

A holographic theory of quantum measurement

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Intro

This presentation will focus on the key concepts that, I believe, are more agreeable. So it is **not** going to be about a particular manuscript nor a particular proposal.

The hope is that at the end of this presentation, you are convinced that **a holographic theory of quantum measurement is a viable project**, especially given the background story in holographic gravity regarding **the relation between theory states and observation states**.

I will start with a review of the recent works in holographic gravity and move onto how they may be linked to a theory of quantum measurement. Moreover, the nature of the link between quantum states and observations is to be heavily scrutinized.

Intro (2)

This is a 37-page presentation slide. So it is worth pointing out that the slides directly relating to quantum measurements start from Slide 26. But very likely, one will end up having to read holographic gravity backgrounds.

What's the problem with black holes?

Well, at least according to basic semiclassical gravity calculations, the same information is in two different 'places,' which is banned by quantum physics. (But, as will be later noted, there are differences between the theory-wise internal and external view,)

Most proposed resolutions for the information paradox try to remove information in one place thereby restoring information monogamy.

Three major post-AMPS ideas for the information paradox

- ▶ Holography of information. Naive spacetime locality ("split property") breaks down. Information is encoded in the boundary in quantum gravity. Semiclassical gravity calculations utilize a naive split of spacetime, so this is where things go wrong. (see: arXiv:2002.02447, arXiv:2110.05470)
- ▶ Holographic map/replica wormhole/quantum error-correcting code/bulk-boundary sub-region duality approach: A way to eliminate information in one location as to restore information monogamy. This is done by some holographic map V that is linear but non-isometric (and thus non-unitary). (see: arXiv:1411.7041, arXiv:1802.01040, arXiv:2207.06536, arXiv:1911.11977, arXiv:1911.12333) **This is the proposal I focus on.**
- ▶ Fuzzballs. Denies that black holes have ordinary event horizons. Black holes have "structure." (see: arXiv:0909.1038, arXiv:2204.13113. But note also Suvrat Raju's criticism of the fuzzball program, arXiv:1804.10616)

A common misunderstanding regarding AdS/CFT?

The traditional understanding of AdS/CFT is that it is an inter-representational duality. That is, **holography without holography** (arXiv:2008.10421).

What this means: information on the bulk is not actually holographically encoded in the boundary, if we work entirely in the bulk theory. What AdS/CFT suggests is that we can transport our works in the bulk theory to the boundary theory and everything will be fine.

Or.. maybe that is not a misunderstanding?

In recent years, holography of information does suggest that information may be holographically encoded in the boundary, in fashion of the gravitational Gauss law. So maybe this is not a misunderstanding.

But holography of information comes with the steep cost: breakdown of usual spacetime locality, even at the semiclassical level. And we could say that information does not ever enter a black hole and everything is in the outside, but we do not expect this to be the picture perceived by observers.

This is why holography of information proponents present ways to construct the picture of the black hole interior from the information outside. But this map is non-unitary, as could easily be expected.

A reconciliation between holography of information and replica wormholes

Despite holography of information and replica wormholes being two different proposals to the information paradox, we could say that they are actually two different representations of the same thing - or at least eventually.

A reconciliation between holography of information and replica wormholes

- ▶ Replica wormholes provide a convenient way to consistently represent the black hole interior and the exterior by modifying (but remaining in) the semiclassical Euclidean path integral procedure. That is, it could be understood to partially provide a map that constructs the black hole interior in holography of information, though not entirely.
- ▶ The reason for 'not entirely': replica wormholes construct the black hole interior in the semiclassical theory. By contrast, holography of information starts from a quantum gravity theory that is mostly unknown at this point.
- ▶ But once we know how an effective semiclassical theory emerges from this quantum gravity theory, then the replica wormholes approach provides a mechanism of how the black hole interior may be constructed despite holography of information.

Two different spirits regarding bulk locality

Holography of information breaks away from traditional bulk locality.

Replica wormholes maintain bulk locality as much as possible. The holographic map proposal could be understood to be a way to maintain both bulk locality and information monogamy.

