

One way to show

sufficiency is using the definition,

which is what we just did with the two examples,

that was not particularly fun.

We can also show sufficiency using a much nicer method.

This is I guess the second key out of today's class.

We can also show sufficiency by using the Neyman-Fisher.

Let me use a different colour,

it means it's fairly important.

We're going to use the Neyman-Fisher Factorisation Criterion.

Instead of applying the definition by computing this distribution of my sample,  
given my sufficient statistic,

we can apply Neyman-Fisher Factorisation Criterion.

Essentially, it fires as this.

If my random variables  $X_i$  have some density function which depend on

$\theta$  and then we want to see a sufficient statistic  $T$  of  $x$ ,

so it's simply evaluated at my sample.

This statistic is efficient for

$\theta$  if and only if.

This implies that if I have this factorisation,

so  $L(x; \theta)$  is just my likelihood.

So what I mean my likelihood is the joint density of my observations,  $X_1$  to  $X_n$ .

We'll say this likelihood function of one.

When we have our ID, it's pretty much just a product of my individual density functions.

Here when my joint density of

my observations can be written as the following factorisation.

I take some function  $g$  of my sufficient statistic and my parameter  $\Theta$ .

We have one function,  $g$ ,

which depends on my sufficient statistic,

my primary  $\Theta$ , multiplied by some function  $h$ ,

which depends only on my data.

$h$  and  $g$  are positive functions here, it's sufficient.

We have this particular factorisation,

it means that  $T$  of  $x$  is sufficient for  $\Theta$ .

Instead of applying the definition,

all we have to do is compute the likelihood,

this thing here, show that it can be factored into this form.

One function that depends on my sufficient statistic and also my parameter,

$\Theta$ , multiplied by some function that only depends on my data  $x$ .

If it has this factorisation,

then it means that  $T$  of  $x$  is sufficient for  $\Theta$ .

Again, that's going to provide much simpler and directly

applying the definition as we did in that first question