

# Reproducing a L<sup>A</sup>T<sub>E</sub>X Document

Put your name here

**Theorem.** *If  $x$  and  $y$  are odd integers then  $x + y$  is an even integer.*

*Proof.* We assume that  $x$  and  $y$  are odd integers and will prove that  $x + y$  is an even integer. Since  $x$  and  $y$  are odd, there exist integers  $m$  and  $n$  such that  $x = 2m + 1$  and  $y = 2n + 1$ . By substitution and algebra we obtain

$$\begin{aligned}x + y &= 2m + 1 + 2n + 1 \\&= 2m + 2n + 2 \\&= 2(m + n + 1).\end{aligned}$$

Define  $q = m + n + 1$ . Since  $m$  and  $n$  are integers and the integers are closed under addition, we conclude that  $q$  is an integer. Since  $x + y = 2q$  for the integer  $q$  we conclude that  $x + y$  is an even integer.  $\square$

## Challenge Typing

Suppose that  $f : (-1, 1) \rightarrow \mathbb{R}$  and  $f$  is differentiable at 0. Let sequences  $(\alpha_n)_{n \geq 1}$  and  $(\beta_n)_{n \geq 1}$  satisfy  $-1 < \alpha_n < \beta_n < 1$  for all  $n \geq 1$  and  $\lim_{n \rightarrow \infty} \alpha_n = \lim_{n \rightarrow \infty} \beta_n = 0$ . Set

$$\lambda_n = \frac{f(\beta_n) - f(\alpha_n)}{\beta_n - \alpha_n}.$$