The Small Corrections Theorem: Untangling Dynamic from Kinematic Corrections in the Black Hole Information Paradox

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(Dated: February 15, 2026)

Abstract

Samir D. Mathur's small corrections theorem represents a foundational constraint on resolutions to the black hole information paradox, yet its physical implications are frequently misunderstood. A common critique invokes the Hilbert space of quantum gravity and the Eigenstate Thermalization Hypothesis (ETH) to argue that since Effective Field Theory (EFT) states receive large static corrections, the theorem is rendered invalid. This paper argues that such critiques fundamentally mischaracterize the theorem's target. The core thesis of this paper is that the true burden of the information paradox lies in dynamic corrections to the Hamiltonian evolution, not merely kinematic or static corrections to the initial EFT state. Even if the initial state undergoes large kinematic corrections, the theorem holds so long as the dynamical evolution remains approximately that of semiclassical EFT. Furthermore, this paper evaluates recent prominent resolutions - including black hole complementarity, AdS/CFT, the ER=EPR conjecture, Hilbert space factorization breakdown and replica wormholes - demonstrating that all physical resolutions to the paradox implicitly concede the theorem by requiring macroscopic dynamical alterations to local effective field theory.

Keywords: small corrections theorem, black hole information problem, kinematic vs. dynamic corrections, eigenstate thermalization hypothesis, AdS/CFT, Hilbert space factorization, black hole complementarity

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I. INTRODUCTION

The black hole information paradox, since its inception by Stephen Hawking [1], has remained a central driver of high-energy and black hole physics. One of the sharpest modern formulation of this paradox is articulated by Samir D. Mathur's small corrections theorem [2]. Mathur mathematically demonstrated that if one assumes the emission of Hawking radiation follows semiclassical Effective Field Theory (EFT) to leading order, small, local ϵ-corrections to the creation of entangled pairs cannot accumulate to restore the purity of the final state.

Despite its clarity, the theorem is often dismissed or conceptually bypassed by researchers appealing to the full apparatus of quantum gravity (QG). A prevalent counter-argument asserts that QG naturally provides large corrections to the initial EFT state. Concurrently, the Eigenstate Thermalization Hypothesis (ETH) [3] is frequently invoked to suggest that while the true QG microstate differs wildly from the EFT state, they yield identical expectation values for "simple" observables. Therefore, the critique goes, any purely EFT-based result - including Mathur's theorem - is invalidated by the presence of large quantum gravitational state corrections.

This paper argues that such dismissals commit a category error, conflating *kinematic* (or static) corrections with *dynamic* corrections. The small corrections theorem does not fail simply because the initial state requires significant alteration. Instead, it asks whether the step-by-step *evolution* of the state requires macroscopic correction. If the dynamic deviation from EFT is small, the theorem holds, and unitarity is lost. Even if the initial state undergoes large kinematic corrections, the theorem holds so long as the dynamical evolution remains approximately that of semiclassical EFT. Furthermore, this paper evaluates recent prominent resolutions - including Black Hole Complementarity, AdS/CFT, the ER=EPR conjecture and replica wormholes - implicitly concede the theorem. They do so by requiring macroscopic dynamical alterations to local effective field theory, or by abandoning the local tensor product structure of spacetime entirely.

II. THE SMALL CORRECTIONS THEOREM AND THE ETH OBJECTION

In the standard EFT description, the vacuum near the horizon produces a pair of quanta: an exterior mode b_1 and an interior mode c_1. To leading order, they are maximally entangled:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{b_1} |0\rangle_{c_1} + |1\rangle_{b_1} |1\rangle_{c_1} \right). \tag{1}$$

The entanglement entropy of the early radiation $S(B_N)$ grows by $\ln 2$ with each emission. Mathur's theorem posits that if the true state includes a small correction, $|\Psi\rangle_{\text{exact}} = |\Psi\rangle_{\text{EFT}} + |\delta\Psi\rangle$ where $||\delta\Psi|| < \epsilon$, the change in entanglement entropy remains bounded:

$$S(B_{N+1}) > S(B_N) + \ln 2 - 2\epsilon. \tag{2}$$

Thus, small corrections cannot induce the Page curve turnaround required for unitary evaporation.

