§5.2: THE SECOND FUNDAMENTAL THEOREM

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April 16, 2021

ANNOUNCEMENTS

PREVIOUSLY

FTC:

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so:

1. ($\rightarrow$) If we have a graph of f and we can compute the area bounded by f on $[a, b]$, we can compute the change in an antiderivative F over the interval
2. ($\leftarrow$) If we can find an algebraic formula for F, we can compute the net signed area bounded by f

erf

PREVIEW ACTIVITY DISCUSSION

FROM INTEGRAL FUNCTIONS TO FTC II

Theorem (Fundamental Theorem of Calculus II)

If f is a continuous function and c is any constant, then f has a unique antiderivative A that satisfies $A(c) = 0$, and that antiderivative is given by the rule

$$A(x) = \int_c^x f(t)\,dt.$$

ACTIVITY 5.2.2

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UNDERSTANDING INTEGRAL FUNCTIONS

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where $F(c) = 0$. Then:

$$\frac{d}{dx}[F(x)] = \frac{d}{dx}\left[\int_c^x f(t)\,dt\right] = f(x).$$

Thus, we can understand the graph of F via f, even if an algebraic formula for F eludes us.

ACTIVITY 5.2.3

INVERSE PROCESSES (ALMOST)

Recall that if

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where $F(a) = 0$, then:

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Since f is an antiderivative of $\frac{d}{dt}[f(t)]$, we can apply FTC I:

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$$\int_a^x \frac{d}{dt}[f(t)]\,dt = f(t)\Big|_a^x = f(x) - f(a).$$

Thus, integration and differentiation are (almost) inverse processes, up to the constant $f(a)$.

ACTIVITY 5.2.4

INTEGRATION: THE BIG PICTURE

- Integrals give us net signed area of the functions we integrate over an interval
- FTC I: We can calculate exact values of the integral using any antiderivative
- FTC II: the integral function gives a unique antiderivative
- FTC consequence: integration and differentiation are almost inverse processes