CS 444: Assignment 2

Due on Tue, Oct. 15, 2013

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Problem 1

Validity

(a) $\models \phi \Rightarrow (\psi \Rightarrow \phi)$

For any model $M = (D, I)$
$\phi \Rightarrow (\psi \Rightarrow \phi)$ iff
$\neg\phi \vee (\psi \Rightarrow \phi)$ iff
$\neg\phi \vee \neg\psi \vee \phi$
Because $\neg\ \phi \vee\ \phi$ is always true, therefore the original formula is always true with regard to all models.

(b) $\models (P(A) \wedge A = B) \Rightarrow P(B)$

For any model $M = (D, I)$
$(P(A) \wedge A = B) \Rightarrow P(B)$ iff
$\neg(P(A) \wedge A = B) \vee P(B)$ iff
$\neg P(A) \vee \neg(A = B) \vee P(B)$
There are two cases:
First, when A=B is true, $\neg P(A) \vee \neg(A = B) \vee P(B)$ is equivalent to $\neg P(A) \vee P(A)$, which is always true.
Second, when A=B is false, $\neg(A = B)$ is true, then the original formula is true with reagard to any model.

(c) $\models (\exists x A = x)$

$\models (\exists x A = x)$ iff for all models $M = (D, I) \models_M A = x$	Truth in all models
iff for all v.a. U and all models M	
$\models_M (\exists x A = x)[U]$	Satisfaction condition
iff for some $d \in D$ $T_{IU_{x:d}} \in (A = x)^I = \{A\}$	Term assignment
iff for some $d \in \{A\} \cup \{D - \{A\}\}, d \in \{A\}$	Definition of term assignment
TRUE	

(d) $\models (\forall x(\forall y((P(x) \wedge x = y) \Rightarrow P(y))))$

Let $M = (D, I) be an arbitrary model$, let $d1, d2 \in D \wedge \neg d1 = d2$ be two arbitrary varialbles.
The original claim is true iff

iff $\models_M (\forall x(\forall y((P(x) \wedge x = y) \Rightarrow P(y))))[U]$	Definition of truth in a model
iff $\models_M (\forall x(\forall y((P(x) \wedge x = y) \Rightarrow P(y))))[U_{x:d_1\ \ y:d_2}]$	Definition of satisfaction condition for $\forall$
iff for all $< d_1, d_2 > \in D^2$ $T_{IU_{x:d1\ \ y:d_2}} \in (P(X) \wedge x = y \Rightarrow P(y))^I$	Term assignment
iff for all $< d_1, d_2 > \in D^2$, $< d_1, d_2 > \in D^2$	Definition of variable assignment
TRUE	

Problem 2

(a) Prove that $P(A), A = B \vDash P(B)$

For all model $M = (D, I)$, any v.a. U,
$\vDash_M P(A), A = B[U]$
$T_{IU}(A) \in P^I$
$T_{IU}(A) = T_{IU}(B)$
Therefore,
$T_{IU}(B) = T_{IU}(A) \in P^I$
$T_{IU}(B) \in P^I$

(b) Prove that $\phi \vDash (\psi \Rightarrow \phi)$

For all M, all v.a. U
$\vDash_M \phi[U]$
$\vDash_M \psi \Rightarrow \phi[U]$
iff $\vDash_M \neg\psi \Rightarrow \phi[U]$
iff $\neg \vDash_M \psi[U] \vee \vDash_M \phi[U]$ which is always true.
Therefore $\phi \vDash (\phi \Rightarrow \psi)$

(c) Prove that $\frac{\phi}{\phi \vee \psi}$ is sound

For all M, all v.a. U
$\vDash_M \phi[U]$
This is equivalent to
$\vDash_M \phi[U] \vee \vDash_M \psi[U]$
All models of ϕ are models of $\phi \vee \psi$.
Therefore this is sound.

(d) Prove that $\frac{\phi \Rightarrow \psi, \psi \Rightarrow \chi}{\phi \vee \psi}$ is sound

For all M, all v.a. U
$\vDash_M \Rightarrow \psi[U]$ iff $\vDash_M \neg\phi[U] \vee \vDash_M \psi[U]$
$\vDash_M \psi \Rightarrow \chi[U]$ iff $\vDash_M \neg\psi[U] \vee \vDash_M \chi[U]$
$\vDash_M \phi \Rightarrow \chi[U]$ iff $\vDash_M \neg\phi[U] \vee \vDash_M \chi[U]$
Therefore
$\vDash_M \phi \Rightarrow \psi, \psi \Rightarrow \chi[U]$
iff $\vDash_M (\neg\phi \vee \psi \vee \neg\phi \vee \chi)[U]$
iff $\vDash_M (\neg\phi \vee \chi)[U]$
i.e. $\vDash_M \phi \Rightarrow \chi\Box$

(e) In what sense is soundness a "good " property of a proof system?

Because "sound" inference gurantees correct conclusions as long as premises are true.

