NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
“IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE”

Faculty of Informatics and Computer Engineering

Department of Computer Engineering

Lab 4 Report

Investigating Neural Network Structure and Simulating a Two-Variable Function

Variant 12

Student, group IM-14
(group code)

in the educational and professional program

“Software Engineering For Computer System”

Specialty 121 "Computer Engineering"

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(position, academic degree, academic status, surname and initials)

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The purpose of the work: To investigate the structure and principle of operation of a neural network. With the help of a neural network, simulate the function of two variables

12.	$y = \cos(x)/x - \sin(x)/x^2$ $z = \sin(x/2) + y \cdot \sin(x)$	12.	19.	7.

1 inner layer with 10 neurons;

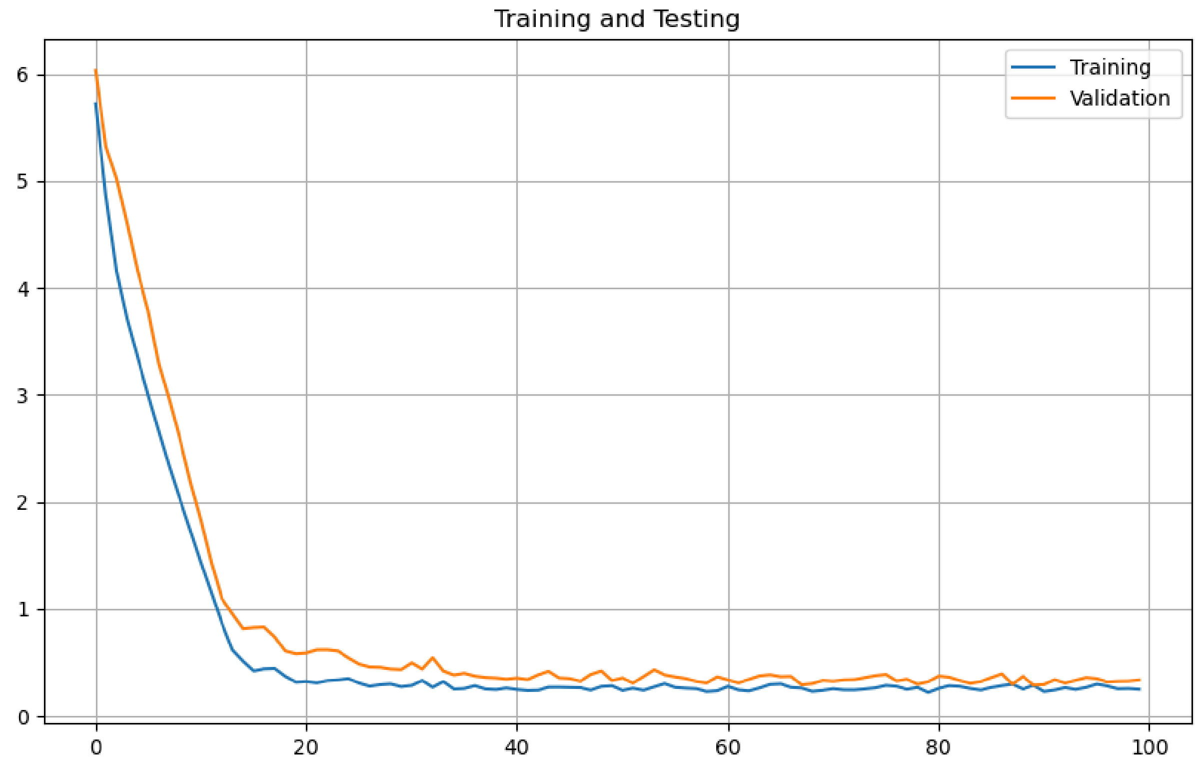
Program listing

```
In [7]: import numpy as np
        import tensorflow as tf
        import matplotlib.pyplot as plt

        # Function of Two Variables
        def func(x):
            y = np.cos(x[0]) / x[1] - np.sin(x[0]) / x[1]**2
            z = np.sin(x[0] / 2) + y * np.sin(x[0])
            return y, z

        # Collecting Dataset
        x = np.linspace(12, 19, 7)
        x0, x1 = np.meshgrid(x, x)
        x0, x1 = x0.ravel(), x1.ravel()
        X = [[x0[i], x1[i]] for i in range(len(x0))]
        y, z = zip(*[func(x) for x in X])

        split = int(len(X) * 0.8)
        x_train, y_train, z_train = X[:split], y[:split], z[:split]
        x_test, y_test, z_test = X[split:], y[split:], z[split:]

        # Function to display
        def visual(his):
            loss = his.history['loss']
            val_loss = his.history['val_loss']
            epochs = range(len(loss))
```

```python
    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
    plt.title('Training and Testing')
    plt.legend()
    plt.grid()
    plt.show()

# Model with 1 inner layer and 10 neurons
model_fl = tf.keras.models.Sequential([
    tf.keras.layers.Dense(10, input_shape=(2,), activation='relu'),
    tf.keras.layers.Dense(2)  # Two outputs for y and z
])

model_fl.summary()

model_fl.compile(
    optimizer=tf.keras.optimizers.SGD(
        learning_rate=tf.keras.optimizers.schedules.ExponentialDecay(0.001, decay_steps=75, decay_rate=0.96)
    ),
    loss='mae',
    metrics=['mae']
)

# Train the model
history_f = model_fl.fit(
    np.array(x_train), np.array([y_train, z_train]).T,  # Two outputs
    epochs=100,
    validation_data=(np.array(x_test), np.array([y_test, z_test]).T),
)

# Display learning curves
visual(history_f)
```

