Lambda Calculus

A very important model of computation

Usually in math, we talk about functions like

Let $f(x) = x^2 + 3x - 4$

But sometimes it's inconvenient to name f

This is much less common than naming, but still happens

$$\left[x \longmapsto x^2 + 3x - 4\right]$$

Why useful? Often useful when dealing w/ spaces of functions

For example, if V is a vector space, V^* is vs of lin funcs from V to $\mathbb{R}$

$V \to V^{**}$ given by $v \longmapsto \left[f \longmapsto f(v)\right]$

$\lambda x.\ x^2 + 3x - 5$

Notion of anonymous functions is a feature of modern pl's: Javascript, lisp/scheme/clojure, haskell, ocaml, ...

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variables: $x, y, z, \ldots$

terms $= v \mid t_1 t_2 \mid \lambda v.t$

$\lambda x.x \qquad (\lambda x.\ x x)$

Rules are: $(\lambda x.\, t_1)\, t_2 = t_1[{}^{t_2}/_x]$

$(\lambda x.\, x^2+3)\,4 = 4^2+3$

Just these terms is universal!

$0 \quad (\lambda f\, \lambda x.\, x)$

$1 \quad (\lambda f\, \lambda x.\, f\, x)$

$2 \quad (\lambda f\, \lambda x\;\; f(f(x)))$

$\vdots$

true $= \lambda x y.\, x$

false $= \lambda x y.\, y$

if $= \lambda b$ then else. b then else

Recursion is via the Y-combinator

(also name of VC firm)

$Y = \lambda f\, (\lambda x\, (f\, x x))\, (\lambda x\, (f\, x x))$

don't show

$f(0) = 1$

$f(n+1) = n \cdot f(n)$

$f = x \longmapsto$ if $x=0$ then 1 else $x \cdot f(x-1)$

$f \longmapsto x \longmapsto$ ________________

Other directions

Types Normally think of

simply typed lambda calculus

$\downarrow$

polymorphic lambda calculus

Resource boundedness - Linear logic

$x \longmapsto (x, x)$ $\qquad \mathbb{R} \to \mathbb{R}^2$