§4.3: THE DEFINITE INTEGRAL

Dr. Mike Janssen

April 5, 2021

ANNOUNCEMENTS

-

PREVIEW ACTIVITY DISCUSSION

MOTIVATION AND DEFINITION

As n grows larger, L_n, M_n, and R_n approach the same value.

As n grows larger, L_n, M_n, and R_n approach the same value.

Definition

The definite integral of a continuous function f on the interval $[a, b]$, denoted $\int_a^b f(x)\,dx$, is the real number given by

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$ (for $i = 0, 1, \dots, n$), and x_i^* satisfies $x_{i-1} \leq x_i^* \leq x_i$ (for $i = 1, 2, \dots, n$).

WHAT DOES THE DEFINITE INTEGRAL GIVE US?

Answer: the integral $\int_a^b f(x)\,dx$ gives net signed area under the curve $y = f(x)$ between $x = a$ and $x = b$.

WHAT DOES THE DEFINITE INTEGRAL GIVE US?

Answer: the integral $\int_a^b f(x)\,dx$ gives net signed area under the curve $y = f(x)$ between $x = a$ and $x = b$.

Let's compute $\int_1^3 (x+2)\,dx$.

ACTIVITY 4.3.2

PROPERTIES OF THE DEFINITE INTEGRAL

If $f(x)$ and $g(x)$ are continuous functions and a, b, c, k are real numbers:

- $\int_a^a f(x)\,dx = 0$

PROPERTIES OF THE DEFINITE INTEGRAL

If $f(x)$ and $g(x)$ are continuous functions and a, b, c, k are real numbers:

- $\displaystyle\int_a^a f(x)\,dx = 0$
- $\displaystyle\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

PROPERTIES OF THE DEFINITE INTEGRAL

If $f(x)$ and $g(x)$ are continuous functions and a, b, c, k are real numbers:

- $\int_a^a f(x)\,dx = 0$
- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$
- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

PROPERTIES OF THE DEFINITE INTEGRAL

If $f(x)$ and $g(x)$ are continuous functions and a, b, c, k are real numbers:

- $\int_a^a f(x)\,dx = 0$
- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$
- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
- (Constant Multiple Rule) $\int_a^b k \cdot f(x)\,dx = k\int_a^b f(x)\,dx$

PROPERTIES OF THE DEFINITE INTEGRAL

If $f(x)$ and $g(x)$ are continuous functions and a, b, c, k are real numbers:

- $\int_a^a f(x)\,dx = 0$
- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$
- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
- (Constant Multiple Rule) $\int_a^b k \cdot f(x)\,dx = k \int_a^b f(x)\,dx$
- (Sum Rule) $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

ACTIVITY 4.3.3

AVERAGE VALUE

Theorem (Average Value of a Function)

If f is a continuous function on $[a, b]$, then its average value on $[a, b]$ is given by the formula

$$f_{AVG[a,b]} = \frac{1}{b-a} \cdot \int_a^b f(x)\,dx$$

ACTIVITY 4.3.4