Skill Mastery Quiz 6

Communicating in Math (MTH 210-01)
Winter 2020

Name:

P3-3 The following statement is incorrect:

For each integer n, if n is odd, then $(n^2 + 1)$ is a prime number.

Show the statement is false using a counterexample. You should clearly explain why the counterexample you found shows the statement is false. (If you don't remember what a prime number is just ask!)

This statement is false and there are many counterexamples. As one, let $n = 3$. Then $n \in \mathbb{Z}$ and n is odd (since $3 = 2(1) + 1$ and $1 \in \mathbb{Z}$). Additionally, $n^2 + 1 = 3^2 + 1 = 10$ which is not prime (since $10 = 5 \cdot 2$). Thus we have found an integer for which the hypothesis is true and the conclusion is false, making the statement false.

P1-2 Consider the following statement:

For all natural numbers p and q, if p and q are twin primes other than 3 and 5 , then $pq + 1$ is a perfect square and 36 divides $pq + 1$.

State what you would assume in a direct proof.

Assume that p and q are natural numbers and p and q are twin primes other than 3 and 5. (Basically, we assume the hypothesis.)

State what you would assume in a proof by contradiction.

Assume that there exist natural numbers p and q such that p and q are twin primes other than 3 and 5 and that $pq + 1$ is not a perfect square or 36 does not divide $pq + 1$.

P4-1 Consider the following proposition and proof. Is the proof correct? If not, explain any major mathematical errors. If so, does the proof meet our writing guidelines?

Theorem 1. *If a is an odd integer then $3a+2$ is an odd integer.*

Proof. We will use a direct proof. For $3a+2$ to be an odd integer there must exist an integer n such that

$$3a + 2 = 2n + 1.$$

By subtracting 2 from both sides of this equation we obtain

$$\begin{aligned} 3a &= 2n - 1 \\ &= 2(n-1) + 1. \end{aligned}$$

By the closure properties of integers, $n-1$ is an integer, and hence, the last equation implies that a is an odd integer. This proves that if a is an odd integer then $3a+2$ is an odd integer. □

Though the statement is true, the proof is wrong for multiple reasons. For one, they start by assuming the conclusion (by saying there exists an integer n such that $3a + 2 = 2n + 1$. Moreover, they only show then that $3a$ is odd, not that a is odd (though showing a is odd so that is the hypothesis anyway, which isn't what they should be trying to show!)