

§1.5: INTERPRETING, ESTIMATING, AND USING THE DERIVATIVE

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ANNOUNCEMENTS

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PREVIEW ACTIVITY DISCUSSION

MEANING OF THE DERIVATIVE

- Derivative measures instantaneous rate of change of the output variable with respect to the input variable, regardless of what they represent—doesn't have to be change of position!
- **Units:** units on $y = f'(x)$ are units of y per unit of x .

EXAMPLE

Let W be the volume of water, in gallons, in a bathtub at time t , in minutes.

- (a) What are the meaning and units of $W'(t)$?
- (b) Suppose the bathtub is full of water at time t_0 , so that $W(t_0) > 0$. Subsequently, at time $t_p > t_0$, the plug is pulled. Is $W'(t)$ positive, negative, or zero:
 - For $t_0 < t < t_p$?
 - After the plug is pulled, but before the tub is empty?
 - When all the water has drained from the tub?

GOAL: MORE ACCURATE ESTIMATES OF THE DERIVATIVE

Two choices:

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The central difference gives a line closer to being parallel to the tangent.

ex: $y = -0.5x^2 + 4$, $a = 2$

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A cup of coffee has its temperature F (in degrees F) at time t given by the function $F(t) = 75 + 110e^{-0.05t}$, where time is measured in minutes.

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1. Use a central difference with $h = 0.01$ to estimate the value of $F'(10)$.
2. What are the units on the value of $F'(10)$? What is the practical meaning of the value of $F'(10)$?
3. Which do you expect to be greater: $F'(10)$ or $F'(20)$? Why?

ACTIVITIES 1.5.2-1.5.4