An Isomorphic non-Rotemberg Flexible-Price Model for the New Keynesian Model

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Abstract

It is shown that if a flexible-price model is generalized to include an inflation wedge in the aggregate production function, it becomes isomorphic to the New Keynesian model without requiring separate inflation-related adjustment costs or sticky prices. This formulation does not require finitely many firms, though an intermediate goods sector with monopolistic competition is utilized for aggregation. Potential problems with the functional form of the inflation wedge are examined. A comparison against the Rotemberg adjustment cost model is provided, highlighting the distinction between unused but wasted monetary resources (Rotemberg) and production efficiency (our model). The paradox of flexibility in a New Keynesian model under the zero lower bound is re-examined in light of this alternative flexible-price framework.

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1 Introduction

The New Keynesian (NK) model has become the workhorse of modern monetary policy analysis. Its core feature—the non-neutrality of money—is typically derived from nominal rigidities, such as Calvo-style staggered price setting or Rotemberg-style adjustment costs. These mechanisms generate the New Keynesian Phillips Curve (NKPC), linking inflation

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to real economic activity and expected future inflation. However, the reliance on sticky prices is not without controversy, especially in the sense that nominal rigidities drive the necessity of monetary policies instead of real economic concerns.

This paper proposes an alternative framework: a flexible-price model that is mathematically isomorphic to the standard New Keynesian (NK) model. Instead of positing that firms cannot change prices (Calvo) or must pay a (wasted) resource cost to do so (Rotemberg), we introduce an inflation wedge directly into the aggregate production function. This wedge captures the idea that high inflation (or deflation) creates frictional inefficiencies in the production process itself—perhaps due to search frictions, misallocation of relative prices, or information processing constraints—thereby reducing effective total factor productivity for final goods.

We show that by carefully specifying this inflation wedge, we can recover the exact log-linearized three-equation system of the standard NK model. This isomorphism allows us to interpret monetary non-neutrality not as a failure of price adjustment, but as a fluctuation in aggregate production efficiency driven by nominal instability.

We further utilize this framework to re-examine the paradox of flexibility at the Zero Lower Bound (ZLB). In standard models, greater price flexibility can deepen a recession during a liquidity trap. In our alternative framework, this parameter maps to the sensitivity of aggregate production efficiency to inflation. We find that while the mathematical results hold, the economic interpretation shifts: reduced sensitivity of the production function to inflation (the analogue to high price flexibility) exacerbates the downturn.

The remainder of the paper is organized as follows. Section 2 presents the alternative flexible-price model and various specifications of the inflation wedge. Section 3 compares this framework to the Rotemberg model. Section 4 concludes. The full derivation of the model and the stability conditions are provided in the Appendix.

2 The Alternative Flexible-Price Models

2.1 The Main Intuition

We begin with the standard three-equation log-linearized model known in the New Keynesian literature.

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) \quad \text{(IS equation)} \tag{1}$$

$$i_t = i_0 + \phi_\pi \pi_t + \phi_x x_t \quad \text{(MP equation)} \tag{2}$$

Here, $x_t \equiv \log(Y_t/Y_t^n)$ is the output gap, where Y_t^n refers to the natural flexible-price output level in the absence of the inflation wedge. i_t is the nominal interest rate, π_t is the inflation rate, and r_t^n is the natural real rate of interest. The microfoundations for the IS and MP equations—derived from the household Euler equation and the central bank's reaction function—remain untouched by the alternative model.

The standard New Keynesian Phillips curve (NKPC) is given by:

$$\pi_t = \beta_\pi E_t[\pi_{t+1}] + \kappa x_t \quad \text{(NKPC)} \tag{3}$$

Typically, $\beta_\pi = \beta$ in a New Keynesian model, originating from the discount factor in the household utility. In our alternative framework, β_π is a parameter of the inflation wedge function. In Calvo pricing models, κ reflects the degree of price flexibility. In this alternative flex-price model, κ refers to the inverse sensitivity of the aggregate production function to inflation deviations.

