# Exercise 4.35

Suppose that $A_i$ is a solution wrt. Defn 4.29; this means

(a) $\hat{S}[\![\alpha]\!](A_i(q_\circ)) \subseteq A_i(q_\bullet)$ for all edges $(q_\circ, \alpha, q_\bullet)$

(b) $\widehat{Mem}_\triangleright \subseteq A_i(q_\triangleright)$

To show that $A$ given by $A(q) = A_1(q) \cap A_2(q)$ is a solution wrt. Defn. 4.29 proceed as follows:

(a) We have $\hat{S}[\![\alpha]\!](A_1(q_\circ) \cap A_2(q_\circ))$

$\subseteq \hat{S}[\![\alpha]\!](A_i(q_\circ))$ as $\hat{S}[\![\alpha]\!]$ monotonic

$\subseteq A_i(q_\bullet)$

for both $i = 1$ and $i = 2$ and hence

$$\hat{S}[\![\alpha]\!](A_1(q_\circ) \cap A_2(q_\circ)) \subseteq A_1(q_\bullet) \cap A_2(q_\bullet)$$

for all edges $(q_\circ, \alpha, q_\bullet)$

(b) We have $\widehat{Mem}_\triangleright \subseteq A_i(q_\triangleright)$ for $i = 1, 2$ and

hence $\widehat{Mem}_\triangleright \subseteq A_1(q_\triangleright) \cap A_2(q_\triangleright)$.