

# **ADCC**

## **Experiment-04 (MPSK)**

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## **1 Aim**

To design discrete time MPSK scheme and analyse the performance.

## **2 Theory**

MPSK is the abbreviation for M-ary Phase Shift Keying. M-ary implies M possible values at a time. Phase shift implies change in phase for carrier signal to represent the information. Keying implies the way to represent the information in another form.

Usually for practical implementations, M would be some exponent of 2 such that  $\log_2 M$  bits can be considered at a time i.e. base of scheme. In this particular scheme, system has M different symbols to modulate. For this task to achieve, it chooses M different phase values for the carrier signal. Chosen phase value would be constant for a particular time interval i.e. width of information symbol. After that system changes phase of carrier signal based on the current symbol. In this way, M-ary system continuously modulates the symbols in carrier signal. That modulated carrier signal propagates through channel/medium and captured by receiver or receivers. Based on phase of the received signal, a receiver is supposed to decode the information symbol. Hence, difference between any two phase angle should be maximum as much as possible.

Generally, phase starts with 0 and increase by  $\frac{2\pi}{M}$  and  $M > 2$ . Consider a constellation diagram as given below.

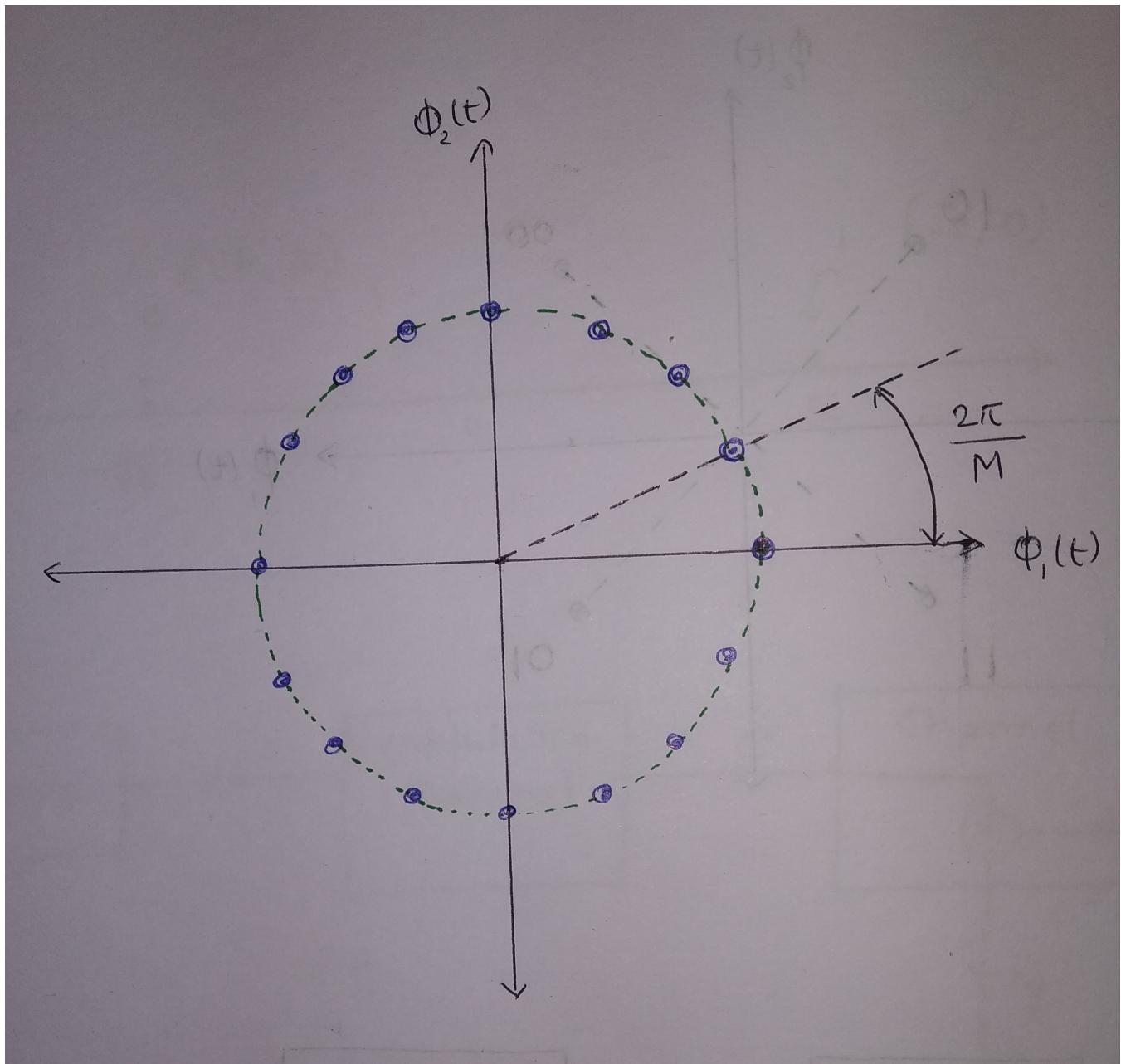


