Assignment 3 Solutions Addendum

CSC 244/444, Fall 2013

The part (b) referenced in my solution to 6(c):

> Propositional resolution is refutation-complete. We prove that this implies as well that propositional (Boolean) satisfiability is decidable via resolution.
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> Let's consider the simple case of deciding the satisfiability of a sentence S with only two variables, x and y. S is satisfiable iff there exists an assignment of truth values to x and y such that S is true. Then S is satisfiable iff from the premises we can prove S given a premise set asserting the truth or falsity of the variables: {x, y}, {¬x, y}, {x, ¬y}, {¬x, ¬y}. To prove satisfiability of S by a refutation, we negate it and try to derive a contradiction from that negation and the premise set, for each premise set.
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> We need to convert each of the negated wffs to clause form. Then we apply resolution as usual. This is clearly not an efficient process in the sense that the denial of the condition for satisfiability contains a number of variables of S (and thus, in the worst case, in the length of S). However, efficiency is hardly to be expected since SAT is known to be NP-complete.
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> From the outline above for the case of sentences with only two variables, you can easily make the generalization to n variables.
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> To have an actual decision procedure, which is guaranteed to terminate, we run the satisfiability proof attempt above in pseudoparallel fashion with a refutation attempt of S. Then either the satisfiability or the unsatisfiability proof attempt will eventualy succeed if we use, say, a breadth-first (or any other completeness-preserving) proof strategy.