Math 304 Extra Board Work

Your name goes here

Due December 12, 2018

Choose **one** of the following. If you do more than one problem, I will grade the first one I see.

Your work will be graded out of 1 point on the scale a board work question would be graded: 1 point for a clear and correct proof, 1/2 point if it is unclear or somewhat incorrect, and 1/4 point if it is in completely the wrong direction. You are welcome to ask for help as you prepare your proof.

Exercise W. Let G be a group and fix an element $g \in G$. Define $\varphi_g : G \to G$ by $\varphi(x) = gxg^{-1}$. Prove that φ_g is a homomorphism, and determine whether or not it is an isomorphism.

Proof. $\square$

Let G be a group and $N \leq G$. We say N is a *normal* subgroup if for all $a \in G$, $aN = Na$, where $aN = \{an : n \in N\}$ and $Na = \{na : n \in N\}$. We denote this by $N \lhd G$.

Theorem X. A subgroup H of G is normal if and only if $xHx^{-1} \subseteq H$ for all $x \in G$.

Proof. $\square$

Theorem Y. Let $f : G \to K$ be a homomorphism. Then $\ker f \lhd G$. [You may use without proof the fact that $f(x^{-1}) = (f(x))^{-1}$.]

Proof. $\square$

Theorem Z. Let G be a group. The *center* of G is the set

$$Z(G) = \{g \in G : gh = hg \forall h \in G\}.$$

Prove that $Z(G) \lhd G$.

Proof. $\square$