

# Network Slicing and Resource Allocation in an Open RAN System

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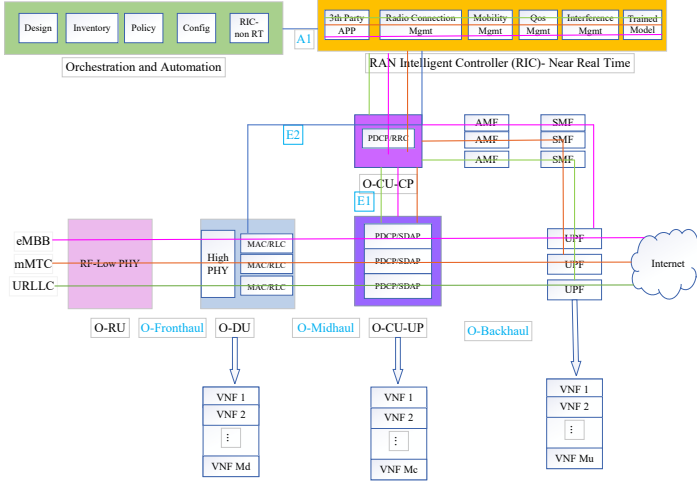


Fig. 1: Network sliced ORAN system

**Abstract—**  
**Index Terms—**

## I. INTRODUCTION

In this paper, as depicted in Figure 1, the downlink of the ORAN system is studied.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, first, we present the system model. Then, we obtain achievable data rates and delays for the downlink (DL) of the ORAN system. Afterward, we discuss about the assignment of physical data center resources. Finally, the main problem is expressed.

### A. System Model

Suppose we have three service types includes mMTC, eMBB and URLLC which support different applications.

Assume we have  $S_1$ ,  $S_2$  and  $S_3$  different applications for the first, second and third service type, respectively ( $S = S_1 + S_2 + S_3$ ). So, we have  $S$  preallocated slices serving these  $S$  services; There are  $S_1$  slices for the first service type (eMBB),  $S_2$  slices for the second service type (URLLC) and  $S_3$  slices for the third service type (mMTC). So each service request  $s$  served by its corresponding slice.

Each Service  $s_j \in \{1, 2, \dots, S_j\}$  consists of  $U_s$  request from the single-antenna UEs which require certain QoS to be able to use the requested program ( $j \in \{1, 2, 3\}$  indicate service type). There are different application request which fall into one of these service categories. Each application request requires specific QoS. Based on the request for the application and QoS, UE may be admitted and allocated to the resources. Each slice  $s_j \in \{1, 2, \dots, S_j\}$ ,  $j \in \{1, 2\}$  consists of preallocated virtual resource blocks that are mapped to the Physical Resource Blocks (PRBs),  $M_s^d$  VNFs for the processing of O-DU,  $M_s^c$  VNFs for the processing of O-CU-UP and  $M_s^u$  VNFs for the processing of UPF.

All  $K$  PRBs can be assigned to the all UE in each service. Also, each VNF instance is running on the virtual machine (VM) that are using resources from the data centers. Each VM, requires enough resources of CPU, memory, storage and network bandwidth.

In addition, there are  $R$  multi-antenna O-RU that are shared between slices. Each O-RU  $r \in \{1, 2, \dots, R\}$  has  $J$  antenna for transmitting and receiving data. Also  $\mathcal{R} = \{r | r \in 1, 2, \dots, R\}$  depicts the set of O-RUs. Moreover, all O-RUs, have access to the all PRBs.

### B. The Achievable Rate

The SNR of  $i^{th}$  UE served at slice  $s$  on PRB  $k$  is obtained from

$$\rho_{r,u(s,i)}^k = \frac{|p_{r,u(s,i)}^k \mathbf{h}_{r,u(s,i)}^{Hk} \mathbf{w}_{r,u(s,i)}^k|^2}{BN_0 + I_{r,u(s,i)}^k}, \quad (1)$$

where  $p_{r,u(s,i)}^k$  represents the transmission power from O-RU  $r$  to  $i^{th}$  UE served at slice  $s$  on PRB  $k$ .  $\mathbf{h}_{r,u(s,i)}^k \in \mathbb{C}^J$  is the vector of channel gain of a wireless link from  $r^{th}$  O-RU to the  $i^{th}$  UE in  $s^{th}$  slice. In addition,  $\mathbf{w}_{r,u(s,i)}^k \in \mathbb{C}^J$  depicts the transmit beamforming vector from  $r^{th}$  O-RU to the  $i^{th}$  UE in  $s^{th}$  slice that is the zero forcing beamforming vector to minimize the interference which is indicated as below

$$\mathbf{w}_{r,u(s,i)}^k = \mathbf{h}_{r,u(s,i)}^k (\mathbf{h}_{r,u(s,i)}^{Hk} \mathbf{h}_{r,u(s,i)}^k)^{-1} \quad (2)$$

