Week4 lecture - Classification

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Classification: Definition

- Given a set of records (called the training set)
- Each record contains a set of attributes. One of the attributes is the class
- Find a model for the class attribute as a function of the values of other attributes
- Goal: Previously unseen records should be assigned to a class as accurately as possible
- Usually, the given data set is divided into training and test set, with training set used to build the model and validating set used to validate it. The accuracy of the model is determined on the validating data set.

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Classification Example: Direct Marketing

- Goal: Reduce cost of mailing by targeting a set of consumers likely to buy a new cell phone product
- Approach:
  - Use the data collected for a similar product introduced in the recent past.
  - Use the profiles of customers along with their {buy, didn't buy} decision. The latter becomes the class attribute.
  - The profile of the information may consist of demographic, lifestyle and company interaction.
    Demographic - Age, Gender, Geography, Salary
    Psychographic - Hobbies
    Company Interaction -Recentness, Frequency, Monetary
  - Use these information as input attributes to learn and build a classifier model

Classification Example: Fraud Detection

- Goal: Predict fraudulent cases in credit card transactions
- Approach:
  - Use credit card transactions and the information on its account holders as attributes (important information: when and where the card was used)
  - Label past transactions as {fraud, fair} transactions. This forms the class attribute
  - Learn a model for the class of transactions
  - Use this model to detect fraud by observing credit card transactions on an account

Classification Example: Customer Churn

- Goal: To predict whether a customer is likely to be lost to a competitor
- Approach:
  - Use detailed record of transaction with each of the past and current customers, to find attributes

  How often does the customer call, Where does he call, What time of the day does he call most, His financial status, His marital status, etc. (Important Information: Expiration of the current contract).
  - Label the customers as {churn, not churn}
  - Find a model for Churn

Classification Example: Sky survey cataloging

- Goal: To predict class {star, galaxy} of sky objects, especially visually faint ones, based on the telescopic survey images (from Palomar Observatory)
  - 3000 images with 23,040 x 23,040 pixels per image
- Approach:
  - Segment the image
  - Measure image attributes (40 of them) per object
  - Model the class based on these features
- Success story: Could find 16 new high red-shift quasars (massive and extremely remote celestial object, emitting exceptionally large amounts of energy; some of the farthest objects that are difficult to find) !!!

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Classification problems

Classification problems occur often, perhaps even more so than regression problems. Some examples include

- A person arrives at the emergency room with a set of symptoms that could possibly be attributed to one of three medical conditions. Which of the three conditions does the individual have
- An online banking service must be able to determine whether or not a transaction being performed on the site is fraudulent, on the basis of the users IP address, past transaction history, and so forth.
- On the basis of DNA sequence data for a number of patients with and without a given disease, a biologist would like to figure out which DNA mutations are disease causing and which are not.

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Classification Continued

Consider a qualitative target or response variable Y and associated predictor variables $X_1, X_2, ...X_p$.

The Classification task is to build a function or a rule set in terms of $X_1, X_2, ...X_p$ that takes as input and predicts its value (or category) for Y

Examples: Qualitative variables take values in an unordered set C, such as:

- Eye color $\in$ {brown, blue, green}
- Species $\in$ {versicolour, verginica, sethosa}.
- Insurance claim $\in$ {fraudulent, legitimate }.

Note that these Qualitative variables take values in an unordered set C:

Use Default Data set

```r
library('ISLR')
attach(Default)
View(Default)
dim(Default)
```

```
## [1] 10000     4
```

```r
head(Default)
```

```
##   default student   balance    income
## 1      No      No  729.5265 44361.625
## 2      No     Yes  817.1804 12106.135
## 3      No      No 1073.5492 31767.139
## 4      No      No  529.2506 35704.494
## 5      No      No  785.6559 38463.496
## 6      No     Yes  919.5885  7491.559
```

