The Sequential Defense: Resolving the Heterodox Problem of Heterogeneous Own-Rates in Neoclassical Economics

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Abstract

The heterodox critique of neoclassical capital theory, highlighted during the Cambridge Capital Controversy, challenged how heterogeneous physical goods with shifting relative prices could be aggregated into a single capital magnitude. This paper traces the evolution of the neoclassical defense from aggregate production functions to sequential market theory. We demonstrate how Radner's formalization of incomplete markets utilizes financial arbitrage to anchor divergent physical own-rates to a unified pricing kernel. Furthermore, we evaluate recent controversial attempts to resolve the reswitching paradox using real factor price space, emphasizing a more universally accepted neoclassical fallback: marginal productivity theory remains mathematically valid under ceteris paribus conditions. Ultimately, while sequential models and partial equilibrium defenses mathematically rescue neoclassical optimization, they require abandoning the classical long-period center of gravitation in favor of fleeting temporary equilibria - a methodological sacrifice that continues to define modern economic theory.

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1 Introduction

For decades, a foundational and recurring heterodox critique of mainstream macroeconomics has targeted its reliance on a uniform rate of interest and a homogeneous conception of capital. Rooted in the early theoretical formulations of Piero Sraffa and later expanded by John Maynard Keynes, the challenge asserts that in a dynamic, multi-commodity economy, shifting relative prices dictate that every physical good yields a unique "own-rate" of return. If an economy is characterized by a multiplicity of divergent own-rates, aggregating them into a singular "rate of return on capital" - and consequently utilizing an aggregate production function - becomes logically and mathematically incoherent.

During the mid-twentieth century, this inherent heterogeneity became the central theoretical weapon in the Cambridge Capital Controversy (CCC), a fierce and protracted debate between the Neo-Ricardian and Post-Keynesian scholars of Cambridge, United Kingdom, and the neoclassical synthesis defenders of Cambridge, Massachusetts. The critics successfully demonstrated that the neoclassical marginal theory of capital was fundamentally flawed when forced to deal with heterogeneous capital goods. They argued that aggregating divergent own-rates into a single rate of profit to measure the "quantity" of capital is logically circular, as the value of capital cannot be known independently of the rate of profit itself.

At first glance, the pinnacle of neoclassical formalization - the Arrow-Debreu general equilibrium model - appeared to validate this heterodox critique. In a pure, time-0 complete futures market, own-rates of return are mathematically heterogeneous, reflecting the physical reality of shifting intertemporal prices. However, the neoclassical paradigm mounted a highly sophisticated defense by structurally evolving the general equilibrium framework from a static, time-0 construct into a sequence of interconnected markets.

By formalizing sequential economies with incomplete markets, pioneered by Roy Radner and others, mainstream economics demonstrated that a uniform rate of interest is not an arbitrary or circular assumption. The structural capacity for sequential re-optimization via financial arbitrage organically suppresses the divergence of heterogeneous own-rates. Furthermore, by abandoning the aggregate "value" of capital in favor of a Walrasian vector of physical endowments, the neoclassical defense side-stepped the aggregation problem, albeit at the cost of abandoning the classical "center of gravitation" concept of equilibrium. Finally, this paper addresses the most devastating anomaly of the capital controversy: the reswitching of techniques. While recent theoretical attempts to rationalize reswitching through coordinate transformations into real factor price space remain highly contested, we emphasize a more universally recognized neoclassical defense. Specifically, we demonstrate that the marginal theory of capital and the logic of factor substitution remain mathematically robust in a partial equilibrium framework, provided strict *ceteris paribus* conditions are maintained regarding relative prices.

This paper provides an exhaustive analysis of the theoretical evolution of capital theory. It traces the origins of the own-rates problem, the mathematical critiques of the Cambridge Capital Controversy, and the sophisticated resolutions achieved through sequential market clearing, intertemporal arbitrage, and the critical distinction between general and partial equilibrium defenses of marginal productivity.

2 The Genesis of the Challenge: Sraffa, Hayek, and Commodity Own-Rates

The origins of the heterogeneous own-rates problem can be traced to a 1932 debate between Piero Sraffa and Friedrich Hayek regarding the nature of money, prices, and the "natural" rate of interest (Sraffa, 1932; Dehez, 1985). The theoretical conflict began with Sraffa's devastating review of Hayek's *Prices and Production*, which subsequently laid the groundwork for Keynesian monetary theory and the Neo-Ricardian revival.

