

*"Per aspera ad astra..."*

---

# Contents

---

<b>Introduction</b>	<b>3</b>
<b>1 Introduction to the stars in high energy</b>	<b>4</b>
1.1 Motivation . . . . .	5
1.2 Aim of this work . . . . .	6
1.3 Observations . . . . .	7
1.3.1 Optical and IR observations . . . . .	8
1.3.2 X-ray observations . . . . .	8
1.3.3 Gamma ray observations . . . . .	9
<b>2 White Dwarfs</b>	<b>10</b>
2.1 First look of white dwarfs interior . . . . .	10
2.2 Fermi energy . . . . .	12
2.3 Degeneracy . . . . .	13
2.4 The Chandrasekhar limit . . . . .	16
2.5 White dwarfs classes . . . . .	18
<b>3 Cataclysmic variable stars</b>	<b>20</b>
3.1 Non magnetic cataclysmic variables . . . . .	20
3.2 Magnetic cataclysmic variables . . . . .	20
3.2.1 Polars . . . . .	20
3.2.2 Intermediate polars . . . . .	20
3.3 Galactic population of cataclysmic variables . . . . .	20
3.4 Others important creatures . . . . .	20
3.5 GXRE . . . . .	20

---

<b>4</b>	<b>Masses of white dwarfs in intermediate polars</b>	<b>21</b>
4.1	Breaking radiation . . . . .	21
4.1.1	Bremsstrahlung . . . . .	21
4.1.2	Thermal bremsstrahlung . . . . .	21
4.2	Synchrotron radiation . . . . .	21
4.3	Post shock region . . . . .	21
4.4	WD mass estimations methods . . . . .	21
<b>5</b>	<b>Data analysis</b>	<b>22</b>
5.1	INTEGRAL . . . . .	22
5.2	XMM-Newton . . . . .	22
5.3	Results . . . . .	22
5.4	Discussion . . . . .	22
<b>6</b>	<b>Conclusions</b>	<b>24</b>
	<b>Bibliography</b>	<b>26</b>
	<b>Apendix</b>	<b>27</b>

---

## Introduction to the stars in high energy

---

Let your imagination soar. By sitting on the old rocker looking at the sky with couple of good old whiskey you can easily start thinking about the universe. You are looking at a heck of a different kinds of cosmic objects, but suddenly you see almost only the stars. Almost all the shiny dots on the sky are stars and these stars are only the closest ones. Yes, you can see few other galaxies by naked eye<sup>1</sup>, but none of the exotic cosmic objects you are imaging about. They are too faint to be observed easily, because they are not only far, far away, but they also usually shine on different wavelengths, not visible by human eye.

Think about distances in the universe. One of the most accurate explanation is that from: Adams (1979) *"Space," it says, "is big. Really big. You just won't believe how vastly, hugely, mindbogglingly big it is. I mean, you may think it's a long way down the road to the chemist's, but that's just peanuts to space..."*

Consider this, sometimes you want to study processes in these extreme, very faint objects, but they are too faint and too far in the universe. You are looking for "laboratory" with similar processes, but located much closer to the observer. The X-ray binary stars can be this kind of laboratories.

There is, of course, many interesting phenomena which could be studied in X-ray binaries or in non-binary X-ray stars. Several of them are mentioned in the motivation section.

I am mentioning many interesting things in this work, but the main effort is taken to study post shock region in the Intermediate Polars (IPs).

---

<sup>1</sup>M31 and M33 in extremely good conditions on northern hemisphere and Magellanic clouds on southern one

## 1.1 Motivation

We can easily find many reasons why to study stars in the high energy bands. We can consider the direct and the most common scientific applications like observations of the supernovae, black holes & neutron stars in X-ray binaries. But for the education purposes I prefer several others, very nice examples closer to topic of this work.

- **Relativistic jet phenomena:** like it was proposed by Mirabel (2002) that universal mechanism should be at work in all the relativistic jet sources in the universe. Better understanding of sources including: microblazars, AGNs and gamma-ray burst will help to gain more comprehensive understanding of these phenomena. Microblazars can play role of “space laboratories”, where interesting processes last on different timescales as is the case with AGNs or GRBs.

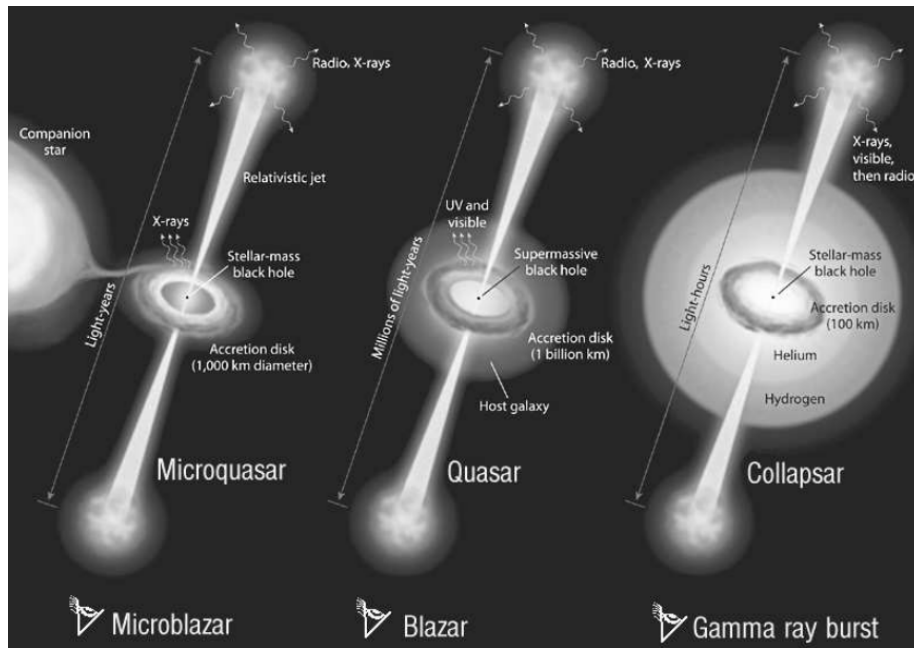


Figure 1.1: NOT in scale diagram, showing current ideas of micro-quasars, AGNs and gamma-ray bursts as space objects driven by same, universal mechanism Mirabel (2002).

