Recent changes in firm dynamics and the nature of economic growth*

Markus Kondziella†
IIES, Stockholm University

Job Market Paper

This version: February 1, 2023. Latest version here

Abstract

In line with the US economy, market concentration and dispersion of revenue productivity within industries, a popular measure of efficiency frictions, increased in Sweden from 1997–2017. I document a novel finding in administrative data that provides important insights about the trends: firm size and revenue productivity growth accelerated starting in the 1990s. I reconcile the trends in a dynamic framework. Firms grow in size by expanding into new product markets and increase markups by distancing competitors within their product markets through R&D. The model rationalizes the empirical trends by reducing the R&D cost of distancing competitors and raising the cost of entering new product markets. Despite lowering the level of aggregate output, firms distancing their competitors through R&D faster increases long-run growth by 0.5pp per year. R&D policies limiting the firms' ability to distance competitors through R&D, e.g., a patent waiver, lower the aggregate growth rate.

*I thank Timo Boppart, Per Krusell and Joshua Weiss for their advice and guidance on this project. I further thank Gualtiero Azzalini, Niklas Engbom, José-Elías Gallegos, Jonas Gathen, Sampreet Goraya, Basile Grassi, Ida Kristine Haavi, John Hassler, Kieran Larkin, Kurt Mitman, José Montalban, Alessandra Peter, Michael Peters, Filip Rozsypal and seminar and conference participants at the European Winter Meeting of the Econometric Society 2022, the Enter Jamboree 2022, the IIES, the Nordic Summer Symposium in Macroeconomics 2022, the Swedish Conference in Economics, the Swedish House of Finance, the Stockholm School of Economics and Stockholm University for their helpful comments.

†Institute for International Economic Studies (IIES), Stockholm University. E-mail: markus.kondziella@iies.su.se

1 Introduction

The US economy has experienced several trends over the past decades. Sales concentration has been on the rise, dispersion of revenue productivity across firms has increased, firm entry has slowed down, and the aggregate labor income share has declined. The trends have raised concerns about the aggregate economy: increasing market concentration and revenue productivity dispersion indicate rising market power and misallocation of production factors across firms with adverse effects on aggregate output (Hsieh and Klenow, 2009; Akcigit and Ates, 2021; Decker, Haltiwanger, Jarmin and Miranda, 2017).[1]

Even though the US economy has received considerable interest,[2] similar trends are observed globally.[3] Using high-quality administrative data on the universe of Swedish firms, I show that, in line with the US industry trends, sales concentration and revenue productivity dispersion increased during 1997–2017. I document a novel result at the *firm* level that provides important insights about the mechanisms behind the trends at the industry level. Life cycle growth of sales, revenue productivity, and employment for firms established after 1997 started to accelerate. This is the first study that documents an acceleration of firm life cycle growth starting in the 1990s.

To explain the trends at the firm and industry level jointly, I build a model where firm size and markup growth give rise to market concentration and misallocation of production factors at the industry level. In the model, firm size growth is the driving force behind market concentration. Firms grow in size by expanding into new product markets. To enter a new product market, firms improve the product of the previous incumbent through R&D (creative destruction) as in Klette and Kortum (2004). Firms constantly compete for product markets through creative destruction. As a result, market concentration rises if firms expand into new product markets more aggressively.

On the other hand, firms increase their markups through R&D within their own product markets (own-innovation). Firms distance themselves through own-innovation from their competitors that produce the same product, allowing them to charge higher markups. Firms with relatively high markups produce inefficiently little, generating a loss in aggregate output. The extent of such misallocation of production factors across firms is reflected by the cross-sectional markup (or revenue productivity) dispersion. Not all firms grow in size and increase their markups at the same rate. Differences in permanent productivity levels lead to firms optimally choosing heterogeneous R&D efforts. More productive firms systematically innovate at faster rates.

I take the model to the data by estimating unobserved model parameters, e.g., the cost of R&D, using Swedish administrative data. The model parameters are informed by firm-level moments. In particular, the estimated model parameters match firm life cycle growth in the model with the

[1] See Bils, Klenow and Ruane (2021) for a discussion about the limitations of revenue productivity dispersion as a measure of misallocation.

[2] For rising sales concentration and revenue productivity dispersion see Andrews, Criscuolo and Gal (2015); Autor, Dorn, Katz, Patterson and Van Reenen (2017); Decker, Haltiwanger, Jarmin and Miranda (2020); Grullon, Larkin and Michaely (2019); Van Reenen (2018). The decline in firm entry is documented in Decker, Haltiwanger, Jarmin and Miranda (2016); Gourio, Messer and Siemer (2014); Karahan, Pugsley and Şahin (2019). For the decline in the aggregate labor income share see Autor, Dorn, Katz, Patterson and Van Reenen (2020); Elsby, Hobijn and Şahin (2013); Karabarbounis and Neiman (2014); Kehrig and Vincent (2021); Lawrence (2015).

[3] See Andrews, Criscuolo and Gal (2016); Autor, Dorn, Katz, Patterson and Van Reenen (2020); Karabarbounis and Neiman (2014)

heterogeneous life cycle profiles that I observe in administrative data. To explain the increase in revenue productivity dispersion, sales concentration, and acceleration in revenue productivity growth, I adopt a general approach that allows for any parameter changes. The model asks for changes in the R&D costs to rationalize the empirical trends. A 31% decline in within-product market R&D costs for firms with a high productivity level and a 13% increase in creative destruction costs for incumbents (11% for entrants) replicate the empirical trends quantitatively. The increase in creative destruction costs and the decrease in own-innovation costs capture the general notion that entering new markets has become more difficult. At the same time, firms find it easier to improve their products in markets where they have already established themselves as the market leader.