At this stage, it may seem that the replica wormhole proposal is superior: we have both bulk locality and information monogamy. But not really so, considering other issues..

On replica wormholes and holographic map/subregion duality view

You may wonder: are 'you' really sure that replica wormholes (arXiv:1911.11977, arXiv:1911.12333) are 'always' about the holographic map view (or equivalently, bulk-boundary subregion duality, arXiv:1411.7041, arXiv:1802.01040, arXiv:2207.06536)?

Well, this is obvious in a way. Replica wormholes modify the semiclassical approach. The problem with the semiclassical theory is that while it internally does not know that information monogamy has broken down, we know externally from the knowledge of black hole evaporation that information is replicated.

So what replica wormholes do is that they systemically eliminate information from one place (or 'entanglement' in the semiclassical theory). This is akin to a non-isometric holographic map from semiclassical theory states to observation states.

On holographic map/subregion duality

So the key idea in the holographic map view (or subregion duality, arXiv:2207.06536) is that information is redundantly encoded in the boundary theory (such as CFT in AdS/CFT or effective semiclassical theory) such that information monogamy is considered to have broken down.

Or equivalently stated, the boundary theory does not respect bulk locality: $[\phi(x), O(X)] \neq 0$ for all reconstructed bulk operators $\Phi(x)$ and boundary operator $O(X)$ for x and X space-like separated is not possible. (see arXiv:1802.01040 that states this point clearly.)

To restore information monogamy, we need to eliminate information redundancy in the boundary theory as to get the actual observational (and bulk) ‘quasi-theory.’ This is done by some non-isometric holographic map V from the boundary theory to the bulk quasi-theory that respects bulk locality.

But what exactly is the bulk theory? (1)

Well, according to arXiv:1802.01040, it seems that we are supposed to think some bulk theory does exist and somehow this non-isometric holographic map V links a unitary quantum theory to another unitary quantum theory.... Or more specifically, V links the boundary CFT to some (string) theory on the AdS bulk...

But V is non-unitary. We are doing something akin to postselection! So should we really think that this bulk theory.. is actually a valid quantum theory?

In fact, this could be considered a restatement of Samir D. Mathur's criticism (and possibly and more generally, the fuzzball community) of replica wormholes. They argue that such small corrections to the original effective theory cannot respect unitary dynamics. ('small corrections theorem' - the original theorem is in arXiv:0909.1038 and its more recent incarnation is in arXiv:2204.13113)

But what exactly is the bulk theory? (2)

You may be horrified by the suggestion that in the boundary-bulk duality, there is no theory on the bulk side. This is especially so, since in AdS/CFT, the AdS side is... string theory,

So am I saying that string theory may not exist according to the holographic map view? Sort of. Recall: the reason why AdS/CFT is considered so important in the history of string theory is that via AdS/CFT, one may provide a non-perturbative definition of string theory. So technically speaking, string theory is yet to have a rigorous definition that does not involve boundary holography.

But what exactly is the bulk theory? (3)

So.. well, why not? We could only have CFT and what we call as string theory may turn out to be sets of calculation tools. For most purposes, that is almost the same as a theory maintaining approximately unitary dynamics.

Now, arXiv:2204.13113 does criticize this perspective toward holographic gravity and suggests that many people use AdS/CFT in a way that is not warranted by the actual Maldacena principle. (View: string theory \gg boundary CFT) But for now, I will brush aside this critique.

Bizarre characteristics of replica wormholes (1)

arXiv:2204.13113 points out this aspect as well: the replica trick (which gives the ‘replica’ part of ‘replica wormholes’) for calculating entanglement entropy involves nothing of dynamics. This should be clear from the fact that these are Euclidean calculations, not Minkowski calculations. **No dynamic (or, ‘theory’) correction is involved in replica wormholes and only state-to-state corrections are involved.**

The authors of arXiv:2204.13113 ask: are we really justified of modifying the replica trick that was originally simply intended to simplify entropy calculations? They say that what replica wormholes actually modify is basic rules of quantum physics (given that we are working in Euclidean), not a specific quantum theory.

