CST 405
Algorithm Analysis & Design

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Chapter 22

Elementary Graph Algorithms

Representations of Graphs

- There are 2 standards ways to represent a graph G = (V,E): as a collection of adjacency lists or as an adjacency matrix.
- Both methods are applicable to directed and undirected graphs.

Un-directed Graph

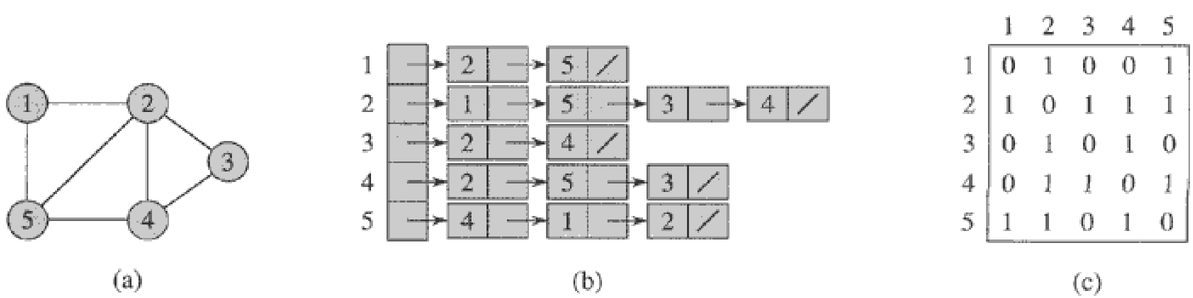
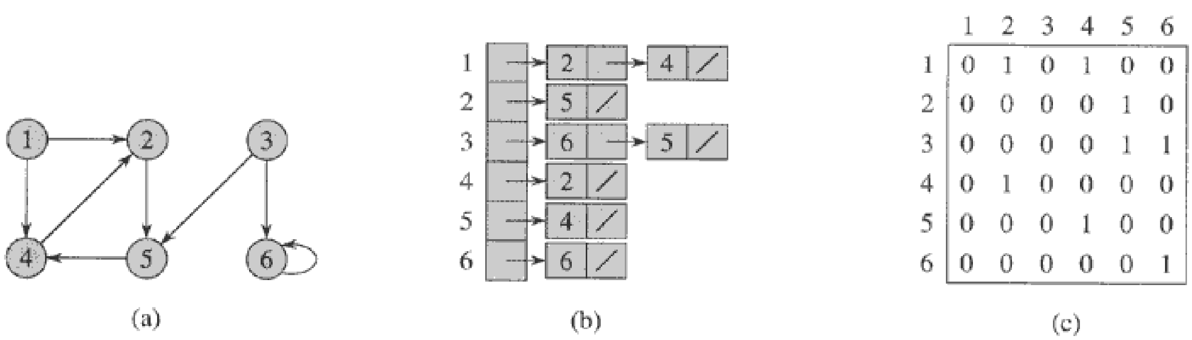


Figure 22.1 Two representations of an undirected graph. **(a)** An undirected graph G having five vertices and seven edges. **(b)** An adjacency-list representation of G. **(c)** The adjacency-matrix representation of G.

Directed Graph



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

Figure 22.2 Two representations of a directed graph. **(a)** A directed graph G having six vertices and eight edges. **(b)** An adjacency-list representation of G. **(c)** The adjacency-matrix representation of G.

Adjacency-list Representation

- **The adjacency-list representation of a graph $G = (V,E)$ consists of an array Adj of $|V|$ lists, one for each vertex in V.**

Breadth-first Search

- Given a graph G = (V,E) and a distinguished source vertex s, breadth-first search systematically explores the edges of G to "discover" every vertex that is reachable from s.
- It computes the distance (smallest number of edges) from s to each reachable vertex.