Moreover,  $g_{u(s,i)}^r \in \{0, 1\}$  is the binary variable that illustrates whether O-RU  $r$  served the  $i^{th}$  UE that is allocated to  $s^{th}$  slice or not. Also,  $BN_0$  denotes the power of Gaussian additive noise, and  $I_{r,u(s,i)}^k$  is the power of interfering signals represented as follow

$$\begin{aligned}
I_{r,u(s,i)}^k &= \underbrace{\sum_{\substack{l=1 \\ l \neq i}}^{U_s} \gamma_1 p_{u(s,l)}^k \sum_{\substack{r'=1 \\ r' \neq r}}^R |\mathbf{h}_{r',u(s,i)}^H \mathbf{w}_{r',u(s,l)}^k g_{u(s,l)}^{r'}|^2}_{\text{(intra-slice interference)}} \\
&+ \underbrace{\sum_{\substack{n=1 \\ n \neq s}}^S \sum_{l=1}^{U_s} \gamma_2 p_{u(n,l)}^k \sum_{\substack{r'=1 \\ r' \neq r}}^R |\mathbf{h}_{r',u(s,i)}^H \mathbf{w}_{r',u(n,l)}^k g_{u(n,l)}^{r'}|^2}_{\text{(inter-slice interference)}} \\
&+ \underbrace{\sum_{j=1}^R \sigma_{q_{rj}}^2 |\mathbf{h}_{r,u(s,i)}|^2}_{\text{(Quantization Noise Interference)}}
\end{aligned} \tag{3}$$

where  $\gamma_1 = e_{u(s,i)}^k e_{u(s,l)}^k$  and  $\gamma_2 = e_{u(s,i)}^k e_{u(n,l)}^k$ .  $e_{u(s,i)}^k$  is the binary variable to show whether the  $k^{th}$  PRB is allocated to the UE  $i$  in slice  $s$ , assigned to  $r^{th}$  o-RU.

To obtain SNR as formulated in (1), let  $y_{u(s,i)}$  be the received signal user  $i$  in  $s^{th}$  service

$$y_{u(s,i)} = \sum_{r=1}^R \sum_{k=1}^{K_s} \mathbf{h}_{r,u(s,i)}^H g_{u(s,i)}^r e_{r,u(s,i)}^k \mathbf{w}_{r,u(s,i)}^k + z_{u(s,i)}, \tag{4}$$

where  $\mathbf{w}_{r,u(s,i)}^k = \mathbf{w}_{r,u(s,i)}^k p_{r,u(s,i)}^{\frac{1}{2}} x_{u(s,i)} + \mathbf{q}_r$  and  $x_{u(s,i)}$  depicts the transmitted symbol vector of UE  $i$  in  $s^{th}$  set of service,  $z_{u(s,i)}$  is the additive Gaussian noise  $z_{u(s,i)} \sim \mathcal{N}(0, N_0)$  and  $N_0$  is the noise power. In addition,  $\mathbf{q}_r \in \mathbb{C}^J$  indicates the quantization noise, which is made from signal compression in O-DU.

The achievable data rate for the  $i^{th}$  UE request in the  $s_1^{th}$  application of service type 1 (eMBB) can be written as  $\mathcal{R}_{u(s_1,i)}$  that is formulated as below.

$$\begin{aligned}
\mathcal{R}_{r,u(s_1,i)}^k &= B \log_2(1 + \rho_{r,u(s_1,i)}^k), \\
\mathcal{R}_{u(s_1,i)}^r &= \sum_{k=1}^K B \log_2(1 + \rho_{r,u(s_1,i)}^k e_{r,u(s_1,i)}^k), \\
\mathcal{R}_{u(s_1,i)} &= \sum_{r=1}^R \mathcal{R}_{u(s_1,i)}^r g_{u(s_1,i)}^r
\end{aligned} \tag{5}$$

where  $B$  is the bandwidth of system.  $\mathcal{R}_{u(s_1,i)}^r$  is the achievable rate of each RU  $r$  to UE  $i$  in slice  $s_1$ . Since the blocklength in URLLC and mMTC is finite, the achievable data rate for the  $i^{th}$  UE request in the  $s_j^{th}$  ( $j \in \{2, 3\}$ ) application of service type 2 (URLLC) and 3 (mMTC) is not achieved from Shannon Capacity formula. So, for the short packet transmission the achievable data rate is approximated from follow

$$\begin{aligned}
\mathcal{R}_{r,u(s_j,i)}^k &= B \log_2(1 + \rho_{r,u(s_j,i)}^k - \zeta_{u(s_j,i)}^k) e_{u(s_j,i)}^k, \\
\mathcal{R}_{u(s_j,i)}^r &= \sum_{k=1}^K B (\log_2(1 + \rho_{u(s_j,i)}^k) - \zeta_{u(s_j,i)}^k) e_{u(s_j,i)}^k \\
\mathcal{R}_{u(s_j,i)} &= \sum_{r=1}^R \mathcal{R}_{u(s_j,i)}^r g_{u(s_j,i)}^r
\end{aligned} \tag{6}$$

