MATH 212 EXAM 2 QUESTIONS

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Instructions: By April 21, you must use this link to sign up for a 15 minute slot during the final week of class.

Forty-eight (48) hours before your scheduled time, you will be notified which three problems you will present.

Twenty-four (24) hours before your scheduled time, you will submit solutions to your three problems. These should represent your **best possible work** and will be graded on a 3-2-1-0 scale (corresponding roughly to our EMRN portfolio scale). Failure to submit any single problem on time will result in that problem's initial assessment being 0/3.

During your exam time, we'll walk through your solutions. You will be asked a series of questions about your work, possibly including asking you to justify your steps, explain why you made certain choices, etc. Your answers to the questions will determine how many additional points you are awarded on each problem:

- 0-3 points: You were unable to provide satisfactory answers to any questions, even with hints and prompting.
- 4-7 points: You could answer some, but not all, questions with sufficient hints and prompting.
- 8-10 points: You could satisfactorily answer all questions, but perhaps required one or more hints to get started.
- 11-12 points: You were able to satisfactorily answer all questions without any hints.

Thus, each problem will be worth 15 points (3 points for the initial assessment, and 12 for the final oral assessment). You will also earn 5 points per exam for making and keeping your appointment while complying with the given deadlines, for a total of 50 points.

If you have questions on these problems before your exam is due (24 hours before the appointment), I am happy to answer them. You may discuss/work on these questions with others in the class and refer to any resources you see fit as you work on your solutions. During our meeting, however, you may **only** use the solutions you submitted.

PROBLEMS

(1) Let $a \in \mathbb{Z}$ and $m \in \mathbb{N}$ such that $m > 1$. Then $a \equiv 0 \mod m$ if and only if $m|a$.
(2) Calculate a closed form for the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$. Then use it to compute a_9.
(3) Use methods from class to find a closed form for the sequence

$$4, 4, 10, 28, 64, 124, 214, 340, 508, 724, \ldots.$$

(4) Let $a, b, c, d \in \mathbb{Z}$ such that $a|b$, $b|c$, and $c|d$. Then $a|d$.
(5) Let $a, b, c, d \in \mathbb{Z}$ such that $a|b$ and $c|d$. Then $ac|bd$.
(6) Let $A_1, A_2, \ldots, A_n$ be sets (where $n \geq 2$). Suppose for *any* two sets A_i and A_j that either $A_i \subseteq A_j$ or $A_j \subseteq A_i$. Prove by induction that one of these n sets is a subset of all of them.
(7) For all $n \geq 1$,

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$$

(8) Let G be the graph whose vertex set is $\{1, 2, 3, 4, 5, 6\}$ with an edge between vertices v and w if and only if $v - w$ is odd (that is, the difference of v and w is odd). Let $K_{3,3}$ be the complete bipartite graph with parts $A = \{a, b, c\}$ and $B = \{d, e, f\}$. Find an isomorphism $\varphi : G \to K_{3,3}$.
(9) Let T be a tree with at least two vertices and let v be a vertex of T. Let $T \setminus v$ denote the graph obtained from T by deleting v and any edges incident to v. If $T \setminus v$ is a tree, prove that v is a leaf.
(10) Consider the graph M_n, $n \geq 3$, with vertices $V = \{1, 2, \ldots, 2n\}$ and two types of edges:
  - For all s, $1 \leq s \leq 2n - 1$ we have the edge $\{s, s+1\}$, as well as the edge $\{2n, 1\}$; that is, these edges form a cycle on the $2n$ vertices of M_n.
  - For all t, $1 \leq t \leq n$, we have the edge $\{t, t+n\}$.

  Calculate, with proof, $\chi(M_n)$.

  [Hint: draw some examples, e.g., M_3, M_4, etc., but remember that an example is not a proof.]