Mixing and the Mirage of Convergence

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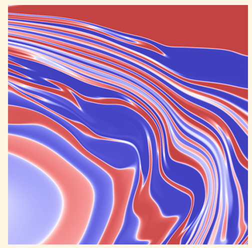
Introduction

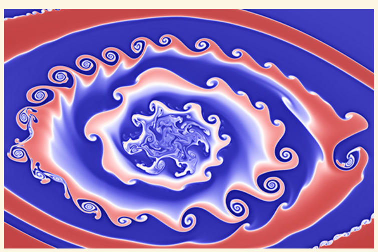
A strange fact of fluid dynamics: **fluids don't actually mix**. The mixing we observe—cream into coffee, dye into water—isn't due to stirring alone, but to *molecular diffusion,* a process absent in the standard fluid equations.



From a fluid dynamics standpoint, when you stir cream into your coffee, all you're doing is folding and stretching the different layers of fluid into thinner and thinner sheets…once the sheets get microscopically thin, the fluid equations break down, diffusion takes over, and you finally mix the two liquids.

Most astrophysical simulations omit diffusion entirely, so when we see "mixing," it's due to **numerical errors**. This raises an unsettling question: *are the beautiful structures in high-resolution simulations real—or artifacts?*

The Kelvin-Helmholtz Instability

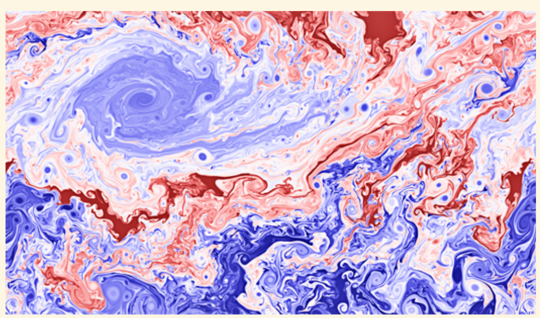


The ***Kelvin–Helmholtz instability*** (KH) arises when two fluid layers slide past each other. It produces iconic rolling billows—seen in the sky, in oceans, and in countless simulation papers.

As resolution increases, KH simulations often reveal intricate *swirls-within-swirls,* which are widely assumed to indicate numerical accuracy and turbulence.

Do These Swirls Converge?

We investigated whether these simulations actually *converge* to a well-defined solution. The result: **they don't**. Simulations with increasing resolution produce ***different answers, not better ones***—more swirls, more chaos, more discrepancy.

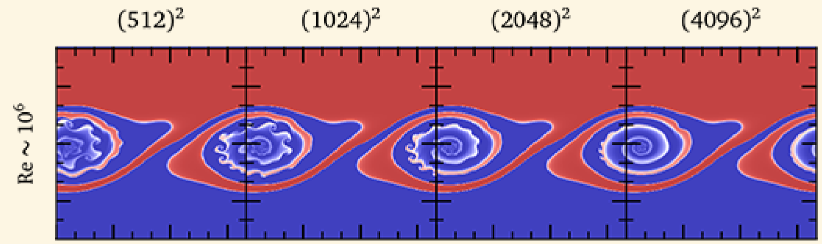


You can often guess the simulation's resolution by counting how many layers of swirls it has.

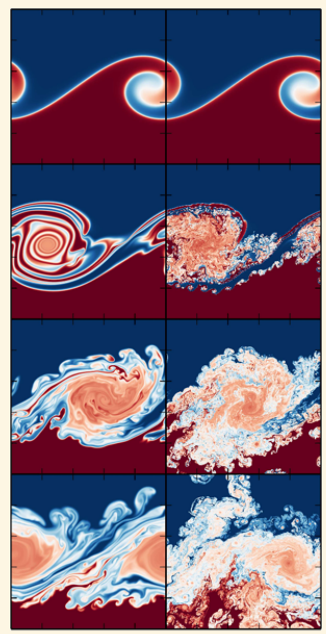
A Modified, Solvable Problem

We designed a slightly altered version of the KH problem—one that includes a small but controlled amount of diffusion. This version has a well-defined solution, allowing us to test convergence rigorously.

The result was striking: **the true solution has no such tiny swirls**. Even more surprising, the tiny swirls are present in lower resolution, un-converged simulations, but **disappear as we approach convergence!**



Implications for Simulation Practice



Astronomers often neglect diffusion from their simulations because the true, physical scale for diffusion is impossible to resolve. We think that, by setting diffusion to zero in our codes, we get the least amount of diffusion. However, this is not the case!

Without any diffusion, the smallest perturbations grow unchecked, generating swirls from *numerical noise, not physics*. Including diffusion—even at unrealistically high levels—**reduces unphysical mixing** and produces more reliable results, *with **less** mixing*.

It's better to have a controlled approximation than uncontrolled chaos.

What's next? We're developing benchmark tests and convergence criteria for mixing in astrophysical simulations.

This work cautions against interpreting small-scale structures as signs of realism. **Sometimes, a prettier picture is a numerical mirage.**

References

Lecoanet, McCourt, Quataert, Burns, et al. (2007)