1a: Neuroanatomy and Perceptrons

Week 1: Overview

In this first lesson, we will look at the historical background of artificial intelligence and deep learning, biological and artificial neurons, the perceptron learning algorithm, and the training of multi-layer neural networks by gradient descent.

We will briefly review certain topics from probability which are essential for deep learning, and we will introduce the issue of generalization and overfitting in supervised learning.

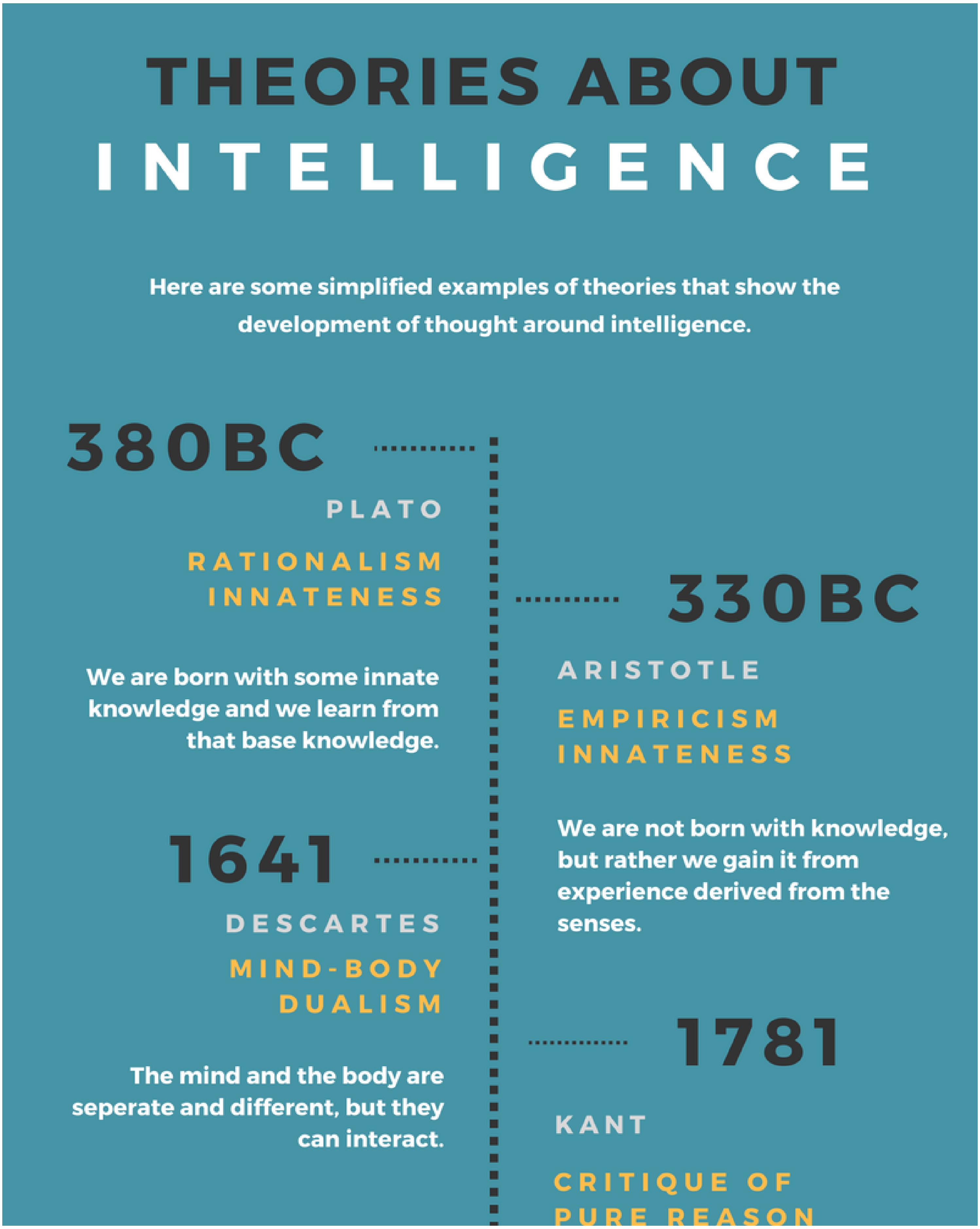
Weekly learning outcomes

By the end of this week, you will be able to:

- construct perceptrons by hand
- apply the Perceptron Learning Algorithm
- construct multi-layer perceptrons by hand
- explain how continuous activation functions allow multi-layer neural networks to be trained by gradient descent

- compute Entropy and KL-Divergence for discrete probability distributions
- compute probabilities using Bayes’ Rule
- describe the basics of supervised learning
- explain how to avoid overfitting in neural networks

History and Neuroanatomy



THEORIES ABOUT INTELLIGENCE

Here are some simplified examples of theories that show the development of thought around intelligence.

380BC

PLATO

RATIONALISM
INNATENESS

We are born with some innate knowledge and we learn from that base knowledge.

330BC

ARISTOTLE

EMPIRICISM
INNATENESS

We are not born with knowledge, but rather we gain it from experience derived from the senses.

1641

DESCARTES

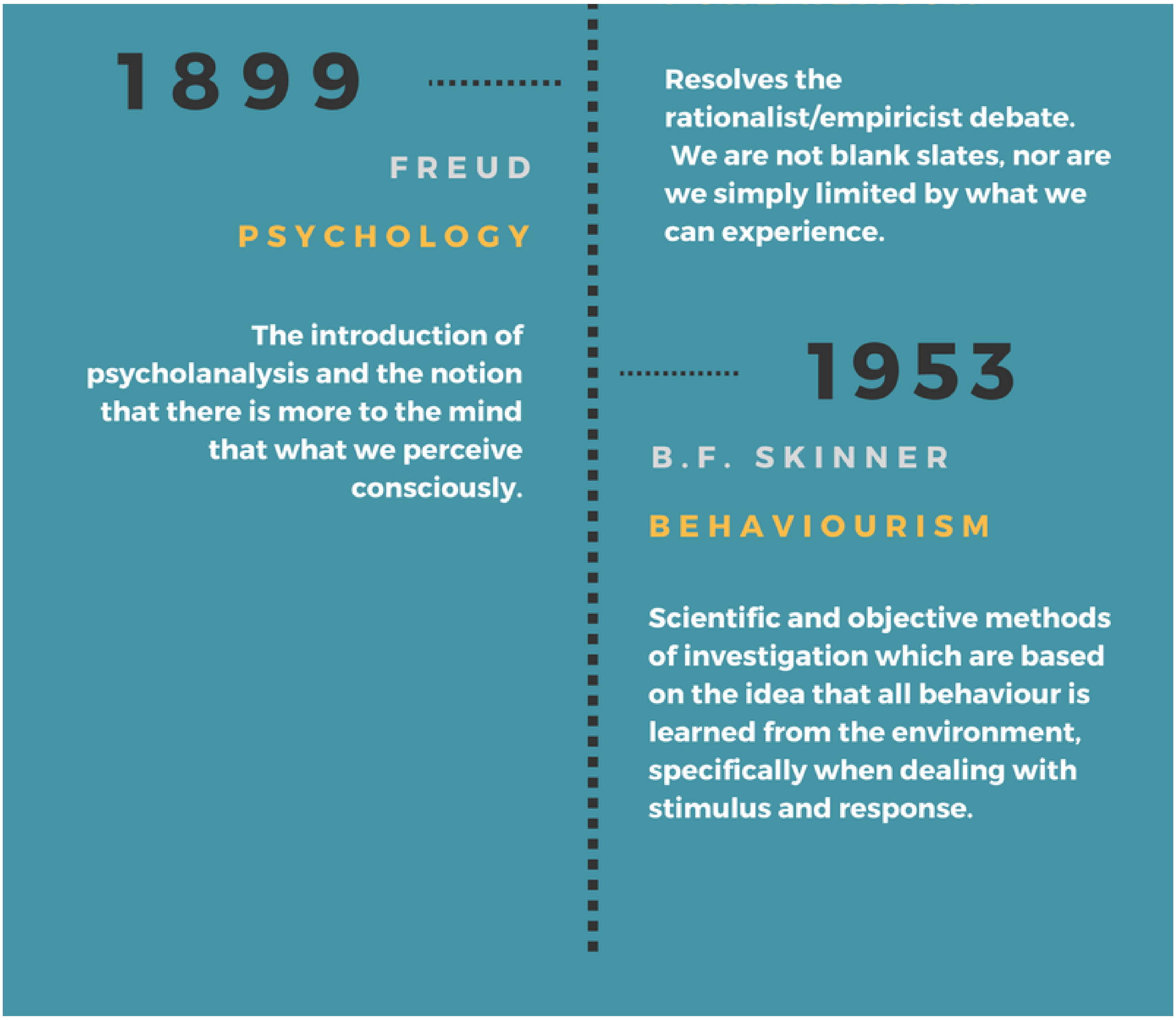
MIND-BODY
DUALISM

The mind and the body are seperate and different, but they can interact.

1781

KANT

CRITIQUE OF
PURE REASON



1899

FREUD

PSYCHOLOGY

The introduction of psycholanalysis and the notion that there is more to the mind that what we perceive consciously.

Resolves the rationalist/empiricist debate. We are not blank slates, nor are we simply limited by what we can experience.

