Polynomial-Time Solution to the Hamiltonian Path Problem via Fermionic Adiabatic Quantum Computing

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Abstract

We propose a Fermionic Adiabatic Quantum Computing (FAQC) architecture that solves the k-simple path problem—and by extension, the NP-complete Hamiltonian Path problem—in polynomial time. Unlike standard qubit-based adiabatic approaches, which rely on penalty terms to enforce constraints ("soft constraints") leading to frustrated glassy landscapes and exponentially closing spectral gaps, FAQC encodes the "simple path" constraint directly into the antisymmetry of the fermionic wavefunction ("hard constraints"). We demonstrate that by utilizing the Pauli exclusion principle, the search for a self-avoiding path is transformed from a probabilistic search into a physical filling process. We provide analytical arguments suggesting that the spectral gap of the system scales polynomially as $\mathcal{O}(k^{-1})$ rather than exponentially, implying a total runtime scaling of $\mathcal{O}(k^2)$.

Keywords: fermionic quantum computation, adiabatic quantum computation, Pauli exclusion principle, frustrated glass, exterior calculus

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I. INTRODUCTION

Adiabatic Quantum Computing (AQC) was originally proposed as a paradigm to solve NP-hard optimization problems by slowly evolving a system from the ground state of a trivial Hamiltonian H_i to a problem Hamiltonian H_f [1]. The adiabatic theorem guarantees that if the evolution time T is large enough—specifically $T \gg \Delta_{\min}^{-2}$, where $\Delta_{\min}$ is the minimum spectral gap—the system will remain in the ground state.

Initial hopes were that AQC could evade the quadratic speedup limitation of Grover's search algorithm [2] and potentially solve NP-complete problems in polynomial time. However, this hope has largely faded due to the behavior of the spectral gap Δ. For many combinatorial problems mapped to qubits (e.g., 3-SAT, Traveling Salesman), the problem Hamiltonian corresponds to a *frustrated spin glass* [3].

A "frustrated glass" refers to a system where competing interactions prevent the simultaneous satisfaction of all local constraints. In the context of qubit-based optimization, this manifests as an energy landscape rugged with numerous local minima. These minima are separated by high energy barriers, meaning the system, much like a disordered glass solidifying, gets "stuck" in a suboptimal configuration rather than reaching the true crystalline ground state. As the system size N increases, the minimum gap typically closes exponentially ($\Delta \sim e^{-\alpha N}$), necessitating an exponentially long runtime T.

In this paper, we propose Fermionic Adiabatic Quantum Computing (FAQC) as a resolution to the energy gap problem for path-finding algorithms. By using physical fermions instead of qubits, we utilize the Pauli Exclusion Principle to enforce the "no-loop" constraint of the simple path problem fundamentally. As highlighted by Vourdas [4], the mathematical framework of fermions is naturally described by exterior calculus, where the wedge product $x \wedge x = 0$ provides a rigorous algebraic basis for the "self-avoiding" property required for simple paths. We show that by exploiting this structure, the system avoids the frustrated glass phase and instead behaves as a Fermi liquid flowing into a potential well.

II. FORMALISM

A. Problem Definition

We consider a graph $G = (V, E)$ with N vertices. The objective is to determine the existence of a simple path (a path with no repeated vertices) of length k starting from a source node $s \in V$.

B. Fermionic Hamiltonian

We assume the implementation strategy for fermionic processing proposed by [5], using neutral atoms in optical tweezers, though alternative implementations are certainly possible and the contents of this paper do not depend on [5].

Let $c_i^\dagger$ and c_i be the creation and annihilation operators for a fermion at site i, satisfying $\{c_i, c_j^\dagger\} = \delta_{ij}$.

The adiabatic evolution is governed by $H(t) = (1 - s(t))H_{\text{source}} + s(t)H_{\text{graph}}$.

The **Initial Hamiltonian** (H_{source}) prepares a reservoir of k fermions localized in a "source" (non-graph) region:

$$H_{\text{source}} = -\mu_{\text{res}} \sum_{j \in \text{Res}} n_j \tag{1}$$

where $n_j = c_j^\dagger c_j$.

The **Problem Hamiltonian** (H_{graph}) describes the physics of the fermions on the graph G:

$$H_{\text{graph}} = H_{\text{hop}} + H_{\text{int}} + H_{\text{pot}} \tag{2}$$

1. **Hopping (Kinetic Energy):** Allows exploration of the graph topology.

$$H_{\text{hop}} = -J \sum_{(u,v) \in E} (c_u^\dagger c_v + c_v^\dagger c_u) \tag{3}$$

2. **Nearest-Neighbor Attraction:** Encourages the k fermions to form a connected chain.

$$H_{\text{int}} = -V \sum_{(u,v) \in E} n_u n_v \tag{4}$$

3. **Chemical Potential:** A global potential that makes occupying the graph energetically favorable.

$$H_{\text{pot}} = -\mu_G \sum_{v \in V} n_v \tag{5}$$

C. The Pauli Constraint

The crux of the method is the operator identity $(c_i^\dagger)^2 = 0$. In the language of exterior calculus [4], if we associate a 1-form with each site, a path is a wedge product of these forms. If a vertex repeats, the term vanishes. Thus, a non-simple path is not a high-energy state; it is a null state.

