Probabilistic beliefs on strategy theories (commitment against discretion) versus beliefs on theory-conditional strategies: on the chainstore paradox

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Abstract

The induction hypothesis (IH) of the chainstore paradox is that if the monopolist always adopts the cooperation strategy for competitor x, then it also has to adopt the same strategy for competitor x-1. IH holds whenever commitments unconditional on competitor actions are always not credible. However, it is exactly the question of whether commitments can ever be credible that the chainstore paradox asks, so we cannot assume it away. The question then shifts to what probabilistic credence competitor x (or the monopolist itself) would assign for the monopolist utilizing the commitment theory against competitor x. The resulting meta induction hypothesis (meta-IH) is different from the original IH and justifies a (probabilistic) use of the commitment theory when the monopolist is far away from the last round. The meta-IH is consistent with the original IH whenever the monopolist invokes the discretionary theory and especially for the last rounds. This does not require incomplete information, resource constraints, epistemic blindspots, subgame perfection breakdown or bounded rationality, and the only assumption needed is that competitor actions are rationally influenced by what they believe as expected monopolist responses.

Keywords: commitment versus discretion, chainstore paradox, backward induction, meta-rationality, ensemble theory, subgame perfection

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I. INTRODUCTION

Suppose that monopolist M owns N stores in different towns. M faces N potential competitors CP_i, with one in each town, with $1 \leq i \leq N$ (with time $t = i$), sequentially. CP_i chooses their action first before M decides.

At each time i (that is M facing CP_i), the payoff goes for each action pair:

$$f(In, A) = (0, 0), \ f(In, C) = (10, 10), \ f(Out, A/C) = (8, 10^8) \tag{1}$$

where In, Out refers to CP_i's action to enter or drop out of the market, and C, A refers to M's action to cooperate or to be aggressive.

The conventional backward induction solution to the above chainstore problem says that CP_i should go for In and M should cooperate at all times. However, the monopolist adopting the deterrence strategy can convince competitors of its action by going aggressive at first few times, thereby getting a significantly higher payoff in the end. This is paradoxical and thus the name 'chainstore paradox.' [1]

In the preceding works, the deterrence strategy is justified in terms of incomplete information [2] (or asymmetric information on the structure of the game [3]), the breakdown of common knowledge of rationality [4, 5] and the KK principle breakdown [6] (that is, limitations of 'knowing means knowing that it is known'), bounded rationality, epistemic blindspots [7] or resource constraints [8]. However, it is *untrue* that backward induction and time consistency (or subgame perfection) are incompatible with the deterrence strategy, which is what this paper sets out to prove. For backward induction, it is the meta backward induction hypothesis that is unreasonable, not the (meta) backward induction itself.

Note that this paper may initially seem similar to [3] but it is not. The main driving logic of [3] is asymmetric information between the monopolist and competitors on the very structure of the game (or the strategy of the monopolist). There is no asymmetric information utilized in this paper and operates perfectly on under perfect information and rational expectations. The formulation in this paper, in particular the meta induction hypothesis (meta-IH) in Equation (3), is actually closer to the sorites paradox.

More precisely, this paper concerns a meta induction hypothesis (meta-IH) against a usual induction hypothesis (IH). The IH of the chainstore problem is consistent with the meta-IH and holds whenever the discretion theory - the monopolist optimizing its decision

conditional on competitor responses - is used, especially at the last rounds. However, the IH does not apply whenever the monopolist commits to a particular strategy profile, which can be probabilistic but is not dependent on agent actions - that is, whenever the monopolist invokes the commitment theory. The meta-IH (Equation (3)) suggests that there can be a boundary where it is no longer credible to choose the commitment theory, and this is why many confusions arise.

II.   THE META INDUCTION HYPOTHESIS AND THE INDUCTION HYPOTHESIS OF THE CHAINSTORE PROBLEM

A.   Conventional induction hypothesis

The conventional induction hypothesis of the chainstore problem goes as follows.

$$R_{Disc,t}(C) \Rightarrow R_{Disc,t-1}(C) \tag{2}$$

where $R_{Disc,t}$ refers to the 'rational under the discretion theory at time t' predicate, and C refers to the cooperation action of the monopolist. Since $R_{Disc,N}(C)$ is true, where N is the last stage, at all t, $R_{Disc}(C)$ holds.

B.   Meta induction hypothesis (meta-IH)

The meta induction hypothesis (meta-IH) instead states:

$$MR_t(P_{Commit,t} = c_t) \Rightarrow MR_{t+1}(P_{Commit,t+1} = \max(0, c_t - \alpha_t)) \quad (\alpha_t > 0) \tag{3}$$

where probability $P_{Disc,t} = 1 - c_t$ and MR refers to meta-rational or credible. The commitment theory, which subscript $Commit$ refers to, has the monopolist simply committing to a particular strategy regardless of the competitor action. For simplification, we can think of the commitment strategy as being A (Aggressive) for the monopolist.