1953

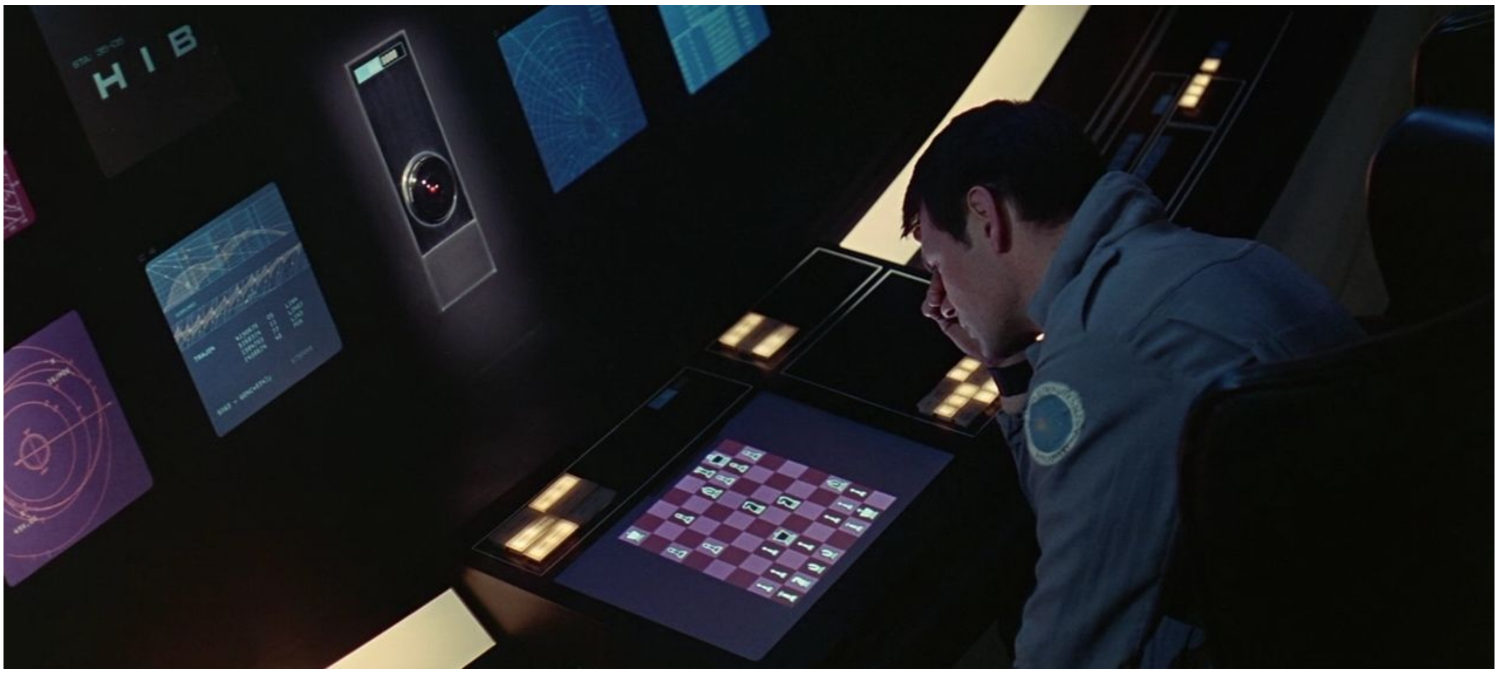
B.F. SKINNER

BEHAVIOURISM

Scientific and objective methods of investigation which are based on the idea that all behaviour is learned from the environment, specifically when dealing with stimulus and response.

> "For the machine is not a thinking being, but simply an automation which acts according to the laws imposed upon it."
> —Ada Lovelace, 1843

The table above summarizes various theories which have been proposed throughout history to explain the workings of the human mind. The idea that human reasoning could be simulated by some kind of mechanical calculator dates back to Blaise Pascal and Gottfried Leibniz in the 1600's, was developed in the 1800's by George Boole, Gottlob Frege, Charles Babbage and Ada Lovelace, and in the early 1900's by Alan Turing.



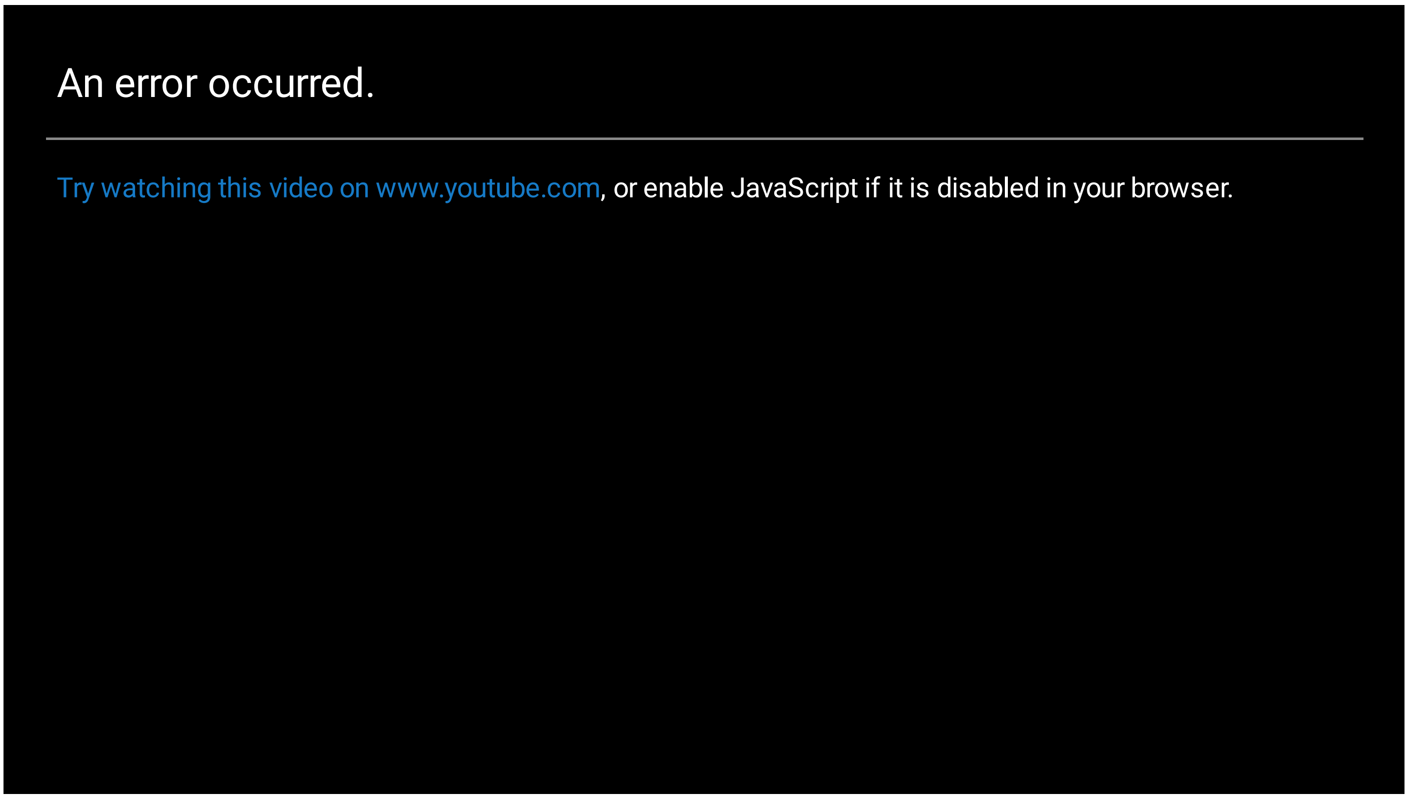
The modern field of Artificial Intelligence (AI) perhaps began in earnest with a workshop at Dartmouth College in the summer of 1956. In the early days of AI, researchers tended to focus on logical reasoning and problem solving. Less attention was paid to perceptual tasks such as vision and language processing, because there was a general expectation that these tasks would be easily solved within a few years. In reality, these seemingly simple tasks turned out to be much more challenging, and were not achieved to any degree of proficiency until the second decade of the 21st Century. The major advances in these areas stem chiefly from a class of models called Deep Neural Networks which are inspired by the structure of the human brain.

Sub-Symbolic Processing

Can you see an animal in this picture? What kind of animal is it? What direction is it facing?

The human brain performs tasks such as object recognition unconsciously and in a highly distributed fashion. Low-level features are recognised in different parts of the image and are hierarchically combined into high level features from which the identity of the object is extracted.

