Math 212 Portfolio

Your name goes here

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Changelog: *List the changes you've made since the last draft, with special attention paid to problems that have received significant revisions since the last draft (i.e., more than fixing typos). If there is any additional information you'd like me to consider as I review this submission, please say so now.*

Instructions: Each of the problems below is/will be presented as a conjecture. Each conjecture asks you to prove or disprove the conjecture, possibly along with some additional directions.

- If the conjecture is true, your job is to write a complete proof for the proposition. If there are multiple parts, you should consider each part in turn.
- If it is false, you should provide a counterexample plus make reasonable modifications to the stated conjecture so that a new proposition is true. Then, write a complete proof of your new proposition. You may want to run your new proposition by me before trying to write a proof–this is allowed and encouraged!

Academic Honesty Policy: The portfolio is an independent project in which no outside resources or collaboration is allowed. You may not ask other professors or discuss the problems with anyone besides me. You should not discuss even which problem you chose. Violation of this policy is grounds for failing the course. The point is that you need to be confident and competent in writing proofs for future courses.

Conjecture I. *Let A and B be subsets of some universe $\mathcal{U}$. Then:*

1. $A \setminus (A \cap \overline{B}) = A \cap B$
2. $\overline{(\overline{A} \cup B)} \cap A = A \setminus B$
3. $(A \cup B) \setminus A = B \setminus A$
4. $(A \cup B) \setminus B = A \setminus (A \cap B)$

Proof. $\square$

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

Conjecture II. *Define* $f : \mathbb{N} \setminus \{0\} \to \mathbb{Z}$ *as follows: for each* $n \in \mathbb{N} \setminus \{0\}$,

$$f(n) = \frac{1 + (-1)^n(2n-1)}{4}.$$

Then f *is a bijection.*

Proof. $\square$

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

Example III. *For the following example, choose two of the four problems to do.Exactly one of your choices should be a combinatorial proof.*

1. *(Combinatorial) For* $n \geq 1$*,*

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}.$$

2. *(Combinatorial) For* $0 \leq k \leq n$*,*

$$\sum_{m=k}^{n} \binom{m}{k} = \binom{n+1}{k+1}$$

3. *Consider the alphabet* $\{a, b, c, d, e, f\}$ *and make words without repetition of letters allowed.*

   (a) *How many six-letter words are there?*

   (b) *How many words begin with d or e?*

   (c) *How many words end in b or a?*

   (d) *How many words begin with d or e and end in b or a?*

   (e) *How many have first letter neither d nor e and last letter nether b nor a?*

4. *We wish to improve upon the ogre's distribution of 43 cupcakes to 12 baby mice by ensuring that every baby mouse gets at least two cupcakes. How many ways are there to accomplish this?*

Proof. □

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

Theorem IV. *Consider the recurrence relation $a_n = sa_{n-1} + d$, where $s \neq 1$. Prove that*

$$a_n = \left(a_0 + \frac{d}{s-1}\right) s^n - \frac{d}{s-1}$$

is a solution. Then use this theorem to solve for a closed formula for the recurrence $a_n = 5a_{n-1} + 3$ where $a_0 = 1$.

Proof. □

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

Conjecture V. *For all $n \geq 1$,*

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

Proof. (By induction.) $\square$

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

For Theorem VI, we will use the following definition.

Definition 1. Let $a, b, \in \mathbb{Z}$ and $m \in \mathbb{N}$ with $m > 1$. We say that a is *congruent to* b *modulo* m if $m|(a-b)$. We write $a \equiv b \mod m$.

Thus, e.g., $11 \equiv 3 \mod 4$, since $4|11-3$, but $9 \not\equiv 3 \mod 4$ since $4 \nmid 9-3$.

Theorem VI. *Suppose $a, b \in \mathbb{Z}$ and $m \in \mathbb{N}$ with $m > 1$ such that $a \equiv b \mod m$. Then $a^2 \equiv b^2 \mod m$.*

Proof. $\square$

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

Conjecture VII. *If a, b, and c are integers, then $ab + ac$ is even.*

Proof. □

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

Conjecture VIII. *If r is any real number and ξ is irrational, then $r + \xi$ is irrational or $-r + \xi$ is irrational.*[1]

Proof. $\square$

Evaluation: __________

Opening: __________

Logical Correctness: __________

Reasons: __________

Use of Notation: __________

Clarity and Writing: __________

LaTeX Formatting: __________

Stating the Conclusion: __________

Other Comments:

[1] You may assume without proof that the sum of two rational numbers is rational.