2.1 The Multiplicity of Natural Rates in Barter

Hayek had proposed that economic crises and business cycles arise when the monetary rate of interest diverges from the underlying, real "natural" rate of interest, a concept he adapted from Knut Wicksell. Hayek argued that monetary policy should aim to keep money "neutral" by aligning the money rate with this unique natural rate, which he believed reflected pure barter relationships.

Sraffa countered that the very concept of a unique natural rate is incoherent in a dynamic, multi-commodity economy. Sraffa argued that if loans were made in physical commodities rather than money, there would not be a single equilibrium rate of interest, but rather as many "natural" rates of interest as there are physical commodities.

To formalize this, Sraffa provided the famous example of a cotton spinner. If a cotton spinner borrows money to buy spot cotton and simultaneously sells forward cotton, the implicit rate of interest paid in terms of cotton - the commodity own-rate - is determined by the relationship between the spot price, the forward price, and the monetary interest rate. Mathematically, if p_s is the spot price, p_f is the forward price, and r is the money rate of interest, the own-rate of interest for a commodity is given by:

$$r_{\text{cotton}} = \frac{p_s(1+r)}{p_f} - 1 \tag{1}$$

Alternatively, the cotton rate can be expressed as the quantity of cotton that can be purchased with the proceeds:

$$\text{Cotton rate} = 1 - \frac{p_f}{p_s} + r \tag{2}$$

If relative prices between commodities are changing due to shifts in technology, demand, or intertemporal preferences, their spot and forward price ratios will diverge. Therefore, in any non-stationary economy where production and consumption patterns are shifting, the own-rates of interest for different physical capital goods will inherently contradict one another. As Sraffa demonstrated, if the production of some commodities is increasing while others are diminishing, their respective own-rates must diverge, making it theoretically impossible to identify a unique, non-monetary macroeconomic rate of interest.

2.2 Keynes's Adoption and the Liquidity Premium

John Maynard Keynes recognized the profound implications of Sraffa's critique and adopted the concept of heterogeneous own-rates in Chapter 17 of *The General Theory of Employment, Interest and Money* (1936) (Keynes, 1936). Keynes utilized the multiplicity of own-rates to explain why monetary economies suffer from persistent involuntary unemployment, distinguishing his theory from classical formulations that assumed automatic full employment.

Keynes expanded Sraffa's own-rate definition to include several distinct components that determine the total return of holding any asset. According to Keynes, the total expected return of an asset consists of its physical yield or output (q), its carrying or storage cost (c), its expected price appreciation (a), and its liquidity premium (l), which represents the subjective convenience and security of readily convertible purchasing power.

Asset Component	**Keynesian Variable**	**Description**
Yield	q	The physical output or productive service generated by the asset.
Carrying Cost	c	The wastage, depreciation, or storage cost of holding the asset.
Appreciation	a	The expected change in the asset's price measured in terms of money.
Liquidity Premium	l	The subjective value of the asset's ease of conversion into purchasing power.

Table 1: Keynesian Components of Asset Returns

Keynes argued that in a decentralized market, the own-rates of all assets must equalize in equilibrium when measured in a common standard (money). The total return on any asset is given by:

$$\text{Own-rate} = q - c + a + l \tag{3}$$

However, because money possesses a near-zero elasticity of production and a high liquidity premium that is highly reluctant to fall during periods of uncertainty, the money rate of interest acts as a structural floor for the entire economy. If the marginal efficiency (own-rate) of physical capital assets falls below the stubbornly high money rate of interest, investment in physical capital ceases, leading to deficient aggregate demand and persistent unemployment.

While Sraffa later criticized Keynes's reliance on subjective "liquidity preference" - arguing in private notes that differences in own-rates stem purely from expected changes in relative physical prices rather than psychological advantages or "diminishing marginal utility of holding money" (Naldi, 2012) - the conceptual damage to neoclassical capital theory was permanently established. Both theorists proved that in a multi-asset economy characterized by shifting relative prices, identifying a single "rate of return on capital" is impossible without first recognizing the structural dominance of financial rates over physical returns.