Bizarre characteristics of replica wormholes (2)

But maybe that view (in arXiv:2204.13113) is too dismissive. I think this is an unfortunate misunderstanding caused by how replica wormholes are presented.

Rather than demonstrating how and why semiclassical entanglement entropy calculations are wrong, replica wormholes provide a simple mechanism in the semiclassical path integral framework that corrects original quantum states into observation states. There is nothing 'wrong' with original semiclassical entanglement entropy calculations, and what is actually wrong is that a theory state does not resemble an observation state.

This again shows how replica wormholes are naturally cast in the language of a non-isometric holographic map (or quantum error-correcting codes).

Notes on replica wormholes (1)

Throughout years of modern physics, states and observations have not been distinguished that much. But we now have a crisis: in the holographic map view of holographic gravity, boundary theory states are mapped to observation states by non-isometric (and non-unitary) map V ... **but we are unsure whether a unitary bulk theory actually exists.**

You could argue that V maps **from an effective theory** and that this effective theory is only approximate. So no assurance of unitarity is not problematic at the hope that with fully complete theories, everything would turn out alright.

But the small corrections theorem suggests that this is likely improbable, though I would not say that it completely eliminates such a possibility. And naturally, doing something close to postselection moves us away from unitary dynamics.

Notes on replica wormholes (2)

Again, note that this is about eliminating information from one place such that information monogamy is restored. In AdS/CFT, (from the perspective of arXiv:1802.01040) CFT has information redundantly encoded and this needs to be rectified - right because it creates issues for black holes.

(See arXiv:2012.05770 for black hole paradoxes in AdS/CFT. Though the way these paradoxes are resolved in the holographic map view and holography of information is at least stylistically different, as we have seen before.)

And since CFT is a **(valid) unitary quantum theory**, it cannot be explained away by holographic map V being a map from an **approximate** theory.

Quantum computational complexity and sub-exponential states?

But don't arXiv:2207.06536 argue by quantum computational complexity protections? That is, for most states being sub-exponential states, $\langle \Psi' | V^\dagger V | \Psi \rangle \approx \langle \Psi' | \Psi \rangle$ for boundary theory states $|\Psi\rangle$ and $|\Psi'\rangle$.

Yes, but this does not change the point that there is no unitary theory on the bulk side yet.

But we know that in pre-AMPS era, black hole complementarity is assumed to work because while information monogamy is broken, there is no way of confirming such information replication. The sub-exponential state proposal restores this observational aspect of original black hole complementarity.

Distinguishing observation states and theory states

The following argument can then go on: even in case the small corrections theorem as in arXiv:2204.13113 fully applies, we can just say that what we see observation-wise are **not boundary theory states but bulk quasi-theory states**, linked by non-isometric holographic map V .

And we can assume that there is only a theory on the boundary and no quantum theory on the bulk. All holographic map V does is to map theory states into states that we actually observe. Thus, there is only a quasi-theory on the bulk, not a valid quantum theory.

And the sub-exponential state analysis helps to clarify why unitarity seems to be preserved in the bulk despite V being non-unitary.

Now comes the question...

So far we assumed that no violation of unitarity can be detected on the bulk quasi-theory side. But what if this is not the case?

Remember: that inner product measure is approximately maintained by holographic map V for most states does not mean such small deviations can never be confirmed. In fact, this partially relates to the question of whether BQP contains NP or not. And most do assume that BQP does not fully contain NP, so sub-exponential state justifications may be considered very likely to be upheld.

(BTW: The assumption that the inner product measure is sufficient for comparisons of theories/quasi-theories justifies the interpretation of the BBBV theorem as almost demonstrating that BQP does not fully contain NP.)

Why even try to uphold holographic map? (1)

Now given complexity of the holographic map proposal despite initially positive prospects, one may wonder why we even try to uphold the holographic map proposal.

But it is also equally very unlikely that the holographic map proposal is just a fluke. Just by modifying semiclassical Euclidean path integrals in a systematic way, we get just right states that restore bulk locality, information monogamy and the Page curve.