Where  $j \in \{1, 2\}$ . Also we have

$$\zeta_{u(s_j,i)}^k = \log_2(e) Q^{-1}(\epsilon) \sqrt{\frac{\mathfrak{C}_{u(s_j,i)}^k}{N_{u(s_j,i)}^k}} \tag{7}$$

Where,  $\epsilon$  is the transmission probability,  $Q^{-1}$  is the inverse of Q- function (Gaussian),  $\mathfrak{C}_{u(s_j,i)}^k = 1 - \frac{1}{(1 + \rho_{u(s_j,i)}^k)^2}$  depicts the channel dispersion of UE  $i$  at slice  $s_j$ , experiencing PRB  $k$  and  $N_{u(s_j,i)}^k$  represents the blocklength of it.  $\mathcal{R}_{u(s_j,i)}^{e,r}$  is the achievable rate of each O-RU  $r$  to UE  $i$  in slice  $s_j$ .

If we replace  $p_{u(s,l)}^k$  and  $p_{u(n,l)}^k$  in (3) by  $P_{max}$ , an upper bound  $\bar{I}_{r,u(s,i)}^k$  is obtained for  $I_{r,u(s,i)}^k$ . Therefore,  $\bar{\mathcal{R}}_{u(s,i)} \forall s, \forall i$  is derived by using  $\bar{I}_{r,u(s,i)}^k$  instead of  $I_{r,u(s,i)}^k$  in (6) and (5).

### C. Power of O-RU and Fronthaul Capacity

Let  $P_r$  denote the power of transmitted signal from the  $r^{th}$  O-RU to UEs served by it. From (4), we have,

$$P_r = \sum_{s=1}^S \sum_{k=1}^{K_s} \sum_{i=1}^{U_s} |\mathbf{w}_{r,u(s,i)}^k|^2 p_{r,u(s,i)}^k g_{u(s,i)}^r e_{r,u(s,i)}^k + \sigma_{q_r}^2. \tag{8}$$

Since we have fiber link between O-RU and O-DU, the rate of users on the fronthaul link between O-DU and the  $r^{th}$  O-RU is formulated as

$$C_r = \log \left( 1 + \frac{\sum_{s=1}^S \sum_{k=1}^{K_s} \sum_{i=1}^{U_s} |\mathbf{w}_{r,u(s,i)}^k|^2 \alpha_{r,u(s,i)}^k}{\sigma_{q_r}^2} \right), \tag{9}$$

Where,  $\alpha_{r,u(s,i)}^k = p_{r,u(s,i)}^k g_{u(s,i)}^r e_{r,u(s,i)}^k$  and  $\sigma_{q_r}^2$  is the power of quantization noise.

### D. Mean Delay

In this part, the end to end mean delay for a service is obtained. Suppose the mean total delay is depicted as  $T_{tot}$ .

$$\begin{aligned}
T_{tot} &= T_{process} + T_{transmission} + T_{propagation} \\
T_{process} &= T_{RU} + T_{DU} + T_{CU} + T_{UPF} \\
T_{transmission} &= T_{front} + T_{mid} + T_{back} + T_{trans2net} \\
T_{propagation} &= T_{front} + T_{mid} + T_{back} + T_{trans2net}
\end{aligned} \tag{10}$$

Total delay is sum of processing delay, transmission delay and propagation delay. The propagation delay is the time takes for a signal to reach to its destination. So it has a constant value based on the length of fiber link ( $T = L/c$ , where  $L$  is the length of link and  $c$  is the speed of signal). Also, the transmission delay is the amount of time required to push all the packets into the fiber link. ( $T = \frac{\alpha}{R}$  Where,  $R$  is the rate of transmission in each link and  $\alpha$  is the mean arrival data rate of the each link which is constant in this model.) Here we assume the value of propagation delay and transmission is negligible compared to the rest.

$$T_{tot} \approx T_{process} \tag{11}$$

1) *Processing Delay*: Assume the packet arrival of UEs follows a Poisson process with arrival rate  $\lambda_{u(s,i)}$  for the  $i^{th}$  UE of the  $s^{th}$  slice. Therefore, the mean arrival data rate of the  $s^{th}$  slice in the UPF layer is  $\alpha_s^1 = \sum_{u=1}^{U_s} a_{u(s,i)} \lambda_{u(s,i)}$ , where  $a_{u(s,i)}$  is a binary variable which indicates whether the  $i^{th}$  UE requested  $s^{th}$  service is admitted or not.

