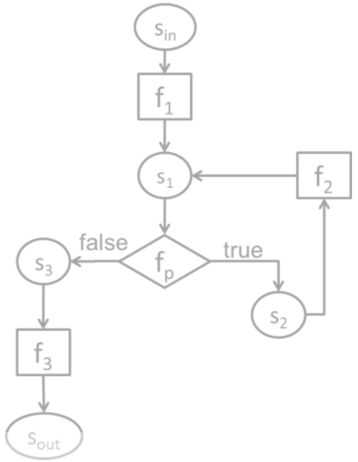
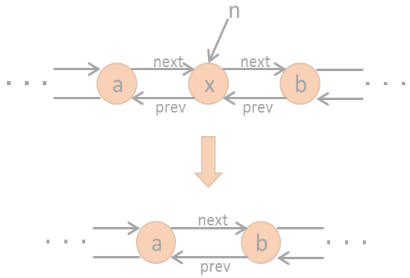
$$\exists c \forall in\ Q(c, in)$$

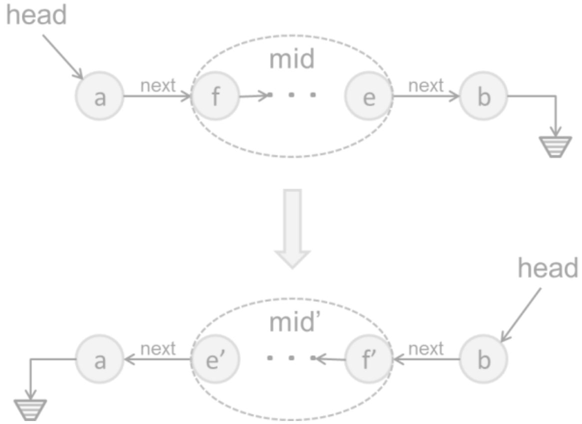
```
/* Average of x and y without using x+y (avoid overflow)*/
int avg(int x, int y){
  int t = expr({x/2, y/2, x%2, y%2, 2 }, {PLUS, DIV});
  assert t == (x+y)/2;
  return t;
}
```

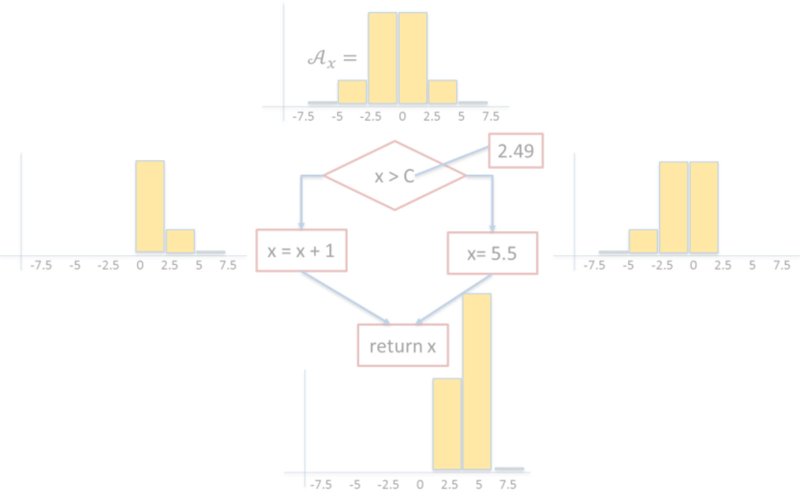





```
{
   s  = n.succ;
   p = n.pred;
   p.succ = s;
   s.pred = p;
}
```

Module II: Synthesis from Specifications





$$\varphi(p)$$

$$Sk[c](in)$$

Lecture 8
Specifications and Reduction to Inductive Synthesis

Nadia Polikarpova

Module I vs Module II

Behavioral constraints

examples

~~`[0] → [0]`~~
~~`[5,1] → [1,5,0]`~~

→ rich specifications

Structural constraints

~~straight-line / conditional programs~~

→ general programs with loops / recursion

Search strategy

- Enumerative
- ~~Stochastic~~
- Representation-based
- Constraint-based
- Deductive

Examples of rich specifications

Reference implementation

Assertions

Pre- and post-condition

Refinement type

Reference Implementation

Easy to compute the result, but hard to compute it efficiently or under structural constraints

```
bit[W] AES_round (bit[W] in, bit[W] rkey)
{
  ... // Transcribe NIST standard
}
bit[W] AES_round _sk (bit[W] in, bit[W] rkey) implements AES_round
{
  ... // Sketch for table lookup
}
```

