Skill Mastery Quiz 7

Communicating in Math (MTH 210-01)
Winter 2020

Name:

P1-3 Consider the following statement:

For all integers a and b, if $a \neq 0$ and a does not divide b, then $ax^3 + bx + (b + a) = 0$ does not have a solution that is a natural number.

State what you would assume in a direct proof.

Assume that a and b are integers with $a \neq 0$ and that a does not divide b. (Basically, we assume the hypothesis.)

State what you would assume in a proof by contradiction.

Assume that there exist integers a and b such that $a \neq 0$ and a does not divide b and $ax^3+bx+(b+a) = 0$ does have a solution that is a natural number. (Basically, we assume the negation.)

P2-1 For which of the following situations is it more appropriate to use induction. Explain.

1. For all $a \in \mathbb{Z}$ the equation $ax^3 + ax + a = 0$ does not have a solution that is a natural number.
2. For each natural number n, 3 divides $4^n - 1$.

Circle one and explain why you chose that.

We choose the second statement because it starts "for each natural number n". Induction works when you have a base case and a way to get from one step to the next.

For the statement you chose, state what your steps would be in a proof by induction.

We let $P(n)$ be the predicate $3 \mid 4^n - 1$. Then we prove $P(1)$ which is $3 \mid 4^1 - 1$. Next we let $k \in \mathbb{N}$ and assume $P(k)$ or that $3 \mid 4^k - 1$. Finally using that we prove that $3 \mid 4^{k+1} - 1$.