TMA4245 Statistikk Innlevering 1, Blokk 1

Tobias Rage Torben

1a) Uordnet utvalg uten tilbakelegging:

$$M = \binom{n}{r} = \binom{100}{4} = \frac{100!}{4!\,96!} = \underline{\underline{3921225}}$$

La T angi loddet uten gevinst.

La oss se på delmengden ABTT.

Her ser på tilfellet at vi trekker A og B, og deretter skal vi trekke to lodd av 96 mulige uten tilbakelegging. Det gir

$$\binom{96}{2} = \frac{96 \cdot 95}{2} = 4560 \text{ mulige kombinasjoner.}$$

Videre har vi seks slike tilfeller som gir 2 premier:

ABTT
ACTT
ADTT
BCTT
BDTT
CDTT

Hver av disse har 4560 mulige kombinasjoner av de to loddene uten premie.

Antall kombinasjoner blir derfor $6 \cdot 4560 = \underline{\underline{27360}}$

b) $P(\text{Ingen gevinst}) = \frac{96}{100} \cdot \frac{95}{99} \cdot \frac{94}{98} \cdot \frac{93}{97} = 0{,}847$

$P(\text{minst 1 gevinst}) = 1 - 0{,}847 = \underline{0{,}1528}$

For at han skal trekke lodd A finnes det $\binom{99}{3}$ gunstige utfall, og $\binom{100}{4}$ mulige utfall.

$P(A) = \binom{99}{3} / \binom{100}{4} = \underline{\underline{0{,}04}}$

c) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\binom{98}{2} / \binom{100}{4}}{0{,}04} = \underline{0{,}0303}$

La C være at per vant minst 1, og D være at han vant alle 4.

$P(E|D) = \frac{P(E \cap D)}{P(D)} = \frac{P(E)}{P(D)} = \frac{\frac{1}{\binom{100}{4}}}{0.1528} = \underline{\underline{1.664 \cdot 10^{-6}}}$

2a) Hendelsene er ikke disjunkte, da $P(E \cap F) > 0$.

$P(E|F) = \frac{P(E \cap F)}{P(F)} = 0.4 \neq P(E)$. Hendelsene er ikke uavhengige.

b) $R = E \cup F$

$P(R) = P(E \cup F) = P(E) + P(F) - P(E \cap F) = \underline{\underline{0{,}08}}$

c) $P(V \mid (E^c \cap F^c)) = 0{,}07$

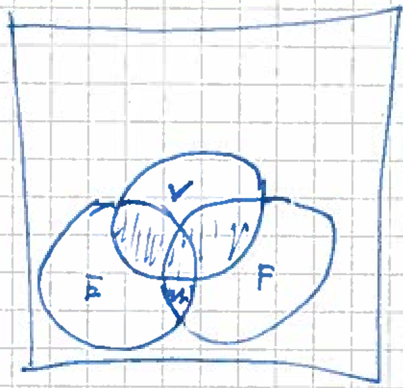
$P(V \mid (E \cup F)) = 0{,}5$

$P(V) = P(E^c \cap F^c) \cdot 0.07$

$+ P(E \cup F) \cdot 0.5$

$= 0.92 \cdot 0{,}07 + 0.08 \cdot 0.5$

$\underline{\underline{= 0.1044}}$



$P(E \cup F \cup V) = P(E) + P(F) + P(V) - P(E \cap F)$

$- P(V \cap (A \cup F))$

$= 0{,}05 + 0{,}05 + 0.1044 - 0{,}02 - 0.08 \cdot 0.5$

$\underline{= 0.1444}$

3a) $F(x) = 1 - e^{-\frac{x^2}{2\alpha}}$

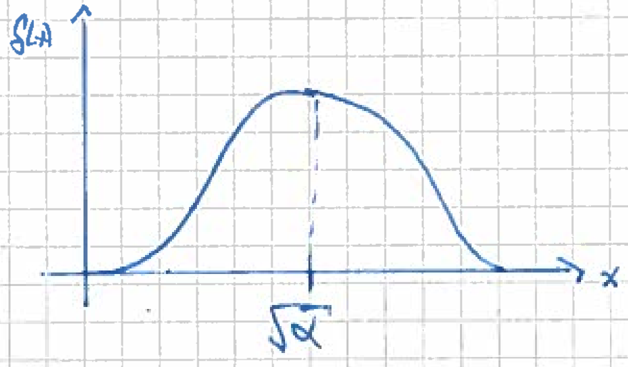
$f(x) = \frac{dF}{dx} = -e^{-\frac{x^2}{2\alpha}} \cdot \frac{-2x}{2\alpha}$

