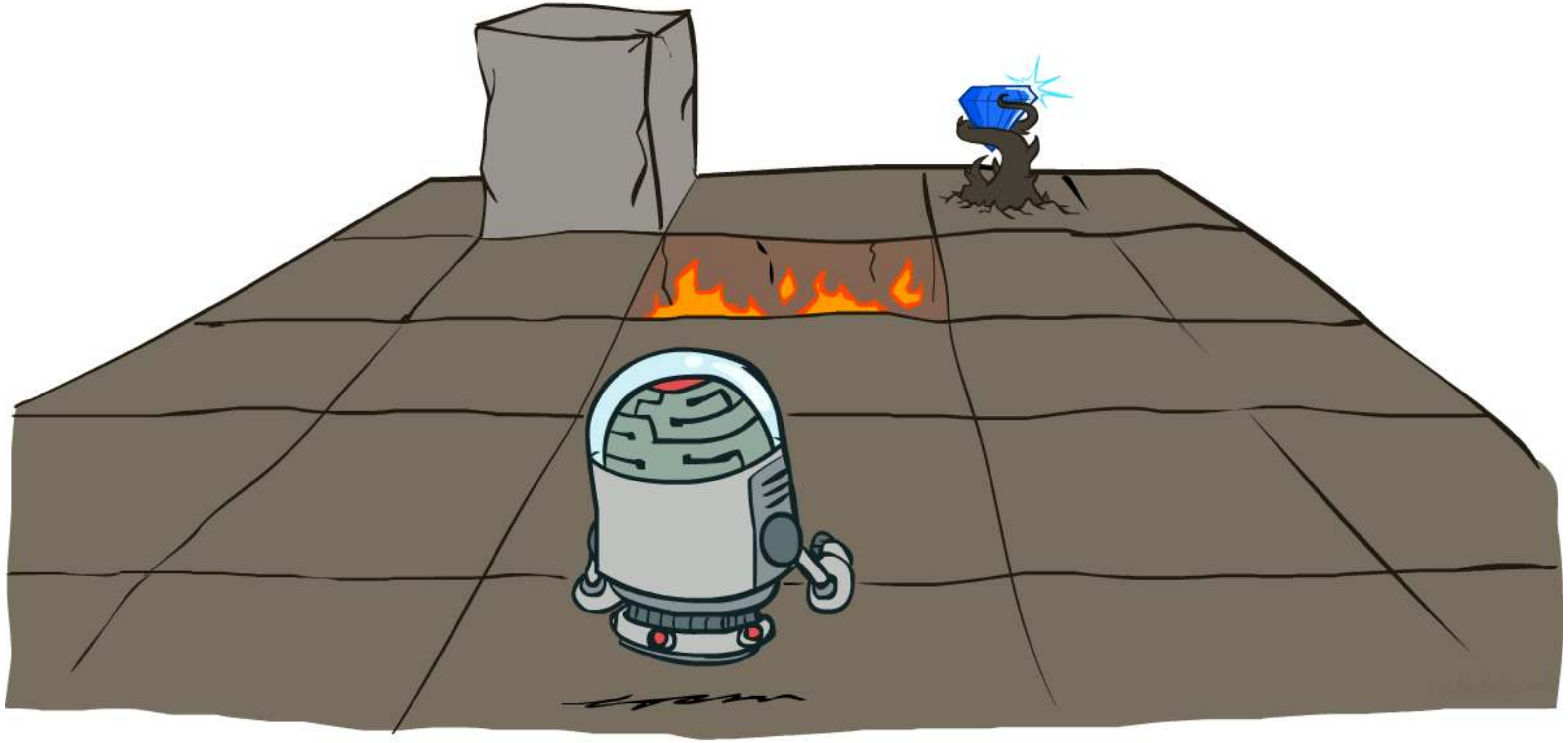
Artificial Intelligence

Markov Decision Processes



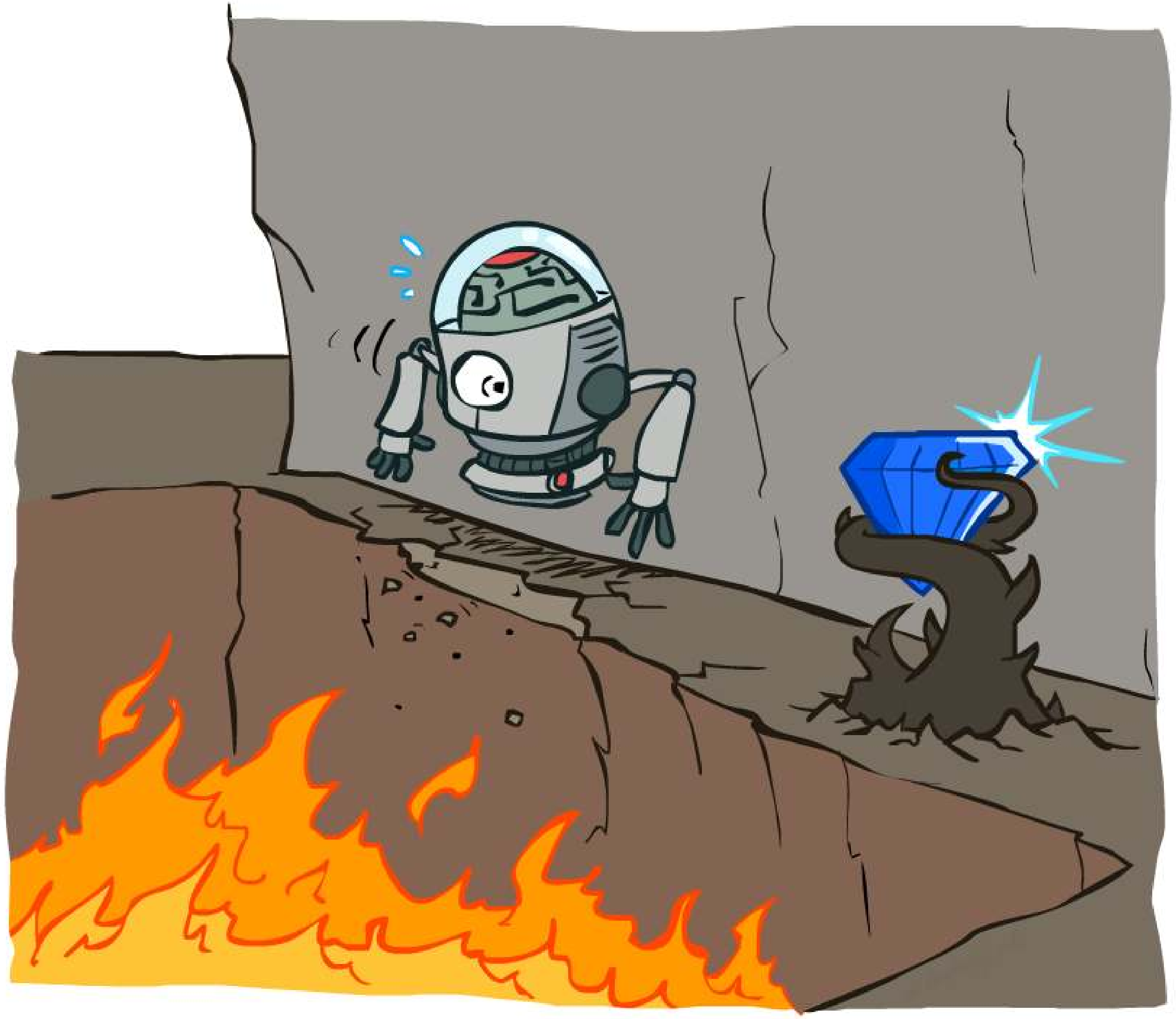
Instructor: Heni Ben Amor

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

What we Learned

- Trees solve discrete search problems
- When ordering is not important we have CSP
- The challenge is computational complexity
- Heuristics can help reduce it
- A* combines efficiency and guarantees
- In adversarial domain use alternation of choices
- Expecti-max allows for uncertain events

Non-Deterministic Search



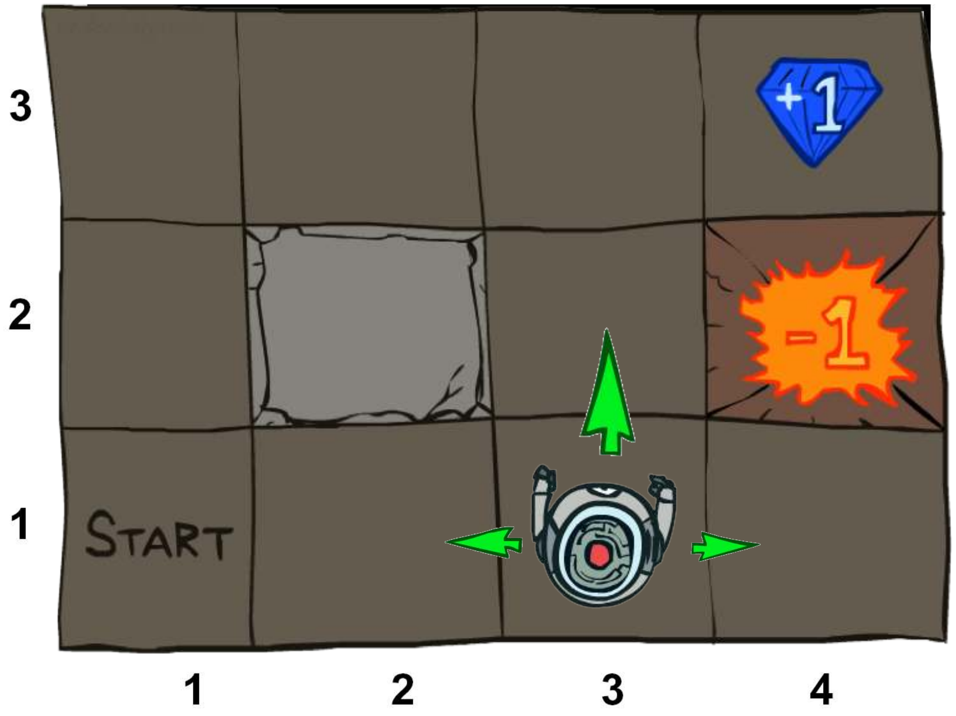
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path

- Noisy movement: actions do not always go as planned
- 80% of the time, the action North takes the agent North
  - (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put

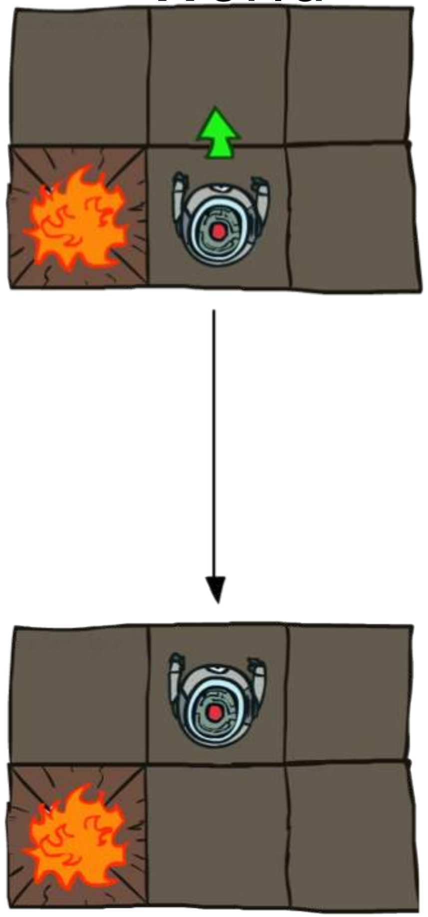
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)