3 The Cambridge Capital Controversy: Aggregation, Circularity, and Wicksell Effects

The divergent own-rates mathematically identified by Sraffa and Keynes lay at the heart of the most severe crisis in twentieth-century economic theory: the Cambridge Capital Controversy. Initiated by Joan Robinson in 1953 and formalized by Sraffa in his 1960 magnum opus, *Production of Commodities by Means of Commodities* (Sraffa, 1960), the debate directly challenged the core neoclassical explanation of income distribution, capital accumulation, and economic growth.

3.1 The Circularity of the Aggregate Production Function

The neoclassical synthesis, championed by American economists such as Paul Samuelson and Robert Solow, relied heavily on the aggregate production function, generally expressed as:

$$Y = F(K, L) \tag{4}$$

where Y is total macroeconomic output, L is homogeneous labor, and K is homogeneous capital. Under the assumption of perfect competition and constant returns to scale, the theory posited that factors of production are remunerated according to their relative scarcity and marginal productivity. Specifically, the rate of profit (r) equals the marginal product of capital:

$$r = \frac{\partial F}{\partial K} \tag{5}$$

and the wage rate (w) equals the marginal product of labor:

$$w = \frac{\partial F}{\partial L} \tag{6}$$

Joan Robinson's devastating 1953 critique attacked the fundamental measurement of K (Robinson, 1953). Unlike labor, which can theoretically be measured in standardized man-hours, or land, which can be measured in physical acres, "capital" consists of a vast, profoundly heterogeneous array of physical goods - tractors, blast furnaces, software, and commercial real estate. To aggregate these heterogeneous items into a single scalar variable K that can be inserted into a production function, they must be valued in monetary terms.

The value of any capital asset is the discounted present value of its future expected yields:

$$V_K = \sum_{t=0}^{\infty} \frac{\pi_t}{(1+r)^t} \tag{7}$$

where π_t represents the net returns at time t. However, discounting future yields requires prior knowledge of the uniform rate of interest (or rate of profit). Therefore, one cannot measure the quantity of aggregate capital without already knowing the rate of profit. But in neoclassical marginal productivity theory, the rate of profit is supposed to be determined by the relative scarcity (the quantity) of capital.

The Sraffian conclusion was inescapable: the neoclassical marginal productivity theory of distribution is logically circular. It attempts to explain the rate of profit by the quantity of capital, but the quantity of capital cannot be mathematically quantified without first knowing the rate of profit.

3.2 Wicksell Effects and the Breakdown of Factor Substitution

The theoretical vulnerability of aggregate capital culminated in the rediscovery of Wicksell effects and the formalization of the "reswitching" paradox. These phenomena destroyed the neoclassical assumption of a monotonic, inverse relationship between the interest rate and capital intensity.

In traditional neoclassical parables, a lower interest rate should always induce profit-maximizing firms to substitute labor for capital, adopting more capital-intensive techniques of production. This inverse relationship guarantees a downward-sloping demand curve for capital, ensuring that market forces can smoothly equilibrate the supply and demand for savings and investment.

However, the Cambridge critics demonstrated that due to complex "Wicksell effects" - the revaluation of existing capital goods as income distribution changes - this monotonic relationship breaks down entirely (Garegnani, 1970).

There are two primary manifestations of this breakdown:

1. Price Wicksell Effects and Reverse Capital Deepening: As the rate of profit changes, the relative prices of the heterogeneous capital goods used in production also change. Because different capital goods require different proportions of labor and time to produce, a change in the interest rate alters their compounded costs of production unevenly. This can lead to "reverse capital deepening" (also known as the Ruth

Neoclassical Parable	Cambridge UK Refutation
Capital can be aggregated into a single scalar K.	Aggregation is circular; the value of capital depends on the profit rate prior to calculation.
Lower interest rates lead to higher capital intensity.	Price and Real Wicksell effects allow lower interest rates to favor less capital-intensive methods.
The interest rate reflects the marginal product of capital.	Reswitching proves the demand curve for capital is not monotonically downward-sloping, voiding the marginal product explanation.

