

§1.3: THE DERIVATIVE OF A FUNCTION AT A POINT

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PREP ACTIVITY DISCUSSION

Recall: given a function f , its average rate of change is

$$AV_{[a,b]} = \frac{f(b) - f(a)}{b - a}$$

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What meaningful sentence did you write that explains how the average rate of change is connected to a slope?

Average \rightarrow Instantaneous

THE INFINITY PRINCIPLE STRIKES BACK

The **BIG IDEA of Calculus**: Do “easy” calculations over infinitesimally small intervals, and stitch the results into a calculation over a larger interval on which more complicated behavior occurs.

The Limit Definition of the Derivative

THE DEFINITION

Definition

Let f be a function and $x = a$ a value in the function's domain. We define the **derivative of f with respect to x evaluated at $x = a$** , denoted $f'(a)$, by the formula

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h},$$

provided this limit exists.

NOTES AND OBSERVATIONS

- If a function has a derivative at $x = a$ we say it is *differentiable* there.

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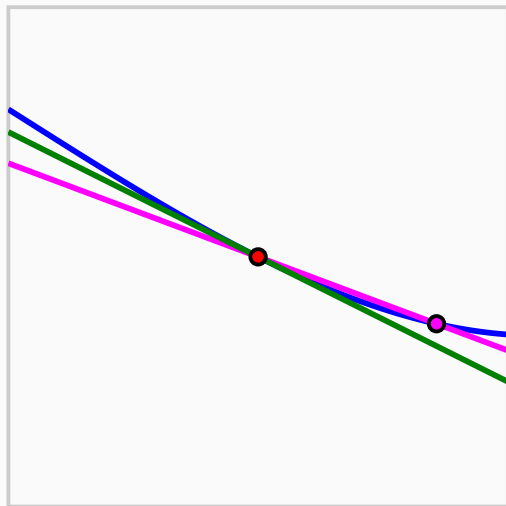
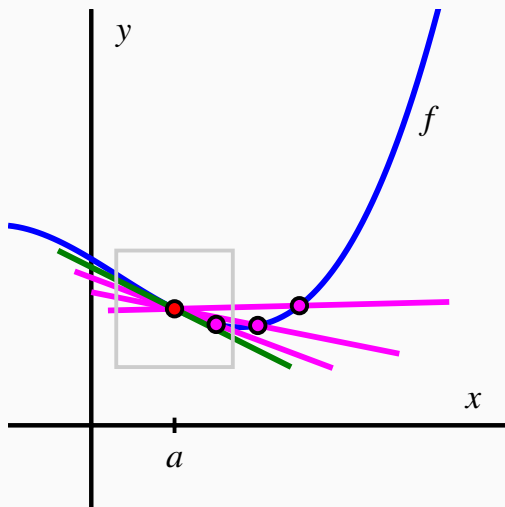
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- If a function has a derivative at $x = a$ we say it is *differentiable* there.
- The derivative at a point generalizes the notion of instantaneous velocity
- What are the units on $f'(a)$?
- When computing $f'(a)$ we are computing a limit of the slopes of lines. Which lines?



A sequence of secant lines approaching a tangent line.

EXAMPLE

Let's compute $g'(2)$ given $g(x) = 3x^2 + x - 1$.

Activities 1.3.2–1.3.4

ACTIVITY 1.3.2

ACTIVITY 1.3.3

ACTIVITY 1.3.4