- **Galactic ridge X-ray emission (GRXE):** various physical processes contribute to brightness of GRXE in different bands, but several studies in 3-20 keV provide evidence that diffuse X-ray radiation originates from huge number of stellar X-ray sources, mostly coronally active stars and white dwarf X-ray binaries. In particular for the energies over 20 keV to 200 keV is

spectrum very similar to spectrum of magnetic white dwarf binaries – e.g. Intermediate polars (IP) and polars (P). Krivonos et al. (2007)

- **White dwarfs masses in Intermediate Polars (IP):** as was proposed in Rothschild et al. (1981), the temperature of the post shock region (PSR) depends on WD mass. Therefore the X-ray spectrum can be used for WD mass determination Suleimanov et al. (2005). The WD mass estimations in cataclysmic stars is in general complicated. Usually the curve of radiation velocities can be used, but it is quite hard to construct and because of . Therefore X-ray spectrum method is very attractive for several reasons. This work is dedicated to this topic.

## 1.2 Aim of this work

To cover the whole topic: “stars in high energies” is far behind capacity of such master thesis, because of that I decide to aim on cataclysmic variable stars (CVs), especially to intermediate polars (IPs).

As it will be mentioned in next sections closely, IPs are magnetized CVs where the compact, primary star is white dwarf with  $B \sim 10^6 - 10^7$  Gauss. The mass accretion is taking place from, mainly low-mass, non-degenerate star through Roche lobe. Accretion disk is in some distance from WD surface destroyed by strong magnetic field and accretion continuous through, so called, accretion curtain across magnetic force-field.

Falling material in some point creates stationary shock near the WD surface where the kinetic energy is converted through thermal bremsstrahlung to radiation. The temperature of such created plasma is typically more than 10 keV with low density. The optically thin hard X-ray<sup>2</sup> emission is taking place and heated gas creates post shock region (PSR) with temperature gradient. The hot gas then descends and cools by X-ray emission while it hits the WD surface.

Because of relatively high temperature of PSR are IPs very good observed in hard X-rays band. IPs are only small fraction  $\sim 15\%$  of all CVs, but they dominate in hard X-ray band over 10keV, and most  $\sim 80\%$  of detected CVs are IPs Landi et al. (2009).

The temperature of PSR depends in first order only on WD mass, which is the most fundamental parameter of WDs. This means, that if we are able to find temperature from fitting thermal bremsstrahlung model to spectrum of IP, we are also able to establish the WDs mass.

An accreting WDs are very important for cosmology, because some of such objects probably cause Type Ia supernovae, when the WD mass reaches Chandrasekhar limit.

As is showed on figure 1.2, IP as NY Lup are well observed by INTEGRAL/IBIS detector which makes them interesting space laboratories for WD basic parameters

<sup>2</sup>In this case, hard X-rays means 10 - 120 keV region.

study. In some cases, it can also be studied if the accretion stream is strong enough.

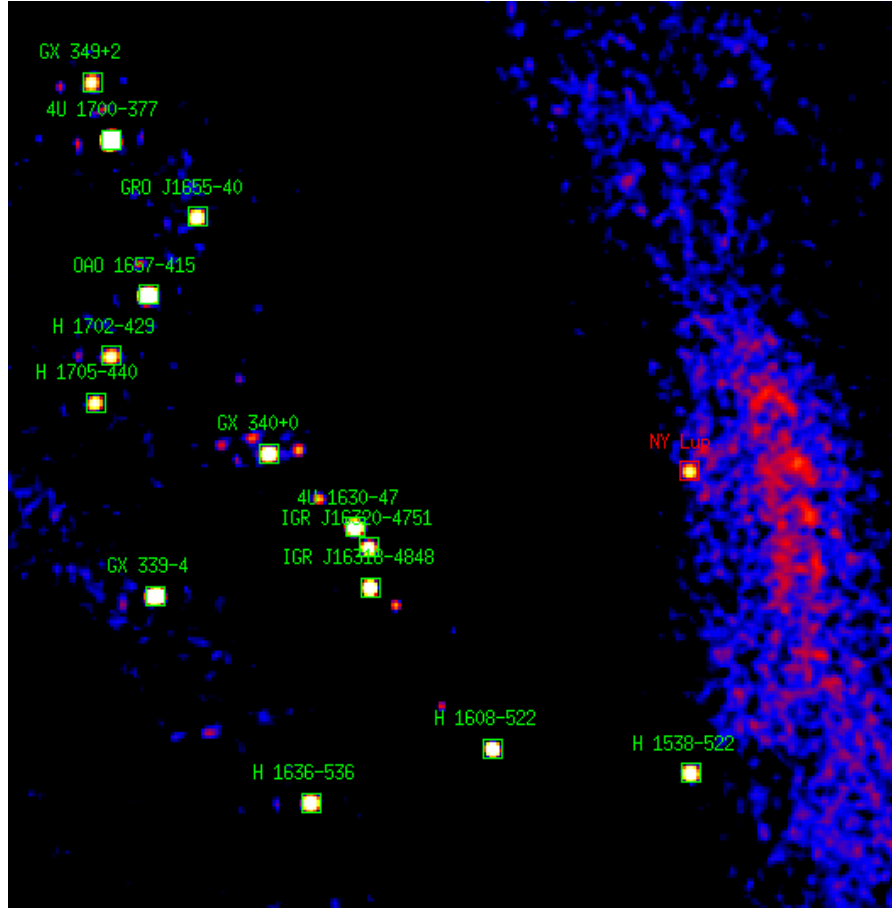


Figure 1.2: 1.2 Msec exposure of NY Lup region in 17-80 keV. The NY Lup is marked by red square. There are many others X-ray sources, mostly HMXBs or LMXBs which is because of .

