02141 COMPUTER SCIENCE MODELLING

SELECTED SOLUTIONS FM - CHAP 2 (GUARDED COMMANDS)

Exercise 2.16.

Extend the syntax and the semantics for new boolean operators false, $a_1 \neq a_2$, $a_1 \leq a_2$, $a_1 < a_2$, $b_1 \vee b_2$

S/ THE NEW OPERATORS ARE A SYNTACTIC SUGAR FOR THE SET OF OPERATORS b DEFINED IN EXERCISE 2.13. WE SEE THEIR SEMANTICS

$$\mathcal{B}[\![\text{false}]\!]\sigma = \neg\mathcal{B}[\![\text{true}]\!]\sigma$$

$$\mathcal{B}[\![a_1 \neq a_2]\!]\sigma = \mathcal{B}[\![\neg(a_1 = a_2)]\!]\sigma$$

$$\mathcal{B}[\![a_1 \leq a_2]\!]\sigma = \mathcal{B}[\![\neg(a_1 > a_2)]\!]\sigma$$

$$\mathcal{B}[\![a_1 < a_2]\!]\sigma = \mathcal{B}[\![\neg(a_1 \geq a_2)]\!]\sigma$$

$$\mathcal{B}[\![b_1 \vee b_2]\!]\sigma = \mathcal{B}[\![\neg(\neg(b_1) \wedge \neg(b_2))]\!]\sigma$$

Essential Exercise 2.20

EXTEND DEFINITION 2.15 TO APPLY TO THE EXTENDED LANGUAGE; USE DEFINITION OF

$$Mem = (Var \cup \{A[i] \mid A \in Arr,\ 0 \leq i \leq size(A)\}) \to \mathbb{Z}$$

and define the semantic function $\mathcal{S}[\![\cdot]\!]$, as well as extensions for $\mathcal{A}[\![\cdot]\!]$ and $\mathcal{B}[\![\cdot]\!]$

S/ The semantic function $\mathcal{S}[\![\cdot]\!] : Act \to (Mem \hookrightarrow Mem)$ is given by

$$\mathcal{S}[\![\text{SKIP}]\!]\sigma = \sigma$$

$$\mathcal{S}[\![x := a]\!]\sigma = \begin{cases} \sigma[x \mapsto \mathcal{A}[\![a]\!]\sigma & \text{if } \mathcal{A}[\![a]\!]\sigma \text{ is defined} \\ \text{undefined} & \text{OTHERWISE} \end{cases}$$

$$\mathcal{S}[\![b]\!]\sigma = \begin{cases} \sigma & \text{if } \mathcal{B}[\![b]\!]\sigma \text{ is defined} \\ \text{undefined} & \text{OTHERWISE} \end{cases}$$

$$\mathcal{S}[\![A[a_1] := a_2]\!]\sigma = \begin{cases} \sigma[A[j] \mapsto v] & \text{if } \left(\begin{array}{l} \mathcal{A}[\![a_1]\!]\sigma \text{ IS DEFINED} \wedge \\ \mathcal{A}[\![a_2]\!]\sigma \text{ IS DEFINED} \wedge \\ \mathcal{A}[\![a_1]\!]\sigma = j \wedge 0 \leq j < size(A) \wedge \\ \mathcal{A}[\![a_2]\!]\sigma = v \end{array}\right) \\ \text{UNDEFINED} & \text{OTHERWISE} \end{cases}$$

$$\mathcal{A}[\![A[a]]\!]\sigma = \begin{cases} \sigma(A[i]) & \text{if } \mathcal{A}[\![a]\!]\sigma = i \wedge 0 \leq i < size(A) \\ \text{undefined} & \text{OTHERWISE} \end{cases}$$

$$\mathcal{B}[\![a_1 = a_2]\!]\sigma = \begin{cases} tt & \text{if } \mathcal{A}[\![a_1]\!]\sigma = \mathcal{A}[\![a_2]\!]\sigma \\ ff & \text{if } \neg(\mathcal{A}[\![a_1]\!]\sigma = \mathcal{A}[\![a_2]\!]\sigma) \\ \text{UNDEFINED} & \text{OTHERWISE} \end{cases}$$