Practice

–Name –

Theorem. *If x and y are odd integers then $x + y$ is an even integer.*

Proof. We assume that x and y are odd integers and will prove that $x + y$ is an even integer. Since x and y are odd, there exist integers m and n such that $x = 2m + 1$ and $y = 2n + 1$. By substitution and algebra we obtain

$$\begin{aligned} x + y &= 2m + 1 + 2n + 1 \\ &= 2m + 2n + 2 \\ &= 2(m + n + 1). \end{aligned}$$

Define $q = m + n + 1$. Since m and n are integers and the integers are closed under addition, we conclude that q is an integer. Since $x+y = 2q$ for the integer q we conclude that $x + y$ is an even integer. $\square$

Challenge Typing

Suppose that $f : (-1, 1) \to \mathbb{R}$ and f is differentiable at 0. Let sequences $(\alpha_n)_{n\geq 1}$ and $(\beta_n)_{n\geq 1}$ satisify $-1 < \alpha_n < \beta_n < 1$ for all $n \geq 1$ and $\lim\limits_{n\to\infty} \alpha_n = \lim\limits_{n\to\infty} \beta_n = 0$. Set

$$\lambda_n = \frac{f(\beta_n) - f(\alpha_n)}{\beta_n - \alpha_n}.$$

Theorem. *The set* $\{x \in \mathbb{Z} : |x - 2.5| = 2\}$ *is the empty set.*

Proof. Let y be an integer such that $y \in \{x \in \mathbb{Z} : |x - 2.5| = 2\}$ Then $y \in \mathbb{Z}$ and $|y - 2.5| = 2$. Since $|y - 2.5| = 2$ then $y = 4.5$ or $y = -.5$. But then y is not an integer. Therefore the set $\{x \in \mathbb{Z} : |x - 2.5| = 2\}$ has no elements and

$$\{x \in \mathbb{Z} : |x - 2.5| = 2\} = \emptyset.$$

□

Theorem. *There exist two positive irrational numbers* s *and* t *such that* s^t *is rational.*

Proof. We will consider two cases. For the first case, suppose that $\sqrt{2}^{\sqrt{2}}$ is rational. Then we may take $s = t = \sqrt{2}$. For the second case, suppose that $\sqrt{2}^{\sqrt{2}}$ is irrational. Let $s = \sqrt{2}^{\sqrt{2}}$ and $t = \sqrt{2}$. Then

$$\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \left(\sqrt{2}\right)^2 = 2.$$

Since 2 is rational, s^t is rational. Therefore, there exists irrational numbers s and t such that s^t is rational. □

Theorem. *Let* n *be a natural number. Then*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Consider the following matrix,

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$