The very reason why the paradox can occur for the chainstore paradox is that agents can 'credibly' believe $c_t \gg 0$ and even $c_t \approx 1$ for early $t \approx 0$ but can only credibly believe $c_t \approx 0$ for late t. The meta-IH is consistent with such an expectation. We can simplify further and assume $\alpha_t = \alpha$. As long as N is not too large, $P_{Commit,t} = 0$ for $t \geq t_{boundary}$ under the meta-IH.

Under $(P_{Commit,t} = 0) \equiv (P_{Disc,t} = 1)$, the conventional IH continues to hold, and therefore the meta-IH is consistent with the conventional IH as well.

C. Probability over theories versus probability over actions

If this were the case, why does the chainstore paradox look so problematic? This is because we are not properly distinguishing strategy theories against strategy profiles or actions under a particular theory. The conventional approach to the chainstore paradox has been similar to stating that central banks can never commit to some rule because they will be compelled to run discretionary actions, with all the monetary policy analysis carried out as if central banks are completely discretionary. But 'never' is too strong - central banks in reality do commit to some rule (even when initially disadvantageous) for some time and then gradually drift toward discretion. Furthermore, it does not make much sense to compare the same action (or policy) under different theories (commitment rules versus discretionary theory [9]) in the same analysis.

In fact, for the chainstore problem, commitments are much more credible than central bank rules. The meta-IH already assumes that $P_{commit,t} = 0$ at late times, so the whole deterrence (or aggression) meta-strategy of monopolist M is already built on the premise of M not being able to commit at late times.

D. A counter backward meta-IH

We can formulate a 'counter' backward meta-IH to see why the chainstore problem seems so problematic and yet it is not:

$$MR_t(P_{Commit,t} = 0) \Rightarrow^? MR_{t-1}(P_{Commit,t-1} = 0) \tag{4}$$

Indeed this is basically the sorites paradox, and if the predicates were $R_{Disc,t}$ instead of MR_t and $P_{Commit,t} = 0$ replaced with C, then we obtain back the same chainstore paradox. However, MR_t and $R_{Disc,t}$ ask different questions. MR_t concerns whether a competitor at time t can credibly believe that monopolists have non-zero chance of deploying the commitment theory over the discretionary theory. And we cannot simply assume away by relying on the discretionary theory to say that there is zero chance, since this is exactly the question we set out to respond to.

According to the meta-IH of Equation (3), the counter-meta-IH of Equation (4) only holds for late times and not for early times. In the language of the sorites paradox, α (or α_t to be more general) allows determination of the boundary $t = t_{boundary}$ where the commitment theory ceases to be credible and discretionary backward induction fully applies.

The breakdown of the counter-meta-IH does not suggest that backward induction has limitations and is consistent with Robert J. Aumann's defense of backward induction under common knowledge of rationality [4]. It is only the 'hypothesis' that does not hold, and backward induction is perfectly fine. The counter-meta-IH simply points out a common trap: some outcome having zero probability does not mean a nearby outcome has zero probability however small differences might be.

In this sense, the whole deterrence meta-strategy may surprisingly be too meta-rational such that in case that epistemic and cognition limitations (or incomplete information, computability limitations) are introduced, the meta-strategy may be difficult to hold onto. If agents infer backward in time and treat small probability as essentially zero probability, then their reasoning will follow the counter-meta-IH to make the meta-rational deterrence strategy impossible. This is an opposite conclusion to the conventional approaches to justifying the deterrence (meta-)strategy.

E.   Imbalance of commitment powers between the monopolist and competitors

The chainstore paradox is actually different from other backward induction paradoxes in the sense of the following property: there is an imbalance between competitor CP_i and M in terms of an ability to commit to some particular action regardless of each other's action. This arises because CP_i only has one chance. Therefore, CP_i alone does not have the power to force M's action to respond to the expected (meta-)behavior of CP_i.

In the single chance CP_i has, CP_i has to make an optimal decision based on the expected (meta-)behavior of M, which means that it cannot simply choose In and expect M to adopt the discretionary policy and respond optimally as in a single-period problem.

1. Temporal directions

Indeed, what CP_i has to do is to learn (or infer) parameters c_t (which subsumes α_t) from the past results at $t < i$. This does not mean that the model is intrinsically backward-looking and not forward-looking. The model only appears backward-looking (inferring from the past) because we have the zero lower bound on $P_{commit,t}$ such that we cannot properly invoke the following backward-meta-IH (b-meta-IH) for all times t including the $c_t = 0$ case:

$$MR_t(P_{Commit,t} = c_t) \Rightarrow MR_{t-1}(P_{Commit,t-1} = c_t + \alpha_{t-1}) \quad (c_t \geq 0, \alpha_t > 0) \tag{5}$$

But if N is close to infinity and α_t is such that $\lim_{t\to\infty} P_{Commit,t} = 0$ but $P_{commit,t} \neq 0$ for all times t, then $c_t \neq 0$ and the b-meta-IH of Equation (5) holds safely such that inference can be done either backward in time or forward in time.