Equation (3) can be rearranged to express the output gap as a function of inflation dynamics:

$$x_t = \frac{\pi_t - \beta_\pi E_t[\pi_{t+1}]}{\kappa} \tag{4}$$

This implies an aggregate production relationship of the form:

$$Y_t = Y_t^n \exp\left[\frac{\pi_t - \beta E_t[\pi_{t+1}]}{\kappa}\right] \tag{5}$$

The term $\exp[(\pi_t - \beta E_t[\pi_{t+1}])/\kappa]$ serves as the inflation wedge in the aggregate production function. Even with fully flexible prices, if aggregate productivity is driven by this wedge, we recover the log-linearized New Keynesian model.

2.2 *Alternative NKPC Specification (1)*

Equation (5) implies that output can increase indefinitely with greater inflation deceleration. To impose a more realistic bound where inflation volatility hurts output, we can specify:

$$Y_t = Y_t^n \exp\left[-\frac{\text{abs}(\pi_t - \beta E_t[\pi_{t+1}])}{\kappa}\right] \tag{6}$$

This specification ensures that deviations from the stable inflation path reduce output. For the domain where $\pi_t - \beta E_t[\pi_{t+1}] < 0$, the standard NKPC holds. For $\pi_t - \beta E_t[\pi_{t+1}] > 0$, the sign flips:

$$\pi_t = \beta_\pi E_t[\pi_{t+1}] - \kappa x_t \tag{7}$$

2.3 Alternative NKPC Specification (2)

Additionally, one might set $\beta_\pi = 1$ in the wedge. Since $\beta_\pi \neq \beta$ is permissible in this framework (as it is a technological parameter, not a preference parameter), we can write:

$$Y_t = Y_t^n \exp\left[\frac{\pi_t - E_t[\pi_{t+1}]}{\kappa}\right] \tag{8}$$

Or, ensuring symmetry:

$$Y_t = Y_t^n \exp\left[-\frac{\text{abs}(\pi_t - E_t[\pi_{t+1}])}{\kappa}\right] \tag{9}$$

This encodes the idea that constant inflation is neutral for production, but changes in the inflation rate create disruptions.

2.4 Alternative NKPC Specification (3)

Setting $\beta_\pi = 0$ implies that any inflation (or deflation) is disruptive, regardless of expectations:

$$Y_t = Y_t^n \exp\left[\frac{\pi_t}{\kappa}\right] \tag{10}$$

Alternatively, to penalize both inflation and deflation:

$$Y_t = Y_t^n \exp\left[-\frac{\text{abs}(\pi_t)}{\kappa}\right] \tag{11}$$

2.5 Alternative NKPC Specification (4)

It is also possible to specify the production function such that it is one-sided:

$$Y_t = Y_t^n \exp\left[\frac{\min(\pi_t - \beta_\pi E_t[\pi_{t+1}], 0)}{\kappa}\right] \tag{12}$$

This yields $Y_t = Y_t^n$ for $\pi_t \geq \beta_\pi E_t[\pi_{t+1}]$, while maintaining the original NKPC dynamics for present deflationary pressures relative to future inflation expectations. Additionally, we can generalize further (both-sided and bounded):

$$Y_t = Y_t^n \exp\left[\frac{\max[\min(\pi_t - \beta_\pi E_t[\pi_{t+1}], k_\Delta), -k_\Delta]}{\kappa}\right] \tag{13}$$

This restores the original NKPC dynamics for the bounded inflation wedge region regardless of inflation and deflation but limits a complete collapse or unbounded overheating.

2.6 Equilibrium Determinacy Conditions

Variations in β_π or the sign of κ are permissible provided the system remains determinate. The full determinacy conditions are provided in Appendix B. In short, determinacy can be assured if we set $\phi_\pi > 1, \phi_y \gg 1$, regardless of the sign flip in κ or variations in β_π. This is consistent with the following conventional existence condition (Taylor principle):

$$\kappa(\phi_\pi - 1) + (1 - \beta_\pi)\phi_y > 0 \tag{14}$$

2.7 Aggregation: Final Goods vs. Intermediate Goods

To micro-found this wedge without altering the IS or MP curves, we adopt a standard monopolistic competition structure. The perfectly competitive representative final goods producer aggregates intermediate goods C_{it} into final good $C_t = Y_t$. The optimization problem produces the equation that connects final goods price P_t with intermediate goods price P_{it} and the demand function for an intermediate goods. Assuming that intermediate goods producers are identical, the price setting formula of an intermediate goods producer can be converted to relate Y_t with π_t by substituting the two equations from the final goods producer, which produce NKPC. The full derivation is given in Appendix A.