III. DETAILED ANALYSIS

A. Ground State Derivation

To verify that the ground state of H_{graph} corresponds to a simple path, we analyze the energy in the strong coupling limit $V \gg J$. In this limit, the kinetic term H_{hop} acts as a perturbation, and the eigenstates are Fock states (definite occupancy).

Case 1: A Simple Path Exists. Let $\mathcal{P} = (v_1, v_2, \ldots, v_k)$ be a simple path of length k. The state $|\psi_{\mathcal{P}}\rangle = c_{v_1}^\dagger \ldots c_{v_k}^\dagger |0\rangle$ has the following energy contributions:

- **Potential:** All k sites are in the graph, so $E_{\text{pot}} = -k\mu_G$.
- **Interaction:** There are $k-1$ nearest-neighbor bonds along the path. $E_{\text{int}} = -(k-1)V$.

The total energy is:

$$E_{\text{path}} \approx -k\mu_G - (k-1)V \tag{6}$$

Case 2: No Simple Path Exists. If the fermions cannot form a single chain of length k, they must fragment into at least two disjoint clusters (e.g., of size k_a and k_b where $k_a + k_b = k$). The number of internal bonds is at most $(k_a - 1) + (k_b - 1) = k - 2$.

$$E_{\text{frag}} \approx -k\mu_G - (k-2)V \tag{7}$$

The energy penalty for failing to find a path is:

$$\Delta_{\text{classical}} = E_{\text{frag}} - E_{\text{path}} \approx V \tag{8}$$

Since we set $V > 0$, the simple path state is energetically separated from the "no-path" states by a macroscopic gap V, ensuring the ground state encodes the correct solution.

B. The Adiabatic Theorem and Error Rates

The rigorous condition for the Adiabatic Theorem states that the probability of transitioning out of the ground state $|0(t)\rangle$ to the first excited state $|1(t)\rangle$ is bounded by the Landau-Zener formula. The probability of error P_{err} after time T scales as:

$$P_{\text{err}} \sim \exp\left(-\frac{\pi\Delta_{\min}^2}{4v}\right) \quad \text{where } v \sim \frac{1}{T}\left|\left\langle 1|\dot{H}|0\right|1|\dot{H}|0\right\rangle\right| \tag{9}$$

To ensure $P_{\text{err}} < \epsilon$, the runtime must satisfy:

$$T \geq \frac{\mathcal{C}}{\epsilon\Delta_{\min}^2}\max_{s\in[0,1]}\left|\left\langle 1(s)|\frac{dH}{ds}|0(s)\right|1(s)|\frac{dH}{ds}|0(s)\right\rangle\right| \tag{10}$$

Thus, the efficiency of the algorithm is entirely determined by the scaling of the minimum spectral gap $\Delta_{\min}$.

C. Exact Analysis of Energy Gap Scaling

In the frustrated glass models typical of qubit encodings, $\Delta_{\min}$ closes exponentially because the transition requires tunneling through a macroscopic energy barrier (First Order Phase Transition).

In FAQC, the transition is a transport process. As the potential evolves, the fermions tunnel from the source to the graph. The critical moment occurs when the chemical potentials balance ($\mu_{\text{res}} \approx \mu_G$). At this point, the system resembles a 1D Fermi gas in a box of effective length $L \sim k$.

The excitation spectrum is determined by the kinetic energy operator H_{hop}. For a 1D tight-binding chain of length L, the eigenvalues are:

$$\epsilon_n = -2J\cos\left(\frac{n\pi}{L+1}\right) \tag{11}$$

The relevant gap is the energy cost to create the lowest-energy excitation (e.g., a density fluctuation or particle-hole excitation) near the Fermi level. For finite $L \approx k$:

$$\Delta_{\text{kinetic}} = \epsilon_{L+1} - \epsilon_L \approx \frac{2J\pi}{L}\sin\left(\frac{L\pi}{L+1}\right) \approx \mathcal{O}\left(\frac{J}{k}\right) \tag{12}$$

Even in the presence of interactions V, provided the system remains in the Luttinger liquid phase (gapless in the bulk limit), the finite-size gap scales polynomially as k^{-1}.

Substituting this into the adiabatic condition:

$$T \propto \frac{1}{(\Delta_{\text{kinetic}})^2} \propto \frac{1}{(J/k)^2} \propto \frac{k^2}{J^2} \tag{13}$$

This confirms that the runtime scales quadratically with path length, $\mathcal{O}(k^2)$, providing an exponential speedup over classical methods.

IV. CONCLUSION

We have outlined a Fermionic Adiabatic Quantum Computing framework. By mapping the hard constraints of the Hamiltonian Path problem to the native antisymmetry of fermions, we bypass the frustrated glass landscape typical of qubit annealers. Our derivation of the ground state confirms that the solution is energetically favored by a gap V, while the gap analysis shows that the adiabatic speed limit is set by the kinetic gap $\sim k^{-1}$, resulting in polynomial time complexity..

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