Proof

Now to find Var(a(Y)), we calculate

$$rac{d^2f(y| heta)}{d heta^2} = [a(y)b''(heta) + c''(heta)]f(y| heta) + [a(y)b'(heta) + c'(heta)]^2f(y| heta).$$

Using (1.5.5), the second term on the right hand side can be written as

$$b'(\theta)^2 \{a(y) - \mathbb{E}[a(Y)]\}^2 f(y|\theta)$$

Consequently, we have

$$0 = \int rac{d^2 f(y| heta)}{d heta^2} dy = b''(heta) \mathbb{E}[a(\mathrm{Y})] + c''(heta) + [b'(heta)]^2 \mathbb{V}ar(a(\mathrm{Y}))]$$

since
$$\int \{a(y) - \mathbb{E}[a(\mathrm{Y})]\}^2 f(y; heta) dy = \mathbb{V}ar[a(\mathrm{Y})]$$
. Hence

$$\mathbb{V}ar[a(Y)] = \frac{b''(\theta)c'(\theta) - c''(\theta)b'(\theta)}{b'(\theta)^3}$$
 (1.5.6)

1 of 2

2 of 2 1/22/24, 8:55 AM