The changes in the innovation costs affect the long-run economy as follows: the fall in the within-product market R&D costs for firms with a high productivity level incentivizes them to raise their within-product market R&D efforts. As a result, firm markup growth and markup dispersion in the cross-section increase. At the same time, faster markup accumulation incentivizes high-productivity firms to expand into new product markets. Together with a fall in firm entry due to rising creative destruction costs for entrants, this leads to sales concentration. The changes in markup dispersion and market concentration affect aggregate output. The rise in markup dispersion introduces a permanent 0.7% loss in aggregate output. In contrast, the reallocation of market shares towards more efficient firms increases aggregate output permanently by 0.1% through a rise in aggregate productivity. Taken together, this results in a permanent 0.6% decline in the level of aggregate output. There is a positive side effect. The rise in within-product market R&D increases the economy's long-run growth rate by almost half a percentage point. The positive growth effect on aggregate output is opposite to the negative level effect. This novel growth effect puts the observed trends at the industry level in a more favorable light.

Creative destruction reduces markup dispersion generating static gains in aggregate output. What are the dynamic consequences of subsidizing creative destruction? I quantify the effects of subsidizing firms' creative destruction costs on static efficiency and economic growth. A concrete example of a creative destruction subsidy is to lift patent protection that has been recently discussed for Covid vaccines or green technologies. Such a policy lowers the cost for firms to adopt and improve the technology of incumbent, patenting firms. I find that a subsidy that lowers the creative destruction costs of firms improves static efficiency but decreases the aggregate growth rate. Incumbents react to the increase in creative destruction by lowering their within-product market R&D efforts. In the estimated model, within-product market R&D by incumbents is the primary source of economic growth. The decline in within-product market R&D by incumbents outweighs the rise in creative destruction resulting in a fall in the aggregate growth rate. A subsidy on firms' own-innovation costs instead successfully generates economic growth. The downside of such a subsidy is that it generates further markup dispersion and aggregate output losses. An own-innovation subsidy that is financed by a 10% profit tax results in a permanent output loss of 1% relative to a subsidy on creative destruction. On the flip side, the aggregate growth rate with the own-innovation subsidy exceeds the growth rate with the creative destruction subsidy by 0.6pp. A subsidy that does not differentiate between the innovation types subsidizing total innovation expenditures of firms has a muted yet positive effect on economic growth. The effect on growth is small as the positive effect of the own-innovation subsidy is almost offset by the creative destruction subsidy.

This paper relates to the literature that documents trends in revenue productivity dispersion and sales concentration. Andrews, Criscuolo and Gal (2016), Decker, Haltiwanger, Jarmin and Miranda (2020), and Van Reenen (2018) find evidence for rising revenue productivity dispersion in the US and selected OECD countries. Autor, Dorn, Katz, Patterson and Van Reenen (2017), Grullon, Larkin and Michaely (2019) and Akcigit and Ates (2021) document rising sales concentration in the US. I confirm the trends of increased revenue productivity and sales concentration for the Swedish economy. Two advantages of my data are its quality and coverage. Balance sheet information comes directly from tax registries covering the universe of Swedish firms. Due to the comprehensiveness of the data, I show that the trends hold for the same set of firms. I further contribute a new finding to the literature. Firm life cycle growth of sales, employment, and revenue productivity accelerated for new firms established after 1997.

The proposed theory is based on Schumpeterian models of endogenous growth in the spirit of Aghion and Howitt (1992), Grossman and Helpman (1991), and Klette and Kortum (2004). These models are analytically tractable yet yield a realistic description of firm dynamics and capture salient features of the data (Lentz and Mortensen, 2008; Akcigit and Kerr, 2018). The framework in this paper particularly relates to Peters (2020) and Aghion, Bergeaud, Boppart, Klenow and Li (2019). Peters (2020) builds a framework of creative destruction and own-innovation.[4] In Peters (2020), all firms make identical innovation decisions as there is no ex-ante heterogeneity across firms. Ex-post shocks drive differences in realized markup, sales, and employment life cycle growth across firms. Sterk, Sedláček and Pugsley (2021) emphasize that ex-ante heterogeneity, rather than ex-post shocks, explains the differences in firm growth rates. I introduce heterogeneity in the permanent productivity level of firms, which gives rise to systematically different innovation and hence growth rates across firms. This heterogeneity is essential to explain the empirical trends and generates a cross-sectional firm size distribution that matches the data. Aghion, Bergeaud, Boppart, Klenow and Li (2019) feature heterogeneity in the permanent productivity level of firms. More productive firms charge higher markups and endogenously choose to operate more product lines. There is no incumbent own-innovation and firm entry such that this model abstracts from the firm life cycle.

The model application in this paper relates to the literature explaining recent trends in the US economy. Proposed drivers for the trends are an increasing importance of intangible capital and information and communications technology (ICT) (Aghion, Bergeaud, Boppart, Klenow and Li, 2019; Crouzet and Eberly, 2019; Chiavari and Goraya, 2020; De Ridder, 2019; Hsieh and Rossi-Hansberg, 2019; Weiss, 2019), demographic changes (Bornstein, 2018; Engbom, 2020; Hopenhayn, Neira and Singhania, 2018; Karahan, Pugsley and Şahin, 2019; Peters and Walsh, 2021), declining interest rates (Chatterjee and Eyigungor, 2019; Liu, Mian and Sufi, 2022), changes in the quality of ideas (Olmstead-Rumsey, 2019) or declining imitation rates (Akcigit and Ates, 2019). While matching trends at the industry level that the above papers focus on, I further explain the acceleration of firm life cycle growth. My analysis identifies a rise in own-innovation as a new driver behind the trends at the industry and firm level generating a novel growth effect on aggregate output.

My theory relates to the literature on misallocation of production factors that has studied the effects of firm-level distortions on aggregate productivity. In Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), misallocation arises from exogenous firm-specific wedges that are correlated with

[4] Similarly, Akcigit and Kerr (2018) features a quality ladder model of endogenous growth with creative destruction and own-innovation.

the firm's productivity. In Edmond, Midrigan and Xu (2018), markups grow with firm size, whereas in Peters (2020), markups increase in firm age. My model captures misallocation from both the age and size dimensions separately. Firms increase their markups as they age, and firms with initially high markups expand faster in size.

