Proof Portfolio Problem 9

Choose only one of 9A, 9B, and 9C.

For problem 9, if the function is not a bijection, but is an injection or surjection, then provide a counterexample, then state your theorem as the one part that's true. You do not need to change the function.

Final drafts of problems 1, 9, and 10 are due by the final (Wednesday, April 22 at 10AM). No drafts for feedback will be accepted after Friday, April 17 at 11:59PM.

Conjecture 9A. Let $\mathcal{M} = \mathcal{M}_2(\mathbb{R})$ be the set of all 2×2 matrices with real number entries. Define the determinant function $\det : \mathcal{M} \to \mathbb{R}$ by the rule

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc.$$

Then $\det : \mathcal{M} \to \mathbb{R}$ is a bijection.

Conjecture 9B. The function $M : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ given by $M(u, v) = 7m + n$ is a bijection.

Conjecture 9C. Let A and B be sets such that $|A| < |B|$. There exists a bijection $f : A \to B$.

TeX Code for 9A:

For $\det : \mathcal{M} \to \mathbb{R}$:

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$\det: \mathcal{M} \to \mathbb{R}$
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For

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc.$$

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\det \left(\begin{bmatrix}  a &b \\ c &d \end{bmatrix} \right) = ad -bc.
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