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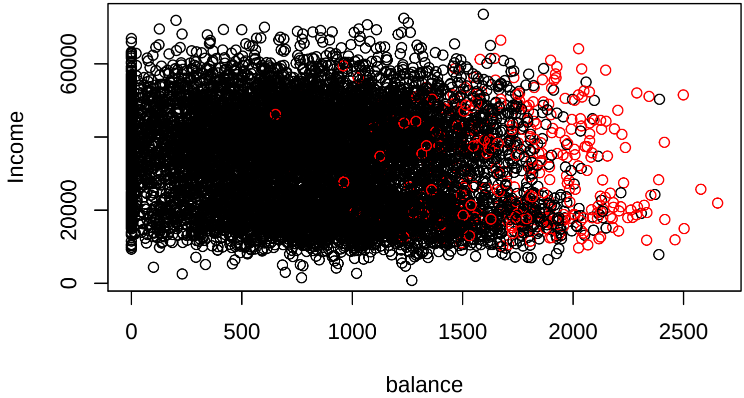
Data Explore

```r
str(Default)
```

```
## 'data.frame':    10000 obs. of  4 variables:
##  $ default: Factor w/ 2 levels "No","Yes": 1 1 1 1 1 1 1 1
##  $ student: Factor w/ 2 levels "No","Yes": 1 2 1 1 1 2 1 2
##  $ balance: num  730 817 1074 529 786 ...
##  $ income : num  44362 12106 31767 35704 38463 ...
```

Data Explore

```
plot(income~balance,col=default, data=Default, ylab= "Income")
```



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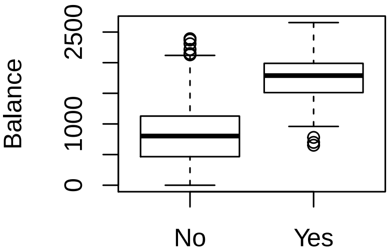
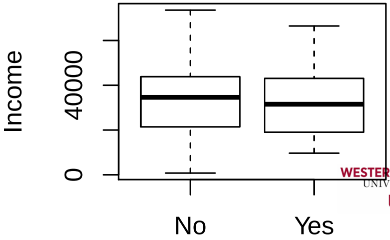
Data Explore

```
par(mfrow=c(1,2))
boxplot(balance~default, data=Default,
        ylab= "Balance", xlab= "Default")
boxplot(income~default, data=Default,
        ylab= "Income", xlab="Default")
```





Can we use Linear Regression when Y is qualitative?

If we are to classify customers according to credit card Default

Set Code

Y = 0, if default is No

Y= 1, if default is Yes

Question? Can we simply perform a linear regression of Y on X and classify as 'Yes' if $\hat{Y}_i > 0.5$?

Limitations of using Linear Regression when Y is binary

Consider Y is default and X is balance,

Then simple linear regression model is

$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$ for $i = 1, 2, ..., n$

In this case of a binary outcome, linear regression does a good job as a classifier Why????

$E(Y_i) = P(Y = 1)1 + P(Y = 0)0 = P(Y = 1) = P(Default)$

and $E(Y_i) = \alpha + \beta X_i$

Therefore

$P(Default) = \alpha + \beta X_i = E(Y_i)$

NOTE: However, linear regression might produce probabilities less than zero or bigger than one. Next few slides we demonstrate this point.

Then we consider Logistic Regression

Simple Linear Regression Model requires a numeric X and Y variables

- How to change a factor variable to a numeric variable?
  - Add another variable named Defcode to table Default
  - check the levels of the new variable (It will be same class as the original variable)

```
Defcode = Default$default
levels(Defcode)
```

```
## [1] "No"  "Yes"
```

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Change the levels as 1 for Yes and 0 for No

```
levels(Defcode)[levels(Defcode)=="No"]=0
levels(Defcode)[levels(Defcode)=="Yes"]=1
levels(Defcode)
```

```
## [1] "0" "1"
```

Still Defcode varibale is a factor variable and cannot use as a numeric variable in regression setting.

To summarise a factor variable

```r
Defcode = as.character(Default$default)
table(Defcode)
```

```
## Defcode
##   No  Yes
## 9667  333
```

```r
Defcode[Defcode=="No"]=0
Defcode[Defcode=="Yes"]=1
table(Defcode)
```

```
## Defcode
##    0    1
## 9667  333
```

Change a factor variable to a numeric variable

```
Defcode = as.numeric(Defcode)
class(Defcode)
```

```
## [1] "numeric"
```

