x_D = # deluxe tables $\sim$ integers — for counting indivisible goods.

Constraints:

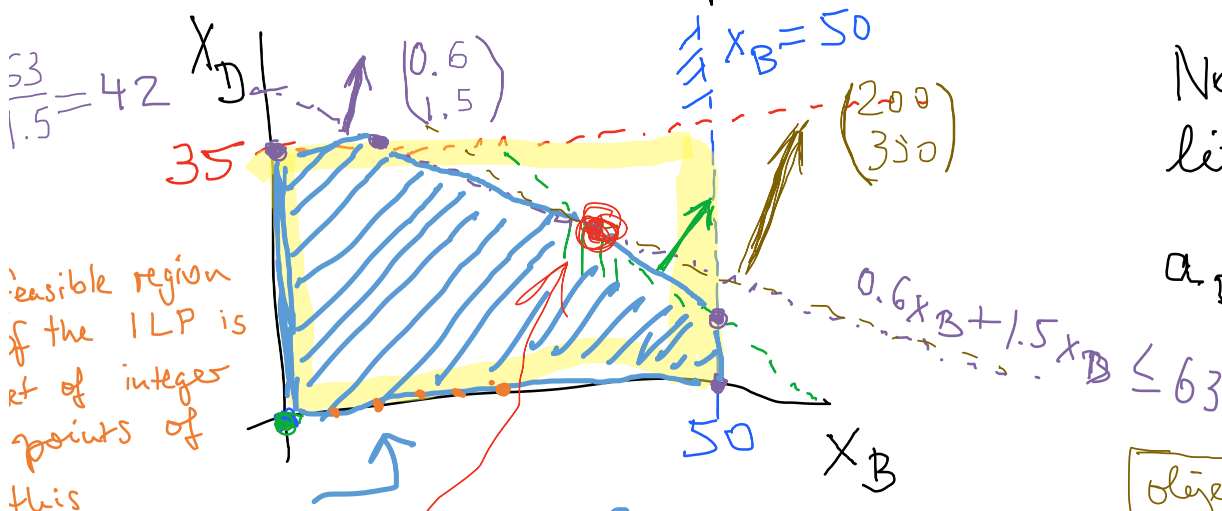
$$\boxed{0.6\, x_B + 1.5\, x_D \leq 63} \quad \text{assembly line}$$

$$\boxed{5\, x_B + 5\, x_D \leq 300} \quad \text{\# legs}$$

$$\boxed{x_B \leq 50} \quad \text{\# wood tops}$$

$$\boxed{x_D \leq 35} \quad \text{\# glass tops.}$$

2 variables — can plot this problem.



convex polyhedron of $\mathbb{R}^2$

Normal form of a line:

$$a_B x_B + a_D x_D = a_0$$

$\begin{pmatrix} a_B \\ a_D \end{pmatrix}$ is normal vector

$$\boxed{\begin{array}{l}\text{objective function} \\ \max\ 200 \cdot x_B + 350\, x_D\end{array}}$$

For the continuous relaxation: unique optimal solution (vertex of the polyhedron)

For a given objective value α:

$$\boxed{\begin{array}{c} 200\,x_B + 350\,x_D = \alpha \\ \text{normal eqn of a line (hyperplane)} \end{array}}$$

We will prove for linear optimization problems in "standard form": If an optimal solution exists, then there exists an optimal vertex solution ("basic" solution).

Generalize normal eqn of a line in $\mathbb{R}^2$: $a_1 x_1 + a_2 x_2 = a_0$

in $\mathbb{R}^3$: normal eqn of a plane (dim. 2): $a_1 x_1 + a_2 x_2 + a_3 x_3 = a_0$.

codimension $= 3 - 2 = 1$

Enter $\mathbb{R}^n$: Given a nonzero vector $a \in \mathbb{R}^n$, $a_0 \in \mathbb{R}$

a hyperplane with normal vector a:

$$\left\{ x \in \mathbb{R}^n : \; a^T x \left(= \sum_{j=1}^{n} a_j x_j \right) = a_0 \right\}.$$

| (codimension-1)

Inequalities for halfspaces:

$$\{x \in \mathbb{R}^n : \quad a^T x \le a_0\}.$$

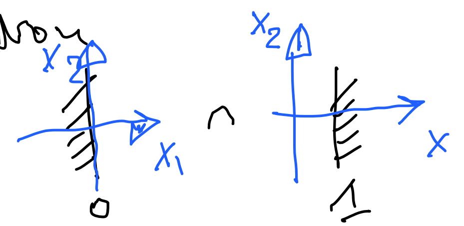
(Convex) polyhedron of $\mathbb{R}^n$:

Intersection of a finite family of halfspaces.

is allowed to be empty

$(\bigcap \emptyset = \mathbb{R}^n.)$ $\mathbb{R}^n$ is a polyhedron.

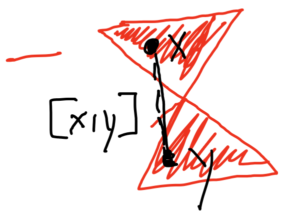
$\emptyset$ is a polyhedron

b/c $\emptyset =$ 

Not a polyhedron:

- Disk in dimension ≥ 2 is not a polyhedron.

If try to write it as an intersection of halfspaces... after any finite ... always finitely many "corners" (vertices) remaining.



non-convex polygon.

$\Rightarrow$ not a convex polyhedron.

Definition: A set $X \subseteq \mathbb{R}^n$ is **convex** if $\forall x, y \in X$, the line segment

$$[x,y] = \{ \lambda x + (1-\lambda) y : \lambda \in \mathbb{R}, \ \lambda \in [0,1] \}$$

$$\subseteq X.$$

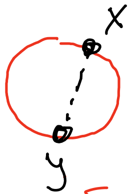
Theorem: Convexity is preserved under (finite or infinite) intersections:

X convex, Y convex

$\Rightarrow X \cap Y$ is convex.

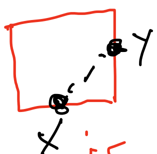
$\Rightarrow$ B/c every halfspace is convex, every (finite) intersection of halfspaces is convex

$\Rightarrow$ Convex polyhedra are convex.



$= \partial$  (the topological boundary of)

is not convex.



$= \partial$ 

is not a convex polyhedron