BFS

```
BFS(G, s)
for each vertex u ∈ V[G] − {s}
    do color[u] ← WHITE
       d[u] ← ∞
       π[u] ← NIL
color[s] ← GRAY
d[s] ← 0
π[s] ← NIL
Q ← Ø
ENQUEUE(Q, s)
while Q ≠ Ø
    do u ← DEQUEUE(Q)
       for each v ∈ Adj[u]
           do if color[v] = WHITE
                 then color[v] ← GRAY
                      d[v] ← d[u] + 1
                      π[v] ← u
                      ENQUEUE(Q, v)
       color[u] ← BLACK
```

BFS on undirected graph

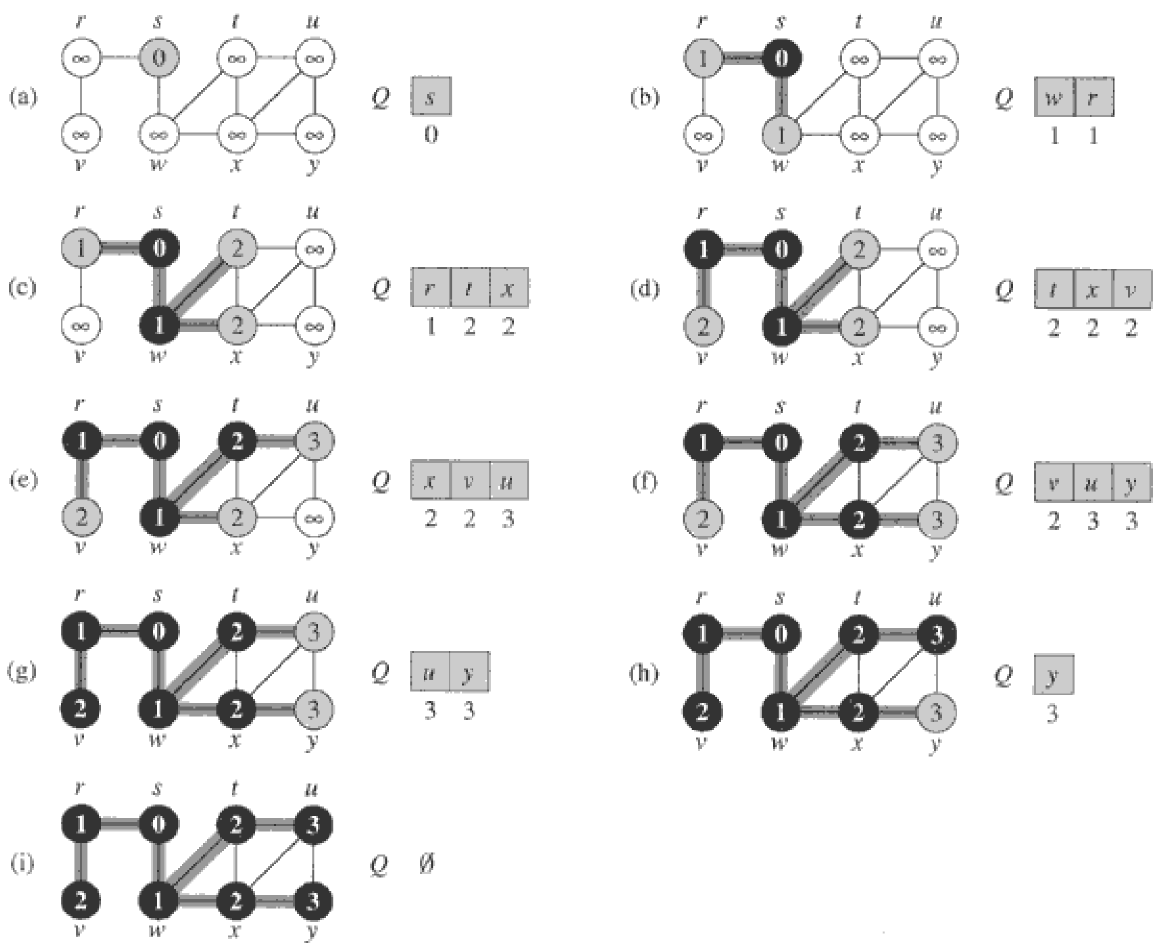


Figure 22.3 The operation of BFS on an undirected graph. Tree edges are shown shaded as they are produced by BFS. Within each vertex u is shown $d[u]$. The queue Q is shown at the beginning of each iteration of the **while** loop of lines 10–18. Vertex distances are shown next to vertices in the queue.

