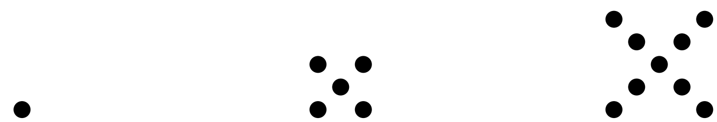
Math 212 **From Dots to Sequences**

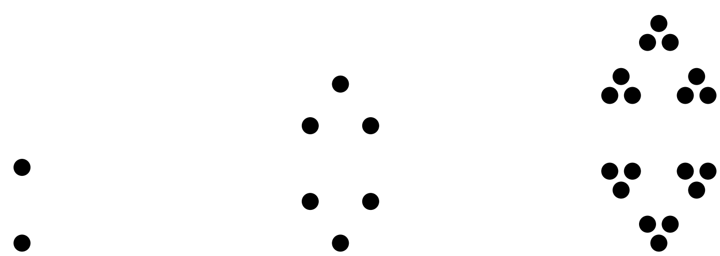
Activity 1: Dots

For the patterns of dots below, draw the next pattern in the sequence. Then give a recursive definition and a closed formula for the number of dots in the nth pattern.



$n = 0$ $n = 1$ $n = 2$

Solution: The next pattern will contain 13 dots. Since each pattern will contain 4 more dots than the previous, the recursive definition will be $a_n = a_{n-1} + 4$; $a_0 = 1$. The closed formula will be $a_n = 1 + 4n$.



$n = 0$ $n = 1$ $n = 2$

Solution: The next figure will have 54 dots. Each figure is created by splitting each dot into 3, so each will be 3 times the previous. This leads directly to the recursive definition: $a_n = 3a_{n-1}$; $a_0 = 2$. The closed formula is $a_n = 2 \cdot 3^n$.

$n = 1$ $n = 2$ $n = 3$ $n = 4$

Solution: The next patter will contain 15 dots. These are the triangular numbers. Each one is found by adding a new row of dots (one longer than the previous row). Thus $a_n = a_{n-1} + n$; $a_1 = 1$ is the recursive definition. A closed formula is $a_n = \frac{n(n+1)}{2}$.

Activity 2: Sequences

For each sequence of numbers, guess the next term in the sequence. Then find a recursive definition and closed formula for the nth term of the sequence. Assume the first term given is a_0.

- 3, 6, 12, 24, ...

> **Solution:** Recursive: $a_n = 2a_{n-1}$; $a_0 = 3$. Closed: $a_n = 3 \cdot 2^n$.

- 2, 5, 8, 11, ...

> **Solution:** Recursive: $a_n = a_{n-1} + 3$; $a_0 = 2$. Closed: $a_n = 2 + 3n$.

- 4, 12, 20, 28, ...

> **Solution:** Recursive: $a_n = a_{n-1} + 8$; $a_0 = 4$. Closed: $a_n = 4 + 8n$.

- 4, 12, 36, 108, ...

> **Solution:** Recursive: $a_n = 3a_{n-1}$; $a_0 = 4$. Closed: $a_n = 4 \cdot 3^n$.

- 2, 5, 10, 17, 26, ...

> **Solution:** Recursive: $a_n = a_{n-1} + 2n + 1$; $a_0 = 2$. Closed: $a_n = (n+1)^2 + 1$ (if you notice that each is one more than a perfect square) or $a_n = \frac{2+(2n+4)n}{2}$ (which is the same, but is what you get if you notice this is the sum of an arithmetic sequence).