However, in terms of parameter estimation, as with even forward-looking rational expectation dynamic stochastic general equilibrium models (forward-looking DSGE models [10]), learning has to be done from the historical data. In case of the chainstore problem, this means competitor CP_i inferring parameters from how monopolist M actually acts at $t < i$.

The topic of parameter estimation gives an illusion that the model relies on asymmetric information. This is not the case, since even if all competitors CP_i already know the parameters perfectly, the same conclusion holds - that the deterrence meta-strategy is possible.

III. CONCLUSION

The analysis of this paper suggests that incomplete information [2] (or asymmetric information on the structure of the game [3]), the breakdown of common knowledge of rationality [4, 5] and the KK principle breakdown [6], bounded rationality, epistemic blindspots [7] or resource constraints [8] are not necessary to justify the deterrence (meta-) strategy in the chainstore paradox. Indeed, a focus on the deterrence 'strategy' gives us misleading pictures, and the commitment theory versus the discretionary theory is the right way to frame the problem. This is also not all-or-nothing, as it is possible for agents to believe that some agent will credibly commit and then switch to discretion later, which happens frequently in the real world by governments or central banks around the world. And agents can have the full information on this theory-switching behavior and respond optimally.

In terms of distinguishing probability over theories against probability over strategies conditional on one theory, a good example exists in black hole physics. The recent Jackiw-Teitelboim/random matrix theory (JT/RMT) duality with replica wormholes [11–13] suggests that some quantum (effective) field theory under classical gravity is an ensemble theory of actual theories instead of an actual theory itself. We do not need to go over the details of the duality here, and a simple example suffices.

Suppose there are multiple theories (which dictate evolution of the black hole state) compatible with same classical gravity, and it is infeasible to distinguish these theories. An ensemble theory gives us an 'average' ('coarse') state when evolved by these theories. However, only one of the theories is the actual theory. This is not problematic when multiple theories produce largely similar states but become problematic under black hole evaporation where states start to diverge for different theories. Under this circumstance, even if each theory has similar black hole entanglement entropy evolution (entropy curves), the entropy results in the ensemble theory will be very different from the actual theory's entanglement entropy, because the ensemble theory calculates the entropy of the probabilistic ensemble, not the actual state. In terms of final black hole entanglement entropy function S of black hole states ρ, where $\mathbb{E}$ is the expectation operator:

$$\mathbb{E}_{theories}[S(\rho_{final})] = 0 \neq S(\mathbb{E}_{theories}[\rho_{final}]) \tag{6}$$

The circumstance is identical here for the chainstore paradox as well, though in a reverse manner. The problem now is that competitor CP_i does not know in advance which theory monopolist M will certainly use at each time. Nor does M know certainly, since it adopts a probabilistic meta-strategy profile. M only knows its probability of choosing a theory at each time t. Therefore, more important probability (entropy) calculations reside in what we can call as the ensemble theory, instead of an actual individual theory.

DATA AVAILABILITY AND DECLARATION OF INTERESTS

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[1] Reinhard Selten, "The chain store paradox," Theory and Decision **9**, 127–159 (1978).

[2] David M Kreps and Robert Wilson, "Reputation and imperfect information," Journal of Economic Theory **27**, 253–279 (1982).

[3] Paul Milgrom and John Roberts, "Predation, reputation, and entry deterrence," Journal of Economic Theory **27**, 280–312 (1982).

[4] Robert J. Aumann, "Backward induction and common knowledge of rationality," Games and Economic Behavior **8**, 6–19 (1995).

[5] Cristina Bicchieri, "Self-refuting theories of strategic interaction: A paradox of common knowledge," Erkenntnis **30**, 69–85 (1989).

[6] Timothy Williamson, *Knowledge and its Limits* (Oxford University Press, 2000).

[7] Roy Sorenson, *Blindspots* (Oxford University Press, 1988).

[8] Jean-Pierre Benoit, "Financially constrained entry in a game with incomplete information," RAND Journal of Economics **15**, 490–499 (1984).

[9] Finn E. Kydland and Edward C. Prescott, "Rules rather than discretion: The inconsistency of optimal plans," Journal of Political Economy **85**, 473–491 (1977).

[10] Finn E. Kydland and Edward C. Prescott, "Time to build and aggregate fluctuations," Econometrica **50**, 1345–1370 (1982).

[11] Geoff Penington, Stephen H. Shenker, Douglas Stanford, and Zhenbin Yang, "Replica wormholes and the black hole interior," Journal of High Energy Physics **2022**, 205 (2022).

[12] Phil Saad, Stephen H. Shenker, and Douglas Stanford, "JT gravity as a matrix integral," arXiv e-prints (2019), arXiv:1903.11115 [hep-th].

[13] Ahmed Almheiri, Thomas Hartman, Juan Maldacena, Edgar Shaghoulian, and Amirhossein Tajdini, "Replica wormholes and the entropy of hawking radiation," Journal of High Energy Physics **2020**, 13 (2020).