§1.7: LIMITS, CONTINUITY, AND DIFFERENTIABILITY

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PREVIEW ACTIVITY DISCUSSION

LEFT/RIGHTHAND LIMITS

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Definition

We say that f has limit L_1 as x **approaches** a **from the left** and write

$$\lim_{x \to a^-} f(x) = L_1$$

provided we can make the value of $f(x)$ as close as we like to L_1 by taking x sufficiently close to a while always having $x < a$. We call L_1 the **left-hand limit** of f as x approaches a.

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$$\lim_{x \to a^+} f(x) = L_2.$$

EXAMPLE/BIG IMPORTANT FACT

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110 110 (-100,-100)(100,100) (-100,-100)(

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Fact: $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$

ACTIVITY 1.7.2

125 125 (-100,-100)(100,100) (-100,-100)

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A function f is **continuous at** $x = a$ provided that

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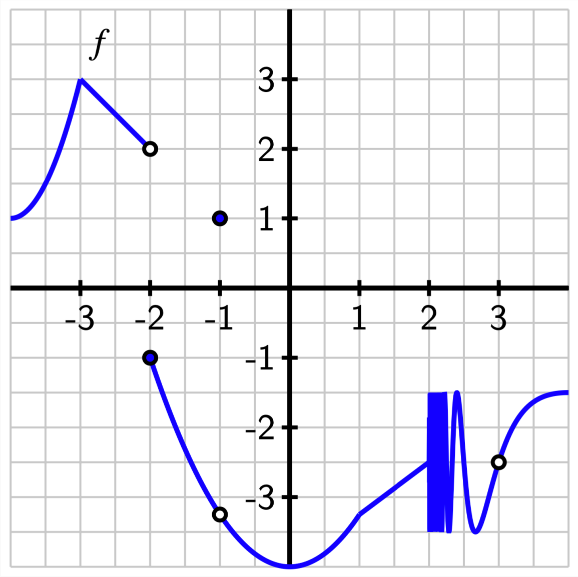
What does this mean?

cont. implies $\lim = $ *fcn*

ACTIVITY 1.7.3



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THE RELATIONSHIP BETWEEN DIFFERENTIABILITY AND CONTINUITY

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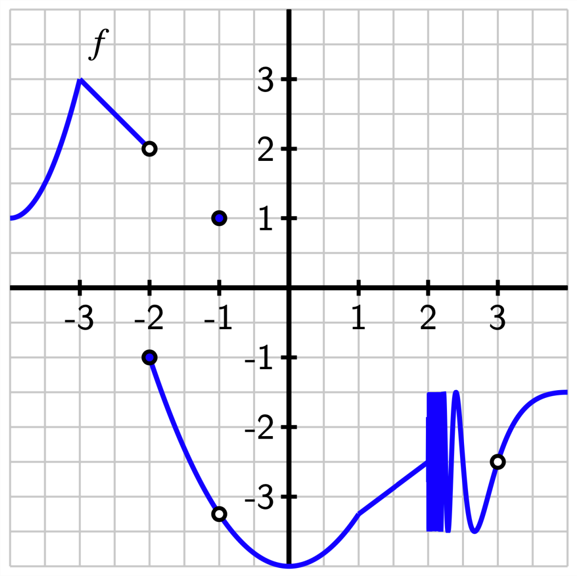
No.

ACTIVITY 1.7.4



REMINDERS

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