976).

[5] Ahmed Almheiri, Donald Marolf, Joseph Polchinski, and James Sully, "Black holes: complementarity or firewalls?" Journal of High Energy Physics **2013**, 62 (2013).

[6] Samir D. Mathur, "The information paradox: a pedagogical introduction," Classical and Quantum Gravity **26**, 224001 (2009).

[7] Wen-Yu Wen, "Hawking radiation as stimulated emission," Physics Letters B **803** (2020), 10.1016/j.physletb.2020.135348.

[8] Maulik K. Parikh and Frank Wilczek, "Hawking radiation as tunneling," Physical Review Letters **85**, 5042–5045 (2000).

[9] Steven B. Giddings, Donald Marolf, and James B. Hartle, "Observables in effective gravity," Physical Review D **74** (2006), 10.1103/physrevd.74.064018.

[10] Tomás Andrade, Donald Marolf, and Cédric Deffayet, "Can hamiltonians be boundary observables in parametrized field theories?" Classical and Quantum Gravity **28** (2011), 10.1088/0264-9381/28/10/105002.

[11] Donald Marolf, "Holography without strings?" Classical and Quantum Gravity **31**, 015008 (2013).

[12] Alok Laddha, Siddharth Prabhu, Suvrat Raju, and Pushkal Shrivastava, "The holographic nature of null infinity," SciPost Physics **10** (2021), 10.21468/scipostphys.10.2.041.

[13] Chandramouli Chowdhury, Olga Papadoulaki, and Suvrat Raju, "A physical protocol for observers near the boundary to obtain bulk information in quantum gravity," SciPost Phys. **10**, 106 (2021).

[14] Suvrat Raju, "Failure of the split property in gravity and the information paradox," Classical and Quantum Gravity **39** (2022), 10.1088/1361-6382/ac482b.

[15] Don N. Page, "Information in black hole radiation," Physical Review Letters **71** (1993), 10.1103/physrevlett.71.3743.

[16] Leonard Susskind, Lárus Thorlacius, and John Uglum, "The stretched horizon and black hole complementarity," Physical Review D **48**, 3743–3761 (1993).

[17] Gopal Yadav, "Multiverse in Karch-Randall Braneworld," Journal of High Energy Physics **2023** (2023), 10.1007/jhep03(2023)103.