The paper proceeds as follows. Section 2 introduces the data, section 3 documents the trends in the Swedish economy, and section 4 contains the theory. In section 5, I apply the model to the data to explain the empirical trends and quantify the implications for the long-run economy. The paper proceeds with policy counterfactuals studying the effect of R&D policies on static efficiency and economic growth. Section 6 concludes the paper.

2 Data

2.1 Data description

For the empirical results I link three data sets provided by Statistics Sweden (SCB). The first is *Företagens Ekonomi* (FEK), which covers information from balance sheets and profit and loss statements for the universe of Swedish firms. The unit of observation is the legal unit at annual frequency covering the period 1997-2017. Before 1997, FEK was a sample covering large Swedish firms. The data further contains information on the legal type and the industry code of the firm. I restrict myself to firms in the private economy with at least five employees. Using the detailed 5-digit industry codes, unless mentioned otherwise, all of the empirical analysis is conducted within industries.

The second data set is the *Producentprisindex* microdata. This data contains producer prices of goods that enter the national producer price index (PPI). The data set includes a representative sample of firms with at least 10 Mio. SEK net sales. In terms of employment, firms in the PPI data cover 22% of the cleaned FEK data in 2017 (28% of value added).

The auxiliary data set *Registerbaserad Arbetsmarknadsstatistik* (RAMS) provides additional information. It contains matched employer-employee data at the establishment level. I use it to define the universe of active employers and to get a measure of employment that is independent of the employment measure in the balance sheet data (measured in full-time equivalents).

Throughout the paper nominal variables are deflated to 2017-SEK values using the GDP deflator. The empirical analysis is carried out within 5-digit industry-years for which any finer deflation of variables (up to the 5-digit industry level if available) would have no effect.

2.2 Revenue productivity

Physical productivity measures a firm's technology, mapping production inputs into physical output quantities. As large-scale data sets on firms' balance sheets became available, the approach of measuring firm "productivity" using balance sheet variables (e.g., value added, the wage bill and the capital stock) gained attraction. Productivity measures using value added or sales instead of physical output quantities as the output measure have been labelled *revenue productivity*.

2.2.1 Measurement

One standard measures of revenue productivity is labor productivity, defined as

$$\text{Labor productivity}_{fst} \equiv \frac{VA_{fst}}{wL_{fst}},$$

where VA denotes value added and wL the wage bill for firm f in industry s. To account for the capital deepening of the firm Hsieh and Klenow (2009) introduced the concept of revenue TFP ("$TFPR$") as a measure of revenue productivity

$$TFPR_{fst} \equiv \frac{VA_{fst}}{K_{fst}^{\alpha_{st}}(wL_{fst})^{1-\alpha_{st}}}, \tag{1}$$

where K denotes capital and α the capital income share. I use $TFPR$ as my baseline measure of revenue productivity. All empirical results are robust to using labor productivity as the revenue productivity measure.[5] For the $TFPR$ computation I use fixed assets less depreciation as a measure of the firm's capital stock. Labor income shares, $(1-\alpha_{st})$, are computed as labor cost shares at the industry level, $\frac{wL_{st}}{wL_{st}+R\cdot K_{st}}$, where R is the rental price of capital set to 10% as in Hsieh and Klenow (2009).

2.2.2 Revenue productivity, misallocation and markups

Whereas *physical productivity* captures the firm's technology to produce output quantities, *revenue productivity*, using the dollar value (quantities × price) to measure output, captures both the firm's technology to produce and the output price. The output price in return is affected by factors related to, e.g., industry competition or product quality. A firm that faces low competition or produces a product that is of higher quality than the product of its competitors potentially charges a relatively high price. Low competition or high product quality therefore translate into high revenue productivity of the firm. In the model in section 4 firms innovate upon the quality of their products, which gives rise to firm life cycle growth of revenue productivity.

One advantage of revenue productivity capturing both the firm's technology and the output price is that it allows for a comparison across firms. The technology of two firms that produce different output goods cannot be compared directly. Using the dollar value instead of physical quantities as the output measure, revenue productivity can easily be compared across firms that produce different products. Further, by capturing the firm's output price, revenue productivity contains information that we can use to study misallocation and markups. In particular, dispersion of revenue productivity across firms is a popular measure for misallocation of production factors. The intuition is that firm-specific distortions affect the firms' optimal input and output choices. If the distortions correlate with firm productivity (or quality) this leads to aggregate output losses. Such distortions can take on the form of input or output taxes that correlate with the firm's productivity (Hsieh and Klenow, 2009), or arise endogenously as in settings where more productive firms optimally charge higher markups. Such distortions are reflected in the marginal revenue productivity of firms as profit maximizing firms equate the marginal revenue product with marginal costs. Dispersion of revenue productivity across firms, as studied in the next section, is therefore a popular indicator for the extent of input and output distortions at the firm level.

[5] The empirical findings are also robust to using sales instead of value added and introducing intermediate inputs as a third production factor to eq. (1).

Table 1: Summary statistics (1997-2017)

	High TFPR			*Low TFPR*		
	Mean	SD	Obs.	Mean	SD	Obs.
*Value Added**	61.5	615.9	117,436	22.4	199.0	1,113,247
*Net Sales**	268.0	2,589.6	117,436	76.0	786.2	1,113,247
Employees	46.9	320.5	117,436	30.3	211.5	1,113,247
wL*	20.4	170.2	117,436	11.5	84.4	1,113,247
K*	38.2	454.5	117,436	29.5	548.0	1,113,247
ln $TFPR$	0.97	0.54	117,436	0.54	0.46	1,113,247

Note: *: in mio 2017-SEK. K is fixed assets minus depreciation, wL labor compensation. Firms that rank in the top decile of the industry $TFPR$ distribution for five years during 1997-2017 are classified as high $TFPR$ firms.