- Goal: maximize sum of rewards

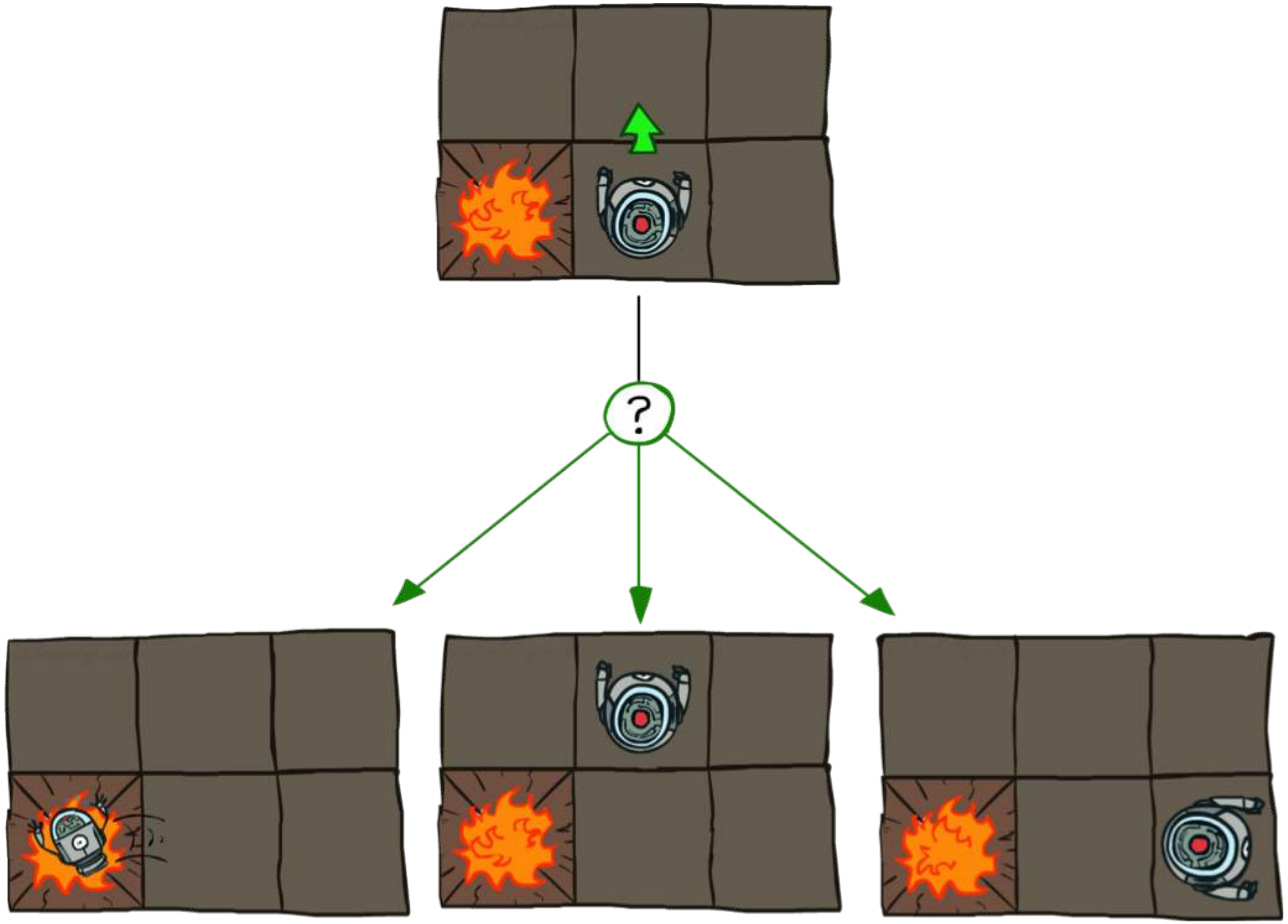


Grid World Actions

Deterministic Grid World

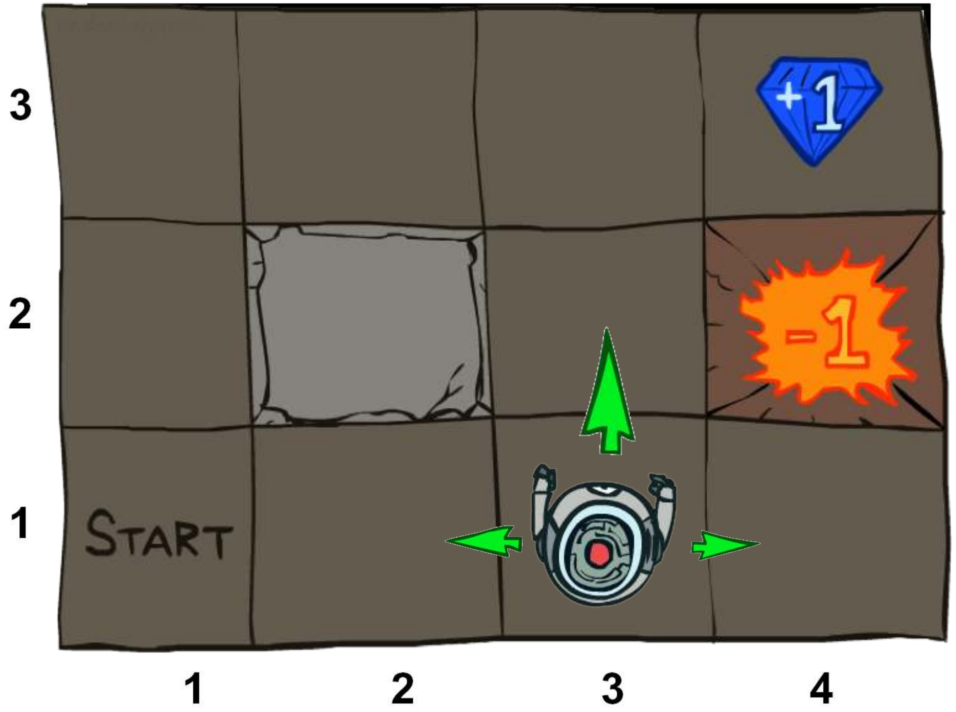


Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
  - A set of states $s \in S$
  - A set of actions $a \in A$
  - A transition function T(s, a, s')
    - Probability that a from s leads to s', i.e., P(s'| s, a)
      Also called the model or the dynamics
  - A reward function R(s, a, s')
    - Or just R(s) or R(s')
  - A start state
  - Maybe a terminal state
- MDPs are non-deterministic search problems
  - One way to solve them is with expectimax search
  - We'll have a new tool soon



What is Markov about MDPs?

- "Markov" generally means that given the present state, the future and the past are independent
- For Markov decision processes, "Markov" means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

=

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

- This is just like search, where the successor function could only depend on the current state (not the history)

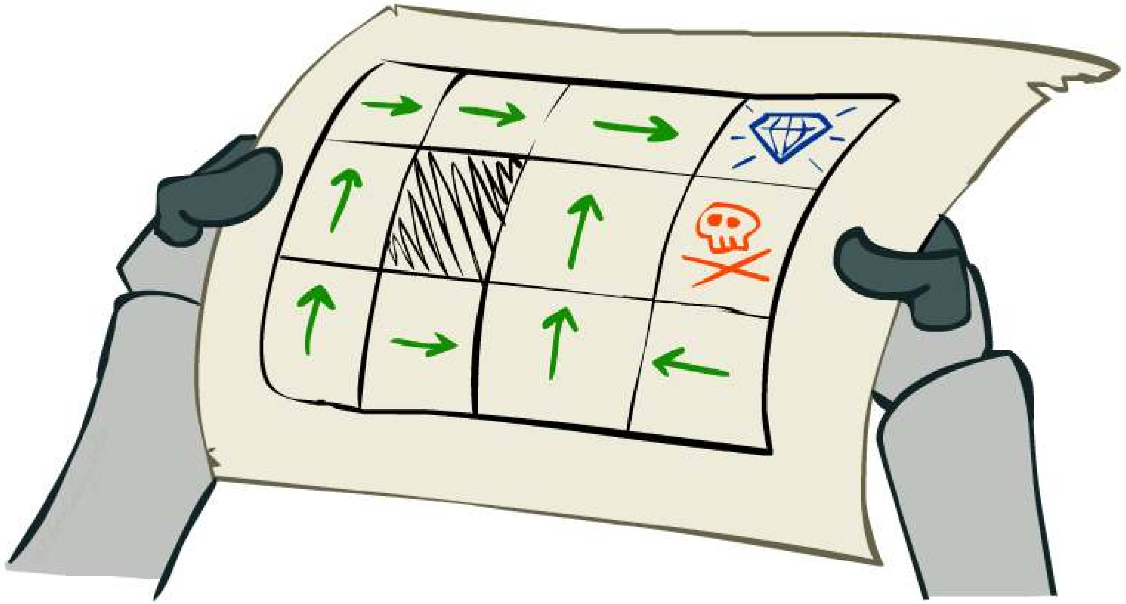
Andrey Markov
(1856-1922)

Policies

- In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal

- For MDPs, we want an optimal policy $\pi^*: S \rightarrow A$
  - A policy π gives an action for each state
  - An optimal policy is one that maximizes expected utility if followed

- Expectimax didn't compute entire policies
  - It computed the action for a single state only
  - MDPs can have infinite length!



Optimal policy when R(s, a, s') = -0.03 for all non-terminals s

Example: Learning to Lift

Example: Learning to Lift

To be presented in 2 weeks at **IROS 2017** in Vancouver.

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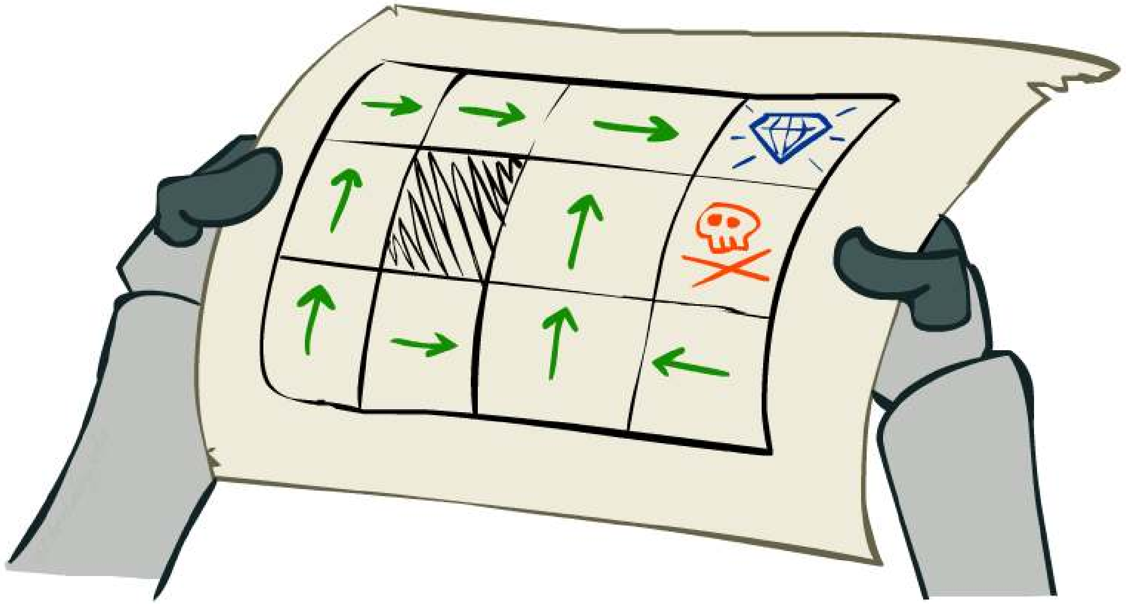
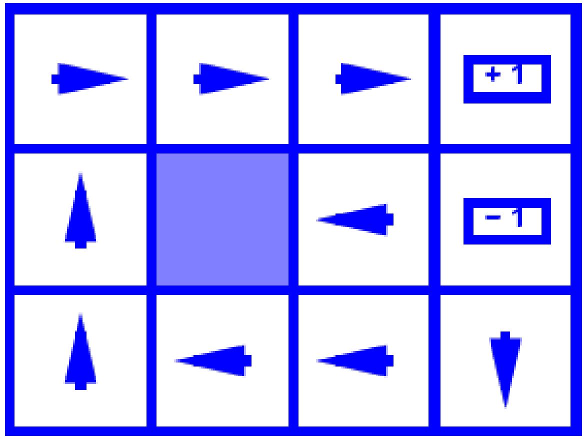
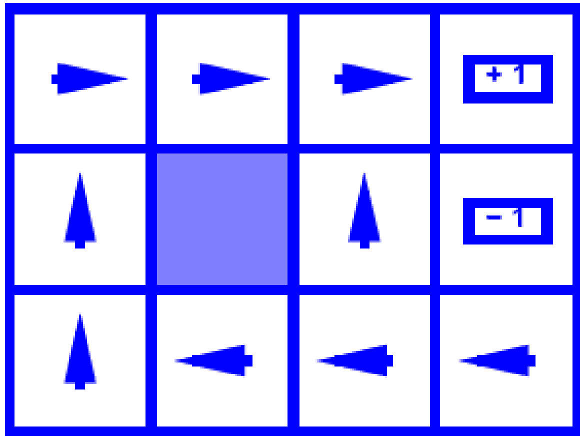