Basic Neuroanatomy



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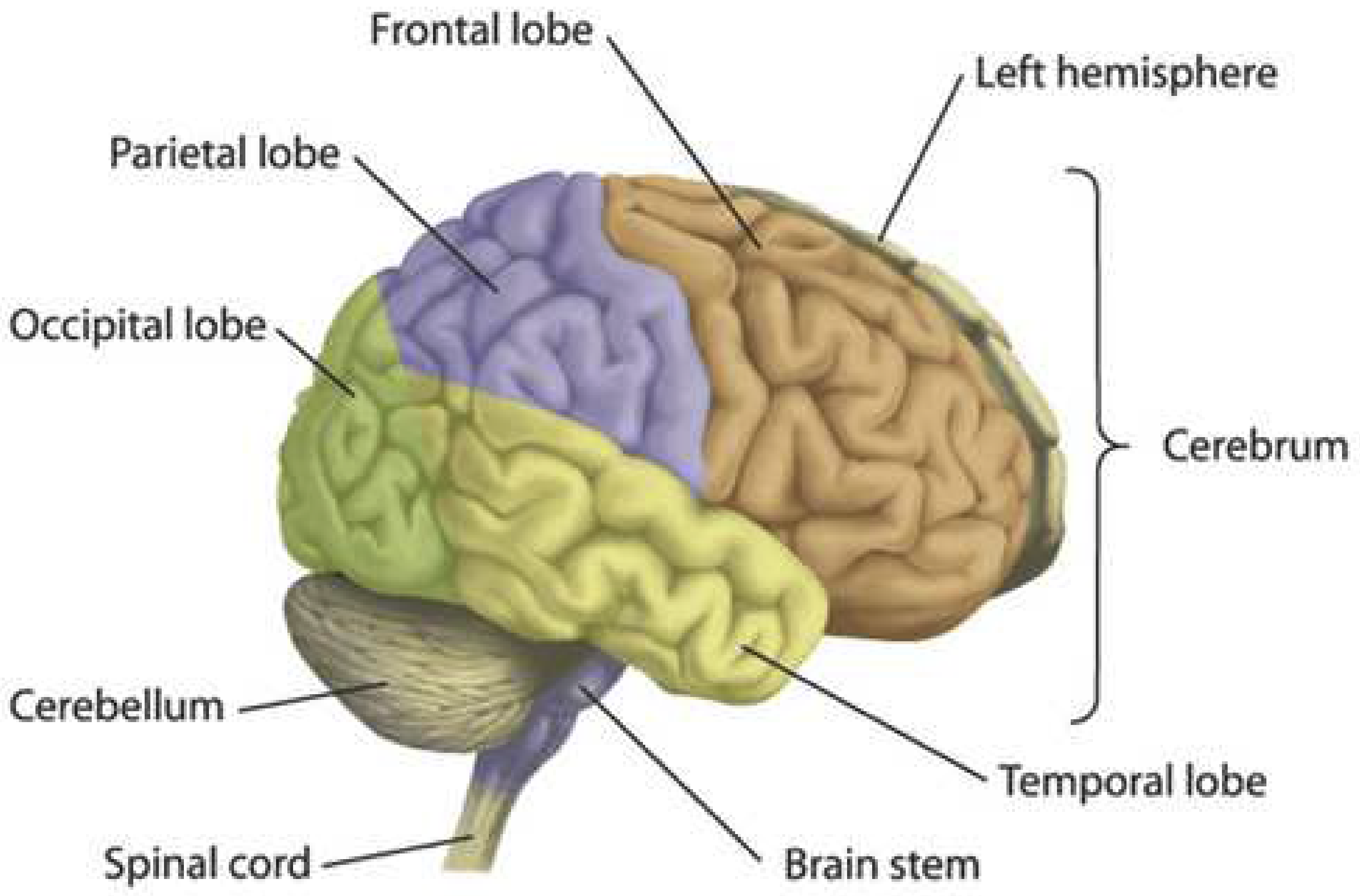
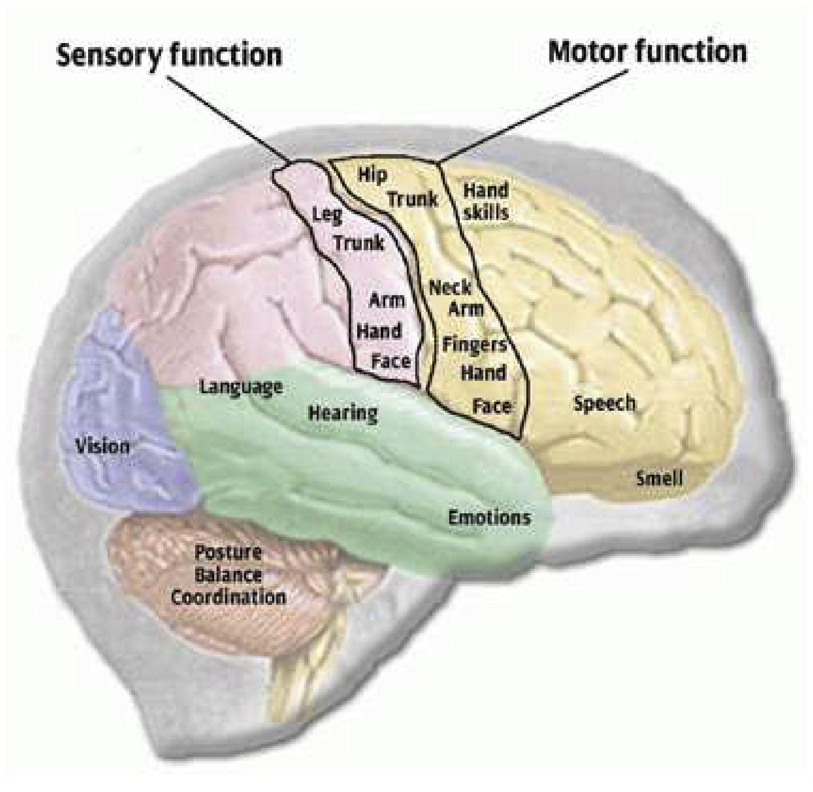
Cerebral Cortex

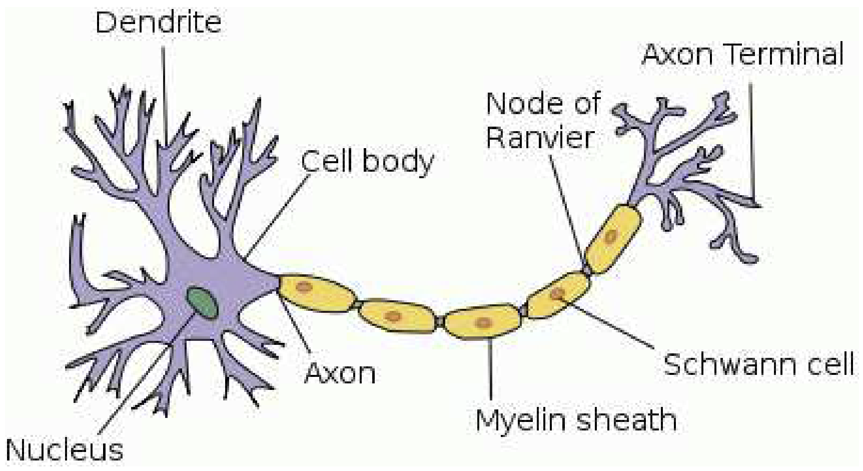


Brain Functions



Neurons

The body is made up of billions of cells. Cells of the nervous system, called neurons, are specialized to carry “messages” through an electrochemical process. The human brain has about 100 billion neurons, and a similar number of support cells called “glia”. Neurons share many functions in common with other cells in the body. For example, they are surrounded by a cell membrane, they have a nucleus containing genes (DNA), and they carry out basic cellular processes like protein synthesis and energy production.



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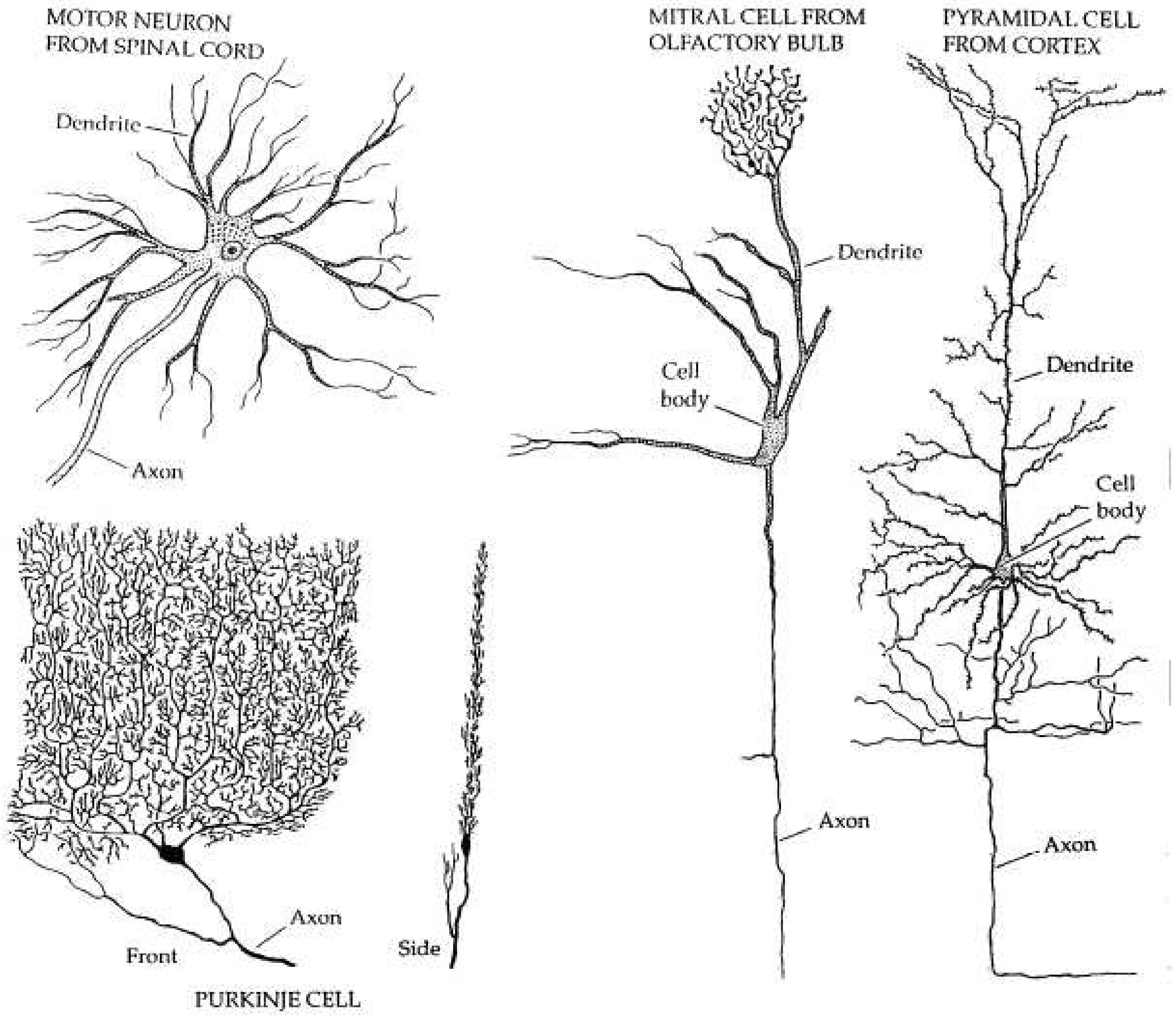
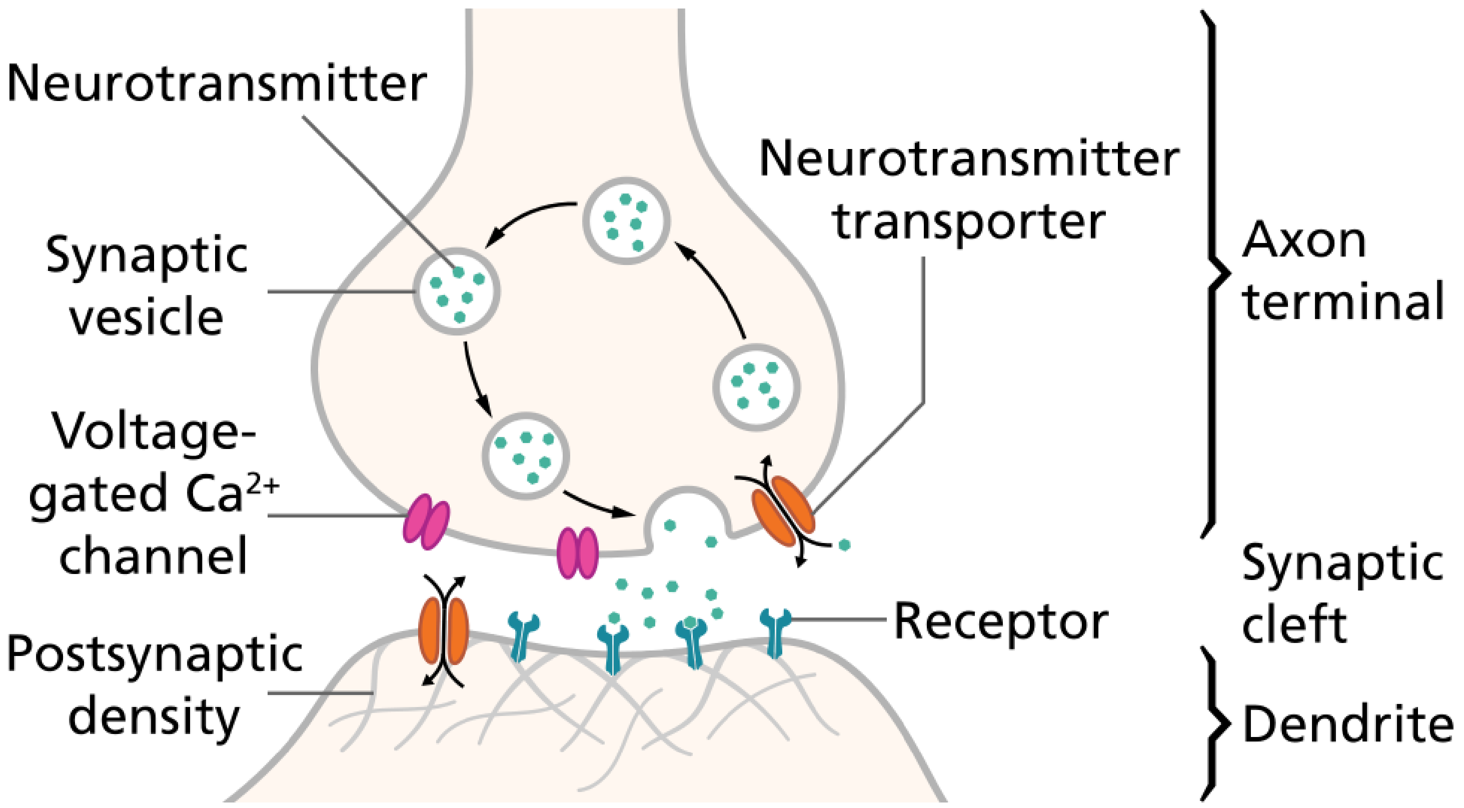
Neurons also have certain features which distinguish them from other body cells. They have specialized extensions called dendrites and axons. Dendrites bring information to the cell body, while axons take information away from the cell body. The axon of one neuron can connect to the dendrite of another neuron through an electrochemical junction called a synapse.