### 1.3 Observations

Cataclysmic Variable stars (CVs) have been, in fact, observed as early as ancient times. In historical records of many civilizations we can find references for various astronomical events. Mostly they are about temporary objects: planets, Moon and Sun. However several are about comets and new stars. Rightly, these new stars are in many cases novae and supernovae. In China, the records date back to 1500 AD.

Many records are saved from medieval time, for example positions of *Nova Vulpecula 1670* and *Nova Cygni 1600* (now known as P Cygni) in Hevelius maps.

With progress of astronomical photography in late 19th century started era of continuous observations and with development of first photo-multipliers in the

mid-1940s CVs were begun attractive targets because of their big variability in different time scales.

It is suitable to mention, that AAVSO has light curve of SS Cyg from 1896 up to date.

### 1.3.1 Optical and IR observations

The very first visual observation was follows by photographic photometry and then spectroscopy, follows by photo-multiplier photometry since mid-1940s. The binary nature of all the CVs was confirm. The flickering was discovered and was assumed that it is somehow connected to stars duplicity Walker (1957), Warner (1995).

Statistical studies by Luyten and Hughes in mid-1960th showed, that novae remnants have  $M_V \approx 4$  and dwarf novae at quiescence have  $M_V \approx 7.5$ . They conclude that the hot primary star in CVs must be WD or hot subdwarf Warner (1995).

The most important contribution of optical astronomy to this work is the discovery of large and variable circular polarization in several CVs. This helped to identify magnetic CVs, which were later divided into two categories, polars and intermediate polars.

There are more important discoveries in optical and IR bands in CVs subject. In case of interest the Warner (1995) is proper book.

### 1.3.2 X-ray observations

The very first CV detected in X-rays was the EX Hya observed by Uhuru X-ray space mission. Uhuru works in 2.0 – 6.0 keV and in spite of its poor sensitivity the well-known 4U catalogue was created Forman et al. (1978) from its observations.

The NASA's HEAO<sup>3</sup> program follows with three space missions. As the X-ray detectors technology evolves, the number of detected CVs grown linearly. EXOSAT provided long and uninterrupted data of many CVs during his operation from May 1983 until April 1986. Similar results were obtained from Soviet mission Kvant 1 and Japan's Ginga. The high hopes were entered into ROSAT which provides all-sky survey in the 0.1 – 2.0 keV but expected huge number of new CVs was not discovered.

The situation slightly changes with RXTE<sup>4</sup> which after years on orbit provides good data for several articles about WD masses Suleimanov et al. (2005). The data from RXTE are used in new articles even  $\sim 15$  years after its launch Butters et al. (2011).

Several others missions were launched in last ten years period. Few of them carried several detectors where one was sensitive in X-rays, like SUZAKU/XIS and Swift/XRT. But for the X-ray astronomy was been the year of 1999 the most important ever. The two major big observatories was launched on the Earth's orbit.

<sup>3</sup>High Energy Astronomy Observatory, The HEAO 2 was also known as The Einstein Observatory

<sup>4</sup>Rossi X-ray Timing Explorer



The Chandra X-ray Observatory onboard STS-93 space shuttle Columbia on July and the XMM-Newton launched onboard ESA's Ariane 5 rocket.

That was the beginning of the X-ray astronomy's golden era. During last decade the combination of Chandra and XMM provides enormous data archives which will be useful for astronomers for another decades.

Sadly, there will not be such big observatory in X-rays for several decades. Only bigger space mission is Japan's ASTRO-H with several X-ray and gamma ray detectors on-board to cover broad high energy bands. The future of big ESA & NASA space mission Athena (formerly: Constellation-X, XEUS, IXO) is questionable because of budget cuts in both space agencies.

Fortunately, there are several data archives with open data for anybody interested. This is big challenge mainly for young astronomers, who are not in any big space mission program but want to do science. In this case, they don't need any special hardware, even modern laptops are powerful enough.

### 1.3.3 Gamma ray observations

In last millennium several space mission observed few CVs in bands from tens of keV to TeV.<sup>5</sup> The biggest breakthrough came with ESA's INTEGRAL space mission which was able with its sensitivity and large field of view observed many CVs. Mostly intermediate polars. Only  $\sim 2\%$  of all CVs are actually magnetic ones, but those ones are only visible in gamma rays. INTEGRAL/IBIS was been used to determine white dwarf masses by Landi et al. (2009).

Two others space missions have on-board detectors similar to INTEGRAL/IBIS with their sensitivity and coverage: the NASA's Swift/BAT and Japan's Suzaku/XRT. Both are widely used to study white dwarf masses in IPs Brunschweiler et al. (2009), Yuasa et al. (2010).

---

<sup>5</sup>The most studied CV from this era is AE Aqr (Meintjes 1990; Bowden et al. 1991)

---

## White Dwarfs

---

White dwarfs born when normal mass stars die. WDs are degenerated, late type stars with typical mass  $\sim 1M_{\odot}$ . Their typical radius is about 5000 km and mean density around  $10^6 \text{ g.cm}^{-3}$  Shapiro & Teukolsky (2004). They no longer burn nuclear fuel and if they don't have any other matter influx e.g. by accretion from close star, they slowly cool as they radiate away residual thermal energy.

WDs support themselves against gravity by the pressure of electron degenerate gas and their interior is in the local thermal equilibrium, except the thin atmosphere.

### 2.1 First look of white dwarfs interior

WDs are a class of the less compact objects among the possible endpoints of the stellar evolution. The mass of the star is the main factor determining whether the star ends up as a WD, neutron star or a black hole. The medium mass stars with masses  $M \lesssim 4M_{\odot}$ <sup>1</sup> in some point of late state of their evolution gently spread mass forming planetary nebulae. The rest of the star becomes the white dwarf.

