Math 212 Proof Portfolio

Your Name here

1 New Proofs

Theorem 1. *If A is a set with n elements, then $|\mathcal{P}(A)| = 2^n$.*

Proof. (By induction.) ■

Theorem 2. *Consider a relation $\sim$ on $Q = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ defined as follows:*

$$(a, b) \sim (c, d) \textit{ if and only if } ad = bc.$$

Then $\sim$ is an equivalence relation on Q.

Proof. ■

Theorem 3. *Let T be a tree with at least two vertices. Then T has at least two leaves.*

Proof. ■

2 Old Proofs

Statement 3.37. If $n \in \mathbb{N}$, then $2 \cdot 6 \cdot 10 \cdot 14 \cdots (4n-2) = \dfrac{(2n)!}{n!}$.

Proof. ■

Exploration 4.122. Find a way to define a family of nonempty sets $\mathcal{F} = \{F_n \ : \ n \in \mathbb{N}\}$, indexed by the natural numbers, so that:

$$\bigcup_{n\in\mathbb{N}} F_n = \mathbb{R} \qquad \text{and} \qquad \bigcap_{n\in\mathbb{N}} F_n = \emptyset.$$

Prove that your family of nonempty sets satisfies these conditions.

Proof. ■

If Statement 5.106 is false, salvage it with a different function $g : \mathbb{Z} \to B$ which *is* one-to-one and onto.

Statement 5.106. Let $B = \{n \in \mathbb{Z} \ : \ 3|(n-1)\}$. Let $g : \mathbb{Z} \to B$ be defined by $g(n) = 3n^2 + 4$. Then g is both one-to-one and onto.

Proof. ■

3 Proof Analysis

Theorem 4. *Let X and Y be sets with $f : X \to Y$ a function, and suppose $A, B \subseteq X$. Then $f[A \setminus B] = f[A] \setminus f[B]$.*

Proof. Let $y \in f[A \setminus B]$. Then there is an $x \in A \setminus B$ such that $f(x) = y$. Since $x \in A$ and $x \notin B$, $f(x) = y \in f[A]$ but $y \notin f[B]$, so $y \in f[A] \setminus f[B]$.

Now let $y \in f[A] \setminus f[B]$. Then there is an $x \in A$ such that $f(x) = y$, but for all $b \in B$, $f(b) \neq y$. Since $x \in A \setminus B$, $y = f(x) \in f[A \setminus B]$. ■