Figure 1: MPSK constellation diagram

### 3 Design

#### 3.1 Block diagram

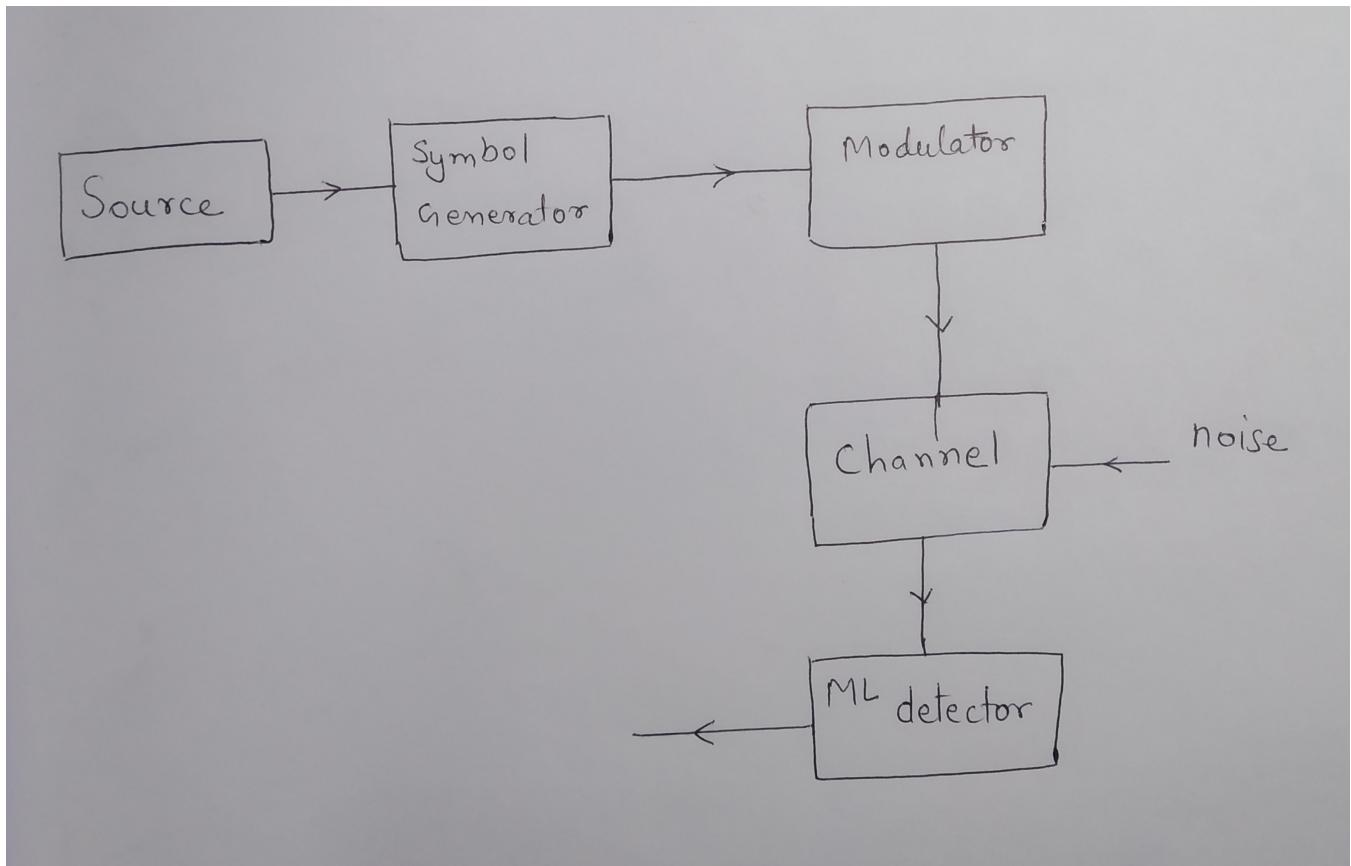


Figure 2: Block diagram

Where source block generates binary bit stream. Symbol generator is the block that takes  $\log_2 M$  bits at a time and convert into symbols. Modulation block does mapping based on constellation diagram figure 1. Channel block is the representation of AWGN (Additive White Gaussian Noise), in that gaussian noise adds up to the signal values.

### 3.2 ML detector

For the ML detector, angular bisector region would be the best choice for decision. For visualization of angular bisector a constellation diagram is given below.

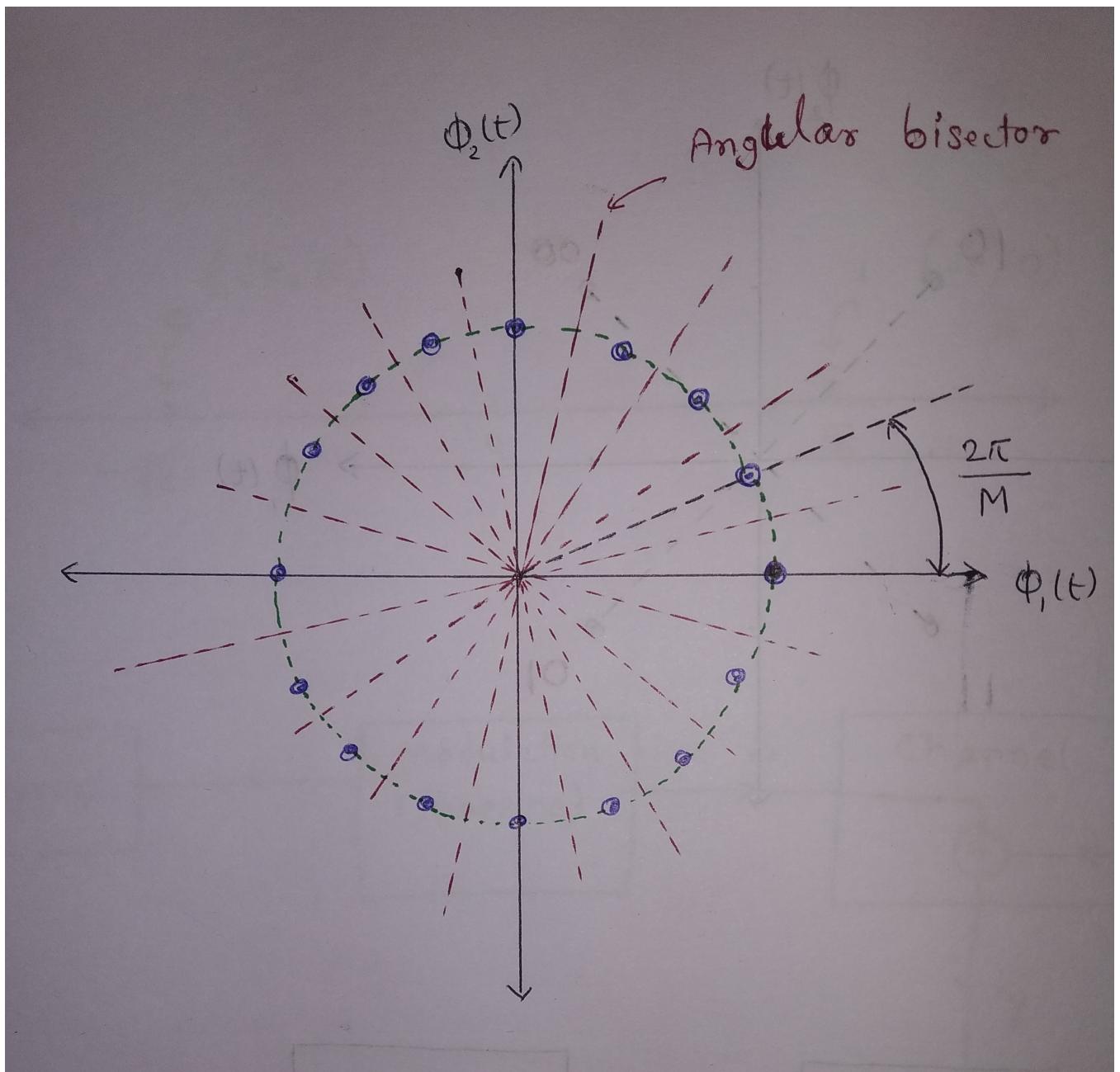


Figure 3: Angular bisector

### 3.3 Probability of error

$$\begin{aligned}
 f_{y|S_0}(y|S_0) &= f_{y_1|S_0}(y_1|S_0) \cdot f_{y_2|S_0}(y_2|S_0) \\
 &\quad \left( \text{where } f_{N_1}(y_1 - \bar{y}_E) \cdot f_{N_2}(y_2) \right) \\
 &= \left( \frac{1}{\sigma \sqrt{2\pi}} \right) e^{-\frac{(y_1 - \bar{y}_E)^2}{2\sigma^2}} \cdot \left( \frac{1}{\sigma \sqrt{2\pi}} \right) e^{-\frac{(y_2)^2}{2\sigma^2}} \\
 &= \left( \frac{1}{\sigma \sqrt{2\pi}} \right)^2 \exp \left( -\frac{(y_1 - \bar{y}_E)^2}{2\sigma^2} - \frac{y_2^2}{2\sigma^2} \right).
 \end{aligned}$$

Now here, we are in 2-D domain of  $(y_1, y_2)$  we  
for simplicity we want to go into  $(r, \theta)$  domain

$$\begin{aligned}
 y_1 &= r \cos \theta \\
 y_2 &= r \sin \theta
 \end{aligned}$$

Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial y_1}{\partial r} & \frac{\partial y_1}{\partial \theta} \\ \frac{\partial y_2}{\partial r} & \frac{\partial y_2}{\partial \theta} \end{bmatrix}$$

$$J = \begin{bmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{bmatrix}.$$

$$|J| = r$$

Figure 4: Probability of error calculation

$$\begin{aligned}
 & \# f_{y|s}(y|s_0) \rightarrow f_{R,\theta|s}(r, \theta | s_0) \\
 & \Rightarrow \left( \frac{1}{\sigma \sqrt{2\pi}} \right)^2 e^{-\left( \frac{1}{2\sigma^2} ((y_1 - rE)^2 + y_2^2) \right)} \\
 & y_1 = r \cos \theta \quad y_2 = r \sin \theta \\
 & \Rightarrow f_{R,\theta}(r|\theta | s_0) = \frac{r}{(\sigma \sqrt{2\pi})^2} e^{-\left( \frac{1}{2\sigma^2} ((r \cos \theta - rE)^2 + r^2 \sin^2 \theta) \right)} \\
 & f_{\theta|s}(\theta | s_0) \quad f_{\text{correct}} = \int_{-\infty}^{\infty} f_{R,\theta|s}(r, \theta | s_0) dr \\
 & \Rightarrow \int_0^{\infty} \frac{r}{(\sigma \sqrt{2\pi})^2} e^{-\left( \frac{1}{2\sigma^2} (r^2 \cos^2 \theta + E^2 - 2r \cos \theta rE + r^2 \sin^2 \theta) \right)} dr \\
 & \Rightarrow \int_0^{\infty} \frac{r}{(\sigma \sqrt{2\pi})^2} e^{-\left( \frac{1}{2\sigma^2} (r^2 + E^2 - 2r \sqrt{E} \cos \theta) \right)} dr
 \end{aligned}$$

Figure 5: Probability of error calculation

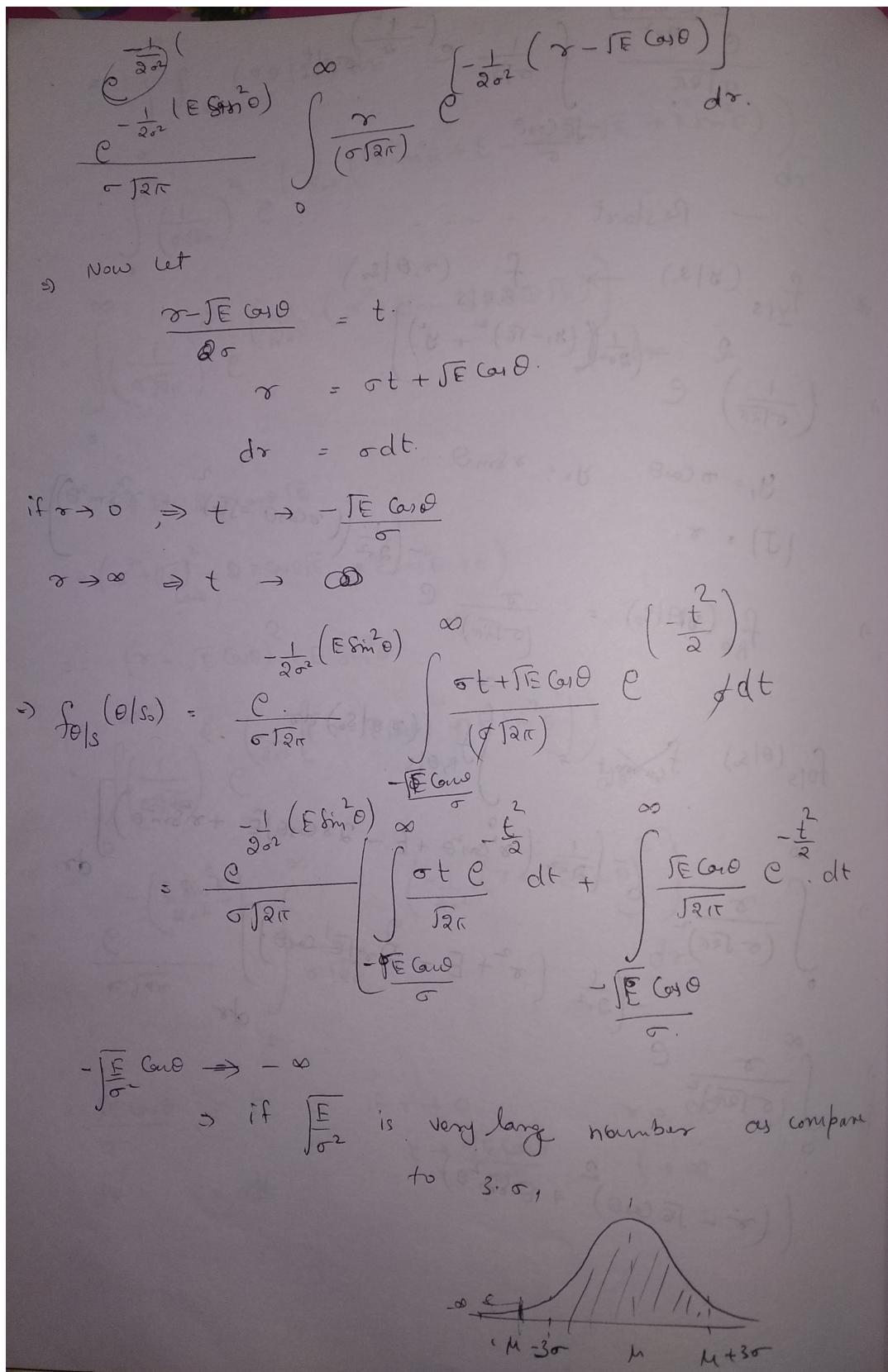


Figure 6: Probability of error calculation

$$\approx \frac{C}{\sigma \sqrt{2\pi}} \left( -\frac{1}{2\sigma^2} (\mathbb{E} \sin^2 \theta) \right)$$

$$\approx \frac{C}{\sigma \sqrt{2\pi}} \left[ \int_{-\infty}^{\infty} \frac{(xt e^{-\frac{t^2}{2}})}{\sqrt{2\pi}} dt + \int_{-\infty}^{\infty} \frac{\sqrt{\mathbb{E} \cos \theta} e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt \right]$$

$$\downarrow$$

$$I = \int_{-\infty}^{\infty} xe^{-\frac{x^2}{2}} dx = \int_{-\infty}^0 xe^{-\frac{x^2}{2}} dx + \int_0^{\infty} xe^{-\frac{x^2}{2}} dx.$$

Now  $x = -t$ .  $x \rightarrow \infty \Rightarrow t \rightarrow -\infty$ .