Firms' revenue productivity is further related to the price-cost markup. To see this, assume firms operate with a general production function taking input prices as given. Cost minimization with respect to an input of production (here labor) leads to the following expression for markups μ

$$\mu_{fst} = \text{Output elasticity labor}_{fst} \times \frac{VA_{fst}}{wL_{fst}}. \tag{2}$$

This approach has been used by De Loecker and Warzynski (2012) (among others) to estimate firm markups. Eq. (2) states that the markup is proportional to revenue (labor) productivity. Most balance sheet data cover value added and labor costs such that revenue productivity of the firm (VA/wL) is straightforward to measure. Since estimation of the output elasticity requires functional form assumptions about the production function and data on both physical input quantities and physical output quantities, the output elasticity is generally hard to identify. With various assumptions one can infer information about markups from revenue productivity without estimating the output elasticity. First, in models where firms produce with a technology that is linear in labor (as in section 4) the output elasticity of labor equals unity. In this case, eq. (2) states that the level of revenue productivity identifies the level of the markup. Alternatively, under the assumption that firms within industries have the same output elasticity, eq. (2) suggests that the dispersion of log revenue productivity is equal to the dispersion of log markups across firms. Dispersion of revenue productivity is hence a measure of markup dispersion. A third alternative is to assume that the output elasticity is fixed over time (but potentially varies across firms). In that case, revenue productivity growth of the firm is equal to its markup growth. In the following section, I study revenue productivity dispersion and revenue productivity growth. Under the assumptions above, one can think of revenue productivity dispersion across firms and revenue productivity growth within firms as proxies for markup dispersion and markup growth.

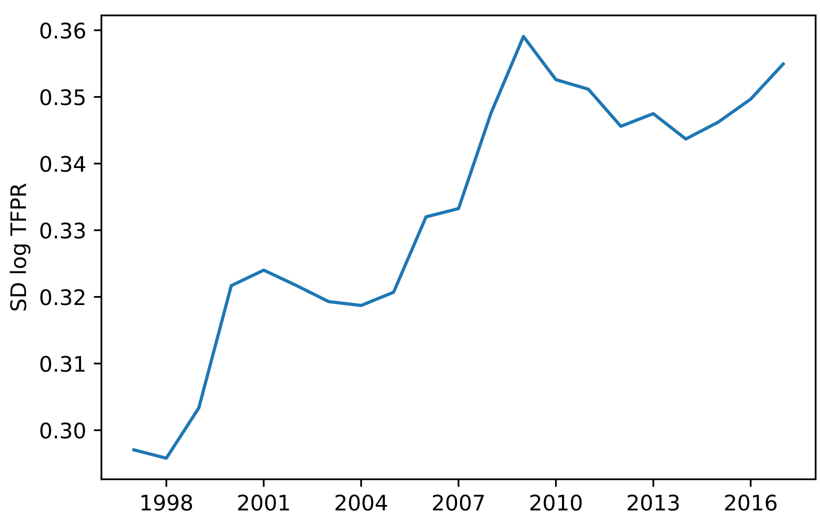
2.3 Descriptive statistics

Table 1 provides descriptive statistics for firms conditional on their revenue productivity. For this table, I classify firms into high and low revenue productivity firms according to their permanent revenue productivity level. A firm is labelled a high $TFPR$ firm if it ranks in the top decile of the industry $TFPR$ distribution for five years during 1997-2017. Table 1 shows that high $TFPR$ firms are on average larger than low $TFPR$ firms in terms of value added and sales, employ more workers (46.9 vs. 30.3), and own a larger capital stock. In this classification, high $TFPR$ firms have on average $e^{0.97-0.54} = 1.54$ higher $TFPR$ than low $TFPR$ firms. Note that those moments are pooled across all years and do not condition on the firm's age or industry.

3 Trends in the Swedish economy

Figure 1 shows the standard deviation of log revenue productivity ($TFPR$) within 5-digit industries during 1997–2017.[6] The standard deviation of log revenue productivity increased by 19.5% over this period. The increase in dispersion is particularly pronounced during 1997–2008, the years before the Great Recession. Despite the two recessions compressing revenue productivity dispersion temporarily a clear positive long-run trend is visible. Through the lens of the theory in section 4, revenue productivity (or markup) dispersion lowers aggregate output. Firms that charge relatively large markups constrain their demand for input factors such that reallocation of input factors from low to high markup firms increases aggregate output (i.e. production factors are misallocated). I show in section 5 that the 19.5% rise in revenue productivity dispersion is associated with a 0.7% permanent loss in aggregate output.

Figure 1: Rise in revenue productivity ($TFPR$) dispersion in Sweden



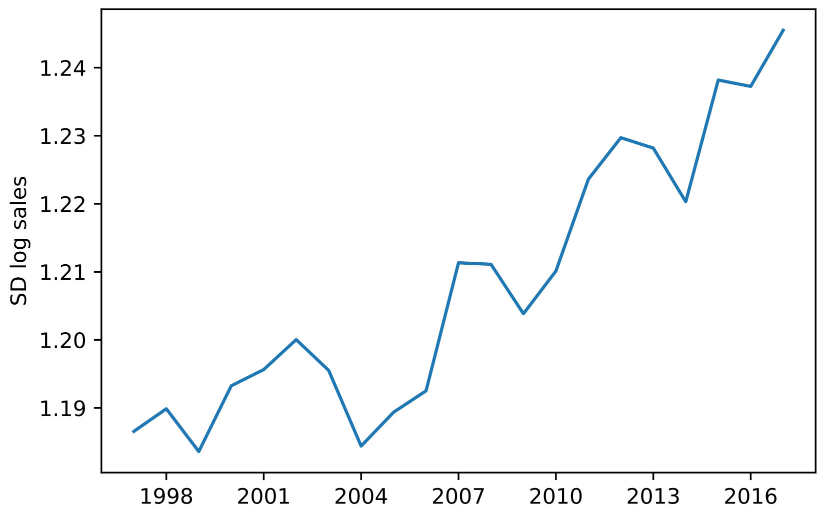
Notes: the figure shows the avg. standard deviation of log $TFPR$ within 5-digit industries with at least ten firms. $TFPR$ computed at the firm level as value added over a Cobb-Douglas composite of the capital stock and the wage bill (see section 2).

