

2) Another method for finite termination:

Bland's rule (smallest index rule)

→ purely of interest in theory,
see textbook.

► Sometimes choices of variables

to enter/leave are not unique

→ "pivot rules" to make a choice.

■ Deterministic pivot rules:

entering: strategic

- Choose variable with largest obj coeff \bar{c}_j
- Steepest edge:

$$\max_{j \in N} \frac{\bar{c}_j}{\|\bar{A}_{\cdot \rightarrow j}\|}$$

↑
j-th col of \bar{A}

- primal lookahead:

Compute ratios &
actual objective increase
Pick the entering/leaving pair
with greatest increase

expensive

to make the rule deterministic

tie-broken

for example

lexico-graphically:

Among maximizers,
choose variable w/
smallest index

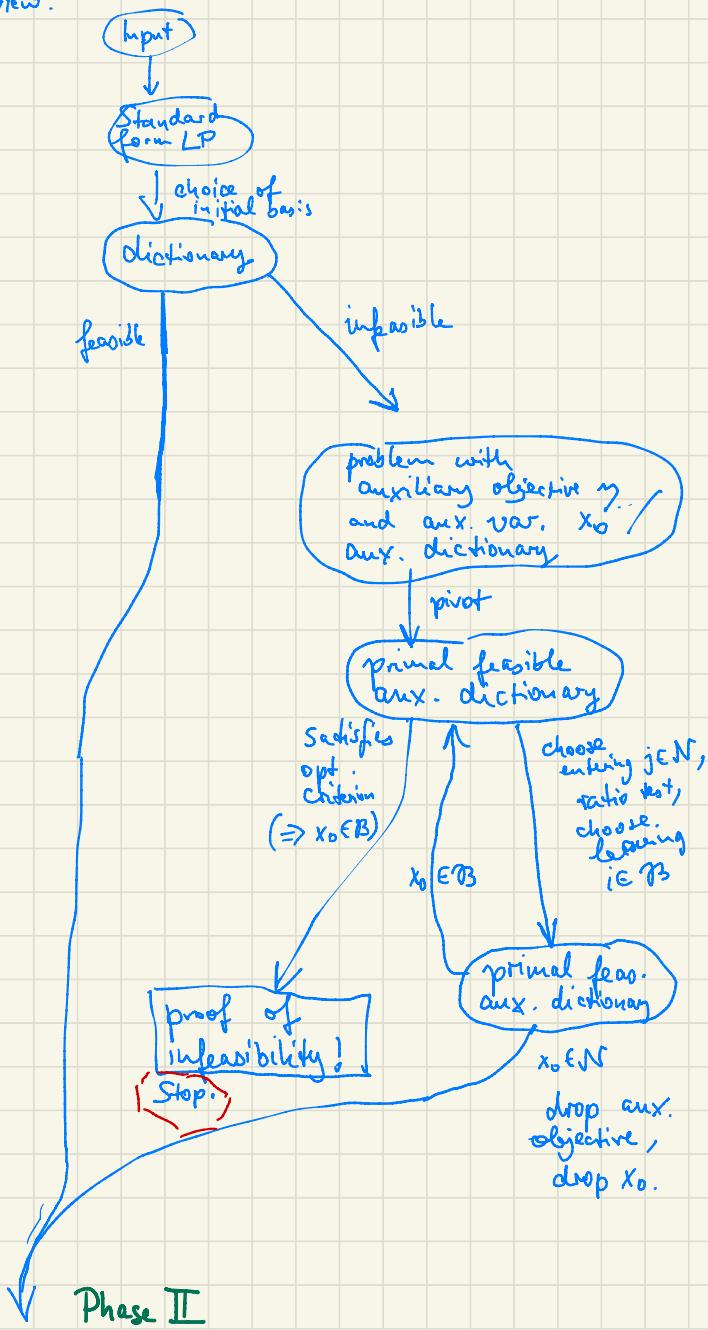
Random pivot rules

- e.g. Choose a nonbasic variable with $\bar{c}_j > 10\% \cdot \max \bar{c}_j$ randomly.
- useful for testing whether a problem can be expected to be solved with some simplex steps.

