

MATH 304 HOMEWORK 7

YOUR NAME GOES HERE

DUE NOVEMBER 2, 2018

Theorem M. Let R be a Euclidean domain with norm δ . Then $u \in R^\times$ if and only if $\delta(u) = \delta(1)$.

Proof. (\Rightarrow)

(\Leftarrow)

□

Let $S = \mathbb{Z}[\sqrt{d}] = \{a + b\sqrt{d} : a, b \in \mathbb{Z}\}$, where $d \in \mathbb{Z}$, $d \neq 1$, and d is not divisible by the square of a prime. Define a norm δ on S by $\delta(a + b\sqrt{d}) = |a^2 - db^2|$ ¹.

Theorem N. Verify the following properties of δ :

1. $\delta(x) = 0$ if and only if $x = 0$;
2. $\delta(xy) = \delta(x)\delta(y)$;
3. $\delta(x) = 1$ if and only if x is a unit;
4. If $\delta(x)$ is prime, then x is irreducible in S .

Proof.

- 1.
- 2.
- 3.
- 4.

□

¹Despite having this norm, $\mathbb{Z}[\sqrt{d}]$ need not be a Euclidean domain, as we will see later.