Variety of Neuron Types


MOTOR NEURON FROM SPINAL CORD
Dendrite
Axon
MITRAL CELL FROM OLFACTORY BULB
Dendrite
Cell body
Axon
PYRAMIDAL CELL FROM CORTEX
Dendrite
Cell body
Axon
Front
Axon
Side
PURKINJE CELL

Most neurons have only one axon, but the number of dendrites can vary widely. Unipolar and Bipolar neurons have only one dendrite, but Purkinje neurons in the cerebellum can have up to 100,000 dendrites. Dendrites are typically less than a millimetre in length. Axons can vary in length from less than a millimetre to more than a metre (for motor neurons). Long axons are sometimes surrounded by a myelinated sheath, which prevents the electrical signal from dispersing, and allows it to travel faster (up to 100 m/s).

Synapses, Neurotransmitter and Ion Channels



[https://commons.wikimedia.org/wiki/File:SynapseSchematic_en.svg]

An electrical pulse reaches the endbulb and causes the release of neurotransmitter molecules from little packets (vesicles) through the synaptic membrane. The transmitter passes through voltage-gated ion channels and diffuses through the synaptic cleft to the other side.

When the neurotransmitter reaches the post-synaptic membrane, it binds to receptors and causes a change in polarisation of the membrane. The change in potential can be excitatiory (moving the potential towards the threshold) or inhibitory (moving it away from the threshold).

In the next section, we will look at a simple mathematical model of the neuron which excludes most of the complex biological, chemical and physiological details and aims instead to capture the essential features of a neuron necessary for distributed computation.

Further Reading

For more on the history of Deep Learning, consider these two perspectives:

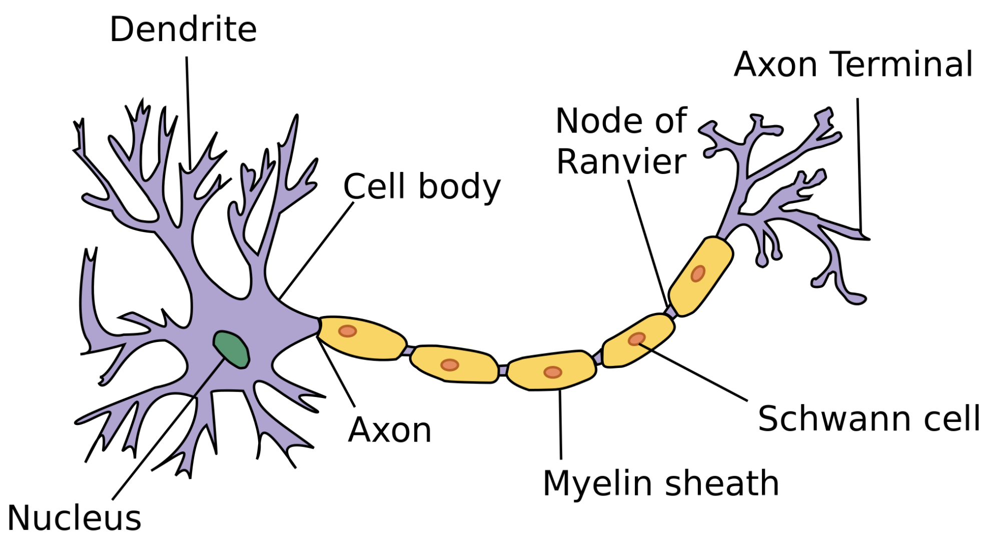
- **Viewpoint 1:** Focusing on recent work (after 2012)
    - https://www.cs.toronto.edu/~hinton/absps/NatureDeepReview.pdf
- **Viewpoint 2:** Focusing on earlier work (before 2012)
    - http://people.idsia.ch/~juergen/deep-learning-overview.html

Textbook Deep Learning (Goodfellow, Bengio, Courville, 2016):

- Historical trends in Deep Learning

Biological vs Artificial Neurons

Biological Neurons



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The brain is made up of neurons (nerve cells) which have:

- a cell body (soma)
- dendrites (inputs)
- an axon (output)
- synapses (connections between cells)

Synapses can be excitatory or inhibitory, and may change over time. When the inputs reach some threshold, an action potential (electrical pulse) is sent along the axon to the output.