$$dx = -dt. \quad x \rightarrow 0 \Rightarrow t \rightarrow 0.$$

$$-\int_0^{\infty} t e^{-\frac{t^2}{2}} dt + \int_0^{\infty} xe^{-\frac{x^2}{2}} dx.$$

$$\approx \frac{C}{\sigma \sqrt{2\pi}} \left( \frac{\sqrt{\mathbb{E} \cos \theta}}{\sqrt{2\pi}} \left[ \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \right] \right).$$

$$f_{\theta|S_0}(\theta|s_0) \approx \frac{C}{\sigma \sqrt{2\pi}} \left( \sqrt{\mathbb{E} \cos \theta} \right).$$

$$P_{\text{correct}} = 1 - \int_{-\pi/M}^{\pi/M} f_{\theta|S_0}(\theta|s_0)$$

Figure 7: Probability of error calculation

$$\begin{aligned}
 &= 1 - \int_{-\pi/M}^{\pi/M} \frac{e^{-\frac{1}{2}\sigma^2 E \sin^2 \theta}}{\sigma \sqrt{2\pi}} \sqrt{E} \cos \theta \, d\theta. \\
 \frac{\sqrt{E} \sin \theta}{\sigma} &= dt \quad \text{if } \theta = \frac{\pi}{M} t \Rightarrow t = \frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}. \\
 \frac{\sqrt{E} \cos \theta}{\sigma} \, d\theta &= dt. \quad \theta = -\frac{\pi}{M} t \Rightarrow t = -\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}. \\
 &= 1 - \int_{-\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}}^{\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt. \\
 &= 1 - \int_{-\infty}^{\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt + \int_{-\infty}^{\frac{-\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt. \\
 \text{Let } I_1 &= \int_{-\infty}^{\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt = 1 - \int_{\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}}^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt. \\
 &= 1 - Q\left(\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}\right) \\
 \Rightarrow I_2 &= \int_{-\infty}^{\frac{-\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt = \int_{\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}}^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt = Q\left(\frac{\sqrt{E} \sin(\frac{\pi}{M})}{\sigma}\right).
 \end{aligned}$$

Figure 8: Probability of error calculation

$$\begin{aligned}
 & \Rightarrow 1 - I_1 + I_2 \\
 & \Rightarrow 1 - Q\left(\frac{\sqrt{E}}{\sigma} \sin\left(\frac{\pi}{M}\right)\right) + Q\left(\frac{\sqrt{E}}{\sigma} \sin\left(\frac{\pi}{M}\right)\right) \\
 & \Rightarrow 2Q\left(\frac{\sqrt{E}}{\sigma} \sin\left(\frac{\pi}{M}\right)\right). \\
 & \Rightarrow 2Q\left(\sqrt{\frac{E}{N_o}} \sin\left(\frac{\pi}{M}\right)\right).
 \end{aligned}$$

Figure 9: Probability of error calculation

**Final result:*****For MPSK probability of error***

$$P(\text{Error}) = 2Q\left(\sqrt{\frac{2E}{N_o}} \sin\left(\frac{\pi}{M}\right)\right)$$

That means, probability of error for MPSK depends on two quantities, one is  $\frac{E}{N_o}$  and second is M.

## 4 Code and result

### 4.1 Code for MPSK in cpp

```
1 //////////////////////////////////////////////////////////////////
2 //////////////////////////////////////////////////////////////////
3 /**
4 // Author:- MANAS KUMAR MISHRA
5 // Organisation:- IIITDM KANCHEPURAM
6 // Topic:- MPSK MODEL
7 //////////////////////////////////////////////////////////////////
8 //////////////////////////////////////////////////////////////////
9
10
11 #include <iostream>
12 #include <cmath>
13 #include <iterator>
14 #include <random>
15 #include <chrono>
16 #include <time.h>
17 #include <fstream>
18 #include <bits/stdc++.h>
19 #include <vector>
20
21 #define one_million 1000000
22 #define pi 3.14159
23
24 using namespace std;
25
26
27 // Function for generating binary bits at source side. Each bit is equiprobable.
28 // Input is nothing
29 // Output is a vector that contains the binary bits of length one_million*1.
30 vector<double> Source()
31 {
32     vector<double> sourceBits;
33
34     // Use current time as seed for random generator
35     srand(time(0));
36
37     for(int i = 0; i<one_million; i++){
38         sourceBits.insert(sourceBits.end(), rand()%2);
39     }
40
41     return sourceBits;
42 }
43
44
45 // Function to generate symbols from the binary bit stream.
46 // Input is the binary bit vector and base for symbol conversion
47 // Output is the symbol vector containing values from (0, 2^(base-1))
48 vector <double> binaryToDecimalConversion(vector<double> sourceBits, const int& base)
49 {
50     vector <double> convertedBits;
```

```

51     int start =0;
52
53     if((sourceBits.size()% base) == 0 ){
54
55         int finalSize = sourceBits.size()/base;
56         start = 0;
57         int conversion;
58
59
60         for(int i=0; i<finalSize; i++){
61             conversion =0;
62             for(int j =base-1; j>-1; j--){
63                 conversion = conversion + (sourceBits[start])*(pow(2, j));
64                 start++;
65             }
66             convertedBits.insert(convertedBits.end(), conversion);
67         }
68
69     }
70     else{
71         int addedBitsNO = base - (sourceBits.size()%base);
72
73         for(int q=0;q<addedBitsNO;q++){
74             sourceBits.insert(sourceBits.end(), 0);
75         }
76
77         int finalSize = sourceBits.size()/base;
78         start = 0;
79         int conversion;
80
81
82         for(int i=0; i<finalSize; i++){
83             conversion =0;
84             for(int j =0; j<base; j++){
85                 conversion = conversion + (sourceBits[start])*(pow(2, j));
86                 start++;
87             }
88             convertedBits.insert(convertedBits.end(), conversion);
89         }
90
91     }
92
93     return convertedBits;
94 }
95
96
97
98
99 // Functions SignalVector1 and SignalVector2 for convert sourceSymbols to 2D-vector
100 // Input are the source Symbols and energy of symbols for mapping
101 // Output is a vector represent signal component
102 vector<double> SignalVector1(vector<double> sourceSymbols, double eneryOfSymbols,
103 const double& M)
104 {
105     vector<double> y1;
106     double theta;