Let's use data from the table 2.1 and try to assume the pressure inside of WD. For very rough estimation, we can use equation of mechanic equilibrium to compare WD with the Sun.

$$dP = -G \frac{M \rho dr}{r^2} \quad (2.1)$$

The ratio between variables  $P, M, \rho$  and  $r$  in the Sun case and  $P', M', \rho', r'$  in the WD's case can be written as follows:

---

<sup>1</sup> Also  $M \lesssim 8M_{\odot}$  can be found in some literature Padmanabhan (2001).

Table 2.1: Basic statistics of the compact objects Shapiro &amp; Teukolsky (2004)

Object	Mass <sup>a</sup> [ $M$ ]	Radius <sup>b</sup> [ $R$ ]	Mean Density [ $r.cm^{-3}$ ]	Surface Potential [ $GM/Rc^2$ ]
Sun	$M_{\odot}$	$R_{\odot}$	1	$10^{-6}$
White Dwarf	$\lesssim M_{\odot}$	$\sim 10^{-2}R_{\odot}$	$\lesssim 10^7$	$\sim 10^{-4}$
Neutron Star	$\sim 1 - 3M_{\odot}$	$\sim 10^{-5}R_{\odot}$	$\lesssim 10^{15}$	$\sim 10^{-1}$
Black Hole	Arbitrary	$2GM/c^2$	$\sim M/R^3$	$\sim 1$

$$^a M_{\odot} = 1.989 \times 10^{33} g$$

$$^b R_{\odot} = 6.9599 \times 10^{10} cm$$

$$\begin{aligned} M' &= M, \\ r' &= 10^{-2}r, \\ \rho' &= 10^6\rho, \end{aligned}$$

From easy calculation we get:

$$dP' = -G \frac{M' \rho' dr'}{r'^2} = -G \frac{M \cdot 10^6 \rho \cdot 10^{-2} dr}{r^2 (10^{-2})^2} = 10^8 dP \quad (2.2)$$

From such results is eminent that there is something wrong with the WDs interior in comparison with central reion of the normal star. As the ionization is increasing by higher temperatures, the bigger pressure otherwise helps recombination. However very big pressure actually increase ionization up to totally ionized atoms. Atoms without their electrons shells are closer to each others which explains very high density.

For densities in range  $10^5 gm.cm^{-3} \lesssim \rho \lesssim 10^9 gm.cm^{-3}$  the WD is made of ideal nondegenerate gas of ions and a degenerate gas of electrons. The system will be degenerate if  $T > T_c$  where  $T_c \approx 3 \times 10^9 K (\rho/\rho_c)^{2/3}$

If the electrons are relativistic or not can be found by comparing  $m_e c$  with a fermi momentum

$$p_F = (3\pi^2)^{1/3} \hbar n_e^{1/3} = (3\pi^2)^{1/3} \hbar (\rho/\mu_e)^{1/3} \quad (2.3)$$

where

$$\mu_e = (\rho/n_e m_p) = 2(1+X)^{-1} \quad (2.4)$$

is the mass per electron. The fermi momentum will be equal to  $m_e c$  at the critical density:

$$\rho_c \equiv \frac{8\pi}{3} m + p \mu_e \frac{m_e c^3}{h} \approx 10^6 \mu_e . gm.cm^{-3} \quad (2.5)$$

If the density is higher than  $\rho \gtrsim 10^9 gm.cm^{-3}$ , the electrons are combined with protons inside of nuclei and create different matter called neutron degenerated gas. This is how neutron stars are made.

## 2.2 Fermi energy

Now we can imagine interior of WDs as an area full of atoms nuclei very close to each other with free electrons around them. But electrons are fermions, which means that they must behave according to Pauli exclusion principle and it allows only at most one fermion per each quantum state.

If we imagine a normal, everyday gas at standard temperature and pressure, only one of every  $10^7$  quantum states is occupied by gas particles, so Pauli exclusion principle limitation is very insignificant. When energy is removed from the gas and it's temperature falls down, an increasingly large fraction of the particles been forced into the lower energy states.

For fermions gas only one particle can take the lowest energy state and others must take another, higher and higher states, thus only one particle per state is allowed. Even in limit  $T \rightarrow 0$  pressure is produced by motions of electrons on excited positions.

At the zero temperature all of the lowest states and none of the higher states are occupied, this kind of fermion gas is called completely degenerated. The max energy of electron in completely degenerate gas at  $T = 0K$  is known as Fermi energy.

For determining the limiting energy we can imagine 3D box where length of each of its sides will be  $L$ . The wavelengths of electrons trapped in the box in each dimension will be

$$\lambda_{xyz} = \frac{2L}{N_{xyz}} \quad (2.6)$$

where  $N_{xyz}$  are integer quantum numbers for each dimension. The momentum can be written through de Broglie wavelength

$$p_{xyz} = \frac{hN_{xyz}}{2L} \quad (2.7)$$

We can now write the total kinetic energy of the electron when  $p^2 = p_x^2 + p_y^2 + p_z^2$  as follows

$$\epsilon = \frac{p^2}{2m} \Rightarrow \frac{h^2 N^2}{8mL^2} \quad (2.8)$$

Total number of electrons is same as total number of unique quantum numbers  $N_x, N_y, N_z$ , multiply by factor two. The two factor comes from the fact that electrons are particles with spin, which can be  $\pm 1/2$ . This means that two electrons are allowed to have the same tree quantum numbers but different spin. The total number of electrons out to radius  $N = \sqrt{N_x^2 + N_y^2 + N_z^2}$  will be

$$N_e = 2 \left( \frac{1}{8} \right) \left( \frac{4}{3} \pi N^3 \right) \Rightarrow N = \left( \frac{3N_e}{\pi} \right)^{1/3}. \quad (2.9)$$

By using Eq. (2.8) and simplifying it, we will get the Fermi energy as follows

$$\epsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \quad (2.10)$$

where  $m$  is the mas of the electron or any other fermion<sup>2</sup> and  $n \equiv N_e/L^3$  is the electron count per volume. The average energy per electron at  $T = 0K$  is  $\frac{3}{5}\epsilon_F$ .