A common philosophical and physical objection leverages ETH and the vastness of the QG Hilbert space. Because the Bekenstein-Hawking entropy dictates an exponentially large number of microstates ($e^{S_{BH}}$), it is argued that the semiclassical EFT vacuum is merely a coarse-grained thermodynamic approximation. ETH implies that many dramatically different pure states in the underlying QG theory will reproduce the same simple observable expectation values as the EFT state. Because the true state involves $\mathcal{O}(1)$ (large) corrections to the EFT description, critics claim the ϵ-bound of the small corrections theorem is fundamentally inapplicable.

III. KINEMATIC VS. DYNAMIC CORRECTIONS

The aforementioned critique misidentifies the operational core of Mathur's argument. To see why, we must establish a rigorous distinction between correcting the state (*kinematics*) and correcting the evolution operator (*dynamics*).

We can readily concede that the initial state of the black hole, $|\Psi_0\rangle$, may be subject to arbitrarily large corrections due to QG microstate structure. We can replace the naive EFT vacuum with any highly complex QG state. However, the critical question is what happens to this state under time evolution.

Let U_{exact} be the true unitary evolution operator of quantum gravity, and U_{EFT} be the semiclassical Hamiltonian evolution governing the horizon. We can write:

$$U_{\text{exact}} \approx U_{\text{EFT}} + \delta U. \tag{3}$$

The ETH objection establishes that $|\Psi_0\rangle_{\text{exact}} \neq |\Psi_0\rangle_{\text{EFT}}$. But Mathur's theorem does not require the initial state to be purely EFT. It requires that the *action* of generating the next Hawking quantum depends on the dynamics of the horizon.

If the dynamic correction is small (at least for the purpose of analyzing entanglement entropy) - that is, if the local Hamiltonian generating the pairs only deviates from EFT by an operator norm $||\delta U|| \sim \epsilon$ over the timescale of one emission - then the bound on the entanglement entropy holds. Even if we completely replace the initial static state to align with the "right" QG microstate, a local evolution that is approximately semiclassical ($\delta U \ll 1$) will continue to generate entanglement monotonically.

Therefore, invoking ETH or QG Hilbert space complexity to argue that the EFT *state* is vastly incorrect is a non-sequitur regarding the information paradox. To save unitarity, the theorem dictates that one must drastically alter the *dynamics* ($\delta U \sim 1$). The power of the small corrections theorem lies precisely in forcing this dichotomy: static corrections are insufficient; local physics must dynamically break down at the horizon.

A. Late-Time Dynamic Divergence and the Linearity Constraint

A more subtle attempt to evade the small corrections theorem involves positing a time-dependent divergence between the exact quantum gravity Hamiltonian and the semiclassical one. One might hypothesize that for an early black hole state, the exact evolution closely tracks the semiclassical prediction: $U_{\text{exact}}|\Psi_{\text{early BH}}\rangle \approx U_{\text{EFT}}|\Psi_{\text{early BH}}\rangle$. However, as the black hole ages - specifically, after the Page time - the exact evolution diverges significantly: $U_{\text{exact}}|\Psi_{\text{late BH}}\rangle \neq U_{\text{EFT}}|\Psi_{\text{late BH}}\rangle$.

This framework is highly appealing because it preserves the success of Hawking's original derivation for young black holes while attempting to salvage unitarity at late times. However, assessing the plausibility of this evasion requires confronting the linearity of quantum mechanics and the nature of macroscopic superpositions.

A late-time black hole state is generically a massive superposition of microstates. Suppose we express this late state in a basis that includes branches resembling smooth, semiclassical

EFT geometries alongside exotic, heavily quantum-gravitational (QG) microstates:

$$|\Psi_{\text{late BH}}\rangle = \alpha|\psi_{\text{EFT-like}}\rangle + \beta|\psi_{\text{QG-like}}\rangle. \tag{4}$$

By the principle of quantum linearity, the evolution operator must distribute over this superposition:

$$U_{\text{exact}}|\Psi_{\text{late BH}}\rangle = \alpha U_{\text{exact}}|\psi_{\text{EFT-like}}\rangle + \beta U_{\text{exact}}|\psi_{\text{QG-like}}\rangle. \tag{5}$$

Herein lies the trap. To preserve the correspondence principle - the requirement that standard physics holds in regimes where semiclassical conditions are met - we must demand that U_{exact} acts on the "normal" EFT branch exactly as U_{EFT} would. That is, $U_{\text{exact}}|\psi_{\text{EFT-like}}\rangle \approx U_{\text{EFT}}|\psi_{\text{EFT-like}}\rangle$. This branch will therefore continue to strictly generate $\ln 2$ of entanglement entropy per emission step.

