## **Proof**

First note that

$$1=\int f(y| heta)dy.$$

If Y takes values in  $\mathbb{Z}$  or  $\mathbb{N}_0$ , replace the integral with a sum. Differentiating both sides with respect to  $\theta$ , we obtain

$$0=rac{d}{d heta}\cdot 1=rac{d}{d heta}\int f(y| heta)dy=\int rac{df(y| heta)}{d heta}dy.$$

Differentiating once again yields

$$\int rac{d^2 f(y| heta)}{d heta^2} dy = 0.$$

Now apply these results to the distribution  $f(y, \theta)$  in the exponential family:

$$f(y|\theta) = \exp[a(y)b(\theta) + c(\theta) + d(y)]$$

Then

$$rac{df(y| heta)}{d heta} = [a(y)b'( heta) + c'( heta)]f(y| heta)$$

which implies

$$0 = \int [a(y)b'( heta) + c'( heta)]f(y| heta)dy = b'( heta)\mathbb{E}[a(\mathrm{Y})] + c'( heta),$$

and hence

$$\mathbb{E}[a(\mathbf{Y})] = -\frac{c'(\theta)}{b'(\theta)} \tag{1.5.5}$$

1 of 2

2 of 2 1/22/24, 8:54 AM