Quantum probability is not a generalization of classical probability

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Abstract

It is well-known that conditionalization properties of quantum probability and classical probability are different. A common argument is that quantum probability is a generalization of classical probability, and this conditionalization difference is just a consequence of quantum probability being more general. We argue this is not true. That fully decohered states allow for classical conditionalization does not mean quantum probability subsumes classical probability, and fully decohered states are nevertheless sustained by quantum entanglements. The real problem with classical probability is that it has no method of distinguishing whether uncertainty arises from epistemic reasons or fundamental ontic reasons, and the conventional use of conditionalization supports the view that uncertainty in classical probability is epistemic. Whenever uncertainty is ontic, classical conditionalization is implausible, and quantum probability offers a way of expressing ontic uncertainty. However, quantum probability cannot properly handle epistemic uncertainty. Quantum probability is not a generalization of classical probability.

Keywords: quantum probability, classical probability, conditionalization, Bayes rule, ontic uncertainty, quantum decoherence

I. INTRODUCTION

It is well-known that in quantum probability, the following conditionalization rule does not hold:

$$P(y) = \sum_k P(y|x_k)P(x_k) \tag{1}$$

A classic example is the double-slit experiment, where y is the location of a particle observed, and x_1, x_2 refer to the two slits. Decoherent contributions from the two slits are such that the above equation does not hold.

This is typically understood to be a consequence of quantum probability generalizing classical probability. Classcal probability is understood to be recovered from quantum probability with fully decohered states ρ_d (of system A) of the following form:

$$\rho_d = \sum_k p_k |\psi_k\rangle\langle\psi_k| \quad (\sum_k p_k = 1,\ \langle\psi_j|\psi_k\rangle = \delta_{jk},\ p_k \geq 0) \tag{2}$$

The reason why ρ_d is said to recover classical probability goes as follows. Suppose some unitary operation U is applied to system A with initial state ρ_d, resulting in ρ'_d. Then we have:

$$\rho'_d = \sum_k p_k\, U|\psi_k\rangle\langle\psi_k|U^\dagger = \sum_k p_k \sum_{a,b} b'_{k,ab}|\psi'_a\rangle\langle\psi'_b| \quad (\langle\psi'_a|\psi'_b\rangle = \delta_{ab}) \tag{3}$$

Consider only the $a = b$ case, thereby transforming ρ'_d to ρ''_d

$$\rho''_d = \sum_k p_k \sum_a p'_{k,a}|\psi'_a\rangle\langle\psi'_a| \quad (\langle\psi'_a|\psi'_b\rangle = \delta_{ab}) \tag{4}$$

where $p'_{k,a} \geq 0$ is real-valued. If we are looking for probability of a then it follows from ρ''_d that

$$P(a) = \sum_k P(a|k)P(k) \quad (P(a|k) = p'_{k,a}, P(k) = p_k) \tag{5}$$

which is consistent with classical probability.

A. Arguments in this paper

1. We argue that fully decohered states ρ_d do not model classical probability in quantum probability. This is mainly due to ρ_d relying on quantum entanglements.

2. Quantum probability has difficulties in modeling epistemic uncertainty.

3. Classical probability has difficulties in modeling ontic uncertainty.

4. Classical and quantum probability serve different purposes, and classical probability is not a specific instance of quantum probability.

We also note that these arguments are unrelated to the question of ψ-epistemic versus ψ-ontic interpretations of quantum mechanics [1], with 'ontic' against 'ψ-ontic' properly distinguished.

II. QUANTUM PROBABILITY IS INSUFFICIENT TO MODEL EPISTEMIC UNCERTAINTY, CLASSICAL PROBABILITY IS INSUFFICIENT TO MODEL ONTIC UNCERTAINTY

A. Reversibly irreducible uncertainty as ontic uncertainty and ψ-ontic against ontic

In this paper, we distinguish ψ-ontic from ontic as follows.

- ψ-ontic refers to the notion that quantum states ψ or ρ are knowledge states - they reflect the knowledge of the universe, instead of the fundamental reality.
- The notion of 'ontic', as used in this paper, describes reversibly irreducible uncertainty.

Reversibly irreducible uncertainty refers to the following case. Suppose we have pure state $|\psi(0)\rangle$ at $t=0$. This state evolves to $|\psi(T)\rangle = \sum_k a_k|\psi_k\rangle$. When $|\psi(T)\rangle$ is measured in the $|\psi_k\rangle$ basis, we get probability $|a_k|^2$.

$$|\psi(0)\rangle \to |\psi(t)\rangle = \sum_k a_k|\psi_k\rangle \tag{6}$$

Now suppose we can go back in time from $t=T$ to $t=0$ with this observation, and assume that the evolution is replayed.

Now different interpretations of quantum mechanics will assign different probabilities to $|\psi_k\rangle$ at $t=T$, with some ψ-epistemic interpretations applying probability of 1 to the previously observed outcome.