To compare magnitudes with the US, using Census data Decker, Haltiwanger, Jarmin and Miranda (2020) report an increase in the standard deviation of log labor productivity by roughly 4% during 1996–2013. Using their measure of revenue productivity, I observe an increase in the dispersion

[6]See section 2 for a detailed description of the data and variable construction.

by 8% in Sweden for the same time period.[7] In Appendix A.1 I show that the rise in revenue productivity dispersion is robust to computing weighted standard deviations, alternative measures of revenue productivity or restricting to large industries. I further show that the increase in the standard deviation is not driven by a subset of firms but is due to a widening of the whole industry revenue productivity distribution.

Figure 2: Rise in sales concentration in Sweden



Notes: the figure shows the avg. standard deviation of log sales (shares) within 5-digit industries with at least ten firms.

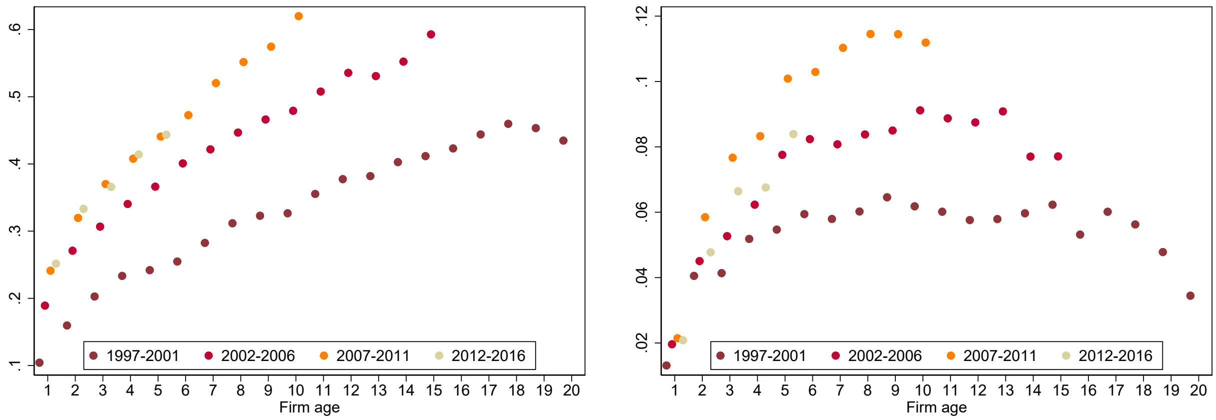
The rise in revenue productivity dispersion is accompanied by an increase in sales concentration. Figure 2 shows the average industry sales concentration over time. I measure sales concentration within 5-digit industries using the standard deviation of log sales (or log sales shares). More dispersed sales shares within industries indicate a higher sales concentration. The standard deviation of log sales increased by 5% from 1997 to 2017, providing evidence for a rise in sales concentration. As for revenue productivity dispersion, the early 2000s recession and the Great Recession compressed sales concentration temporarily; however, the long-run trend is positive. The increase in sales concentration in Sweden is quantitatively similar to the one in the US. Sales shares of top-4 firms within 4-digit industries in US Census data increased by roughly four percentage points (pp) during 1997–2012 (Akcigit and Ates, 2021; Autor, Dorn, Katz, Patterson and Van Reenen, 2020). Using this metric, sales concentration in Sweden equally increased by 4pp over the same period (also measured at the 4-digit industry level). In Appendix A.2, I provide further robustness for the rise in sales concentration by computing weighted standard deviations of sales shares or focusing on large industries.

To understand the rise in sales concentration and revenue productivity dispersion[8] at the industry

[7] Industry classifications in the U.S. Census data are at the six-digit level and at the five-digit level in Swedish administrative data.

[8] Using the same data source as I do in this paper, Engbom (2020) further documents a decline in the firm entry rate in Sweden.

Figure 3: Cumulative firm life cycle growth (by cohort)



(a) Sales growth

(b) Revenue productivity growth

Notes: the figure shows life cycle growth of sales and revenue productivity ($TFPR$) measured as the log difference to age zero. Both measures are net of industry $\times$ year fixed effects. Cohorts pooled as indicated in the legend.

level, I shift the focus to sales and revenue productivity growth at the firm level. Figure 3 shows firms' life cycle growth of sales (left panel) and revenue productivity (right panel) for different cohorts, as indicated in the legend. The main observation is that sales and revenue productivity growth rates have accelerated. This translates into a steepening of the sales and revenue productivity age gradient for the more recent cohorts. For example, when looking at sales and revenue productivity growth during the first four years of the firm, growth has increased by 78% (sales) and 25% (revenue productivity). A similar trend can be observed for employment life cycle growth, see A.3 (Appendix). This is the first paper that documents an acceleration in firm life cycle growth of sales, revenue productivity and employment. To obtain the life cycle growth of firms, I regress log sales and log revenue productivity (net of industry $\times$ year fixed effects) on age dummies of the firm (age zero left out) and a constant. The age coefficients are displayed in the figure. In Appendix A.3, I provide robustness for the cohort grouping.

4 Model

In the following section, I build a framework that jointly addresses the trends in market concentration, misallocation of production factors and firm growth.