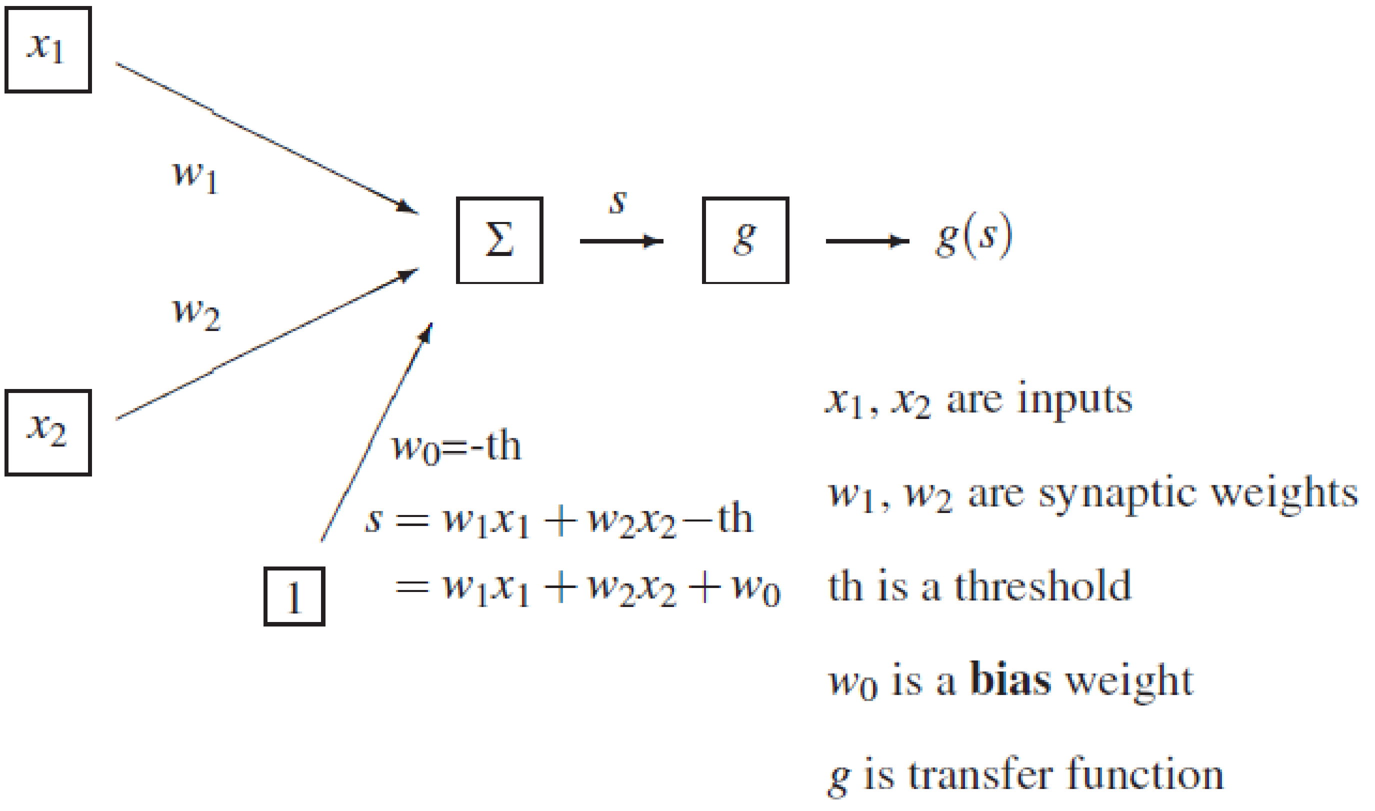
McCulloch and Pitts Neuron

In 1943, McCulloch and Pitts published a simplified mathematical model of a neuron. Each input x_i is multiplied by a corresponding **weight** w_i and the results are added together. An extra constant w_0 called the **bias** is also added.

$$s = w_0 + \sum_i w_i x_i$$

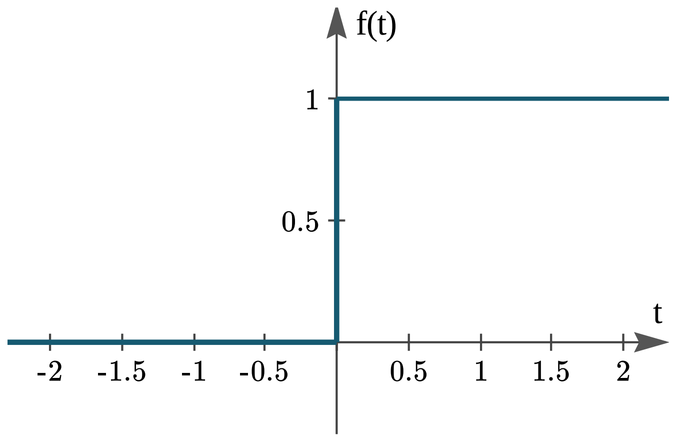
Finally, a non-linear **transfer function** is applied to this linear combination to produce the output of the neuron.

$$z = g(s) = g(w_0 + \sum_i w_i x_i)$$


x_1
x_2
w_1
w_2
Σ
s
g
$g(s)$
1
w_0=-th
$s = w_1x_1 + w_2x_2 - \text{th}$
$= w_1x_1 + w_2x_2 + w_0$
x_1, x_2 are inputs
w_1, w_2 are synaptic weights
th is a threshold
w_0 is a **bias** weight
g is transfer function

Transfer function

The original transfer function used by McCulloch and Pitts was a discontinuous step function (also called the Heaviside function).



$$g(s) = \begin{cases} 1, & \text{if} \quad s \geq 0 \\ 0, & \text{if} \quad s < 0 \end{cases}$$

An artificial neuron with this transfer function is called a **Perceptron**.

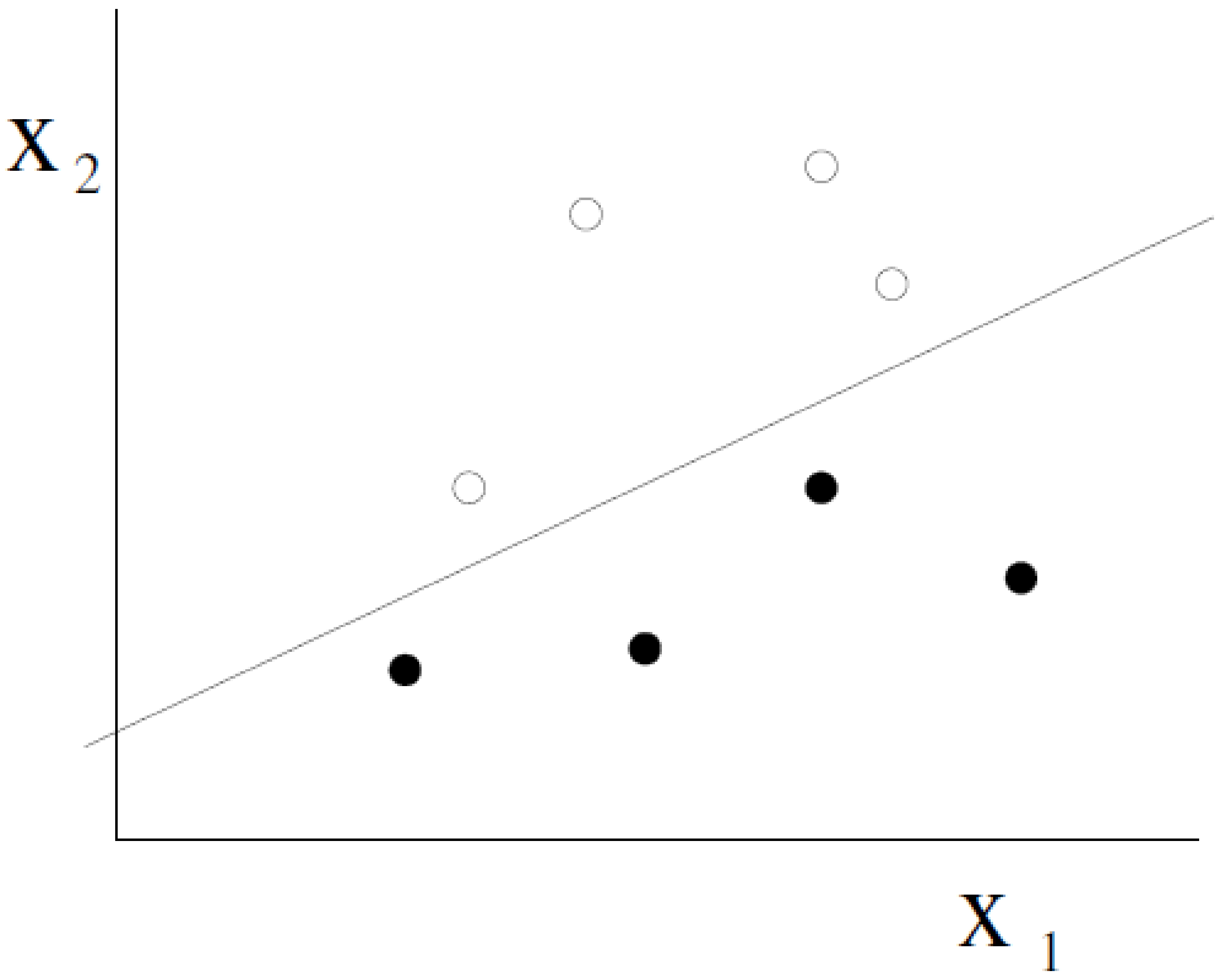
In the early days, the negative of the bias $(-w_0)$ was thought of as a kind of **threshold**. If the combination $\sum_i w_i x_i$ exceeded this threshold, the neuron would "fire" (its output would be 1); otherwise, the neuron would not fire (its output would be 0). The subtracted threshold was changed to an additive bias for mathematical convenience. The higher the bias, the more likely the neuron is to fire.

Later on, alternative transfer functions were introduced which are continuous and (mostly) differentiable.

