Skill Mastery Quiz 8

Communicating in Math (MTH 210-01)
Winter 2020

Name:

P2-2 For which of the following situations is it more appropriate to use induction (circle one).

1. For all integers a and b,
$$(a+b)^2 \equiv (a^2+b^2) \pmod 2.$$
2. For each natural number n,
$$1+3+5+\cdots+(2n-1)=n^2.$$

.

Explain why you chose that statement to prove by induction.

The second statement makes more sense because it starts "For each natural number n."

For the statement you chose, state what your steps would be in a proof by induction.

Let $P(n)$ be the predicate $1+3+5+\cdots+(2n-1)=n^2$. We would first prove $P(1)$ or that $1=1^2$. Then we would let $k \in \mathbb{N}$ and assume $P(k)$, or that

$$1+3+5+\cdots+(2k-1)=k^2$$

and show that $P(k+1)$ is true, or that

$$1+3+5+\cdots+(2(k+1)-1)=(k+1)^2.$$

S1-1 Let $A = \{1, \{2\}, \{3, 4\}, 5\}$. From the list $\in, \notin, =, \neq, \subseteq, \not\subseteq, \subset, \not\subset$, fill in a correct symbol for each of the following:

- $\{1\}$____A There's a bunch of answers, one could use $\notin$, $\neq$, $\subseteq$, or $\subset$. I would have gone for $\subset$ since that is phrased positively and is "stronger" than $\subseteq$. Note that this is a set for which every element is an element of A.

- $\{2\}$____A A bunch of answers here too, one could use $\in$, $\neq$, $\not\subseteq$, or $\not\subset$. I would have gone for $\in$ since this is phrased positively. Note this looks a lot like the first one, but in this case, one of the elements of A is $\{2\}$. We could say $\{\{2\}\} \subset A$ since that's a set containing an element of A.

- $\{1, 2, 3, 4, 5\}$____A There are lots of correct answers here, but I would choose $\neq$. Though the order of elements doesn't affect what the set is, and repeated elements don't affect what the set is, the set $\{1, 2, 3, 4, 5\}$ has different elements than the set A.

S2-1 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the universal set. Let $A = \{3, 4, 5, 6, 7\}$ and $B = \{1, 5, 7, 9\}$.

1. Find $A \cap B$
   $A \cap B = \{5, 7\}$. This is the intersection, or "and" - so all of the elements that are both in A and in B.

2. Find $A \cup B$
   $A \cup B = \{1, 3, 4, 5, 6, 7, 9\}$ This is the union, so "or" - we take all of the elements that are in one or the other or both.

3. Find A^C
   $A^c = \{1, 2, 8, 9, 10\}$. This is the complement, so everything that is in U that is not in A. Note $U = \{1, 2, \ldots, 10\}$.

4. Find $A - B$.
   $A - B = \{3, 4, 6\}$. This is A "minus" B, or all of the elements that are in A that aren't also in B.