Performance results

```
Model: "sequential_6"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_12 (Dense)            (None, 10)                30

 dense_13 (Dense)            (None, 2)                 22

=================================================================
Total params: 52 (208.00 Byte)
Trainable params: 52 (208.00 Byte)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
Epoch 1/100
2/2 [==============================] - 0s 146ms/step - loss: 5.7178 - mae: 5.7178 - val_loss: 6.0310 - val_mae: 6.0310
Epoch 2/100
2/2 [==============================] - 0s 31ms/step - loss: 4.8520 - mae: 4.8520 - val_loss: 5.3169 - val_mae: 5.3169
Epoch 3/100
2/2 [==============================] - 0s 31ms/step - loss: 4.1556 - mae: 4.1556 - val_loss: 5.0190 - val_mae: 5.0190
```



1 inner layer with 20 neurons;

Program listing

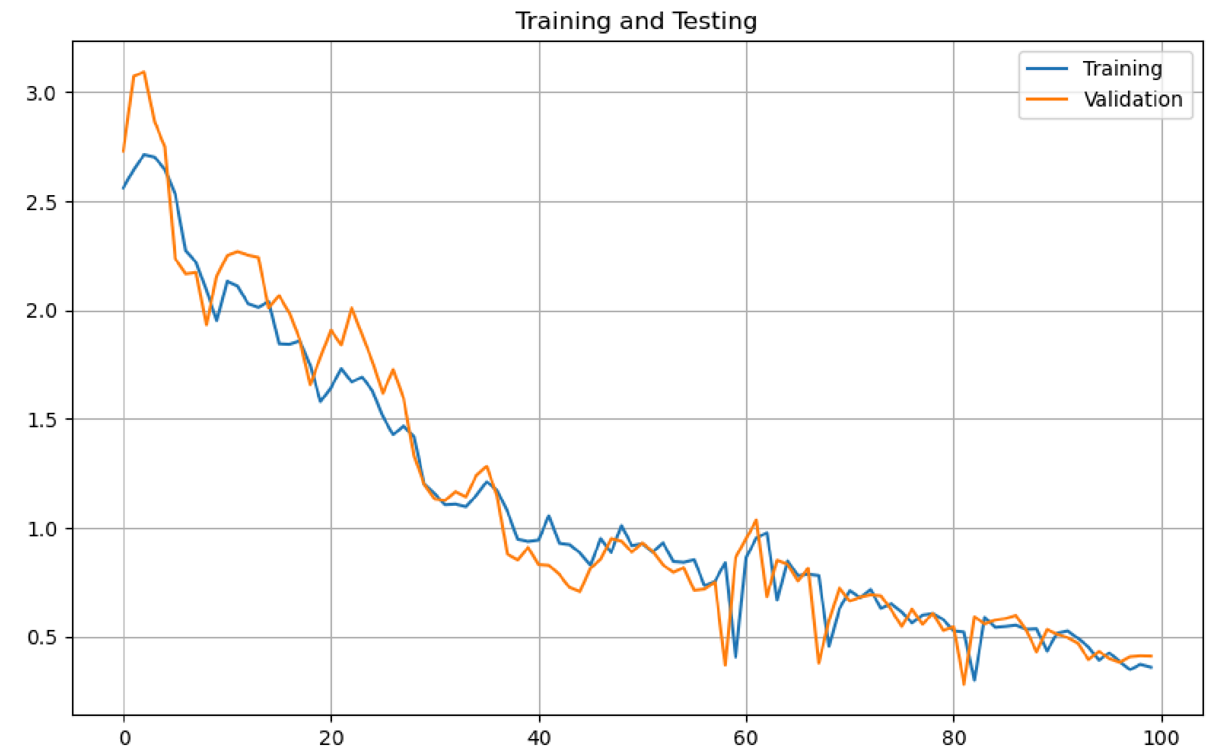
```
In [2]: import numpy as np
        import tensorflow as tf
        import matplotlib.pyplot as plt

        # Function of Two Variables
        def func(x):
            y = np.cos(x[0]) / x[1] - np.sin(x[0]) / x[1]**2
            z = np.sin(x[0] / 2) + y * np.sin(x[0])
            return y, z

        # Collecting Dataset
        x = np.linspace(12, 19, 7)
        x0, x1 = np.meshgrid(x, x)
        x0, x1 = x0.ravel(), x1.ravel()
        X = [[x0[i], x1[i]] for i in range(len(x0))]
        y, z = zip(*[func(x) for x in X])

        split = int(len(X) * 0.8)
        x_train, y_train, z_train = X[:split], y[:split], z[:split]
        x_test, y_test, z_test = X[split:], y[split:], z[split:]

        # Function to display
        def visual(his):
            loss = his.history['loss']
            val_loss = his.history['val_loss']
            epochs = range(len(loss))
```