4.1 Preferences and aggregate economy

The model is formulated in continuous time. The economy consists of a representative household that chooses the path of consumption C_t and wealth A_t to maximize lifetime utility

$$U = \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

subject to the budget constraint $\dot{A}_t = r_t A_t + w_t L_t - C_t$ and a standard no-Ponzi game condition. ρ denotes the discount factor, r_t the interest rate and w_t the real wage. The household supplies one unit of labor inelastically such that $L_t = 1$. The optimality condition (Euler equation) for the household problem reads

$$\frac{\dot{C}_t}{C_t} = r_t - \rho.$$

Aggregate output is produced competitively using a Cobb-Douglas technology[9] over a continuum of different products indexed by i (time subscripts suppressed)

$$Y = \exp\left(\int_0^1 \ln\left[q_i y_i\right] di\right),$$

where y_i and q_i denote the quantity and quality of product i. Output is consumed entirely such that $Y = C$. Expenditure minimization leads to the standard demand function for product y_i

$$y_i = \frac{YP}{p_i}.$$

Here P is defined as the aggregate price index

$$P \equiv \exp\left(\int_0^1 \ln\left[p_i/q_i\right] di\right),$$

which is normalized to 1.

4.2 Production

Firms can produce in every product market i with the following technology

$$y_{ij} = \varphi_j l_{ij},$$

where y_{ij} is the amount of product i produced by firm j, l_{ij} is the amount of labor hired, and φ_j denotes the physical productivity of firm j producing product i. A firm active in multiple markets produces the products using the same productivity, i.e., φ_j varies with j, but not with i. The firm's productivity is fixed over time, which captures the notion that some firms are persistently more efficient at producing than others. For simplicity, firms are either of the high or low productivity type, i.e., $\varphi_j \in \{\varphi^h, \varphi^l\}$ with $\varphi^h/\varphi^l > 1$.

In previous models of creative destruction and own-innovation, the notion of the firm is a collection of random product lines. With differences in productivity across firms, a firm is a collection of product lines, among which it produces with a technology that is potentially distinct from the technology of its competitors. Apart from changing the interpretation of the firm, permanent differences in productivity across firms have implications for firm dynamics. More productive firms will charge higher markups and therefore have a larger incentive to expand into new product lines: more productive firms endogenously grow faster in size. Without permanent productivity differences, firms choose identical innovation rates. In this case, firm size growth is purely determined by ex-post shocks. Sterk, Sedláček and Pugsley (2021) emphasize that ex-ante heterogeneity rather than ex-post shocks explain differences in firm growth rates. Permanent productivity differences that lead to differences in the optimal innovation rates across firms introduce a deterministic component to firm growth.

[9] With a CES aggregator, own-innovation generates both cross-sectional markup dispersion *and* market concentration. Rising own-innovation (as highlighted later) plays an even more prominent role in explaining the empirical trends in a CES setting than under the Cobb-Douglas technology. The model is formulated with the Cobb-Douglas technology because I find only weak evidence for markups increasing in size *within* firms as I discuss in section 4.8.

4.3 Static allocation

Taking the distribution of product qualities and the number of firms as exogenous in this section, I characterize static allocations at the product, firm and aggregate levels.

4.3.1 Product level

Firms within a product market i compete in prices (Bertrand competition). In equilibrium, only the firm with the highest quality-adjusted productivity $q_{ij}\varphi_j$ produces product i.

Under Bertrand competition, the leader (the firm with the highest quality-adjusted productivity) engages in limit pricing and sets its price equal to the quality-adjusted marginal costs of the follower (the firm with the second highest quality-adjusted productivity). The leader's price in equilibrium is hence given by

$$p_{ij} = \frac{q_{ij}}{q_{ij'}} \frac{w}{\varphi_{j'}}, \tag{3}$$

where j' indexes the follower in a market. According to eq. (3), the price that the leader sets is increasing in the quality gap between the leader and the follower.

The equilibrium price-cost markup in market i for producer j is defined as the output price over marginal costs, hence

$$\mu_{ij} \equiv \frac{p_{ij}}{w/\varphi_j} = \frac{q_{ij}}{q_{ij'}} \frac{\varphi_j}{\varphi_{j'}}. \tag{4}$$

The leader's markup for product i is increasing in the quality and productivity gap between the leader and the follower.

The price setting of the leader gives rise to the following equilibrium profits for product i

$$\pi_{ij} = p_{ij}y_{ij} - wl_{ij} = Y\left(1 - \frac{1}{\mu_{ij}}\right),$$

with labor demand for product i given by

$$l_{ij} = \frac{Y}{w}\mu_{ij}^{-1}.$$

Employment in product line i is decreasing in the markup.

To see the relationship between markups and revenue productivity, I define $TFPR_{ij}$ of the incumbent[10] in line i as

$$TFPR_{ij} \equiv \frac{p_{ij}y_{ij}}{wl_{ij}} = \frac{p_{ij}}{w}\varphi_j = \frac{q_{ij}}{q_{ij'}} \frac{\varphi_j}{\varphi_{j'}} = \mu_{ij}. \tag{5}$$

Revenue productivity in line i is equal to the markup, which results from an output elasticity of labor equal to unity. Eq. (5) highlights the distinction between revenue productivity and physical productivity. Physical productivity φ_j, measuring the firm's technology, enters revenue productivity; however, revenue productivity further reflects the output price captured by the physical productivity of the second-best firm and the quality gap.

[10]In the following, I refer to leader and incumbent equivalently.

4.3.2 Firm level

Summing employment per product over the set of products that firm j is producing, N_j, gives employment at the firm level:

$$l_j = \sum_{i \in N_j} l_{ij} = \frac{Y}{w} \left(\sum_{i \in N_j} \mu_{ij}^{-1} \right).$$

Employment at the firm level is increasing in the number of products the firm produces.

At the firm level, I define revenue productivity as firm sales over the firm's wage bill:

$$TFPR_j \equiv \frac{\sum_{i \in N_j} p_{ij} y_{ij}}{w \sum_{i \in N_j} l_{ij}} = \frac{\sum_{i \in N_j} p_{ij} y_{ij}}{w l_j} = \left(\frac{1}{n_j} \sum_{i \in N_j} \mu_{ij}^{-1} \right)^{-1}, \tag{6}$$

where n_j denotes the cardinality of N_j. $TFPR_j$ is a harmonic mean of the markups the firm charges for its products.

