§1.6: THE SECOND DERIVATIVE

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PREVIEW ACTIVITY DISCUSSION

Definition

Given a function $f(x)$ on an interval (a, b), we say that f is **increasing on** (a, b) provided that for all $x < y$ on (a, b) we have $f(x) < f(y)$.
Similarly, we say f is **decreasing on** (a, b) provided that for all $x < y$ on (a, b) we have $f(x) > f(y)$.

INCREASING, DECREASING AND THE BIG QUESTION

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The derivative is a function: how does *it* change? If it's increasing, what does that mean for the original function? *How* is it increasing?

THE SECOND DERIVATIVE

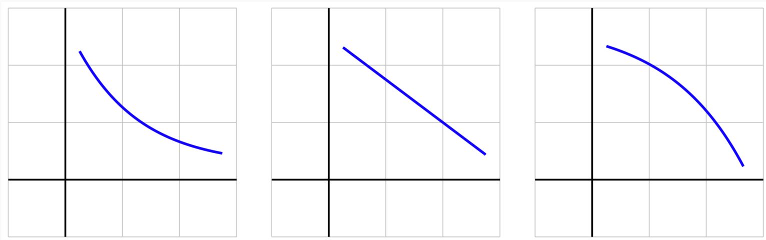
Definition

Given a function f, the **second derivative of** f is the function

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

for all x for which the limit exists.

CHANGE OF CHANGE



All three graphs show a decreasing function. How would you describe the rate of decrease for each: increasing, decreasing, or neither?

Definition

Let f be differentiable on (a, b). Then f is **concave up** on (a, b) if and only if f' is increasing on (a, b). We say f is **concave down** on (a, b) if and only if f' is decreasing on (a, b).

Activities 1.6.2-1.6.4

ACTIVITY 1.6.2

ACTIVITY 1.6.3

ACTIVITY 1.6.4