

We have exponentially many  
(strengthened) cutset inequalities.

Need to solve (in a constraint  
generation scheme) the subproblem of  
finding for a solution  $\tilde{x}$  (to a  
relaxation) a violated cutset  
inequality! Again: an optimization  
problem:

$$\min \sum_{\substack{\{i,j\} \\ \in \delta(S)}} \tilde{x}_{\{i,j\}}$$

parameters,  
not decision  
variables

$$\text{s.t. } \emptyset \neq S \subset V$$

(a combinatorial optimization problem:  
"undirected min cut").

Observation:

- Feasible Solutions to these  
subproblems can already provide

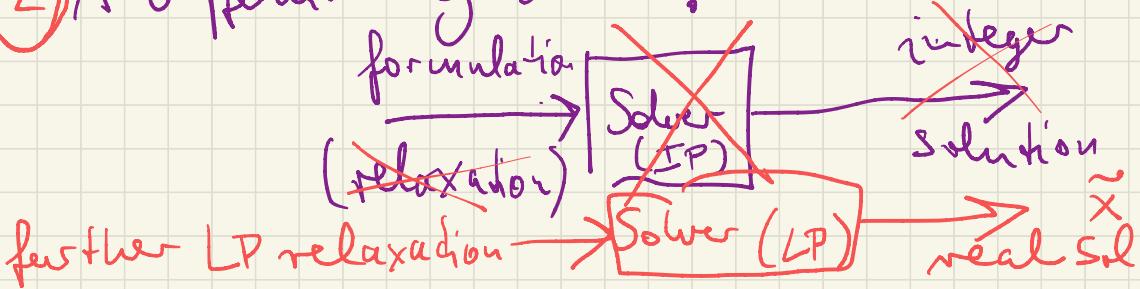
violated inequalities!

- Often heuristics for solving the subproblem are used:
- "Separation subproblems"  
"flip" primal heuristics to dual, proof devices.

Observation:

$\tilde{x}$  can be any vector —  
if does not have to be  
an integer solution.

② A different algorithm:



Run the same subproblem,  
find an inequality from the  
exponential-sized formulation  
that is violated by  $\tilde{x}$ .

Add this inequality to the LP  
and re-run the solver.

(-) Weaker relaxation  
(further relaxation  
by forgetting integrality)

(+) much faster to solve LP

"Solve the "root relaxation"  
of the problem using  
cutting planes."

LP-based cutting plane algorithm - for solving a linear optimization problem with exponentially many constraints.

(2(b))

Observation: We can implement this cutting plane algorithm using the dual simplex method.

- Formulation (LP) was solved using the simplex method  
→ have optimal dictionary:

$$\text{Max } \sum_i - \sum_{j \in N} (\bar{z}_j) x_j \geq 0 \quad \text{dual feasible}$$

s.t.

$$x_i = b_i + \sum_{j \in N} \bar{a}_{ij} x_j \geq 0 \quad i \in \mathcal{B}$$

primal feasible

$$x_i, x_j \geq 0$$

Add a constraint:

$$\sum_{j=1}^n d_j x_j \leq d_0$$

Original variables.

new slack

$$\sum_{j=1}^n d_j x_j + s = d_0$$

current basic & nonbasic

Idea : Make the slack variable  
 $s$  basic.

Eliminate the basic variables in

$$s = d_0 - \sum_{\substack{j=1 \\ j \in N}}^m d_j x_j - \sum_{\substack{i=1 \\ i \in B}}^m d_i x_i$$

by using the eqns of  
 the dictionary!

$$x_i = \bar{b}_i + \sum_{j \in N} \bar{a}_{ij} x_j$$

$$s = d_0 - \sum_{\substack{j=1 \\ j \in N}}^m d_j x_j - \sum_{\substack{i=1 \\ i \in B}}^m d_i \left( \bar{b}_i + \sum_{j \in N} \bar{a}_{ij} x_j \right)$$

$$= \underbrace{\left( d_0 - \sum_{\substack{i=1 \\ i \in B}}^m d_i \bar{b}_i \right)}_{=: \bar{s}} - \sum_{j \in N} \underbrace{\left( \dots \right)}_{=: \bar{d}_j} x_j$$

If we added an inequality that was violated by the current basic solution  $\tilde{x}$ , then the value of  $s$  for  $x = \tilde{x}$  is the right-hand side constant

$$\bar{s} = \left( d_0 - \sum_{i=1, i \in B}^m d_i \bar{b}_i \right)$$

So  $\bar{s} < 0$ .

$\Rightarrow$  New dictionary with the additional basic variable  $\bar{s}$

- is primal infeasible  
(one negative RHS const.)
- still dual feasible

⇒ Use the dual simplex method starting from this dictionary.

In practice, "a few" dual simplex steps suffice to restore primal feasibility

— fast update of the optimal dictionary after adding a violated constraint !!

Refinement:

When a constraint becomes loose (some later optimal basic solution has its slack in

the basis, positive value),  
then drop the constraint  
from the dictionary,  
 $\Rightarrow$  Method to keep the  
size of a problem  
during a cutting plane  
method small!