Proof

First note that

$$1=\int f(y| heta)dy.$$

If **Y** takes values in \mathbb{Z} or \mathbb{N}_0 , replace the integral with a sum. Differentiating both sides with respect to θ , we obtain

$$0=rac{d}{d heta}\cdot 1=rac{d}{d heta}\int f(y| heta)dy=\int rac{df(y| heta)}{d heta}dy.$$

Differentiating once again yields

$$\int rac{d^2 f(y| heta)}{d heta^2} dy = 0.$$

Now apply these results to the distribution f(y, heta) in the exponential family:

$$f(y|\theta) = \exp[a(y)b(\theta) + c(\theta) + d(y)]$$

Then

$$rac{df(y| heta)}{d heta} = [a(y)b'(heta) + c'(heta)]f(y| heta)$$

which implies

$$0 = \int [a(y)b'(heta) + c'(heta)]f(y| heta)dy = b'(heta)\mathbb{E}[a(\mathrm{Y})] + c'(heta),$$

and hence

$$\mathbb{E}[a(Y)] = -\frac{c'(\theta)}{b'(\theta)}$$
 (1.5.5)

1 of 2

2 of 2 8/3/23, 4:47 pm