The problem at hand involves designing an efficient supply chain for producing bioethanol in the state of Texas. This supply chain consists of suppliers (counties providing biomass), hubs (consolidation and preprocessing centers), and biorefineries (conversion facilities). The truck is the transportation mode utilized to move the biomass from the counties to the hubs. the transportation mode utilized to move the raw material from the hubs to the biorefineries is train. The objective is to minimize investment and transportation costs by determining the optimal number of hubs and biorefineries, as well as the biomass flows between suppliers, hubs, and biorefineries.

Sets

V_1 = {1,2,3, ……….,254} is the set of the suppliers

V_2 = {1,2,3, ……….,33} is the set of the hubs

V_3 = {1,2,3, ……….,167} is the set of the plants

Parameters

C_{ij} = Transportation cost of trucks from i to j ($/Mg)

C_{jk} = Transportation cost of train from j to k ($/Mg)

l_{jk} = Loading and unloading cost of train from j to k

O_i = Maximum capacity of supplier i

q_j = Maximum capacity of hub j (300000 Mg for all hubs)

p_k = Maximum capacity of plant k (152063705/232=655447 Mg for all plants)

t_j = Maximum capacity of train j (338000 Mg for all trains)

f_j = investment cost of hub j ($3476219 for all hubs)

g_k = investment cost of plant k ($130956797 for all plants)

D= Total demand (1476310602/232=6363408 Mg)

G = third-party cost (for sensitivity analysis)

Variables

S_{ij} = Quantity of biomass transported from supplier i to hub j (Mg)

S_{jk} = Quantity of biomass transported from hub j to plant k (Mg)

y_j = Binary decision variable, for each j ∈ V_2, set equal to 1 if a potential hub j is opened, 0 otherwise

z_j = Binary decision variable, for each k ∈ V_3, set equal to 1 if a potential plant k is opened, 0 otherwise

S_t = Quantity of biomass transported from third-party (Mg)

Objective Function:

Minimize the total cost, including investment and transportation costs. The first part of the objective function is to minimize the transportation cost from Supplier i to Potential Hub j. The second part of the function for minimizing the transportation cost including the loading and unloading cost of the train used for traveling from potential Hub j to potential plant k. The third and fourth parts of the equation are used for minimizing the investment cost of hubs and plants respectively. The last part of the equation has minimized the unmet demand through third-party. If third-party costs are lower it can minimize the high-cost supplier.

$$\min(\sum_{i\in v_1}\sum\nolimits_{j\in v_2} S_{ij}C_{ij} + \sum_{j\in V_2}\sum_{k\in V_3}\left(S_{jk}C_{jk} + l_{jk}\right) + \sum_{j\in V_2} f_j * y_j + \sum_{k\in V_3} g_k * z_k + (\, S_t * G)$$

Constraints

Supplier capacity Constraint: Biomass flow from suppliers to hubs cannot exceed supplier capacity (O_i)

$$\sum_{j\in V_2} S_{ij} \le O_i \qquad i \in V_1$$

Hub Capacity Constraint: Biomass flow from hubs to plants cannot exceed the potential hub's preprocessing capacity (q_j)

$$\sum_{j \in V_2} S_{jk} \leq q_j \, y_j \qquad k \in V_3$$

Plant Capacity Constraint: Biomass flow from hubs to plants cannot exceed the potential plant's capacity (p_k)

$$\sum_{j \in V_2} S_{jk} \leq p_k \, z_k \qquad k \in V_3$$

Demand Constraint: demand must be met through the flow of biomass from all potential hubs to potential plants.

$$\sum_{j \in V_2} S_{jk} \, z_k = d \qquad k \in V_3$$

Train capacity constraint: biomass flow from hub j to plant k can't be greater than from train capacity(t_j)

$$S_{jk} \leq t_j \qquad j \in V_2, k \in V_3$$

Third-party Constraint: The unmet demand must be satisfied through third-party

$$S_t = \boldsymbol{D} - \sum_{j \in V_2} S_{ij} \quad i \in V_1$$

Flow Conservation Constraint: Biomass flows from suppliers to hubs must be consistent

$$S_t + \sum_{j \in V_2} S_{ij} \quad i \in V_1 = \sum_{j \in V_2} S_{jk} \quad k \in V_3$$

y_j is a Binary decision variable, for each j ∈ V_2, set equal to 1 if a potential hub j is opened, 0 otherwise

$$y_j \in \{0, 1\}, \quad \mathrm{j} \in V_2$$

z_j is a Binary decision variable, for each k ∈ V_3, set equal to 1 if a potential plant k is opened, 0 otherwise

$$z_k \in \{0, 1\}, \quad \mathrm{k} \in V_3$$

Biomass flow from supplier i to hub j must be positive

$$S_{ij} \geq 0 \quad \mathrm{i} \in V_1, \mathrm{j} \in V_2$$

Biomass flow from hub j to plant k must be positive

$$S_{jk} \geq 0 \quad \mathrm{j} \in V_2, \mathrm{k} \in V_3$$

Discussion:

When I ran this on GUROBI, an error message encountered, "Model too large for a size-limited license," indicates that the size of the optimization problem I have formulated exceeds the limitations of the Gurobi free trial license. To fix this issue have tried to simplify the optimization model by reducing the number of decision variables, constraints, or the complexity of the expressions involved because this could involve aggregating data, eliminating non-essential variables, or finding ways to represent the problem with fewer terms. But I didn’t get rid of it. Another thing that can happen optimization problem is complex and requires a Gurobi license. This would allow me to solve larger and more complex optimization problems without the size limitations of the free trial or consider exploring open-source optimization libraries like CPLEX, or scipy.optimize.