# §1.7: LIMITS, CONTINUITY, AND DIFFERENTIABILITY

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# PREVIEW ACTIVITY DISCUSSION

# LEFT/RIGHTHAND LIMITS

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**Definition**

We say that $f$ has limit $L_1$ as $x$ **approaches** $a$ **from the left** and write

$$\lim_{x \to a^-} f(x) = L_1$$

provided we can make the value of $f(x)$ as close as we like to $L_1$ by taking $x$ sufficiently close to $a$ while always having $x < a$. We call $L_1$ the **left-hand limit** of $f$ as $x$ approaches $a$.

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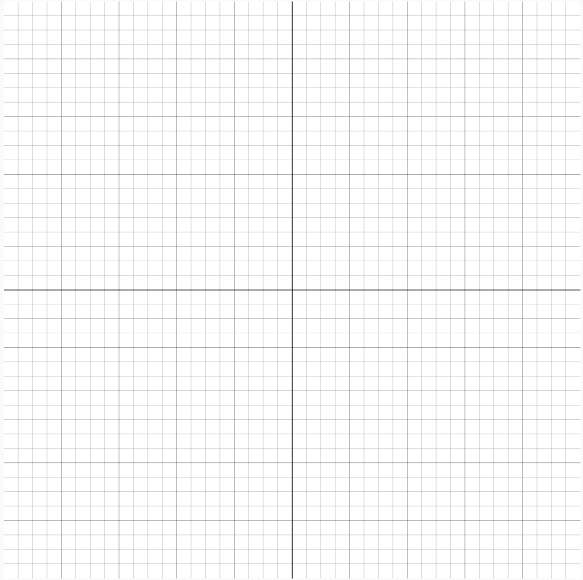
$$\lim_{x \to a^+} f(x) = L_2.$$

## EXAMPLE/BIG IMPORTANT FACT

**Example:**



**Fact:** $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$

# ACTIVITY 1.7.2

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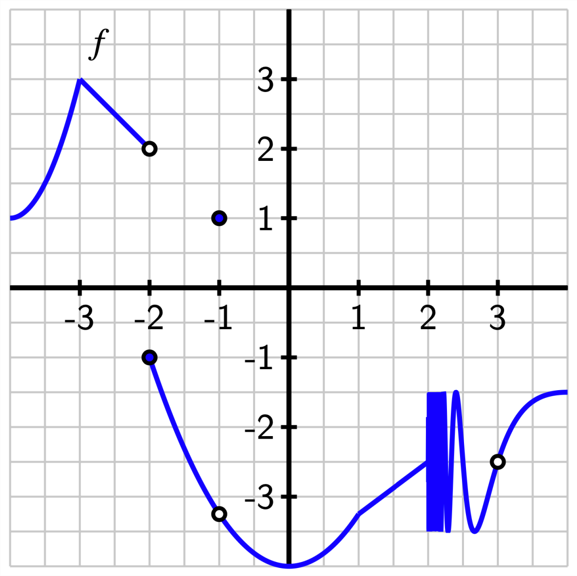

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A function $f$ is **continuous at** $x = a$ provided that

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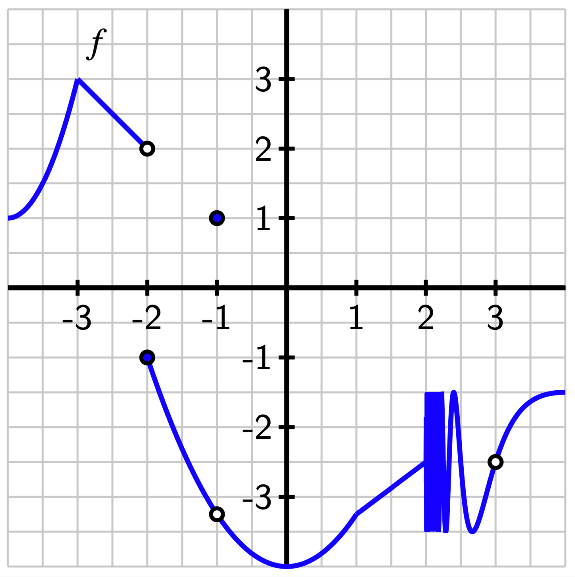
What does this mean?

*cont. implies* $\lim = $ *fcn*

## ACTIVITY 1.7.3



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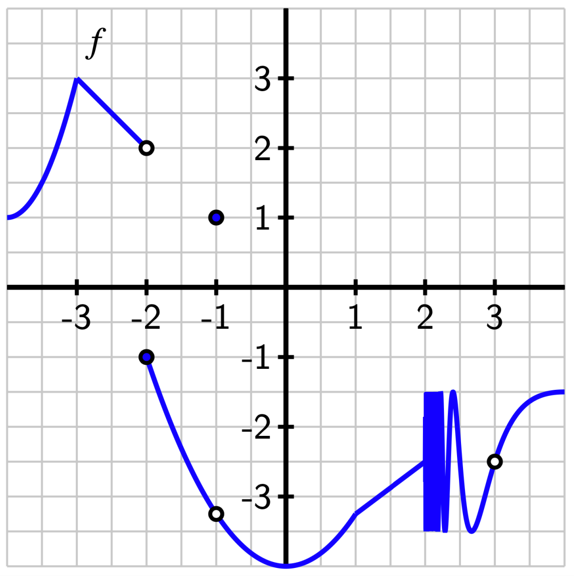
**No.**

# ACTIVITY 1.7.4



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