Polynomial-Time Solution to the Hamiltonian Path Problem via Reptation-Based Fermionic Adiabatic Quantum Computing

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Abstract

We propose a Fermionic Adiabatic Quantum Computing (FAQC) architecture that solves the Hamiltonian Path problem in polynomial time. Unlike qubit-based approaches that rely on soft energy penalties leading to frustrated glassy landscapes and exponentially closing gaps, FAQC encodes the simple path constraint directly into the antisymmetry of the fermionic wavefunction (Pauli Exclusion Principle). However, standard fermionic models suffer from a fundamental trade-off: weak interactions lead to path fragmentation, while strong interactions lead to dynamical freezing (the "sticky chain" problem). We resolve this dilemma by introducing a **Correlated Hopping (Reptation) Hamiltonian**. Inspired by polymer physics, this 3-body interaction term facilitates zero-energy "corner-turning" moves, allowing the fermionic chain to reconfigure its geometry without breaking bonds. We argue that this restores ergodicity within the manifold of connected paths, effectively opening a "kinetic tunnel" through the energy barriers and recovering polynomial runtime scaling.

Keywords: fermionic quantum computation, adiabatic quantum computation, reptation, correlated hopping, Pauli exclusion principle

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I. INTRODUCTION

Adiabatic Quantum Computing (AQC) offers a potential route to solving NP-hard optimization problems by evolving a system from a trivial ground state to a problem-encoding ground state [1]. However, for NP-complete problems mapped to spin glasses (e.g., Ising models), the minimum spectral gap $\Delta_{\min}$ typically vanishes exponentially with system size N, requiring exponential runtime $T \sim \mathcal{O}(e^N)$ [2].

This exponential slowdown is often due to frustration: competing constraints create a rugged energy landscape full of local minima. In this paper, we propose a fundamentally different encoding using fermions. We exploit the Pauli Exclusion Principle (PEP) to enforce the no-loop constraint of the Simple Path problem as a hard physical constraint, utilizing the exterior calculus identity $c_i^\dagger \wedge c_i^\dagger = 0$ [3].

We identify a critical failure mode in naive fermionic implementations - the Sticky Chain dilemma - and propose a resolution based on Reptation Dynamics (correlated hopping), enabling the system to find the solution via a fluid-like flow rather than a glassy search.

II. FORMALISM

A. Problem Definition

Given a graph $G = (V, E)$, we seek a simple path of length k starting from a source s. We utilize spinless fermions on the graph and a connected reservoir.

B. The Hamiltonian

We employ a Constant-Kinetic Schedule. The Hamiltonian is $H(t) = H_{\text{kin}} + H_{\text{pot}}(t)$. Crucially, the kinetic term is augmented with a reptation term to prevent glassiness.

1. 1. Standard Kinetic Terms

$$H_{\text{std}} = -J \sum_{(u,v)\in E} (c_u^\dagger c_v + h.c.) - V \sum_{(u,v)\in E} n_u n_v \tag{1}$$

Here, J is the hopping amplitude, and $V > 0$ is a strong attractive interaction ensuring the fermions form a connected chain rather than fragmented clusters.

2. 2. The Reptation Term (Correlated Hopping)

To allow the chain to reconfigure without breaking bonds, we introduce a 3-body interaction acting on all 2-paths (triangles or open wedges) $i - j - l$:

$$H_{\text{rep}} = -K \sum_{j \in V} \sum_{i,l \in \mathcal{N}(j)} \left(c_l^\dagger c_i n_j + h.c. \right) \tag{2}$$

This term annihilates a particle at i and creates one at l *only if the common neighbor* j *is occupied*. This represents a pivot move around the anchor j.

3. 3. Driving Potential

The evolution is driven by chemical potentials:

$$H_{\text{pot}}(t) = -\mu_{\text{res}}(t) \sum_{j \in \text{Res}} n_j - \mu_G(t) \sum_{v \in V} n_v \tag{3}$$

following the linear ramp $\mu_{\text{res}} \to 0$ and $0 \to \mu_G$.

III. PHYSICAL ANALYSIS

A. The Sticky Chain Dilemma

Without the reptation term ($K = 0$), the system faces a fatal trade-off:

- **Fragmentation Regime ($J \gg V$):** The gap is large (polynomial), but the ground state is a gas of disconnected particles, not a path.
- **Glassy Regime ($V \gg J$):** The ground state is a connected path, but the dynamics freeze. To escape a local minimum (e.g., a wrong path entering a dead end), the chain must retract, breaking a bond. This costs energy V. The transition rate becomes Arrhenius-like, $\Gamma \sim e^{-V}$, leading to an exponentially closing gap.

B. Resolution via Reptation

The reptation term H_{rep} resolves this by creating a kinetic tunnel through the energy barrier. Consider a chain segment $\cdots - j - i$. Moving particle i to a neighbor l usually breaks the $i - j$ bond. However, if l is also connected to j, the operator $c_l^\dagger c_i n_j$ moves the particle while *preserving* the bond with j ($\cdots - j - l$).

- **Energy Conservation:** Since the number of bonds is preserved, $\Delta E_{\text{int}} = 0$.
- **Ergodicity:** This pivot move allows the chain to slither (reptate) through the graph. The head of the path can retract and turn corners without ever paying the energy penalty V.

IV. GAP SCALING AND RUNTIME

A. Connectivity as a Conserved Quantity

In the limit $V \to \infty$ and $K > 0$, the low-energy Hilbert space $\mathcal{H}_{\text{connected}}$ is separated from fragmented states by a gap $\sim V$. The effective Hamiltonian within this manifold is governed by K.

B. Mixing Time Scaling

The adiabatic runtime is related to the inverse of the spectral gap, which is physically linked to the mixing time of the underlying stochastic process. According to the reptation theory of polymers (de Gennes), the relaxation time τ of a polymer of length k confined in a network scales as a power law of its length:

$$\tau \sim k^3. \tag{4}$$

Unlike the frustrated spin glass, where reconfiguration requires global tunneling ($T \sim e^N$), the fermionic chain reconfigures via local pivot moves. Provided the graph topology does not essentially knot the path (topological jamming), the spectral gap Δ is expected to scale polynomially:

$$\Delta \sim \frac{K}{k^\alpha} \quad (\alpha \approx 3). \tag{5}$$

This implies a total adiabatic runtime $T \sim \mathcal{O}(k^{2\alpha})$, placing the solution within P.

V. CONCLUSION

We have presented a reptation-based FAQC architecture. By identifying the sticky chain failure mode of standard fermionic embeddings and correcting it with correlated hopping, we utilize the Pauli Exclusion Principle to enforce hard constraints while maintaining dynamical ergodicity. This suggests that quantum mechanical snakes can solve path-finding problems exponentially faster than classical random walkers or frustrated quantum magnets.

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