Math 304 Homework 6

Your name goes here

Due October 26, 2018

Theorem K. Let R be a commutative ring with identity, and $I, J \subseteq R$ ideals.

1. The *sum* $I + J$ of I and J is an ideal of R, where
$$I + J := \{x + y : \ x \in I, \ y \in J\}.$$
2. The *product* IJ of I and J is an ideal of R, where
$$IJ := \{x_1y_1 + x_2y_2 + \cdots + x_ny_n : \ n \geq 0, \ x \in I, \ y \in J\}.$$
3. The nth power of I, I^n is an ideal of R, where
$$I^n = \underbrace{I \cdot I \cdots I}_{n \text{ times}}.$$

Proof. □

In the following theorem, fill in the blank, and then complete the proof.

Theorem L. Let $m, n \in \mathbb{Z}$. If a is _____, then $\langle a \rangle = \langle m \rangle + \langle n \rangle$.

Proof. □