Optimal policy when R(s, a, s') = -0.03 for all non-terminals s

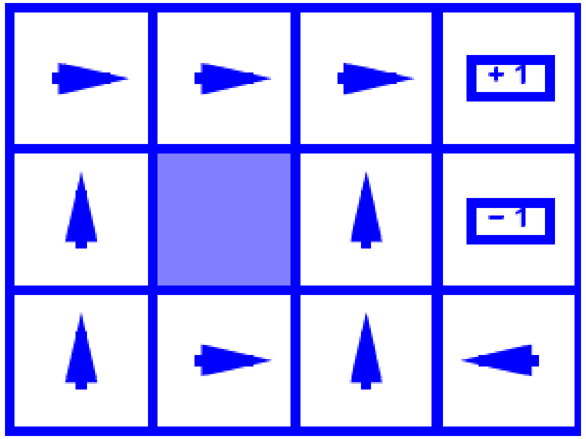
Optimal Policies



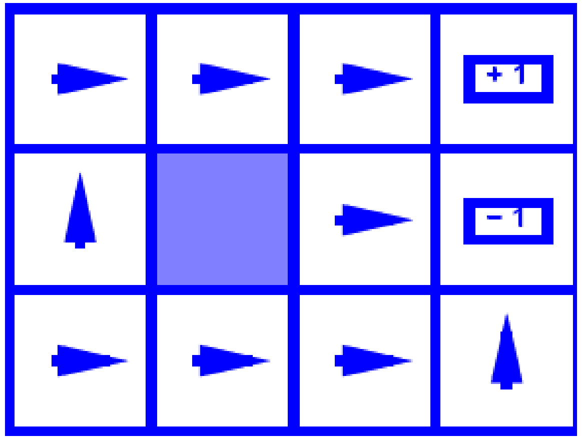
R(s) =
-0.01



R(s) = -0.03



R(s) =
-0.4

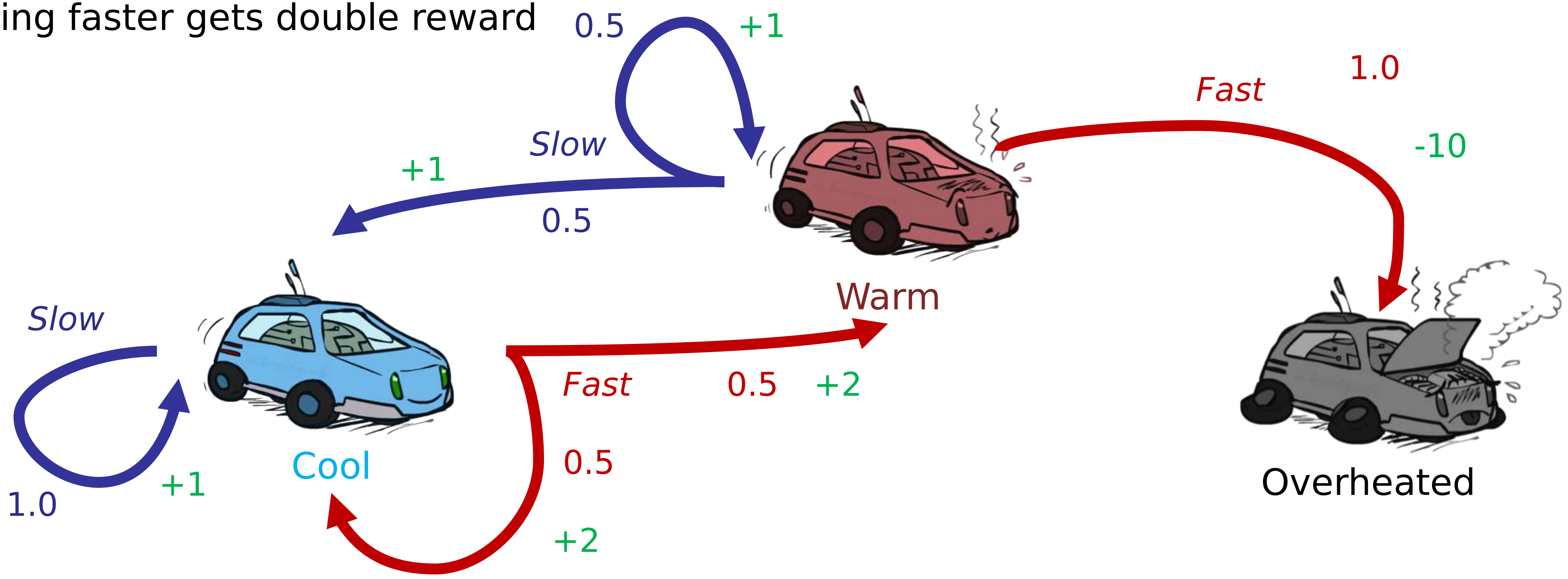


R(s) =
-2.0

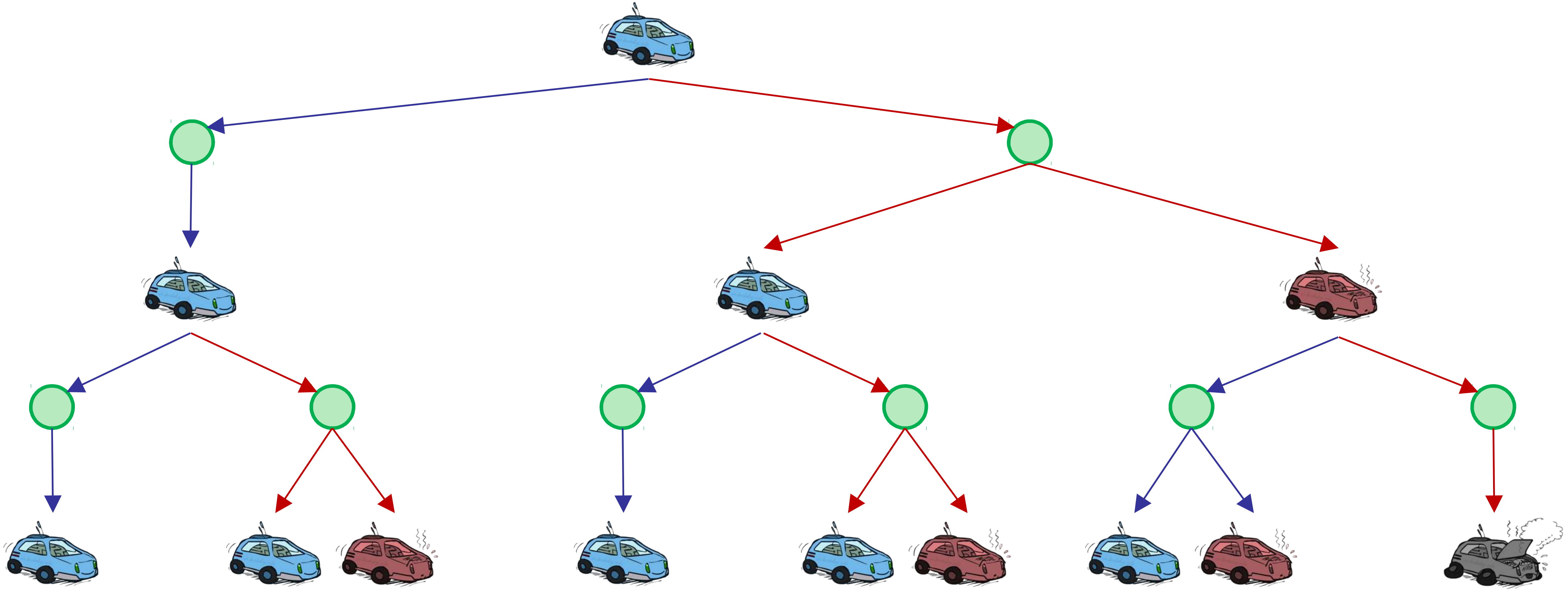
Example: Racing

Example: Racing

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
- Going faster gets double reward

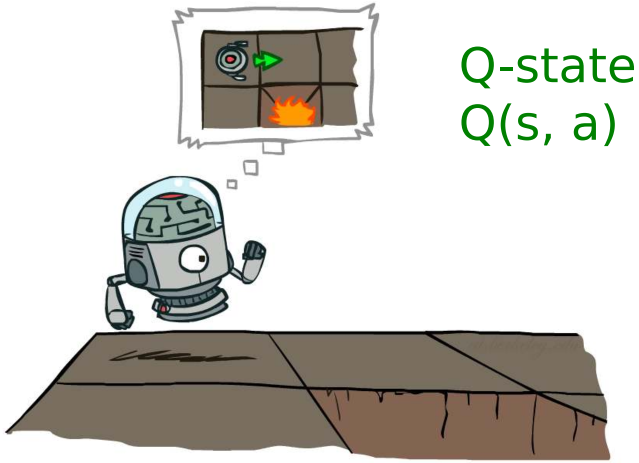


Racing Search Tree

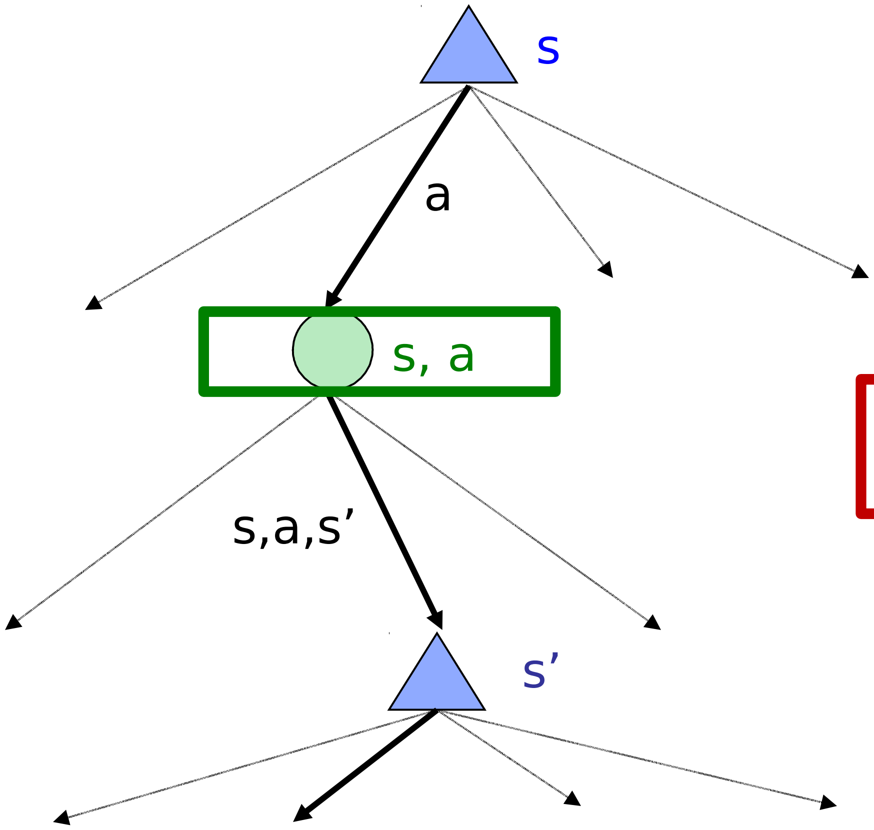


MDP Search Trees

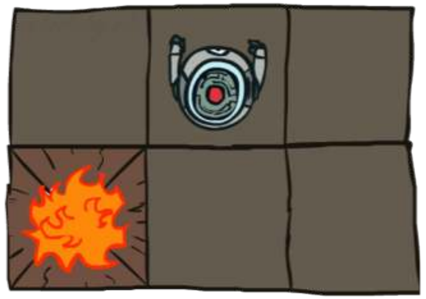
- Each MDP state projects an expectimax-like search tree



Q-state
Q(s, a)

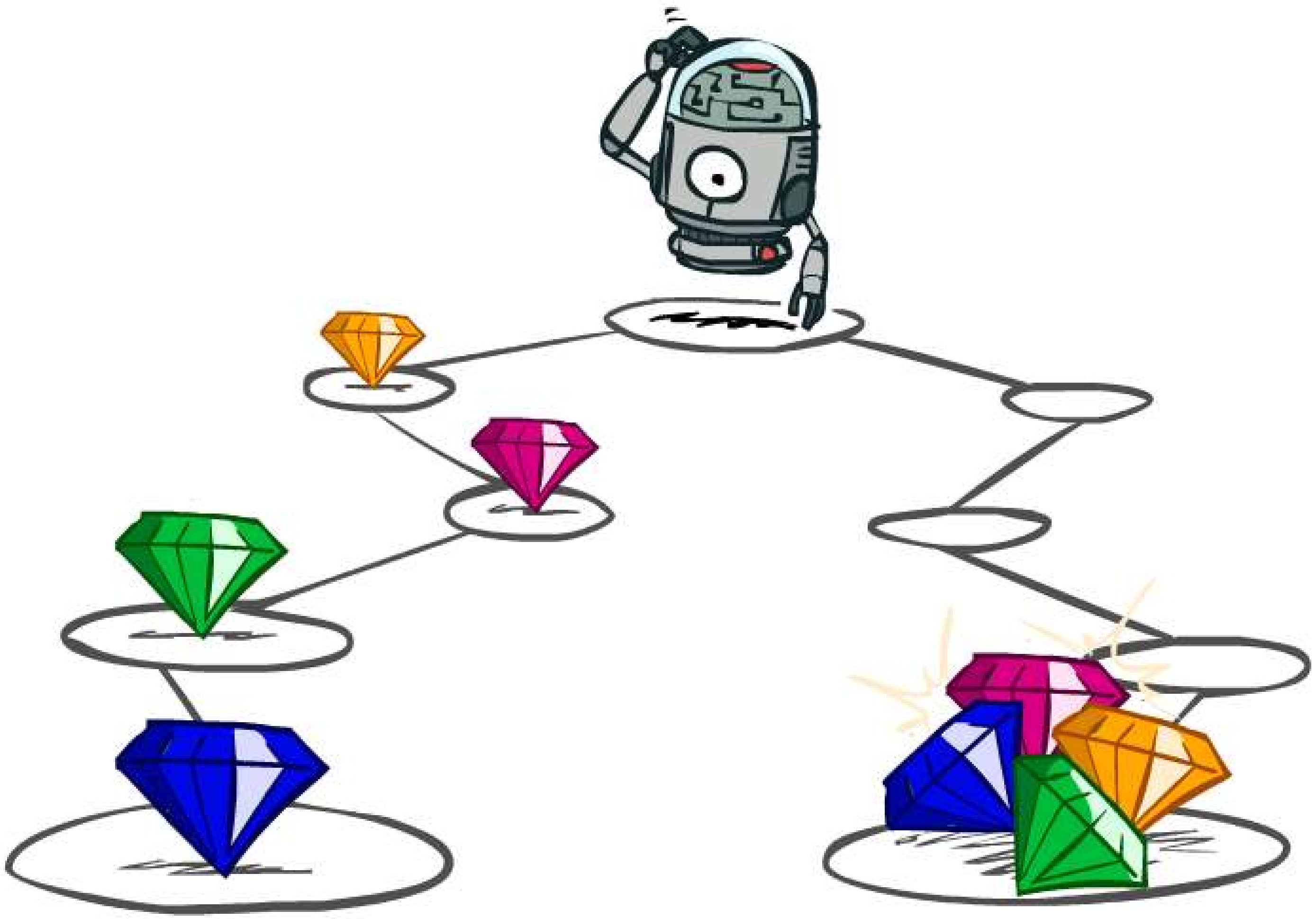


s is a
state



(s,a,s') called a
transition
T(s,a,s') = P(s'|s,a)
R(s,a,s')

Utilities of Sequences

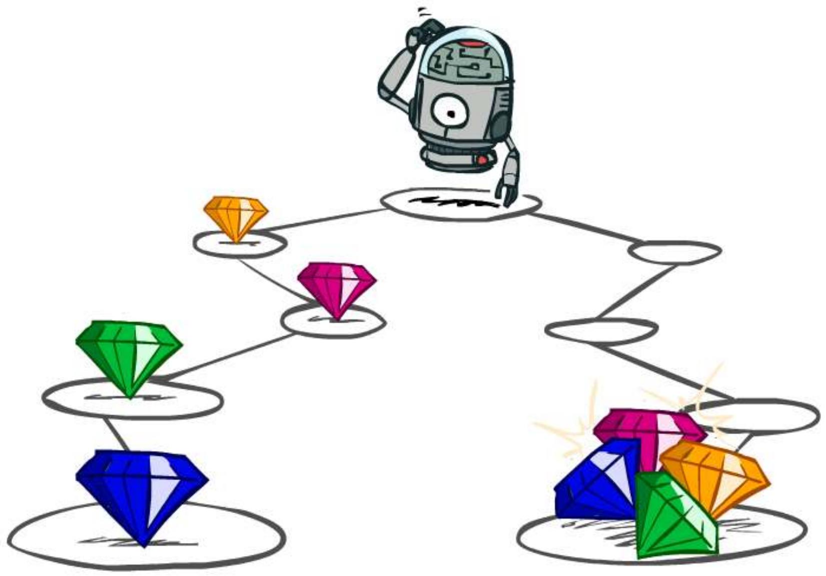


Utilities of Sequences

- What preferences should an agent have over reward sequences?