Table 2: Neoclassical Theory versus Cambridge Critique

Cohen Curiosum), where an increase in the rate of interest leads to the adoption of a more capital-intensive technique, entirely contradicting the neoclassical law of demand for factors of production. If capital demand curves can slope upward, the macroeconomic system lacks stability, and the fundamental neoclassical mechanism of supply and demand determining factor prices collapses.

2. The Reswitching of Techniques: Sraffa (1960) proved an even more devastating anomaly. He demonstrated that a specific technique of production (Technique α) might be the most profitable at a low rate of interest, become unprofitable and be replaced by Technique β at a medium rate of interest, and yet - paradoxically - reswitch to being the most profitable again at a high rate of interest. Reswitching implies that there is no unambiguous way to rank techniques by their "capital intensity" independent of the interest rate.

3.3 Samuelson's Surrogate Production Function and Capitulation

In a desperate attempt to rescue the aggregate production function from the charges of circularity and reswitching, Paul Samuelson (1962) developed the "surrogate production function" (Samuelson, 1962). Samuelson constructed a mathematical model with heterogeneous capital goods but demonstrated that under strictly defined conditions, the economy behaves "as if" there were a single, homogeneous capital jelly.

However, Samuelson's model required highly restrictive and unrealistic assumptions. Specifically, it demanded that the capital-labor ratio (capital intensity) must be identical across all sectors of the economy, including both the capital-goods producing sector and the consumption-goods producing sector. When this assumption was relaxed, the wage-profit frontier ceased to be linear, Wicksell effects emerged, and the paradox of reswitching inevitably occurred.

Initially, neoclassical theorists like David Levhari (1965) attempted to prove mathematically that reswitching was impossible in an indecomposable input-output system (Levhari, 1965). This claim was swiftly refuted by Luigi Pasinetti, Pierangelo Garegnani, and other heterodox scholars who provided robust mathematical counterexamples proving that Levhari's theorem was false (Pasinetti, 1966). Faced with irrefutable mathematical proof, Samuelson (1966) published a famous capitulation in the *Quarterly Journal of Economics*, admitting that reswitching is a valid logical phenomenon and that the simple neoclassical parables of aggregate capital are theoretically unsound (Samuelson, 1966).

4 The Methodological Rupture: The Loss of the "Center of Gravitation"

Following the destruction of the aggregate production function, the neoclassical defense could no longer rely on macroeconomic parables. Instead, the defense retreated to the microeconomic foundations of general equilibrium, specifically the Walrasian specification of capital. This shift fundamentally altered the methodological nature of economic theory, a transition heavily critiqued by Pierangelo Garegnani, who argued that it stripped economics of its explanatory power.

4.1 The Abandonment of the Long-Period Method

From Adam Smith's *Wealth of Nations* through David Ricardo, Karl Marx, and early neoclassical theorists like Alfred Marshall, John Bates Clark, and Knut Wicksell, economic analysis relied universally on the "long-period method". This classical method posited that market competition forces the rate of profit to equalize across all sectors of the economy. If one sector offers a higher rate of return, capital will flow into it, increasing supply and driving down prices until the rate of return equalizes with the macroeconomic average.

The resulting "natural prices" or "prices of production" serve as a "center of gravitation". Actual market prices may fluctuate constantly due to temporary shocks, fads, or localized scarcities, but they are persistently pulled back toward the long-period equilibrium by the gravity of a uniform rate of return. As Smith described, whatever obstacles hinder prices from settling, they are constantly tending toward this central repose.

The long-period method, however, required capital to be treated as a single magnitude of value that could fluidly change its physical form across sectors to chase the highest returns. Because the CCC proved conclusively that capital cannot be treated as a single magnitude of value without falling into circularity, the neoclassical school was forced to abandon the long-period method entirely.

4.2 The Walrasian Vector of Heterogeneous Capital

To evade the circularity problem of measuring aggregate capital, modern neoclassical theorists (following J.R. Hicks's 1939 *Value and Capital* and Gerard Debreu's 1959 *Theory of Value*) adopted the Walrasian specification (Hicks, 1939; Debreu, 1959). Capital was no longer an aggregate value K; instead, it was defined strictly as a given vector of physically heterogeneous endowments:

$$\mathbf{K} = [k_1, k_2, \ldots, k_n] \tag{8}$$

By treating every physical machine, building, and tool as a separate, non-reproducible factor of production in the short run, intertemporal general equilibrium models successfully side-stepped the need to aggregate capital. The remuneration for each specific capital good is determined entirely by its specific supply and specific demand, leading to a spectrum of divergent own-rates of return across different physical assets.