The firm's sales are $n_j Y$, which follows from the fact that revenue per line is equalized.

4.3.3 Aggregate level

Summing firm employment over all firms yields the total workforce in production:

$$L_P = \int_j l_j dj = \frac{Y}{w} \int_0^1 \mu_{ij}^{-1} di. \tag{7}$$

An expression for the wage can be found from the markup equation (4):

$$w = \frac{p_{ij}}{\mu_{ij}} \varphi_j = \frac{p_{ij}}{q_{ij}} \frac{q_{ij} \varphi_j}{\mu_{ij}}.$$

Taking logs and integrating over all products gives

$$w = \exp\left(\int_0^1 \ln \left[p_{ij}/q_{ij} di \right] \right) \times \exp\left(\int_0^1 \ln q_{ij} di \right) \times \exp\left(\int_0^1 \ln \varphi_{j(i)} di \right) \times \exp\left(\int_0^1 \ln \mu_{ij}^{-1} di \right).$$

As the final good is the numeraire, the wage expression simplifies to

$$w = \exp\left(\int_0^1 \ln q_{ij} di \right) \times \exp\left(\int_0^1 \ln \varphi_{j(i)} di \right) \times \exp\left(\int_0^1 \ln \mu_{ij}^{-1} di \right).$$

To find an expression for aggregate output, rearrange equation (7):

$$Y = \frac{w}{\int_0^1 \mu_{ij}^{-1} di} L_P.$$

Inserting the wage expression gives

$$Y = Q \Phi \mathcal{M} L_P,$$

where

$$Q = \exp\left(\int_0^1 \ln q_{ij} di\right)$$
$$\Phi = \exp\left(\int_0^1 \ln \varphi_{j(i)} di\right)$$
$$\mathcal{M} = \frac{\exp\left(\int_0^1 \ln \mu_{ij}^{-1} di\right)}{\int_0^1 \mu_{ij}^{-1} di}$$

Aggregate output Y depends on geometric averages of quality Q and productivity Φ as well as on the dispersion of markups $\mathcal{M}$ and the total labor force L_P. Aggregate TFP is captured by $Q\Phi\mathcal{M}$. Markup dispersion and average productivity are constant along a balanced growth path (defined below). Changes in $\mathcal{M}$ and Φ across balanced growth paths introduce permanent level effects on aggregate output. I, therefore, refer to $\mathcal{M}$ and Φ capturing static efficiency. The measure of markup dispersion, $\mathcal{M}$, is less (or equal) than unity as a geometric mean (numerator) is weakly lower than an arithmetic mean (denominator). The aggregate output loss from markup dispersion is minimized if markups are equalized across product lines ($\mathcal{M}$ equals unity). Any dispersion of markups drives $\mathcal{M}$ below unity lowering aggregate output, i.e., production inputs (labor) are misallocated across product lines.

Using again equation (7), monopoly power affects factor prices by reducing labor demand. The aggregate labor income share is given by

$$\Lambda \equiv \frac{wL_P}{Y} = \int_0^1 \mu_{ij}^{-1} di.$$

Aggregate TFP depends on the dispersion of markups. The aggregate labor income share depends on the level of markups.

Due to the Cobb-Douglas demand, the sales-weighted average markup equals the unweighted markup

$$E^{sales}[\mu] \equiv \int_0^1 \mu_i \frac{p_i y_i}{Y} di = \int_0^1 \mu_i di.$$

The cost-weighted average markup is equal to the inverse of the aggregate labor income share

$$E^{cost}[\mu] \equiv \int_0^1 \mu_i \frac{w\ell_i}{wL_p} di = \int_0^1 \mu_i \frac{\ell_i}{L_p} di = \frac{Y}{wL_P} = \Lambda^{-1}.$$

It is easy to show that the cost-weighted average markup is weakly lower than the sales-weighted one.[11] Intuitively, a high markup in a given line reduces the labor demand and hence the labor cost in that line.

4.4 Dynamic firm problem

Firms continuously improve the quality of products, q_i, in the economy through two different types of innovations. Own-innovation raises the quality of an item that the firm produces, whereas,

[11] Using Jensen's inequality: $\frac{1}{\int_0^1 \mu_i di} \leq \int_0^1 \mu_i^{-1} di \Leftrightarrow E^{cost} \equiv \frac{1}{\int_0^1 \mu_i^{-1} di} \leq \int_0^1 \mu_i di \equiv E^{sales}$.

through creative destruction, the firm improves the quality of a competitor's product. Item quality is improved step-wise such that every time quality is improved (either through creative destruction or through own innovation), quality increases by a factor of λ. As Aghion, Bergeaud, Boppart, Klenow and Li (2019) I assume $\lambda > \varphi^h/\varphi^l$. This assumption guarantees that the firm with the highest quality version in a product line is always the active producer.[12] Denote by λ^{Δ_i} the ratio of product qualities between the active producer and the second best firm (firm with second highest value of $q_{ij}\varphi_j$) in product line i such that

$$\lambda^{\Delta_i} = \frac{q_{ij}}{q_{ij'}}.$$

The markups determine firm profits in each product line, which depend on the productivity and quality gap to the follower. To infer the firm's current profits, the markups per line are sufficient; however, for the dynamic problem of the firm, one needs to keep track of the firm's productivity separately. The (expected) productivity gap in a new line depends on the productivity type of the firm. To save on notation, denote by $[\mu_i]$ the set of markups in the firm's product lines. Firm profits are given by

$$\pi_t(n, [\mu_i]) = \sum_{k=1}^{n} Y_t \left(1 - \frac{1}{\mu_{kjt}}\right) = \sum_{k=1}^{n} Y_t \left(1 - \frac{1}{\lambda^{\Delta_{kt}}} \frac{1}{\frac{\varphi_{kj}}{\varphi_{kj'}}}\right) \equiv \sum_{k=1}^{n} \pi(\mu_{kt}).$$

Whereas $\pi_t(n, [\mu_i])$ denotes total firm profits, $\pi(\mu_{kt})$ denotes product line profits.

