Proof

Using the general result on the distribution of quadratic forms, we need to see what is the noncentrality parameter and find the rank of matrix $\mathbb{I} - \mathbf{H}$.

The distributions of residuals is $\mathbf{r} \sim \mathcal{N}(\mathbf{0}, \sigma^2(\mathbb{I} - \mathbf{H}))$.

Additionally, since $\mathbb{E}(\mathbf{r}) = \mathbf{0}$ the noncentrality parameter $\lambda = 0$. Moreover,

$$\begin{aligned} \operatorname{rank}(\mathbb{I} - \mathbf{H}) = \operatorname{tr}(\mathbb{I} - \mathbf{H}) = \operatorname{tr}(\mathbb{I}) - \operatorname{tr}(\mathbf{H}) &= N - \operatorname{tr}(\mathbf{X}(\mathbf{X}^\top \mathbf{X})^{-1}\mathbf{X}^\top) \\ &= N - \operatorname{tr}(\mathbb{I}_p) = N - p \end{aligned}$$

This is true because $\operatorname{rank}(\mathbf{H}) = \operatorname{tr}(\mathbf{H})$ and $\mathbf{H}$ has rank p (which is the $\operatorname{rank}(\mathbf{X})$): therefore the $\operatorname{tr}(\mathbf{H})$ can be seen as the $\operatorname{tr}(\mathbb{I}_p)$.

Which concludes the proof of this theorem.