Depth-first search

- **A depth-first search will search 'deeper' in the graph whenever possible.**

DFS

```
DFS(G)
    for each vertex u ∈ V[G]
            do color[u] ← WHITE
               π[u] ← NIL
    time ← 0
    for each vertex u ∈ V[G]
            do if color[u] = WHITE
                  then DFS-VISIT(u)
```

DFS on directed graph

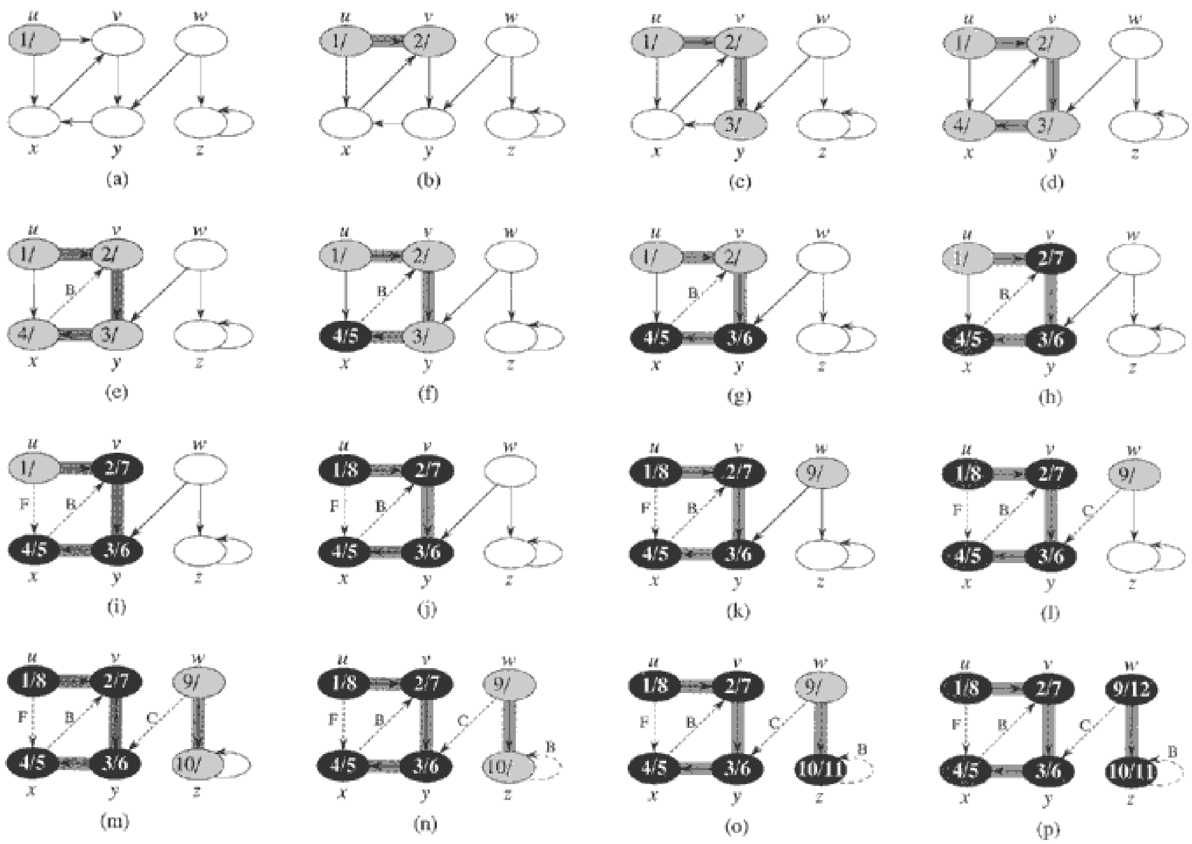


Figure 22.4 The progress of the depth-first-search algorithm DFS on a directed graph. As edges are explored by the algorithm, they are shown as either shaded (if they are tree edges) or dashed (otherwise). Nontree edges are labeled B, C, or F according to whether they are back, cross, or forward edges. Vertices are timestamped by discovery time/finishing time.