```

```
107     double angDiv = (2*pi)/M;
108
109    for(int i=0; i<sourceSymbols.size(); i++){
110        theta = sourceSymbols[i]*angDiv;
111
112        y1.insert(y1.end(), sqrt(eneryOfSymbols)*cos(theta));
113    }
114
115    return y1;
116}
117
118
119 vector<double> SignalVectors2(vector<double> sourceSymbols, double eneryOfSymbols,
120 const double& M)
121{
122    vector<double> y2;
123    double theta;
124    double angDiv = (2*pi)/M;
125
126    for(int i=0; i<sourceSymbols.size(); i++){
127        theta = sourceSymbols[i]*angDiv;
128
129        y2.insert(y2.end(), sqrt(eneryOfSymbols)*sin(theta));
130    }
131
132
133    return y2;
134}
135
136
137
138 // Function for generating random noise based on gaussian distribution N(mean, variance).
139 // Input mean and standard deviation.
140 // Output is the vector that contain gaussian noise as an element.
141 vector<double> GnoiseVector(double mean, double stddev)
142{
143    vector<double> noise;
144
145    // construct a trivial random generator engine from a time-based seed:
146    unsigned seed = std::chrono::system_clock::now().time_since_epoch().count();
147    std::default_random_engine generator (seed);
148
149    std::normal_distribution<double> dist (mean, stddev);
150
151    // Add Gaussian noise
152    for (int i =0; i<=one_million; i++) {
153        noise.insert(noise.end(), dist(generator));
154    }
155
156    return noise;
157}
158
159
160
161 // Function for model the channel
162 // Input sourcesymbols and AWGN
```

```
163 // Output is a vector that represent the addition of two vectors
164 vector<double> channelModel(vector<double> sourceSymbols, vector<double> AWGnoise)
165 {
166     vector<double> received;
167
168     for(int i=0; i<sourceSymbols.size(); i++){
169         received.insert(received.end(), sourceSymbols[i]+AWGnoise[i]);
170     }
171
172     return received;
173 }
174
175
176 double arcTan(double yvalue, double xvalue)
177 {
178     double theta;
179
180     if(yvalue==0 && xvalue ==0){
181         theta =0;
182     }else{
183         theta = atan2(yvalue, xvalue)*(180/pi);
184     }
185
186     if(yvalue <0){
187         theta = theta +360;
188     }
189
190     theta = theta*(pi/180);
191     return theta;
192 }
193
194
195
196 // Function for take decision after receiving the vector
197 // Input is the 2D receiver vector
198 // Output is the decision {0, 1, 2, ..., M-1}
199 vector <double> decisionBlock(vector<double> receive1, vector<double> receive2,
200 double& M)
201 {
202     vector<double> decision;
203
204     double theta;
205     double upperLimit;
206     double lowerLimit;
207     double increment;
208
209     increment = 360/M;
210     lowerLimit = 180/M;
211
212
213     for( int index =0; index < receive1.size(); index++){
214         theta = arcTan(receive2[index], receive1[index]);
215
216         theta = theta*(180/pi);
217
218         increment = 360/M;
```

```

219     lowerLimit = 180/M;
220     upperLimit = 180/M;
221
222
223     double j = 1;
224     while(j<=M){
225         if(j==1 && (theta<=lowerLimit || theta-360>-lowerLimit)){
226             decision.insert(decision.end(), j-1);
227             j = j+1;
228             upperLimit = lowerLimit + increment;
229             continue;
230         }
231         if(theta<= upperLimit && theta> lowerLimit){
232             decision.insert(decision.end(), j-1);
233         }
234         j = j+1;
235         lowerLimit = upperLimit;
236         upperLimit = lowerLimit+increment;
237
238
239     }
240
241 }
242 cout << endl;
243
244 return decision;
245 }
246
247
248
249 // Function for counting errors in the symbols
250 // Input are the source symbols and decided bits
251 // Output is the error count
252 int errorCount(vector <double> sourceSymbols, vector<double> decisionSymbols)
253 {
254     int count =0;
255
256     for(int i=0; i<sourceSymbols.size(); i++){
257         if(sourceSymbols[i] != decisionSymbols[i]){
258             count++;
259         }
260     }
261
262     return count;
263 }
264
265
266
267
268 // Function to store the data in the file (.dat)
269 // Input is the SNR per bit in dB and calculated probability of error
270 // Output is the nothing but in processing it is creating a file and writing data into it.
271 void datafile(vector<double> SNR, vector<double> Prob_error)
272 {
273     ofstream outfile;
274

```

```

275     outfile.open("MPSK(m=8)Data.dat");
276
277     if(!outfile.is_open()){
278         cout<<"File opening error !!!"<<endl;
279         return;
280     }
281
282     for(int i =0; i<SNR.size(); i++){
283         outfile<< SNR[i] << " " <<"\t" << " " << Prob_error[i]<< endl;
284     }
285
286     outfile.close();
287 }
288
289
290
291
292 // Function to compute Q function value.
293 // Input is the x.
294 // Output is Q function output.
295 double Qfunc(double x)
296 {
297     double Qvalue = erfc(x/sqrt(2))/2;
298     return Qvalue;
299 }
300
301
302
303 vector<double> Qfunction(vector <double> SNR_dB, const double& M)
304 {
305     vector <double> Qvalue;
306     double po, normalValue, extra;
307     for (int k =0; k<SNR_dB.size(); k++) {
308
309         normalValue = pow(10, (SNR_dB[k]/10));
310         extra = sin(pi/M);
311         po = (2*Qfunc(sqrt(2*normalValue)*extra)); // Defination of Pe from calculations.
312         Qvalue.insert(Qvalue.end(), po);
313     }
314
315     return Qvalue;
316 }
317
318
319 void qvalueInFile(vector <double> SNR, vector <double> Qvalue)
320 {
321     ofstream outfile;
322
323     outfile.open("MPSK(m=8)_Qvalue.dat");
324
325     if(!outfile.is_open()){
326         cout<<"File opening error !!!"<<endl;
327         return;
328     }
329
330     for(int i =0; i<SNR.size(); i++) {

```

```
331     outfile<< SNR[i] << " " <<"\t" << " " << Qvalue[i]<< endl;
332 }
333
334 outfile.close();
335 }
336
337
338
339 int main()
340 {
341     // source defination
342     vector<double> sourceBits;
343
344     // Symbols formation
345     vector<double> Symbols;
346
347     // Base for conversion
348     int base =3; //3, 4, 5, 6 ...
349     double M = pow(2, base);
350
351     // Transmiited vectors
352     vector<double> transl;
353     vector<double> trans2;
354
355     int SizeOfTransmission = transl.size();
356
357     // Noise vector
358     vector<double> gnoise1;
359     vector<double> gnoise2;
360
361     // Receive bits
362     vector<double> receivedBits1;
363     vector<double> receivedBits2;
364
365
366     // Decision block
367     vector<double> decodedBits;
368
369     vector<double> EnergyVector;
370
371     vector<double> p_error;
372
373     double errors=0;
374     double pe;
375     double N_o = 8;
376     double stddev = sqrt(N_o/2);
377
378
379     // SNR in dB
380     vector<double> SNR_dB;
381     for(float i =0; i<=20; i=i+0.125)
382     {
383         SNR_dB.insert(SNR_dB.end(), i);
384     }
385
386     double normalValue;
```

```
387
388     for(int i =0; i<SNR_dB.size(); i++){
389         normalValue = pow(10, (SNR_dB[i]/10));
390         EnergyVector.insert(EnergyVector.end(), normalValue*N_o);
391     }
392
393
394     for(int step =0; step <SNR_dB.size(); step++){
395
396         sourceBits = Source();
397
398         Symbols = binaryToDecimalConversion(sourceBits, base);
399         // cout<< "Done symbols"<<endl;
400
401         transl = SignalVectors1(Symbols, EnergyVector[step], M);
402         trans2 = SignalVectors2(Symbols, EnergyVector[step], M);
403
404         gnoise1 = GnoiseVector(0.0, stddev);
405         gnoise2 = GnoiseVector(0.0, stddev);
406
407         receivedBits1 = channelModel(transl, gnoise1);
408         receivedBits2 = channelModel(trans2, gnoise2);
409
410
411         decodedBits = decisionBlock(receivedBits1, receivedBits2, M);
412
413
414         errors = errorCount(Symbols, decodedBits);
415         double sizeOfsymbols = Symbols.size();
416         pe = errors/ sizeOfsymbols;
417
418         p_error.insert(p_error.end(), pe);
419
420         cout<< errors<< "\n";
421         cout<< pe << "\n";
422
423         cout<< "\n";
424     }
425
426
427
428     datafile(SNR_dB,p_error);
429
430     vector<double> qvalue = Qfunction(SNR_dB, M);
431     qvalueInFile(SNR_dB, qvalue);
432
433     return 0;
434
435
436 }
```