## 2.3 Degeneracy

Matter inside of WD has very symmetric spherical distribution, we can calculate the mass interior to radius  $r$  like<sup>3</sup>

$$\frac{dm_{(r)}}{dr} = 4\pi r^2 \rho \quad (2.11)$$

In normal case the WD is in steady state and gravitation force is balanced by the pressure at every point. For deriving the hydrostatic equilibrium equation we need to consider an infinitesimal element laying between  $r$  and  $r + dr$  with an area  $dA$ . The element is lying perpendicular to the radial direction, should looks like fig.(2.1).

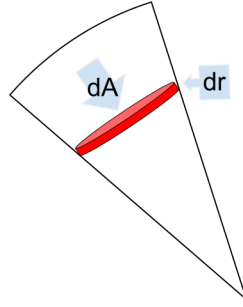


Figure 2.1: How could be infinitesimal fluid element laying between  $r$  and  $r + dr$  with an area  $dA$  imagine

The gravitation attraction between mass  $dm = \rho dA dr$  and  $m_{(r)}$  is the same as if  $m_{(r)}$  were only point at the center with the same mass. While the outside mass exerts no force on  $dm$ . Then the net outward pressure forced on  $dm$  is  $-[P(r + dr) - p(r)]dA$ . In equilibrium

$$\frac{dP}{dr} = -\frac{Gm_{(r)}\rho}{r^2} \quad (2.12)$$

In such equilibrium, the gradient of degeneracy pressure is balanced by gravitation:

$$\nabla P = -\rho \nabla \Phi, \quad (2.13)$$

<sup>2</sup>Can by applies for any fermion, not only electrons

<sup>3</sup>More elaborate and precises approach can by find Shapiro & Teukolsky (2004), Padmanabhan (2001), Kleczek (1957), Camenzind (2007), Nauenberg (1972) and simpler explanation with less math in Carroll & Ostlie (2007)

where  $\Phi$  means gravitation potential. Consequence of the Eq.(2.12) is the viral theorem. The gravitation potential energy of the star is then

$$W = - \int_0^R \frac{Gm(r)}{r} \rho 4\pi r^2 dr = \int_0^R \frac{dP}{dr} 4\pi r^3 dr = -3 \int_0^R P 4\pi r^2 dr \quad (2.14)$$

We can characterize the gas by an adiabatic equation of state, where  $K$  and  $\Gamma$  are constants

$$P = K \rho_0^\Gamma \quad (2.15)$$

Then Eq.(2.14) can be rewritten as follows

$$W = -3(\Gamma - 1)U, \quad (2.16)$$

Where  $U$  is the total star's internal energy

$$U = \int_0^R \epsilon' 4\pi r^2 dr \quad (2.17)$$

and  $\epsilon'$  comes from:

$$\epsilon' \equiv \epsilon - \rho_0 c^2 \quad (2.18)$$

where

$$\epsilon = \rho_0 c^2 + \frac{P}{\Gamma - 1} \quad (2.19)$$

Now we can see that the energy density, of the gas (excluding the rest mass energy) can be also written as

$$\epsilon' = \frac{P}{\Gamma - 1} \quad (2.20)$$

Assuming adiabatic changes, the Eq.(2.20) follows from the first law of thermodynamics

$$d\left(\frac{\epsilon}{\rho_0}\right) = -Pd\left(\frac{1}{\rho_0}\right). \quad (2.21)$$

The equation of state for ideal Fermi gas reduce to the simple polytropic form Eq.(2.15) in the two limiting cases Shapiro & Teukolsky (2004), Padmanabhan (2001):

- Nonrelativistic electrons,  $\rho_0 \ll 10^6 g.cm^{-3}$ ,  $x \ll 1$ ,  $\Phi_{(x)} \rightarrow x^5/15\pi^5$

$$\Gamma = \frac{5}{3} \implies K = \frac{3^{\frac{2}{3}} \pi^{\frac{4}{3}}}{5} \frac{\hbar^2}{m_e m_u^{\frac{5}{3}} u_e^{\frac{5}{3}}} = \frac{1.0036 \times 10^{13}}{u_e^{\frac{5}{3}}} cgs. \quad (2.22)$$

- Extremely relativistic electrons,  $\rho_0 \gg 10^6 g.cm^{-3}$ ,  $x \gg 1$ ,  $\Phi_{(x)} \rightarrow x^4/12\pi^2$

$$\Gamma = \frac{4}{3} \implies K = \frac{3^{\frac{1}{3}} \pi^{\frac{2}{3}}}{4} \frac{\hbar c}{m_u^{\frac{4}{3}} u_e^{\frac{4}{3}}} = \frac{1.2435 \times 10^{15}}{u_e^{\frac{4}{3}}} cgs. \quad (2.23)$$

This kind of the equilibrium configurations such these are called polytropes. We can use Eq.(2.13), take the divergence of it and by using  $\nabla^2\Phi = 4\pi G\rho$  in spehrically symmetric case we get

$$\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2}{\rho} \frac{dP}{dr} \right) = -4\pi G\rho. \quad (2.24)$$

Using few trics, Eq.(2.24) can be rewrite and reduce to dimension less form

$$\frac{1}{\xi^2} \frac{d}{d\xi} \xi^2 \frac{d\theta}{d\xi} = -\theta^n \quad (2.25)$$

use Eq.(2.15) with  $\Gamma \equiv 1 + \frac{1}{n}$  where  $n$  is called the polytropic index

$$\rho = \rho_c \theta^n \quad (2.26)$$

$$r = a\xi \quad (2.27)$$

$$a = \left[ \frac{(n+1)K\rho_c^{(1/n-1)}}{4\pi G} \right]^{1/2} \quad (2.28)$$

where  $\rho_c$  is the central density,  $\rho_c = \rho$  when  $r = 0$ .