Assume the mean arrival data rate of the UPF layer for slice  $s$  ( $\alpha_s^U$ ) is approximately equal to the mean arrival data rate of the O-CU-UP layer ( $\alpha_s^C$ ) and O-DU ( $\alpha_s^D$ ). so  $\alpha_s = \alpha_s^U \approx \alpha_s^C \approx \alpha_s^D$ . since, by using Burkes Theorem, the mean arrival data rate of the second and third layer which are processed in the first layer is still Poisson with rate  $\alpha_s$ . It is assumed that there are load balancers in each layer for each service to divide the incoming traffic to VNFs equally. Suppose the baseband processing of each VNF is depicted as M/M/1 processing queue. Each packet is processed by one of the VNFs of a slice. So, the mean delay for the  $s^{th}$  slice in the first and the second layer, modeled as M/M/1 queue, is formulated as follow, respectively

$$\begin{aligned} T_{DU}^s &= \frac{1}{\mu_s^d - \alpha_s/M_s^d}, \\ T_{CU}^s &= \frac{1}{\mu_s^c - \alpha_s/M_s^c}, \\ T_{UPF}^s &= \frac{1}{\mu_s^u - \alpha_s/M_s^u} \end{aligned} \quad (12)$$

Where  $M_s^d$ ,  $M_s^c$  and  $M_s^u$  are the variables that depict the sum of VNFs in O-DU, O-CU-UP and UPF, respectively. Moreover,  $1/\mu_s^d$ ,  $1/\mu_s^c$  and  $1/\mu_s^u$  are the mean service time of the O-DU, O-CU and the UPF layers respectively. Besides,  $\alpha_s$  is the arrival rate which is divided by load balancer before arriving to the VNFs. The arrival rate of each VNF in each layer for each slice  $s$  is  $\alpha_s/M_s^i$   $i \in \{d, c, u\}$ .

In addition,  $T_{RU}^{u(s,i)}$  is the mean transmission delay of  $i^{th}$  UE in  $s^{th}$  service on the wireless link. The arrival data rate of wireless link for each UE  $i$  in service  $s$  is  $\lambda_{u(s,i)}$ . As a result we have,  $\sum_{i=1}^{U_s} \lambda_{u(s,i)} = \alpha_s$ . Moreover, The service time of transmission queue for UE  $i$  requesting service  $s$  has an exponential distribution with mean  $1/(R_{u(s,i)})$  and can be modeled as a M/M/1 queue.

Therefore, the mean delay of the transmission layer for UE  $i$  in slice  $s$  is

$$T_{RU}^{u(s,i)} = \frac{1}{R_{u(s,i)} - \lambda_{u(s,i)}}; \quad (13)$$

So the mean processing delay for each UE  $i$  in slice  $s$  is

$$T_{process}^{u(s,i)} = T_{RU}^{u(s,i)} + T_{DU}^s + T_{CU}^s + T_{UPF}^s \quad (14)$$

Hence,  $T_{tot}^{u(s,i)} \approx T_{process}^{u(s,i)}$

#### E. VNF Power

Assume the power consumption of baseband processing at each DC  $d$  that is connected to VNFs of a slice  $s$  is depicted as  $\phi_s$ . So the total power of the system for all active DCs that are connected to slices can be represented as

$$\phi_{tot} = \sum_{s=1}^S \phi_s.$$

Where,  $\phi_s$  is obtained from below

$$\phi_s = M_s^u \phi_s^u + M_s^c \phi_s^c + M_s^d \phi_s^d \quad (15)$$

Moreover,  $\phi_s^u$ ,  $\phi_s^c$  and  $\phi_s^d$  are the static cost of energy in UPF, O-CU and O-DU, respectively.

#### F. Problem Statement

The optimization problem is formulated as follow. The aim of this paper is to maximize the sum rate of all UEs with the presence of constraints which is written as follow,

$$\max_{\mathbf{G}} \sum_{r=1}^R g_{u(s,i)}^r \bar{\mathcal{R}}_{u(s,i)} \quad (16a)$$

$$\text{subject to } P_r \leq P_{max} \quad \forall r \quad (16b)$$

$$p_{r,u(s,i)}^k \geq 0 \quad \forall i, \forall r, \forall s, \forall k, \quad (16c)$$

$$\bar{\mathcal{R}}_{u(s,j,i)} \geq \mathcal{R}_{min}^{s,j} \quad \forall s, j \in \{1, 2, 3\}, \quad (16d)$$

$$C^r \leq C_{max}^r \quad \forall r, \quad (16e)$$

$$T_{tot}^{u(s,i)} \leq T_{max}^s \quad \forall i, \forall s, \quad (16f)$$

$$\mu_s \geq \alpha_s/M_s \quad \forall s, \quad (16g)$$

$$\bar{\mathcal{R}}_{u(s,i)} \geq \lambda_{u(s,i)} \quad \forall i, \forall s, \quad (16h)$$