To achieve a unitary Page curve turnaround, the burden falls entirely on the β branches. These non-EFT outcomes must evolve in such a radically different way that their quantum interference perfectly cancels out the monotonically growing entanglement generated by the α branch.

Mathur's effective small corrections theorem [4] explicitly addresses this exact scenario, though the original theorem appended with Section III A is largely sufficient for this purpose. The theorem demonstrates that if the amplitude of the semiclassical branch is dominant ($|\alpha|^2 \approx 1$), no amount of exotic evolution in the heavily suppressed β branches can mathematically overturn the entropy growth. The strict subadditivity inequalities bound the influence of the β terms.

Therefore, for this time-dependent divergence to successfully purify the black hole, one of three highly radical physical realities seems to be required:

1. **Violation of Linearity:** The operator U_{exact} acts non-linearly, allowing the presence of the β branches to suppress the entanglement generation of the α branch without ordinary unitary interference.

2. **Macroscopic Dynamic Phase Transition:** The coefficient α must become small post-Page time. In other words, the late-time black hole state must possess small probability amplitude of being in a state that looks like a normal EFT geometry.

3. **Breakdown of Bulk Hilbert Space Factorization:** The foundational assumption that the tensor product structure holds for the bulk Hilbert space fails entirely at late

times. There is no well-defined entanglement entropy of a black hole, and thus no well-defined Page curve in the bulk.

The first option requires abandoning fundamental quantum mechanics. The third option is particularly relevant in the context of the AdS/CFT correspondence. If the bulk Hilbert space factorization breaks down, there are currently two prevailing interpretations regarding its relationship to the unitary boundary theory:

Interpretation A (Matching Factorizations): The boundary and bulk Hilbert space factorizations are completely equivalent. If the boundary CFT does not permit a clean, local tensor factorization that isolates the 'interior' degrees of freedom from the early radiation, then the bulk quantum gravity theory simply cannot either. The apparent local factorization in the bulk is an illusion of the low-energy approximation, and U_{exact} inherently acts on a fundamentally non-factorizable space.

Interpretation B (Non-equivalent Factorizations via Subregion Holography): The boundary and bulk factorizations are fundamentally not equivalent. As demonstrated by Seiji Terashima's results on subregion holography [5], the localization of states differs drastically between the bulk and the boundary. A completely localized state within a boundary subregion does not map to a strictly localized state within the corresponding bulk entanglement wedge. Because the tensor product structure of the boundary conformal field theory does not isometrically translate to a localized tensor product structure in the bulk quantum gravity theory, the foundational assumption that the bulk Hilbert space cleanly factorizes - e.g., $\mathcal{H} = \mathcal{H}_{\text{early}} \otimes \mathcal{H}_{\text{out}} \otimes \mathcal{H}_{\text{in}}$ - breaks down completely. Consequently, an operator that appears perfectly local in the bulk (such as a local emission operator acting on the EFT branch) corresponds to a highly non-local, delocalized state on the boundary. This non-equivalence demonstrates that the strict spatial locality assumed by U_{EFT} is a flawed kinematic starting point for late-time black holes. This is not a completely negative result - it suggests that while the Page curve is ill-defined in the bulk, it can be imposed in the boundary. The $\mathcal{O}(1)$ turnaround in the Page curve now becomes more acceptable in the boundary, which contrasts with the circumstance in the bulk.

In all three scenarios, the attempt to smoothly evade the theorem fails. Even if we rely on the breakdown of bulk factorization, we are conceding that the local semiclassical operator U_{EFT} is structurally incapable of governing the system. One must still introduce a catastrophic, $\mathcal{O}(1)$ dynamic breakdown of the local EFT evolution to salvage information, maintaining the unyielding boundary drawn by the small corrections theorem.

IV. EVALUATING PROPOSED RESOLUTIONS THROUGH THE DYNAMIC LENS

The small corrections theorem forces a stark choice: either accept information loss, or accept a macroscopic breakdown of local effective field theory dynamics at the horizon. It is instructive to evaluate various proposed resolutions to the firewall paradox [6] through this lens of kinematic versus dynamic corrections. As we will see, the most prominent physical resolutions implicitly concede the theorem's core argument by introducing precisely the kind of $\mathcal{O}(1)$ dynamic alterations to semiclassical evolution that the theorem demands.