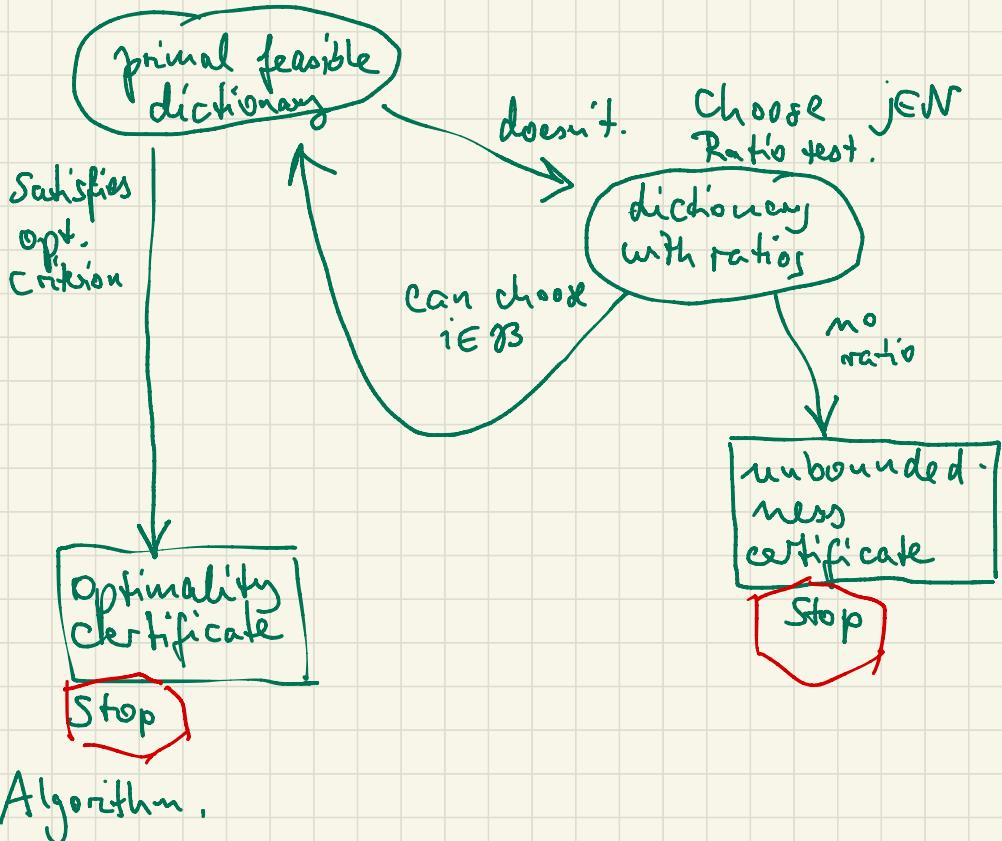
We focus on deterministic rules.

procedure
+ finiteness
+ determinism } \Rightarrow algorithm.

Review:



Phase II:



Theorem (Fundamental theorem of linear optimization)

Take an LP in standard form.

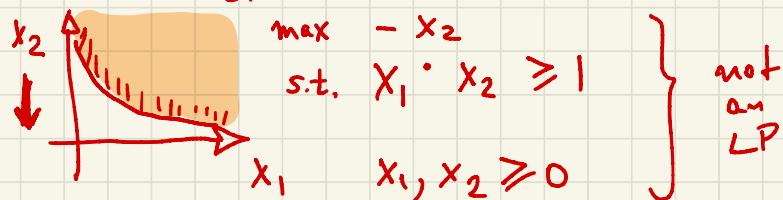
- 1) If there exists a feasible solution, then there exists a basic feasible solution.
- 2) If there exists an optimal solution, then there exists a feasible basis.

that satisfies the optimality criterion.

3) The problem is either infeasible, unbounded, or it has an optimal solution.

Why is 3) a nontrivial statement?

Consider



problem w/ topologically closed
but unbounded feasible region

has no optimal solution.

$$\sup \{-x_2 : (x_1, x_2) \in F\} = 0$$

⇒ sequence of solutions whose
obj value goes to 0.

∅ a feasible solution of obj value 0.

3) says: this cannot happen for LP.

Preview of LP theory development (later in class)

- Duality ... theory of optimality certificates
 - theory of bounds
 - theory of valid inequality.

→ Performance of the simplex method

- How many pivots are needed?
 - worst-case
 - average-case
 - smoothed analysis
 - data-dependent

• Numerical issues —

how to implement the steps fast

- revised simplex method
(avoid to rework the dictionary explicitly)

- factorization & factorization updates

→ Parametric & post-optimality analysis

How does the optimal solution change when we make changes to the input?