Furthermore, remember that we can think of black holes as thermodynamic objects. When the black hole exterior is also thought of as a thermodynamic system, we realize that thermodynamic calculations at some point have to be broken down.

Why even try to uphold holographic map? (2)

Now it is true that in de Sitter-Schwarzschild space, it is difficult to think of a black hole as an ordinary thermodynamic object, so that may be used as an excuse for breakdown of a purely thermodynamic understanding. But we can still assume that a black hole is a thermodynamic object and make it interact with the exterior that is also thermodynamic. But this thermodynamic calculation must break down eventually due to black hole evaporation.

It is typically believed that the semiclassical view should hold up to some limit. So we prefer to see how things break down in the semiclassical picture directly. 'How' is made clear in the holographic map proposal - an important advantage.

Why even try to uphold holographic map? (3)

By contrast, in holography of information, since bulk locality is denied in general, the quest very likely is to be how an effective theory respecting bulk locality emerges from a quantum gravity theory... which is of course a very difficult quest. And AdS/CFT is already about quantum gravity, so it gives little insights to this question.

Holographic map and quantum measurements

'Finally,' the final topic of this presentation. Can a holographic map be used to construct a theory of quantum measurements?

Holographic map V is often criticized on the ground of being non-unitary, but we know that in reality, non-unitary evolution does happen due to quantum measurements. So maybe, non-unitary V actually supports the view that observation-wise, non-unitary evolution does happen, just as what quantum measurements seem to suggest.

Well, many-worlds interpretations allow unitary evolution to be preserved despite quantum measurements 'sort of', along with help of the principle of delayed measurements and et cetera. For now, we brush aside the many-worlds interpretation until later.

Holographic map and quantum measurements (2)

Essentially, the idea is: there are many (boundary) theory states $|\Psi_{in,k}\rangle$ mapped via V to the same (bulk) observation state $|\psi_{in}\rangle$, with k being an index. Then we could say that unitary evolution U happens in the boundary theory such that:

$$|\Psi_{fi,k}\rangle = U|\Psi_{in,k}\rangle$$

And this becomes the catch: there is no assurance that $V|\Psi_{fi,k}\rangle$ has the same $|\psi_{fi}\rangle$. And this indeed is a good thing for a theory of quantum measurement. Instead, what we have is distinct $|\psi_{fi,k}\rangle = V|\Psi_{fi,k}\rangle$.

Born rule and holographic map

The Born rule can now be re-interpreted. The Born rule provides how probability must be assigned to each $|\Psi_{in,k}\rangle$ that is all equivalent observationally to $|\psi_{in}\rangle$. That is, probability $|\langle\psi_{in}|\psi_{fi,k}\rangle|^2$ is to be used to assign probability to $|\Psi_{in,k}\rangle$.

Note that this involves only unitary operations, probability measure assignments, holographic map V and nothing else. We may need to introduce other concepts such as amplitude amplification when specifying U in conjunction with V , but as for outlining basic ideas, **this is not needed**.

As with the many-worlds interpretation, this leaves the question as to how the measure problem may be resolved. Why is it that probability must be assigned this way and not the other?

Issues?

Maybe, we are resolving the problem too easily. One could say that since we already know that postselection can resolve all complicated problems in quantum physics easily, we should not rely too much on postselection.

Indeed, we know that a quantum theory but the one that allows general linear operations instead of just unitary ones leads to some information and probability theory-wise issues, as pointed out in quantum reconstruction.

Holographic map V avoids this line of criticisms by maintaining a unitary quantum theory on the theory side but changing how the observation side may perceive that particular quantum state.

