FM Chapter 6 – Model Checking – Exercise 6.20

We have the set of initial states as $I = \{a0, a1, a2, a3\}$, and the set of reachable states is $\mathsf{Reach(I)} = \{an, bn, cn \mid n \in \{0,1,2,3\}\} \setminus \{b0, c3\}$. For each formula Φ, we need to determine

1. the set of states where it holds ($\{\varsigma \mid \varsigma \models \Phi\}$),
2. the set of *reachable* states where it holds ($\{\varsigma \mid \varsigma \models \Phi\} \cap \mathsf{Reach(I)}$),
3. whether or not it holds for the transition system ($\forall \varsigma \in \mathsf{I} : \varsigma \models \Phi$, or determine if $\mathsf{I} \subseteq \{\varsigma \mid \varsigma \models \Phi\} \cap \mathsf{Reach(I)}$)

First we determine the set of states where the formula holds

Case (1) $\Phi = @_a$ That is all the states ς where $@_a \in \mathsf{L}(\varsigma)$, which is $a0, a1$, $a2$, and $a3$.

Case (2) $\Phi = \#_2$ That is all the states ς where $\#_2 \in \mathsf{L}(\varsigma)$, which is $a2$, $b2$, $c2$, and $d2$.

Case (3) $\Phi = EF(@_a \wedge \#_2)$ First we look at the inner formula $@_a \wedge \#_2$, which holds at all the states ς where $@_a \in \mathsf{L}(\varsigma)$ and $\#_2 \in \mathsf{L}(\varsigma)$, which only amounts to $a2$. Then we have to figure out the states ς_0 where there exists a path $\varsigma_0\varsigma_1 \cdots \varsigma_n \cdots$, where $\varsigma_i = a2$ for some i (e.g. a path to $a2$). We can see that from all the reachable states we can get to $a2$. The states where the formula holds is therefore $\mathsf{Reach(I)}$.

Case (variation of 4) $\Phi = AG((@_c \wedge \#_2) \Rightarrow AX(@_a \wedge \#_3))$ We break up the formula into smaller parts: The states where $@_c \wedge \#_2$ holds amounts to $c2$. The states where $@_a \wedge \#_3$ holds amounts to $a3$. We can see that the only path to $a3$ is from $c2$, all the sucessor states of $c2$ is therefore $a3$, and the states where $AX(@_a \wedge \#_3))$ is then $c2$. If we are at $c2$ then the next state will always be $a2$, and $(@_c \wedge \#_2) \Rightarrow AX(@_a \wedge \#_3)$ therefore holds. As there is no way to escape the property that if we are in $c2$ then the next state is $a3$ e.g. Φ, then all states S satisfy Φ.

It is now easy to determine the set of reachable states where the formula holds, and weather it holds for the transition system, summarised below:

Φ	$H = \{\varsigma \mid \varsigma \models \Phi\}$	$H \cap \mathsf{Reach(I)}$	$\mathsf{I} \subseteq H \cap \mathsf{Reach(I)}$
$@_a$	$\{a0, a1, a2, a3\}$	$\{a0, a1, a2, a3\}$	$true$
$\#_2$	$\{a2, b2, c2, d2\}$	$\{a2, b2, c2\}$	$false$
$EF(@_a \wedge \#_2)$	$\mathsf{Reach(I)}$	$\mathsf{Reach(I)}$	$true$
$AG((@_c \wedge \#_2) \Rightarrow AX(@_a \wedge \#_3))$	S	$\mathsf{Reach(I)}$	$true$