Math 304 Homework 10

Your name goes here

Due November 30, 2018

Let $R = \mathbb{R}[x,y]$ and let $f(x,y), g(x,y) \in \mathbb{R}[x,y]$ be non-constant polynomials. Define the *zero set* of f and g, $Z(f,g) \subseteq \mathbb{R}^2$, by:

$$Z(f,g) := \left\{(a,b) \in \mathbb{R}^2 : \ f(a,b) = g(a,b) = 0\right\}.$$

Thus, for example, $Z(y - x^3, y - x) = \{(-1,-1), (0,0), (1,1)\}$, while $Z(y - x^2 + 5x - 4)$ contains all the points on the graph of the parabola given by $y = x^2 - 5x + 4$.

Define $I(Z(f,g)) = \{p \in R : \ p(a,b) = 0 \text{ for all } (a,b) \in Z(f,g)\}$.

Theorem S. The set $I(Z(f,g))$ is an ideal of R containing $\langle f, g \rangle$.

Proof. □

In general, $I(Z(f,g)) \neq \langle f, g \rangle$, though if we replace $\mathbb{R}$ by $\mathbb{C}$ and assume an additional technical condition on f and g, equality does hold.

Theorem T. Let $S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : \ a, b \in \mathbb{R} \right\}$. Prove that $\phi : \mathbb{C} \to S$ given by

$$\phi(a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

is a ring isomorphism.

Proof. □