§1.2-1.3: LIMITS AND THE DERIVATIVE AT A POINT

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January 20, 2021

RECALL THE DEFINITION OF A LIMIT

Definition

Given a function f, a fixed input $x = a$, and a real number L, we say that f **has limit** L **as** x **approaches** a, and write

$$\lim_{x \to a} f(x) = L$$

provided we can make $f(x)$ *as close to* L *as we like* by taking x *sufficiently close (but not equal to)* a.

ACTIVITY 1.2.2

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- Recall our discussion of the **Infinity Principle**

ACTIVITIES 1.2.3–1.2.4

Bonus Activities

PREP ACTIVITY DISCUSSION

Recall: given a function f, its average rate of change is

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What meaningful sentence did you write that explains how the average rate of change is connected to a slope?

Average → Instantaneous

The **BIG IDEA of Calculus**: Do “easy” calculations over infinitesimally small intervals, and stitch the results into a calculation over a larger interval on which more complicated behavior occurs.

The Limit Definition of the Derivative

THE DEFINITION

Definition

Let f be a function and $x = a$ a value in the function's domain. We define the **derivative of f with respect to x evaluated at** $x = a$, denoted $f'(a)$, by the formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided this limit exists.

- If a function has a derivative at $x = a$ we say it is *differentiable* there.

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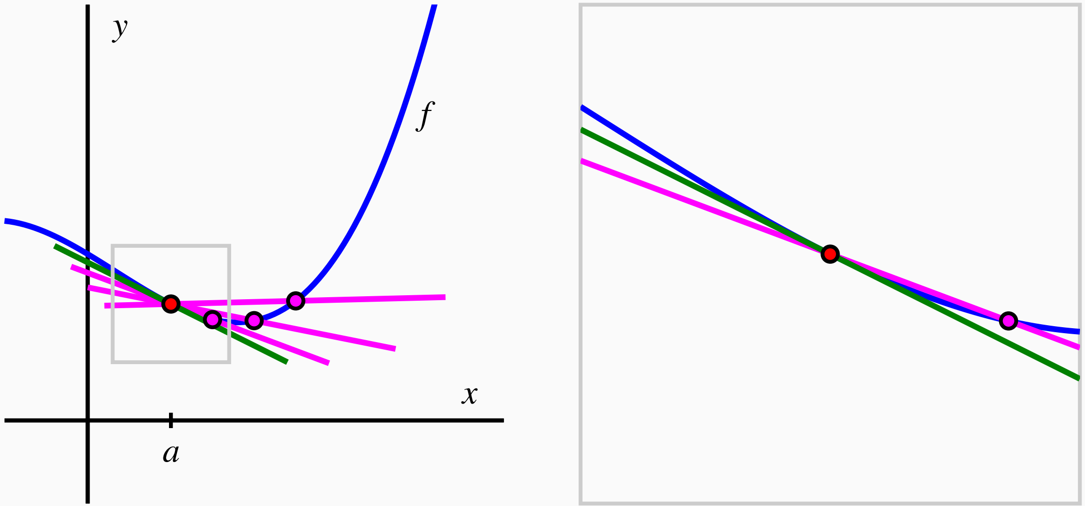
- What are the units on $f'(a)$?
- When computing $f'(a)$ we are computing a limit of the slopes of lines. Which lines?



A sequence of secant lines approaching a tangent line.

EXAMPLE

Let's compute $g'(2)$ given $g(x) = 3x^2 + x - 1$.

Activities 1.3.2–1.3.4

ACTIVITY 1.3.2

FOR NEXT TIME

- Buy the calculus bundle from the campus store
- Edfinity Section 1.1 due today, Section 1.2 due Friday
- Do the Section 1.4 Prep assignment by 8am Monday