A. Black Hole Complementarity and the Petz Map

Black hole complementarity (BHC), proposed by Susskind, Thorlacius, and 't Hooft [7], offers a framework that attempts to preserve both unitarity and the equivalence principle. BHC postulates that information is both reflected at the horizon (for the distant observer) and passes smoothly through the horizon (for the infalling observer), avoiding the no-cloning theorem simply because no single observer can access both descriptions simultaneously.

However, Mathur's theorem fundamentally challenges this peaceful coexistence. If we apply the axiom of BHC that local, semi-classical EFT correctly describes the dynamics outside the stretched horizon, then the evolution operator U_{EFT} dictates a continuous monotonic increase in entanglement. Thus, without introducing a macroscopic $\mathcal{O}(1)$ dynamic breakdown to the local Hamiltonian, unitarity cannot be satisfied. BHC attempts to bypass the contradiction by declaring the states kinematically complementary, but the small corrections theorem demonstrates that the *dynamic evolution* required by the two descriptions is mathematically incompatible.

Recently, critics have attempted to revive and rigorously formalize BHC through the lens of quantum error correction (QEC) [8] and the Petz recovery map [9]. In this modern view, the black hole interior is treated as a logical subspace encoded within the physical degrees of freedom of the exterior radiation and the remaining black hole. Even if local unitarity (or final purity) appears to break down in the full Hilbert space (e.g., after the black hole has completely evaporated), restricting the code space strictly to the exterior subspace allows the Petz map to reconstruct interior operators perfectly. Critics argue that this mathematical reconstruction vindicates BHC: unitarity is restored from the perspective of the exterior observer without needing to explicitly modify the semiclassical dynamics of the infalling observer.

Viewed through the kinematic versus dynamic distinction, however, the Petz map defense falls short of evading the small corrections theorem. The Petz map is a quantum channel - a purely kinematic recovery operation applied to a specific, highly entangled state. To physically enact this recovery, the actual time-evolution operator U_{exact} governing the Hawking emission process must implement this highly non-local mapping.

If the true dynamics at the horizon inherently map interior operators to exterior ones via QEC, it represents a massive dynamic deviation from the local U_{EFT}, which strictly and independently creates exterior (b) and interior (c) quanta. The success of the Petz map fundamentally relies on the exact state belonging to a restricted code space. If local EFT holds dynamically at the horizon, Hawking pair creation continuously expands the physical Hilbert space dimension in a way that breaks the required code space isometry. Therefore, invoking the Petz map does not smoothly bypass Mathur's theorem; rather, it provides a mathematical characterization of the macroscopic, non-local dynamical corrections ($\delta U \sim 1$) required to save information. It is an acknowledgment that the strict spatial locality assumed by U_{EFT} must be dynamically overthrown. Section III A also provided why $U_{\text{exact}} \approx U_{\text{EFT}}$ for early times (Petz map correction is minimal for early times) and $U_{\text{exact}} \neq U_{\text{EFT}}$ for late times (Petz map correction is significant for late times) do not evade the small corrections theorem.

B. AdS/CFT Correspondence

A generalized invocation of the AdS/CFT correspondence [10] is frequently utilized to dismiss the information paradox entirely. The correspondence posits a one-to-one equivalence between entities in a gravitational string theory in an Anti-de Sitter (AdS) bulk space and a conformal field theory (CFT) on its boundary. Because the boundary of this space is locally Minkowski, the boundary CFT is a manifestly unitary quantum field theory. Therefore, a black hole represented in the AdS bulk must evolve unitarily. Within this framework, some argue that firewalls simply cannot form, and the paradox is inherently resolved by the duality.

Viewed through the kinematic/dynamic distinction, citing AdS/CFT as a resolution without detailing the bulk mechanism is a purely kinematic declaration. It mathematically guarantees that the global, exact state $|\Psi\rangle_{\text{exact}}$ remains pure over time. However, knowing that the boundary evolution is unitary does not answer the localized, dynamical question posed by Mathur's theorem: *how* does the local bulk Hamiltonian U_{exact} near the horizon deviate from U_{EFT} to permit this escape of information? The AdS/CFT dictionary may also suggest a mismatch between boundary and bulk localization [5] - if AdS/CFT guarantees unitarity, it simultaneously guarantees that the local bulk dynamics must suffer a macroscopic correction from standard semiclassical EFT, potentially breaking the local tensor product structure of the bulk entirely.

