02141 COMPUTER SCIENCE MODELLING

SELECTED SOLUTIONS FM - CHAP 1 (PROGRAM GRAPHS)

## Exercise 1.19 ✓

Let A, B be arrays corresponding to two vectors of size n and m, respectively. Construct program graphs for the following operations:

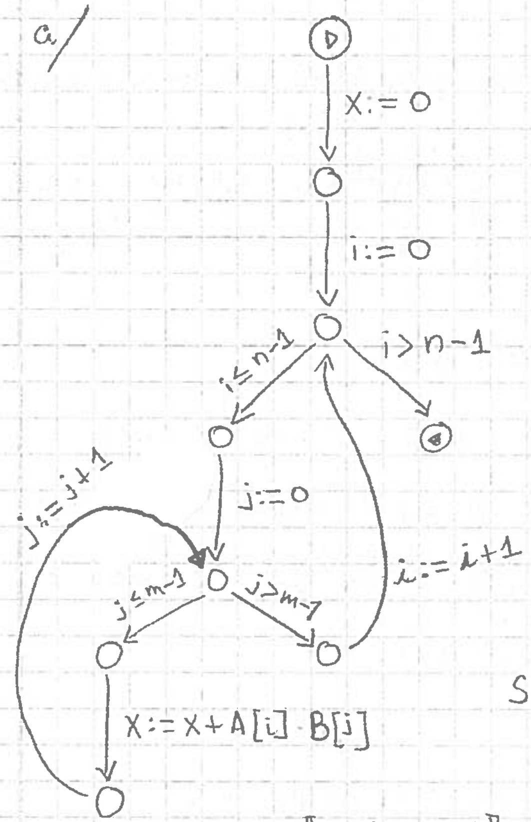
a. The inner product being the number defined by

$$\sum_{i=0}^{n-1} \sum_{j=0}^{m-1} A[i] \cdot B[j]$$

b. The outer product being an $n \times m$ matrix C with (i,j)'th entry given by $C[i,j] = A[i] \cdot B[j]$.

Define the corresponding semantics.

a/




The memory of the semantics will specify the elements of arrays A and B as well as the values of variables x, i and j and the sizes of A and B, denoted by n and m.

We shall define the semantic function $S[\![\cdot]\!]$ for each of the actions in the PROGRAM GRAPH.

$Var = \{x, i, j, n, m\}$

$Arr = \{A, B\}$

$\sigma : (Var \cup \{C[i] \mid C \in Arr, 0 \le i \le size(C)\} \to Val$

$$S[\![ z = v ]\!]\sigma = \begin{cases} \sigma[z \mapsto v] & \text{if } z \in dom(\sigma) \\ & \quad v \in Val \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$S[\![ z \le y - v ]\!]\sigma = \begin{cases} \sigma & \text{if } \sigma(z) \le \sigma(y) - v \wedge \\ & \quad \{z, y\} \subseteq dom(\sigma) \wedge v \in Val \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$S[\![ z > y - v ]\!]\sigma = \begin{cases} \sigma & \text{if } \sigma(z) > \sigma(y) - v \wedge \{z, y\} \subseteq dom(\sigma) \\ & \quad \wedge\ v \in Val \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$S[\![ z := z + A[i] \cdot B[j] ]\!]\sigma = \begin{cases} \sigma[z \mapsto a + (b \times c)] & \text{if } \sigma(i) = I \wedge \sigma(j) = J \\ & \wedge \{z, A[I], B[J]\} \subseteq \mathrm{dom}(\sigma) \\ & \wedge a = \sigma(z) \wedge b = \sigma(A[I]) \\ & \wedge c = \sigma(B[J]) \\ \text{undefined} & \text{otherwise} \end{cases}$$

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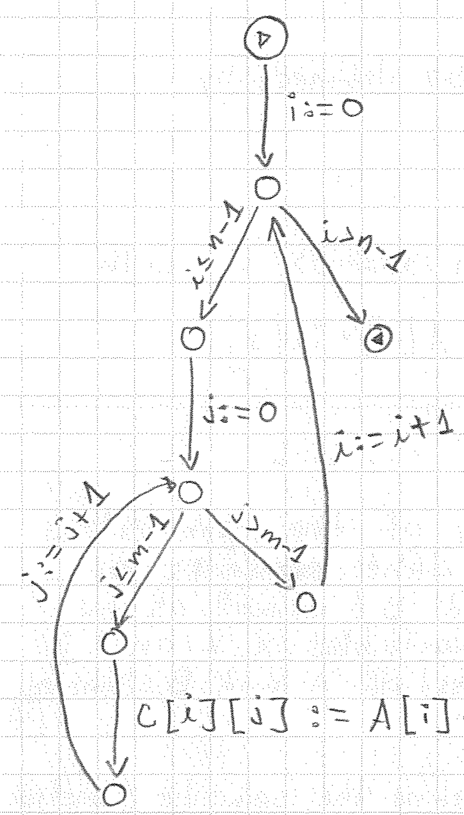
b. OUR PROGRAM GRAPH:




Apart from the sets of values, variables and arrays defined in point a, now we need to specify a set of matrixes, denoted by Mat.

We shall then take a new definition of memory:

$$\sigma : \left( \mathrm{Var} \cup \{ A[i] \mid A \in \mathrm{Arr},\ 0 \le i \le \mathrm{size}(A) \} \cup \left\{ C[i][j] \mid C \in \mathrm{Mat},\ 0 \le i \le \mathrm{WIDTH}(C),\ 0 \le j \le \mathrm{HEIGHT}(C) \right\} \right) \rightarrow \mathrm{Val}$$

The semantics will specify the meaning of the actions, and most of these are defined in a fairly straightforward way from the semantics of point a. We will concentrate on the semantics for matrix manipulation

$$S[\![ C[i][j] := A[i] \cdot B[j] ]\!]_\sigma = \begin{cases} \sigma[C[i][j] \mapsto v] \\ \quad \text{if } \sigma(i) = I \wedge \sigma(j) = J \\ \quad\quad \wedge A[I], B[J], C[I][J] \in \mathrm{dom}(\sigma) \\ \quad\quad \wedge v_A = \sigma(A[I]) \wedge v_B = \sigma(B[J]) \\ \quad\quad \wedge v = v_A * v_B \\ \text{undefined} \quad \text{otherwise} \end{cases}$$