basic

Repeat the same procedure. Unique choice for entering:

x_3. Ratio test:

$$
\begin{array}{lcccccl}
x_5 & = & 1 & + 5x_2 & + 0x_3 & + 2x_4 & \longrightarrow \text{ no restriction} \\
w_3 = x_6 & = & \frac{1}{2} & + \frac{1}{2}x_2 & - \frac{1}{2}x_3 & + \frac{3}{2}x_4 & \longrightarrow \lambda \le 1 \quad \text{leaving.} \\
x_1 & = & \frac{5}{2} & - \frac{3}{2}x_2 & - \frac{1}{2}x_3 & - \frac{1}{2}x_4 & \longrightarrow \lambda \le 5.
\end{array}
$$

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9:09 AM

Rewrite the dictionary so as to correspond to the new basis

$\mathcal{B} = \{x_1, x_5, x_3\}$. $\mathcal{N} = \{x_2, x_4, x_6\}$.

Any equivalent rewrite of the dictionary (System of linear equations) will give "a" (unique up to order) correct new dictionary.

Using pivot tool (Java version of the simple tool)

https://vanderbei.princeton.edu/JAVA/pivot/simple.html

New dictionary:

$$\max \ \zeta = 13 \underbrace{- w_1 - 3x_2 - w_3}_{\text{nonpositive!!}}$$

s.t.

$$x_1 = 2 - 2w_1 - 2x_2 + w_3$$

$$w_2 = 1 + 2w_1 + 5x_2$$

$$x_3 = 1 + 3w_1 + x_2 - 2w_3$$

Every feas. solution has nonneg. variable values.

equivalent to the original problem

Current basic solution is feasible.

$$\bar{w}_1 = \bar{x}_2 = \bar{w}_3 = 0$$

$$\bar{x}_1 = 2$$
$$\bar{w}_2 = 1$$
$$\bar{x}_3 = 1$$

$$\boxed{\bar{\zeta} = 13.}$$

bar is Notation for current basic solution

As a vector:

$$\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{w}_1 \\ \bar{w}_2 \\ \bar{w}_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Cannot continue with (the same) step b/c there is no positive obj coefficient.

Claim: The nonpositivity of the obj coeffs proves "certifies" that the current basic (feasible) solution is optimal.

Take any feasible solution $x \in \mathbb{R}^6$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ w_1 \\ w_2 \\ w_3 \end{pmatrix}$,
What is its obj function value?

$x \geq 0.$

$$\zeta = 13 \underbrace{- w_1 - 3x_2 - w_3}_{\leq 0} \leq 13. = \bar{\zeta}$$

obj value of basic solution.

Algorithm* terminates with a proof (certificate) for optimality.

General notation for dictionaries

$\max \sum^{n} c_i x_i$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i$$
$$x_j \ge 0$$

standard inequality form.

$x_{n+1}, \ldots, x_{n+m}$
$\quad \| \qquad\qquad \|$
$\; w_1 \qquad\qquad w_m$

After introducing slack variables: $\mathcal{B} \sqcup \mathcal{N} = \{1, \ldots, n+m\}$.

$$\boxed{\begin{aligned} \max \quad & \zeta = \bar{\zeta} + \sum_{j \in \mathcal{N}} \bar{c}_j x_j \\ & x_i = \bar{b}_i - \sum_{j \in \mathcal{N}} \bar{a}_{ij} x_j \qquad \forall i \in \mathcal{B} \\ & x \ge 0. \end{aligned}}$$

Dictionary

Dictionary is primal feasible if $\bar{b}_i \ge 0$ for all $i \in \mathcal{B}$.

- satisfies the primal optimality condition if $\bar{c}_j \le 0$ for all $j \in \mathcal{N}$.

(The case that some $\bar{b}_i = 0$ or $\bar{c}_j = 0$ is "degeneracy".)

How to initialize the algorithm if the basis of the slack variables is not (primal) feasible?

Need to find a basis first that is feasible.

This is the "Phase I" of the simplex method.

Idea: The problem of finding a feasible solution is an optimization problem.

$$\boxed{\begin{array}{ll} \max & \cancel{\sum c_j x_j} \\ \text{s.t.} & \sum a_{ij} x_j \le b_i \\ & x_j \ge 0 \end{array}}$$

replace it by a new objective function . . .

that drives to feasibility!