C. Non-locality: ER=EPR and Islands

Another major class of resolutions attacks the strict tensor product factorization of the Hilbert space. The ER=EPR conjecture [11] of Maldacena and Susskind proposes that quantum entanglement (Einstein-Podolsky-Rosen) and geometrical wormholes (Einstein-Rosen bridges) are fundamentally identical. In this framework, the early Hawking radiation is geometrically connected to the black hole interior. Therefore, when a new Hawking quantum is emitted, its interior partner is not strictly local to the horizon, but is non-locally connected to the distant early radiation.

Recent advancements have formalized this non-local geometry via the calculation of the Page curve using replica wormholes and the "island" formula [12]. In this formulation, the gravitational path integral includes topologically connected wormhole saddles that dominate after the Page time. The consequence is that the entanglement wedge of the outgoing radiation includes an "island" situated deep inside the black hole interior.

Both ER=EPR and the islands proposal operate on the same philosophical premise: information escapes because the degrees of freedom inside and outside the black hole do not dynamically evolve independently. The local, semiclassical Hamiltonian U_{EFT} assumes that spacelike-separated regions (the interior and the distant radiation) commute and evolve autonomously. By introducing wormholes and islands, these models declare that the true

QG evolution operator U_{exact} involves macroscopic, non-local dynamic interactions. Mathur's theorem is thus vindicated: local EFT dynamics must break down to save unitarity. Invoking non-locality is not a refutation of Mathur's theorem; it is a concession to it.

D. Operational Limitations and Computational Complexity

Some approaches seek to resolve the paradox not by altering fundamental physics, but by questioning the operational realizability of the paradox itself. Harlow and Hayden argued [13] that for an infalling observer to verify the entanglement between the early radiation and the late Hawking quantum, they would need to perform a quantum computation that takes longer than the black hole's evaporation time. Specifically, the time required scales exponentially with the number of qubits, approximately $\mathcal{O}(2^{2n})$. Similarly, it has been shown that an infalling observer's causal patch cannot encompass the entire interior of the black hole, making it impossible to collect the information required to verify the interior entanglement.

While these arguments provide an epistemic shield against the firewall - preventing any single observer from ever experiencing the contradiction - they do not address the ontological question of the state's dynamic evolution. Even if the calculation is practically impossible, the objective physical evolution of the state must still be described. Thus, computational complexity arguments bypass the kinematic/dynamic distinction rather than resolving it.

V. DEFENDING THE THEOREM'S ARCHITECTURE

A. The Qubit Approximation

Critics occasionally target the theorem's idealization of field modes as qubits. In reality, the Hawking emission involves continuous phase spaces, greybody factors, and wavepackets. Does the restriction to a finite-dimensional Hilbert space for the emissions oversimplify the physics?

Philosophically, the qubit approximation is a robust abstraction. Near the horizon, the relevant modes undergo a dimensional reduction to a two-dimensional effective conformal field theory [14, 15]. Mapping the presence or absence of a mode excitation to a $\{|0\rangle, |1\rangle\}$

basis captures the exact entangling dynamics of the Bogoliubov transformations without loss of generality regarding the unitarity problem.

A modern holographic critique might target this approximation by arguing that subregion holography and the non-equivalence of bulk/boundary factorizations invalidate the strict $\mathcal{H} = \mathcal{H}_{\text{early}} \otimes \mathcal{H}_{\text{out}} \otimes \mathcal{H}_{\text{in}}$ tensor product structure required to define these independent qubits. However, this critique misapprehends the theorem's logical structure, which takes the form of a *reductio ad absurdum.* The qubit approximation does not assert that quantum gravity fundamentally factorizes; rather, it assumes the semiclassical EFT limit, which *does* strictly factorize locally. If holographic principles dictate that bulk factorization fails, this is not a debunking of Mathur's theorem, but a confirmation of its central thesis: local EFT dynamics (and its inherent tensor product structure) must macroscopically break down to preserve unitarity.

B. The Effective (Extended) Small Corrections Theorem

Further defense of the theorem's applicability is found in its extended version [4] - though the original theorem combined with Section III A is sufficient for the context of this paper. Recognizing that a singular unique vacuum or a rigid emission basis might be too restrictive, the extended small corrections theorem relaxes these assumptions. It accounts for scenarios where the vacuum might be degenerate, where there are macroscopic superpositions of different semiclassical geometries, or where the emission happens in a basis different from the standard Hawking derivation.

The effective theorem demonstrates that as long as the probability of emitting something highly deviant from the semiclassical prediction is small, the rigorous bound on entropy growth persists. This directly shields the theorem from critiques suggesting that minor ambiguities in defining the field theory basis could serve as a loophole for information escape.