But suppose that, following traditional interpretations of quantum mechanics, including many-worlds or Everettian interpretations [2], that the same $P(|\psi_k\rangle) = |a_k|^2$ must still be used for the replay. Then we have cases of reversibly irreducible uncertainty - uncertainty is fundamental. Even if we replay the same evolution, while keeping all the observations, uncertainty is not reduced.

1. *In terms of a vague heap in the sorites paradox*

We can think of some vague heap (in the sorites paradox) instead. Suppose that vague heap VH is assumed to be 30% (likely to be a) heap and 70% (likely to be a) non-heap. Now at one point, suppose that VH is referred to as being a heap. Does this mean VH is now forever a heap? This is likely not the case, and it would still be possible that VH may be referred to as a non-heap. That is, 30%-70% uncertainty is not really reduced by one observation of VH called as a heap.

2. *Ontic uncertainty within ψ-epistemic interpretations*

That $|\psi(T)\rangle$ is observed as $|\psi_k\rangle$ does not reveal that $|\psi(0)\rangle = U^\dagger|\psi_k\rangle$, where U refers to the unitary propagator from $t = 0$ to $t = T$. This can even be understood in terms of ψ-epistemic interpretations.

Recall that in ψ-epistemic interpretations, ψ is a knowledge state. Therefore that ψ has reversibly irreducible uncertainty only implies that knowledge states obtained by replays are never the same. After each replay, knowledge I have about the universe is refreshed to something different, though observations might actually share the same fundamental reality.

Unless ψ-epistemic interpretations additionally demand that every such replay results in the same knowledge state, ψ-epistemic interpretations of quantum mechnics are compatible with ontic uncertainty (= reversibly irreducible uncertainty).

3. *Epistemic uncertainty*

In this paper, epistemic uncertainty refers to the case where an observation of $|\psi(T)\rangle$ as $|\psi_k\rangle$ establishes that evolution replays also produce $|\psi_k\rangle$ as observation results. In other words, epistemic uncertainty is irreversibly reducible uncertainty.

The notions of epistemic uncertainty and ontic uncertainty used in this paper are in the far extremes, and it is definitely possible to have a notion of uncertainty in between. Because such a generalization does not serve much purpose in this paper, we do not discuss it further.

B. Fully decohered states do not model classical probability

$$\rho_d = \sum_k p_k |\psi_k\rangle\langle\psi_k|$$

$$\rho''_d = \sum_k p_k \sum_a p'_{k,a} |\psi'_a\rangle\langle\psi'_a|$$

Despite ρ''_d (which arises from a unitary operation applied to ρ_d in Equation (2)) in Equation (4) replicating classical conditionalization, we now argue that ρ''_d cannot be used as justifying quantum probability subsuming classical probability.

The reason is extremely basic. Suppose that at $t \ll 0$ (faraway past), system A is not entangled with rest of the universe. Over time, entanglement develops such that A comes to have fully decohered ρ_d state at $t = 0$. This implies that from the universe point of view, there exists some universe-wise (though not A-wise) unitary operation U' that can reverse entanglements, thereby reversing decoherence as well. Therefore, classical conditionalization holds for ρ_d only because we assumed that unitary operations are restricted to A. Whenever this is not the case, then classical conditionalization is no longer ensured.

That is, classical probability is only ensured whenever system A does not interact with rest of the universe. The condition is often ensured in quantum mechanical contexts, and this is one reason why Everettian interpretations can (ever attempt to) claim to explain illusions of wavefunction collapse without worrying about superpositions destroying past measurement outcomes or world branching. However, this is still very restrictive from the general uncertainty modeling point of view - the recovered 'classical probability' applies restrictively for conditionalization on each $|\psi_k\rangle$ at $t = T$ *only*. For other questions, classical probability rules cannot be invoked, and quantum calculations have to be carried out.

C. Quantum probability has difficulties in modeling epistemic uncertainty

Consider mixed state ρ_d of system A in Equation (2). Some form of classical conditionalizaton holds for those states though not exactly as we have seen in Section II B.

$$\rho_d = \sum_k p_k |\psi_k\rangle\langle\psi_k|$$

Technically ρ_d is still within some quantum superposition universe-wise, and this is why classical conditionalization does not always work. This creates an additional issue: we cannot really think of ρ_d as reflecting epistemic uncertainty.

If ρ_d reflects epistemic uncertainty, and the actual state of A is one of $|\psi_k\rangle$, then the evolution from ρ_d should not be inconsistent with the evolution from $|\psi_k\rangle$. Yet whenever A does interact with rest of the universe, inconsistencies will be inevitable. Therefore, quantum probability cannot be used to model epistemic uncertainty, and we need to use classical probability. We revisit this in Section II E.

D. Classical probability has difficulties in modeling ontic uncertainty

By contrast, classical probability cannot model ontic uncertainty as we have seen in Section II A. This part is largely well-understood and is part of the general consensus. Again, suppose we have:

$$|\psi(T)\rangle = \sum_k a_k |\psi_k\rangle$$

Whenever $|\psi(T)\rangle$ is viewed as reflecting ontic uncertainty, we cannot assume classical conditionalization.