- More or less? [1, 2, 2] or [2, 3, 4]

- Now or later? [0, 0, 1] or [1, 0, 0]



Discounting

- It’s reasonable to maximize the sum of rewards
- It’s also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially

1

Worth Now

γ

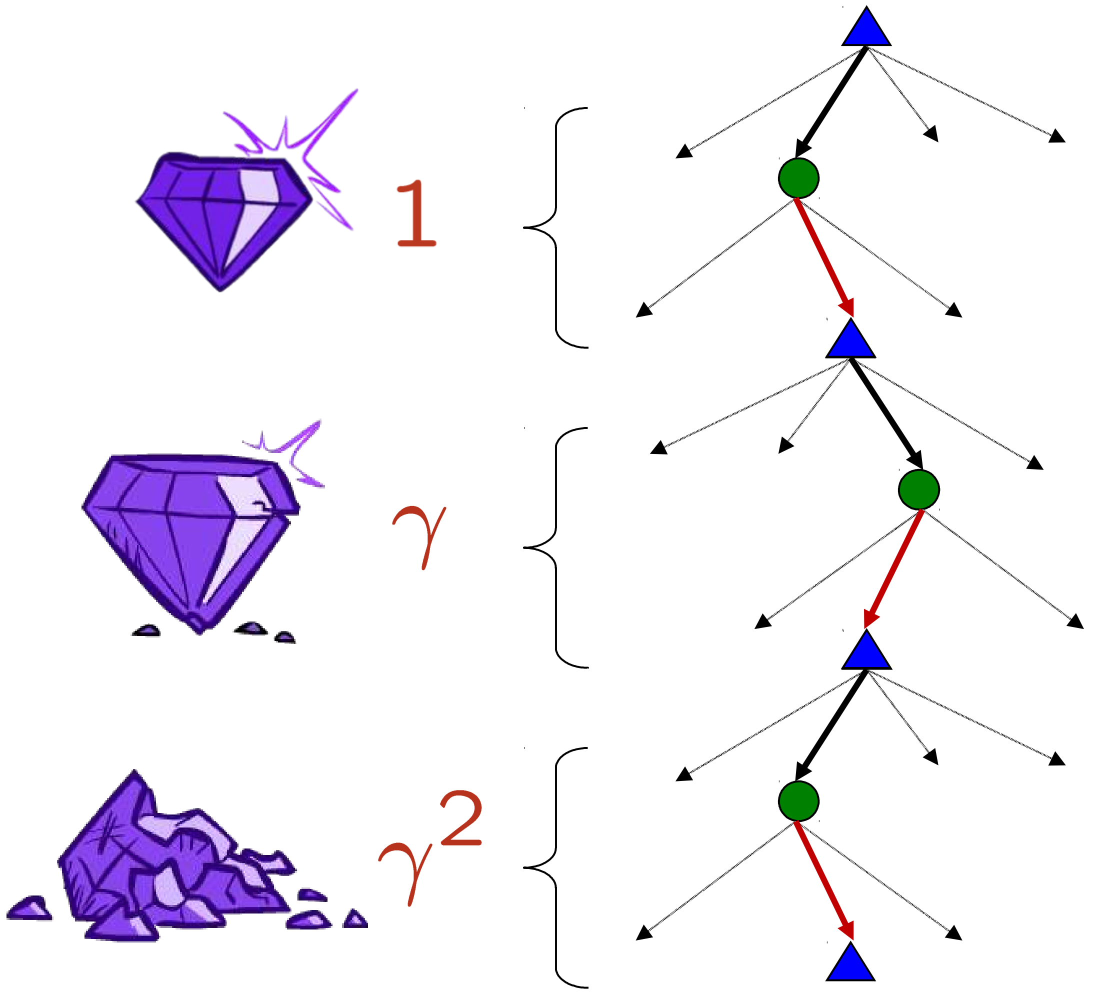
Worth Next Step

γ^2

Worth In Two Steps

Discounting

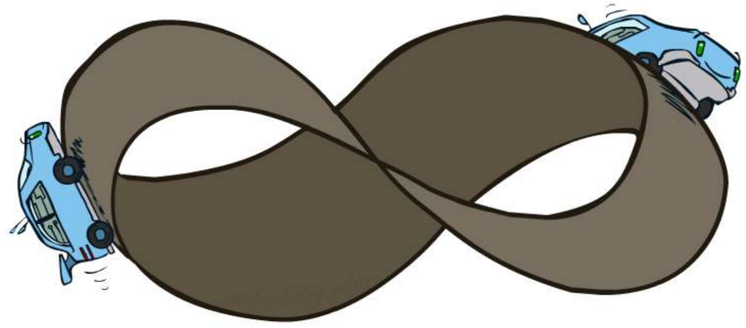
- How to discount?
  - Each time we descend a level, we multiply in the discount once
- Why discount?
  - Sooner rewards probably do have higher utility than later rewards
  - Also helps our algorithms converge
- Example: discount of 0.5
  - U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3
  - U([1,2,3]) < U([3,2,1])



Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?

- Solutions:
  - Finite horizon: (similar to depth-limited sear
    - Terminate episodes after a fixed T steps (e.g. life)
    - Gives nonstationary policies (π depends on time lef



  - Discounting: use $0 < \gamma < 1$

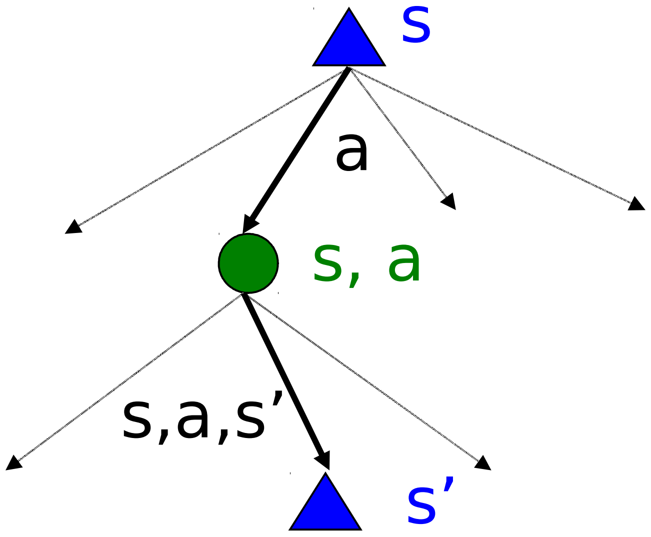
$$U([r_0, \ldots r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\text{max}}/(1-\gamma)$$

    - Smaller γ means smaller "horizon" – shorter term focus

  - Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "overheated" for racing)

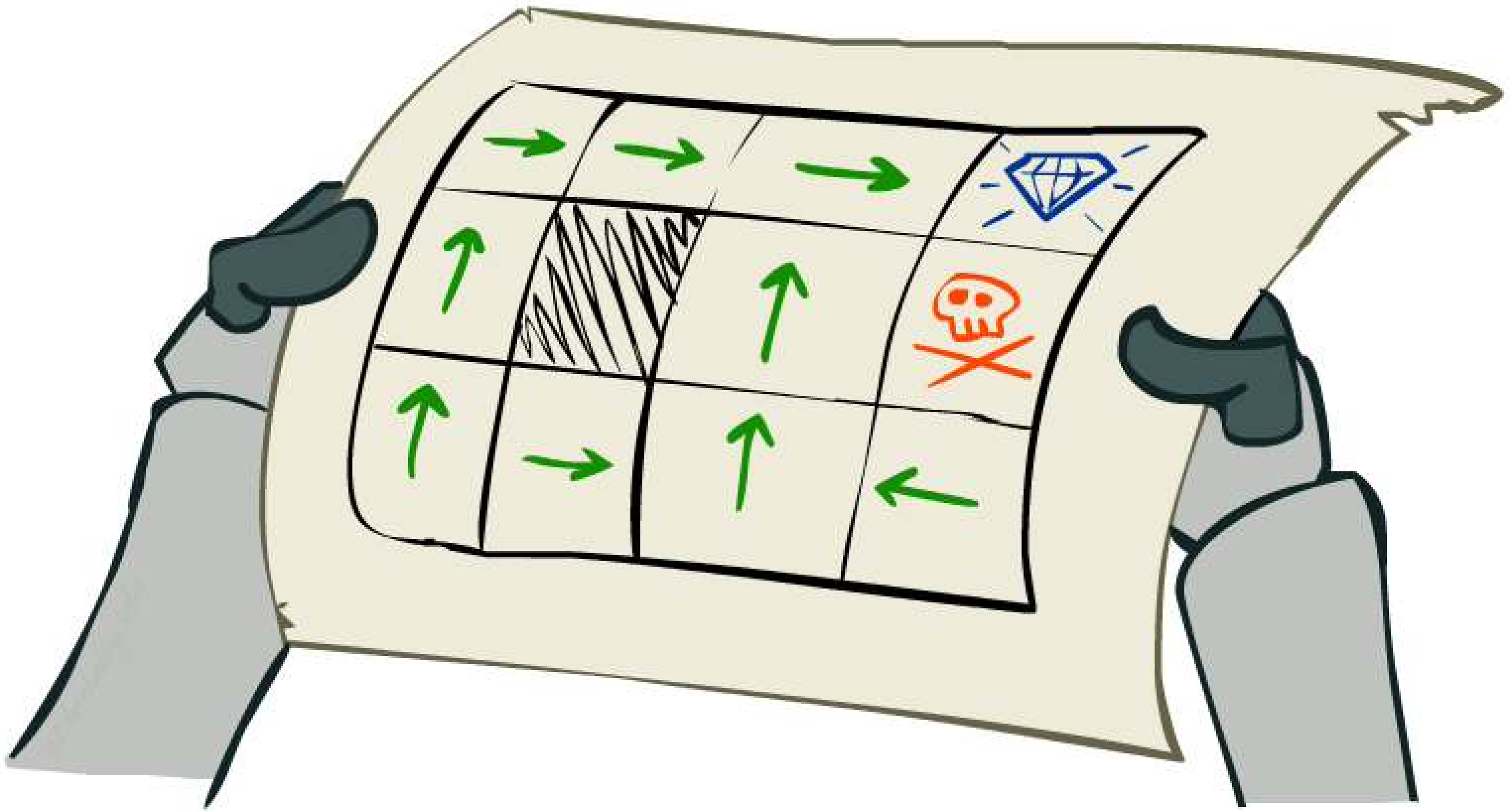
Recap: Defining MDPs

- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s’|s,a) (or T(s,a,s’))
  - Rewards R(s,a,s’) (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards

Solving MDPs



Optimal Quantities

- The value (utility) of a state s:

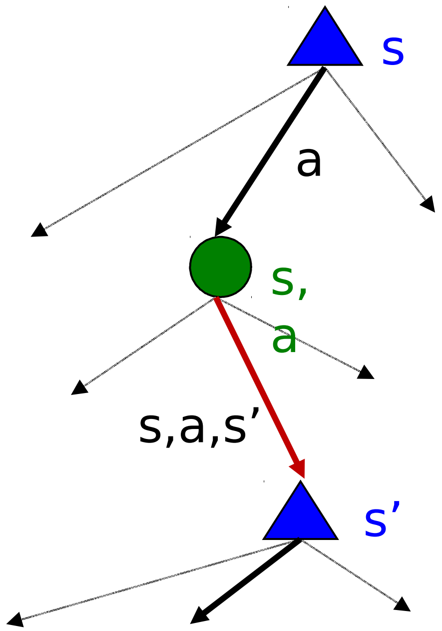
$V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a q-state (s,a):

$Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:

$\pi^*(s)$ = optimal action from state s

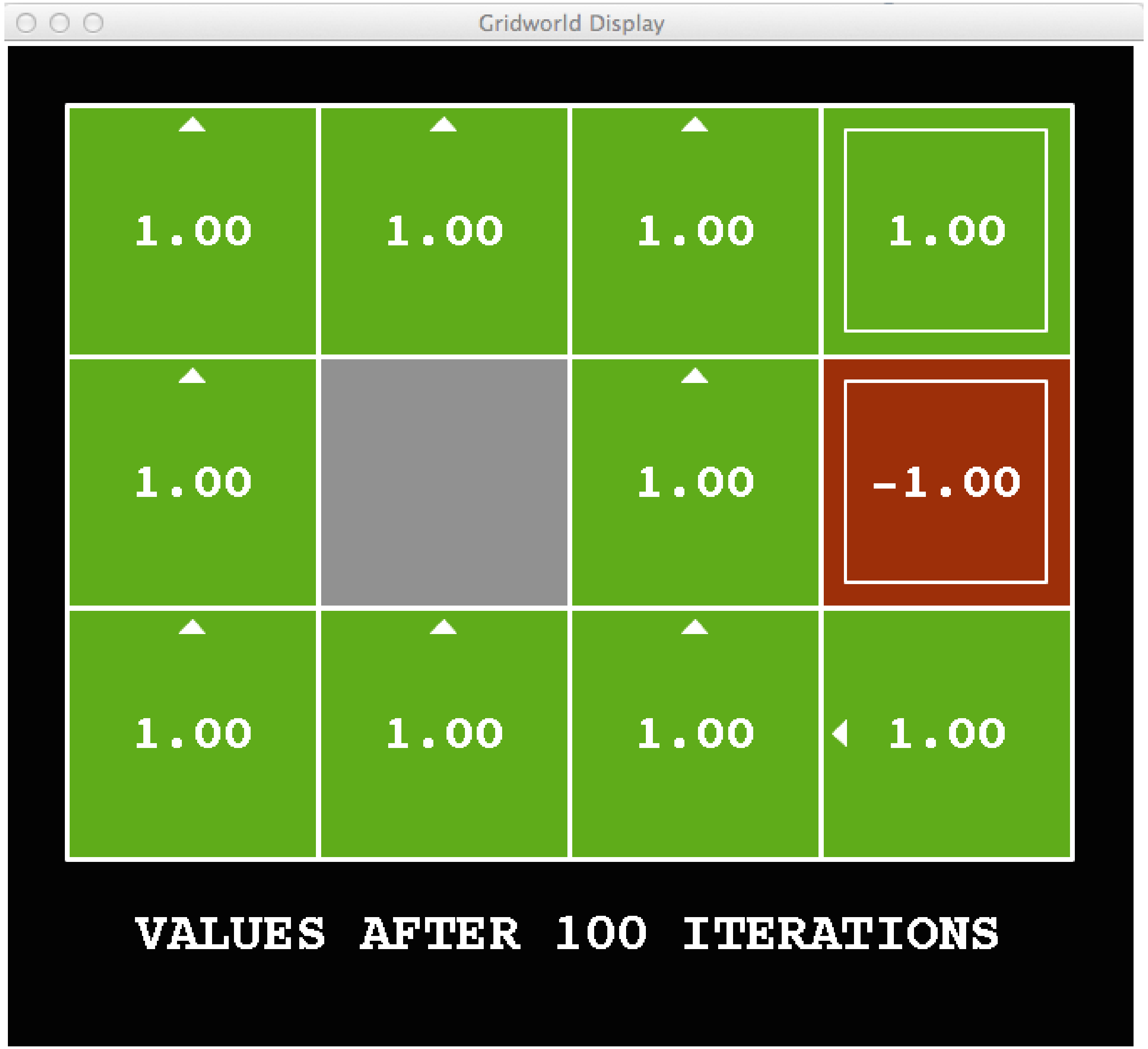


s is a *state*

(s, a) is a *q-state*

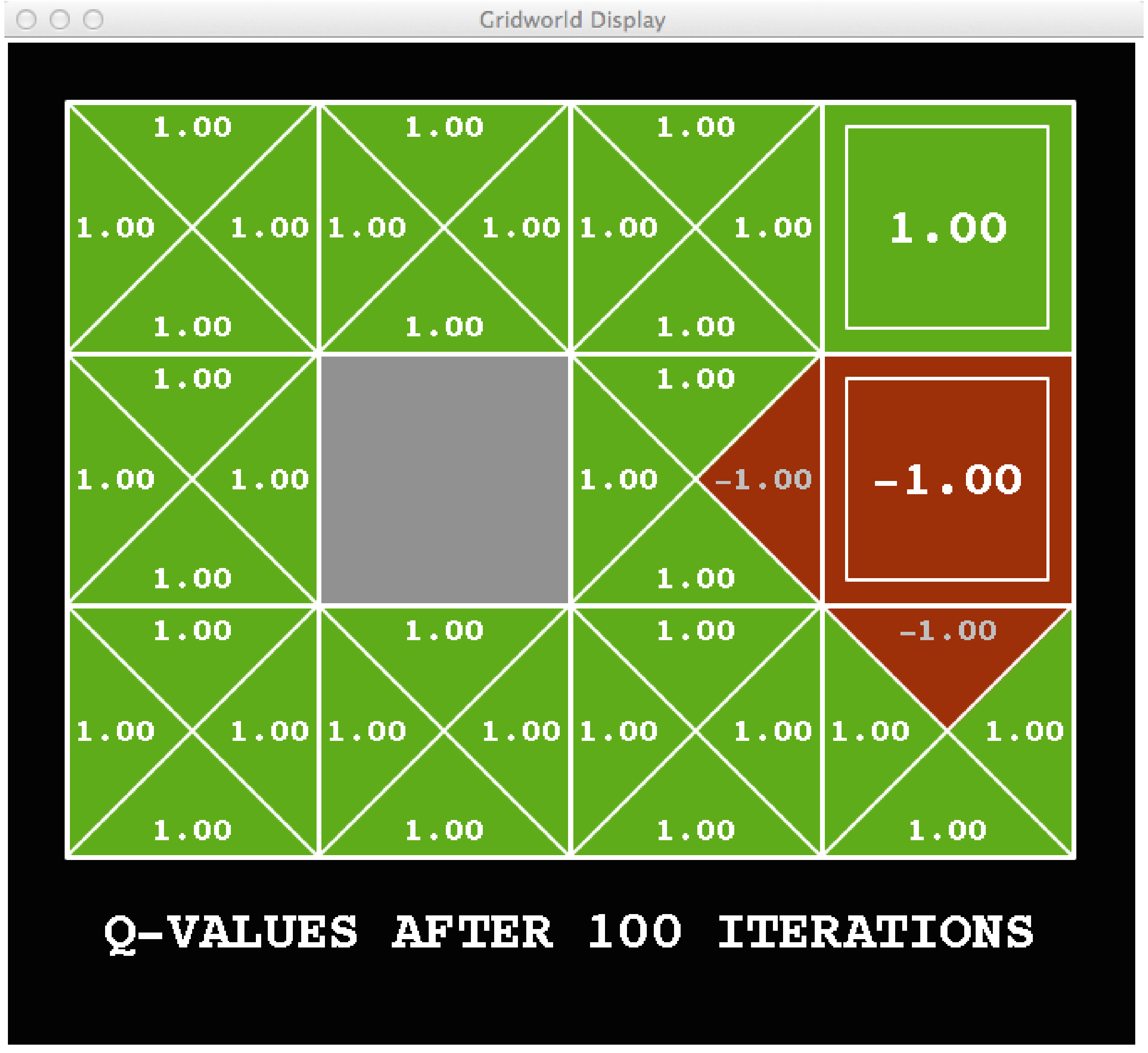
(s,a,s’) is a *transition*

Snapshot of Demo – Gridworld V Values



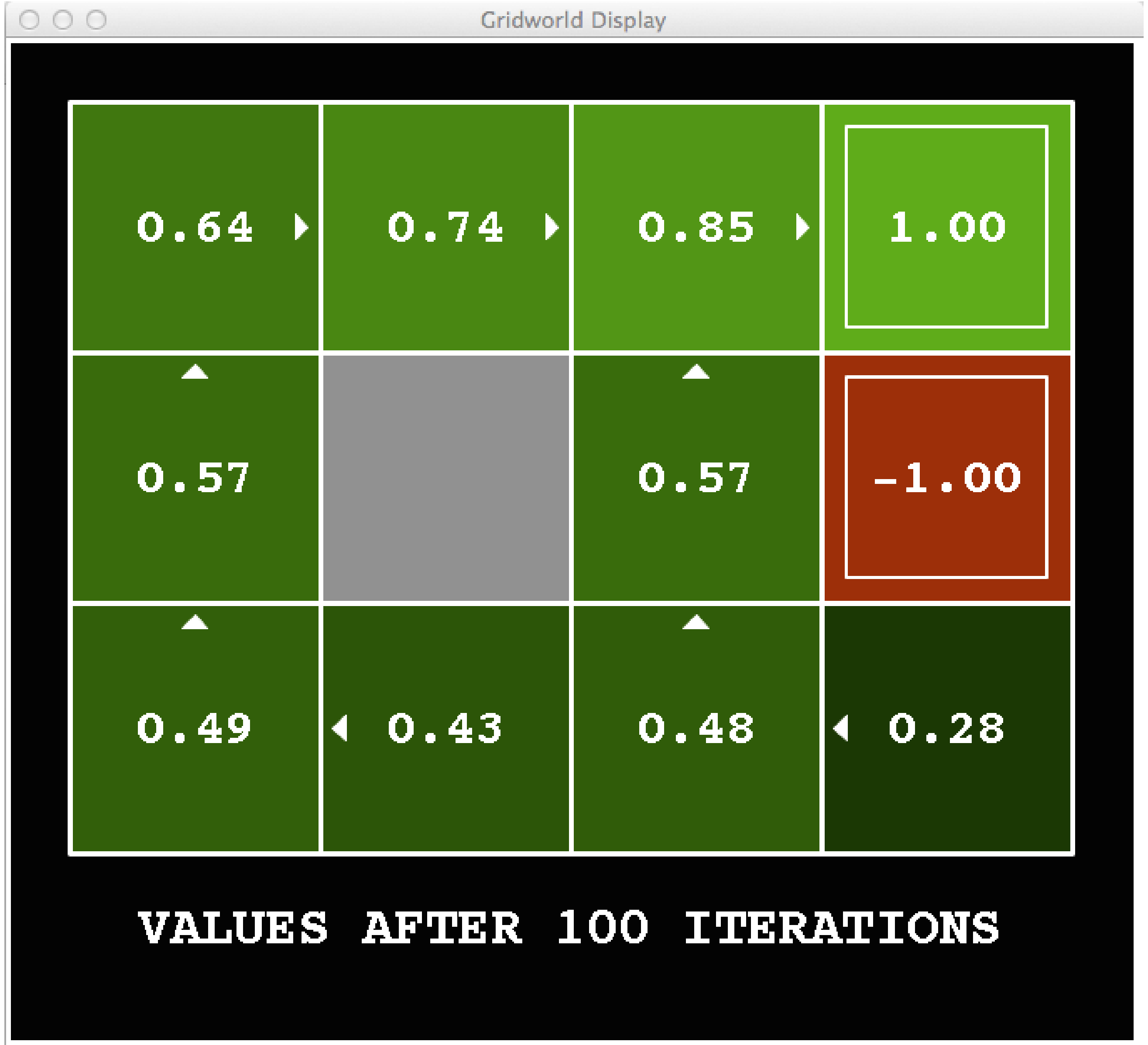
Noise = 0
Discount = 1
Living reward = 0

Snapshot of Demo – Gridworld Q Values



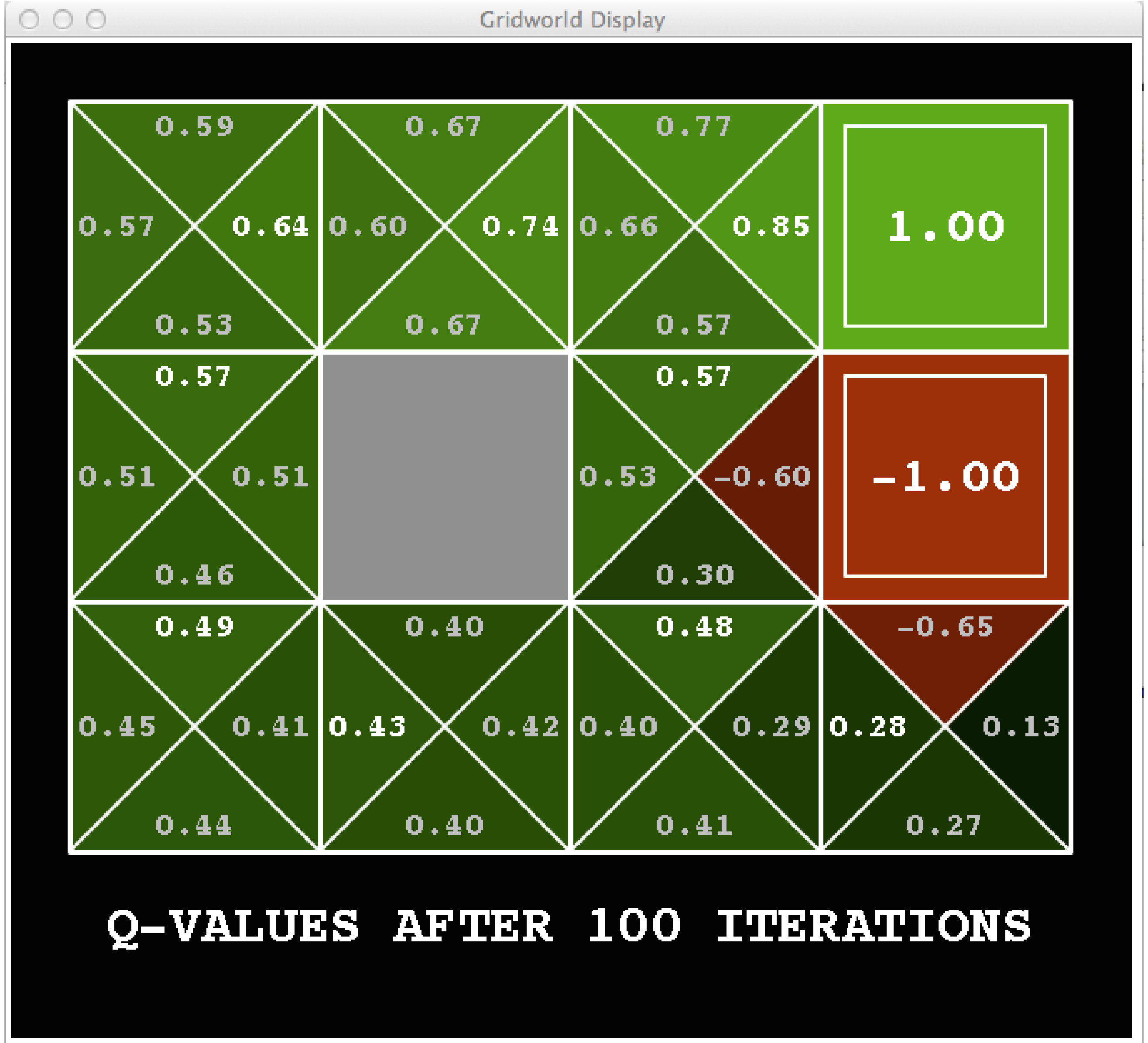
Noise = 0
Discount = 1
Living reward = 0

Snapshot of Demo – Gridworld V Values



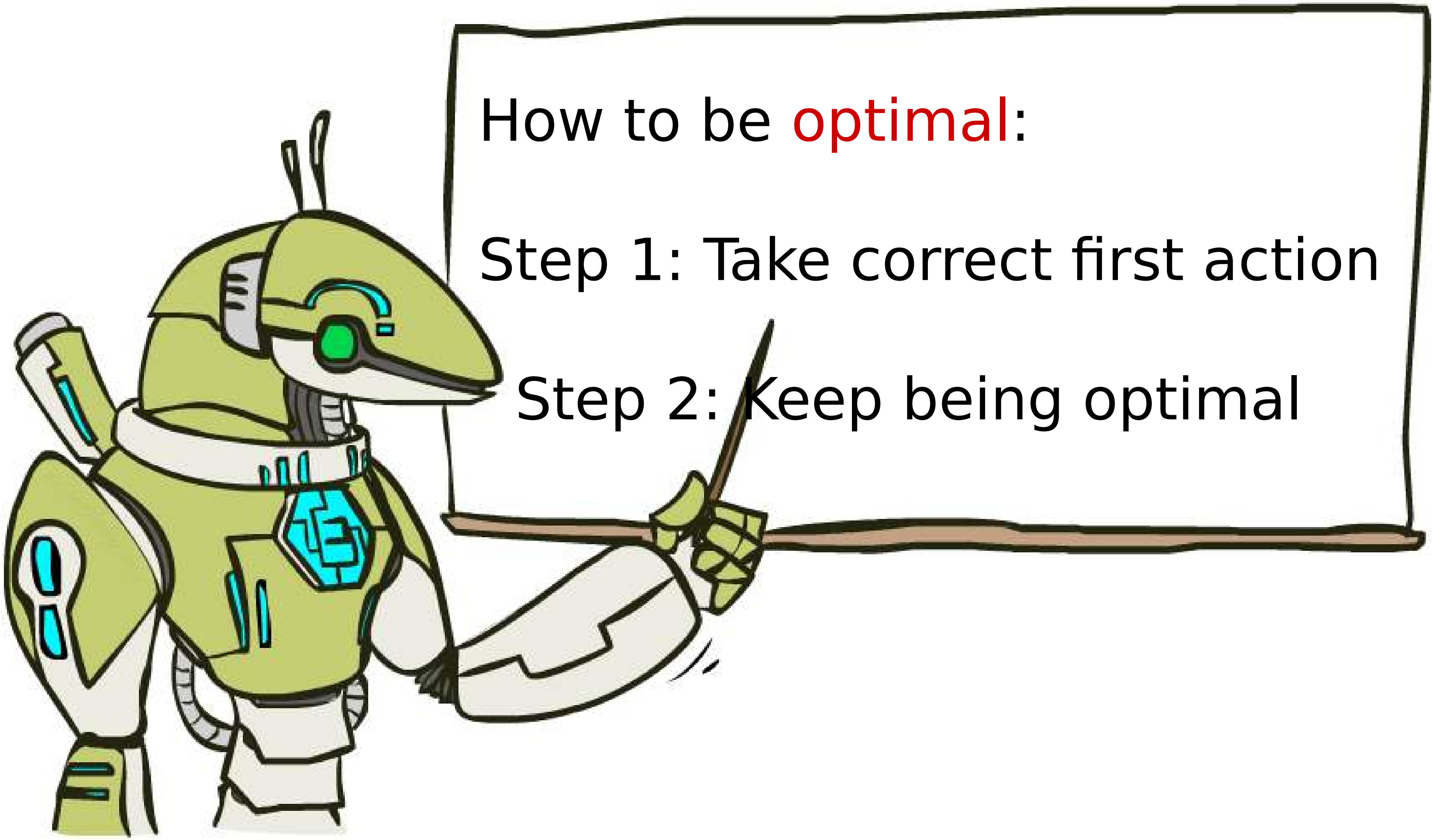
Noise = 0.2
Discount = 0.9
Living reward = 0

Snapshot of Demo – Gridworld Q Values



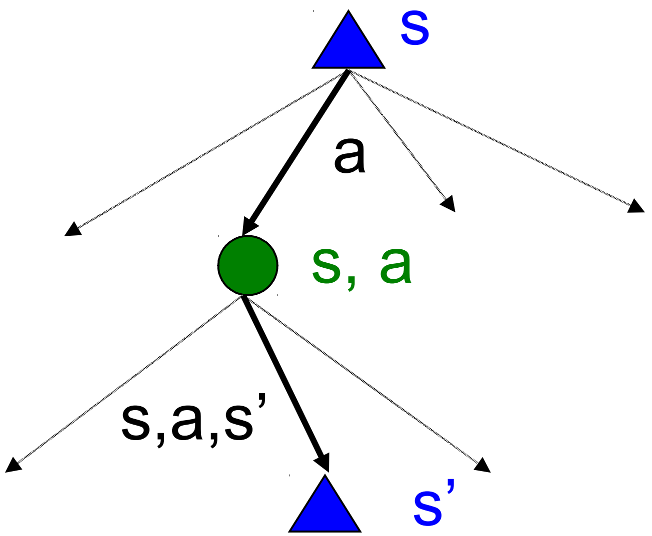
Noise = 0.2
Discount = 0.9
Living reward = 0

The Bellman Equations



Bellman Equations

- Fundamental operation: compute the (expectimax) value of a state
  - Expected utility under optimal action
  - Average sum of (discounted) rewards
  - This is just what expectimax computed!