Program listing

```python
    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
    plt.title('Training and Testing')
    plt.legend()
    plt.grid()
    plt.show()

# Model with 1 inner layer and 20 neurons
model_fl2 = tf.keras.models.Sequential([
    tf.keras.layers.Dense(20, input_shape=(2,), activation='relu'),
    tf.keras.layers.Dense(1)
])

model_fl2.summary()

model_fl2.compile(
    optimizer=tf.keras.optimizers.SGD(
        learning_rate=tf.keras.optimizers.schedules.ExponentialDecay(0.005, decay_steps=75, decay_rate=0.96)
    ),
    loss='mae',
    metrics=['mae']
)

# Train the model
history_f2 = model_fl2.fit(
    np.array(x_train), np.array(y_train),  # Assuming y_train is a single array
    epochs=100,
    validation_data=(np.array(x_test), np.array(y_test)),  # Assuming y_test is a single array
)

# Display learning curves
visual(history_f2)
```

Performance results

```
Model: "sequential_1"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_2 (Dense)             (None, 20)                60

 dense_3 (Dense)             (None, 1)                 21

=================================================================
Total params: 81 (324.00 Byte)
Trainable params: 81 (324.00 Byte)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
Epoch 1/100
2/2 [==============================] - 0s 131ms/step - loss: 2.5616 - mae: 2.5616 - val_loss: 2.7308 - val_mae: 2.7308
Epoch 2/100
2/2 [==============================] - 0s 30ms/step - loss: 2.6446 - mae: 2.6446 - val_loss: 3.0731 - val_mae: 3.0731
Epoch 3/100
2/2 [==============================] - 0s 32ms/step - loss: 2.7143 - mae: 2.7143 - val_loss: 3.0941 - val_mae: 3.0941
```



Network type: cascade-forward backprop:

1 inner layer with 20 neurons;

