Probabilistic Beliefs on Strategy Theories and Indexical Uncertainty: An Ensemble Approach to the Chainstore and Sleeping Beauty Paradoxes

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Abstract

The induction hypothesis (IH) of the chainstore paradox relies on the assumption that commitments unconditional on competitor actions are never credible. However, this assumes away the very question the paradox asks. We propose a solution where competitors assign probability not to specific strategies, but to the decision-theoretic model (Commitment vs. Discretion) the monopolist employs. This results in a Meta Induction Hypothesis (Meta-IH) that governs the decay of commitment credibility over time. Unlike the reputation models of Milgrom and Roberts (1982), our approach does not rely on exogenous asymmetric information or irrational agent types. We then extend this framework to the Sleeping Beauty Problem (SBP), demonstrating that the Thirder and Halfer positions correspond to an ensemble theory view (time-averaged uncertainty) versus an actual theory view (post-collapse realized state), analogous to entanglement entropy debates in black hole physics.

Keywords: commitment versus discretion, chainstore paradox, sleeping beauty problem, ensemble theory, backward induction, subgame perfection, reputation equilibrium

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I. INTRODUCTION

Suppose that a monopolist M owns N stores in different towns. M faces N potential competitors CP_i, with one in each town, indexed sequentially $1 \leq i \leq N$ (where time t corresponds to the index i). The game unfolds sequentially: in each period t, competitor CP_t moves first, deciding whether to enter the market, followed by M's response.

The payoff structure for each stage game is defined explicitly for each action pair:

$$\Pi(In, A) = (0, 0), \quad \Pi(In, C) = (10, 10), \quad \Pi(Out, \cdot) = (8, 10^8) \tag{1}$$

where In and Out denote CP_t's decision to enter or drop out of the market, respectively. For the monopolist M, C refers to the action to Cooperate (accommodate the entrant), and A refers to the action to be Aggressive (fight the entrant).

The conventional backward induction solution dictates that CP_i should always play In and M should always play C. However, empirical intuition suggests that M can deter entry by playing A in the first few rounds to build a reputation, yielding a significantly higher payoff in the end. This contradiction is the 'chainstore paradox' [1].

In preceding works, deterrence is justified via incomplete information [2] (or asymmetric information on the structure of the game [3]), the breakdown of common knowledge of rationality [4, 5], the KK principle breakdown [6] (limitations of 'knowing means knowing that it is known'), bounded rationality, epistemic blindspots [7], or resource constraints [8]. However, it is *untrue* that backward induction and subgame perfection are incompatible with the deterrence strategy. For backward induction, it is the meta backward induction hypothesis that is unreasonable, not the backward induction itself.

A. The Ensemble Approach: Probability over Theories

Standard economic modeling assumes agents optimize a specific utility function subject to constraints. However, in contexts involving long time horizons, we adopt an approach analogous to ensemble theories in statistical mechanics or quantum field theory.

In this framework, the monopolist operates as a superposition of two distinct decision-theoretic modes:

1. **The Discretionary Theory (T_{Disc}):** The agent optimizes payoffs at every node t conditional on competitor responses.

2. **The Commitment Theory (T_{Commit}):** The agent commits to a strategy profile ex-ante (e.g., always play A) and ignores sequential optimality constraints.

Unlike a mixed strategy over actions, the agent operates under a specific *theory* with probability c_t.

B. The Benchmark: 'Crazy Types' vs. Structural Uncertainty

The dominant resolution [3] relies on assuming a non-zero prior that M is a 'crazy type' - an exogenous, irrational automaton that always plays A. While this generates deterrence, it is fragile: if the probability of the 'crazy type' is exactly zero, deterrence completely collapses.

Our formulation does not utilize asymmetric information about irrational types. It operates perfectly under complete information and rational expectations. The uncertainty lies purely in the *decision rule* being employed by a rational actor. The 'Meta-IH' proposed below is formally closer to the Sorites paradox than to a signaling game.

II. THE META INDUCTION HYPOTHESIS VS. THE STANDARD INDUCTION HYPOTHESIS

A. Conventional Induction Hypothesis (IH)

The conventional induction hypothesis of the chainstore paradox is:

$$R_{Disc,t}(C) \Rightarrow R_{Disc,t-1}(C) \tag{2}$$

where $R_{Disc,t}$ is the 'rational under the discretion theory at time t' predicate. Since $R_{Disc,N}(C)$ is true at the final stage N, $R_{Disc}(C)$ holds for all t.

B. Meta Induction Hypothesis (Meta-IH)

The Meta-IH instead governs the credibility of the commitment theory itself:

$$MR_t(P_{Commit,t} = c_t) \Rightarrow MR_{t+1}(P_{Commit,t+1} = \max(0, c_t - \alpha_t)) \quad (\alpha_t > 0) \tag{3}$$

where probability $P_{Disc,t} = 1 - c_t$, and MR refers to meta-rational (or credible). α_t represents the decay rate of commitment credibility.

The paradox occurs because agents can credibly believe $c_t \approx 1$ for early $t \approx 0$ but can only believe $c_t \approx 0$ for late t. As long as N is not infinite, there exists a boundary $t_{boundary}$ such that $P_{Commit,t} = 0$ for $t \geq t_{boundary}$. Crucially, when $P_{Commit,t} = 0$, the conventional IH (Equation 2) continues to hold.