- Recursive definition of value:

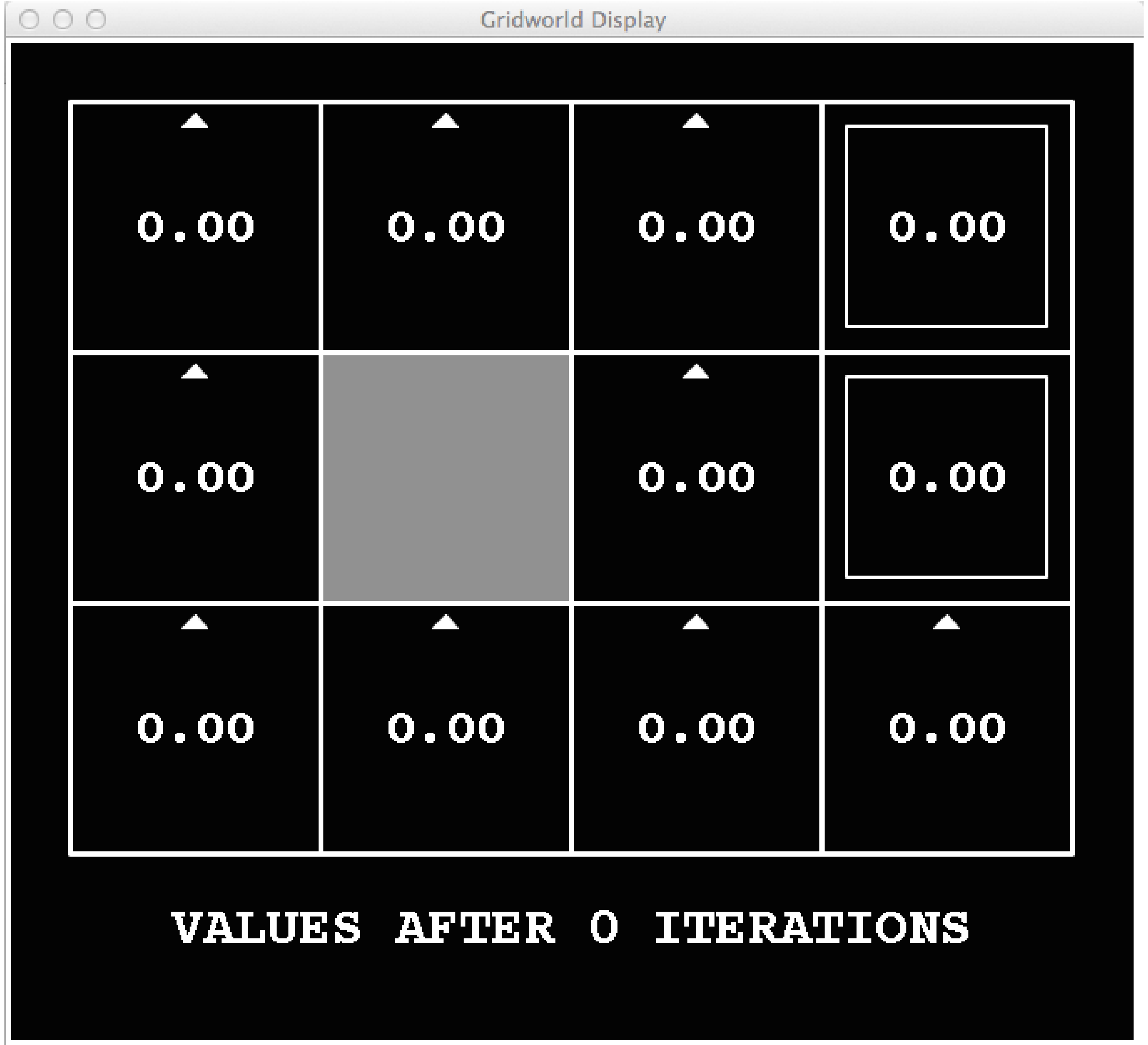
$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

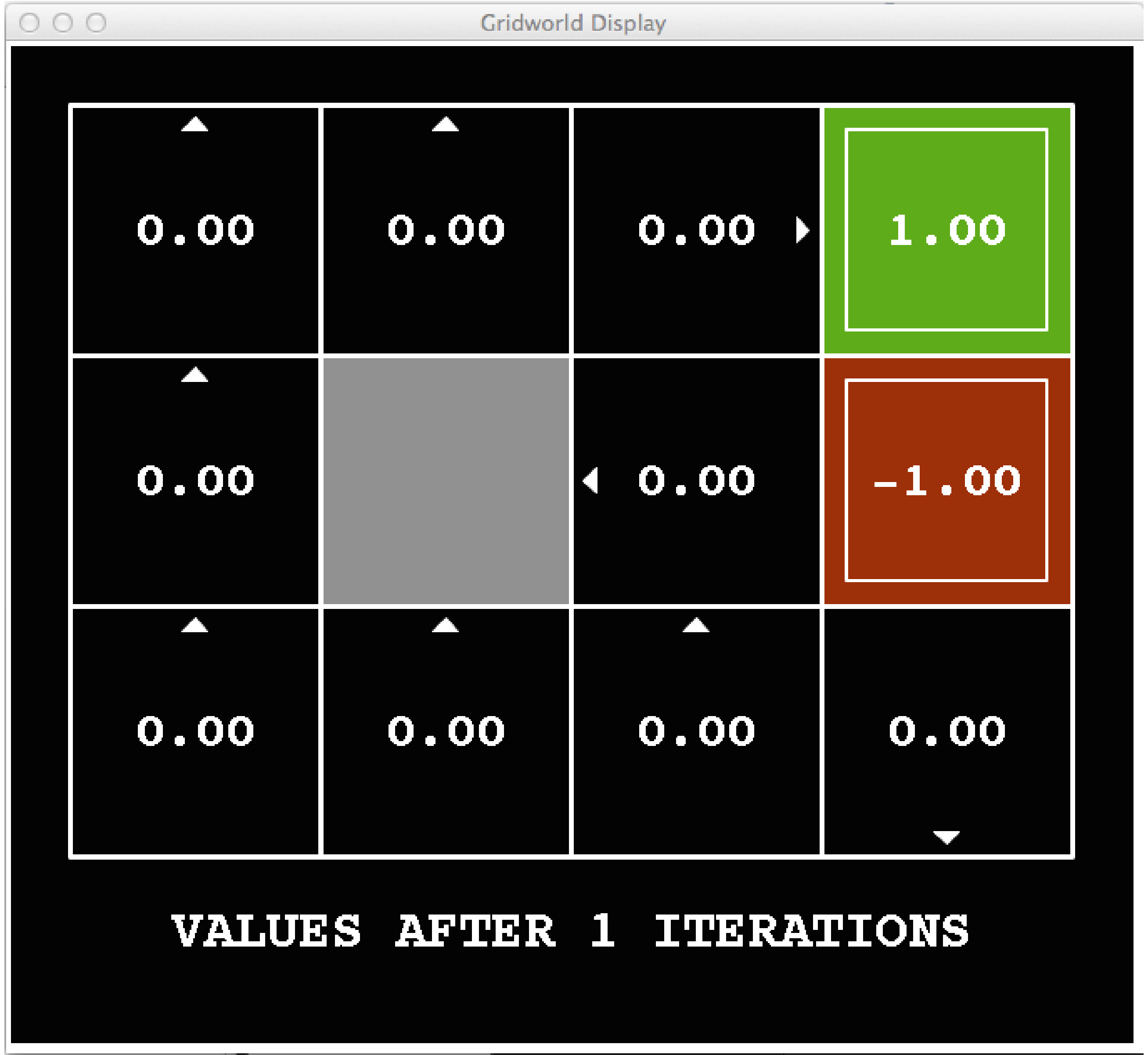
- An important step is calculating the utilities (Value) of state
- The Bellman equations define these utilities
- Missing: how to efficiently compute them
- Iterative algorithm: Value Iteration

k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

k=1

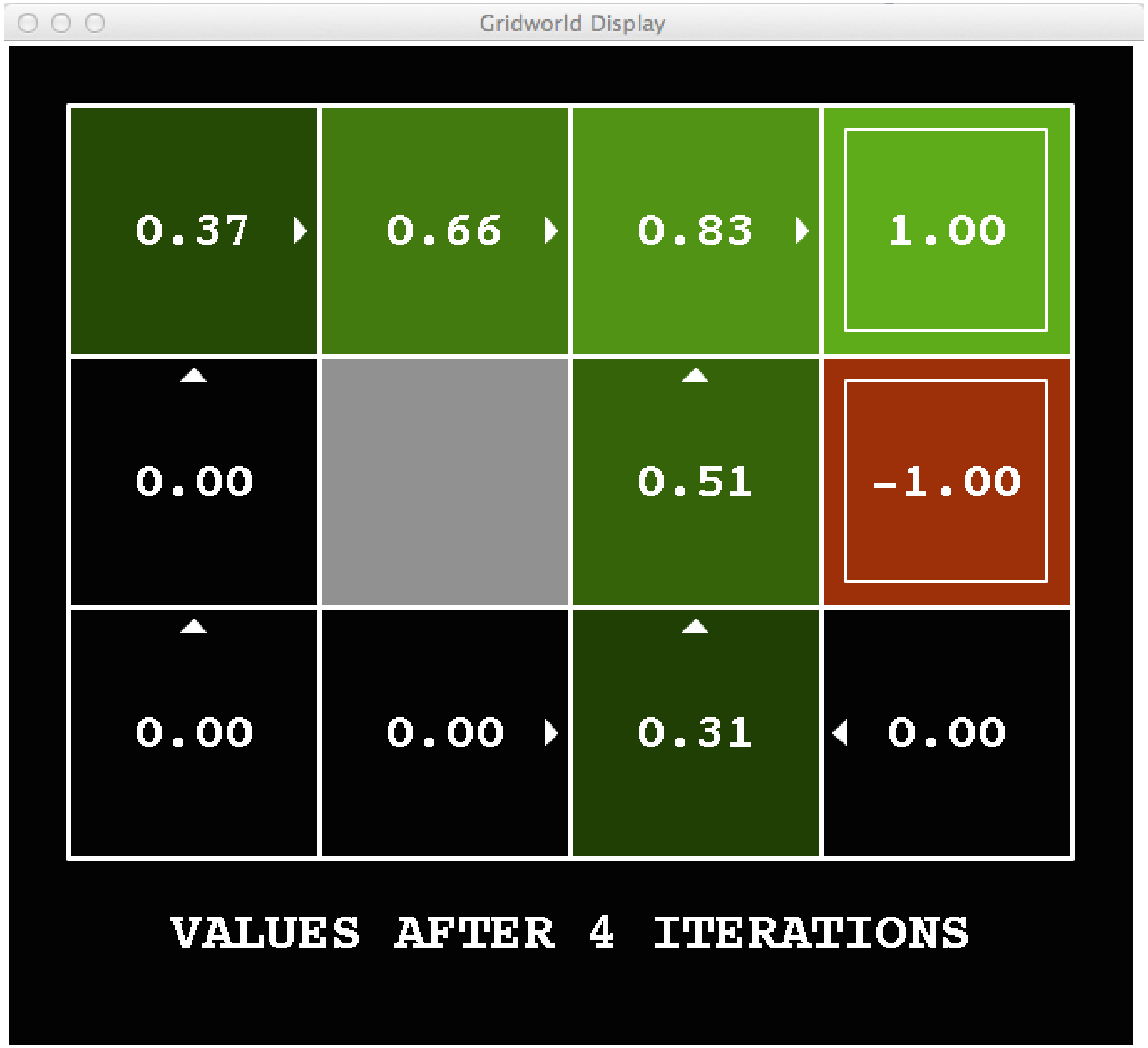


Noise = 0.2
Discount = 0.9
Living reward = 0

k=2

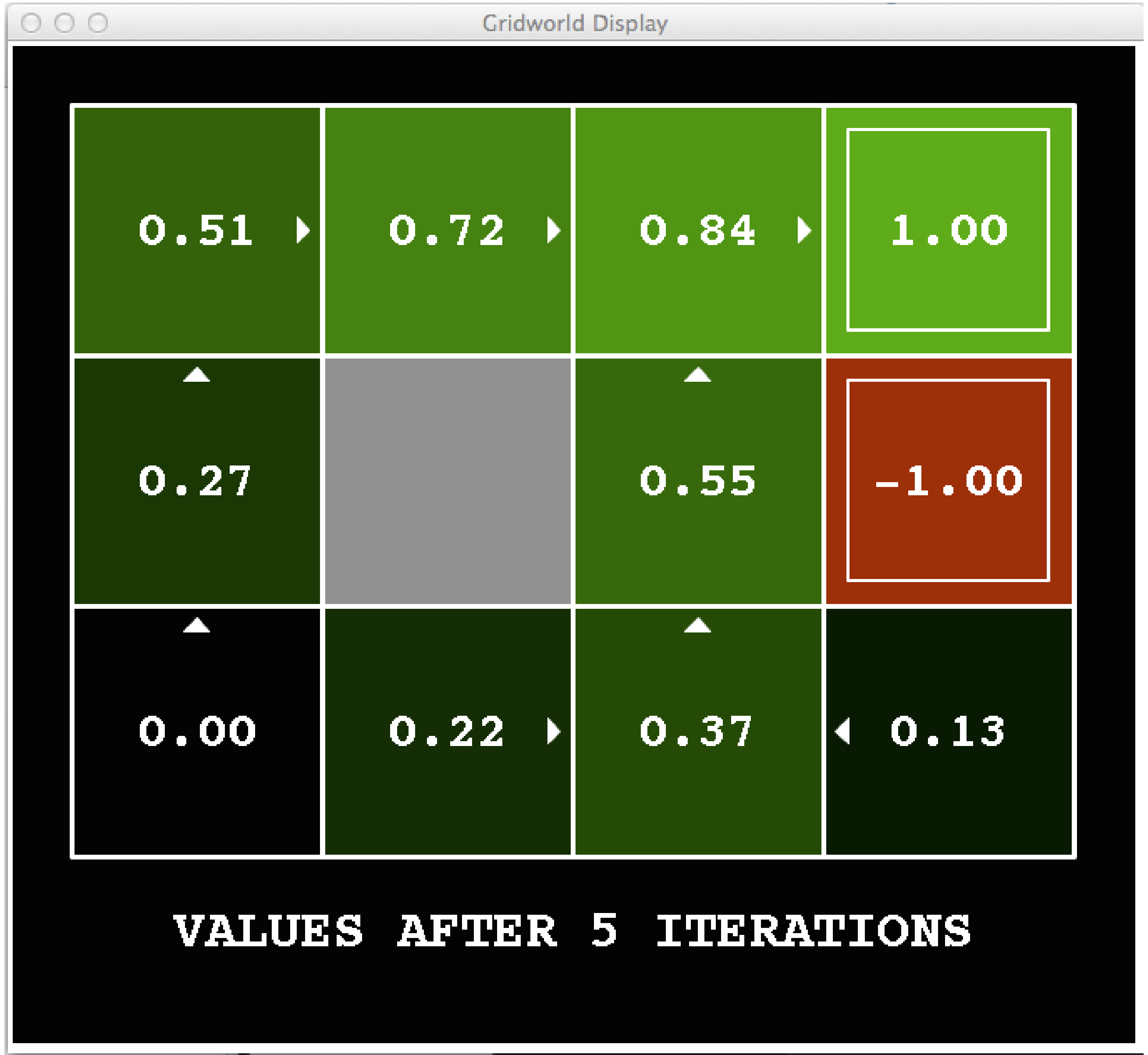
Noise = 0.2
Discount = 0.9
Living reward = 0

