Bias-Variance Tradeoff

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Bias-variance trade-off

$$MSE(x_0) = E(y_0 - \hat{f}(x_0))^2$$

$$= E(f(x_0) + \varepsilon - \hat{f}(x_0))^2$$

$$= E(f(x_0) - \underbrace{E(\hat{f}(x_0))}_{\text{variable}} + E(\hat{f}(x_0)) - \hat{f}(x_0) + \varepsilon)^2$$

$$= E(f(x_0) - E(\hat{f}(x_0)) + E(\hat{f}(x_0)) - \hat{f}(x_0))^2$$

independence of ε and x_0

$E(\varepsilon) = 0$

$$+ 2\,\cancel{E(\varepsilon)}^{0}\, E(f(x_0) - E(\hat{f}(x_0)) + E(\hat{f}(x_0)) - \hat{f}(x_0))$$

$$+ E(\varepsilon^2) \quad Var(\varepsilon)$$

$(f(x_0) - E(\hat{f}(x_0)))$

$\times E(E(\hat{f}(x_0)) - \hat{f}(x_0))$

0

constant

$$= E[(f(x_0) - E(\hat{f}(x_0)))]^2 + 2\,\cancel{E[(f(x_0) - E(\hat{f}(x_0))) \times (E(\hat{f}(x_0)) - \hat{f}(x_0))]} + E(E(\hat{f}(x_0)) - \hat{f}(x_0))^2 + Var(\varepsilon)$$

$$= E(f(x_0) - E(\hat{f}(x_0)))^2 + E(\hat{f}(x_0) - E(\hat{f}(x_0)))^2$$

$$+ Var(\varepsilon) = Bias^2(\hat{f}(x_0)) + Var(\hat{f}(x_0)) + Var(\varepsilon)$$

Binomial distribution(Exponential Family)

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$$f(y) = \binom{n}{y} p^{y} (1-p)^{n-y} = \exp\left[\log\left(\binom{n}{y} p^{y}(1-p)^{n-y}\right)\right)$$

$$= \exp\left[\log\binom{n}{y} + y\log p + (n-y)\log(1-p)\right]$$

$$= \exp\left[\log\binom{n}{y} + y\log p - y\log(1-p) + n\log(1-p)\right]$$

$$= \exp\left[\underbrace{\log\binom{n}{y}}_{d(y)} + \underbrace{y}_{a(y)}\underbrace{\log\frac{p}{1-p}}_{b(p)} + \underbrace{n\log(1-p)}_{c(p)}\right]$$

Canonical form

$\log\frac{p}{1-p}$: natural parameter

Poisson and Weibull Distributions

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$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \exp\left(\log\left(\frac{\lambda^x e^{-\lambda}}{x!}\right)\right)$$

$$= \exp(x \underbrace{\log\lambda}_{\text{natural parameter}} - \lambda - \log x!)$$

canonical form

Weibull (θ, λ)

$$f(y) = \frac{\lambda y^{\lambda-1}}{\theta^\lambda} \exp\left(-\left(\frac{y}{\theta}\right)^\lambda\right)$$

$$= \exp\left[\log\left(\frac{\lambda y^{\lambda-1}}{\theta^\lambda} \exp\left(-\left(\frac{y}{\theta}\right)^\lambda\right)\right)\right]$$

$$= \exp\left[\log\lambda + (\lambda-1)\log y - \lambda\log\theta - \left(\frac{y}{\theta}\right)^{\lambda}\right]$$

$\exp(a(y)b(\cdot\cdot) + c(\cdot\cdot) + d(y))$

$\lambda \rightarrow$ nuisance parameter

$$\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \qquad V = Var(X) = \begin{pmatrix} Var(X_1) & Cov(X_1, X_2) \\ Cov(X_1, X_2) & Var(X_2) \end{pmatrix}$$

V^{-1} : V is positive definite $\rightarrow$ invertable