Safety distances in airsoft with proper consideration of atmospheric drag

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ABSTRACT

In this paper the relationship between safety distances, projectile mass and muzzle velocities in airsoft is reviewed. The physical background is studied, especially the Reynolds number, the drag coefficient and the quadratic drag model of air resistance. A physical model is presented and then evaluated with empirical testing. From these evaluating tests an empirical value of the drag coefficient is presented and the drag model is determined to be sufficiently accurate for use in safety table design. Based on the physical model two safety table examples are then presented to serve as a basis for further discussion about more subjective factors in airsoft safety table design.

Contents

1 Introduction

In airsoft, we want all shots impact energy to not exceed a safety threshold to ensure the health and safety of all game participants. To allow for a greater range of the airsoft weapons the energy of the projectile is not necessarily counted by the muzzle, but can instead be counted at a farther distance where atmospheric drag has brought down the energy of the projectile to safe levels. The shooter is not allowed to fire at anything within this "safety distance" to ensure that the impact energy of his shots are always below an energy level deemed safe. The energy of the shots are routinely measured before any airsoft game.

While one could imagine the projectiles energy being measured at the expected safety distance, which would simplify the calculations to triviality by not having to deal with modeling the atmospheric drag, it is not practical to do with normal measuring equipment at games. Instead the projectile energy is measured directly at the muzzle of the weapon and then referenced against a precalculated table with various values of projectile weights, safety distances and muzzle velocities. While the primary objective of this report is to construct and evaluate a good model for drag resistance in airsoft the end goal is to use this model to design a good safety table. An important part of designing such a table is a good understanding of the underlying physics of the ballistic arc of the airsoft projectile, but it is far from the only factor to consider. The physics model provide the ability to say with confidence how hard a projectile will hit at a certain distance given its initial conditions, but the question of how hard a hit is appropriate is in large part subjective and depend on factors such as safety, comfort, fairness, legality and game balance. The physics model given in this report should serve as a basis for discussion about these other factors. In order to contribute to further discussions this report will provide two examples of alternative safety tables based on the same physics model but with slightly different choices for the other factors.

Fluid mechanics is hard. There is a reason why companies still use wind tunnels rather than computation and simulation only and for our purposes in airsoft we will always have to resort to simplifications and estimations unless we make the effort to do real measurements at the safety distance for all table values, eliminating the need to model atmospheric drag. That said, the model proposed in this paper is probably good enough for our purposes. Common assumptions and estimations in fluid mechanics include linear drag for slow speeds with laminar flow and quadratic drag for higher speeds with turbulent flow. Computing the Reynolds number can help determine which flow regime a problem resides in. The Reynolds number for airsoft shots is outside the regime of laminar flow, indicating that the problem is turbulent and that the quadratic drag model should be used.

The drag coefficient is an important factor that incorporates most of the complexities of fluid mechanics for the drag problem, including the shape of the object. It depends only on the Reynolds number with the relationship, determined empirically, illustrated in figure 1. Note that airsoft shots are found firmly within the constant region, which supports our use of the quadratic drag model and allows us to assume a constant drag coefficient value for all relevant parameter sets found in airsoft.

If we where interested in computing the ballistic arc of the projectile, considering both the drag force and the gravitational force, we would not be able to find an analytic solution to the equations of motion and would have to fall back on a numerical solution. However, we are not interested in computing the arc but rather the projectile energy as a function of distance traveled and for that energy the contribution of the gravitational force is small enough to be neglected. This is great, because it simplifies the equations and allows us to find a closed, analytical solution.

One factor that is ignored in this paper is the rotation of the airsoft projectiles. Most airsoft weapons use a device called a "hop-up", giving the projectiles a backspin. This spin takes advantage of another fluid mechanical process called the "Magnus effect", producing an upward force balancing the gravitational pull and extending the airsoft weapons range considerably. This spin obviously affects the aerodynamic properties of the projectile, but in this paper it is assumed that it does not have a significant impact on the breaking force of the atmospheric drag and therefore does not contribute considerably to the impact energy of the projectile. In our model an effect on the drag force from spin would show up as a shift in the drag coefficient value, so in order to test this assumption we determine the coefficient empirically for airsoft conditions.

