§2.6: DERIVATIVES OF INVERSE FUNCTIONS

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ANNOUNCEMENTS

PREVIEW ACTIVITY DISCUSSION

Notation: by $f : A \to B$ we mean that f is a function with *domain* A and *codomain* B.

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Big Important Fact: $f : A \to B$ has an inverse if and only if f passes the vertical and horizontal line tests on A

EXAMPLES OF INVERSES

On appropriate domains:

- $y = a^x$ if and only if $x = \log_a(y)$
- $y = x^n$ if and only if $x = \sqrt[n]{y}$
- $y = \sin x$ if and only if $x = \sin^{-1} y = \arcsin y$
- etc

Let $f(x) = \ln x$. Let's find $f'(x)$. We know $e^{f(x)} =$

ACTIVITY 2.6.2

THE DERIVATIVE OF ARCSIN X

Question: On what domain is $f(x) = \sin x$ invertible?

ACTIVITIES 2.6.3-2.6.4

Theorem (Inverse Functions)

Suppose that f is differentiable with inverse g and that (x_0, y_0) *is a point that lies on the graph of f at which* $f'(x_0) \neq 0$. *Then*

$$g'(y_0) = \frac{1}{f'(x_0)}$$

More generally, for any z in the domain of g', *we have* $g'(z) = 1/f'(g(z))$.