# The variance-covariance matrix of the score

The variance-covariance matrix of the score is

$$\mathcal{I}_{jk} = \mathbb{E}[\mathrm{U}_j \mathrm{U}_k] \tag{3.1.5}$$

which represents the information matrix

$$\begin{aligned}\mathcal{I}_{jk} &= \mathbb{E}\left\{\sum_{i=1}^{N}\left[\frac{(\mathrm{Y}_i - \mu_i)}{\mathbb{V}\mathrm{ar}(\mathrm{Y}_i)}\mathrm{X}_{ij}\left(\frac{d\mu_i}{d\eta_i}\right)\right]\sum_{i=1}^{N}\left[\frac{(\mathrm{Y}_i - \mu_i)}{\mathbb{V}\mathrm{ar}(\mathrm{Y}_i)}\mathrm{X}_{ik}\left(\frac{d\mu_i}{d\eta_i}\right)\right]\right\} \\ &= \sum_{i=1}^{N}\frac{\mathbb{E}[(\mathrm{Y}_i - \mu_i)^2 \mathrm{X}_{ij}\mathrm{X}_{ik}]}{[\mathbb{V}\mathrm{ar}(\mathrm{Y}_i)]^2}\left(\frac{d\mu_i}{d\eta_i}\right)^2\end{aligned}$$

because $\mathbb{E}[(\mathrm{Y}_i - \mu_i)(\mathrm{Y}_l - \mu_l)] = 0$ for $i \neq l$.

Since $\mathbb{E}[(\mathrm{Y}_i - \mu_i)^2] = \mathbb{V}\mathrm{ar}(\mathrm{Y}_i)$

$$\mathcal{I}_{jk} = \sum_{i=1}^{N}\frac{\mathrm{X}_{ij}\mathrm{X}_{ik}}{\mathbb{V}\mathrm{ar}(\mathrm{Y}_i)}\left(\frac{d\mu_i}{d\eta_i}\right)^2 \tag{3.1.6}$$