2.8 Flexibility and the Paradox of Flexibility

The alternative model is highly flexible; the aggregate production function can be nonlinear to incorporate various dynamics. However, we focus on the behavior around steady state $\pi = 0$.

This framework offers a new interpretation of the paradox of flexibility. In standard NK models, as prices become more flexible ($\kappa \to \infty$), a deflationary shock at the zero lower bound (ZLB) leads to a deeper recession. Consider the ZLB condition ($i_t = 0$) with the system:

$$x_t = E_t x_{t+1} + \frac{1}{\sigma}(E_t \pi_{t+1} + r_t^n) \tag{15}$$

$$\pi_t = \beta_\pi E_t[\pi_{t+1}] + \kappa x_t \tag{16}$$

If a negative real rate shock ($r_t^n < 0$ for $0 \leq t < T$) occurs, causing $x_t < 0$, a large κ implies a massive drop in π_t. In our model, $\kappa \to \infty$ corresponds to a low sensitivity of the production wedge to inflation (i.e., the wedge is small unless inflation is massive). The dynamics can be verified by setting $x_T = 0, \pi_T = 0$ and evolving backward.

Note that $\pi_{T-1} - \beta_\pi E_t[\pi_T] < 0$ and therefore effective κ remains positive even when the aggregate production logic of penalizing all forms of inflation deviations is followed. The paradox of flexibility cannot be avoided unless the $x_T = 0, \pi_T = 0$ equilibrium selection is questioned.

In this isomorphic model, a low sensitivity of real production to inflation (high κ) worsens the crisis. Thus, under ZLB, some sensitivity of aggregate production efficiency to nominal variables helps recovery.

3 A Comparison Against the Rotemberg Model

The Rotemberg (1982) model of quadratic adjustment costs is the standard alternative to Calvo pricing for generating New Keynesian dynamics with flexible-price firms. In the Rotemberg framework, the firm faces a real cost to change prices given by $\frac{\psi}{2}(\frac{P_{it}}{P_{it-1}} - 1)^2 Y_t$.

While the Rotemberg model yields a log-linearized Phillips curve identical to the Calvo model (and our model), it suffers from a distinct conceptual deficiency when compared to the production-wedge approach. In the Rotemberg model, price adjustment costs are treated as a deduction from final output—effectively, monetary resources are wasted to change price tags without corresponding to production factors or goods. This creates a wedge between aggregate output produced (Y_t) and aggregate consumption (C_t):

$$C_t = Y_t \left[1 - \frac{\psi}{2}(\pi_t)^2\right] \tag{17}$$

This implies that $C_t \neq Y_t$ unless $\pi_t = 0$. To rigorously justify this "menu cost," one technically requires a sector that produces these menu-adjustment services, utilizing labor and capital. If we abstract away from that sector, modeling the cost as simply lost monetary resources is ad-hoc.

Furthermore, if the menu cost actually involves hiring labor to change prices (managerial time, printing, etc.), then strictly speaking, labor is being diverted from production to price administration. This is exactly what our ”inflation wedge” model captures. Instead of treating the cost as a complete fiscal waste with no corresponding use, our model treats it as a technological regression: for the same unit of labor, less final output is realized because the technology of aggregation is hampered by inflation.

Thus, if the micro-foundation of adjustment costs is the diversion of labor/resources, the inflation-wedge production function is the more structurally consistent representation (Y_t is produced efficiently or inefficiently) rather than the Rotemberg model. We could even say that our model properly fixes the Rotemberg model without changing NKPC for small deviations from zero inflation rate. Far away from the zero inflation rate, our model, when fit with the realistic aggregate production function, should produce a different policy analysis from the Rotemberg model.

