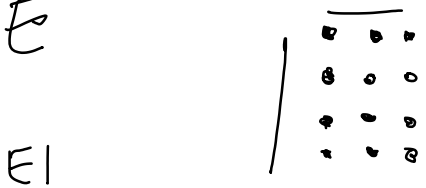
Recap: Assignment problem

Wednesday, October 7, 2020 8:48 AM

$\{i,j\}$ — i driver $\in I$
$\{i,j\}$ — j bus $\in J$

$$x_e = \begin{cases} 1 & \text{if } e \in M \\ 0 & \text{o/w.} \end{cases} \quad \text{for } e \in E$$

$$(x_e)_{e \in E} \subseteq \{0,1\}^{|E|} \subseteq \mathbb{R}^{|E|}$$



Express the constraint "at most 1 set to 1" as an inequality:

$$\sum_{i \in I} x_{\{i,j\}} \leq 1.$$

for fixed $j \in J$

a linear inequality

fixed index set of summation.

Revise the model in response to clarification

$$\sum_{i \in I} x_{\{i,j\}} = 1.$$

This with the other constraint

$$\sum x_{i\cdot\cdot} \leq 1 \quad \text{for fixed } i \in I.$$

$j \in J$ $\{i,j\}$

This completely models what vectors correspond to "feasible" assignments.

$x \in \{0,1\}^E$

Objective function:

Driver satisfaction is to be maximized.

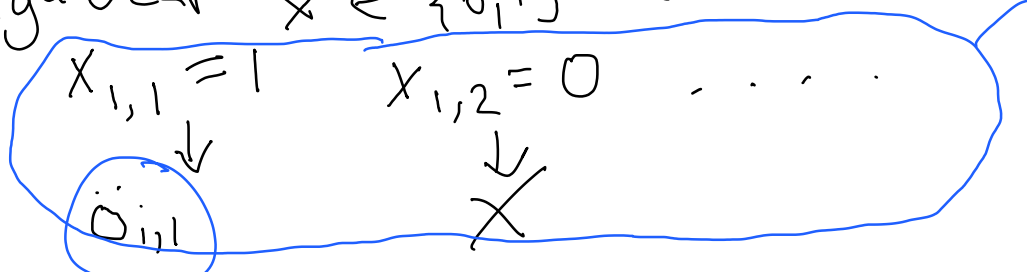
Focus on drivers assigned to buses.

Assume have satisfaction coefficient

$\ddot{\theta}_{\{i,j\}} \in \mathbb{R}$ ~~$\in \mathbb{R}$~~ given parameter (constant)

$\in [1,5]$ for every driver i every bus j.

What is total driver satisfaction given an assignment $x \in \{0,1\}^E$?



an linear function

$$\sum_{\{i,j\} \in E} \ddot{\theta}_{\{i,j\}} x_{\{i,j\}}$$

Have an opt. problem of the form

max linear fn. of x

s.t. linear fn. of $x \leq$ const.

linear fn of $x =$ const.

$x \in X = \{0,1\}^{\in}$ ||| integer linear optimization problem (ILP).

Solve using Gurobi (or other IP solvers).

> ... Company sells two models of ... five-leg tables. The basic version uses a wood top, requires 0.6 hours to assemble, and sells for a profit of $200. The deluxe model takes 1.5 hours to assemble and sells for a profit of $350. Over the next week ... 300 legs, 50 wood tops, 35 glass tops, and 63 hours of assembly available. ... determine a maximum profit production plan ...

Variables: x_B = # basic tables

x_D = # deluxe tables

} integers $\in \mathbb{Z}_{\geq 0}$ ← for counting indivisible goods. ||| Domain.

Constraints:

$0.6\,x_B + 1.5\,x_D \leq 63$ assembly time

$5\,x_B + 5\,x_D \leq 300$ # legs

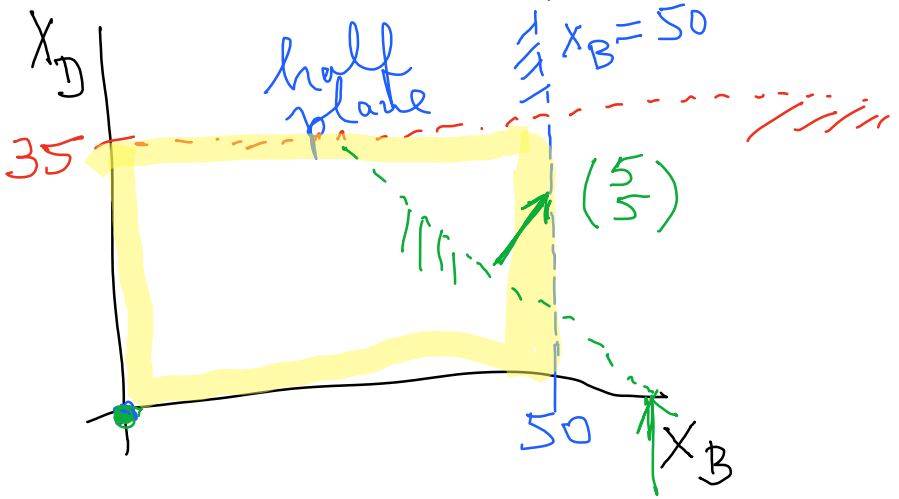
$x_B \leq 50$

$x_D \leq 35$

wood tops

glass tops.

2 variables — can plot this problem.



$\frac{300}{5} = 60.$

Normal form of a line:

$5x_B + 5x_D = 300$

$a_B x_B + a_D x_D = a_0$

$\binom{a_B}{a_D}$ is normal vector