Simple Linear Regression Model for Default data set with Coded Y

Set the codes for variable default

```
model1=lm(Defcode~balance)
summary(model1)
```

```
##
## Call:
## lm(formula = Defcode ~ balance)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.23533 -0.06939 -0.02628  0.02004  0.99046
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.519e-02  3.354e-03  -22.42   <2e-16 ***
## balance      1.299e-04  3.475e-06   37.37   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## Residual standard error: 0.1681 on 9998 degrees of freedom
## Multiple R-squared:  0.1226, Adjusted R-squared:  0.1225
## F-statistic:  1397 on 1 and 9998 DF,  p-value: < 2.2e-16
```

Plot to illustrate how Linear Regression Model produce probabilities less than zero or bigger than one.

```
plot(balance,Defcode,
     ylab= "Probability of Default",xlab= "Balance")
abline(a= -7.519e-02, b=1.299e-04)
```

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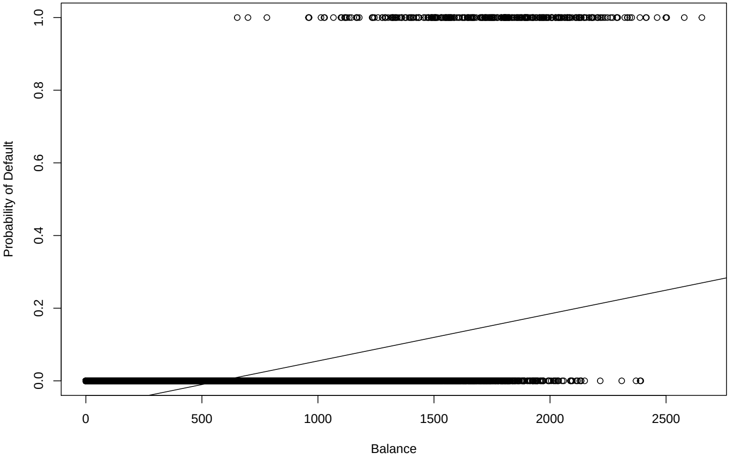

1.0
0.8
0.6
0.4
0.2
0.0
Probability of Default
0
500
1000
1500
2000
2500
Balance

Logistic Regression NOTE:

It's useful to treat simple logistic regression amd Multiple Logistic Regresson separately

- simple logistic regression, with only one independent variable
- multiple logistic regression, which has more than one independent variable

Classification Estimating the Probabilities

Often we are more interested in estimating the probabilities of Y assuming each of its category.

For example, it is more valuable to have:

- an estimate of the probability a customer with a given balance will default , than a classification default or not.
- an estimate of the probability an insurance claim is fraudulent, than a classification fraudulent or not.

Logistic Regression: Odds and LogOdds

$Odds(Default) = \frac{P(Y=1/X)}{P(Y=0/X)} = \frac{P(Y=1/X)}{1-P(Y=1/X)} = \frac{P(X)}{1-P(X)}$
$log(Odds(Default)) = log\frac{P(X)}{1-P(X)} = \alpha + \beta X$

which is called the ***logit transformation*** of $P(X)$. Therefore by rearranging we get the logistic regression form:

$P(X) = \frac{e^{\alpha+\beta X}}{1+e^{\alpha+\beta X}}$

$e = 2.71828$ is a mathematical constant ***Euler's number***. It is easy to show $P(x)$ will always have values between 0 and 1 irrespective of the value of X.

Note that P(x) is the probability that a person with balance X will default.

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Contrast Between Logistic and Linear Regression

– In linear regression, the expected value of Y_1 given X_1 is

$E(Y_i) = \alpha + \beta X_i$ for $i = 1, 2, ..., n$

Y_i has a ***normal distribution*** with ***standard deviation*** σ. It is the ***random component*** of the model, which has a ***normal distribution***.

$\alpha + \beta X_i$ is the ***linear predictor***.

- In logistic regression, the ***Target Variable*** is the ***logit of the expected value of*** Y_i ***given*** X_i and the model takes the form

$logit(E(Y_i)) = \alpha + \beta X_i$ for $i = 1, 2, ..., n$

$logit(E(Y_i))$ is the random component of the model

logit is the ***link function*** that relates the expected value of the random component to the linear predictor.

Note: $logit(\pi) = log\frac{\pi}{(1-\pi)}$

Maximum Likelihood Estimation

- In linear regression we used the method of least squares to estimate regression coefficients.
- In generalized linear models we use another approach called ***maximum likelihood*** estimation.
- The maximum likelihood estimate of a parameter is that value that maximizes the probability of the observed data.
- We estimate $\hat{\alpha}$ and $\hat{\beta}$ by those values and that maximize the probability of the observed data under the logistic regression model.