$$0 \leq M_s \leq M^{max} \quad \forall s, \quad (16i)$$

$$\sum_r g_{u(s,i)}^r \geq 1 \quad \forall s, \forall i, \quad (16j)$$

$$\sum_{k=1}^{K_s} e_{r,u(s,i)}^k \geq 1 \quad \forall s, \forall i, \forall r \quad (16k)$$

$$\phi_{tot} \leq \phi_{max}, \quad (16l)$$

$$g_{u(s,i)}^r \in \{0, 1\} \quad \forall s, \forall i, \quad (16m)$$

$$e_{r,u(s,i)}^k \in \{0, 1\} \quad \forall s, \forall i, \quad (16n)$$

where  $\mathbf{P} = [p_{r,u(s,i)}^k] \quad \forall s, \forall i, \forall r, \forall k$ , is the matrix of power for UEs,  $\mathbf{E} = [e_{r,u(s,i)}^k] \quad \forall s, \forall i, \forall r, \forall k$  indicate the binary variable for PRB association. Moreover,  $\mathbf{G} = [g_{u(s,i)}^r] \quad \forall s, \forall i, \forall r$  is a binary variable for O-RU association. Furthermore,  $\mathbf{M} = [M_s^d, M_s^c, M_s^u] \quad \forall s$  is the matrix that shown the number of VNFs in each layer of slice. (16b), and (16c), indicate that the power of each RU do not exceed the maximum power, and the power of each UE is a positive integer value, respectively. Also (16d) shows that the rate of each UE requesting eMBB, URLLC and mMTC is more than a threshold, respectively. (16e) and (16f) expressed the limited capacity of the fronthaul link, and the limited delay of receiving signal, respectively. (16g) and (16h) denoted the stability of the M/M/1 queue model. (16i) restricted the number of VNF in each slice due to the limited resources. (16j) and (16k) guarantee that O-RU and PRB is associated to the UE, respectively. In addition, (16l) indicate that the static cost of energy of VNFs in each slice do not exceed from the threshold. Moreover, (16m) and (16n) depict that  $\mathbf{E}$  and  $\mathbf{G}$  are matrix of binary variables.

### III. PROPOSED ALGORITHM SCHEME

In this section, we first apply some simplifications to the system; Solving problem (16) is complicated due to

the fact that this problem is a non-convex problem and it is a mixed integer non-linear problem (MINLP) with a binary variable and an integer variable. In the following, we apply the simplifications to reformulate MINLP parts and use iterative heuristic algorithm to solve the reformulated problem. We solve this problem in two level iteratively until it converges; In the first level, parameters  $(\mathbf{P}, \mathbf{E}, \mathbf{M})$  are obtained by relaxing and reformulating parameters and turn it to convex problem; Afterward we solve it by dual optimization problem. In the second level, finding optimal O-RU association  $(\mathbf{G})$  is concerned with the fixed parameter of power, PRB allocation and number of VNFs. We repeat this procedure until the algorithm converges.

### A. Sub-Problem 1

Suppose that  $\mathbf{G}$  is fixed, we want to obtain  $\mathbf{P}$ ,  $\mathbf{E}$  and  $\mathbf{M}$ . Here, we first simplify and relax the parameters to convexify the problem.

As we mentioned before, by replacing  $p_{u(s,i)}^k$  and  $p_{u(n,i)}^k$  in (3) by  $P_{max}$ , an upper bound  $\bar{I}_{r,u(s,i)}^k$  for  $I_{r,u(s,i)}^k$ , the lower bound  $\bar{\rho}_{u(s,i)}^k$  for  $\rho_{u(s,i)}^k$  and the lower bound  $\bar{\mathcal{R}}_{u(s,i)} \forall s, \forall i$  for  $\mathcal{R}_{u(s,i)}$  is obtained by replacing with  $\bar{I}_{r,u(s,i)}^k$  in (6) and (5) and make them concave.

Suppose  $\hat{\rho}_{r,u(s,i)}^k = \frac{|P_{max} \mathbf{h}_{r,u(s,i)}^{H,k} \mathbf{w}_{r,u(s,i)}^k g_{u(s,i)}^r|^2}{BN_0}$ . To convexify (6) (for the short packet transmission), we replace  $\rho_{r,u(s,i)}^k$  with  $\hat{\rho}_{r,u(s,i)}^k$  in (7). So, a lower bound for (6) is given that is a concave function.