2 Theory

2.1 Reynolds number and a constant drag coefficient

Reynolds number is an important property of any fluid mechanical problem

$$Re = \frac{\rho L u}{\mu}$$

and can be computed from the density ρ, the characteristic length L (the diameter for a sphere), the flow velocity u and the dynamic viscosity μ. It gives a ratio between inertial forces and viscous forces in a fluid flow and can help determine if it is laminar, transient or turbulent.

- Laminar: $Re < 2400$
- Transient: $2400 < Re < 4000$
- Turbulent: $Re > 4000$

An important result in fluid dynamics is that the drag coefficient is a function only of the Reynolds number of the fluid flow about the object. That is,

$$C_D = C_D(Re)$$

This functional relationship has no closed form. However, the relationship has been established numerically based on experimental data. See figure 1 for a schematic diagram of the drag coefficient's dependence on the Reynolds number. Airsoft shots typically have a Reynolds number of about $4 \cdot 10^4$, placing them in the constant region of the drag coefficient with a value of approximately 0.5 and validating our assumption of a quadratic drag model.

2.2 An analytical solution of the quadratic drag model in one dimension

The forces acting on the projectile are described by

$$F_D = \frac{1}{2} C_D \rho A u^2 = C u^2 \qquad F = m u'(t) = -F_D = -C u^2(t)$$

We substitute $C = m\lambda$ for now in order to clean up the calculations and arrive at the fundamental differential equation defining our problem.

$$u'(t) = -\lambda u^2(t)$$

We note that this differential equation is separable and thus relatively easy to solve.

$$-\int \frac{du}{\lambda u^2} = \int dt = t + c = \frac{1}{\lambda u} \qquad u(t) = \lambda^{-1}(t+c)^{-1} \qquad u(0) = (\lambda c)^{-1} = u_0 \implies c = (\lambda u_0)^{-1}$$

$$u(t) = (\lambda t + u_0^{-1})^{-1}$$

Having expressed the velocity as a function of time we now want to use it to express the kinetic energy as a function of time.

$$E = \frac{mu^2}{2} \qquad E(t) = \frac{m}{2}(\lambda t + u_0^{-1})^{-2}$$

We note here that the energy decrease with time, as expected. We now want to calculate the time t_1 when the energy has reached the level E_1.

$$E(t_1) = E_1 \qquad t_1 = \lambda^{-1}\left(\sqrt{\frac{m}{2E_1}} - u_0^{-1}\right)$$

Let us now instead consider the distance traveled x. Knowing the velocity as a function of time $u(t)$ we can integrate and get the distance as a function of time $x(\tau)$ as well.

$$x(\tau) = \int_0^\tau u(t)dt = \int_0^\tau \frac{dt}{\lambda t + u_0^{-1}} = \left[\lambda^{-1}\ln(\lambda t + u_0^{-1})\right]_0^\tau = \lambda^{-1}\left(\ln(\lambda\tau + u_0^{-1}) - \ln(u_0^{-1})\right) = \lambda^{-1}\ln(\lambda u_0 \tau + 1)$$

Now we can use the time t_1 that we calculated earlier to calculate how far the projectile has traveled when it reaches the energy level E_1.

$$x_1 = x(t_1) = \lambda^{-1}\ln\left(\lambda u_0\left(\lambda^{-1}\left(\sqrt{\frac{m}{2E_1}} - u_0^{-1}\right)\right) + 1\right) = \lambda^{-1}\ln\left(u_0\sqrt{\frac{m}{2E_1}}\right)$$

In the safety table we want to fix the values of impact energy E_1, safety distance x_1 and mass m to get the initial velocity u_0, so lets solve for u_0.