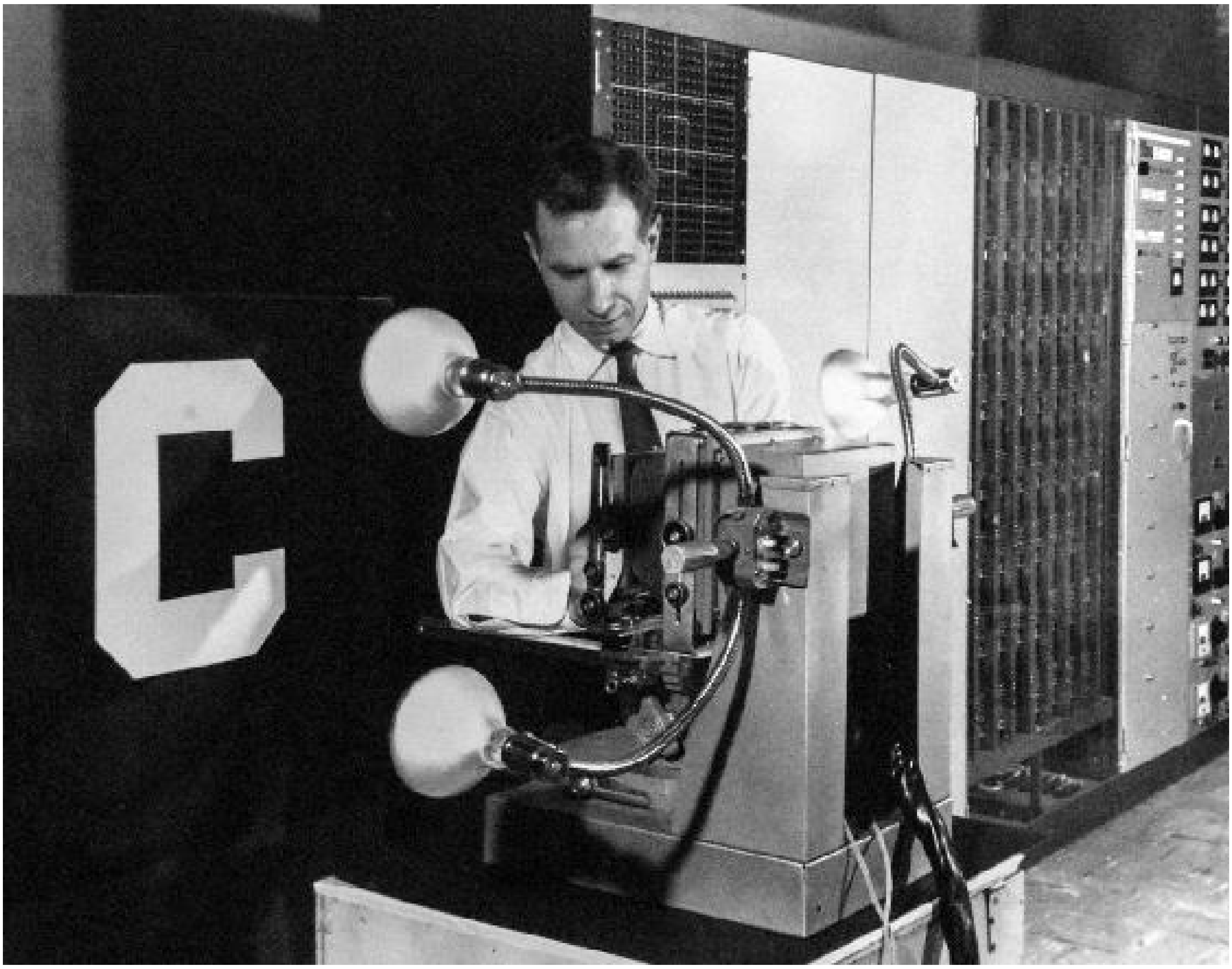
Optional video

Linear separability

The weights and bias of a Perceptron determine a hyperplane which divides the space of inputs into two regions. For inputs on one side of this hyperplane, the output will be 0; for inputs on the other side, the output will be 1. Functions which can be computed in this way are called **Linearly Separable**.



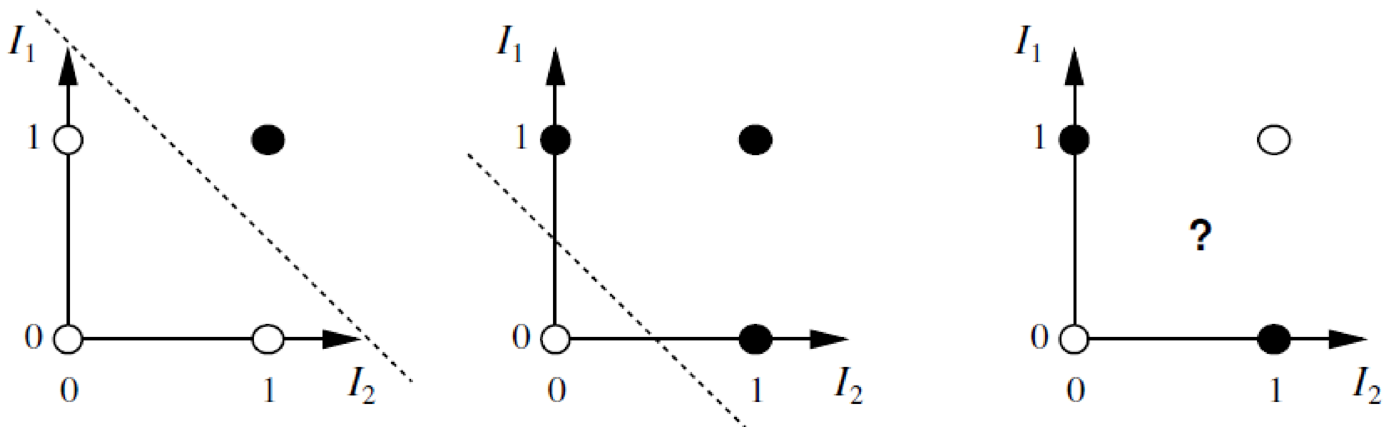
Rosenblatt Perceptron



The Perceptron was introduced by Frank Rosenblatt in 1957. One of the tasks he was interested in was taking the pixels from black and white images of characters and using them to classify the images as $0, 1, \ldots, 9$ or $\mathrm{A}, \mathrm{B}, \ldots, \mathrm{Z}$. For this task, we would need 26 different perceptrons: one to classify each character as "A" or "not A", another for "B" or "not B", etc. up to "Z" or "not Z". If the images are, for example, 20×20, each perceptron would have 400 inputs $x_1, \ldots, x_{400}$.

Logical Functions

If we adopt the convention that $0 = \mathrm{False}$ and $1 = \mathrm{True}$, many simple logical functions like AND, OR and NOT become linearly separable. We can use our knowledge of plane geometry to build Perceptrons to compute these functions. This diagram shows the AND an (inclusive) OR functions, which can be computed by Perceptrons, as well as the Exclusive OR (XOR) which cannot be computed by a Perceptron, because it is not linearly separable.



(a) I_1 and I_2 (b) I_1 or I_2 (c) I_1 xor I_2

These are the weights for Perceptrons to compute the AND and OR function, and the opposite of OR, sometimes called NOR which stands for "Not OR".

$$\begin{array}{lrlr}
\text{AND} & w_1 = w_2 = & 1.0, & w_0 = -1.5 \\
\text{OR} & w_1 = w_2 = & 1.0, & w_0 = -0.5 \\
\text{NOR} & w_1 = w_2 = & -1.0, & w_0 = 0.5
\end{array}$$

Perceptron Learning Algorithm

Supervised Learning

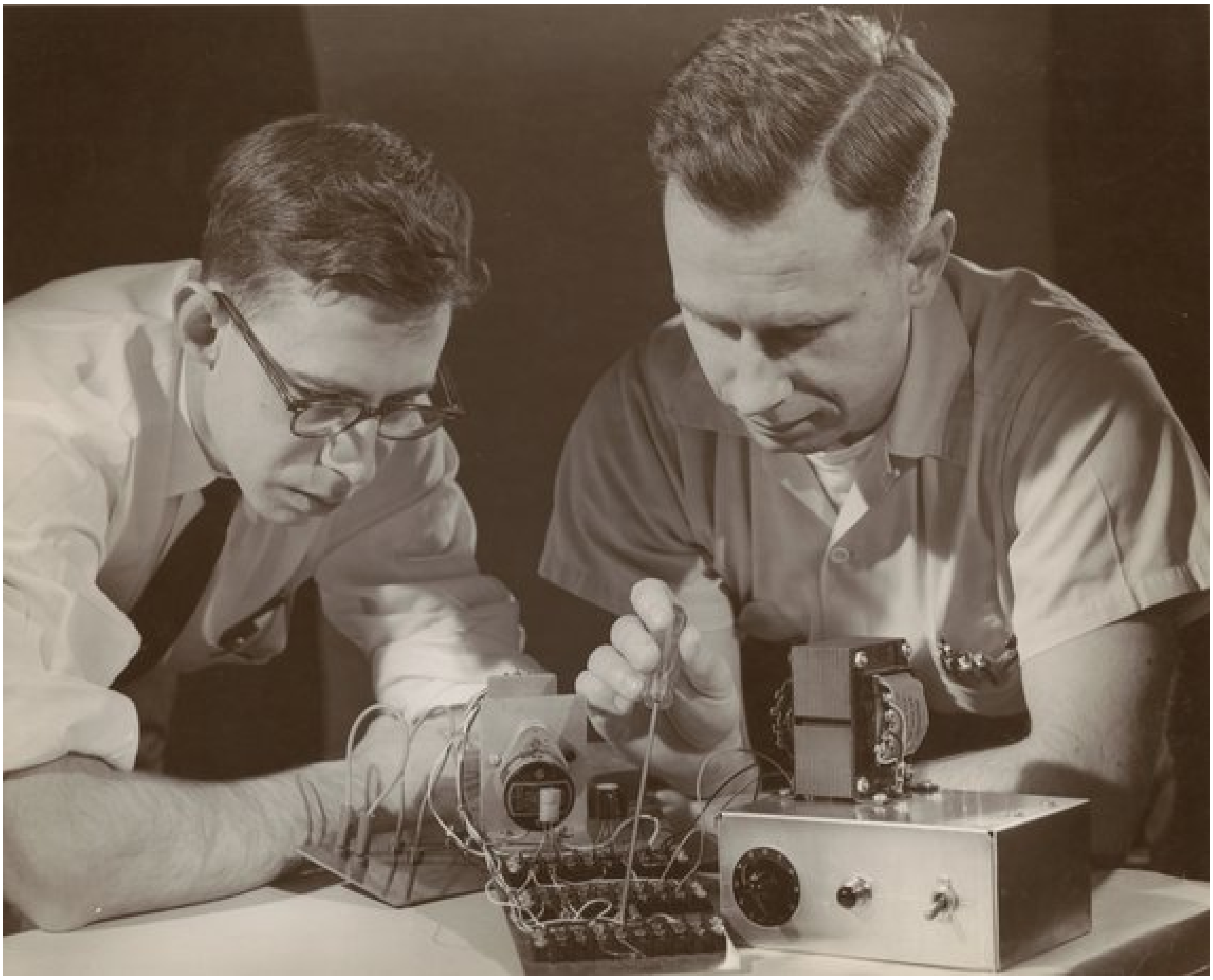
We have seen how simple Perceptrons can be constructed by hand to compute logical functions in 2 dimensions. For larger tasks, such as the character recognition task with 400 inputs extracted from a 20×20 image, the weights clearly cannot be computed by hand. Instead, we seek an automated procedure which, given a set of labeled **data**, will automatically determine the weights and biases of a Perceptron to classify those data.

This is an example of **Supervised Learning**, which will be the focus of this course in Weeks 1-4. Reinforcement Learning will be covered in Week 5, and Unsupervised Learning in Week 6.

For Supervised Learning, we have a training set and a test set, each consisting of a set of items; for each item, a number of input attributes and a target value are specified. The aim is to predict the target value, based on the input attributes. The system is presented with the input and target output for each item in the training set; it must then predict the output for each item in the test set.