Digression: many-worlds interpretation (1)

Many-worlds interpretations do, indeed, provide a compelling story as to why unitary evolution may be consistent with quantum measurements, though as expected, some issues may not have been clearly resolved - which provides a rationale for exploring other theories of quantum measurements. Here, we mainly follow the Zurek variant of the many-worlds interpretation. (See arXiv:0903.5082)

- ▶ While re-coherence between different 'worlds' may not be eliminated a priori, re-coherence is practically impossible - each quantum outcome information tends to get copied, generating entanglements between what may be called as qubits. Other complex quantum interactions are relatively rarer relative to semiclassical 'copying' interactions. In other words, continuous decoherence is the norm and this is the reason why classical theories seem to work so well in most parts of reality. This is also why a quantum computer is difficult to build.

Digression: many-worlds interpretation (2)

- ▶ There is a problem of preferred basis, which is about why we get to observe outcomes only in one particular set of basis. But whatever criteria go for selecting the basis that each world would be branched out, as long as we do not deny the possibility of recoherence between worlds, as long as our measurement basis is consistent with selection criteria, there may not be a problem.
- ▶ And then there is this question of how each branch gets to be probabilistically selected. This is the measure problem, which is yet to be well-resolved - at least no consensus exists yet.

Digression: many-worlds interpretation (3)

In recent years, we have thought experiments like the Frauchiger-Renner thought experiment (arXiv:1604.07422) that generalizes Wigner's friend.

In quantum cryptography, we exploit the fact that quantum measurements lead to wavefunction collapse, allowing us to detect whether eavesdropping was there. Via the many-worlds interpretation argument where this is considered to be copying operations between qubits, we can eliminate wavefunction collapse from consideration.

But what if there indeed are the cases where wavefunction collapse is inconsistent with the many-worlds interpretation? This is what Frauchiger-Renner demonstrated.

Digression: Frauchiger-Renner (1)

Jeffrey Bub made an excellent review on Frauchiger-Renner in arXiv:2008.08538, and I will largely follow the review.

The point mainly is that if we follow the many-worlds argument logic (though this is not confined to many-worlds interpretations), where all measurements can be replaced with unitary operations in enlarged Hilbert spaces, then usual inference rules suggest that we get to observe states that are logically contradictory.

But clearly, these operations are all unitary, and of course unitary operations cannot be inconsistent. So the problem either is in inference rules or as the authors state, a quantum theory cannot consistently describe the use of itself.

Digression: Frauchiger-Renner (2)

And as the authors write, 'which inference rule is wrong' varies depending on interpretations. In the many-worlds interpretation, the issue is that we cannot simply infer from past measurements, given that recoherence between worlds happens as in the experiment. So we do get to measure states with contradictions that are not actually contradictory.

By contrast, in a more traditional interpretation where wavefunction collapse is there during measurements, one has to distinguish a unitary operation and a measurement operation. If a measurement is described as a unitary operation, then we obtain the same understanding with the many-worlds interpretation, though we would not call unitary evolution 'recoherence of the worlds.' **But if** a measurement is described in terms of wavefunction collapse, then such seemingly contradictory states are never observed in the first place.

Digression: Frauchiger-Renner (3)

In other words, Frauchiger-Renner provides the limit as to how the principle of delayed measurements can be used to justify the many-worlds interpretation view via ‘measurements supposedly do not matter that much anyway until we get the final observation.’ Different interpretations have different consequences, however rare such settings are, and we cannot pretend as if these interpretations are practically all equivalent.

Somewhat funnily, if my memory stands correct, Frauchiger-Renner originally intended their paper as providing a strong case for the many-worlds interpretation. This turned out not to be the case but their paper nevertheless has become a part of canon in quantum foundation.

Back to the holographic theory of quantum measurement

So we could say that Frauchiger-Renner provides a justification to pursue an approach outside the many-worlds interpretation, given that wavefunction collapse is not fully captured by the many-worlds interpretation.

The holographic theory of quantum measurement then works beautifully for justifying full wavefunction collapse, not just an illusion.

- ▶ There are several initial theory states that are equivalent to the initial observation state.
- ▶ When these initial theory states undergo unitary measurement process U , they are mapped to different final observation states via holographic map V . It is then postulated that the probability of initial theory states is governed by the Born rule, equivalent to the probability of the corresponding final observation states, assuming these final observation states are all distinct.

The End

Thank You!