

## Runtime Analysis

a.  $T(n) = \Theta(1) \cdot \sum \Theta(1)$

k	i
1	$2^1$
2	$2^2$
3	$2^3$
4	$2^4$
	$2^5$
	$2^6$

$x = 2$   
 $\log_x = \log_2(2^k)$

$\log x = \log_2(\log 2)$

$\log \log x = k \log_2 \log_2$       $\log_2 2 = 1$

$k = \log \log x$

$\Theta(\log \log n)$

b.  $\text{for}(int i = 1; i \leq n; i++) \rightarrow \Theta(n)$  loops n times

$\text{for}(int k = 0; k < \text{len}(i); k++) \{$

$\sum_{i=1}^n \Theta(i^2)$

$\sum_{i=1}^n = \Theta(n) + \sum_{i=1}^n i^2$

$\sum_{i=1}^n = \Theta(n^3)$

$\Theta(n) + \Theta(n^3 \sqrt{n}) = \Theta(n^{\frac{7}{2}})$

c.

k	m
1	1
2	2
3	4
4	8
5	16
6	32
$\log m$	$2^k$

$$T(n) = \sum_{i=1}^n \left( \sum_{k=1}^i (\theta(1) + O(\sum_{j=1}^k \theta(1))) \right)$$

$$\underline{\theta(n^2 + n \log n)}$$

where worst case of the if statement is n times, array index can only equal i once per iteration of k.

d.

k	L
1	10
2	15
3	22
4	33
5	50
6	75
7	112

$$(10)(\frac{3}{2})^k \log_2 \frac{n}{10}$$

$$\text{Outer loop} = \theta(n)$$

$$\text{Inner loop: } O(10 \sum_{j=0}^{\log_2 \frac{n}{10}} (\frac{3}{2})^j)$$

$$= 10 \theta(\frac{3}{2} \log_2 \frac{n}{10})$$

$$10 \theta(\frac{n}{10})$$

$$\theta(n)$$

$$\theta(n) + \theta(n) = \theta(n)$$