4 Conclusion

This paper demonstrates that the New Keynesian framework does not strictly require sticky prices or adjustment costs to hold. An isomorphic flexible-price model exists where nominal non-neutrality arises from an ”inflation wedge” in the aggregate production function. This wedge represents the efficiency losses associated with inflation.

This isomorphism suggests that the central insights of New Keynesian economics — specifically the management of aggregate demand to stabilize inflation and output—are robust to the specific micro-foundation of the friction. Whether the friction is the inability to change a price tag (Calvo) or a loss of production efficiency due to price instability (this model), the policy prescriptions remain largely similar. However, the interpretation of parameters, particularly regarding the ”paradox of flexibility” at the Zero Lower Bound, changes. We find that in a flexible-price world with efficiency wedges, extreme insensitivity of production to inflation can actually destabilize the economy during liquidity traps.

Appendix A: The Model Derivation and Stability Analysis

This appendix provides the full derivation of the isomorphic flexible-price model, ensuring that the price gap between final and intermediate goods is correctly accounted for. It concludes with the derivation of the Bullard-Mitra existence condition.

A.1 Households

The representative household maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{A.1}$$

subject to the budget constraint:

$$P_t C_t + B_t = W_t N_t + (1+i_{t-1}) B_{t-1} + \Pi_t \tag{A.2}$$

The first-order conditions yield the standard Euler equation and labor supply:

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} (1+i_t) \right] \tag{A.3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \tag{A.4}$$

Log-linearizing around the steady state (with $c_t = y_t$) yields the IS equation and labor supply relation:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \tag{A.5}$$

$$w_t - p_t = \sigma y_t + \varphi n_t \tag{A.6}$$

A.2 Firms, Aggregation, and the Price Wedge

This section derives the relationship between the final goods price P_t and the intermediate goods price P_{it} in the presence of the inflation wedge.

The Final Goods Producer

The representative final goods producer aggregates intermediate goods $i \in [0,1]$ using a Dixit-Stiglitz aggregator modified by the efficiency wedge Δ_t:

$$Y_t = \frac{1}{\Delta_t} \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.7}$$

Let $\tilde{Y}_t \equiv Y_t \Delta_t$ be the "raw" aggregate input. The producer minimizes the cost $\int_0^1 P_{it} Y_{it} di$ of producing $\tilde{Y}_t$. This yields the standard price index for the raw aggregate:

$$P_{\text{index}} = \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \tag{A.8}$$

The zero-profit condition in the competitive final goods sector requires Total Revenue = Total Cost:

$$P_t Y_t = P_{\text{index}} \tilde{Y}_t \tag{A.9}$$

Substituting $\tilde{Y}_t = Y_t \Delta_t$:

$$P_t Y_t = P_{\text{index}} (Y_t \Delta_t) \implies P_t = P_{\text{index}} \Delta_t \tag{A.10}$$

Under the assumption of symmetry ($P_{it} = P_i$ for all i), $P_{\text{index}} = P_i$. Thus, the price of the intermediate good is strictly less than the final good price when $\Delta_t > 1$:

$$P_i = \frac{P_t}{\Delta_t} \tag{A.11}$$

The Intermediate Goods Producer

Intermediate firms produce using linear technology $Y_{it} = A_t N_{it}$. They set prices flexibly to maximize profit given the demand curve. The optimal price is a constant markup $\mathscr{M} = \frac{\varepsilon}{\varepsilon - 1}$ over nominal marginal cost:

$$P_i = \mathscr{M} \frac{W_t}{A_t} \tag{A.12}$$

Substituting Equation (A.11) into the pricing rule:

$$\frac{P_t}{\Delta_t} = \mathscr{M} \frac{W_t}{A_t} \tag{A.13}$$

Rearranging for the real wage in terms of final goods prices:

$$\frac{W_t}{P_t} = \frac{A_t}{\mathscr{M} \Delta_t} \tag{A.14}$$

Taking logs ($w_t - p_t$, a_t, μ, $\delta_t = \log \Delta_t$):

$$w_t - p_t = a_t - \mu - \delta_t \tag{A.15}$$

This equation demonstrates that the inflation wedge lowers the real wage paid by firms through two channels: the direct efficiency loss and the price wedge.