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Maximum Likelihood Estimates

- We use maximum likelihood to estimate the parameters

$L(\alpha, \beta) = \prod_{i=0}^{n}[P(X_i)] \prod_{i=1}^{n}[(1 - P(X_{i'}))]$

X_i when $Y_i = 1$ and $X_{i'}$ when $Y_i = 0$

- This likelihood gives the probability of the observed zeros and ones in the data.
- We pick $\hat{\alpha}$ and $\hat{\beta}$ to ***maximize the likelihood*** of the observed data.
- Most statistical packages can fit linear logistic regression models by maximum likelihood.
- In R we use the ***glm function***.
- Maximum likelihood is a very general approach that is used to fit many of the non-linear models. In the linear regression setting, the least squares approach is in fact a special case of maximum likelihood

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Logistic Regression model using glm function

```
model2= glm(Defcode~balance,data=Default,family=binomial)
summary.glm(model2)
```

```
##
## Call:
## glm(formula = Defcode ~ balance, family = binomial, data =
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.2697  -0.1465  -0.0589  -0.0221   3.7589
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.065e+01  3.612e-01  -29.49   <2e-16 ***
## balance      5.499e-03  2.204e-04   24.95   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2920.6  on 9999  degrees of freedom
```

Making Predictions

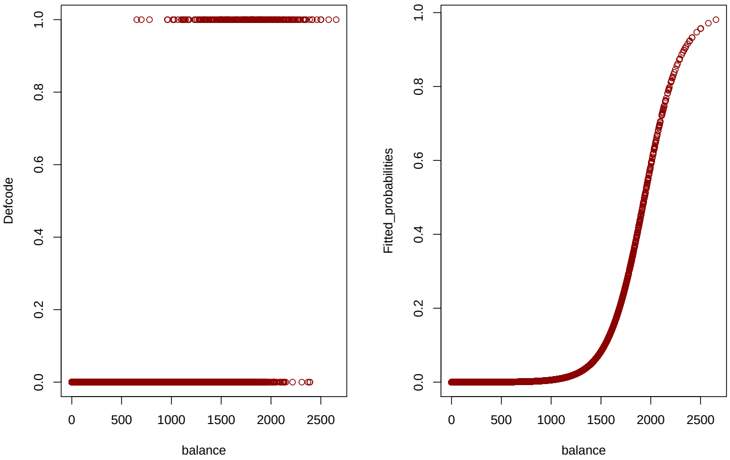
What is our ***estimated probability*** of default for someone with a balance of dollars 1000?

$P(\hat{X}) = \frac{e^{\hat{\alpha}+\hat{\beta}X}}{1+e^{\hat{\alpha}+\hat{\beta}X}} = \frac{e^{-10.65+0.005*1000}}{1+e^{-10.65+0.005*X*1000}} = 0.003505$

What is our ***estimated probability*** of default for someone with a balance of dollars 2000?

$P(\hat{X}) = \frac{e^{\hat{\alpha}+\hat{\beta}X}}{1+e^{\hat{\alpha}+\hat{\beta}X}} = \frac{e^{-10.65+0.005*2000}}{1+e^{-10.65+0.005*2000}} = 0.342989$

Plot of Fitted Propabilities of Default



Lets do it again, using student as the predictor

```
model3= glm(Defcode~student,data=Default,family=binomial)
summary.glm(model3)
```

```
##
## Call:
## glm(formula = Defcode ~ student, family = binomial, data =
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.2970  -0.2970  -0.2434  -0.2434   2.6585
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.50413    0.07071  -49.55  < 2e-16 ***
## studentYes   0.40489    0.11502    3.52 0.000431 ***
## ---
```

Predict Probability of Default using Variable Student

$\hat{P}rob(Default = Yes/Student = Yes) = \frac{e^{-3.50413+0.40489*1}}{1+e^{-3.50413+0.40489*1}} =$
0.04313862

$\hat{P}rob(Default = Yes/Student = No) = \frac{e^{-3.50413+0.40489*0}}{1+e^{-3.50413+0.40489*0}} =$
0.02919495