k=4



Noise = 0.2
Discount = 0.9
Living reward = 0

k=5

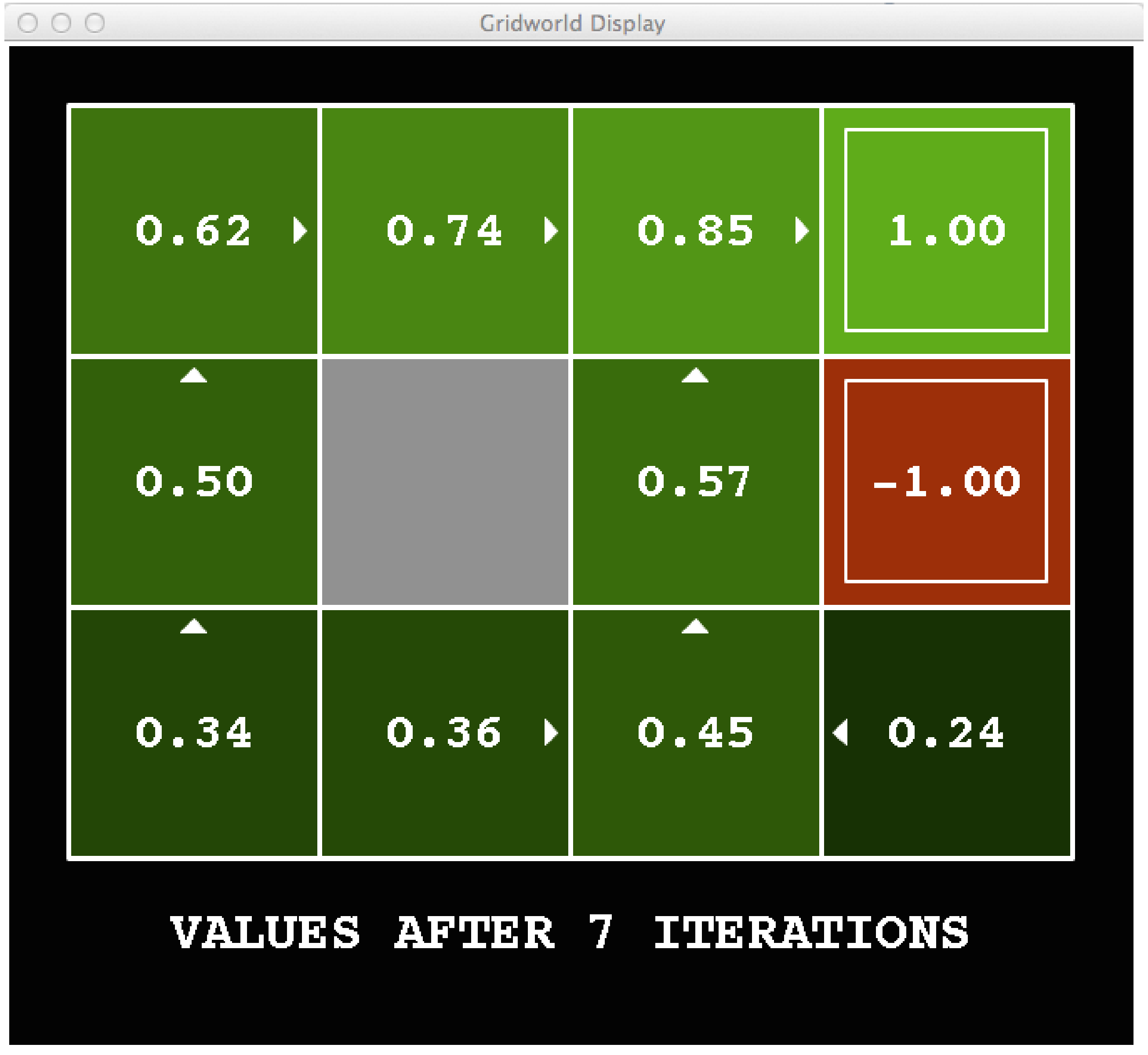


Noise = 0.2
Discount = 0.9
Living reward = 0

k=6

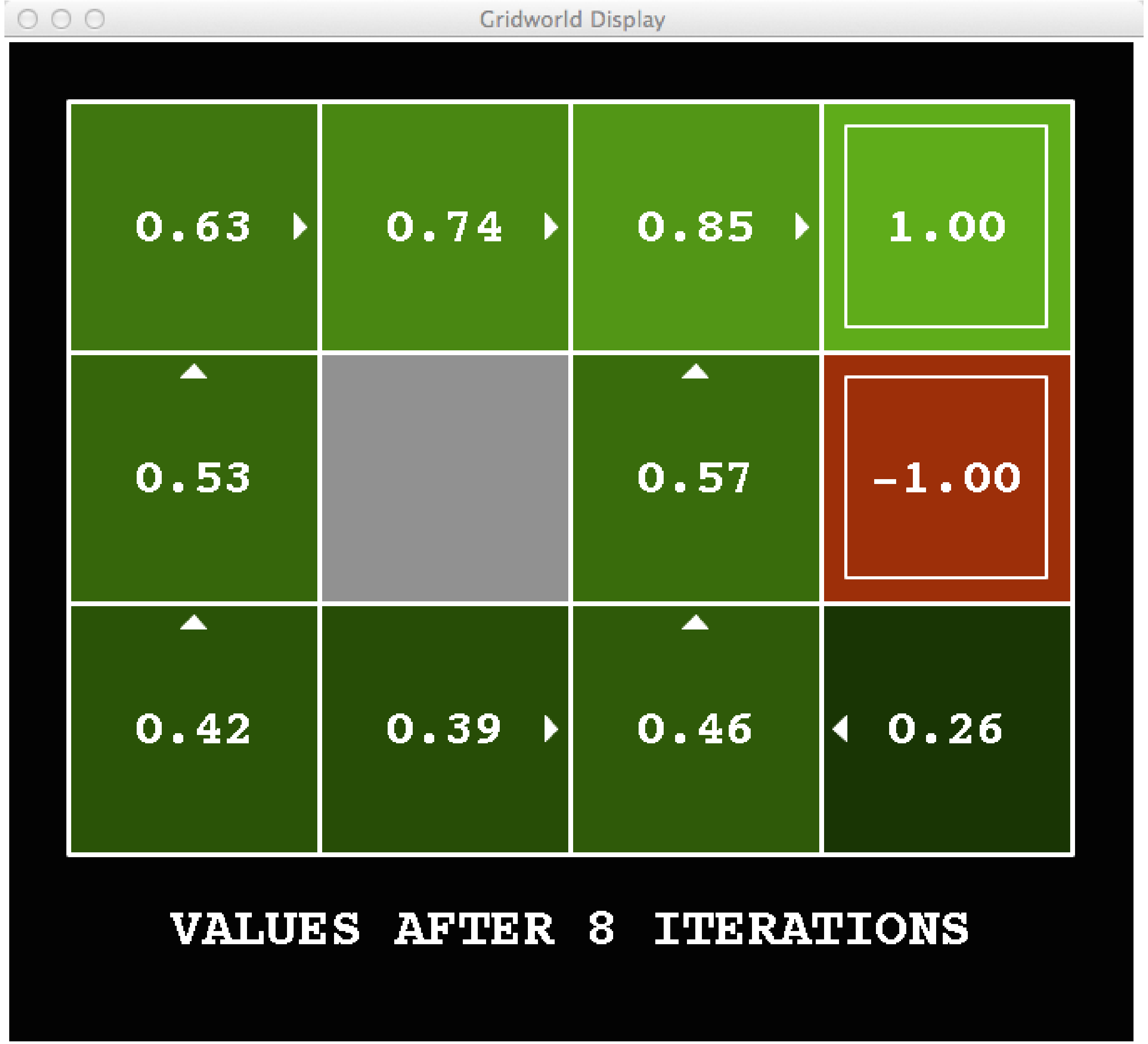
Noise = 0.2
Discount = 0.9
Living reward = 0

k=7



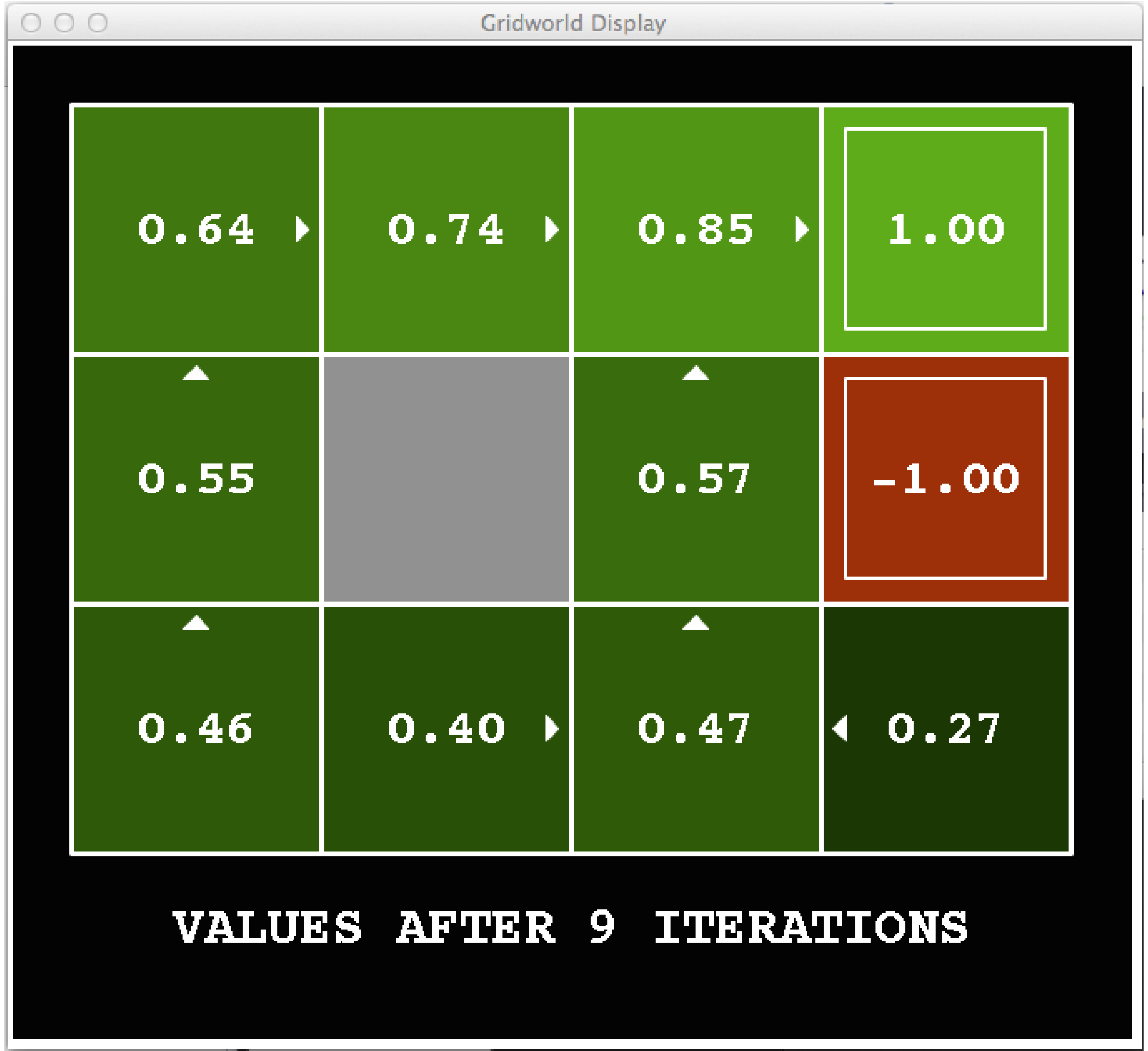
Noise = 0.2
Discount = 0.9
Living reward = 0