$$\begin{aligned} \bar{\mathcal{R}}_{u(s,j,i)}^r &= \sum_{k=1}^{K_{s,j}} B(\log_2(1 + \bar{\rho}_{u(s,j,i)}^k) - \hat{\zeta}_{u(s,j,i)}^k) e_{u(s,j,i)}^k \\ \bar{\mathcal{R}}_{u(s,j,i)} &= \sum_{r=1}^R \bar{\mathcal{R}}_{u(s,j,i)}^r \\ \hat{\zeta}_{u(s,j,i)}^k &= \log_2(e) Q^{-1}(\epsilon) \sqrt{\frac{\hat{\mathcal{C}}_{u(s,j,i)}^k}{N_{u(s,j,i)}^k}} \\ \hat{\mathcal{C}}_{u(s,j,i)}^k &= 1 - \frac{1}{(1 + \hat{\rho}_{u(s,j,i)}^k)^2} \end{aligned} \quad (17)$$

Consider UPF, O-CU and O-DU have the same processor (for simplification), so we have  $\mu_s = \mu_s^u \approx \mu_s^c \approx \mu_s^d$ . Moreover, as mentioned before, the mean arrival data rate of the UPF layer for a service  $s$  ( $\alpha_s^U$ ) is approximately equal to the mean arrival data rate of the O-CU-UP layer ( $\alpha_s^C$ ) and O-DU ( $\alpha_s^D$ ). so  $\alpha_s = \alpha_s^U \approx \alpha_s^C \approx \alpha_s^D$ . As a result of these assumption, for simplicity, we can assume that  $M_s = M_s^u = M_s^c = M_s^d$ . Using the above assumption, we have  $T_{DU}^s = T_{CU}^s = T_{UPF}^s$

$$\begin{aligned} T_{process}^s &= T_{RU}^s + T_{DU}^s + T_{CU}^s + T_{UPF}^s \\ T_{process}^s &= T_{RU}^s + 3 \times T_{DU}^s. \end{aligned} \quad (18)$$

**Lemma 1.** In problem (16), the constraint (16f) can be reformulated as below  $\forall i, \forall s$

$$\begin{aligned} T_{max}^s &\geq \frac{1}{R_{u(s,i)} - \lambda_{u(s,i)}} + \frac{3}{\mu_s - \alpha_s/M_s} \\ M_s &\geq \frac{\alpha_s(T_{max}^s R_{u(s,i)} - T_{max}^s \lambda_{u(s,i)} - 1)}{(T_{max}^s \mu_s - 3)(R_{u(s,i)} - \lambda_{u(s,i)}) - \mu_s} \end{aligned} \quad (19)$$

Also from equation (16l), (16g) and (16i) we have

$$0 \leq M_s \leq \min\{M^{max}, \alpha_s/\mu_s, \phi_{max}/3\phi_s\} \quad (20)$$

We denote  $\mathfrak{M}_s = \min\{M^{max}, \alpha_s/\mu_s, \phi_{max}/3\phi_s\}$ . Thus, if we restrict (16f) to equality we have

$$0 \leq \frac{\alpha_s(T_{max}^s R_{u(s,i)} - T_{max}^s \lambda_{u(s,i)} - 1)}{(T_{max}^s \mu_s - 3)(R_{u(s,i)} - \lambda_{u(s,i)}) - \mu_s} \leq \mathfrak{M}_s \quad (21)$$

In (21),  $0 \leq \frac{\alpha_s(T_{max}^s R_{u(s,i)} - T_{max}^s \lambda_{u(s,i)} - 1)}{(T_{max}^s \mu_s - 3)(R_{u(s,i)} - \lambda_{u(s,i)}) - \mu_s}$  is established due to the fact that the numerator and the denominator will be both negative. Using (16h), in numerator,  $\alpha_s \geq 0$ ,  $R_{u(s,i)} - \lambda_{u(s,i)} \geq 0$  and  $(R_{u(s,i)} - \lambda_{u(s,i)})T_{max}^s \leq 1$ . So the numerator is negative. In denominator, it can be said approximately that  $(T_{max}^s \mu_s - 3)(R_{u(s,i)} - \lambda_{u(s,i)}) - \mu_s \leq 0$ , since,  $(R_{u(s,i)} - \lambda_{u(s,i)}) \leq 1/T_{max}^s$ . Therefore, we just need to have constraint below

$$\frac{\alpha_s(T_{max}^s R_{u(s,i)} - T_{max}^s \lambda_{u(s,i)} - 1)}{(T_{max}^s \mu_s - 3)(R_{u(s,i)} - \lambda_{u(s,i)}) - \mu_s} \leq \mathfrak{M}_s \quad (22)$$

So by reformulating the equation (22), we have a new constraint  $\forall i, \forall s$  as below,

$$\begin{aligned} \mathcal{R}_{u(s,i)} &\geq \frac{\mathfrak{M}_s((T_{max}^s \mu_s - 3)\lambda_{u(s,i)} + \mu_s) - \alpha_s(T_{max}^s \lambda_{u(s,i)} + 1)}{\mathfrak{M}_s(T_{max}^s \mu_s - 3) - \alpha_s T_{max}^s}, \\ \varpi_{u(s,i)} &= \frac{\mathfrak{M}_s((T_{max}^s \mu_s - 3)\lambda_{u(s,i)} + \mu_s) - \alpha_s(T_{max}^s \lambda_{u(s,i)} + 1)}{\mathfrak{M}_s(T_{max}^s \mu_s - 3) - \alpha_s T_{max}^s}, \\ \mathcal{R}_{u(s,i)} &\geq \varpi_{u(s,i)}. \end{aligned} \quad (23)$$