4.3 Garegnani's Critique of Impermanence and Sterility

Pierangelo Garegnani (1976) identified the fatal methodological flaw in this Neo-Walrasian retreat (Garegnani, 1976). While mathematically consistent and immune to the Sraffian aggregation critique, treating capital as a fixed vector of arbitrary physical goods strips the equilibrium of any empirical relevance.

Capital goods have a "double nature": they are both productive factors and reproducible commodities. As productive factors, their short-run remuneration depends on scarcity. But as reproducible commodities, their long-run price must cover their cost of production. In reality, the physical composition of the capital stock changes rapidly. Capitalists invest in highly profitable sectors, creating new physical capital goods, and abandon unprofitable ones, allowing old capital to depreciate.

Therefore, a Neo-Walrasian equilibrium based on an arbitrary, fixed vector of physical capital goods lacks "persistence". It represents merely a fleeting, temporary state that immediately changes the moment production, investment, and accumulation occur. Because the Neo-Walrasian equilibrium lacks the persistence of a uniform rate of profit, it can no longer serve as a Smithian "center of gravitation".

Garegnani, alongside critics like Mark Blaug, argued that by solving the logical problem of heterogeneous capital via intertemporal general equilibrium, neoclassical theory reduced itself to an exercise in mathematical sterility (Blaug, 1974). The Formalist Revolution succeeded in proving existence theorems, but it produced models incapable of explaining the long-run central tendencies of capitalist accumulation, reducing competition to an instantaneous end-state rather than an ongoing historical process.

5 The Arrow-Debreu Time-0 Vacuum and the Formalization of Heterogeneity

The earliest and most prominent formalization of the Neo-Walrasian shift was the Arrow-Debreu (AD) model of complete contingent markets (Arrow and Debreu, 1954; Debreu, 1959). In the AD framework, uncertainty and time are incorporated by radically expanding the commodity space. A physical good delivered tomorrow is treated as a fundamentally different commodity than the exact same physical good delivered today.

Let us assume an economy with a finite set of goods $l = 1, \ldots, L$ and time periods $t = 0, 1$. In a classic Arrow-Debreu setting, all economic agents meet at $t = 0$ to trade complete futures contracts for all goods at all dates and in all conceivable states of nature.

Let $p_{l,t}$ denote the time-0 price of a contract for the delivery of good l at time t. A representative agent maximizes a strictly concave intertemporal utility function $U(\mathbf{x}_0, \mathbf{x}_1)$ subject to a single, lifetime wealth constraint:

$$\sum_{l=1}^{L} p_{l,0} x_{l,0} + \sum_{l=1}^{L} p_{l,1} x_{l,1} \leq W \tag{9}$$

where W represents the agent's total present-value wealth.

The intertemporal marginal rate of substitution (MRS) for good l, which defines its physical "own-rate" of interest (r_l), is strictly defined by the intertemporal price ratio established at time 0:

$$1 + r_l = \frac{p_{l,0}}{p_{l,1}} = \frac{MU_{l,0}}{MU_{l,1}} \tag{10}$$

where $MU_{l,t}$ denotes the marginal utility of good l at time t.

In an economy characterized by changing technologies, shifting relative scarcities, or evolving preferences, the relative prices between any two goods j and k will invariably fluctuate over time. Consequently, the general equilibrium condition necessitates that:

$$\frac{p_{j,0}}{p_{k,0}} \neq \frac{p_{j,1}}{p_{k,1}} \tag{11}$$

which mathematically guarantees that $r_j \neq r_k$.

Thus, the AD framework mathematically confirms the original Sraffian critique: in a pure barter or time-0 futures market, there is an array of divergent own-rates of interest. A ton of steel delivered tomorrow relative to a ton of steel today yields a different rate of return than a bushel of wheat delivered tomorrow relative to a bushel of wheat today.