Confounding

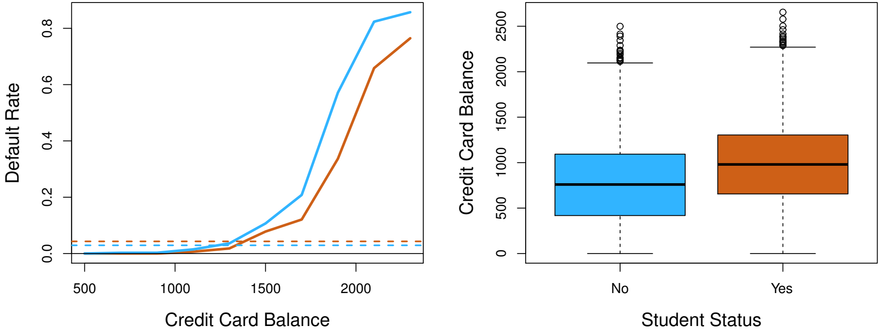


Figure 1

Source:An Introduction to Statistical Learning: with Applications in R

Summary

- Students tend to have higher balances than non-students, so their marginal default rate is higher than for non-students.
- But for each level of balance, students default less than non-students.
- Multiple logistic regression can tease this out.

Logistic regression with several variables

Consider the Qualitative Binary Target Variable Y and several predictor X Variables X_1, X_2, X_3,..., X_p

Then the logistic regression model for the expected value of Yi given X_1, X_2, X_3,..., X_p is

$logit(E(Y)) = logit(P(X)) = log\frac{P(X_i)}{1-P(X_i)} =$
$\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta_p X_{pi}$ for i=1,2,...,n

$logit(E(Y))$ is the random component of the model

Then

$P(X) = \frac{e^{\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+...+\beta_p X_{pi}}}{1+e^{\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}+...+\beta_p X_{pi}}}$

Logistic regression with several variables

```
model4=glm(Defcode~student+balance+income, data=Default,
           family=binomial)
summary(model4)
```

```
##
## Call:
## glm(formula = Defcode ~ student + balance + income, family
##     data = Default)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.4691  -0.1418  -0.0557  -0.0203   3.7383
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.087e+01  4.923e-01 -22.080  < 2e-16 ***
```

```
##
## Call:
## glm(formula = Defcode ~ student + balance + income, family
##     data = Default)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.4691  -0.1418  -0.0557  -0.0203   3.7383
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.087e+01  4.923e-01 -22.080  < 2e-16 ***
## studentYes  -6.468e-01  2.363e-01  -2.738  0.00619 **
## balance      5.737e-03  2.319e-04  24.738  < 2e-16 ***
## income       3.033e-06  8.203e-06   0.370  0.71152
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
```

```
##
## Call:
## glm(formula = Defcode ~ student + balance, family = binomia
##     data = Default)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.4578  -0.1422  -0.0559  -0.0203   3.7435
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.075e+01  3.692e-01 -29.116  < 2e-16 ***
## studentYes  -7.149e-01  1.475e-01  -4.846 1.26e-06 ***
## balance      5.738e-03  2.318e-04  24.750  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## (Dispersion parameter for binomial family taken to be 1)
```

```r
model5=glm(Defcode~student+balance,data=Default, family=binomi
anova(model5)
```

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: Defcode
##
## Terms added sequentially (first to last)
##
##
##         Df Deviance Resid. Df Resid. Dev
## NULL                     9999     2920.7
## student  1    11.97      9998     2908.7
## balance  1  1337.00      9997     1571.7
```

Example 2 - Heart data

