§2.8: USING DERIVATIVES TO EVALUATE LIMITS

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ANNOUNCEMENTS

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PREVIEW ACTIVITY DISCUSSION

MOTIVATION

Observe: the limit definition of the derivative yields an indeterminate form, $\frac{0}{0}$.

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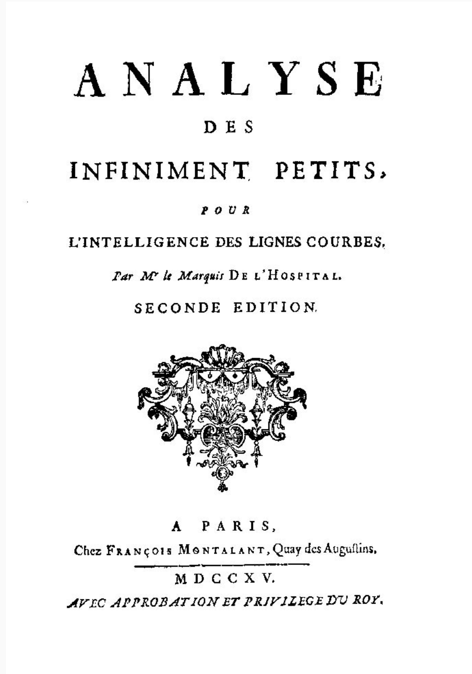
$$\lim_{x \to 0} \frac{\sin x}{x}$$

also have the form $\frac{0}{0}$.

GUILLAUME DE L'HÔPITAL

- 1661-1704
- First calculus textbook: *Infinitesimal calculus with applications to curved lines*



ANALYSE

DES

INFINIMENT PETITS,

POUR

L'INTELLIGENCE DES LIGNES COURBES,

Par Mr le Marquis DE L'HOSPITAL.

SECONDE EDITION.

A PARIS,

Chez FRANÇOIS MONTALANT, Quay des Augustins.

MDCCXV.

AVEC APPROBATION ET PRIVILEGE DU ROY.

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L'HÔPITAL'S RULE

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Theorem (L'Hôpital's Rule I)

Let f and g be differentiable at a, with $f(a) = g(a) = 0$ and $g'(a) \neq 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

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Warning: you need to make sure the hypotheses of the theorem are met before applying it!

ACTIVITIES 2.8.2-2.8.3

LIMITS INVOLVING $\pm\infty$

Consider $f(x) = \frac{1}{x^2}$; what is happening as $x \to 0^+$?

WHAT WE MEAN

What do we mean by the statements

$$\lim_{x\to 0^+} \frac{1}{x^2} = \infty, \quad \lim_{x\to\infty} \frac{1}{x^2} = 0, \quad \text{and} \quad \lim_{x\to\infty} e^x = \infty?$$

end behavior

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Theorem (L'Hôpital's Rule II)

If f and g are differentiable and both approach 0 or both approach $\pm\infty$ as $x \to a$ (where $-\infty \leq a \leq \infty$), then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right exists.

ACTIVITY 2.8.4

A FINAL NOTE

Suppose $\frac{f(x)}{g(x)}$ has the form ∞/∞ as $x \to \infty$. What is meant by:

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- $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \infty$?
- $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = c$, where c is some finite number?