Problem 3

Given that:
1. $(\forall x M(x) \Rightarrow (\exists y A(y) \land H(x, y)))$
2. $(\forall x M(x) \Rightarrow L(x))$
3. $(\exists x M(x))$

Goal:
$L(S) \land A(C) \land H(S, C)$

1. $(\forall x M(x) \Rightarrow (\exists y A(y) \land H(x, y)))$	Δ
2. $(\forall x M(x) \Rightarrow L(x))$	Δ
3. $(\exists x M(x))$	Δ
4. $M(S) \Rightarrow (\exists y A(y) \land H(S, y))$	UI, 1
5. $M(S) \Rightarrow L(S)$	UI, 2
6. $M(S)$	EI, 3
7. $M(S) \Rightarrow A(C) \land H(S, C)$	EI, 4
8. $L(S)$	MP, 5
9. $A(C) \land H(S, C)$	MP, 4, 6
10.$L(S) \land A(C) \land H(S, C)$	AI, 8 , 9

Problem 4

Given that:
1. $(\forall x M(x) \Rightarrow (\exists y A(y) \land H(x, y)))$
2. $(\forall x M(x) \Rightarrow L(x))$
3. $(\forall x L(x) \Rightarrow \neg(\exists y A(y) \land H(x, y)))$

Goal:
$\neg(\exists x M(x))$

1. *Show $\neg(\exists x M(x))$	
2. $(\exists x M x)$	Assume
3. $M(z)$	2,EI
4. $(\forall x M(x) \Rightarrow (\exists y A(y) \land H(x, y)))$	Prem
5. $(\forall x M(x) \Rightarrow L(x))$	Prem
6. $(\forall x L(x) \Rightarrow \neg(\exists y A(y) \land H(x, y)))$	Prem
7. $M(z) \Rightarrow A(c) \land H(z, c)$	4, UI, EI
8. $M(z) \Rightarrow L(k)$	5, UI, EI
9. $L(K) \Rightarrow \neg(A(c) \land H(z, c))$	6. UI, EI
11. *Show $\neg A(c) \land H(z, c)$	
12. $A(c) \land H(z, c)$	3, 7, MP
13. $\square$	

Problem 5

(a)

(b)

(c)

Problem 6

(a) Find the m.g.u. of $\neg Q(x, z, g(x))$ and $Q(f(A), h(y), g(f(y)))$

1. $x/f(A)$ we get $\neg Q(f(A), z, g(x))$ and $Q(f(A), h(y), g(f(y)))$
2. $z/h(y)$ we get $\neg Q(f(A), h(y), g(x))$ and $Q(f(A), h(y), g(f(y)))$
3. y/A we get $\neg Q(f(A), h(A), g(x))$ and $Q(f(A), h(A), g(f(A)))$

The m.g.u for this problem is

$$((x/f(A)),\ (z/h(y)),\ (\text{y/A}))$$

(b) Find 2 resolvents of $\neg P(x) \vee \neg Q(f(x), y)$, $Q(u, f(v)) \vee Q(f(u), v)$

1. r[1b, 2b]: $\neg P(u),\ Q(f(u), v)$ with m.g.u $((x/u),\ (y/v))$.
2. r[1b, 2a]: $\neg P(x),\ Q(f(u), v)$ with m.g.u $((u/f(x),\ (y/f(v))))$.

(c) Find 2 factors of $Q(f(x), y) \vee Q(y, f(x)) \vee Q(x, f(y))$

1.f[1 a, c]: $Q(f(x), f(x)) \wedge Q(x, f(f(x)))$ with m.g.u (y/f(x))
2.f[1 a, b]: $Q(f(x), f(x)) \wedge Q(f(x), f(x))$ with m.g.u (y/f(x))

Problem 7

(a)

1. $(\forall x \forall y S(x, y) \Rightarrow (\exists z parent(z, x) \wedge parent(z, y)))$
2. $(\forall x \forall y M(x, y) \Rightarrow (\exists z P(z, x) \Rightarrow \neg P(z, y)))$
3. $M(Jocasta, Oedipus)$
4. $S(Jocasta, Oedipus)$ (Reject Goal)

(b)

For 1. $(\forall x \forall y S(x, y) \Rightarrow (\exists z parent(z, x) \wedge parent(z, y)))$
First we eliminate $\Rightarrow$
$\exists x \exists \neg S(x, y) \vee (\forall z parent(z, x) \wedge parent(z, y))$
Then we move $\neg$ inward, which leaves equation the same
Then we skolemize, let $(x/J)(y/O)(z/K)$
$\neg S(J, O) \vee (P(K, J) \wedge P(K, O))$, distribute $\vee$ over $\wedge$ we have
$(\neg S(J, O) \vee P(K, J)) \wedge (\neg S(J, O) \vee P(K, O))$
Similarly for 2.$(\forall x \forall y M(x, y) \Rightarrow (\exists z P(z, x) \Rightarrow \neg P(z, y)))$
we have
$\neg M(J, O) \vee \neg P(K, J) \vee \neg P(K, O)$ For 3.
$M(J, O)$
For 4.
$S(J, O)$

(c)

1. $(\neg S(J, O) \vee P(K, J)) \wedge (\neg S(J, O) \vee P(K, O))$	From (b)
2. $\neg M(J, O) \vee \neg P(K, J) \vee \neg P(K, O)$	From (b)
3. $M(J, O)$	From (b)
4. $S(J, O)$	From (b)
5. $\neg P(K, J) \vee \neg P(K, O)$	r[2a, 3]
6. $P(K, J) \wedge P(K, O)$	r[1a, 4]
7. $\neg P(K, O)$	r[5a, 6a]
8. $P(K, O)$	r[7a 8]
9. $\square$	

(d)

A written solution for this problem is attached at the end of this assignment print-out.