VI. CONCLUSION

The small corrections theorem is not merely a mathematical curiosity within an outdated semiclassical paradigm; it is a stringent diagnostic tool. By distinguishing between static/kinematic corrections to the state and dynamic corrections to the evolution, we dismantle the critique that quantum gravitational state complexity trivially resolves the paradox. Eigenstate thermalization may explain why different microstates look the same to a low-energy observer, but it does not explain how information escapes an approximately semiclassical Hamiltonian.

Regardless of whether one accepts the theorem as the final word on black hole evaporation, it successfully isolates the true cost of solving the information problem. It proves that to recover unitarity, the dynamic evolution at the horizon cannot simply be EFT with an ϵ-perturbation. Whether through gravitational shockwaves, non-local replica wormholes, fuzzball horizon replacements, or the wholesale breakdown of bulk Hilbert space factorization via subregion holography, the dynamics themselves must be radically altered. The theorem stands as a rigorous proof that preserving unitarity forbids any smooth, linear ϵ-correction to the semiclassical state; it demands a fundamental, macroscopic rewriting of local spacetime physics.

DATA AVAILABILITY AND DECLARATION OF INTERESTS

The author(s) have no funding source to declare. Furthermore, there is no conflict of interests.

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[1] Stephen W. Hawking, "Particle Creation by Black Holes," Commun. Math. Phys. **43**, 199–220 (1975).

[2] Samir D. Mathur, "The Information paradox: A Pedagogical introduction," Class. Quant. Grav. **26**, 224001 (2009), arXiv:0909.1038 [hep-th].

[3] Mark Srednicki, "Chaos and quantum thermalization," Phys. Rev. E **50**, 888–901 (1994), arXiv:cond-mat/9403051.

[4] Bin Guo, Marcel R. R. Hughes, Samir D. Mathur, and Madhur Mehta, "Contrasting the fuzzball and wormhole paradigms for black holes," Turkish Journal of Physics **45** (2021).

[5] Seiji Terashima, "Bulk locality in the AdS/CFT correspondence," Phys. Rev. D **104**, 086014 (2021), arXiv:2104.12067 [hep-th].

[6] Ahmed Almheiri, Donald Marolf, Joseph Polchinski, and James Sully, "Black Holes: Complementarity or Firewalls?" JHEP **02**, 062 (2013), arXiv:1207.3123 [hep-th].

[7] Leonard Susskind, Larus Thorlacius, and John Uglum, "The Stretched horizon and black hole complementarity," Phys. Rev. D **48**, 3743–3761 (1993), arXiv:hep-th/9306069 [hep-th].

[8] Ahmed Almheiri, Xi Dong, and Daniel Harlow, "Bulk Locality and Quantum Error Correction in AdS/CFT," JHEP **04**, 163 (2015), arXiv:1411.7041 [hep-th].

[9] Jordan Cotler, Patrick Hayden, Geoffrey Penington, Grant Salton, Brian Swingle, and Michael Walter, "Entanglement Wedge Reconstruction via Universal Recovery Channels," Phys. Rev. X **9**, 031011 (2019), arXiv:1704.05839 [hep-th].

[10] Juan Martin Maldacena, "The Large N limit of superconformal field theories and supergravity," Int. J. Theor. Phys. **38**, 1113–1133 (1999), arXiv:hep-th/9711200 [hep-th].

[11] Juan Maldacena and Leonard Susskind, "Cool horizons for entangled black holes," Fortsch. Phys. **61**, 781–811 (2013), arXiv:1306.0533 [hep-th].

[12] Ahmed Almheiri, Netta Engelhardt, Donald Marolf, and Henry Maxfield, "The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole," JHEP **12**, 063 (2019), arXiv:1905.08762 [hep-th].

[13] Daniel Harlow and Patrick Hayden, "Quantum Computation vs. Firewalls," JHEP **06**, 085 (2013), arXiv:1301.4504 [hep-th].

[14] Steven Carlip, "Near-horizon conformal symmetry and black hole entropy," Phys. Rev. Lett. **88** (2002), 10.1103/PhysRevLett.88.241301.

[15] Horacio E. Camblong and Carlos R. Ordóñez, "Black hole thermodynamics from near-horizon conformal quantum mechanics," Phys. Rev. D **71** (2005), 10.1103/PhysRevD.71.104029.