# §1.3: THE DERIVATIVE OF A FUNCTION AT A POINT

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# PREP ACTIVITY DISCUSSION

Recall: given a function $f$, its average rate of change is

$$AV_{[a,b]} = \frac{f(b) - f(a)}{b - a}$$

What meaningful sentence did you write that explains how the average rate of change is connected to a slope?

# Average $\to$ Instantaneous

The **BIG IDEA of Calculus**: Do “easy” calculations over infinitesimally small intervals, and stitch the results into a calculation over a larger interval on which more complicated behavior occurs.

# The Limit Definition of the Derivative

**Definition**

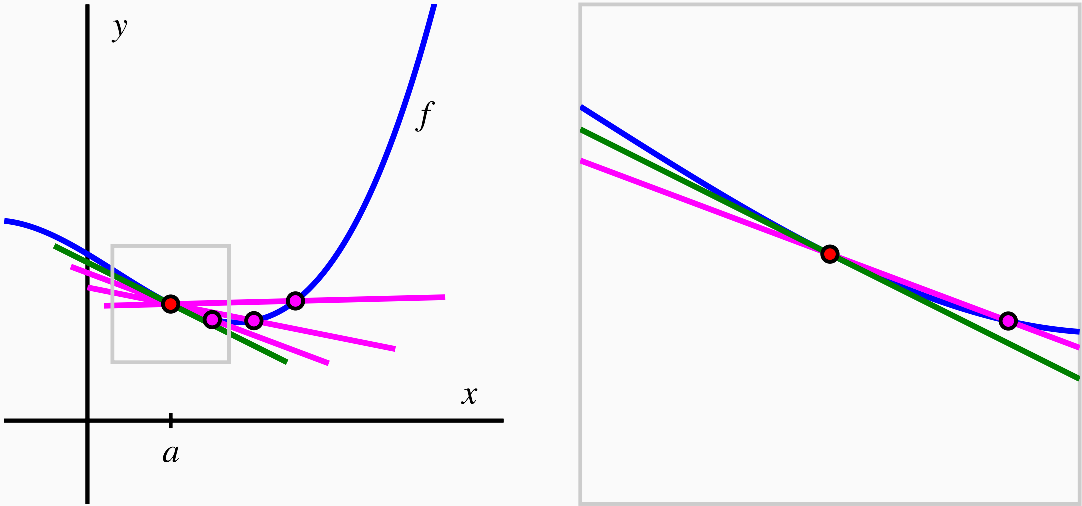
Let $f$ be a function and $x = a$ a value in the function's domain. We define the **derivative of** $f$ **with respect to** $x$ **evaluated at** $x = a$, denoted $f'(a)$, by the formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided this limit exists.

## NOTES AND OBSERVATIONS

- If a function has a derivative at $x = a$ we say it is *differentiable* there.
- The derivative at a point generalizes the notion of instantaneous velocity
- What are the units on $f'(a)$?
- When computing $f'(a)$ we are computing a limit of the slopes of lines. Which lines?




A sequence of secant lines approaching a tangent line.

# EXAMPLE

Let's compute $g'(2)$ given $g(x) = 3x^2 + x - 1$.

# ACTIVITY 1.3.2

# ACTIVITY 1.3.3

# ACTIVITY 1.3.4