Rosenblatt Perceptron

The Perceptron Learning Algorithm was introduced by Frank Rosenblatt in 1957, and is one of the earliest examples of Supervised Learning.



Perceptron Learning Algorithm

The algorithm works as follows. The bias w_0 and weights $\{w_i\}$ of the Perceptron are at first initialised to (small) random values. Then, the training items are presented one at a time, and appropriate changes are made to w_0 and $\{w_i\}$.

Recall that the output of the Perceptron is $g(s)$ where $s = w_0 + \sum_i w_i\, x_i$.

Suppose an input $\{x_i\}$ is presented for which the target output is 1 but the actual output of the Perceptron is 0. In this case, we want to adjust w_0 and $\{w_i\}$ in a way that will make s larger for this particular choice of $\{x_i\}$. We do this by making these adjustments, where $\eta > 0$ is a constant called the **learning rate**:

$$\begin{aligned} w_0 &\leftarrow w_0 + \eta \\ w_i &\leftarrow w_i + \eta\, x_i \\ \text{Consequently,}\quad s &\leftarrow s + \eta(1 + \sum_k x_k^2) \end{aligned}$$

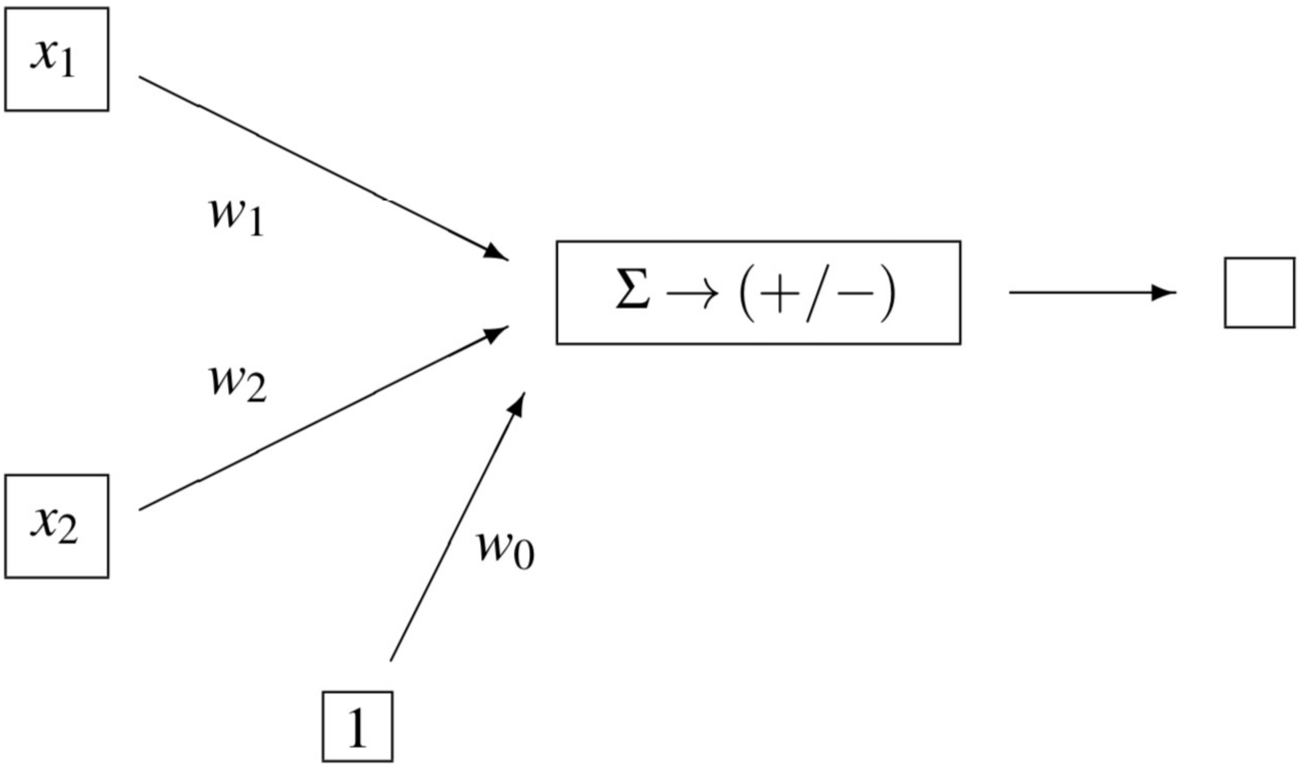
If the target output is 0 but the actual output is 1, we do the opposite, namely:

$$\begin{aligned} w_0 &\leftarrow w_0 - \eta \\ w_i &\leftarrow w_i - \eta\, x_i \\ \text{Consequently,} \quad s &\leftarrow s - \eta(1 + \sum_k x_k^2) \end{aligned}$$

If the actual output is equal to the target output, no change is made to the weights. We may need to cycle through the data several times but, if we reach a point where all the data are correctly classified, the algorithm terminates.

Rosenblatt proved that this algorithm will always learn to classify the training data successfully, provided the data are linearly separable.

Perceptron Learning Example:



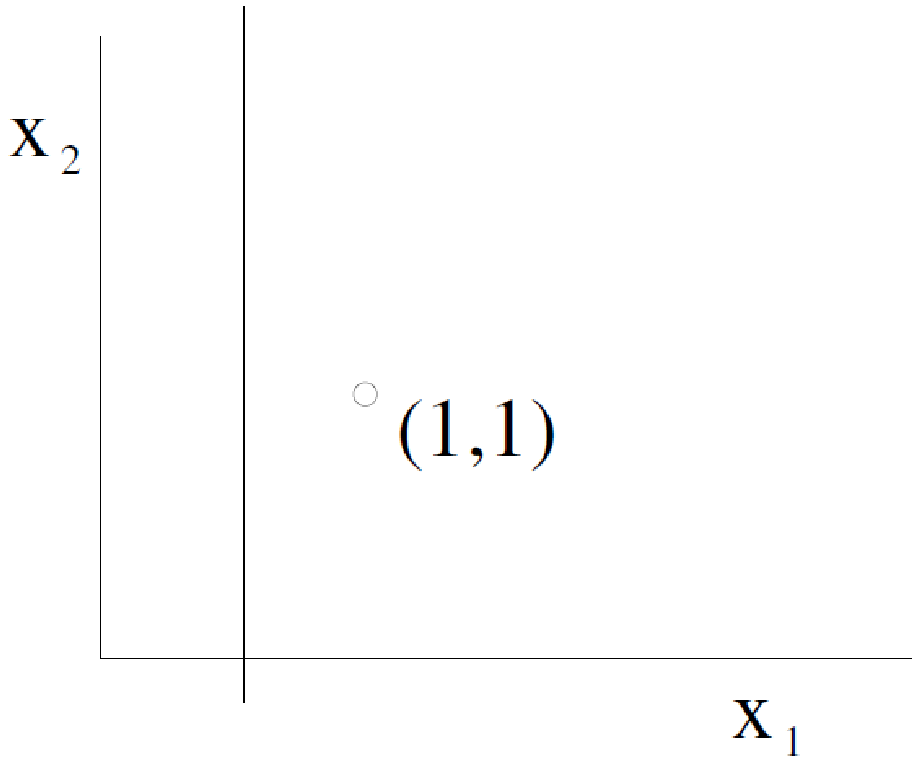
Let's assume a learning rate of $\eta = 0.1$, with these initial weights:

$$\begin{aligned} w_0 &= -0.1 \\ w_1 &= 0.2 \\ w_2 &= 0.0 \end{aligned}$$

Remember that the perceptron will classify as positive those items (x_1, x_2) for which

$$s = w_0 + w_1 x_1 + w_2 x_2 > 0$$

Training step 1



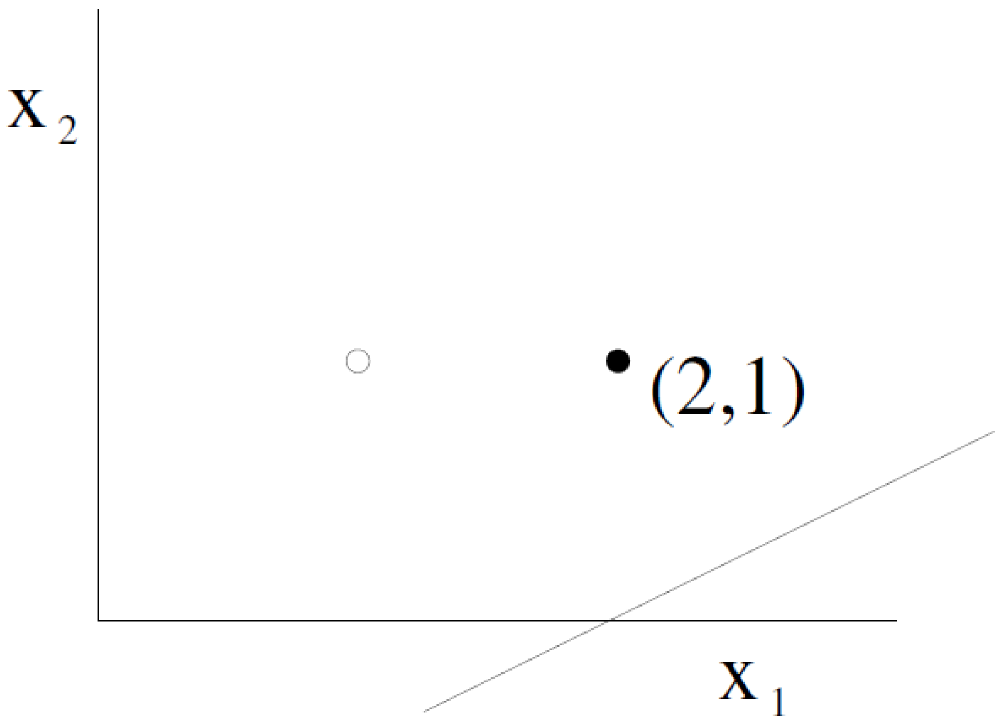
Suppose the first training item is $(1, 1)$ and that it is Negative. We have

$$\begin{aligned} s = &\ \ w_0 \ + \ w_1\, x_1 \ + \ w_2\, x_2 \\ &= -0.1 + 0.2 \times 1 + 0.0 \times 1 = 0.1 > 0 \end{aligned}$$