$\underline{\underline{= \frac{x}{\alpha} e^{-\frac{x^2}{2\alpha}}}}$

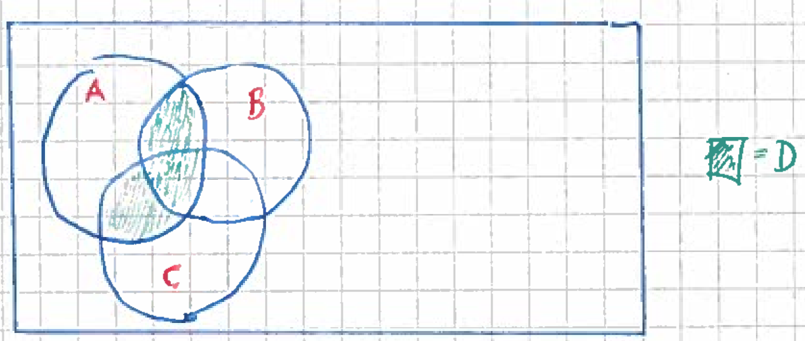
$\frac{df}{dx} = \frac{1}{\alpha} e^{-\frac{x^2}{2\alpha}} + \frac{x}{\alpha} \cdot e^{-\frac{x^2}{2\alpha}} \cdot \frac{-2x}{2\alpha}$

$= \frac{e^{-\frac{x^2}{2\alpha}}}{\alpha} \left(1 - \frac{x^2}{\alpha}\right)$

$f'(x) = 0 \Leftrightarrow \underline{x = \sqrt{\alpha}}$



b)



$$P(D) = P(A \cap (B \cup C))$$

$$P(A) = P(B) = P(C) = 1 - F(2) = 0.1353$$

$$P(B \cup C) = P(B) + P(C) - P(B)P(C) = 0{,}2524$$

$$P(A \cap (B \cup C)) = P(A) \cdot P(B \cup C) = 0.1353 \cdot 0.2524 = \underline{\underline{0.034}}$$

4a) Utfall som tillfredställer $X > Y$:

$(1,0)$, $(2,0)$, $(2,1)$

$$P(X > Y) = f(1,0) + f(2,0) + f(2,1) = \underline{\underline{0.2}}$$

$$g(x) = \sum_{y=0}^{2} f(x,y) = f(x,0) + f(x,1) + f(x,2)$$

$$h(y) = \sum_{x=0}^{2} f(x,y) = f(0,y) + f(1,y) + f(2,y)$$

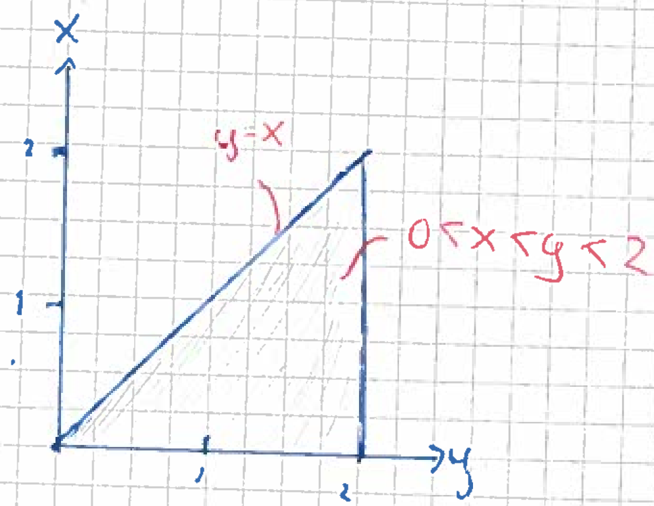
x/y	0	1	2	g(x)
0	0.1	0.25	0.15	0.5
1	0.06	0.15	0.09	0.3
2	0.04	0.1	0.06	0.2
h(y)	0.2	0.5	0.3	

Vi ser at $f(x,y) = g(x)h(y) \; \forall \; (x,y)$,

av definisjonen på statistisk uavhengighet er derfor X og Y uavhengige.

5a) $f(x,y) = \frac{1}{2}xy \; , \; 0 < x < y < 2$

$f(x,y) = 0$ ellers



$$h(y) = \int_0^y f(x,y)\,dx$$

$$= \int_0^y \frac{1}{2}xy\,dx$$

$$= \frac{1}{4}y\left[x^2\right]_0^y = \underline{\underline{\frac{y^3}{4}}}$$

$$g(x) = \int_x^2 f(x,y)\,dy = \int_x^2 \frac{1}{2}xy\,dy$$

$$= \frac{1}{4}x\left[y^2\right]_x^2 = \underline{\underline{\frac{1}{4}x(4-x^2)}}$$

$$f(y|x) = \frac{f(x,y)}{g(x)} = \frac{\frac{1}{2}xy}{\frac{1}{4}x(4-x^2)} = \underline{\underline{\frac{2y}{4-x^2}}}$$

b) $$P(Y>1 \mid X=\tfrac{1}{2}) = \int_1^2 f(y|\tfrac{1}{2})\,dy$$

$$= \int_1^2 \frac{2y}{4-(\frac{1}{2})^2}\,dy = \int_1^2 \frac{8y}{15}\,dy = \frac{8}{30}\left[y\right]_1^2 = \underline{0{,}8}$$