Assertions

Hard to compute the result, but easy to check its desired properties

```
split_seconds (int totsec) {
  int h := ??;
  int m := ??;
  int s := ??;
  assert totsec == h*3600 + m*60 + s;
  assert 0 <= h && 0 <= m < 60 && 0 <= s < 60;
}
```

Pre-/post-conditions

Hard to compute the result but easy to express its properties in logic

```
sort (int[] in, int n) returns (int[] out)
  requires  n ≥ 0
  ensures   ∀i j. 0 ≤ i < j < n ⇒ out[i] ≤ out[j]
            ∀i . 0 ≤ i < n ⇒ ∃j. 0 ≤ j < n ∧ in[i] = out[j]
{
  ??
}
```

Refinement types

Same as pre-/post-conditions but logic goes inside the types

binary search tree

red nodes have black children

```
data RBT a where
  Empty :: RBT a
  Node  :: x: a ->
    black: Bool ->
    left:  { RBT {a | _v < x} | !black ==> isBlack _v } ->
    right: { RBT {a | x < _v} | (!black ==> isBlack _v) &&
                (blackHeight _v == blackHeight left)} ->
    RBT a

insert :: x: a -> t: RBT a -> {RBT a | elems _v == elems t + [x]}
insert = ??
```

same number of black nodes on every path to leaves

Why go beyond examples?

Might need too many

- **Example:** Myth needs 12 for `insert_sorted`, 24 for `list_n_th`
- Examples contain *too little* information
- Successful tools use domain-specific ranking

Output difficult to construct

- **Example:** AES cypher, RBT
- Examples also contain *too much* information (concrete outputs)

Need strong guarantees

- **Example:** AES cypher

Reasoning about non-functional properties

- **Example:** security protocols

Why is this hard?

```
gcd (int a, int b) returns (int c)
```

requires $a > 0 \wedge b > 0$

ensures $a \% c = 0 \wedge b \% c = 0$

$$\forall c \,.\, 0 < d < c \Rightarrow \mathrm{a} \% \mathrm{d} \neq 0 \vee b \% v \neq 0$$

```
{
  int x , y := a, b;
  while (x != y) {
    if (x > y) x := ??;
    else y := ??;
}}
```

infinitely many inputs

cannot validate by testing

infinitely many paths!

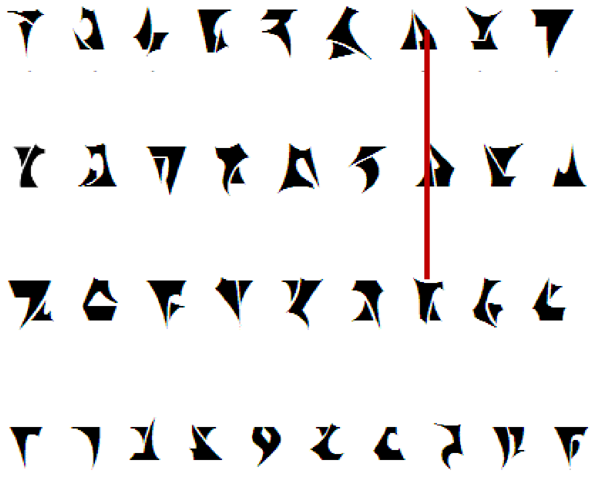
hard to generate constraints

Why is this hard?

Synthesis from examples

validation was easy!

Synthesis from specifications



SEE IF YOU CAN FIND ANY KLINGON FRUIT!

validation is hard!
(and search is still hard)

Map of the module

Constraint-based synthesis

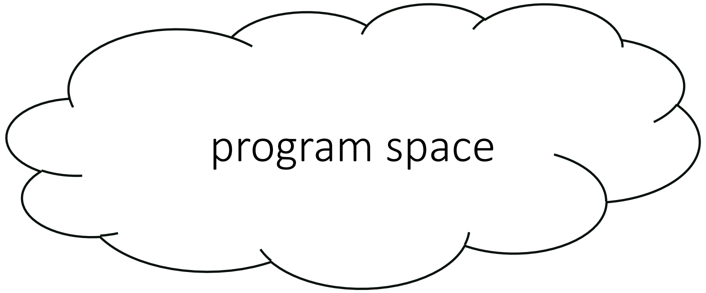
- How to solve constraints about infinitely many inputs? CEGIS
- How to encode semantics of looping / recursive programs?
  - Bounded reasoning
  - Unbounded / deductive reasoning

Enumerative and deductive synthesis

- How to use deductive reasoning to guide the search?

