Skill Mastery Quiz 9

Communicating in Math (MTH 210-01)
Winter 2020

Name:

P2-3 For which of the following situations is it more appropriate to use induction (circle one).

1. For all $a \in \mathbb{Z}$ the equation $ax^3 + ax + a = 0$ does not have a solution that is a natural number.
2. For each natural number n,

$$3 + 6 + 9 + \cdots + 3n = \frac{3n(n+1)}{2}.$$

.

Explain why you chose that statement to prove by induction.

The second statement makes more sense because it starts "For each natural number n."

For the statement you chose, state what your steps would be in a proof by induction.

Let $P(n)$ be the predicate $3+6+9+\cdots+3n = \frac{3n(n+1)}{2}$. We would first prove $P(1)$ or that $3 = \frac{3(1)(1+1)}{2}$. Then we would let $k \in \mathbb{N}$ and assume $P(k)$, or that

$$3 + 6 + 9 + \cdots + (3k) = \frac{3k(k+1)}{2}$$

and show that $P(k+1)$ is true, or that

$$3 + 6 + 9 + \cdots + 3(k+1) = \frac{3(k+1)(k+1+1)}{2}.$$

S1-2 Let $A = \{1, 2, 4\}$ and $B = \{1, 2, 4, 5\}$. From the list $\in, \notin, =, \neq, \subseteq, \not\subseteq, \subset, \not\subset$, fill in a correct symbol for each of the following:

– A____B
– $\emptyset$____A
– $\{4, 2, 1\}$____B

– $A \subset B$ adn $A \subseteq B$ would both work.
– I would use $\emptyset \subset A$ here.
– I would use $\{4, 2, 1\} \subset B$ here. Remember order in sets doesn't matter so it's also the case that $\{4, 2, 1\} = A$.

S2-2 Let $U = \mathbb{Z}$. Let $A = \{x \in \mathbb{Z} : x \geq 7\}$ and $B = \{x \in \mathbb{Z} : x \text{ is odd}\}$. (Roster method is okay for your answers, but make sure the pattern is clear.)

1. Find $A \cap B$ $A \cap B$ is the set of all integers that are both odd and at least 7. In roster notation this is $\{7, 9, 11, \dots\}$

2. Find $A \cup B$ $A \cup B$ is the set of all numbers that are at least 7 or are odd. In roster notation this is $\{\dots, -3, -1, 1, 3, 5, 7, 8, 9, 10, 11, \dots\}$

3. Find A^C A^c is the set of all numbers less than 7. In roster notation this is $\{\dots, 4, 5, 6\}$.

4. Find $A - B$ This is the set of all even numbers that are at least 7 (because they are in A but not in B. So $\{8, 10, 12, 14 \dots\}$.

S3-1 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - 2$.

1. State the domain, codomain, and range of f. (Clearly state which one is which. You can graph this if it helps you.)

   The domain and codomain are each given as $\mathbb{R}$. The range is the set of all real numbers that are at least -2, which one can see by graphing, these are all the outputs (or y values).

2. Find the image(s) of 3 under f. The image of 3 under f is $f(3) = 3^2 - 2 = 7$.

3. Find the preimage(s) of 0. To find preimages we solve $0 = x^2 - 2$ which gives $x = \sqrt{2}$ and $x = -\sqrt{2}$.