So, we need to Subtract from the weights, as follows:

$$\begin{aligned} w_0 &\leftarrow w_0 - \eta &= -0.2 \\ w_1 &\leftarrow w_1 - \eta x_1 &= \ \ 0.1 \\ w_2 &\leftarrow w_2 - \eta x_2 &= -0.1 \end{aligned}$$

Training step 2



Suppose the next training item is $(2, 1)$ and is Positive. We have

$$s = -0.2 + 0.1 \times 2 - 0.1 \times 1 = -0.1 < 0$$

So, we need to Add to the weights:

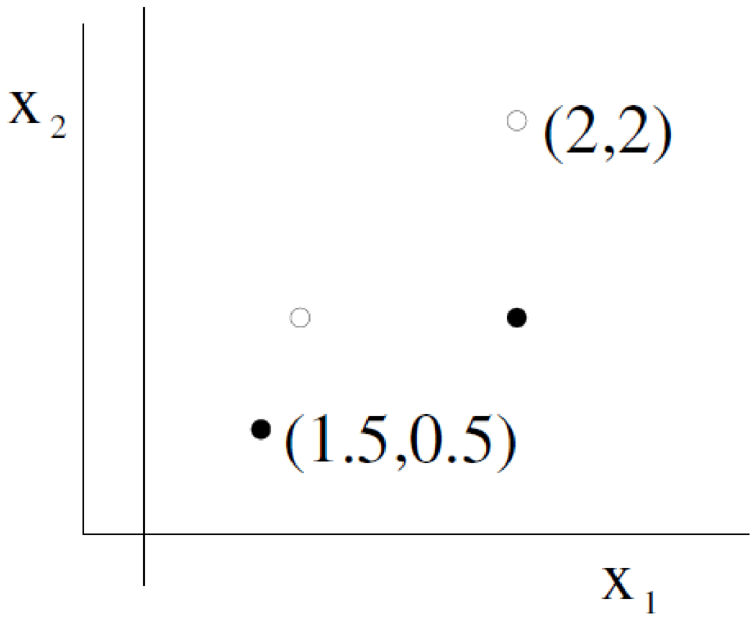
$$\begin{aligned} w_0 &\leftarrow w_0 + \eta &= -0.1 \\ w_1 &\leftarrow w_1 + \eta x_1 &= \ \ 0.3 \\ w_2 &\leftarrow w_2 + \eta x_2 &= \ \ 0.0 \end{aligned}$$

Training step 3

Suppose the next training item is $(1.5, 0.5)$ and is Positive. In this case,

$$s = -0.1 + 0.3 \times 1.5 + 0.0 \times 0.5 = 0.35 > 0$$

So, we do not need to change the weights.



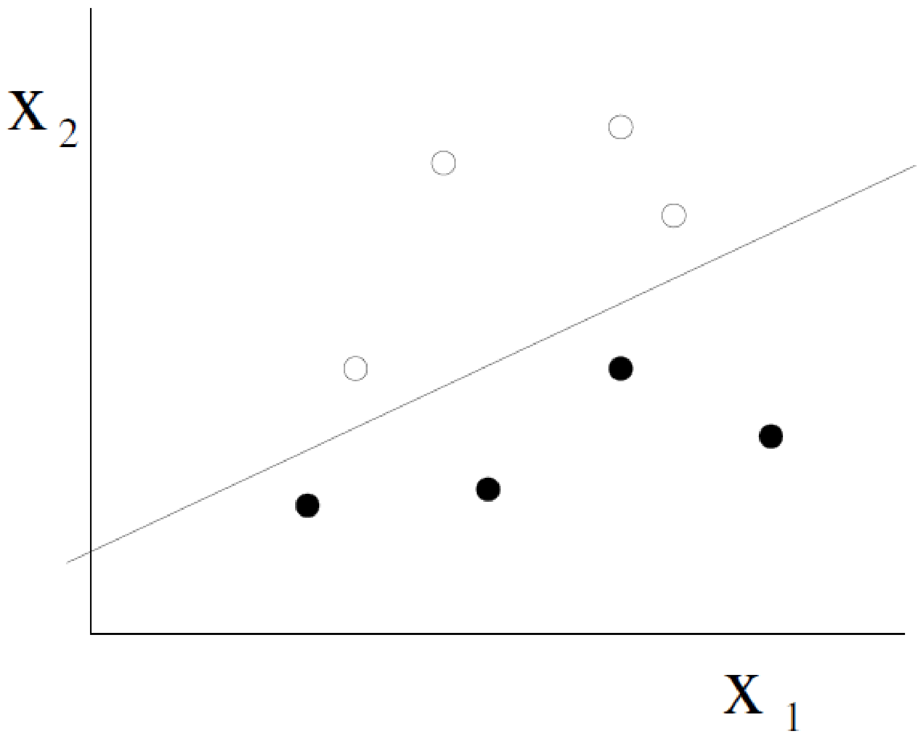
Training step 4

Suppose the next item is $(2, 2)$ and is Negative.

$$s = -0.1 + 0.3 \times 2 + 0.0 \times 2 = 0.5 > 0$$

$$\begin{aligned} w_0 &\leftarrow w_0 - \eta &&= -0.2 \\ w_1 &\leftarrow w_1 - \eta x_1 &&= 0.1 \\ w_2 &\leftarrow w_2 - \eta x_2 &&= -0.2 \end{aligned}$$

Final outcome



Continuing in this way, we will eventually reach a point where all the data are correctly classified (provided they are linearly separable).

Optional video

Further reading

https://news.cornell.edu/stories/2019/09/professors-perceptron-paved-way-ai-60-years-too-soon

Exercise: Perceptrons

Question 1 *Submitted Sep 2nd 2022 at 7:30:04 pm*

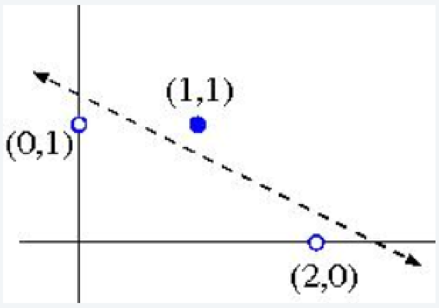
Construct by hand a Perceptron which correctly classifies the following data; use your knowledge of plane geometry to choose appropriate values for the weights w_0, w_1 and w_2.

Training example	x_1	x_2	Class
a.	0	1	-1
b.	2	0	-1
c.	1	1	$+1$

Identify the equation of the line and the weights of the Perceptron (including bias).

Explanation

The first step is to plot the data on a 2-D graph, and draw a line which separates the positive from the negative data points:



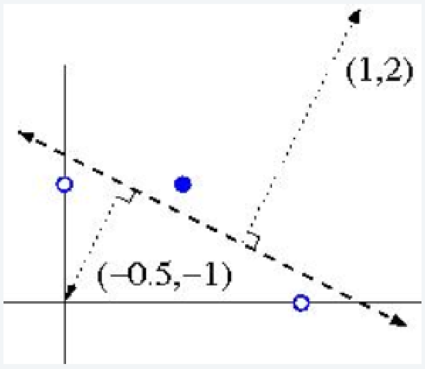
This line has slope -½ and x_2-intersect 5/4, so its equation is:

x_2 = 5/4 - x_1/2, i.e. 2x_1 + 4x_2 - 5 = 0.

Taking account of which side is positive, this corresponds to these weights:

w_0 = -5
w_1 = 2
w_2 = 4

Alternatively, we can derive weights w_1=1 and w_2=2 by drawing a vector normal to the separating line, in the direction pointing towards the positive data points:



The bias weight w_0 can then be found by computing the dot product of the normal vector with a perpendicular vector from the separating line to the origin. In this case w_0 = 1(-0.5) + 2(-1) = -2.5

(Note: these weights differ from the previous ones by a normalizing constant, which is fine for a Perceptron)

Question 2 *Submitted Sep 2nd 2022 at 7:30:49 pm*

Demonstrate the Perceptron Learning Algorithm on the above data, using a learning rate of 1.0 and initial weight values of

$$\begin{aligned} w_0 &= \ -1.5 \\ w_1 &= \ 0 \\ w_2 &= \ 2 \end{aligned}$$

The first three steps are shown below. You should continue until all items are correctly classified.

Iteration	w_0	w_1	w_2	Item	x_1	x_2	Class	$s = w_0 + w_1x_1 + w_2x_2$	Action
1	-1.5	0	2	a.	0	1	-1	$+0.5$	Subtract
2	-2.5	0	1	b.	2	0	-1	-2.5	None
3	-2.5	0	1	c.	1	1	$+1$	-1.5	Add

Explanation

Iteration	w_0	w_1	w_2	Item	x_1	x_2	Class	$s = w_0 + w_1x_1 + w_2x_2$	Action
1	-1.5	0	2	a.	0	1	-1	$+0.5$	Subtract
2	-2.5	0	1	b.	2	0	-1	-2.5	None
3	-2.5	0	1	c.	1	1	$+1$	-1.5	Add
4	-1.5	1	2	a.	0	1	-1	$+0.5$	Subtract
5	-2.5	1	1	b.	2	0	-1	-0.5	None
6	-2.5	1	1	c.	1	1	$+1$	-0.5	Add
7	-1.5	2	2	a.	0	1	-1	$+0.5$	Subtract
8	-2.5	2	1	b.	2	0	-1	$+1.5$	Subtract
9	-3.5	0	1	c.	1	1	$+1$	-2.5	Add
10	-2.5	1	2	a.	0	1	-1	-0.5	None
11	-2.5	1	2	b.	2	0	-1	-0.5	None
12	-2.5	1	2	c.	1	1	$+1$	$+0.5$	None