Solution

Let $\tilde{\boldsymbol{y}}$ be equal to $\boldsymbol{y}$ with $\boldsymbol{y}_i$ replaced by $\hat{y}_i^{-i}$. Then $\boldsymbol{H}\tilde{\boldsymbol{y}} = \hat{\boldsymbol{y}}^{-i}$, and

$$\hat{\boldsymbol{y}} - \hat{\boldsymbol{y}}^{-i} = \boldsymbol{H}(\boldsymbol{y} - \tilde{\boldsymbol{y}}) = \boldsymbol{H}(y_i - \hat{y}_i^{-i})\boldsymbol{e_i}$$

and looking at the $\boldsymbol{i}$-th entry we find

$$\hat{y}_i - \hat{y}_i^{-i} = h_{ii}(y_i - \hat{y}_i^{-i}).$$

or equivalently

$$(y_i - \hat{y}_i^{-i})(1 - h_{ii}) = y_i - \hat{y}_i.$$

Then we find

$$p\hat{\sigma}^2 D_i = |\hat{\boldsymbol{y}} - \hat{\boldsymbol{y}}^{-i}|^2 = (y_i - \hat{y}_i^{-i})^2 \boldsymbol{e_i}^\top \boldsymbol{H}^2 \boldsymbol{e_i} = \frac{h_{ii}}{(1 - h_{ii})^2} r_i^2 = \frac{h_{ii}}{1 - h_{ii}} \hat{\sigma}^2 r_{0i}^2$$

and done. One sees that Cook's distance statistic D_i is unusually large at high leverage points with large residuals.