§1.2-1.3: LIMITS AND THE DERIVATIVE AT A POINT

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PREVIEW ACTIVITY 1.2.1 DISCUSSION

WHY DO WE NEED LIMITS?

- Limits give a precise way to talk about trends in function values.
- They answer the question: what is $f(x)$ (output) doing as x (input) approaches some value?
- Example: What is $AV_{[a,a+h]}$ doing as $h \to 0$?
- **Note:** $f(a)$ need not be defined for f to have a limit at $x = a$.

THE DEFINITION

Definition

Given a function f, a fixed input $x = a$, and a real number L, we say that f **has limit** L **as** x **approaches** a, and write

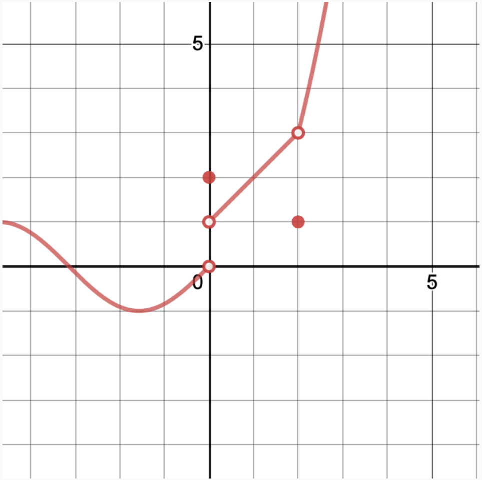
$$\lim_{x \to a} f(x) = L$$

provided we can make $f(x)$ *as close to* L *as we like* by taking x *sufficiently close (but not equal to)* a.

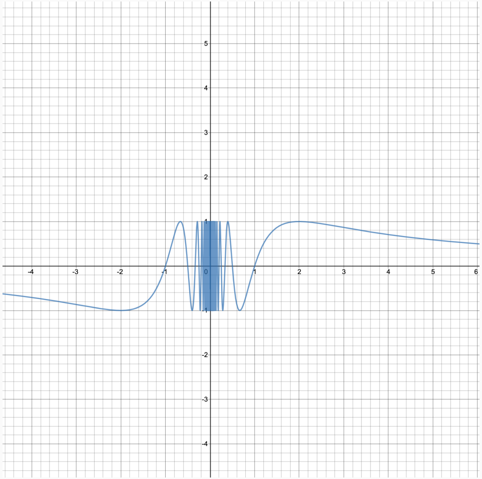
EXPLORATION

For the function $f(x)$ pictured, find the following limits, if they exist:

- $\lim_{x \to 2} f(x)$
- $\lim_{x \to 0} f(x)$



ANOTHER APPROACH



The function is $f(x) = \sin \frac{\pi}{x}$

$10^{-k}, 3 \cdot 10^{-k}$

ACTIVITY 1.2.2

INSTANTANEOUS VELOCITY IS THE LIMIT OF THE AVERAGES

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- Recall our discussion of the **Infinity Principle**

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ACTIVITIES 1.2.3–1.2.4