## 4.2 Results

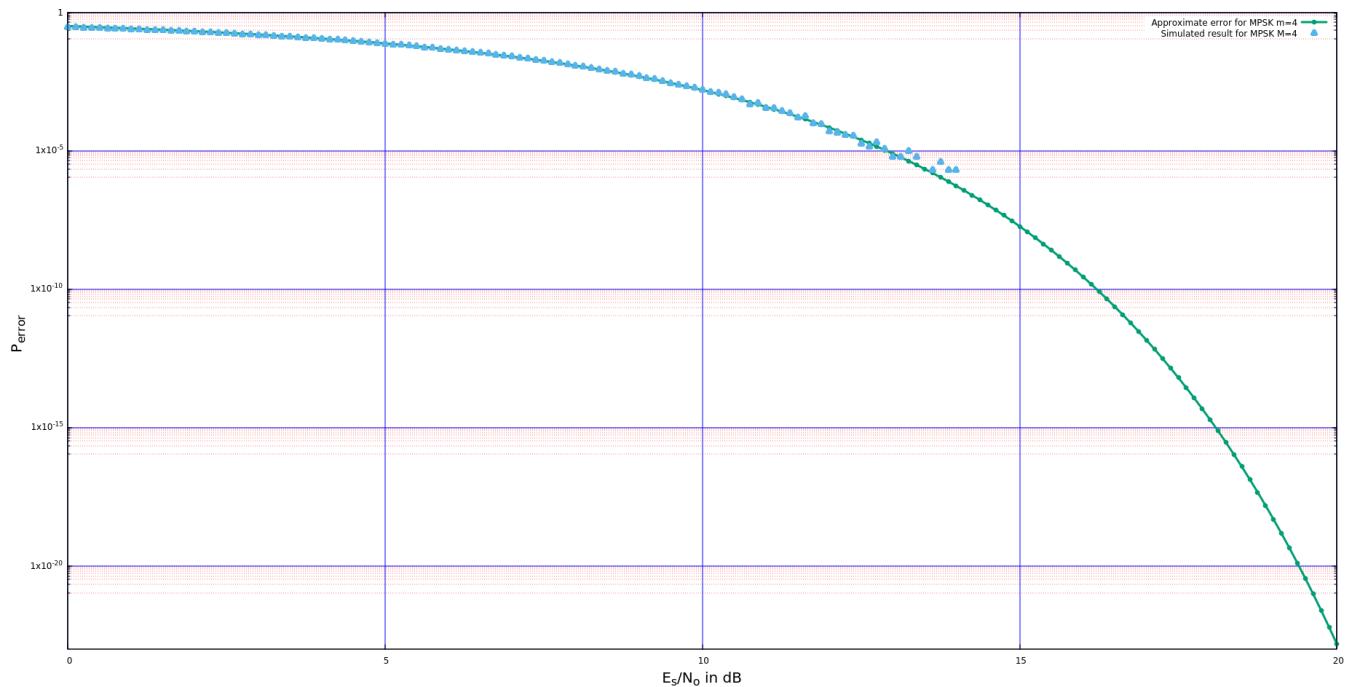


Figure 10:  $M = 4$ , Result of simulation (4-PSK)