In addition, we denote  $\mathbf{M}_{u(s,i)} = \frac{\alpha_s(T_{max}^s R_{u(s,i)} - T_{max}^s \lambda_{u(s,i)} - 1)}{(T_{max}^s \mu_s - 3)(R_{u(s,i)} - \lambda_{u(s,i)}) - \mu_s}$ . So we have,

$$M_s = \max\{\mathbf{M}_{u(s,i)} | i \in 1, 2, \dots, U_s\} \quad \forall s. \quad (24)$$

Despite simplifying the problem (16), it is still non-convex and hard to be solved. So the simplest approach is to relax  $\mathbf{E}$  into continuous value  $e_{r,u(s,i)}^k \in [0, 1] \forall s, \forall i, \forall r, \forall k$ . Furthermore, the problem can be solved using the Lagrangian function and iterative algorithm.

In order to make (16) as a standard form of a convex optimization problem, it is required to change the variable of equations (9) to  $P_r = \sigma_{q_r}^2 \times 2^{C_r}$  so the constraint (16e) is changed to  $P_r \leq \sigma_{q_r}^2 \times 2^{C_{max}^r}$ . The combination of equations (16d) and (16e) leads to the following equation

$$\begin{aligned} \zeta_r &= \min\{P_{max}, \sigma_{q_r}^2 \times 2^{C_{max}^r}\}, \\ P_r &\leq \zeta_r. \end{aligned} \quad (25)$$

Moreover, the combination of equations (16d), (16h) and (23) leads to the following equation

$$\begin{aligned} \eta_{u(s,i)} &= \max\{\mathcal{R}_{u(s,i)}^{max}, \lambda_{u(s,i)}, \varpi_{u(s,i)}\}, \\ \bar{\mathcal{R}}_{u(s,i)} &\geq \eta_{u(s,i)}. \end{aligned} \quad (26)$$

Assume  $\mathbf{v}$ ,  $\mathbf{m}$ ,  $\mathbf{h}$ ,  $\xi$ ,  $\chi$  and  $\kappa$  are the matrix of Lagrangian multipliers that have non-zero positive elements.

The Lagrangian function is written as follow

$$\mathcal{L}(\mathbf{P}, \mathbf{E}; \mathbf{v}, \chi, \mathbf{h}, \xi, \kappa, \mathbf{m}) = \sum_{s=1}^S \sum_{i=1}^{U_s} \bar{\mathcal{R}}_{u(s,i)} \quad (27a)$$

$$+ \sum_{s=1}^S \sum_{i=1}^{U_s} \mathbf{h}_{u(s,i)} (\bar{\mathcal{R}}_{u(s,i)} - \eta_{u(s,i)}) \quad (27b)$$

$$- \sum_{r=1}^R \mathbf{m}_r (P_r - \zeta_r) \quad (27c)$$

$$+ \sum_{s=1}^S \sum_{i=1}^{U_s} \sum_{k=1}^K \sum_{r=1}^R \kappa_{r,u(s,i)}^k p_{r,u(s,i)}^k \quad (27d)$$

$$+ \sum_{r=1}^R \sum_{s=1}^S \sum_{i=1}^{U_s} \chi_{r,u(s,i)} \left( \sum_{k=1}^{K_s} e_{r,u(s,i)}^k - 1 \right) \quad (27e)$$

$$- \sum_{s=1}^S \sum_{i=1}^{U_s} \sum_{k=1}^K \sum_{r=1}^R \mathbf{v}_{r,u(s,i)}^k (e_{r,u(s,i)}^k - 1) \quad (27f)$$

$$+ \sum_{s=1}^S \sum_{i=1}^{U_s} \sum_{k=1}^K \sum_{r=1}^R \xi_{r,u(s,i)}^k e_{r,u(s,i)}^k. \quad (27g)$$

**Lemma 2.** By taking derivatives of (27) (the lagrangian function), with respect to the  $\mathbf{P}$  and the  $\mathbf{E}$ , these two variables are obtained. Assume,  $e_{r,u(s,i)}^k = 1$

$$\frac{\partial \mathcal{L}}{\partial p_{r,u(s,i)}^k} = (1 + \mathbf{h}_{u(s,i)}) \mathfrak{B}_{r,u(s,i)}^k + (\kappa_{r,u(s,i)}^k - \mathbf{m}_r \mathfrak{D}_{r,u(s,i)}^k) = 0 \quad (28)$$