k=8



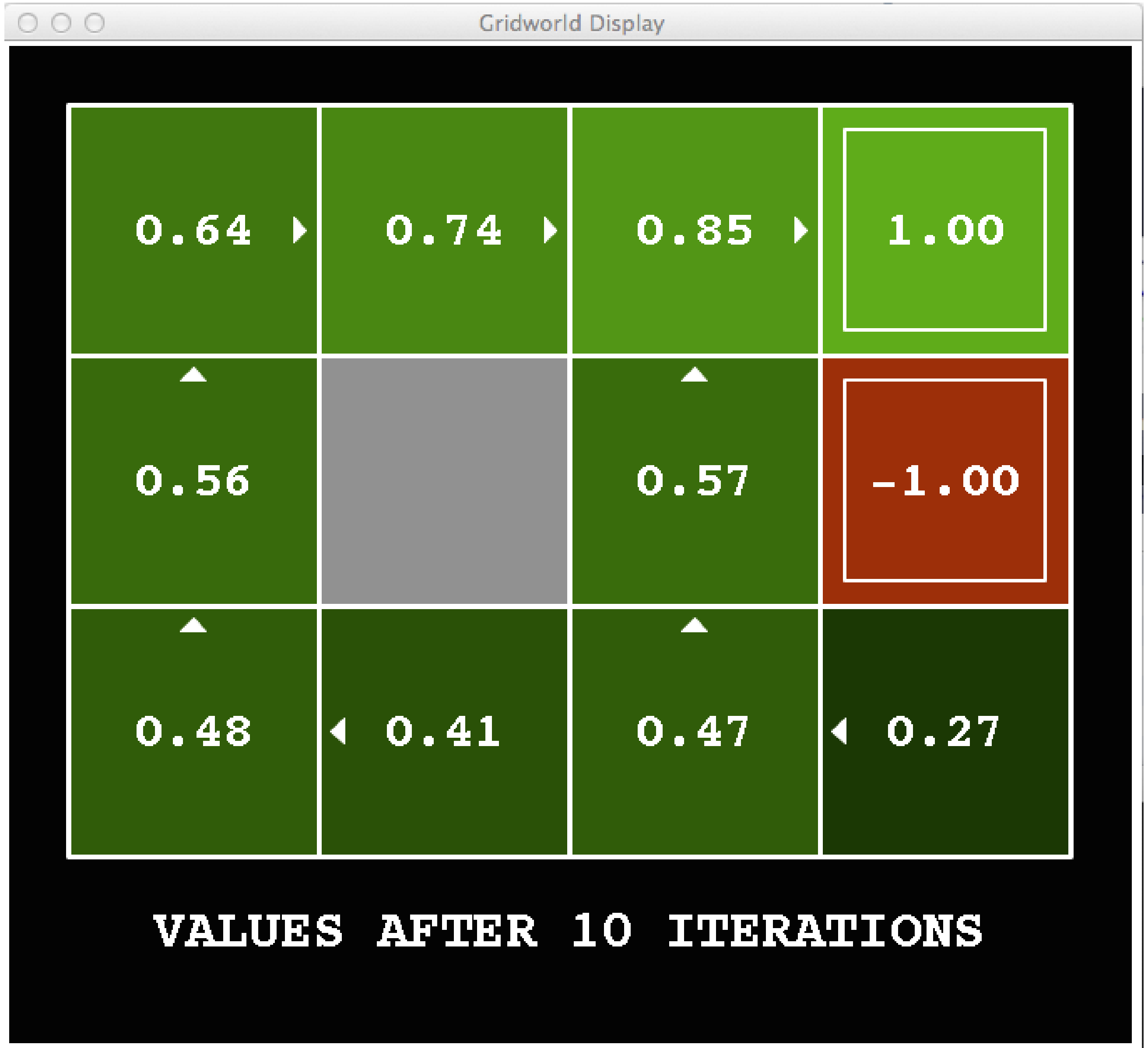
Noise = 0.2
Discount = 0.9
Living reward = 0

k=9



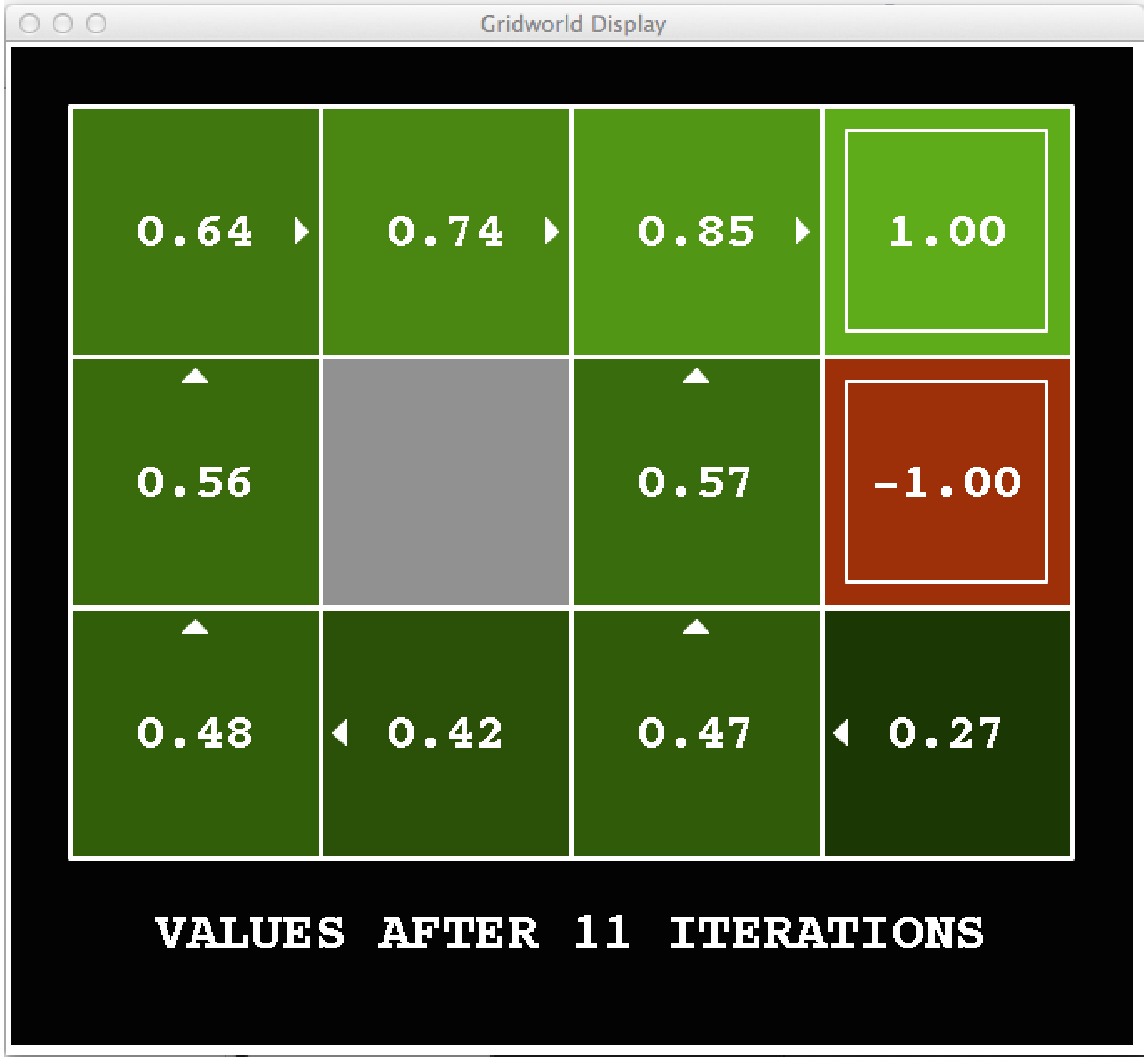
Noise = 0.2
Discount = 0.9
Living reward = 0

k=10



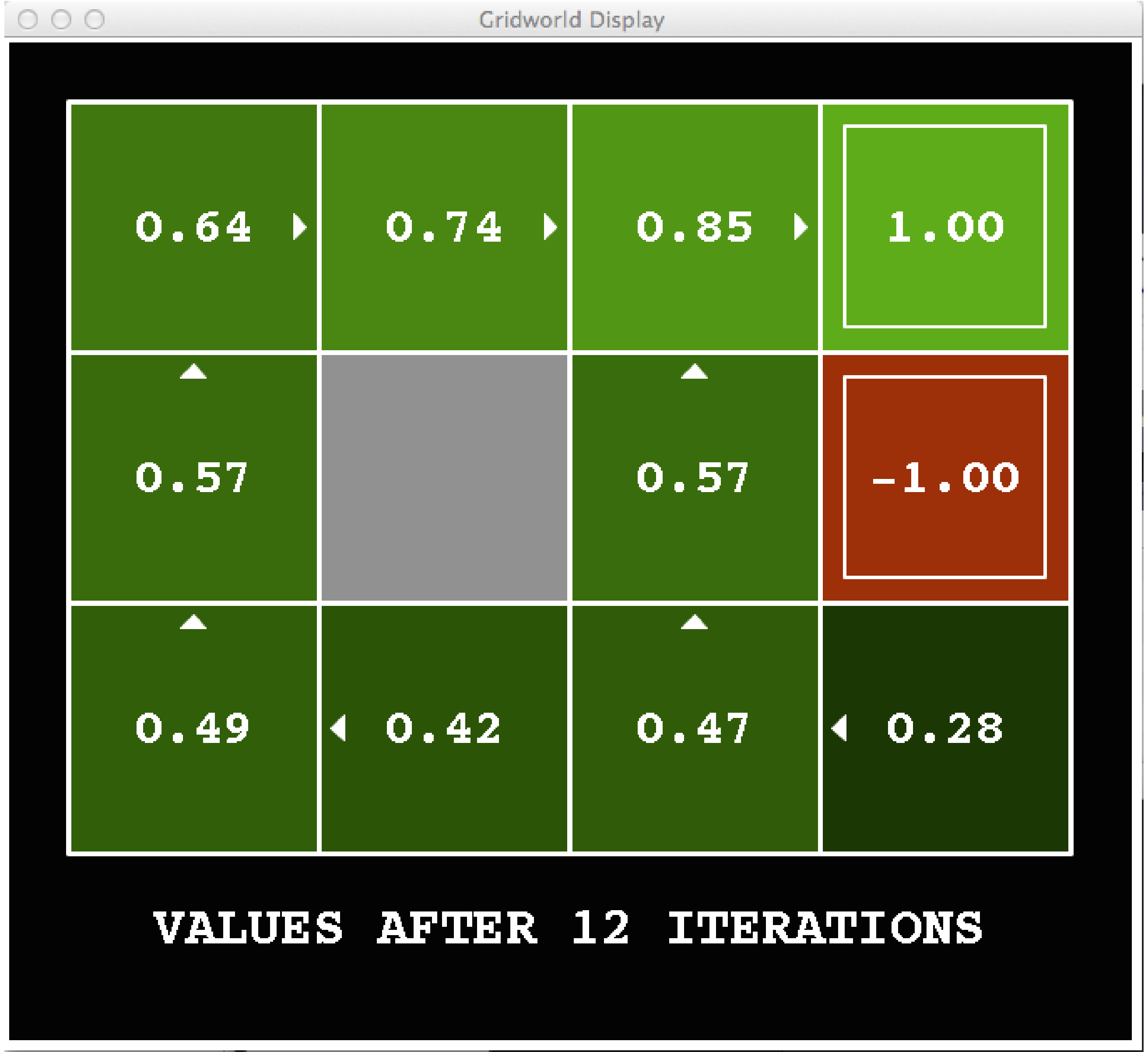
Noise = 0.2
Discount = 0.9
Living reward = 0

k=11



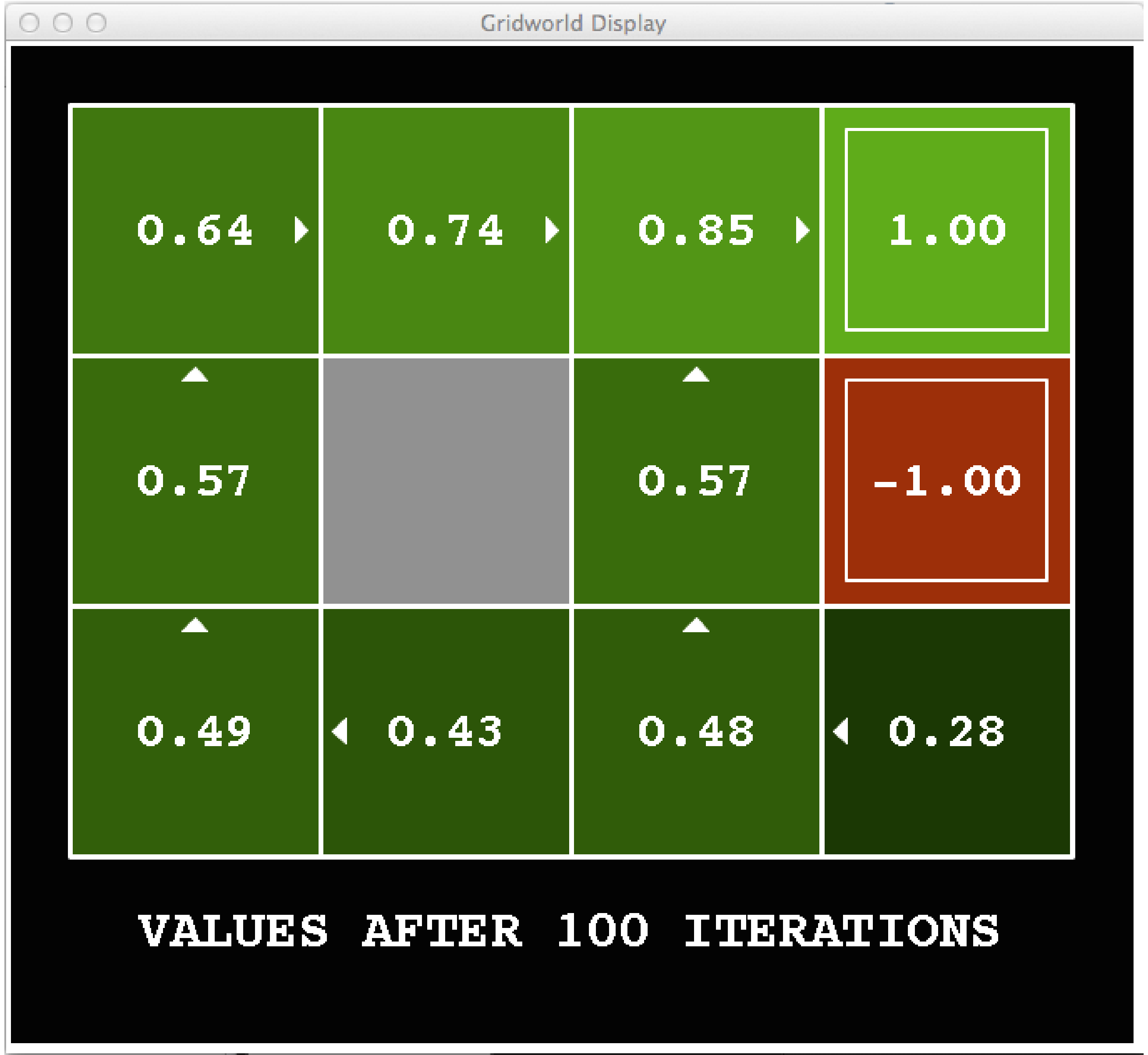
Noise = 0.2
Discount = 0.9
Living reward = 0

k=12



Noise = 0.2
Discount = 0.9
Living reward = 0

k=100



Noise = 0.2
Discount = 0.9
Living reward = 0

Next Time: Value Iteration