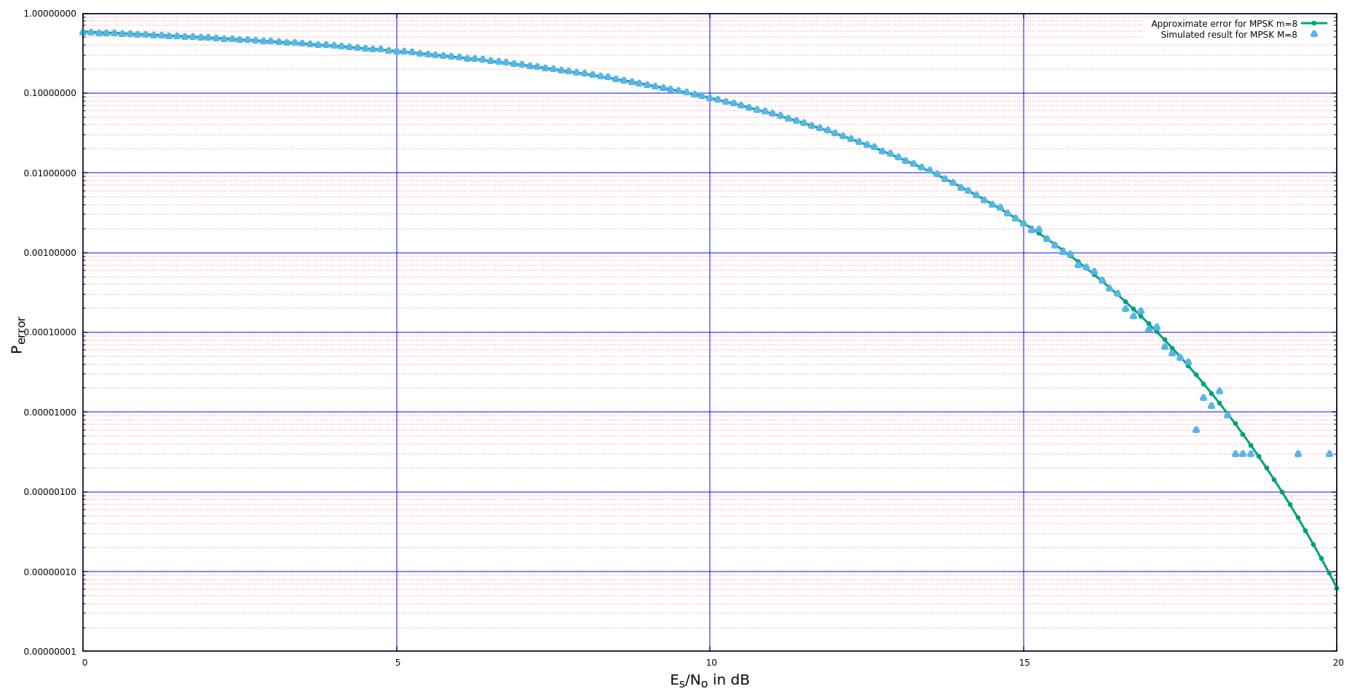
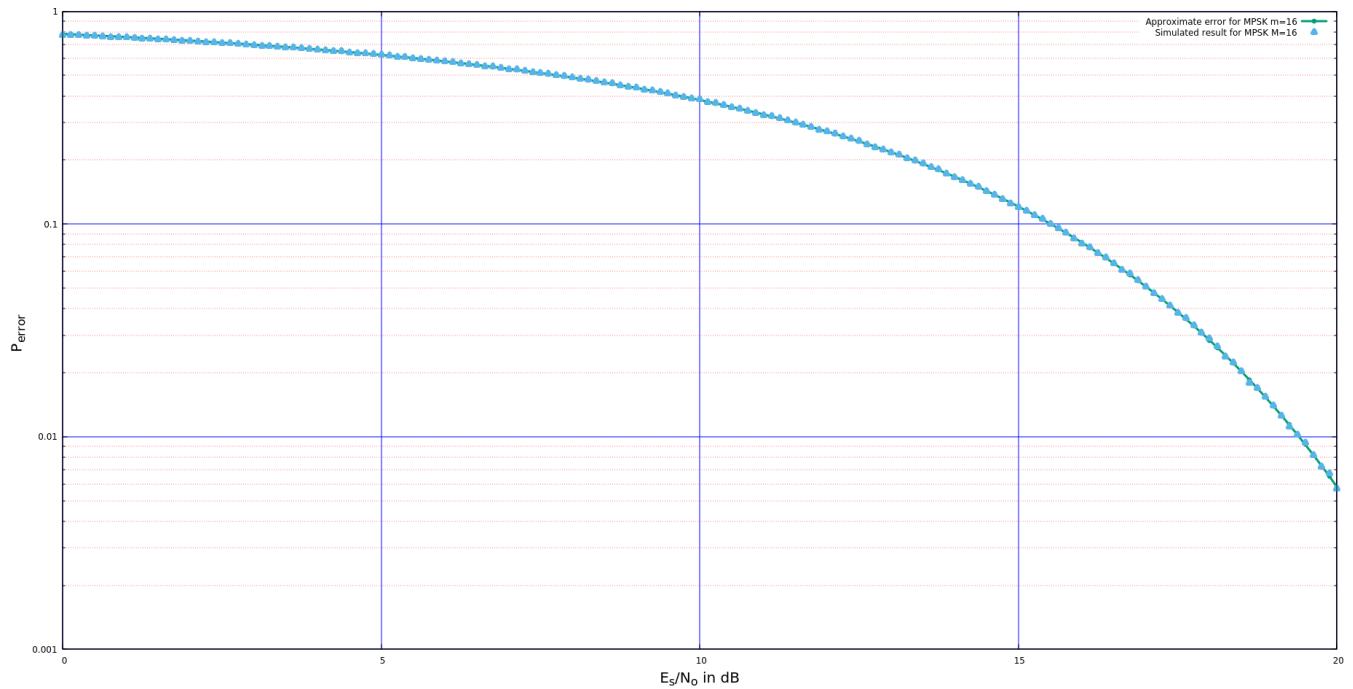
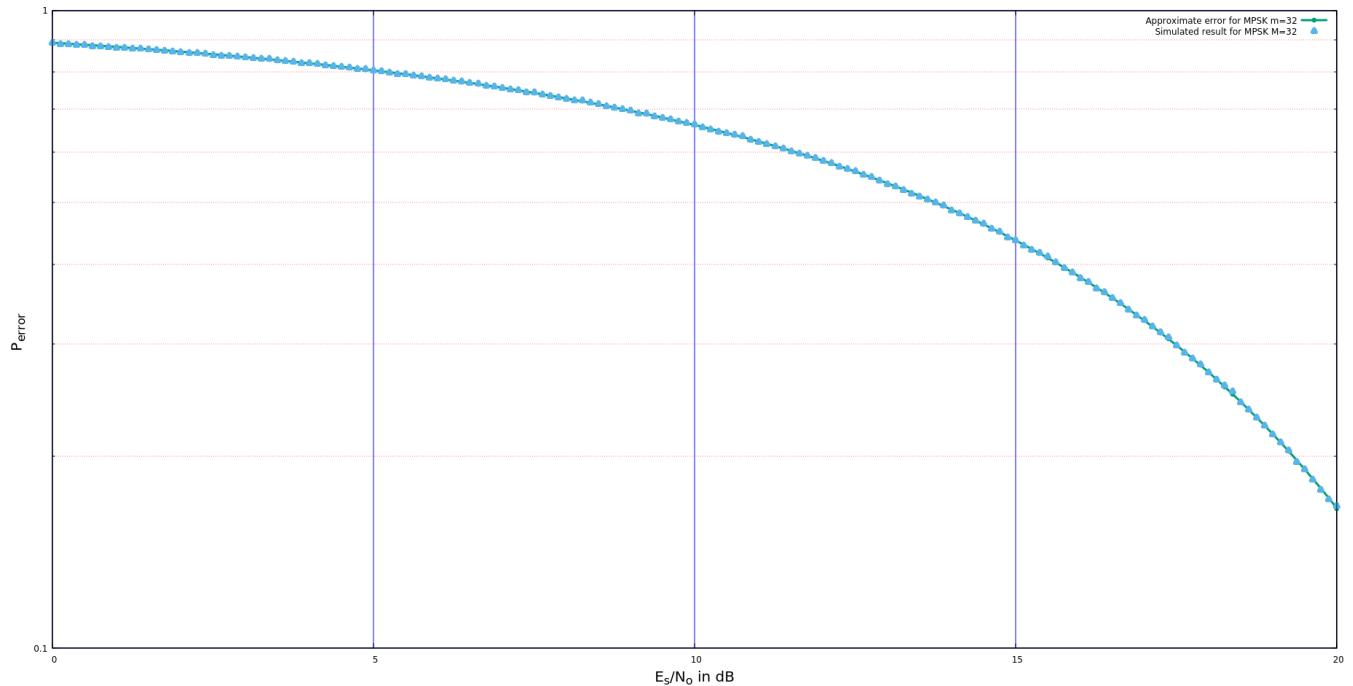