Where

$$\begin{aligned} \mathfrak{D}_{r,u(s,i)}^k &= |\mathbf{w}_{r,u(s,i)}^k|^2 g_{u(s,i)}^r e_{r,u(s,i)}^k, \\ \mathfrak{B}_{r,u(s,i)}^k &= \frac{B |\mathbf{h}_{r,u(s,i)}^H \mathbf{w}_{r,u(s,i)}^k|^2 g_{u(s,i)}^r e_{r,u(s,i)}^k}{\ln(2)} \mathfrak{S}_{r,u(s,i)}^k, \\ \mathfrak{S}_{r,u(s,i)}^k &= \frac{1}{|\mathbf{h}_{r,u(s,i)}^H \mathbf{w}_{r,u(s,i)}^k|^2 \mathfrak{P}_{r,u(s,i)}^k + BN_0 + I_{r,u(s,i)}^k}. \end{aligned} \quad (29)$$

where  $\mathfrak{P}_{r,u(s,i)}^k = g_{u(s,i)}^r e_{r,u(s,i)}^k p_{r,u(s,i)}^k$ . Thus, from equation (28), optimal power is obtained and power is allocated.

We denote  $\mathfrak{J}_{r,u(s,i)}^k = g_{u(s,i)}^r e_{r,u(s,i)}^k$ .

$$p_{r,u(s,i)}^k = \left[ \frac{(1 + \mathbf{h}_{u(s,i)}) B \mathfrak{J}_{r,u(s,i)}^k}{\kappa_{r,u(s,i)}^k - \mathbf{m}_r \mathfrak{D}_{r,u(s,i)}^k} - \frac{BN_0 + I_{r,u(s,i)}^k}{|\mathbf{h}_{r,u(s,i)}^H \mathbf{w}_{r,u(s,i)}^k|^2 \mathfrak{J}_{r,u(s,i)}^k} \right]^+. \quad (30)$$

Also  $[a]^+ = \max(0, a)$ . In addition, PRB assignment is obtained as follow

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_{r,u(s,i)}^k} &= \bar{\mathcal{R}}_{r,u(s,i)}^k (1 + \mathbf{h}_{u(s,i)}) \\ &\quad - \mathbf{m}_r |\mathbf{w}_{r,u(s,i)}^k|^2 p_{r,u(s,i)}^k g_{u(s,i)}^r \\ &\quad + (\xi_{r,u(s,i)}^k - \mathbf{v}_{r,u(s,i)}^k + \chi_{r,u(s,i)}) = 0. \end{aligned} \quad (31)$$

Using KKT conditions, we have

$$e_{r,u(s,i)}^k \times (\bar{\mathfrak{F}}_{r,u(s,i)}^k - \mathbf{v}_{r,u(s,i)}^k - \mathbf{m}_r |\mathbf{w}_{r,u(s,i)}^k|^2 p_{r,u(s,i)}^k g_{u(s,i)}^r) = 0. \quad (32)$$

Where  $\bar{\mathfrak{F}}_{r,u(s,i)}^k = \bar{\mathcal{R}}_{r,u(s,i)}^k (1 + \mathbf{h}_{u(s,i)}) + (\xi_{r,u(s,i)}^k + \chi_{r,u(s,i)})$ . Hence, from equation (31) and (32), PRB assignment is performed as follow.

$$e_{r,u(s,i)}^k = \begin{cases} 1 & u(s,i) = \text{argmax}_k \bar{\mathfrak{F}}_{r,u(s,i)}^k \forall s, \forall r, \forall k \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

Thus the user in each slice  $s$  that have the largest value of  $\bar{\mathfrak{F}}_{r,u(s,i)}^k$ , should be allocated to the PRB  $k$ ; Due to the fact that just one PRB can be allocated to a UE between those UEs (regardless to the services) that are associated to the same O-RU.

## B. Sub-Problem 2

After power allocation and PRB assignment, the remaining problem is to assign O-RU to the UE in each service. The problem can be solved by two different method. Next we introduce the first method which is a greedy algorithm.

1) *Greedy Algorithm:* The problem can be simplified and reformulated as follow

$$\max_{\mathbf{G}} \sum_{r=1}^R g_{u(s,i)}^r \bar{\mathcal{R}}_{u(s,i)}^r \quad (34a)$$

$$\text{subject to } g_{u(s,i)}^r P_r \leq \zeta_r \quad \forall r \quad (34b)$$

$$\sum_r g_{u(s,i)}^r \geq 1 \quad \forall s, \forall i, \quad (34c)$$

$$g_{u(s,i)}^r \in \{0, 1\} \quad \forall s, \forall i, \quad (34d)$$

Since we obtained (26) in (III-A), we can ignore this constraint in (34). So (34), is a NP-complete 0-1 multiple knapsack problem

## REFERENCES

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