$$\lambda x_1 = \ln\left(u_0\sqrt{\frac{m}{2E_1}}\right) \qquad e^{\lambda x_1} = \left(u_0\sqrt{\frac{m}{2E_1}}\right) \qquad u_0 = \sqrt{\frac{2E_1}{m}} e^{\lambda x_1}$$

Finally we arrive at our final expression by re-substituting $C = m\lambda$.

$$u_0 = \sqrt{\frac{2E_1}{m}} e^{C\frac{x_1}{m}}$$

This expression is very useful for computing the values for the safety table, but we may take the opportunity now to also derive another expression that will be useful later for validating the model with measurement data.

$$u_1 = u_0 e^{-C\frac{x_1}{m}}$$

The properties we measure directly are velocity and distance, so this expression shows velocity decay as a function of distance.

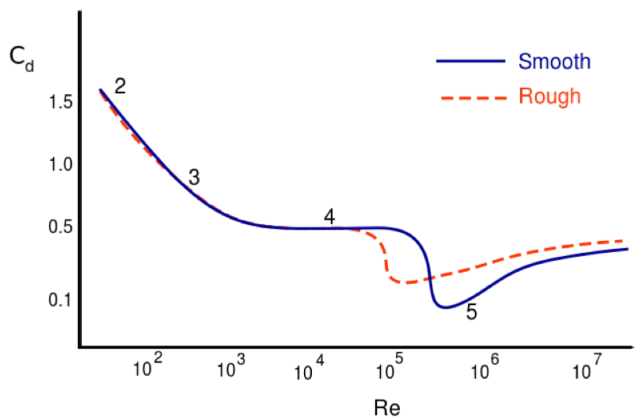

Figure 1. The drag coefficient C_D as a function of the Reynolds number Re. The relationship is determined empirically. Note that the Reynolds number for an airsoft shot is typically around $4 \cdot 10^4$, placing it firmly in the constant region of the drag coefficient. Image credit: NASA[1]

3 Method

We have two questions to answer with empirical testing:

1. Is our assumed value of $C_D = 0.5$ for the drag coefficient good, or is it failing to incorporate something like the spin induced by the hop-up?
2. Is the model good? Stated more precisely: how much does its predictions deviate from measurements?

The kinetic energy of a projectile is determined by the simple equation

$$E = \frac{mu^2}{2}$$

In airsoft we know the mass m of our projectile and can easily measure the velocity u by the muzzle with a chronograph, giving us a simple way to determine the kinetic energy. It is in principle possible shoot through a chronograph at a distance as well to determine how much energy the projectile has left at that point in its ballistic arc. Measuring at a distance like this is unpractical to do before games, but the effort can be made and this is the method used to collect data for the investigation in this report. When we have the data we can compare the predictions of our model with the real world measurements and determine its accuracy. By collecting measurements at several distances with several masses and several initial energies we get a good spread in the parameter space relevant to airsoft.

In the tests a chronograph was used to measure projectile speed, a laser range finder was used to measure distance and the mass and diameter of the projectiles was specified by the manufacturer. The temperature on the day of

(a) Measurements at 7 meters. **(b)** Measurements at 27 meters.

Figure 2. The figure shows the chronograph used and illustrates the conditions on the day of testing.

testing was about $25°C$ and there was no wind. Figure 2 illustrates the measurement procedure. Five measurements where performed for each mass-distance-gun combination. The parameters of the measurement sets can be found in table 1, the raw measurements along with the data analysis software can be found in this projects github repository[2] and a visualization of the data can be seen in figure 3.

