$$y_i = \begin{cases} 1 & \text{if batch } i \text{ is run} \\ 0 & \text{o/w.} \end{cases}$$

No-good constraints:

Write down inequalities that cut off one by one all values of (y_5, y_6, y_7) that are not allowed:

X	$(0, 0, 0)$	$y_5 + y_6 + y_7 \geq 1$
X	$(1, 1, 0)$	⋮
X	$(1, 0, 1)$	
X	$(0, 1, 1)$	

New approach:

Allowed are all 0/1 vectors (y_5, y_6, y_7) that are either $(0, 0, 1)$

or $(0, 1, 0)$

or $(1, 0, 0)$

or $(1, 1, 1)$

Treat them as vertices.

Model:

Solution

- (y_5, y_6, y_7) lies in the convex hull of (polyhedron with vertices)

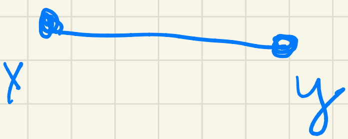
$(0, 0, 1)$
$(0, 1, 0)$
$(1, 0, 0)$
$(1, 1, 1)$

- and is an integer vector

$\mu = 1-\lambda$

Recall: Set S is convex if $\forall x, y \in S$, $\forall \lambda, \mu \geq 0$, $\lambda + \mu = 1$,

$$\lambda x + \mu y \in S$$



points on the line segment

More general convex combinations:

Take $x^1, \ldots, x^m \in S$

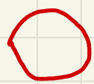
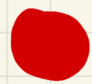
$$\lambda_1 + \cdots + \lambda_m = 1$$
$$\lambda_1, \ldots, \lambda_m \geq 0.$$

$$\Rightarrow \quad \lambda_1 x^1 + \cdots + \lambda_m x^m \in S.$$

Convex hulls of finitely many points: $G = \{x^1, \dots, x^m\} \subseteq \mathbb{R}^n$.

The convex hull (smallest convex set containing all of $x^1, \dots, x^m$) is

$$\mathrm{conv}\, G = \left\{ \sum_{i=1}^{m} \lambda_i x^i : \lambda_i \geq 0, \sum \lambda_i = 1 \right\},$$

(More generally: $G =$  $\mathrm{conv}\, G =$ 

Take any subset $G \subseteq \mathbb{R}^n$.

Then the convex hull of G is

$$\mathrm{conv}\, G = \left\{ \sum_{i=1}^{m} \lambda_i x^i : \exists m,\ \lambda_i \geq 0, \sum \lambda_i = 1, x^i \in G \right\}$$

Stronger — Carathéodory's theorem:

$$\mathrm{conv}\, G = \left\{ \sum_{i=1}^{n+1} \lambda_i x^i : \lambda_i \geq 0, \sum \lambda_i = 1, x^i \in G \right\}$$

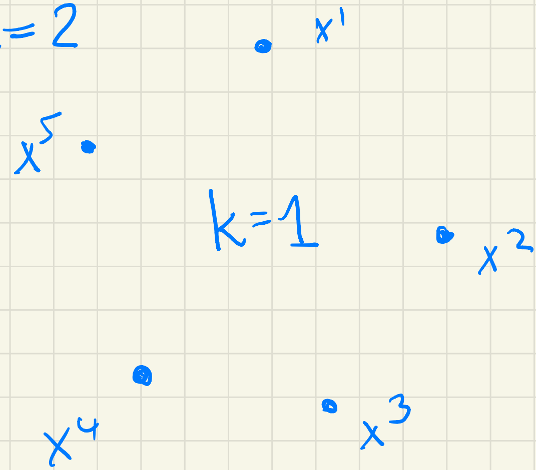
$n=2$

$G = \{x^1, \dots, x^5\}$

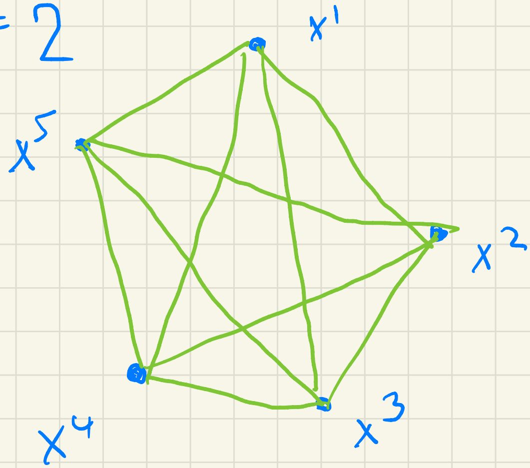
What does $\sum_{i=1}^{m} \lambda_i x^i$ look like? if $\sum \lambda_i = 1$, $\lambda_i \geq 0$.

Define $k := \#\{\lambda_i > 0\}$

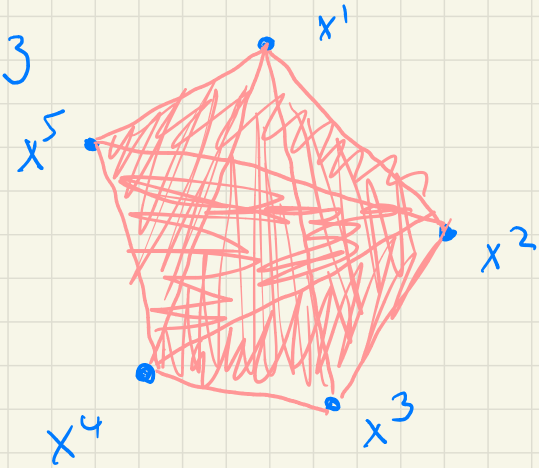
$k=1$



$k=2$



$k=3$



by Caratheodory, $k = 3 = 2+1$

Covered by triangles.

$\check{y} = (y_5, y_6, y_7)$ lies in the convex hull of (polyhedron with vertices)

$$\check{y}^1 = (0,0,1) \qquad \lambda_1$$
$$\check{y}^2 = (0,1,0) \qquad \lambda_2$$
$$\check{y}^3 = (1,0,0) \qquad \lambda_3$$
$$\check{y}^4 = (1,1,1) \qquad \lambda_4$$

$$\begin{aligned}\check{y} = {} & \lambda_1 (0,0,1) \\ & + \lambda_2 (0,1,0) \\ & + \lambda_3 (1,0,0) \\ & + \lambda_4 (1,1,1) = \sum_{i=1}^{4} \lambda_i \check{y}^i\end{aligned}$$

Write it component by component:

$$\boxed{\begin{aligned} y_5 &= \phantom{\lambda_1 + \lambda_2 +{}} \lambda_3 + \lambda_4 \\ y_6 &= \phantom{\lambda_1 +{}} \lambda_2 \phantom{{}+\lambda_3} + \lambda_4 \\ y_7 &= \lambda_1 \phantom{{}+\lambda_2+\lambda_3} + \lambda_4 \end{aligned}}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$

$$\lambda_1, \ldots, \lambda_4 \geq 0.$$

$$y_5, y_6, y_7 \in \{0, 1\}.$$

NB: Could also choose the multipliers to be $\lambda_i \in \{0,1\}$ instead of $\lambda_i \geq 0$.

This formulation has extra variables $\lambda_1, \ldots, \lambda_4$ in addition to the existing variables. "extended formulation"

This extended formulation is in general very strong! Projects to the convex hull.