Matricks

Michael Komodromos

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Abstract

While working with matrices I've picked up some tricks, this document is a collection of them. Very much a WIP.

1 Notation

- $X \in \mathbb{R}^{n \times p}$
- $\beta \in \mathbb{R}^p$
- $\Sigma \in \mathbb{R}^{p \times p}$ is symmetric and positive-definite with eigenvalues $\lambda_1, \dots, \lambda_p$.
- $A = \text{diag}(a_1, \dots, a_n) \in \mathbb{R}n \times n$

2 General

$$\sum_{i=1}^{n} a_i X_{i \cdot} X_{i \cdot}^\top = X^\top A X \tag{1}$$

3 Traces

$$\|X\beta\|_2^2 = \beta^\top X^\top X\beta = \mathrm{tr}(X^\top X\beta\beta^\top) \tag{2}$$

$$\mathrm{tr}(\Sigma\Sigma) = \sum_{i,j} \Sigma_{ij}^2 \tag{3}$$

4 Cholesky Decomposition

Write the Cholesky decomposition of $\Sigma = U^\top U = LL^\top$.

$$\det(\Sigma) = \det(U^\top)\det(U) = \left(\prod_{i=1}^{p} U_{ii}\right)^2 \tag{4}$$

$$\mathrm{tr}(\Sigma) = \sum_{i,j} U_{ij}^2 = \sum_{i=1}^{p} \lambda_i \tag{5}$$

$$\Sigma^{-1} = U^{-1}\left(U^{-1}\right)^\top \tag{6}$$