The Eq.(2.25) is called the *Lane-Emden equation* for the structure of  $n$  index, the boundary conditions at the center such polytropic white dwarf are

$$\theta(0) = 1, \quad (2.29)$$

$$\theta'(0) = 0. \quad (2.30)$$

The condition (2.29) come dirrectly from Eq.(2.26) and condition (2.30) is derived from the fact that near the center of the star is  $m_{(r)} \approx 4\pi\rho_c \frac{r^3}{3}$ . If we use Eq.(2.12), we get

$$\frac{dP_{(\rho)}}{dr} = \frac{d\rho}{dr} = 0. \quad (2.31)$$

If we numerically integrate Eq.(2.25) using Eq.(2.29) and Eq. (2.30) as the boundary conditions, starting at  $\xi = 0$ . We will find that for  $n < 5$  ( $\Gamma > \frac{6}{5}$ ) this solution decrease monotonically with a zero at finite value  $\theta(\xi_1) = 0$ , which corresponds to the star's surface where  $P = \rho = 0$  and we get the equation for white dwarf radius

$$R = a\xi_1 = \left[ \frac{(n+1)K}{4\pi G} \right]^{1/2} \rho_c^{\frac{1-n}{2n}} \xi_1. \quad (2.32)$$

The mass of the WD can be found as follows

$$\begin{aligned}
 M &= \int_0^R 4\pi r^2 \rho dr \\
 &= 4\pi a^3 \rho_c \int_0^{\xi_1} \xi^2 \theta^n d\xi \\
 &= -4\pi a^3 \rho_c \int_0^{\xi_1} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) d\xi \\
 &= 4\pi a^3 \rho_c \xi_1^2 |\theta'(\xi_1)| \\
 &= 4\pi \left[ \frac{(n+1)K}{4\pi G} \right]^{3/2} \rho_c^{\frac{(3-n)}{2\pi}} \xi_1^2 |\theta'(\xi_1)|,
 \end{aligned} \tag{2.33}$$

then  $\rho_c$  can be eliminated between Eq.(2.32) and Eq.(2.33), which gives the mass-radius relations for polytropes Shapiro & Teukolsky (2004):

$$M = 4\pi R^{\frac{3-n}{1-n}} \left[ \frac{(n+1)K}{4\pi G} \right]^{\frac{n}{n-1}} \xi_1^{\frac{3-n}{1-n}} \xi_1^2 |\theta'(\xi_1)| \tag{2.34}$$

## 2.4 The Chandrasekhar limit

What will happen if  $\rho_c \rightarrow \infty$ ? This is one of the most important questions in the universe. Because how central density of WD increases by increase of the WD mass, the electrons become more and more relativistic throughout the star. As  $R \rightarrow 0$  also the space for free electrons moving between atoms nuclei become smaller and smaller. With increasing mass must also increase the speed of electrons to support the degeneracy pressure. While the mass of WD is larger the radius become smaller, mass-volume relations implies that  $\rho \propto M_{WD}^2$ . But the electrons can't have speed larger than speed of light and in some point where  $\rho \sim 10^9 \text{ g cm}^{-3}$  there will be no space left for moving electrons and they will be pressed into the nuclei. This will lead to explosion of such WD.

This all means, that there is a maximum mass for WDs. This mass is called *Chandrasekhar limit* <sup>4</sup>  $M_{ch}$ .

The classical theory of type 1A supernovae assumes explosions of such WDs reaching the  $M_{ch}$ , the very same mass of the WDs when they explode makes Type 1A supernovae perfectly suitable objects for measuring the long range space distances. That's why they are so important objects for cosmology and astronomers called them standard candles.

The Chandrasekhar limit for relativistic electron case is

$$M = 1.457 \left( \frac{2}{\mu_e} \right)^2 M_{\odot}, \tag{2.35}$$

---

<sup>4</sup>Named after Indian physicist Subrahmanyan Chandrasekhar who received the Nobel Prize in Physics in 1983 for his work on stellar structure and evolution. Chandrasekhar made his great discovery at the age of 21 in the 1931.



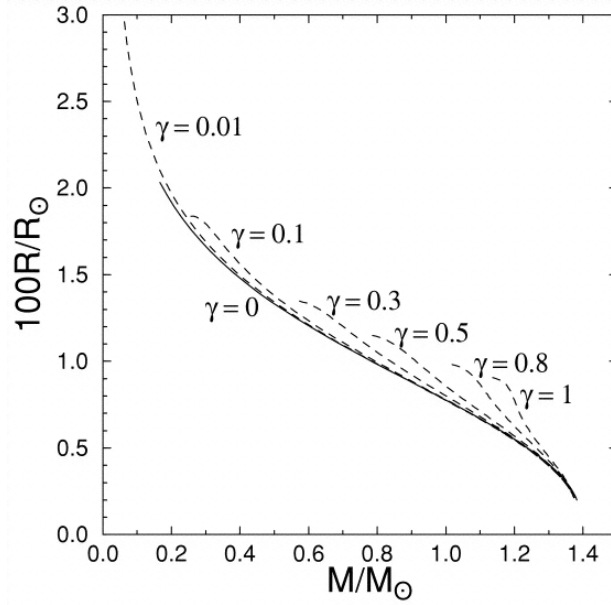


Figure 2.2: Relation between the mass  $M$  and radius  $R$  of a  $^{12}\text{C}$  magnetic white dwarf for the indicated magnetic field strengths. The solid line denotes the Hamada & Salpeter (1961) model for nonmagnetic white dwarfs ( $\gamma = 0$ ). The dashed lines are magnetic white dwarfs. Suh & Mathews (2000)

where  $\mu_e$  is the average molecular weight per electron, which depends on the chemical composition of the star and is typically sets to  $\mu_e = 2$ .

