Math 212 Homework 6

Your name goes here

Due April 10, 2019

List of collaborators:

For Theorem P, prove one of parts 1–2, and one of parts 3–4. You will need to chase elements.

Theorem P. Let I be an index set and let $\mathcal{F} = \{T_i\}_{i \in I}$ be a family of sets in a universal set U. Let $X \subseteq U$. Then:

1. $\left(\bigcap_{i \in I} T_i \right)^c = \bigcup_{i \in I} T_i^c$
2. $\left(\bigcup_{i \in I} T_i \right)^c = \bigcap_{i \in I} T_i^c$
3. $X \cap \left(\bigcup_{i \in I} T_i \right) = \bigcup_{i \in I} (X \cap T_i)$
4. $X \cup \left(\bigcap_{i \in I} T_i \right) = \bigcap_{i \in I} (X \cup T_i)$

Proof. $\square$

For Theorem Q, after you have proved that $\mathcal{R}$ is an equivalence relation, you need to state what the equivalence (relation) classes are, describe them in more familiar terms, and prove that you have found them all.

Theorem Q. The relation $\mathcal{R}$ defined on $\mathbb{Z}$ by $(m, n) \in \mathcal{R}$ if and only if $m + n$ is even is an equivalence relation. The relation classes are __________.

Proof. $\square$

For Theorem R, we make the following definition.

Definition. Let S be a set and $\mathcal{R}$ a relation on S. We say that $\mathcal{R}$ is a *partial ordering* on S if it is reflexive, transitive, and *anti-symmetric*: if $x, y \in S$ and both $(x, y) \in \mathcal{R}$ and $(y, x) \in \mathcal{R}$, then $x = y$. Furthermore, we say that two elements $x, y \in S$ are *comparable* under $\mathcal{R}$ if either $(x, y) \in \mathcal{R}$ or $(y, x) \in \mathcal{R}$. If any two elements of S are comparable under $\mathcal{R}$, we say $\mathcal{R}$ is a *total ordering*.

Theorem R. The relation $\subseteq$ on the set $\mathcal{P}(\mathbb{Z})$ is a partial ordering that is not a total ordering.

Proof. $\square$