Form of an optimization problem (for this class)
(Finite-dimensional deterministic optimization problem):

Finitely many variables $x_1, \dots, x_n$.
Real-valued: $x_1, \dots, x_n \in \mathbb{R}$.
space $\mathbb{R}^n$.

Vector: $x = (x_1, \dots, x_n) \in \mathbb{R}^n$.

Want to maximize or minimize a function ("objective function") $f(x)$.
$x = (x_1, \dots, x_n)$ "decision variables" model a decision. Domain of variables:

$x \in \mathbb{R}^n$ needs to be restricted to a domain that makes sense.

Simple domains:

an optimization problem with real (continuous) variables.

$$x \in X = \mathbb{R}^n$$

positive & negative real values

$$X = \mathbb{R}^n_{\geq 0}$$

nonneg.

an integer optimization problem (integer program)

$$X = \mathbb{Z}^n$$

$$X = \mathbb{Z}^n_{\geq 0}$$

only integer values are allowed.

x is subject to integrality

If we need both continuous & integer variables, this is a <u>mixed integer optimization problem</u>:

e.g. $x_1 \in \mathbb{Z}$, $x_2 \in \mathbb{R}$, $x_3 \in \mathbb{R}_{\geq 0}$, $x_4 \in \mathbb{Z}_{\geq 0}$.

$$x = (x_1, x_2, x_3, x_4) \in X = \mathbb{Z} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}.$$

Domain ✓.

Constraints. e.g. from budgets regarding time, material, workforce, limits on emissions etc.

After expressing decisions as values of variables x, we can express constraints as inequalities / equations.

Public transport case study...

$\longrightarrow$ Not just one optimization problem.

Discuss different aspects / subproblems that need solving.

Day to day basis: Assigning drivers to buses.

maximize convenience for student drivers.

Let $x \in \mathbb{R}^n$ model assignments of drivers to buses for the 9-12 shift. Drivers have a numerical preference for each bus! Data: Available buses. (might come in on short notice).

Availability of drivers: Data might come in in real time.

Preference data — less of a real time aspect.

Drivers perhaps update their preferences after having driven the bus . . . should factor into later decision.

How to model a decision?

Let $i \in I = \{1, \dots, m\}$ be the index set of all drivers.

Let $j \in J = \{1, \dots, n\}$ be the index set of all busses.

- What variables should I use to express an assignment of drivers to buses?

- What constraints hold?

Variables:

Drivers & buses as variables?

4 drivers : $\{1, \ldots, 4\}$

3 buses : $\{1, \ldots, 3\}$

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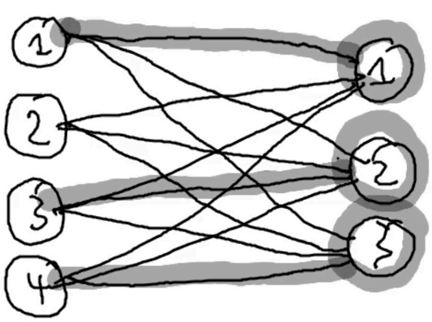
Simplification:
Fixed shift 9–12.
1 driver drives 1 bus for entire shift

Drivers | Buses



matching → graph theoretic view.

Bipartite graph:

$V = \{1,2,3\} \sqcup \{1,2,3,4\}$

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