C. Probability over Theories versus Probability over Actions

Why does the paradox look so problematic? Because we fail to distinguish strategy theories from actions under a particular theory. The conventional approach is akin to stating that central banks can *never* commit to a monetary rule because they will eventually be compelled to use discretionary actions [9]. But 'never' is too strong; central banks do commit to rules for some time before drifting toward discretion. Furthermore, it is a category error to compare an action under a commitment rule directly against an action under a discretionary theory within the exact same analytical layer.

D. A Counter Backward Meta-IH and the Sorites Paradox

We can formulate a 'counter' backward Meta-IH to see why the problem appears paradoxical:

$$MR_t(P_{Commit,t} = 0) \Rightarrow^{?} MR_{t-1}(P_{Commit,t-1} = 0) \tag{4}$$

This is effectively the Sorites paradox (the paradox of the heap). MR_t asks whether a competitor can credibly believe M has a non-zero chance of deploying the commitment theory.

The breakdown of this counter-Meta-IH does not mean backward induction is flawed. This is consistent with Aumann's (1995) defense of backward induction under common knowledge of rationality [4]. It is only the *hypothesis* (that zero probability at t implies zero probability at $t-1$) that fails. The parameter α_t determines the boundary where commitment ceases to be credible. If agents possess cognitive limitations and treat small probabilities as strictly zero, they will falsely follow the counter-Meta-IH, making deterrence impossible.

E. Imbalance of Commitment Powers and Temporal Directions

Unlike M, the competitor CP_i only plays once and thus lacks the power to force M to respond to a meta-strategy. CP_i must make an optimal decision based on M's expected meta-behavior.

What CP_i must do is infer the parameters c_t (and α_t) from M's past actions at $t < i$. This does not make the model intrinsically backward-looking. As with forward-looking dynamic stochastic general equilibrium (DSGE) models [10], rational expectation parameter estimation must be done from historical data. The need for parameter estimation gives the illusion of asymmetric information, but it is not. Even if all CP_i know the parameters perfectly, the deterrence meta-strategy remains viable.

III. THE SLEEPING BEAUTY PROBLEM: ENSEMBLE VS. ACTUAL THEORY

The distinction between theories versus conditional actions maps directly onto the Sleeping Beauty Problem (SBP) [11], providing a resolution between Halfers and Thirders.

A. Setup and the Ensemble View (Thirder)

A fair coin is tossed on Sunday ($P(H) = 1/2$). If Heads (H), Beauty wakes Monday. If Tails (T), she wakes Monday and Tuesday with amnesia.

The Thirder argument ($P(H) = 1/3$) corresponds to the **Ensemble Theory** view. The ensemble theory ignores that the coin toss has already collapsed into a single outcome, tracking the expected evolution of the system over time. In the bra-ket quantum formalism, the ensemble theory Hamiltonian H_{ens} goes as:

$$H_{ens}(t) = |H\rangle\langle H| \otimes H_{head}(t) + |T\rangle\langle T| \otimes H_{tail}(t)$$

Calculating the time-averaged probability of state $|H\rangle$ conditioned on $|Awake\rangle$:

$$P_{time}(H|Awake) = \lim_{N\to\infty} \frac{\#HeadMon}{(\#AwakeMon + \#AwakeTues)} = \frac{1}{3}$$

This is the correct credence for the time-averaged entanglement of the experimental design.

B. The Actual Theory View (Halfer)

The Halfer argument ($P(H) = 1/2$) corresponds to the **Actual Theory** view [12]. The coin toss on Sunday selects exactly one actual theory (H_{head} or H_{tail}). Because there is no entanglement between the coin and Beauty's current mind (due to amnesia), the observation $|Awake\rangle$ provides no information to update her prior.

$$P(H|Awake) = 1/2$$

C. Black Hole Physics Inspiration

This distinction is inspired by the Jackiw-Teitelboim/Random Matrix Theory (JT/RMT) duality [13, 14]. In quantum gravity, an ensemble theory calculates the entropy of a probabilistic ensemble of universes, which differs from the actual state:

$$\mathbb{E}_{theories}[S(\rho_{final})] \neq S(\mathbb{E}_{theories}[\rho_{final}]) \tag{5}$$

In SBP, observers on Sunday (before the toss) should use the ensemble view (1/3). But Beauty, living inside the realized universe post-toss, should use the actual theory view (1/2). Previous quantum treatments of SBP (e.g., Many-Worlds approaches) force the coin to remain quantum, mathematically demanding the Thirder view. However, in the standard macroscopic setup where the coin collapses on Sunday, the Halfer view correctly describes the actual theory.

IV. CONCLUSION

By distinguishing between probability over theories (Ensemble) and probability over actions within a theory (Actual), we resolve two major paradoxes. In the Chainstore Paradox, competitors face an ensemble of decision theories, validating deterrence without requiring irrational types or abandoning backward induction. In Sleeping Beauty, the conflict disappears once we clarify whether the agent is measuring the time-averaged ensemble of the experimental design (Thirder) or the post-collapse actualized state of the universe (Halfer).

DATA AVAILABILITY AND DECLARATION OF INTERESTS

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