Multi-Depot Vehicle Routing Problem with Heterogeneous Fleet

A Mathematical Formulation and Computational Study

Abstract

This technical report presents a computational framework for solving the Multi-Depot Heterogeneous Fleet Vehicle Routing Problem (MDHFVRP) in last-mile delivery logistics. The system optimizes delivery routes across K = 3 distribution centers serving N = 69 geographically dispersed customers using a mixed fleet of Electric Vehicles (EVs) and Gasoline Vehicles (GVs). Our results demonstrate that strategic fleet electrification achieves a 9.1% cost reduction while maintaining service levels.

1. Problem Formulation

1.1 Sets and Indices

Symbol	Description
D = {1, 2, 3}	Set of depots
Ck	Set of customers assigned to depot k
Vk	Set of vehicles at depot k
VkEV ⊆ Vk	Subset of electric vehicles
VkGV ⊆ Vk	Subset of gasoline vehicles

1.2 Parameters

Parameter	Description	Value
dij	Distance from node i to node j	miles
tij	Travel time from node i to node j	minutes
QEV	Maximum range for EVs	200 miles
QGV	Maximum range for GVs	300 miles
Tmax	Maximum working time	540 minutes
cEV	EV operating cost	$0.25-0.30/mile

cGV	GV operating cost	$0.50-0.60/mile
cL	Labor cost	$0.60/minute

1.3 Objective Function

The objective is to minimize total operational cost across all depots, comprising distance-based vehicle operating costs and time-based labor costs:

$$\min Z = \Sigma_{k\in D} \Sigma_{v\in V_k} \Sigma_{(i,j)\in A} (c_v \cdot d_{ij} + c_L \cdot t_{ij}) \cdot x_{ijkv}$$

where x_{ijkv} is a binary decision variable equal to 1 if vehicle v from depot k traverses arc (i,j), and c_v is the vehicle-specific cost coefficient (c_{EV} for electric vehicles, c_{GV} for gasoline vehicles).

1.4 Constraints

Customer Visit Constraint: Each customer must be visited exactly once by exactly one vehicle from its assigned depot.

Flow Conservation: For each vehicle, the number of arrivals at a node must equal the number of departures.

Vehicle Range Constraint: The total distance traveled by each vehicle must not exceed its maximum range (Q_{EV} = 200 mi for EVs, Q_{GV} = 300 mi for GVs).

Working Time Constraint: The total time for each vehicle route must not exceed the maximum working time (T_{max} = 540 minutes).

2. Customer Assignment via K-Means Clustering

Customers are partitioned into K = 3 regions using K-means clustering, which minimizes the within-cluster sum of squared distances:

$$\min \Sigma_{k=1}^{K} \Sigma_{i \in C_k} || p_i - \mu_k ||^2$$

where $p_i = (x_i, y_i)$ denotes the coordinates of customer i, and μ_k is the centroid of cluster k. This approach ensures geographically compact service regions for each depot.

3. Distance and Time Computation

The travel distance between nodes incorporates both inter-node travel and intra-node service distances:

$$d_{ij} = 2 \times 0.621 \times 10^{-3} \times \sqrt{[(x_i - x_j)^2 + (y_i - y_j)^2]} + \delta_i \times \rho$$

where δ_i is the demand at node i and ρ is the distance between drops within a location. Travel time includes inter-node travel, intra-node travel, and service time components.

4. Computational Results

4.1 Configuration Comparison

Configuration	EVs	GVs	Total Cost ($)	Distance (mi)	Active Vehicles
All GV	0	30	6,568	2,908	18
Mixed (30% EV)	9	21	6,200	2,931	19
Mixed (50% EV)	15	15	5,971	2,939	19
Higher costs	9	21	6,394	2,918	18

4.2 Cost Reduction Analysis

The percentage cost reduction from fleet electrification (50% EV deployment) compared to an all-GV fleet:

$$\Delta Z = (Z_{GV} - Z_{mixed}) / Z_{GV} \times 100\% = (6568 - 5971) / 6568 \times 100\% \approx 9.1\%$$

4.3 Per-Depot Performance

Depot	Customers	Cost ($)	Avg Distance (mi)	Utilization
1	30	2,219	167.3	55.8%
2	23	1,964	156.0	52.0%
3	16	1,788	138.7	46.2%

5. Solution Methodology

The problem is solved using a decomposition approach where each depot's subproblem is optimized independently. The solution procedure consists of:

1. Initial Solution Generation: The PATH_CHEAPEST_ARC heuristic constructs an initial feasible solution by iteratively adding the cheapest available arc.

2. Solution Improvement: Guided Local Search (GLS) metaheuristic improves the solution by penalizing frequently used arcs: $c'_{ij} = c_{ij} + \lambda \times p_{ij}$

3. Termination: The solver terminates after $\tau = 5$ seconds per depot.

6. Key Findings

1. Fleet electrification (50% EV deployment) reduces total operational costs by approximately 9.1% compared to an all-GV fleet.

2. K-means clustering effectively partitions customers into geographically compact service regions, enabling efficient multi-depot operations.

3. Average vehicle range utilization of 51.3% indicates potential for further fleet size optimization.

4. The Guided Local Search metaheuristic achieves near-optimal solutions within 5 seconds per depot, demonstrating computational efficiency.