Try It Out 4.22

There are three cases to consider.

Case $\hat{S}(\![b\!]\!)(M) = \{(\hat{\sigma_1}, \hat{\sigma_2}) \mid (\sigma_1, \sigma_2) \in M \land tt \in \hat{B}(\![b\!]\!)(\hat{\sigma_1}, \hat{\sigma_2})\}$

Assume that $M_1 \subseteq M_2$

Then $\{(\hat{\sigma_1}, \hat{\sigma_2}) \mid (\sigma_1, \sigma_2) \in M_1 \land tt \in \hat{B}(\![b\!]\!)(\hat{\sigma_1}, \hat{\sigma_2})\}$
$\subseteq \{(\hat{\sigma_1}, \hat{\sigma_2}) \mid (\sigma_1, \sigma_2) \in M_2 \land tt \in \hat{B}(\![b\!]\!)(\hat{\sigma_1}, \hat{\sigma_2})\}$

Hence $\hat{S}(\![b\!]\!)(M_1) \subseteq \hat{S}(\![b\!]\!)(M_2)$ as was to be shown.

In short, $\hat{S}(\![b\!]\!)(M)$ is monotonic in M because $\hat{S}(\![b\!]\!)(M)$ is defined pointwise on the elements of M.

Case $\hat{S}(\![x := a\,]\!)(M)$ is similar.

Case $\hat{S}(\![A[a_1] := a_2\,]\!)(M)$ is similar.