§4.2: DETERMINING DISTANCE TRAVELED FROM VELOCITY

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ANNOUNCEMENTS

PREVIEW ACTIVITY DISCUSSION

FIRST: SIGMA NOTATION

Intent: develop a compact way of writing a sum of many terms.

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Examples:

$$\sum_{k=1}^{100} k = 1 + 2 + 3 + \cdots + 99 + 100$$

$$\sum_{n=1}^{50} \frac{n(n+1)}{2} = \frac{1(1+1)}{2} + \frac{2(2+1)}{2} + \cdots + \frac{49(49+1)}{2} + \frac{50(50+1)}{2}$$

ACTIVITY 4.2.2

Riemann Sums

BERNHARD RIEMANN

- 1826-1866
- Son of a German Lutheran pastor; saw mathematics as a service to God
- Riemannian geometry is the foundation of general relativity
- First to suggest dimensions greater than 3 or 4 to describe Creation
- Riemann Hypothesis – most famous unsolved math problem

GOAL: CALCULATE THE AREA UNDER A CURVE

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Suppose we want to approximate the area under a general (positive) curve given by $y = f(x)$ on the interval $[a, b]$. How do we start? *equal width*

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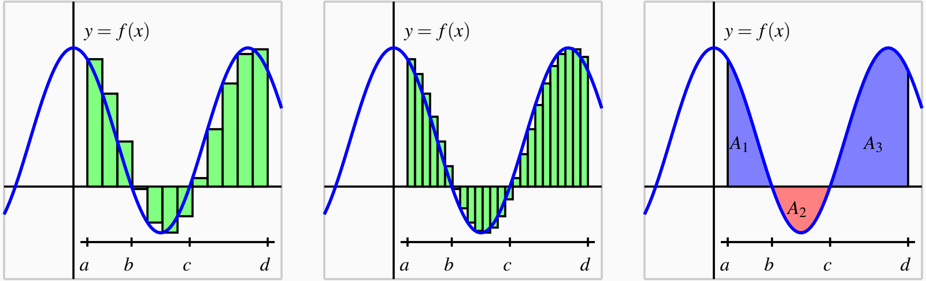
Another option: choose random points in the subintervals. This is more useful for theoretical discussions/proofs, while the above formulae are more useful in practice.

ACTIVITY 4.2.3

Helpful: The applet at `https://www.gvsu.edu/s/a9`

WHAT IF THE FUNCTION IS NEGATIVE?



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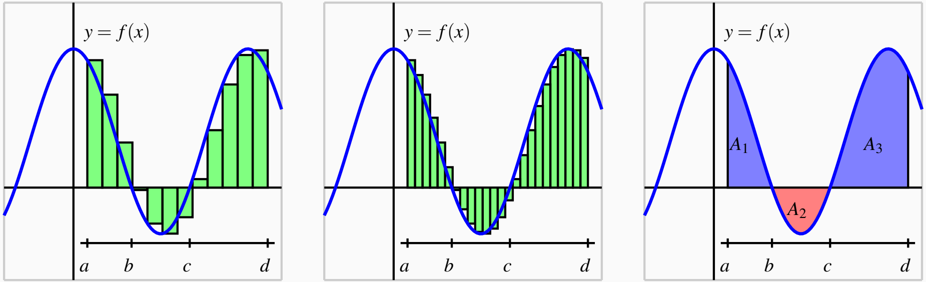

This is the **net signed area bounded by** f **over the interval** $[a, d]$.

ACTIVITY 4.2.4

Name: ______________________________ Section: ________

Math 152 Base Grade Progress

<table>
<tr><th rowspan="2">Base Grade</th><th colspan="2">Level 1</th><th colspan="2">Level 2</th><th rowspan="2">Calculate</th><th rowspan="2">Reflect</th><th rowspan="2">EPs</th><th rowspan="2">EH</th></tr>
<tr><th>Interpret, Approximate</th><th>Utilize</th><th>Interpret, Approximate</th><th>Utilize</th></tr>
<tr><td>A</td><td rowspan="2">Last two I/A Targets:
___
___</td><td rowspan="3">Last U-Target:
___</td><td>One more I/A-Target:
___</td><td>Last U-Target:
___</td><td rowspan="3">CD @ 80%
☐</td><td rowspan="3">The other of RH, RB:
___</td><td>≥ 88: ☐</td><td>[0.92,1]</td></tr>
<tr><td>B</td><td>One more I/A Target:
___</td><td>One more U-Target:
___</td><td>≥ 77: ☐</td><td>[0.8,0.92)</td></tr>
<tr><td>C</td><td>One more I/A Target
___</td><td>Two I/A Targets:
___, ___</td><td>Two U-Targets:
___, ___</td><td>≥ 66: ☐</td><td>[0.67,0.8)</td></tr>
<tr><td>D</td><td>Two I/A Targets:
___, ___</td><td>Three U-Targets:
___, ___, ___</td><td>X</td><td>X</td><td>CD @ 50%
☐</td><td>One of RH, RB:
___</td><td>≥ 55: ☐</td><td>[0.55,0.67)</td></tr>
</table>

	I-targets	A-targets	U-targets
Level 2	**ID** ☐ **II** ☐	**AD** ☐ **AI** ☐ **AG** ☐	**UF** ☐ **UM** ☐ **UC** ☐ **UR** ☐
Level 1	**ID** ☐ **II** ☐	**AD** ☐ **AI** ☐ **AG** ☐	**UF** ☐ **UM** ☐ **UC** ☐ **UR** ☐