A.3 Deriving the NKPC

We combine the household and firm conditions to derive the Phillips Curve.

1. Labor Market Equilibrium: Equating the household labor supply (A.6) with the firm's labor demand (A.15):

$$\sigma y_t + \varphi n_t = a_t - \mu - \delta_t \tag{A.16}$$

2. Aggregate Production Function: In logs, the production function is $y_t = a_t + n_t - \delta_t$. Solving for labor:

$$n_t = y_t - a_t + \delta_t \tag{A.17}$$

Note that for a given output y_t, a positive wedge δ_t requires higher labor input n_t.

3. The Output Gap: Substitute n_t back into the labor market equilibrium:

$$\sigma y_t + \varphi(y_t - a_t + \delta_t) = a_t - \mu - \delta_t \tag{A.18}$$

Rearranging terms:

$$(\sigma + \varphi)y_t = (1 + \varphi)a_t - \mu - (1 + \varphi)\delta_t \tag{A.19}$$

Define the natural level of output y_t^n as the output level when the wedge is zero ($\delta_t = 0$):

$$(\sigma + \varphi)y_t^n = (1 + \varphi)a_t - \mu \tag{A.20}$$

Subtracting y_t^n from y_t gives the output gap $x_t \equiv y_t - y_t^n$:

$$(\sigma + \varphi)x_t = -(1 + \varphi)\delta_t \implies x_t = -\frac{1 + \varphi}{\sigma + \varphi}\delta_t \tag{A.21}$$

4. Recovering the NKPC: To recover the standard form $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$, or equivalently $x_t = \frac{1}{\kappa}(\pi_t - \beta E_t \pi_{t+1})$, we must specify the log-wedge δ_t as:

$$\delta_t = -\frac{\sigma + \varphi}{\kappa(1 + \varphi)}(\pi_t - \beta E_t \pi_{t+1}) \tag{A.22}$$

With this specification, the flexible-price model with an efficiency wedge is isomorphic to the New Keynesian model.

Appendix B: Derivation of the Bullard-Mitra Stability Condition

In this section, we derive the conditions for a unique rational expectations equilibrium (determinacy). We here express current variables as a function of expectations:

$$\mathbf{z}_t = \mathbf{A} E_t \mathbf{z}_{t+1} \tag{B.1}$$

where $\mathbf{z}_t = [\pi_t, x_t]^\top$. Because both inflation and the output gap are non-predetermined (jump) variables, a unique equilibrium requires that the system is stable in the backward dynamics. Therefore, we require both eigenvalues of the matrix $\mathbf{A}$ to lie **inside** the unit circle.

B.1 The Linear System

The reduced-form New Keynesian model consists of the IS curve, the Phillips curve, and the policy rule, where we ignore r_t^n and i_0 as they do not affect the stability and existence analysis, along with $\beta_\pi = \beta$ for convenience (β is therefore not necessarily related to time preference):

1. **IS Equation:**
$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \tag{B.2}$$
2. **Phillips Curve (NKPC):**
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{B.3}$$
3. **Policy Rule:**
$$i_t = \phi_\pi \pi_t + \phi_y x_t \tag{B.4}$$

B.2 Matrix Representation

We substitute the policy rule (B.4) into the IS equation (B.2) and rearrange terms to place all time-t variables on the left-hand side and expectations on the right-hand side.

From the IS equation:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(\phi_\pi \pi_t + \phi_y x_t - E_t \pi_{t+1}) \tag{B.5}$$

$$\sigma x_t = \sigma E_t x_{t+1} - \phi_\pi \pi_t - \phi_y x_t + E_t \pi_{t+1} \tag{B.6}$$

$$\phi_\pi \pi_t + (\sigma + \phi_y) x_t = E_t \pi_{t+1} + \sigma E_t x_{t+1} \tag{B.7}$$

From the NKPC, rearranging to isolate current variables:

$$\pi_t - \kappa x_t = \beta E_t \pi_{t+1} \tag{B.8}$$

We write the system of (B.7) and (B.8) in matrix form $\mathbf{M}\mathbf{z}_t = \mathbf{N}E_t\mathbf{z}_{t+1}$:

$$\begin{bmatrix} 1 & -\kappa \\ \phi_\pi & \sigma + \phi_y \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} \beta & 0 \\ 1 & \sigma \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} \tag{B.9}$$