Figure 11:  $M = 8$ , Result of simulation (8-PSK)

Figure 12:  $M = 16$ , Result of simulation (16-PSK)Figure 13:  $M = 32$ , Result of simulation (32-PSK)

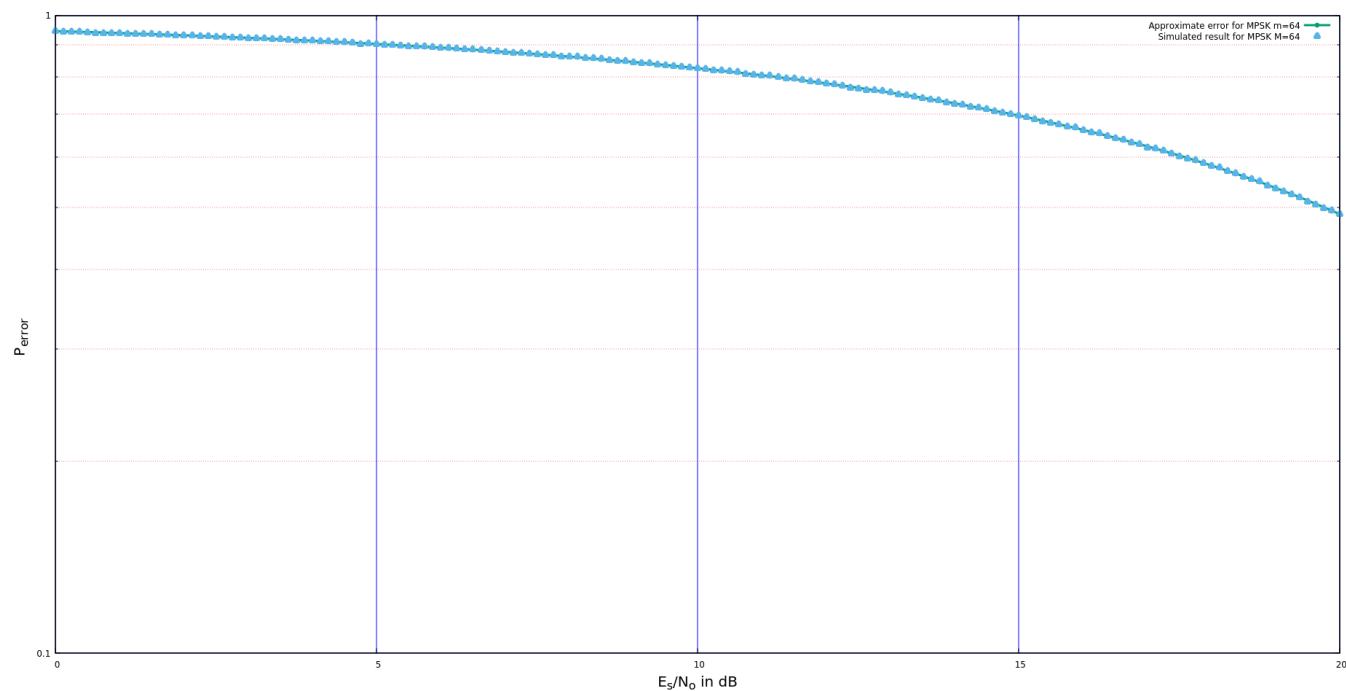
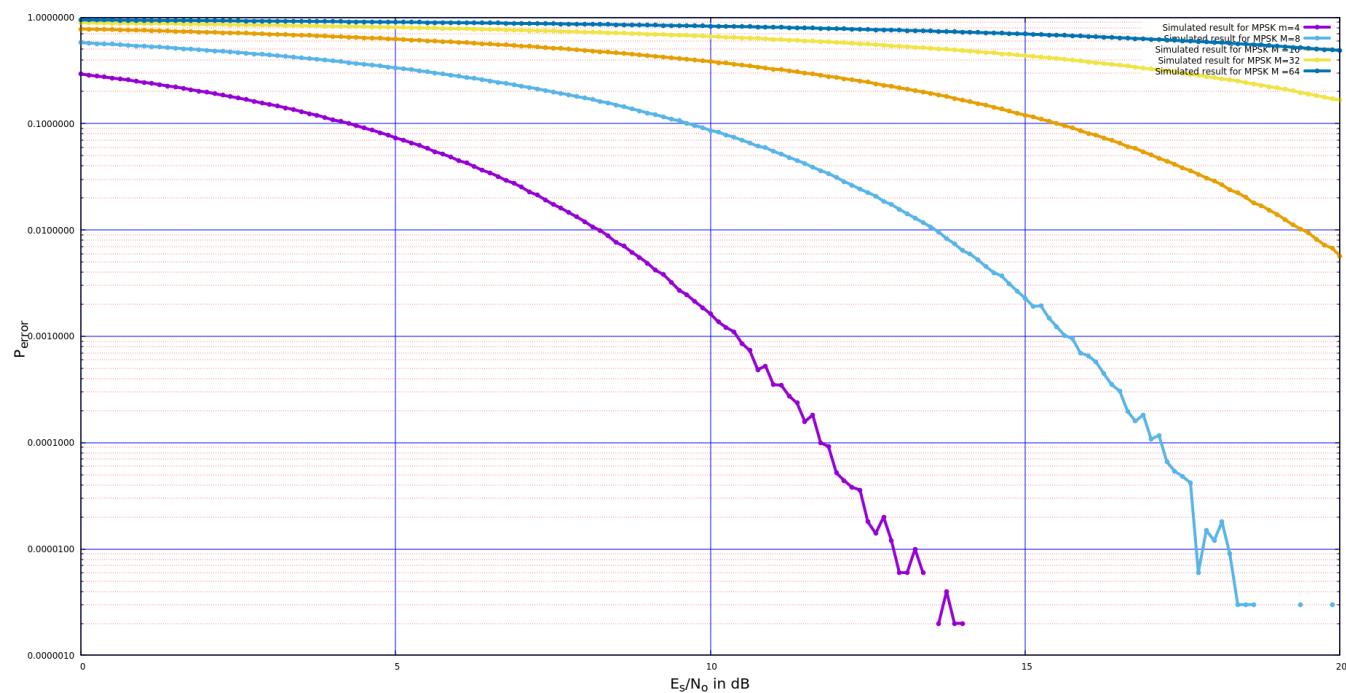
Figure 14:  $M = 64$ , Result of simulation (64-PSK)

Figure 15: Comparison of all results(4-PSK, 8-PSK, 16-PSK, 32-PSK and 64-PSK)

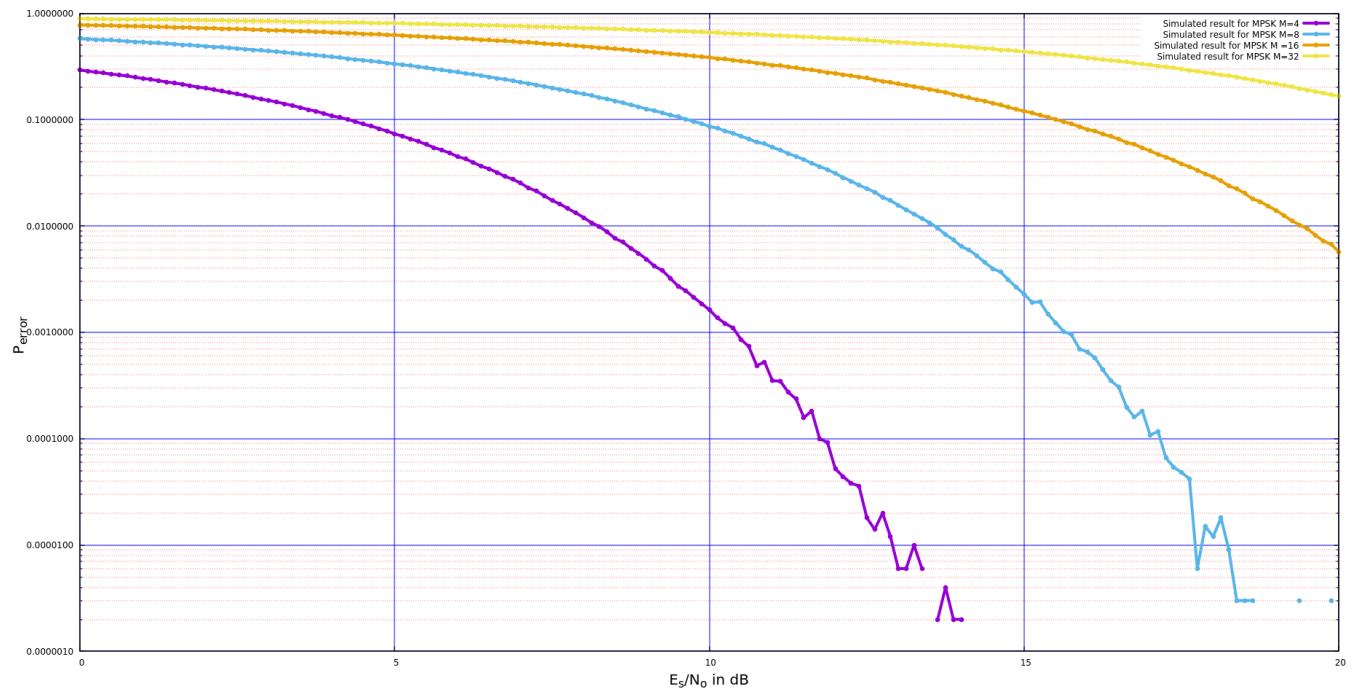


Figure 16: 4-PSK, 8-PSK, 16-PSK and 32-PSK

## 5 Inferences

1. As  $\frac{E_s}{N_o}$  increase, probability of error decreases. But the slope of decrease is different for different  $M$  values in MPSK.
2. As  $M$  value increases, number of phase requires to modulate message, increases. Therefore, difference between two consecutive phase decreases, that makes difficult to distinguish two consecutive symbols by receiver. Hence, more probability of error.
3. Since  $\log_2 M$  ( $M > 2$ ) bits can be transmit through this scheme. That implies data rate is higher as compare to BPSK and QPSK. But it comes with high probability of error.
4. Since, simulated results are exactly same as calculated results. That implies the approximations in the derivation of probability of error in any MPSK are valid.