Consider the double-slit experiment. If $|\psi_0\rangle$ and $|\psi_1\rangle$ are considered two slits, then a_0, a_1 reflect ontic uncertainty. Even if we go back in time from T to $t = 0$ and then repeat the experiment, assuming ψ reflecting ontic uncertainty, then preceding observation results are useless for new replays and observations. This feature is conceptually and philosophically why classical conditionalizaton does not work in quantum probability.

E. Examples: black hole evaporation example and others

We could think of how epistemic uncertainty plays out in black hole evaporation. The framework is not restricted to black hole evaporation and is fairly general.

Suppose that there is epistemic uncertainty over quantum theories H_i that govern black hole evaporation. We can incorporate this uncertainty into the quantum formalism as an ensemble theory [3–5]. This ensemble theory works almost as an actual theory in normal contexts, except in special contexts such as black hole evaporation. Now suppose that every H_i shares the following black hole entanglement entropy S_{BH} characteristic (inspired from the Page curve [6]):

$$\frac{dS_{BH}}{dt} < 0, \quad S_{BH}(t_{final}) \approx 0 \quad (t > t_{page}, \forall H_i) \tag{7}$$

On average, calculated with classical probability after each S_{BH} is computed with quantum calculations for each H_i, $S_{BH}(t_{final})$ is also approximately zero. However, when ensemble theory calculations are carried out, since the entropy of the ensemble state is computed, we get an entirely different result in case the final state ($|\psi_{final,k}\rangle$) of black hole evaporation for each theory H_i is quite different to the point of being orthogonal.

$$S_{ensemble,BH}(t_{final}) \gg 0 \tag{8}$$

This can be seen from the following final ensemble state:

$$|\psi_{ensemble,final}\rangle = \sum_k a_k |\psi_{final,k}\rangle, \quad |a_k| \gg 0 \tag{9}$$

So quantum probability actually gives us a misleading picture whenever epistemic uncertainty is involved, and either we correct the rules of quantum probability to incorporate epistemic uncertainty ('replica wormholes' [3]) or we have to do proper classical probability calculations. In case of black hole evaporation, it is usually costly to probe each unknown actual theory, so the former is carried out.

1. *Brief digression: sleeping beauty problem, chainstore paradox*

It is also reasonable to ask whether uncertainty in different problems and paradoxes are ontic or epistemic in nature. Consider the sleeping beauty problem [7] as an example. Is the uncertainty experienced by the Beauty epistemic or ontic in nature? Is the Beauty epistemically uncertain about a determined state and theory? Or is there fundamentally irreducible (ontic) uncertainty from the Beauty point of view? These questions affect how the problem is answered and resolved, especially with regards to whether quantum mechanics is a good tool to be utilized in this context. (For the use of quantum mechanics in the sleeping beauty problem, see, though not exclusive to, [2, 8–10].)

The same goes with backward induction paradoxes like the chainstore paradox [11]. Is the monopolist allowed to have ontic uncertainty in its action - probability and uncertainty on whether commitments to deterrence regardless of competitor actions or discretionary responses to competitor actions are made? Or more generally, impose quantum entanglement with competitor actions such that commitments can be more credible [12]? Whether a use of quantum mechanics is appropriate depends on whether ontic uncertainty is actually there or not.

Since this paper does not intend to propose new solutions to these problems, we stop short of suggesting ones. However, it should be fairly obvious why a use of quantum probability for these questions must be carefully considered and re-evaluated in light of the proper distinction between epistemic uncertainty and ontic uncertainty.

III. CONCLUSION: QUANTUM PROBABILITY IS NOT A GENERALIZATION OF CLASSICAL PROBABILITY

It has conventionally been misunderstood that quantum probability is a generalization of classical probability. Quantum probability and classical probability serve different purposes - ontic uncertainty and epistemic uncertainty.

That classical probability cannot model ontic uncertainty has been well-understood. Quantum probability not modeling epistemic uncertainty has not been so recognized. Quantum probability fails epistemic uncertainty modeling, because it is not equipped to do so. Its transition to classical probability was supposed to rely on the collapse principle, and classical probability is only recovered at the moment of collapse and not before and after.

The fully decohered states ρ_d (Equation (2)) cannot fully serve as providing cases of classical probability within quantum probability, because the calculations assume that there is no future interactions between the system and rest of the universe. In case of Everettian interpretations of quantum mechanics, this can be tolerated, because 'world branching or split' can be defined as the moments when quantum coherence and interactions simultaneously die out. In the uncertainty (and statistical) setup, this luck cannot be defined into anything general.

The black hole example in Section II E further suggests why quantum probability can fail miserably in light of epistemic uncertainty. Quantum probability is therefore not a generalization of classical probability.

DATA AVAILABILITY AND DECLARATION OF INTERESTS

The author(s) have no funding source to declare. Furthermore, there is no conflict of interests.

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