To obtain the form $\mathbf{z}_t = \mathbf{A}E_t\mathbf{z}_{t+1}$, we define $\mathbf{A} = \mathbf{M}^{-1}\mathbf{N}$. The determinant of $\mathbf{M}$ is $\Omega = \sigma + \phi_y + \kappa\phi_\pi$. Assuming standard parameter signs, $\Omega > 0$. The inverse is:

$$\mathbf{M}^{-1} = \frac{1}{\Omega} \begin{bmatrix} \sigma + \phi_y & \kappa \\ -\phi_\pi & 1 \end{bmatrix} \tag{B.10}$$

Multiplying $\mathbf{M}^{-1}\mathbf{N}$:

$$\mathbf{A} = \frac{1}{\Omega} \begin{bmatrix} \sigma + \phi_y & \kappa \\ -\phi_\pi & 1 \end{bmatrix} \begin{bmatrix} \beta & 0 \\ 1 & \sigma \end{bmatrix} \tag{B.11}$$

$$= \frac{1}{\Omega} \begin{bmatrix} \beta(\sigma + \phi_y) + \kappa & \sigma\kappa \\ 1 - \beta\phi_\pi & \sigma \end{bmatrix} \tag{B.12}$$

B.3 General Stability Conditions

For determinacy, the eigenvalues of $\mathbf{A}$ must lie inside the unit circle. The necessary and sufficient conditions (from the characteristic polynomial) are:

1. $|\det(\mathbf{A})| < 1$
2. $|\mathrm{Tr}(\mathbf{A})| < 1 + \det(\mathbf{A})$

Calculating the Trace and Determinant:

$$\mathrm{Tr}(\mathbf{A}) = \frac{\beta(\sigma + \phi_y) + \kappa + \sigma}{\sigma + \phi_y + \kappa\phi_\pi} = \frac{\beta(\sigma + \phi_y) + \kappa + \sigma}{\Omega} \tag{B.13}$$

$$\det(\mathbf{A}) = \frac{\sigma}{\Omega^2}\left[(\beta(\sigma + \phi_y) + \kappa) - \kappa(1 - \beta\phi_\pi)\right] = \frac{\sigma\beta}{\Omega} \tag{B.14}$$

B.4 Case Analysis

The lesson of this section can be summarized as follows: regardless of 'pathological' cases, with sufficiently large $\phi_y \gg 1$ and $\phi_\pi > 1$, equilibrium determinacy can be assured. ($\Omega > 0$ is assured with this configuration, as well as rest of the conditions even if κ is negative.)

Case 1: No Discounting ($\beta = 0$)

In this case, $\det(\mathbf{A}) = 0$. The conditions reduce to $|\mathrm{Tr}(\mathbf{A})| < 1$. Substituting $\beta = 0$ into the trace:

$$\frac{\kappa + \sigma}{\sigma + \phi_y + \kappa\phi_\pi} < 1 \tag{B.15}$$

Assuming parameters are positive:

$$\kappa + \sigma < \sigma + \phi_y + \kappa\phi_\pi \tag{B.16}$$

$$\kappa < \phi_y + \kappa\phi_\pi \tag{B.17}$$

$$0 < \kappa(\phi_\pi - 1) + \phi_y \tag{B.18}$$

This recovers the standard Taylor Principle: the policy response to inflation must exceed unity unless there is a strong output response.