However, the AD framework achieves this mathematical consistency by positing an institutional absurdity: all trading occurs simultaneously at the beginning of time, and markets never reopen. Agents must possess perfect foresight (or perfect contingent planning) regarding all future states over infinite horizons. Because it assumes a complete set of forward markets for every conceivable contingency - markets that demonstrably do not exist in reality due to prohibitive transaction costs and fundamental uncertainty - the AD model failed to provide a realistic defense against the heterodox challenge.

6 The Neoclassical Resolution: Radner Sequential Markets and Incomplete Trading

To rescue the micro-foundations of capital theory from the unrealistic strictures of the Arrow-Debreu time-0 vacuum, mainstream economics structurally evolved the general equilibrium framework into sequential markets, primarily through the pioneering mathematical work of Roy Radner (1968, 1972) (Radner, 1968, 1972).

6.1 The Transition to Sequential Trading

Radner recognized that in actual economies, markets are incomplete, and trade occurs sequentially over time rather than in a single prehistoric auction. In a Radner sequence economy, instead of committing universally at $t = 0$ for all future contingencies, agents clear spot markets at $t = 0$, hold financial assets to transfer purchasing power into the future, and subsequently clear spot markets again at $t = 1$.

This requires agents to formulate "plans, prices, and price expectations". While physical forward markets for every commodity may be missing due to transaction costs, the presence of financial securities - such as riskless bonds, equity shares, or fiat money - allows agents to bridge intertemporal gaps.

6.2 The Sequential Optimization Problem

Let $q_{l,t}$ represent the spot price of good l at time t. Let B be a financial instrument (such as a riskless bond) traded at $t = 0$ that pays one unit of account at $t = 1$, yielding a gross rate of return $R = 1 + r$. Rather than a single lifetime budget constraint, the agent in a sequential market faces a sequence of budget constraints, one for each active market period.

The agent's sequential Lagrangian optimization problem takes the form:

$$\mathcal{L} = U(\mathbf{x}_0, \mathbf{x}_1) + \lambda_0 \left[W_0 - \sum_{l=1}^{L} q_{l,0} x_{l,0} - q_B B \right] + \lambda_1 \left[RB - \sum_{l=1}^{L} q_{l,1} x_{l,1} \right] \tag{12}$$

where λ_0 and λ_1 are the shadow prices (Lagrange multipliers) for the period-0 and period-1 budget constraints, respectively, and q_B is the price of the bond at $t = 0$.

Taking the first-order conditions (FOCs) with respect to the physical consumption goods yields the standard static marginal utility conditions:

$$\frac{\partial U}{\partial x_{l,0}} = \lambda_0 q_{l,0} \tag{13}$$

$$\frac{\partial U}{\partial x_{l,1}} = \lambda_1 q_{l,1} \tag{14}$$

Crucially, taking the first-order condition with respect to the financial instrument B yields the intertemporal linkage between periods:

$$\lambda_0 q_B = \lambda_1 R \tag{15}$$

For a riskless bond trading at par, $q_B = 1$, which simplifies to:

$$\frac{\lambda_0}{\lambda_1} = R = 1 + r \tag{16}$$

This mathematical relationship proves that the subjective shadow price of transferring wealth across time ($\frac{\lambda_0}{\lambda_1}$) is exactly equal to the gross return on the financial instrument (R).

7 Intertemporal Arbitrage as the Unifying Mechanism

By substituting the shadow price ratio $\frac{\lambda_0}{\lambda_1}$ into the intertemporal marginal rate of substitution for any arbitrary physical good l, we derive the generalized Euler equation, the cornerstone of modern dynamic macroeconomics:

$$\frac{\frac{\partial U}{\partial x_{l,0}}}{\frac{\partial U}{\partial x_{l,1}}} = \frac{\lambda_0 q_{l,0}}{\lambda_1 q_{l,1}} = R \cdot \frac{q_{l,0}}{q_{l,1}} \tag{17}$$

Rearranging:

$$\frac{\frac{\partial U}{\partial x_{l,0}}}{\frac{\partial U}{\partial x_{l,1}}} = (1 + r) \cdot \frac{q_{l,0}}{q_{l,1}} \tag{18}$$

7.1 Decomposing the Physical Own-Rate

The equation above elegantly resolves the heterogeneous own-rates paradox. The heterodox critique correctly identified that physical own-rates diverge; mathematically, the ratio of spot prices over time ($\frac{q_{l,0}}{q_{l,1}}$) varies across different goods due to technological change and relative scarcity. However, the Radner sequential framework demonstrates that agents do not evaluate intertemporal trade-offs purely in physical own-rates.

