§1.8: THE TANGENT LINE APPROXIMATION

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ANNOUNCEMENTS

PREVIEW ACTIVITY DISCUSSION

When $g(x) = -x^2 + 3x + 2$, $g'(x) = -2x + 3$. What do we need to write the equation of a line?

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IP: let's find the tangent line of $f(x)$ at $a = 2$ and find an approximate answer

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- Often called the **local linearization** and denoted by $L(x)$

EXAMPLE

Let's approximate $\sqrt{4.01}$.

$f'(2) = 1/4$

ACTIVITY 1.8.2

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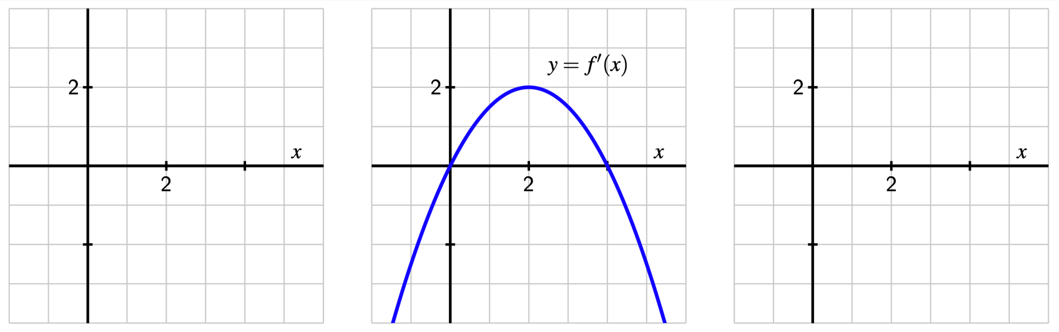
- $f''(a) < 0$: f is concave down, so the tangent line lies entirely above the curve ($L(x) > f(x)$)
- $f''(a) = 0$: the concavity is changing, and it depends on how (up to down, or down to up)

ACTIVITY 1.8.3



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- Don't lose sight of what it all means. We'll try to sprinkle in some contextual problems as we go.