Case 2: Perfect Smoothing ($\beta = 1$)

Here, $\det(\mathbf{A}) = \frac{\sigma}{\sigma + \phi_y + \kappa\phi_\pi} < 1$ (since $\phi_y, \kappa, \phi_\pi > 0$). We examine the second condition: $1 - \mathrm{Tr}(\mathbf{A}) + \det(\mathbf{A}) > 0$.

$$1 - \frac{\sigma + \phi_y + \kappa + \sigma}{\Omega} + \frac{\sigma}{\Omega} > 0 \tag{B.19}$$

$$\frac{\Omega - (2\sigma + \phi_y + \kappa) + \sigma}{\Omega} > 0 \tag{B.20}$$

Expanding the numerator ($\Omega = \sigma + \phi_y + \kappa\phi_\pi$):

$$(\sigma + \phi_y + \kappa\phi_\pi) - \sigma - \phi_y - \kappa > 0 \tag{B.21}$$

$$\kappa\phi_\pi - \kappa > 0 \tag{B.22}$$

$$\kappa(\phi_\pi - 1) > 0 \tag{B.23}$$

For $\beta = 1$, determinacy strictly requires $\phi_\pi > 1$.

Case 3: Negative Slope ($\kappa = -k$) and the Role of Ω

Let $\kappa = -k$ where $k > 0$. The transition matrix is $\mathbf{A} = \mathbf{M}^{-1}\mathbf{N}$. The existence of $\mathbf{A}$ and the stability of the system depend on the determinant of $\mathbf{M}$:

$$\Omega \equiv \mathrm{Det}(\mathbf{M}) = \sigma + \phi_y - k\phi_\pi \tag{B.24}$$

Because of the negative channel ($-k\phi_\pi$), the sign of Ω is ambiguous. We must analyze the stability conditions accounting for the denominator Ω.

The stability conditions (eigenvalues inside the unit circle) require:

$$\text{(C1)} \quad |\mathrm{Det}(\mathbf{A})| < 1 \tag{B.25}$$

$$\text{(C2)} \quad P(1) = 1 - \mathrm{Tr}(\mathbf{A}) + \mathrm{Det}(\mathbf{A}) > 0 \;\; (\mathrm{Tr}(\mathbf{A}) \geq 0) \tag{B.26}$$

$$\text{(C3)} \quad P(-1) = 1 + \mathrm{Tr}(\mathbf{A}) + \mathrm{Det}(\mathbf{A}) > 0 \;\; (\mathrm{Tr}(\mathbf{A}) < 0) \tag{B.27}$$

Substituting the derived Trace and Determinant expressions (where Ω is the common denominator):

$$\mathrm{Det}(\mathbf{A}) = \frac{\sigma\beta}{\Omega} \tag{B.28}$$

$$P(1) = \frac{(1-\beta)\phi_y - k(\phi_\pi - 1)}{\Omega} \tag{B.29}$$

$$P(-1) = \frac{(1+\beta)(2\sigma + \phi_y) - k(\phi_\pi + 1)}{\Omega} \tag{B.30}$$

Scenario 3.1: The Singularity ($\Omega = 0$) If $k\phi_\pi = \sigma + \phi_y$, then $\Omega = 0$. The matrix $\mathbf{M}$ is singular and cannot be inverted. The model has no reduced-form solution $\mathbf{z}_t = \mathbf{A}E_t\mathbf{z}_{t+1}$. This represents a bifurcation point where the system's dynamics change structurally.

Scenario 3.2: The Standard Region ($\Omega > 0$) $\text{Det}(\mathbf{A}) = \frac{\sigma\beta}{\Omega} > 0$. Since the determinant is positive, the stability condition $|\text{Det}(\mathbf{A})| < 1$ requires:

$$\sigma\beta < \Omega \implies k\phi_\pi < \sigma(1-\beta) + \phi_y \tag{B.31}$$

For the remaining conditions, we split the analysis based on the sign of the Trace.

3.2.1 Case: Non-Negative Trace ($\mathbf{Tr}(\mathbf{A}) \geq 0$) This case occurs when $k \leq \beta(\sigma + \phi_y) + \sigma$. The **binding constraint** is $P(1) > 0$.

$$P(1) > 0 \implies 1 - \text{Tr}(\mathbf{A}) + \text{Det}(\mathbf{A}) > 0 \tag{B.32}$$
$$\implies \Omega - [\beta(\sigma + \phi_y) - k + \sigma] + \sigma\beta > 0 \tag{B.33}$$
$$\implies (\sigma + \phi_y - k\phi_\pi) - \beta\sigma - \beta\phi_y + k - \sigma + \sigma\beta > 0 \tag{B.34}$$
$$\implies (1-\beta)\phi_y - k(\phi_\pi - 1) > 0 \tag{B.35}$$

Result: The system requires $(1-\beta)\phi_y > k(\phi_\pi - 1)$, which can be satisfied by $\phi_\pi > 1, \phi_y \gg 1$.