The Eq.(2.35) can be obtained by solving Eq.(2.34) with parameters<sup>5</sup>:

$$\Gamma = \frac{4}{3}, n = 3, \xi_1 = 6.89685, \xi_1^2 |\theta'(\xi_1)| = 2.01824. \quad (2.36)$$

Another, illustrativ aproach to estimating the Chandrasekhar limit is mention in Carroll & Ostlie (2007). An approximate value for  $M_{ch}$  may by obtain by setting the estimate of the central pressure

$$P_c \approx \frac{2}{3} \pi G \rho^2 R_{wd}^2 \quad \text{with} \quad \rho = M_{wd} / \left( \frac{4}{3} \pi R_{wd}^3 \right) \quad (2.37)$$

equal to electron degeneracy pressure equation

$$P = \frac{(3\pi^2)^{2/3}}{4} \hbar c \left[ \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{4/3}, \quad (2.38)$$

with  $\frac{Z}{A} = 0.5$ , the  $R$  cancels from the equation leving for the greatest possible mass:

$$M_{ch} \sim \frac{3\sqrt{2\pi}}{8} \left( \frac{\hbar c}{G} \right)^{3/2} \left[ \left( \frac{Z}{A} \right) \frac{1}{m_H} \right]^2 M_{\odot}. \quad (2.39)$$

<sup>5</sup>Extensive list of polytropic parameters can be find in Chandrasekhar (1939)

It is important to notice that Eq.(2.39) contains three fundamental constants  $\hbar, c$  and  $G$  representing the combined effects of quantum mechanics, relativity and Newtonian gravitation on the WD structure.

The Chandrasekhar limit slightly depends upon the chemical composition (2.35), but in the literature is well known as  $M_{ch} = 1.44M_{\odot}$ .

The similar limit exists for neutron stars, where pressure of degenerate neutron gas support the stars against its own gravity. This limit is called Tolman - Oppenheimer - Volkoff limit (or TOV limit). But it is not so easy in neutron stars case. Their limit mass depends strongly on type of matter in star center. The limit varies from  $0.7M_{\odot}$  up to  $\sim 3M_{\odot}$  in extreme case, when inner section of neutron star is partially filled up by quark - gluon matter.

## 2.5 White dwarfs classes

Previous sections describe interior and fundamental parameters of white dwarfs, but it is also important to tell something about their classes. WDs occupy narrow sliver line in the left bottom corner of H-R diagram fig.(2.3), slightly parallel with the main sequence.

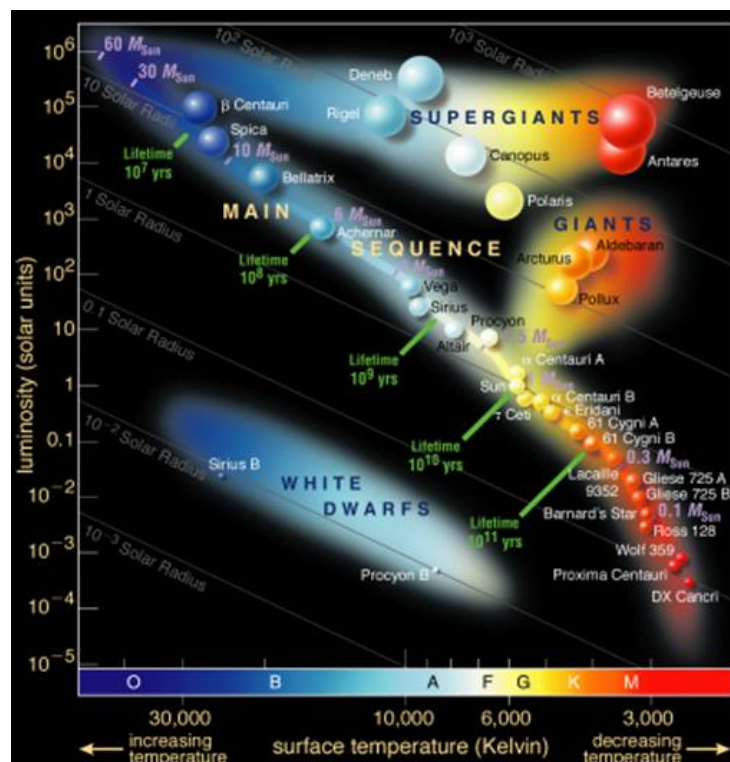


Figure 2.3: WD lies in right bottom corner of HD diagram, under main sequence, which exactly means, they are very small (dwarf) stars with high surface temperature.

---

## Cataclysmic variable stars

---

### **3.1 Non magnetic cataclysmic variables**

### **3.2 Magnetic cataclysmic variables**

#### **3.2.1 Polars**

#### **3.2.2 Intermediate polars**

### **3.3 Galactic population of cataclysmic variables**

### **3.4 Others important creatures**

### **3.5 GXRE**

---

## Masses of white dwarfs in intermediate polars

---

### 4.1 Breaking radiation

#### 4.1.1 Bremsstrahlung

$$a_{\parallel} = \dot{v}_x = -\frac{eE_x}{m_e} \frac{\gamma Z_e^2 v t}{4\pi\epsilon_0 m_e \left[ b^2 + (\gamma v t)^2 \right]^{2/3}} \quad (4.1)$$

