Math 304 Homework 4

Your name goes here

Due September 28, 2018

Let $K = \{2^r \ : \ r \in \mathbb{R}\}$, and define binary operations on K as follows:

- Addition: $2^r \oplus 2^s := 2^{r+s}$
- Multiplication: $2^r \otimes 2^s := 2^{rs}$

Prove or disprove the following conjecture.

Conjecture G. With the operations $\oplus$ and $\otimes$, K is a field. [Note that your solution should include either a careful verification of each of the field axioms, or a specific counterexample of a field axiom that is not satisfied.]

Solution: $\Diamond$

A *Boolean ring* R is a ring in which $x^2 = x$ for all $x \in R$.

Theorem H. Let R be a Boolean ring. Then:

- For all $r \in R$, $r + r = 0_R$. [Hint: Square a convenient element of R.]
- R is commutative. [Hint: Square a different convenient element of R, then use the previous part.]

Proof. $\square$