3.1 Determining the drag coefficient

When we have collected the data it is time to analyze it with respect to our two questions and we begin with determining the drag coefficient C_D. As we derived in the theory section, our model predicts an exponential decay of velocity as

$$u_1 = u_0 e^{-C\frac{x_1}{m}} \qquad C = \frac{1}{2} C_D \rho A$$

so we would like to fit this function to our data using x as the variable, C_D as the optimization parameter and the rest of the values as knowns. We fit the function using the *curve_fit* function in the python packet *scipy*, which in turn uses the Levenberg-Marquardt non-linear optimization algorithm. A fit is computed for each gun-ammo-pair data set separately giving us the optimal values of C_D and the resulting averages and variances are then weighed together, creating a single result backed by all our available evidence. Care should be taken when combining the sub-results to do it properly. If we assume that the parameter results for C_D is normally distributed we find that the combined distribution's mean is the weighted average of means (weighted by precision) and its variance is the "harmonic sum" of variances[3].

3.2 Evaluating our model and computing an error measure

When we have a value for C_D we can use it in our model and can proceed to focus on the second question of how accurate our model predicts the measurements. For each gun-ammo-pair we average the velocity measurements at the 0 meter distance and use this mean value as the initial condition when we compute predictions for velocities at longer distances. We then compute a prediction for each measurement distance and compare this with every measurement value at that distance. The mean square error is computed for the set of all measurements and this error is taken as the accuracy for the model at large.

3.3 Using our model to compute safety tables

The choices of the two tables provided by this report corresponds to 2020tabellen[4] as of June 2020 and a proposed updated version of VSAF-tabellen[5]. Both of our examples use the same safety distances and masses. Both examples will cap the muzzle energy differentiated between weapon classes, but at slightly different values. Both examples cap the impact energy, but while 2020tabellen have a single common cap for all classes VSAF-tabellen differentiates the impact energy between classes. All relevant input values to the table computations can be found in tables 2 and 3.

To compute the values for the tables we simply use the expression we derived in the theory section

$$u_0 = \sqrt{\frac{2E_1}{m}} e^{C\frac{x_1}{m}}$$

and insert our chosen input values of impact energy, distance and mass. If the resulting muzzle velocity corresponds to a too high muzzle energy, it will be capped to the highest valid value instead. The resulting safety tables can be found in tables 4 and 5.

4 Results

4.1 Summary of analytical results

The important expressions derived in this paper can be summarized as

$$u_0 = \sqrt{\frac{2E_1}{m}} e^{C\frac{x_1}{m}} \qquad x_1 = \frac{m}{C} \ln\left(u_0 \sqrt{\frac{m}{2E_1}}\right) \qquad C = \frac{1}{2} C_D \rho A$$

4.2 Drag coefficient

Taking a typical speed of an airsoft projectile to be 100 m/s the Reynolds number of a typical airsoft shot is $Re \approx 4 \cdot 10^4$. This puts it in the center of the constant region of the drag coefficient with a value of approximately 0.5, as can be seen in figure 1. The resulting empirical value derived from the measurements in this report is 0.47745 ± 0.000017. This result is close to the expected value from the approximate readout from figure 1. No apparent pattern could be seen in the computed drag coefficients between the different gun-ammo-sets, which supports the assumption that the projectile spinn induced by the hop-up is negliable with respect to kinetic energy dissipation from drag. This empirical drag coefficient value was used to compute the MSE of the model and the safety table examples.

4.3 Model evaluation and error measure

All the measurement points can be seen together with the predicted curves for each gun-ammo-set in figure 3, with the parameter space for the measurements described in table 1 and the constants in table 2. The mean square error for the model predictions compared with the measurements was 1.443 m/s. Much of this error should probably be contributed to gun inconsistency and imperfect measurement equipment accuracy. If we look at the results in figure 3 we note the the predictions align nicely with the data without any obvious bias to always over- or undershoot. The error size is small enough to support the conclusion that the drag model in this report is useful for the purpose of creating airsoft safety tables.

Property	Symbol	Value	Unit	Comment
Projectile mass	m	0.25, 0.3, 0.45	g	Specified by manufacturer
Distance	x_1	0, 4, 7, 16, 27	m	Measured by laser range finder
Gun	-	SSG24, Krytac Trident, TM HK416	-	Mentioned from harder to softer.

Table 1. The parameters used in the measurements. The manufacturer of the two ligher ammunition types were Green Devil and the manufacturer of the heavier ammunition was Geoff.