Program listing

In [16]:

```python
import numpy as np
import tensorflow as tf
import matplotlib.pyplot as plt

# Function of Two Variables
def func(x):
    y = np.cos(x[0]) / x[1] - np.sin(x[0]) / x[1]**2
    z = np.sin(x[0] / 2) + y * np.sin(x[0])
    return y, z

# Collecting Dataset
x = np.linspace(12, 19, 7)
x0, x1 = np.meshgrid(x, x)
x0, x1 = x0.ravel(), x1.ravel()
X = [[x0[i], x1[i]] for i in range(len(x0))]
y, z = zip(*[func(x) for x in X])

split = int(len(X) * 0.8)
x_train, y_train, z_train = X[:split], y[:split], z[:split]
x_test, y_test, z_test = X[split:], y[split:], z[split:]

# Function to display
def visual(his):
    loss = his.history['loss']
    val_loss = his.history['val_loss']
    epochs = range(len(loss))

    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
```


```python
    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
```


```python
    plt.title('Training and Testing')
    plt.legend()
    plt.grid()
    plt.show()

# Model with 1 inner layer and 20 neurons
inputs = tf.keras.Input(shape=(2,))
x = tf.keras.layers.Dense(20, activation='relu')(inputs)
outputs = tf.keras.layers.Dense(2)(tf.keras.layers.concatenate([inputs, x]))  # Two outputs for y and z
model_cf1 = tf.keras.Model(inputs, outputs)

model_cf1.summary()

model_cf1.compile(
    optimizer=tf.keras.optimizers.SGD(
        learning_rate=tf.keras.optimizers.schedules.ExponentialDecay(0.001, decay_steps=75, decay_rate=0.96)
    ),
    loss='mae',
    metrics=['mae']
)

# Train the model
history_cf1 = model_cf1.fit(
    np.array(x_train), np.array([y_train, z_train]).T,  # Two outputs
    epochs=100,
    validation_data=(np.array(x_test), np.array([y_test, z_test]).T),
)

# Display learning curves
visual(history_cf1)
```

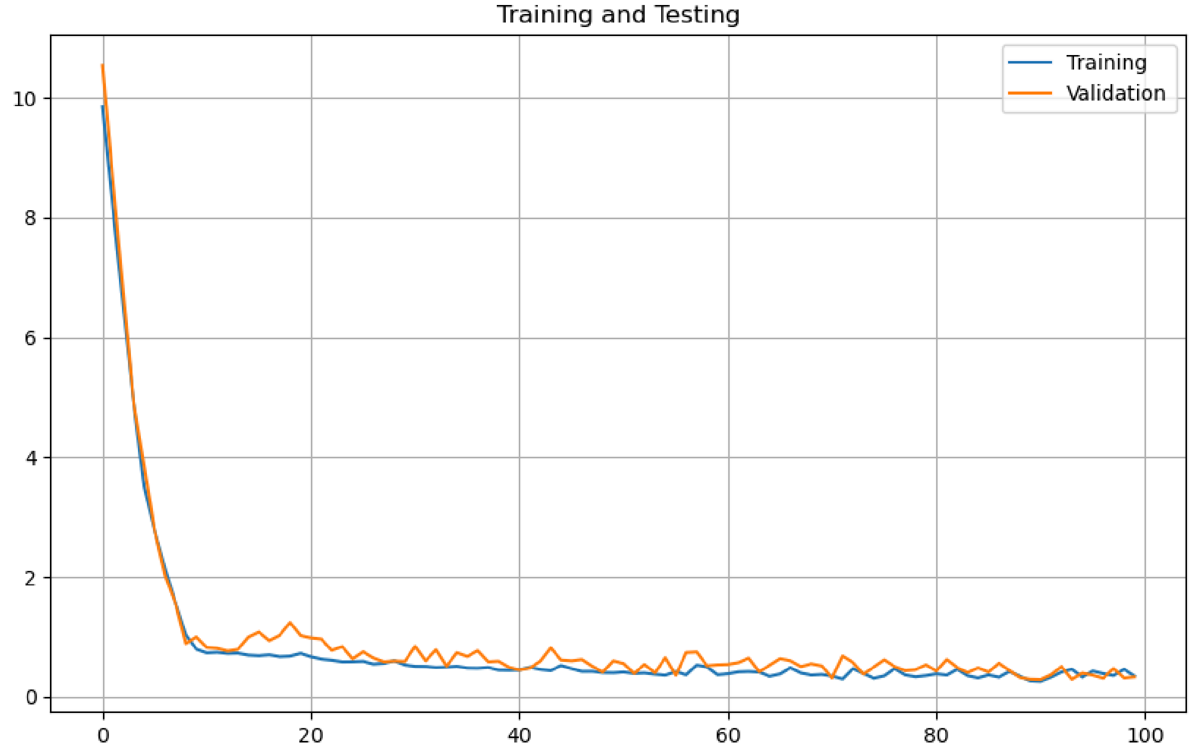
Creating a model

```
Model: "model_1"
__________________________________________________________________________________________________
 Layer (type)                   Output Shape         Param #     Connected to
==================================================================================================
 input_2 (InputLayer)           [(None, 2)]          0           []

 dense_6 (Dense)                (None, 20)           60          ['input_2[0][0]']

 concatenate_1 (Concatenate     (None, 22)           0           ['input_2[0][0]',
 )                                                                'dense_6[0][0]']

 dense_7 (Dense)                (None, 2)            46          ['concatenate_1[0][0]']

==================================================================================================
Total params: 106 (424.00 Byte)
Trainable params: 106 (424.00 Byte)
Non-trainable params: 0 (0.00 Byte)
__________________________________________________________________________________________________
Epoch 1/100
```