```
heart <- read.csv("heart.csv")
attach(heart)
head(heart)
```

```
##   X Age Sex    ChestPain RestBP Chol Fbs RestECG MaxHR ExAn
## 1 1  63   1      typical    145  233   1       2   150
## 2 2  67   1 asymptomatic    160  286   0       2   108
## 3 3  67   1 asymptomatic    120  229   0       2   129
## 4 4  37   1   nonanginal    130  250   0       0   187
## 5 5  41   0   nontypical    130  204   0       2   172
## 6 6  56   1   nontypical    120  236   0       0   178
##   Ca       Thal AHD
## 1  0      fixed   0
## 2  3     normal   1
## 3  2 reversable   1
## 4  0     normal   0
```

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```
## 'data.frame':    303 obs. of  15 variables:
##  $ X        : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ Age      : int  63 67 67 37 41 56 62 57 63 53 ...
##  $ Sex      : int  1 1 1 1 0 1 0 0 1 1 ...
##  $ ChestPain: Factor w/ 4 levels "asymptomatic",..: 4 1 1 2
##  $ RestBP   : int  145 160 120 130 130 120 140 120 130 140
##  $ Chol     : int  233 286 229 250 204 236 268 354 254 203
##  $ Fbs      : int  1 0 0 0 0 0 0 0 0 1 ...
##  $ RestECG  : int  2 2 2 0 2 0 2 0 2 2 ...
##  $ MaxHR    : int  150 108 129 187 172 178 160 163 147 155
##  $ ExAng    : int  0 1 1 0 0 0 0 1 0 1 ...
##  $ Oldpeak  : num  2.3 1.5 2.6 3.5 1.4 0.8 3.6 0.6 1.4 3.1
##  $ Slope    : int  3 2 2 3 1 1 3 1 2 3 ...
##  $ Ca       : int  0 3 2 0 0 0 2 0 1 0 ...
##  $ Thal     : Factor w/ 3 levels "fixed","normal",..: 1 2 3
##  $ AHD      : int  0 1 1 0 0 0 1 0 1 1 ...
```

Heart data

```
##
## Call:
## glm(formula = AHD ~ ., family = binomial, data = heart)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.7629  -0.5101  -0.1494   0.3460   2.7301
##
## Coefficients:
##                          Estimate Std. Error z value Pr(>|z|)
## (Intercept)             -4.739690   2.931081  -1.617  0.10587
## X                        0.002676   0.002221   1.205  0.22811
## Age                     -0.013183   0.024785  -0.532  0.59479
## Sex                      1.486941   0.519796   2.861  0.00423 *
## ChestPainnonanginal     -1.755328   0.493018  -3.560  0.00037 *
## ChestPainnontypical     -0.951481   0.560165  -1.699  0.08940 .
```

ANOVA

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: AHD
##
## Terms added sequentially (first to last)
##
##
##           Df Deviance Resid. Df Resid. Dev
## NULL                         296     409.95
## X          1    0.800        295     409.15
## Age        1   15.661        294     393.49
## Sex        1   31.018        293     362.47
## ChestPain  3   72.702        290     289.77
## RestBP     1    5.391        289     284.38
```

Mis-classification Matrix

	Actual Positive	Actual Negative
Predicted Positive	TP	FP
Predicted Negative	FN	TN

Misclassification rate=(False Positive+ False Negative)/Total

True Positive rate=True Positive/Total Positive

False Positive rate= False Positive/ Total Negative (Type I error)

False Negative rate= False Negative/ Total Positive (Type II error)

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Why Not Linear Regression for Y with more than two classes?

- Suppose that we are trying to predict the medical condition of a patient in the emergency room on the basis of her symptoms. In this simplified example, there are three possible diagnoses: stroke, drug overdose, and epileptic seizure. We could consider encoding these values as a quantitative response variable, Y , as follows:

Y = 1 if Stroke; Y = 2 if if drug overdose; Y = 3 if epileptic seizure.

Using this coding, least squares could be used to fit a linear regression model to predict Y on the basis of a set of predictors X1, . . .,Xp. Unfortunately, this coding implies an ***ordering*** on the outcomes, putting drug overdose in between stroke and epileptic seizure, and insisting that the difference between stroke and drug overdose is the same as the difference between drug overdose and epileptic seizure.

Continued

If the response variable's values did take on a natural ordering, such as mild, moderate, and severe, and we felt the gap between mild and moderate was similar to the gap between moderate and severe, then a 1, 2, 3 coding would be reasonable. Unfortunately, in general there is no natural way to convert a qualitative response variable with more than two levels into a quantitative response that is ready for linear regression.

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Logistic Regression for >2 Response Classes

The ***two-class*** logistic regression models discussed have ***multiple-class*** extensions, but in practice they tend not to be used all that often. One of the reasons is that the method ***discriminant analysis***, is popular for multiple-class classification.

So we do not go into the details of multiple-class logistic regression here, but simply note that such an approach is possible, and that software for it is available in R

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TEXTBOOK

Lecture notes are based on the textbook,

for further reference refer chapter 4;

Prescribed Textbook

– James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An Introduction to Statistical Learning: with Applications in R Springer.