4.4 Safety table examples

The safety tables were computed with the code in this reports projects GitHub repository[2] using the constants in table 2 and subjective values from table 3 and can be seen as tables 4 and 5. We note that the values in 2020tabellen is more squeezed together and in VSAF-tabellen they are more stretched out between weapon classes, as would be expected from the different parameter choices.

5 Discussion

In this paper the physical background of the airsoft safety table has been reviewed. It is interesting to note how much impact the different factors have on the allowed muzzle velocity, where we can note a very strong dependence

Property	Symbol	Value	Unit	Comment
Air density[6]	ρ	1.225	kg/m^3	15°C, 1 atm (safety tables)
Air density[6]	ρ	1.184	kg/m^3	25°C, 1 atm (data analysis)
Air dynamic viscosity[6]	μ	$1.802 \cdot 10^{-5}$	N s/m^2	15°C, 1 atm
Characteristic length (projectile diameter)	L	0.006	m	Specified by manufacturer
Area	A	$1.131 \cdot 10^{-4}$	m^2	Cross section of the projectile
Drag coefficient	C_D	0.48	unitless	Empirical value

Table 2. The constant values used in both data analysis and in safety table computations. A lower temperature was assumed for the safety tables to better match normal nordic weather conditions throughout the year.

Property	Symbol	Value	Unit	Comment
Impact energy	E_1	1.2	J	2020tabellen
Muzzle energy	E_0	$1.2, 1.45, 1.7, 2.2, 2.2, 3, 4$	J	2020tabellen
Impact energy	E_1	$1, 1, 1, 1.08, 1.16, 1.16, 1.16$	J	VSAF-tabellen
Muzzle energy	E_0	$1, 1.34, 1.76, 2.11, 2.51, 3.34, 4.55$	J	VSAF-tabellen
Projectile mass	m	$20, 25, 28, 30, 34, 36, 40, 43, 45, 48, 50$	g/100	Both tables
Safety distance	x_1	$0, 5, 10, 20, 20, 30, 40$	m	Both tables

Table 3. The subjective values used to compute the two safety table examples. Note that for non-license weapons in Sweden the muzzle energy must be below 3 joules for automatic weapons and below 10 joules non-automatic weapons.

on the projectile mass, a strong dependence on the safety distance and a weak dependence on the impact energy. The exponential nature of the energy decay over distance lead to that when both mass and distance simultaneously promote a high muzzle velocity it gets unreasonably high, making it necessary to introduce a cap for both safety and legal reasons.

The empirical evidence supports the analytically derived drag model well with the empirical drag coefficient close to the hypothesized one, the model predictions mean square error fairly small and a nice fit of predictions to data. We thus conclude that the model is useful for constructing safety tables for airsoft. The two provided example safety tables should provide a good basis for further discussion about the remaining more subjective factors in safety table design.

Hopefully this review has clarified some of the confusion that has previously existed around the safety table and the author hope that this review will contribute to safety, fairness and joy in the airsoft community.

