Logical AND

$$y_{ij} = x_i \wedge x_j$$
$$= x_i \cdot x_j$$

$y_{ij} \in \{0,1\}$

$x_i, x_j \in \{0,1\}$.

x_i	x_j	y_{ij}
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

No-good inequalities to cut off bad solutions:

Correct formulation:

0 0 1 $\quad x_i + x_j + (1 - y_{ij}) \geq 1$

$\Leftrightarrow \quad x_i + x_j \geq y_{ij}$

0 1 1 $\quad x_i + (1-x_j) + (1-y_{ij}) \geq 1$

$\Leftrightarrow \quad x_i - x_j - y_{ij} \geq -1$

1 0 1 $\quad (1-x_i) + x_j + (1-y_{ij}) \geq 1$

1 1 0 $\quad (1-x_i) + (1-x_j) + y_{ij} \geq 1$

$\Leftrightarrow \quad y_{ij} \geq x_i + x_j - 1$

Interpret $x_i + x_j \geq y_{ij}$:
(for 0/1 solutions) :

If $y_{ij} = 0 \implies$

$x_i + x_j \geq 0$
is implied by $x_i, x_j \geq 0$.

If $y_{ij} = 1 \implies$

$\boxed{x_i + x_j} \geq 1$

At least one of x_i & x_j
is 1.

Actually know a stronger
inequality! Know: If $y_{ij} = 1$,
both x_i and x_j are 1.

Consider $\boxed{\begin{cases} x_i \geqslant y_{ij} \\ x_j \geqslant y_{ij} \end{cases}}$

If $y_{ij} = 0 \Rightarrow$ both inequalities reduce to

$x_i, x_j \geqslant 0$

If $y_{ij} = 1 \Rightarrow x_i \geqslant 1,$

$x_j \geqslant 1.$

Is this system already a correct formulation?

No — b/c

$x_i, x_j = 1, \quad y_{ij} = 0$

is still a solution.

But together with

$y_{ij} \geqslant x_i + x_j - 1$

we have a correct IP formulation.

Modeling relations between 0/1 variables and real variables.

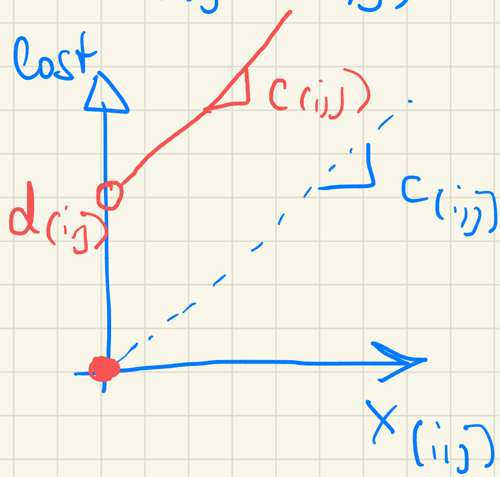
<u>Fixed charge</u>:

Cost: per unit

$x_{(i,j)}$

$c_{(i,j)} \, x_{(i,j)}$

additional cost if $x_{(i,j)} > 0$, ($\boxed{\text{fixed charge}}$) $d_{(i,j)}$

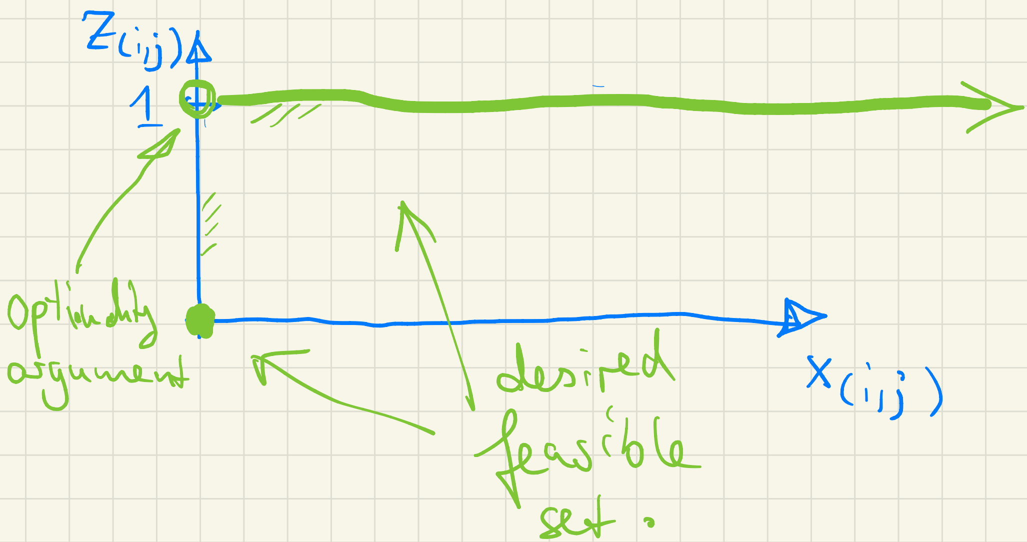


This nonlinear cost function can be modeled using an additional variable

$$z_{(i,j)} = \begin{cases} 1 & \text{if the fixed cost is charged} \\ 0 & \text{o/w.} \end{cases}$$

$$\text{Cost} = d_{(i,j)} z_{(i,j)} + c_{(i,j)} x_{(i,j)}$$

Link $x_{(i,j)}$ and $z_{(i,j)}$:



Bad news:

There are no linear inequalities that can model this in the space $(x_{(i,j)}, z_{(i,j)})$

Good news:

- Assume that fixed charge $d_{ij} > 0$.

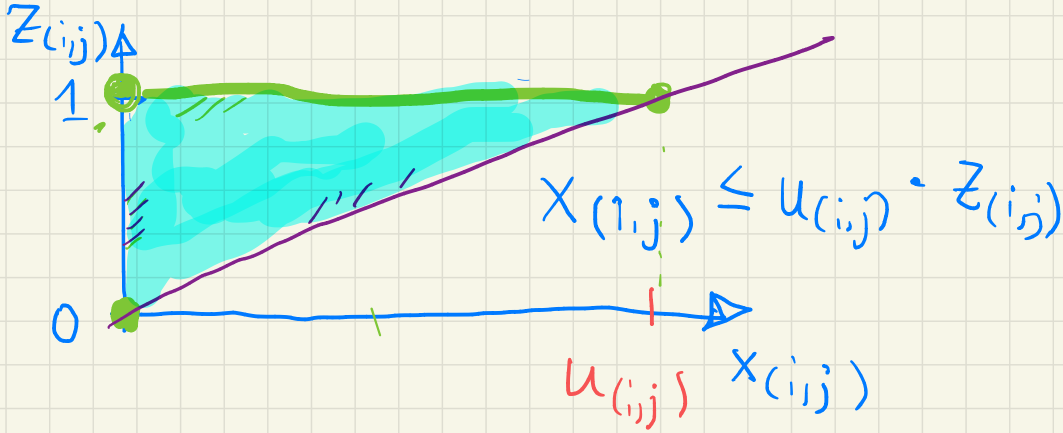
Optimality argument.

- In a minimization problem, no optimal solution can have

$$x_{i,j} = 0 \quad \text{and} \quad z_{(i,j)} = 1.$$

⇒ Include this point in the feasible region of the model.

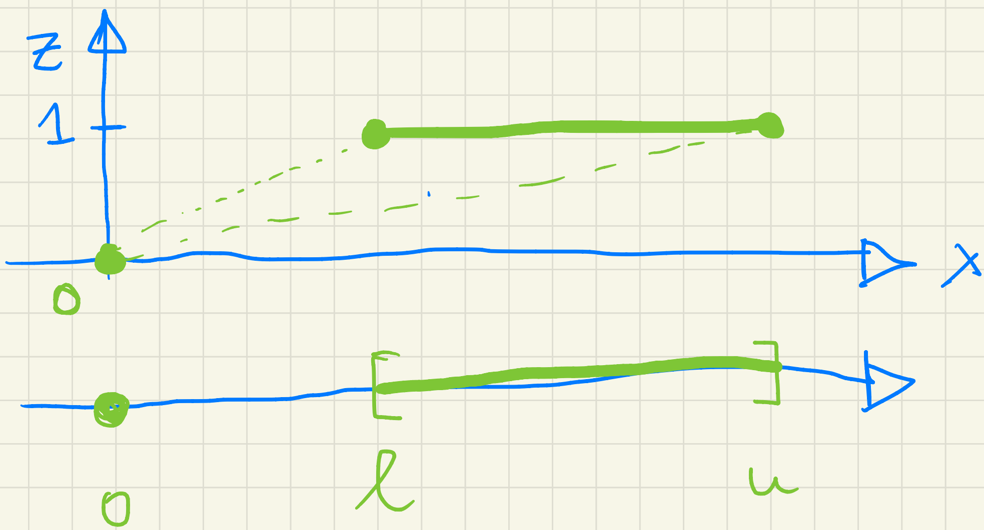
☐ Assume that the variable $x_{(i,j)}$ has a (finite) upper bound $u_{(i,j)}$



⇒ Feasible set is the intersection of triangle with $\{z_{ij} \in \{0,1\}\}$.

Model semicontinuous variables:

$$x \in \{0\} \cup [l, u]$$



Additional variable $z \in \{0,1\}$.

Inequalities: $x \leq u \cdot z$

$$l \cdot z \leq x$$

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