The results of execution

Model training:

```
__________________________________________________________________________________________________
Epoch 1/100
2/2 [==============================] - 1s 337ms/step - loss: 9.8506
- mae: 9.8506 - val_loss: 10.5331 - val_mae: 10.5331
Epoch 2/100
2/2 [==============================] - 0s 30ms/step - loss: 8.1475
- mae: 8.1475 - val_loss: 8.6281 - val_mae: 8.6281
Epoch 3/100
2/2 [==============================] - 0s 29ms/step - loss: 6.5130
- mae: 6.5130 - val_loss: 6.7371 - val_mae: 6.7371
Epoch 4/100
2/2 [==============================] - 0s 31ms/step - loss: 4.8978
- mae: 4.8978 - val_loss: 4.9008 - val_mae: 4.9008
Epoch 5/100
2/2 [==============================] - 0s 31ms/step - loss: 3.5046
- mae: 3.5046 - val_loss: 3.8428 - val_mae: 3.8428
Epoch 6/100
2/2 [==============================] - 0s 36ms/step - loss: 2.7711
```



Training and Testing
Training
Validation
0
2
4
6
8
10
0
20
40
60
80
100

2 inner layers of 10 neurons each:

Program listing

```
In [1]: import numpy as np
import tensorflow as tf
import matplotlib.pyplot as plt

# Function of Two Variables
def func(x):
    y = np.cos(x[0]) / x[1] - np.sin(x[0]) / x[1]**2
    z = np.sin(x[0] / 2) + y * np.sin(x[0])
    return y, z

# Collecting Dataset
x = np.linspace(12, 19, 7)
x0, x1 = np.meshgrid(x, x)
x0, x1 = x0.ravel(), x1.ravel()
X = [[x0[i], x1[i]] for i in range(len(x0))]
y, z = zip(*[func(x) for x in X])

split = int(len(X) * 0.8)
x_train, y_train, z_train = X[:split], y[:split], z[:split]
x_test, y_test, z_test = X[split:], y[split:], z[split:]

# Function to display
def visual(his):
    loss = his.history['loss']
    val_loss = his.history['val_loss']
    epochs = range(len(loss))

    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
    plt.title('Training and Testing')
    plt.legend()
```


```
    plt.plot(epochs, val_loss, label='Validation')
    plt.title('Training and Testing')
    plt.legend()
```


```
    plt.grid()
    plt.show()

# Model with 2 inner layers of 10 neurons each
inputs = tf.keras.Input(shape=(2,))
x0 = tf.keras.layers.Dense(10, activation='relu')(inputs)
x1 = tf.keras.layers.Dense(10, activation='relu')(tf.keras.layers.concatenate([inputs, x0]))
outputs = tf.keras.layers.Dense(2)(tf.keras.layers.concatenate([inputs, x0, x1]))  # Two outputs for y and z
model_cf2 = tf.keras.Model(inputs, outputs)

model_cf2.summary()

model_cf2.compile(
    optimizer=tf.keras.optimizers.SGD(
        learning_rate=tf.keras.optimizers.schedules.ExponentialDecay(0.005, decay_steps=75, decay_rate=0.96)
    ),
    loss='mae',
    metrics=['mae']
)

# Train the model
history_cf2 = model_cf2.fit(
    np.array(x_train), np.array([y_train, z_train]).T,  # Two outputs
    epochs=100,
    validation_data=(np.array(x_test), np.array([y_test, z_test]).T),
)

# Display learning curves
visual(history_cf2)
```