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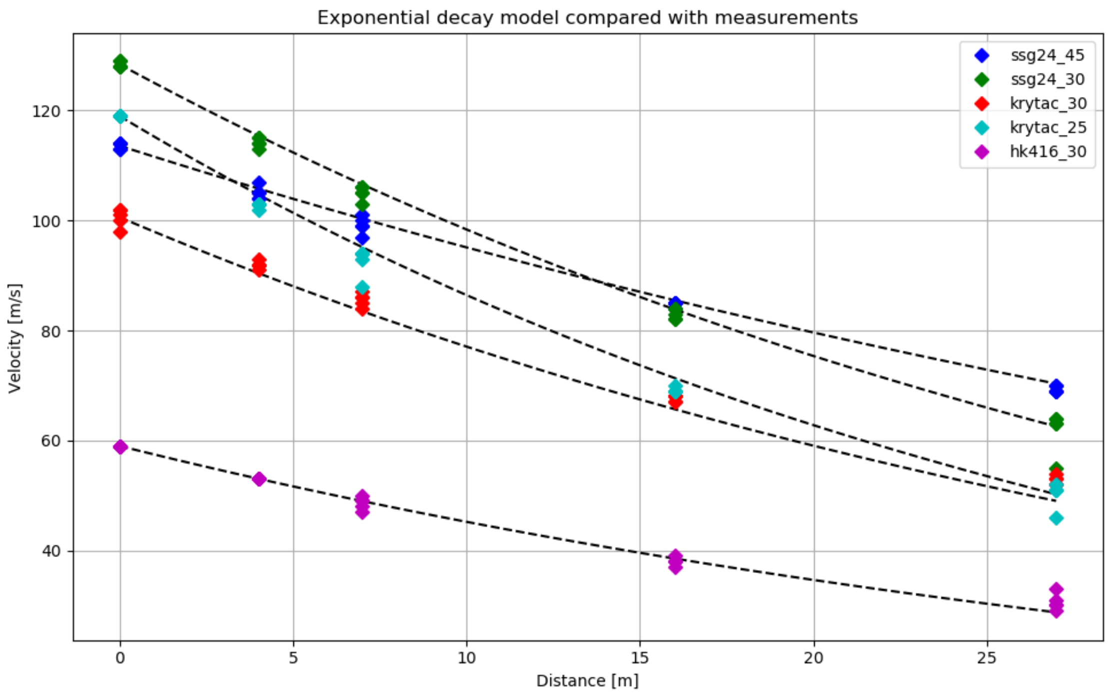

Figure 3. The figure shows the model predictions expected from the measured initial conditions as dashed lines and the measured values as dots. Note that the predictions align well with the measurements.

Class	Safety distance	.20	.25	.28	.30	.34	.36	.40	.43	.45	.48	.50
CQB 2	0	109.5	98.0	92.6	89.4	84.0	81.6	77.5	74.7	73.0	70.7	69.3
Assault 1	5	120.4	107.7	101.8	98.3	92.4	89.8	85.1	82.1	80.0	77.1	75.2
Assault 2	10	130.4	116.6	110.2	106.5	100.0	97.2	92.2	88.9	86.9	84.0	81.7
Support 3	20	148.3	132.7	125.4	121.1	113.8	110.6	104.9	101.2	98.9	95.7	93.8
DMR	20	148.3	132.7	125.4	121.1	113.8	110.6	104.9	101.2	98.9	95.7	93.8
Sniper 1	30	173.2	154.9	146.4	141.4	132.8	129.1	122.5	118.1	115.5	111.8	109.5
Sniper 2	40	200.0	178.9	169.0	163.3	153.4	149.1	141.4	136.4	133.3	129.1	126.5

Table 4. A safety table example with a constant impact energy, corresponding to 2020tabellen.

Class	Safety distance	.20	.25	.28	.30	.34	.36	.40	.43	.45	.48	.50
CQB	0	100.0	89.4	84.5	81.6	76.7	74.5	70.7	68.2	66.7	64.5	63.2
AutoA	5	115.8	103.5	97.8	93.7	86.6	83.6	78.4	75.1	73.1	70.4	68.7
AutoB	10	132.7	118.7	112.1	107.5	97.8	93.8	86.9	82.6	80.1	76.7	74.6
HMG	20	145.3	129.9	122.8	118.6	111.4	108.3	102.7	99.1	96.8	93.8	91.5
Semi	20	158.4	141.7	133.9	129.4	121.5	118.1	112.0	107.9	103.7	98.1	94.8
BoltA	30	182.8	163.5	154.5	149.2	140.2	136.2	129.2	124.6	121.8	116.5	111.8
BoltB	40	213.3	190.8	180.3	174.2	163.6	159.0	150.8	145.5	142.2	137.7	131.9

Table 5. A safety table example with a differentiated impact energy, corresponding to VSAF-tabellen.