Constraint-based synthesis from specifications

CBS from examples



E = [i → o]

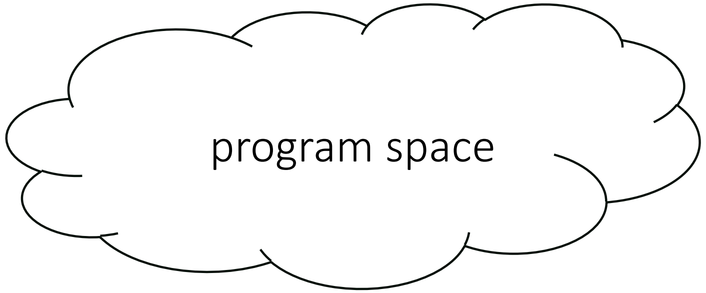
encoding

Parameters (control vector)

“*C* denotes a program that gives output *o* on input *i*”

$$\exists C\ .\ \bigwedge_{\langle i,o\rangle \in E} \varphi(C, i, o)$$

CBS from specifications



~~E = [i → o]~~

$$\forall\, i\ o\, .\, \psi(i,o)$$

encoding

"*C* denotes a program that gives output *o* on input *i*"

$$\cancel{\exists C\, .\, \bigwedge_{\langle i,o\rangle \in E} \varphi(C,i,o)}$$

$$\boxed{\exists C\, .\, \forall i\ o}\, .\, \boxed{\varphi(C,i,o)} \Rightarrow \psi(i,o)$$

doubly-quantified constraint: not solver-friendly

hard to define for programs with loops / recursion

Example

$$\exists C\,.\,\forall i\ o\,.\,\varphi(C,i,o) \Rightarrow \psi(i,o)$$

```
harness void main(int x) {
  int y := ?? * x + ??;
  assert y - 1 == x + x;
}
```

encoding

$$\exists c_1 c_2\,.\,\forall x\ y\,.\,y = c_1 * x + c_2 \Rightarrow y - 1 = x + x$$

simplify

$$\exists c_1 c_2\,.\,\forall x\,.\,c_1 * x + c_2 - 1 = x + x$$

How do we solve this constraint?

CEGIS

$$\exists c\,.\,\forall x\,.\,Q(c, x)$$

Idea 1: Bounded Observation Hypothesis

- Assume there exists a small set of inputs $\mathrm{X} = \{x_1, x_2, \ldots x_n\}$ such that whenever c satisfies

$$\bigwedge_{i \in 1..n} Q(c, x_i)$$

No quantifiers here, can give to SAT / SMT

it also satisfies

$$\forall x.\, Q(c, x)$$

Example

This is a linear constraint, two inputs are enough!

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$

$$\mathrm{X} = \{0, 1\}$$

$$Q(c_1, c_2, 0) \equiv \; c_2 - 1 = 0$$
$$Q(c_1, c_2, 1) \equiv \; c_1 + c_2 - 1 = 2$$

