Mandatory exercise: "Bing or Ding" (from the exam E15)   At the Christmas party there will be a game called "Bing or Ding". There is a row of n red socks, each with an integer on it, which can be positive, negative, or zero. Let s_i denote the number on sock i. The rules are as follows:

- You have to consider all socks in order from sock 1 to n.

- At each sock you *must* say either "Bing!" or "Ding!".

- If you say "Bing!" at the ith sock you get s_i Christmas cookies.

- If you say "Ding!" at the ith sock you must pay s_i Christmas cookies.

- It is possible to end up with a negative number of Christmas cookies.

- It is forbidden to say the same word more than two times in a row. For example if you say "Bing!" at sock 5 and 6 you *must* say "Ding!" at sock 7.

Let $L(1, i)$ denote the maximum number of Christmas cookies you can have after the first i socks if you say "Bing!" at sock i. Similarly, let $L(0, i)$ denote the maximum number of Christmas cookies you can have after the first i if you say "Ding!" at sock i. Let $L(b, 0) = 0$ for $b \in \{0, 1\}$.

Question 1   Fill out the table below for $L(b, i)$ when $S = [s_1, s_2, s_3, s_4] = [3, 4, 8, -4]$.

$L(b, i)$	0	1	2	3	4
0					
1					

Question 2   Which of the following recurrences correctly computes $L(b, i)$ (remember to argue for your choice):

$$\boxed{A} \qquad L(b, i) = \begin{cases} 0 & \text{if } i = 0 \\ L(0, i-1) + s_i & \text{if } b = 1 \\ L(1, i-1) - s_i & \text{if } b = 0 \end{cases}$$

$$\boxed{B} \qquad L(b, i) = \begin{cases} 0 & \text{if } i = 0 \\ s_1 & \text{if } b = 1 \text{ and } i = 1 \\ -s_1 & \text{if } b = 0 \text{ and } i = 1 \\ \max\{L(1, i-2) - s_{i-1}, L(0, i-2) + s_{i-1}\} + s_i & \text{if } b = 1 \text{ and } i \geq 2 \\ \max\{L(1, i-2) + s_{i-1}, L(0, i-2) - s_{i-1}\} - s_i & \text{if } b = 0 \text{ and } i \geq 2 \end{cases}$$

$$\boxed{C} \qquad L(b, i) = \begin{cases} 0 & \text{if } i = 0 \\ s_1 & \text{if } b = 1 \text{ and } i = 1 \\ -s_1 & \text{if } b = 0 \text{ and } i = 1 \\ \max\{L(0, i-1), L(0, i-2) + s_{i-1}\} + s_i & \text{if } b = 1 \text{ and } i \geq 2 \\ \max\{L(1, i-1), L(1, i-2) - s_{i-1}\} - s_i & \text{if } b = 0 \text{ and } i \geq 2 \end{cases}$$

Question 3   Write pseudocode for an algorithm *based on dynamic programming and the recurrence from Question 2* that finds the maximum number of Christmas cookies you can end up with given n socks. Analyze the space usage and running time of your algorithm in terms of n.