§4.4: THE FUNDAMENTAL THEOREM OF CALCULUS

Dr. Mike Janssen

April 9, 2021

ANNOUNCEMENTS

-

PREVIEW ACTIVITY DISCUSSION

THE CONNECTION

Given a 1-dimensional velocity described by $v(t)$ corresponding to a displacement function $s(t)$, we have seen that the distance traveled between $t = a$ and $t = b$ is given by

$$s(b) - s(a)$$

THE CONNECTION

Given a 1-dimensional velocity described by $v(t)$ corresponding to a displacement function $s(t)$, we have seen that the distance traveled between $t = a$ and $t = b$ is given by

$$s(b) - s(a) = \int_a^b v(t)\,dt$$

THE CONNECTION

Given a 1-dimensional velocity described by $v(t)$ corresponding to a displacement function $s(t)$, we have seen that the distance traveled between $t = a$ and $t = b$ is given by

$$s(b) - s(a) = \int_a^b v(t)\,dt$$

Theorem (Fundamental Theorem of Calculus)

If f is continuous on $[a, b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

THE CONNECTION

Given a 1-dimensional velocity described by $v(t)$ corresponding to a displacement function $s(t)$, we have seen that the distance traveled between $t = a$ and $t = b$ is given by

$$s(b) - s(a) = \int_a^b v(t)\, dt$$

Theorem (Fundamental Theorem of Calculus)

If f is continuous on $[a, b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)\, dx = F(b) - F(a) = F(x)\Big|_a^b$$

THE CONNECTION

Given a 1-dimensional velocity described by $v(t)$ corresponding to a displacement function $s(t)$, we have seen that the distance traveled between $t = a$ and $t = b$ is given by

$$s(b) - s(a) = \int_a^b v(t)\, dt$$

Theorem (Fundamental Theorem of Calculus)

If f is continuous on $[a, b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)\, dx = F(b) - F(a) = F(x)\Big|_a^b$$

The Antiderivative Problem: given a function f, find all antiderivatives F of f.

ACTIVITY 4.4.2-4.4.3

THE TOTAL CHANGE THEOREM

In the context of an object moving with velocity v, the integral gives the total displacement of the object. That is, it gives the total change in the antiderivative.

In the context of an object moving with velocity v, the integral gives the total displacement of the object. That is, it gives the total change in the antiderivative.

Theorem (Total Change Theorem)

If f is continuously differentiable on $[a, b]$ with derivative f' then

$$f(b) - f(a) = \int_a^b f(x)\,dx.$$

THE TOTAL CHANGE THEOREM

In the context of an object moving with velocity v, the integral gives the total displacement of the object. That is, it gives the total change in the antiderivative.

Theorem (Total Change Theorem)

If f is continuously differentiable on $[a, b]$ with derivative f' then

$$f(b) - f(a) = \int_a^b f(x)\,dx.$$

That is, the definite integral of the rate of change of a function on $[a, b]$ is the total change of the function itself on $[a, b]$.

ACTIVITY 4.4.4