3.2.2 Case: Negative Trace ($\mathbf{Tr}(\mathbf{A}) < 0$) This case occurs when $k > \beta(\sigma + \phi_y) + \sigma$. The **binding constraint** is $P(-1) > 0$.

$$P(-1) > 0 \implies 1 + \text{Tr}(\mathbf{A}) + \text{Det}(\mathbf{A}) > 0 \tag{B.36}$$
$$\implies \Omega + [\beta(\sigma + \phi_y) - k + \sigma] + \sigma\beta > 0 \tag{B.37}$$
$$\implies (\sigma + \phi_y - k\phi_\pi) + \beta\sigma + \beta\phi_y - k + \sigma + \sigma\beta > 0 \tag{B.38}$$
$$\implies (1+\beta)(2\sigma + \phi_y) - k(\phi_\pi + 1) > 0 \tag{B.39}$$

Result: This case can be eliminated by setting $\phi_\pi > 1, \phi_y \gg 1$, which gives us the $\text{Tr}(\mathbf{A}) \geq 0$ case instead.

Scenario 3.3: The "Flipped" Region ($\Omega < 0$) We can avoid this scenario by setting $\phi_\pi > 1, \phi_y \gg 1$.

Rest of the analysis assumes that this is not the case, which gives us $k\phi_\pi > \sigma + \phi_y$. In this case, $\text{Det}(\mathbf{A}) = \frac{\sigma\beta}{\Omega} < 0$.

Since $\mathrm{Det}(\mathbf{A})$ is negative, the stability condition $|\mathrm{Det}(\mathbf{A})| < 1$ becomes $\mathrm{Det}(\mathbf{A}) > -1$.

$$\frac{\sigma\beta}{\Omega} > -1 \implies \sigma\beta < -\Omega \implies \sigma\beta < k\phi_\pi - (\sigma + \phi_y) \tag{B.40}$$

$$k\phi_\pi > \sigma(1+\beta) + \phi_y \tag{B.41}$$

This is a necessary baseline condition for any stable equilibrium in this region.

We now split the analysis based on the sign of the Trace. Note that because $\Omega < 0$, the sign of the fraction is opposite to the sign of the numerator.

3.3.1 Case: Non-Negative Trace ($\mathbf{Tr}(\mathbf{A}) \geq 0$) This case occurs when the numerator of the Trace is **non-positive** (since $\Omega < 0$):

$$\beta(\sigma + \phi_y) - k + \sigma \leq 0 \implies k \geq \sigma(1+\beta) + \beta\phi_y \tag{B.42}$$

When $\mathrm{Tr} \geq 0$, the binding constraint is $P(1) > 0$. To satisfy $P(1) > 0$ with $\Omega < 0$, we require the **numerator of** $P(1)$ **to be negative**:

$$(1-\beta)\phi_y - k(\phi_\pi - 1) < 0 \tag{B.43}$$

$$k(\phi_\pi - 1) > (1-\beta)\phi_y \tag{B.44}$$

3.3.2 Case: Negative Trace ($\mathbf{Tr}(\mathbf{A}) < 0$) This case occurs when the numerator of the Trace is **positive** (since $\Omega < 0$):

$$\beta(\sigma + \phi_y) - k + \sigma > 0 \implies k < \sigma(1+\beta) + \beta\phi_y \tag{B.45}$$

When $\mathrm{Tr} < 0$, the binding constraint is $P(-1) > 0$. To satisfy $P(-1) > 0$ with $\Omega < 0$, we require the **numerator of** $P(-1)$ **to be negative**:

$$(1+\beta)(2\sigma + \phi_y) - k(\phi_\pi + 1) < 0 \tag{B.46}$$

$$k(\phi_\pi + 1) > (1+\beta)(2\sigma + \phi_y) \tag{B.47}$$

Data availability and declaration of interests

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