

Matricks

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Abstract

While working with matrices I've picked up some tricks, this document is a collection of them. Very much a WIP.

1 Notation

- $X \in \mathbb{R}^{n \times p}$
- $\beta \in \mathbb{R}^p$
- $\Sigma \in \mathbb{R}^{p \times p}$ is symmetric and positive-definite.
- $A = \text{diag}(a_1, \dots, a_n) \in \mathbb{R}^{n \times n}$

2 General

$$\sum_{i=1}^n a_i X_i X_i^\top = X^\top A X \tag{1}$$

3 Traces

$$\|X\beta\|_2^2 = \beta^\top X^\top X \beta = \text{tr}(X^\top X \beta \beta^\top) \quad (2)$$

$$\text{tr}(\Sigma \Sigma) = \sum_{i,j} \Sigma_{ij}^2 \quad (3)$$

4 Cholesky Decomposition

Write the Cholesky decomposition of $\Sigma = U^\top U = LL^\top$.

$$\det(\Sigma) = \det(U^\top) \det(U) = \left(\prod_{i=1}^p U_{ii} \right)^2 \quad (4)$$

$$\text{tr}(\Sigma) = \sum_{ij} U_{ij}^2 \quad (5)$$