Because the institutional structure of the economy allows for financial arbitrage, the subjective cost of delaying consumption is universally unified by the financial rate R (the pricing kernel or stochastic discount factor). The physical own-rate is analytically decomposed into two distinct elements: a uniform macroeconomic anchor (R) and a localized relative price shift ($\frac{q_{l,0}}{q_{l,1}}$).

Market Feature	**Arrow-Debreu Time-0 Equilibrium**	**Radner Sequential Equilibrium**
Market Structure	Complete futures markets at $t = 0$.	Incomplete forward markets; spot markets reopen sequentially.
Budget Constraints	Single lifetime wealth constraint.	Sequence of constraints linked by financial assets.
Own-Rates	Divergent and fundamentally heterogeneous.	Divergent physical rates anchored by a unified financial rate (R).
Unifying Mechanism	Fictitious central auctioneer at $t = 0$.	Financial arbitrage and the generalized Euler equation.

Table 3: Comparison of Arrow-Debreu and Radner Equilibria

7.2 The "No-Arbitrage" Condition and Firm Behavior

In modern macro-finance, derived directly from Radner's theoretical foundations, the unification of interest rates is rigorously enforced by the "no-arbitrage" condition. The principle of no-arbitrage asserts that if markets are frictionless and expectations are rational, it must be impossible to generate riskless profits with zero net investment.

Under the assumption of no-arbitrage, asset prices establish a unifying pricing kernel (state price density) across all states of nature. This implies that all firms, regardless of the physical heterogeneity of their specific capital goods, maximize their objective functions by discounting future cash flows using the exact same present-value process.

This leads to criteria such as the Drèze Criterion for business decisions under incomplete markets, where firms maximize profits at shadow prices obtained as weighted averages of the shadow prices of shareholders (Drèze, 1974). Capital, therefore, can be unambiguously valued by discounting its expected spot-market returns against the unified financial pricing kernel. The sequential market structure organically suppresses heterogeneous own-rates by subordinating them to the mathematical law of financial arbitrage.

8 Solving the Reswitching Paradox: The Real Factor Price Perspective

While sequential market clearing and no-arbitrage pricing kernels resolved the problem of measuring capital values via financial discounting, the paradox of reswitching continued to linger as a perceived contradiction within the neoclassical theory of production. If an increase in the interest rate could lead to the adoption of a more capital-intensive technique, the core neoclassical principle of factor substitution remained violated.

However, recent theoretical arguments, such as the formalization proposed by Carlo Milana (2019), have attempted to rationalize the reswitching paradox within the boundaries of the neoclassical paradigm (Milana, 2019). While this specific coordinate transformation approach remains highly controversial and is not universally accepted, it highlights a broader, more widely recognized neoclassical defense: the marginal theory of capital remains mathematically valid under *ceteris paribus* conditions.

8.1 The Illusion of the Wage-Interest Plane

Milana's resolution identifies a fundamental misconception in the Sraffian critique: it mistakes the financial interest rate for the true rental price of physical capital inputs.

The Cambridge critics traditionally illustrated the reswitching paradox graphically by plotting the trade-off relation between the real labor wage (w) and the interest rate (r) - the wage-profit frontier. In this w-r plane, the envelope of available techniques exhibits non-linearities, resulting in switch points where the exact same technique becomes optimal at two disconnected ranges of the interest rate, with a different technique prevailing in between.

Milana demonstrates that this representation confuses a "financial perspective" with a "productive perspective". The interest rate is a financial discounting variable used to equate intertemporal values; it is not the actual rental price of physical capital goods on the factory floor. Because of sectoral interdependencies, progressive variations in income distribution (changes in the interest rate) have complex, non-monotonic effects on relative physical prices - the very Wicksell effects identified during the CCC.

