# Skill Mastery Quiz 3

Communicating in Math (MTH 210-01)
Winter 2020

Name:

L1-3 Consider the following conditional statement:

If $p$ is a prime number then $p = 2$ or $p$ is an odd number.

Identify the hypothesis and the conclusion of the conditional satement. The hypothesis is "$p$ is a prime number" and the conclusion is "$p = 2$ or $p$ is an odd number".

Assume the above conditional statement is true. Assuming *only* the conditional statement and that a given $p$ is odd what can you conclude (if anything)? Explain your answer.

Given that $p$ is odd we know the conclusion of the conditional statement is true. But, if we look in a truth table, knowing the conclusion is true says nothing about the hypothesis.

L2-3 State the definition of even integer precisely:

An integer $n$ is even provided that...

there exists an integer $q$ such that $n = 2q$.

Outline a proof that if $x, y \in \mathbb{Z}$ are even integers then $xy$ is even. (Make sure to include key details - like what things are integers.)

Suppose $x$ is even and $y$ is even. Then there exist integers $a$ and $b$ such that $x = 2a$ and $y = 2b$. Then $xy = (2a)(2b) = 4ab$ by substitution and algebra. By the distributive property $xy = 2(2ab)$. Let $q = 2ab$. Note that $q$ is an integer because $a$ and $b$ are integers and the integers are closed under addition. Then $xy = 2q$ for the integer $q$ and so $xy$ is an even integer.

L3-2 Construct a truth table for $P \implies (Q \wedge R)$.

| $P$ | $Q$ | $R$ | $Q \wedge R$ | $P \to (Q \wedge R)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | T | F | T |
| T | T | F | F | F |
| T | F | F | F | F |
| F | T | F | F | T |
| F | F | F | F | T |

L4-2 Describe what the set $\{x \in \mathbb{R} \mid 3 \leq x \leq 5\}$ is in words. Then write what the set is in roster notation or explain why you can not.

This is the set of all real numbers between 3 and 5 (including 3 and 5). We can not write this set in roster notation since between any two real numbers there are infinitely many real numbers.

L5-1 Write a useful negation of the following statement:

$$\text{There exists } x \in \mathbb{Z} \text{ such that if } y \in \mathbb{Z} \text{ then } \frac{y}{x} \in \mathbb{Z}.$$

Useful negations don't start with "It is not true that..." and avoid the word not in cases where it could be replaced (e.g., don't use "not even").

The negation is: "for all $x \in \mathbb{Z}$, $y \in \mathbb{Z}$ and $\frac{y}{x} \notin \mathbb{Z}$.