The results of execution

Model training:

```
WARNING:tensorflow:From C:\Users\mehme\anaconda3\Lib\site-packages\keras\src\losses.py:2976: The name tf.losses.sparse_softma
x_cross_entropy is deprecated. Please use tf.compat.v1.losses.sparse_softmax_cross_entropy instead.

WARNING:tensorflow:From C:\Users\mehme\anaconda3\Lib\site-packages\keras\src\backend.py:1398: The name tf.executing_eagerly_o
utside_functions is deprecated. Please use tf.compat.v1.executing_eagerly_outside_functions instead.

Model: "model"
__________________________________________________________________________________________________
 Layer (type)                Output Shape                 Param #   Connected to
==================================================================================================
 input_1 (InputLayer)        [(None, 2)]                  0         []

 dense (Dense)               (None, 10)                   30        ['input_1[0][0]']

 concatenate (Concatenate)   (None, 12)                   0         ['input_1[0][0]',
                                                                     'dense[0][0]']

 dense_1 (Dense)             (None, 10)                   130       ['concatenate[0][0]']
```

Network type:elman backprop::

1 inner layer with 15 neurons;

Program listing

In [1]:
```python
import numpy as np
import tensorflow as tf
import matplotlib.pyplot as plt

# Function of Two Variables
def func(x):
    y = np.cos(x[0]) / x[1] - np.sin(x[0]) / x[1]**2
    z = np.sin(x[0] / 2) + y * np.sin(x[0])
    return y, z

# Collecting Dataset
x = np.linspace(12, 19, 7)
x0, x1 = np.meshgrid(x, x)
x0, x1 = x0.ravel(), x1.ravel()
X = [[x0[i], x1[i]] for i in range(len(x0))]
y, z = zip(*[func(x) for x in X])

split = int(len(X) * 0.8)
x_train, y_train, z_train = X[:split], y[:split], z[:split]
x_test, y_test, z_test = X[split:], y[split:], z[split:]

# Function to display
def visual(his):
    loss = his.history['loss']
    val_loss = his.history['val_loss']
    epochs = range(len(loss))

    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
    plt.title('Training and Testing')
    plt.legend()
```


```python
    plt.plot(epochs, val_loss, label='Validation')
    plt.title('Training and Testing')
    plt.legend()
```


```python
    plt.grid()
    plt.show()

# Model with 1 inner layer and 15 neurons for Elman backprop
model_elman1 = tf.keras.models.Sequential([
    tf.keras.layers.SimpleRNN(15, activation='relu', input_shape=(1, 2)),
    tf.keras.layers.Dense(2)  # Two outputs for y and z
])

model_elman1.summary()

x_train = np.reshape(x_train, (np.shape(x_train)[0], 1, np.shape(x_train)[1]))
x_test = np.reshape(x_test, (np.shape(x_test)[0], 1, np.shape(x_test)[1]))

model_elman1.compile(
    optimizer=tf.keras.optimizers.SGD(
        learning_rate=tf.keras.optimizers.schedules.ExponentialDecay(0.005, decay_steps=75, decay_rate=0.96)
    ),
    loss='mae',
    metrics=['mae']
)

# Train the model
history_elman1 = model_elman1.fit(
    x_train, np.array([y_train, z_train]).T,  # Two outputs
    epochs=100,
    validation_data=(x_test, np.array([y_test, z_test]).T),
)

# Display learning curves
```

Creating a model:

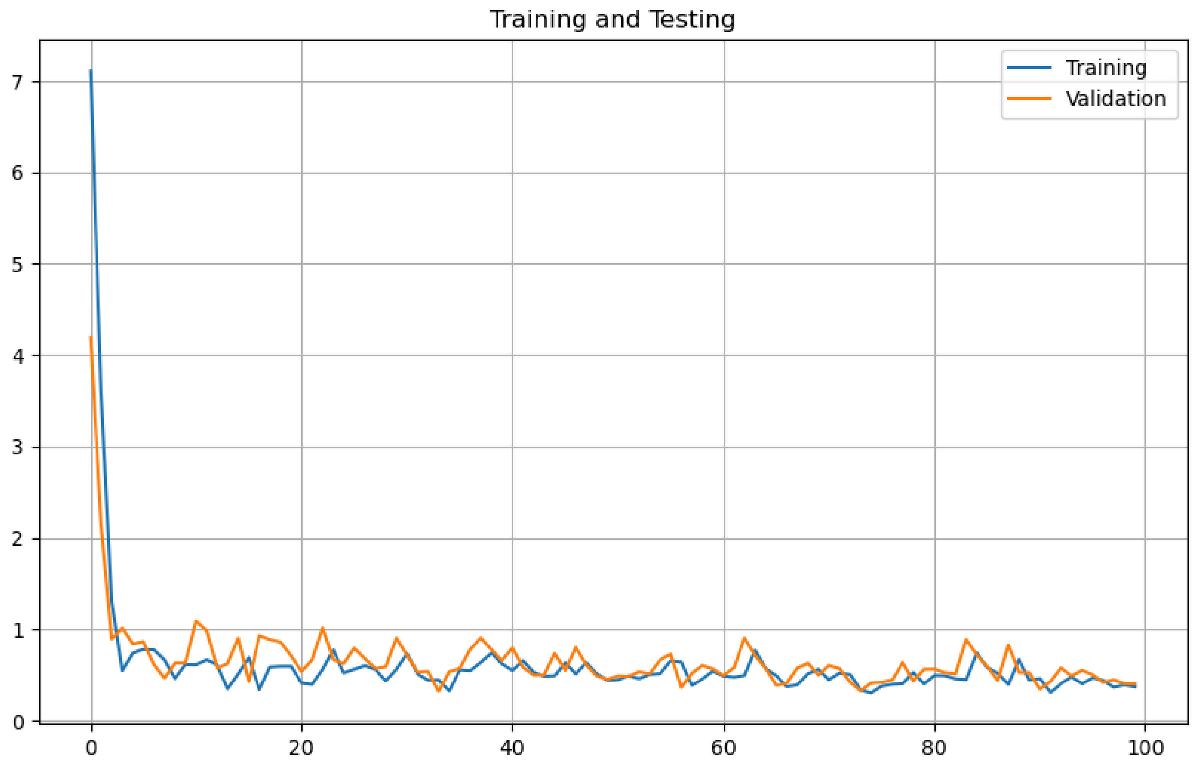
```
WARNING:tensorflow:From C:\Users\mehme\anaconda3\Lib\site-packages\keras\src\losses.py:2976: The name tf.losses.sparse_softma
x_cross_entropy is deprecated. Please use tf.compat.v1.losses.sparse_softmax_cross_entropy instead.

WARNING:tensorflow:From C:\Users\mehme\anaconda3\Lib\site-packages\keras\src\layers\rnn\simple_rnn.py:130: The name tf.execut
ing_eagerly_outside_functions is deprecated. Please use tf.compat.v1.executing_eagerly_outside_functions instead.

Model: "sequential"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 simple_rnn (SimpleRNN)      (None, 15)                270

 dense (Dense)               (None, 2)                 32

=================================================================
Total params: 302 (1.18 KB)
Trainable params: 302 (1.18 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

The results of execution

Model training:

```
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
Epoch 1/100
WARNING:tensorflow:From C:\Users\mehme\anaconda3\Lib\site-packages\keras\src\utils\tf_utils.py:492: The name tf.ragged.Ragged
TensorValue is deprecated. Please use tf.compat.v1.ragged.RaggedTensorValue instead.

WARNING:tensorflow:From C:\Users\mehme\anaconda3\Lib\site-packages\keras\src\engine\base_layer_utils.py:384: The name tf.exec
uting_eagerly_outside_functions is deprecated. Please use tf.compat.v1.executing_eagerly_outside_functions instead.

2/2 [==============================] - 1s 243ms/step - loss: 7.1083 - mae: 7.1083 - val_loss: 4.1945 - val_mae: 4.1945
Epoch 2/100
2/2 [==============================] - 0s 32ms/step - loss: 3.6021 - mae: 3.6021 - val_loss: 2.1551 - val_mae: 2.1551
Epoch 3/100
2/2 [==============================] - 0s 30ms/step - loss: 1.3091 - mae: 1.3091 - val_loss: 0.8961 - val_mae: 0.8961
Epoch 4/100
2/2 [==============================] - 0s 31ms/step - loss: 0.5506 - mae: 0.5506 - val_loss: 1.0157 - val_mae: 1.0157
Epoch 5/100
2/2 [==============================] - 0s 30ms/step - loss: 0.7419 - mae: 0.7419 - val_loss: 0.8406 - val_mae: 0.8406
Epoch 6/100
2/2 [==============================] - 0s 31ms/step - loss: 0.7827 - mae: 0.7827 - val_loss: 0.8672 - val_mae: 0.8672
```

In [ ]:



3 inner layers with 5 neurons each;

Program listing

In [1]:

```
import numpy as np
import tensorflow as tf
import matplotlib.pyplot as plt

# Function of Two Variables
def func(x):
    y = np.cos(x[0]) / x[1] - np.sin(x[0]) / x[1]**2
    z = np.sin(x[0] / 2) + y * np.sin(x[0])
    return y, z

# Collecting Dataset
x = np.linspace(12, 19, 7)
x0, x1 = np.meshgrid(x, x)
x0, x1 = x0.ravel(), x1.ravel()
X = [[x0[i], x1[i]] for i in range(len(x0))]
y, z = zip(*[func(x) for x in X])

split = int(len(X) * 0.8)
x_train, y_train, z_train = X[:split], y[:split], z[:split]
x_test, y_test, z_test = X[split:], y[split:], z[split:]

# Function to display
def visual(his):
    loss = his.history['loss']
    val_loss = his.history['val_loss']
    epochs = range(len(loss))

    plt.figure(figsize=(10, 6))
    plt.plot(epochs, loss, label='Training')
    plt.plot(epochs, val_loss, label='Validation')
```

```
    plt.grid()
    plt.show()

# Model with 3 inner layers and 5 neurons each for Elman backprop
model_elman2 = tf.keras.models.Sequential([
    tf.keras.layers.SimpleRNN(5, activation='relu', input_shape=(1, 2)),
    tf.keras.layers.Dense(5, activation='relu'),
    tf.keras.layers.Dense(5, activation='relu'),
    tf.keras.layers.Dense(2)  # Two outputs for y and z
])

model_elman2.summary()

x_train = np.reshape(x_train, (np.shape(x_train)[0], 1, np.shape(x_train)[1]))
x_test = np.reshape(x_test, (np.shape(x_test)[0], 1, np.shape(x_test)[1]))

model_elman2.compile(
    optimizer=tf.keras.optimizers.SGD(
        learning_rate=tf.keras.optimizers.schedules.ExponentialDecay(0.001, decay_steps=75, decay_rate=0.96)
    ),
    loss='mae',
    metrics=['mae']
)

# Train the model
history_elman2 = model_elman2.fit(
    x_train, np.array([y_train, z_train]).T,  # Two outputs
    epochs=100,
    validation_data=(x_test, np.array([y_test, z_test]).T),
)

# Display learning curves
visual(history_elman2)
```

The results of execution

Model training:

```
uting_eagerly_outside_functions is deprecated. Please use tf.compat.v1.executing_eagerly_outside_functions instead.

2/2 [==============================] - 1s 284ms/step - loss: 0.2806 - mae: 0.2806 - val_loss: 0.3443 - val_mae: 0.3443
Epoch 2/100
2/2 [==============================] - 0s 34ms/step - loss: 0.2651 - mae: 0.2651 - val_loss: 0.3247 - val_mae: 0.3247
Epoch 3/100
2/2 [==============================] - 0s 32ms/step - loss: 0.2551 - mae: 0.2551 - val_loss: 0.3086 - val_mae: 0.3086
Epoch 4/100
2/2 [==============================] - 0s 33ms/step - loss: 0.2484 - mae: 0.2484 - val_loss: 0.2974 - val_mae: 0.2974
Epoch 5/100
2/2 [==============================] - 0s 33ms/step - loss: 0.2464 - mae: 0.2464 - val_loss: 0.3051 - val_mae: 0.3051
Epoch 6/100
2/2 [==============================] - 0s 32ms/step - loss: 0.2453 - mae: 0.2453 - val_loss: 0.3026 - val_mae: 0.3026
Epoch 7/100
2/2 [==============================] - 0s 32ms/step - loss: 0.2429 - mae: 0.2429 - val_loss: 0.2983 - val_mae: 0.2983
Epoch 8/100
2/2 [==============================] - 0s 31ms/step - loss: 0.2404 - mae: 0.2404 - val_loss: 0.2838 - val_mae: 0.2838
Epoch 9/100
2/2 [==============================] - 0s 30ms/step - loss: 0.2383 - mae: 0.2383 - val_loss: 0.2950 - val_mae: 0.2950
```