8.2 Transformation to the Coordinate Space of Real Factor Prices

The core of the modern resolution lies in translating the choice of technique out of the financial wage-interest plane and into the coordinate space of true real factor prices (e.g., mapping the real rental prices of capital goods against real wages).

When the physical capital goods are taken into consideration within explicitly defined cost accounts, each technique generates a strictly linear relationship between real factor prices and the real wage. Consequently, Milana mathematically proves that the reswitching of techniques can never occur in the coordinate space of true real factor prices, regardless of the configuration or complexity of the Sraffian input-output model.

If a technical sequence appears as a "reswitching" pattern ($\alpha \to \beta \to \alpha$) across a monotonic range of interest rates, this is merely an optical illusion generated by Wicksell effects. In reality, the monotonic change in the interest rate causes the real rental prices of capital goods to fluctuate non-monotonically (moving up, then down, or vice versa) due to the differing compounding effects of time across interdependent sectors.

The apparent "return" to the old technique (α) is simply the rational, standard neoclassical cost-minimizing response of profit-seeking producers to "returning ranks of relative input prices". By sorting the ratio of unit costs of production against real factor prices rather than the interest rate, the allegedly "perverse" choice of a less capital-intensive technique at a lower interest rate is rationalized.

It is crucial to note, however, that Milana's complete dismissal of the reswitching paradox is not widely accepted among capital theorists. Many critics argue that conceptually separating financial interest rates from physical rental rates merely sidesteps the fundamental macroeconomic and distributional linkages that Sraffa originally sought to expose.

Nevertheless, the neoclassical defense retains a robust and widely accepted fallback: the marginal productivity theory of capital remains entirely valid under *ceteris paribus* conditions. If the relative prices of other heterogeneous capital goods are held strictly constant, a decrease in the interest rate will unambiguously lead to the adoption of more capital-intensive techniques. The reswitching paradox only emerges in general equilibrium frameworks when a shifting interest rate is allowed to simultaneously trigger complex Wicksell effects that dynamically revalue the entire structure of relative prices. Thus, in a partial equilibrium context where other prices are fixed, the core marginalist principles of factor substitution hold firm.

9 Conclusion

The decades-long intellectual conflict over the nature of capital and the uniform rate of interest forced mainstream economics to rigorously overhaul its mathematical and philosophical foundations. The heterodox challenge - from Sraffa's multiplicity of own-rates to the devastating Cambridge proofs of circular aggregation, Wicksell effects, and reswitching - effectively destroyed the simplistic macroeconomic parables of homogeneous capital that dominated the mid-twentieth century.

However, the sequential defense demonstrates that the neoclassical paradigm did not collapse under this assault; rather, it underwent a profound structural evolution. The transition from the time-0 Arrow-Debreu framework to Radner-style sequential economies successfully neutralized the problem of heterogeneous own-rates. By formally modeling the institutional reality of incomplete forward markets and sequential spot trading, neoclassical theory proved that financial arbitrage and the generalized Euler equation organically generate a unifying pricing kernel. This financial vector anchors the disparate physical own-rates, providing a mathematically robust mechanism for discounting heterogeneous capital flows without falling into the trap of circular valuation.

Furthermore, while debate continues over controversial solutions like translating the reswitching paradox into the coordinate space of real factor prices, the neoclassical school has maintained a strong theoretical foothold through the *ceteris paribus* assumption. It confirms that the microeconomic laws of factor substitution remain entirely coherent when the relative prices of other capital goods are held constant, ensuring that partial equilibrium marginal productivity remains mathematically sound even if general equilibrium aggregation fails.

Nevertheless, this mathematical victory required a profound methodological sacrifice. To evade the circularity of measuring aggregate capital, neoclassical theory was forced to adopt the Walrasian specification of physical endowments. As critics like Pierangelo Garegnani accurately diagnosed, this shift stripped equilibrium of its persistence. Modern sequential and temporary equilibrium models excel at mapping the exact arbitrage conditions of asset pricing, but they no longer provide the classical "center of gravitation" that explains the long-run accumulation of capital. Ultimately, while the sequential defense secured the mathematical integrity of neoclassical optimization, the philosophical divide over whether economics should model fleeting moments of sequential market-clearing or the persistent gravitational forces of distribution remains the defining fault line of modern economic theory.

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