#### 4.1.2 Thermal bremsstrahlung

### 4.2 Synchrotron radiation

### 4.3 Post shock region

### 4.4 WD mass estimations methods

---

## Data analysis

---

### 5.1 INTEGRAL

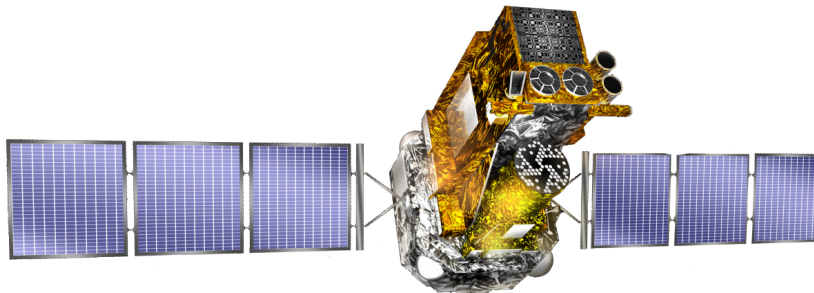


Figure 5.1: INTEGRAL

### 5.2 XMM-Newton

### 5.3 Results

### 5.4 Discussion

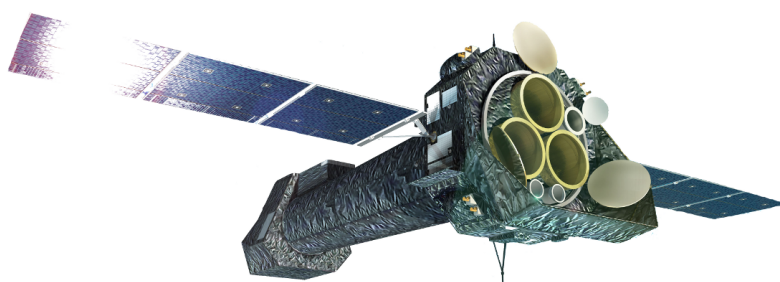


Figure 5.2: XMM-Newton

---

## Conclusions

---



---

## Bibliography

---

- Adams, D. 1979, *The Hitchhiker's Guide to the Galaxy* (Great publishing house of Ursa Minor Beta)
- Aizu, K. 1973, *Progress of Theoretical Physics*, 49, 1184
- Arnaud, Smith, S. 2011, *Handbook of X-ray Astronomy* (Cambridge University Press)
- Brunschweiler, J., Greiner, J., Ajello, M., & Osborne, J. 2009, *A&A*, 496, 121
- Butters, O. W., Norton, A. J., Mukai, K., & Tomsick, J. A. 2011, *A&A*, 526, A77
- Camenzind, M. 2007, *Compact Objects in Astrophysics* (Springer)
- Carroll, B. W. & Ostlie, D. A. 2007, *An Introduction to Modern Astrophysics*, second edition (Pearson Inc.)
- Chandrasekhar, S. 1939, *MNRAS*, 99, 673
- Forman, W., Jones, C., Cominsky, L., Julien, P., Murray, S., Peters, G., Tananbaum, H., & Giacconi, R. 1978, *ApJS*, 38, 357
- Frank, J., King, A., & Raine, D. 2002, *Accretion Power in Astrophysics Third Edition* (University Press, Cambridge)
- Hamada, T. & Salpeter, E. E. 1961, *ApJ*, 134, 683
- Kitchin, C. R. 2009, *Astrophysical Techniques fifth edition* (Taylor & Francis Group)
- Kleczek, J. 1957, *Interior of Stars* (Press of Czechoslovak Academy of Science)

- Krivonos, R., Revnivitsev, M., Churazov, E., Sazonov, S., Grebenev, S., & Sunyaev, R. 2007, *A&A*, 463, 957
- Landi, R., Bassani, L., Dean, A. J., Bird, A. J., Fiocchi, M., Bazzano, A., Nousek, J. A., & Osborne, J. P. 2009, *MNRAS*, 392, 630
- Mirabel, I. F. 2002, *ASP Conference Series*
- Nauenberg, M. 1972, *ApJ*, 175, 417
- Padmanabhan, T. 2001, *Theoretical Astrophysics Volume II: Stars and Stellar Systems* (Cambridge University Press)
- Revnivitsev, M., Sazonov, S., Krivonos, R., Ritter, H., & Sunyaev, R. 2008, *A&A*, 489, 1121
- Rothschild, R. E., Gruber, D. E., Knight, F. K., Matteson, J. L., Nolan, P. L., Swank, J. H., Holt, S. S., Serlemitsos, P. J., Mason, K. O., & Tuohy, I. R. 1981, *ApJ*, 250, 723
- Rybicki, G. B. & Lightman, A. P. 1979, *Radiative Processes in Astrophysics* (John Wiley & Sons, Inc.)
- Sazonov, S., Revnivitsev, M., Gilfanov, M., Churazov, E., & Sunyaev, R. 2006, *A&A*, 450, 117
- Shapiro, S. L. & Teukolsky, S. A. 2004, *Black holes, white dwarfs, and neutron stars: The physics of compact objects*, ed. Shapiro, S. L. & Teukolsky, S. A.
- Suh, I.-S. & Mathews, G. J. 2000, *ApJ*, 530, 949
- Suleimanov, V., Revnivitsev, M., & Ritter, H. 2005, *A&A*, 435, 191
- Warner, B. 1995, *Cataclysmic Variable Stars* (University Press, Cambridge)
- Yuasa, T., Nakazawa, K., Makishima, K., Saitou, K., Ishida, M., Ebisawa, K., Mori, H., & Yamada, S. 2010, *A&A*, 520, A25+

## **Appendix**

this will be the appendix

Table 1: Estimated WD masses from previous reports ...

System	Suzaku XIS+HXD $M_{WD}$	Swift BAT $M_{WD}$	RXTE PCA+HEXTE $M_{WD}$	RXTE PCA $M_{WD}$	Ginga LAC $M_{WD}$	ASCA SIS $M_{WD}$	This work XMM & Integral $M_{WD}$
FO Aqr							
XY Ari							
MU Cam							
BG CMi							
V709 Cas							
TV Col							
TX Col							
YY Dra							
PQ Gem							
EX Hya							
NY Lup							
V2400 Oph							
AO Psc							
V1223 Sgr							
RX J2133							
IGR J17303							