FINAL EXAM REVIEW

Disclaimer: These practice problems are based on what the class requested to review for the first two exams plus the most recent material and what my impression of what the class struggled with. You should not assume the final will be exactly like this, but these are good practice problems to try.

Ways to study:

- Go through the "self quiz" questions on Blackboard and make sure you can answer them all.
- For any section you struggled with do the practice problems from that section (on Blackboard).
- The problems with ** are the potential proofs for the exam. You should definitely do these.
- Make sure you know how to do the questions on previous exams.

Note for all of the above you should be *doing* the mathematics. Reading over worksheet solutions has little to no value - studies have shown this will not improve your performance on the exam.

1. Induction

(1) ** For each $n \in \mathbb{N}$, 5^n is odd.

(2) For all $n \in \mathbb{N}$, $\sum_{i=1}^{n}(2i-1) = n^2$. (This is "sigma notation" which you may have seen in Calculus 2. We could rewrite this statement as "For all $n \in \mathbb{N}$, $1+3+5+\cdots+(2n-1) = n^2$.")

(3) For all natural numbers n with $n \geq 3$, $n^2 \geq 2n+3$.

(4) For any natural number n, $3 \mid n^3 + 2n$.

2. Using Cases in Proofs

(5) ** Let U be a set and A, B, and C subsets of some universal set. Then $(A \cup B) - C = (A - C) \cup (B - C)$.

(6) ** Prove that for each integer n, if n is odd then $8 \mid n^2 - 1$.

(7) For all integers a, b, and d with $d \neq 0$, if $d \mid a$ or $d \mid b$ then $d \mid ab$.

3. Functions

(8) Define $f : \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 5x + 3$. State the domain, codomain and range. Is f an injection? surjection? Prove/disprove.

(9) ** Define $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $g(a, b) = a$. State the domain, codomain and range. Is g an injection? Surjection? Prove/disprove.

(10) ** Let A, B, and C be nonempty sets and $f : A \to B$ and $g : B \to C$. Prove that if f and g are bijections then $g \circ f$ is a bijection.

4. Equivalence Relations and Equivalence Classes

(11) ** For $x, y \in \mathbb{Z}$ define $x \sim y$ if and only if $x+y$ is even. Prove that $\sim$ is an equivalence relation and find the equivalence class of 2 (denoted $[2]_\sim$).

(12) ** For (w, x) and (y, z) in $\mathbb{Z} \times \mathbb{Z}$ define $(w, x) \sim (y, z)$ if and only if $w \leq y$ and $x \leq z$. Is $\sim$ an equivalence relation?

(13) Define a function $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 + 1$. For $a, b \in \mathbb{R}$, say $a \sim b$ if and only if $f(a) = f(b)$. Show this is an equivalence relation and then find $[3]_\sim$.

(14) Write the addition and multiplication tables for $\mathbb{Z}_5$ and $\mathbb{Z}_6$. Somewhere you will have written $[0]$ in each table. Does this mean something different in $\mathbb{Z}_5$ and $\mathbb{Z}_6$ or is it the same?