$$\{c_1 \to 2, c_2 \to 1\}$$

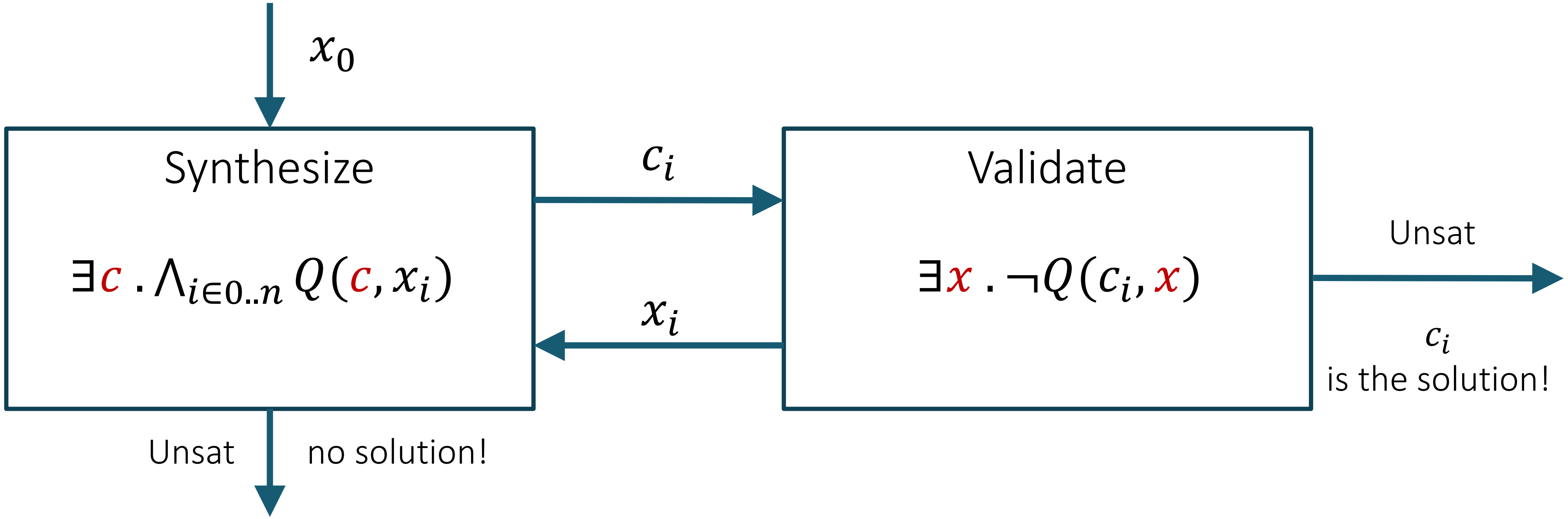
```
harness void main(int x) {
  int y := 2 * x + 1;
  assert y - 1 == x + x;
}
```

How do we find X in a general case?

CEGIS

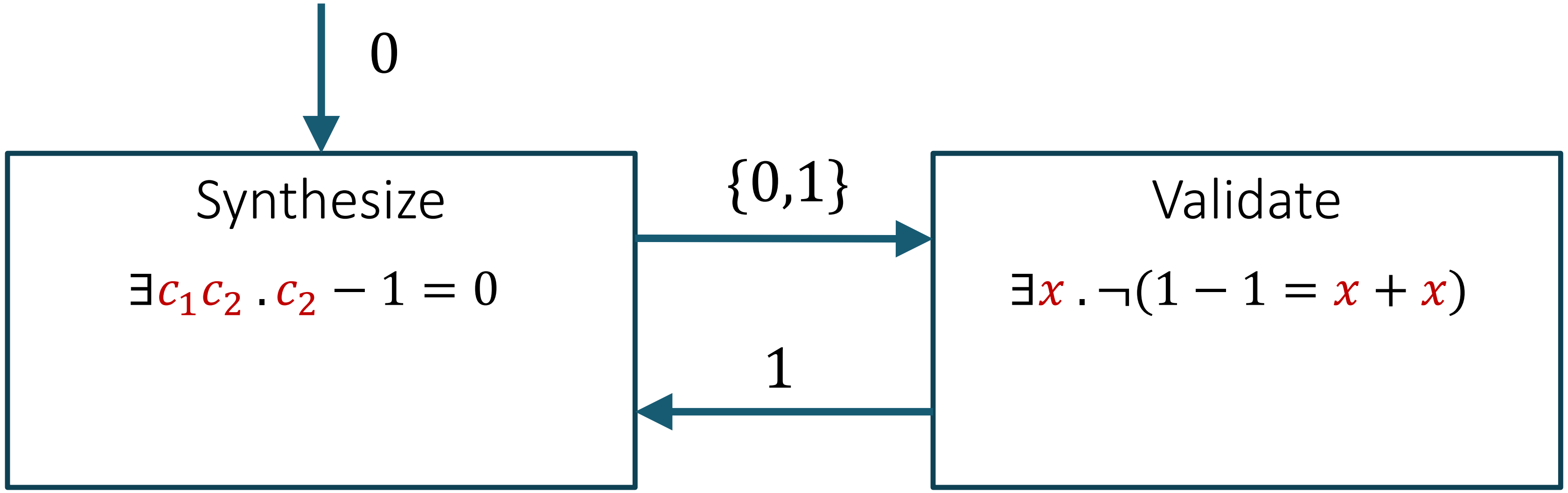
$$\exists c\,.\,\forall x\,.\,Q(c,x)$$

Idea 2: Rely on a validation oracle to generate counterexamples



Example

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$



Example

$$\exists c_1 c_2 \,.\, \forall x \,.\, c_1 * x + c_2 - 1 = x + x$$