§1.1-1.2: VELOCITY/LIMITS

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WHAT IS THE ACTIVITIES UPLOAD ASSIGNMENT?

Each day we'll work on activities from the workbook you bought at the campus store.

At the end of the week, you scan and upload your workbook pages from the week (if participating in class on Fridays, include that day's activities as well).

Thinking about Velocity

PA 1.1.1

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- Units: t is time, $s(t)$ is distance
- Average velocity from $t = a$ to $t = b$:

$$AV_{[a,b]} = \frac{s(b) - s(a)}{b - a}$$

ACTIVITY 1.1.2

> *To shed light on any continuous shape, object, motion, process, or phenomenon—no matter how wild and complicated it may appear—reimagine it as an infinite series of simpler parts, analyze those, and then add the results back together to make sense of the original whole.*

–Steven Strogatz, *Infinite Powers*

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- Question: What can we measure in an instant (e.g., in a photograph)?
- **However:** velocity is a *rate of change*–at any given *instant*, I am not moving!
- The Infinity Principle gives us a way of approximating something like “instantaneous velocity”

...BUT WE CAN DEFINE IT ANYWAY

- The idea: to approximate instantaneous velocity at $t = a$, compute the average velocity over the interval $[a, a + h]$, where h is some arbitrarily (infinitesimally?) small number that is allowed to vary

$h < 0$?

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- The idea: to approximate instantaneous velocity at $t = a$, compute the average velocity over the interval $[a, a + h]$, where h is some arbitrarily (infinitesimally?) small number that is allowed to vary

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- That is:

$$IV_{t=a} \approx AV_{[a,a+h]} = \frac{s(a+h) - s(a)}{a + h - a} = \frac{s(a+h) - s(a)}{h}$$

EXAMPLE

In a time of t seconds, a particle moves a distance of $s(t) = 4t^2 + 3$ meters from its starting point. Find an expression for the average velocity on $[1, 1+h]$, and use it to estimate the instantaneous velocity of the particle at $t = 1$.

ACTIVITY 1.1.4

Section 1.2: The Notion of Limit

PREVIEW ACTIVITY 1.2.1 DISCUSSION

WHY DO WE NEED LIMITS?

- Limits give a precise way to talk about trends in function values.
- They answer the question: what is $f(x)$ (output) doing as x (input) approaches some value?
- **Note:** $f(a)$ need not be defined for f to have a limit at $x = a$.

Definition

Given a function f, a fixed input $x = a$, and a real number L, we say that f **has limit** L **as** x **approaches** a, and write

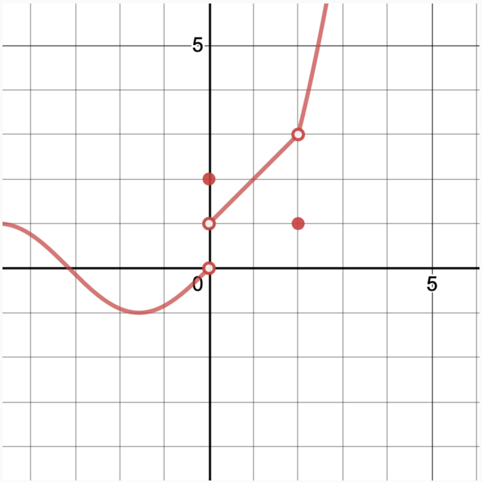
$$\lim_{x \to a} f(x) = L$$

provided we can make $f(x)$ *as close to* L *as we like* by taking x *sufficiently close (but not equal to)* a.

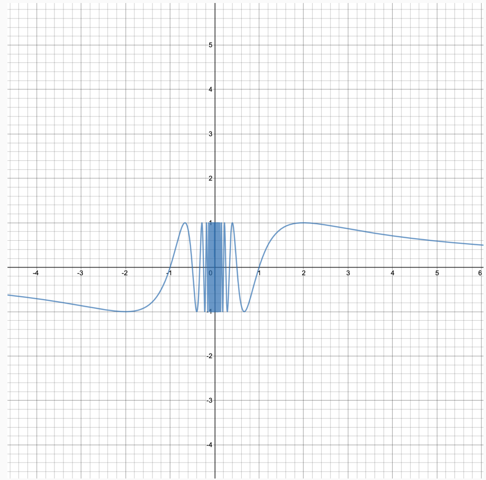
EXPLORATION

For the function $f(x)$ pictured, find the following limits, if they exist:

- $\lim_{x \to 2} f(x)$
- $\lim_{x \to 0} f(x)$



ANOTHER APPROACH



The function is $f(x) = \sin \frac{\pi}{x}$

$10^{-k}, 3 \cdot 10^{-k}$

ACTIVITY 1.2.2

INSTANTANEOUS VELOCITY IS THE LIMIT OF THE AVERAGES

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- Recall our discussion of the **Infinity Principle**

ACTIVITIES 1.2.3–1.2.4

FOR NEXT TIME

- Pass the ALEKS!
- Buy the calculus bundle from the campus store
- Edfinity Section 1.1 due Wednesday