§3.1: USING DERIVATIVES TO IDENTIFY EXTREME VALUES

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ANNOUNCEMENTS

RECALL: CRITICAL NUMBERS AND FDT

Definition

We say that a function f has a critical number at $x = c$ provided that

1. c is in the domain of f, and
2. $f'(c) = 0$ OR $f'(c)$ is undefined.

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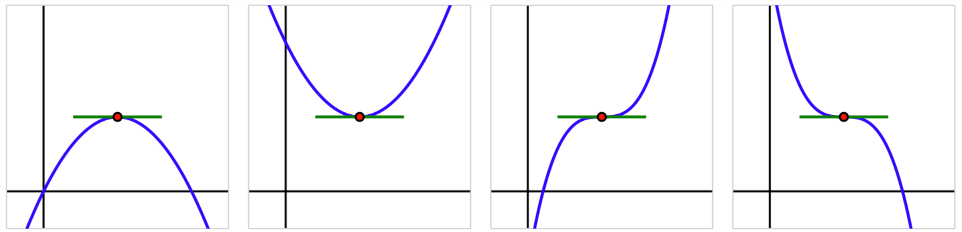
Theorem (First Derivative Test)

If p is a critical number of a continuous function f that is differentiable near p (except possibly at $x = p$), then f has a relative maximum at p if and only if f' changes sign from positive to negative at p, and f has relative minimum at p if and only if f' changes sign from negative to positive at p.

ACTIVITY 3.1.2

SECOND DERIVATIVES



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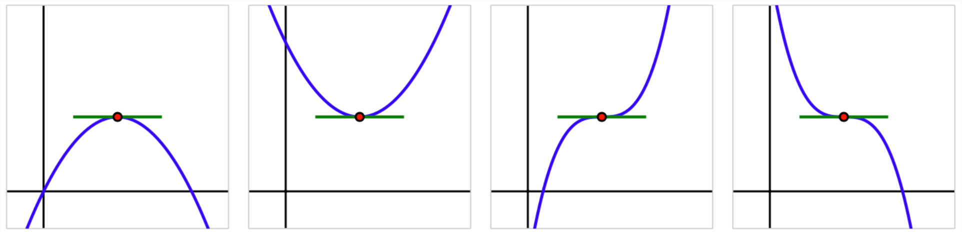

Theorem (Second Derivative Test)

If p is a critical number of a continuous function f such that $f'(p) = 0$ and $f''(p) \neq 0$, then f has a relative maximum at p if $f''(p) < 0$, and f has a relative minimum at p if $f''(p) > 0$.

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Question: What is required for a function to *change* concavity?

ACTIVITIES 3.1.3-3.1.4

§3.2: Using Derivatives to Describe Families of Functions

PREVIEW ACTIVITY DISCUSSION

Goal: Examine *families* of functions in terms of certain *parameters*, certain constants that repeatedly appear in applications of a given function.

EXAMPLE

Consider the function

$$f(x) = a\sin(bx), \quad a, b \neq 0$$

on the interval $[0, 2\pi]$. Let's:

- find the critical numbers of f and construct a first derivative sign chart, taking into account the possible signs of a and b
- Find p'' and construct a second derivative sign chart
- Sketch and label typical graphs of f

ACTIVITY 3.2.2-3.2.4