Firms choose the rate of own-innovation I_i and the rate of expansion x_i (own creative destruction) for their products. When choosing optimal own-innovation and expansion rates, firms take aggregate output Y_t, the real wage w_t, the share of lines operated by high productivity producers S_t, the interest rate r_t and the rate of creative destruction τ_t as given. Denoting the time derivative by $\dot{V}_t^h()$, the value function of a high process efficiency firm indexed by h satisfies the following HJB equation:

$$\begin{aligned}
r_t V_t^h(n, [\mu_i]) - \dot{V}_t^h(n, [\mu_i]) = & \\
& \sum_{k=1}^{n} \underbrace{\pi(\mu_k)}_{\text{Flow profits}} + \sum_{k=1}^{n} \tau_t \underbrace{\Big[V_t^h\left(n-1, [\mu_i]_{i \neq k}\right) - V_t^h(n, [\mu_i])\Big]}_{\text{Agg. creative destruction}} \\
& + \max_{[x_k, I_k]} \Bigg\{ \sum_{k=1}^{n} I_k \underbrace{\Big[V_t^h\left(n, [[\mu_i]_{i \neq k}, \mu_k \times \lambda]\right) - V_t^h(n, [\mu_i])\Big]}_{\text{Own-innovation}} \\
& + \sum_{k=1}^{n} x_k \underbrace{\Big[S_t V_t^h\left(n+1, [[\mu_i], \lambda]\right) + (1 - S_t) V_t^h\Big(n+1, \Big[[\mu_i], \lambda \times \varphi^h/\varphi^l\Big]\Big) - V_t^h(n, [\mu_i])\Big]}_{\text{Product expansion}} \\
& - \underbrace{w_t \Gamma^h\left([x_i, I_i]; n, [\mu_i]\right)}_{\text{Research costs}} \Bigg\}.
\end{aligned}$$

[12] Without this assumption, multiple firms compete within a product line. Similarly, as in my setting, generating markup dispersion and market concentration requires the leading firm within a product line to zoom ahead of its followers. The same mechanisms I suggest later work in such an extended setting.

As in Akcigit and Kerr (2018) and Peters (2020), the value of a firm consists of flow profits, research costs and three parts related to own-innovation, expansion and creative destruction. At rate τ_t (determined in equilibrium), the firm loses one of its n products, in which case $n-1$ of its products remain. The firm chooses own-innovation rates I_k and expansion rates x_k for each product, in which case the firm charges a λ times higher markup (own-innovation), or the firm acquires a new product (expansion). When acquiring a new product, the probability of facing a high-type second-best firm is S, in which case the high-type entrant charges a markup of λ. With probability $1-S$, the second-best firm is of the low type and the high-type entrant charges a markup of $\lambda \times \varphi^h/\varphi^l$. The HJB equation of a low process efficiency firm follows the same expression with the product expansion term replaced by $x_k\Big[S_t V_t^l\Big(n+1,\Big[[\mu_i], \lambda \times \varphi^l/\varphi^h\Big]\Big)+(1-S_t)V_t^l\left(n+1,[[\mu_i],\lambda]\right)-V_t^l(n,[\mu_i])\Big]$, which reflects that a low productivity type firm charges a different markup upon entering a new product line.

The formulation of the value function differs from previous models in the literature that feature own-innovation and creative destruction along two dimensions. First, the value function (and the resulting firm policies) are specific to the productivity type of the firm. Second, the value function depends on the distribution of firm productivity types across product lines through the product expansion term. The expected value of acquiring a new product line depends on the probability of facing a high-productivity incumbent S_t. Firms take S_t as given when making optimal decisions; however, they affect it through their R&D efforts in equilibrium.

$\Gamma^h\left([x_i, I_i]; n, [\mu_i]\right)$ is the productivity type-specific cost of researching for both own-innovation and expansion. I assume the following flexible, functional form

$$\Gamma^h\left([x_i, I_i]; n, [\mu_i]\right) = \sum_{k=1}^{n} c\left(x_k, I_k; \mu_k\right) = \sum_{k=1}^{n}\left[\mu_k^{-1}\frac{1}{\psi_I^h}(I_k)^\zeta + \frac{1}{\psi_x}(x_k)^\zeta\right],$$

with $\zeta > 1$. Profits within a product line are concave in the markup. The incentives to own-innovate therefore decrease with the quality gap that the firm has accumulated. I scale the own-innovation costs by the inverse markup, which renders the product line-specific own-innovation rate independent of the product markup in equilibrium. I introduce a productivity-type specific own-innovation cost shifter (ψ_I^h, ψ_I^l) to match markup life cycle growth specific to the productivity type of the firm in the data. The cost shifter for creative destruction ψ_x is the same for both productivity types. This does not, however, imply that all firms choose equal expansion rates in equilibrium. High-productivity firms have a larger incentive to expand into new product markets as they charge higher markups upon entry. Heterogeneity in the cost shifter is unnecessary to generate productivity-specific expansion rates x^h and x^l. The R&D cost parameters are estimated by matching firm size and markup life cycle growth.

Firm entry is determined as follows: using a linear production technology, potential entrants produce a flow of marketable ideas ψ_z per unit of labor that improves the quality of a randomly selected product line. Entrants start with a one-step quality gap. I assume that after entering, firms get assigned the high productivity type with probability p^h. Denoting by z_t the equilibrium flow rate of entry, the free entry condition reads

$$p^h \left(\underbrace{S_t V_t^h(1,\lambda) + (1-S_t) V_t^h(1, \lambda \times \varphi^h/\varphi^l)}_{\text{Expected value of entering with } \varphi^h} \right) + (1-p^h) \left(\underbrace{S_t V_t^l(1, \lambda \times \varphi^l/\varphi^h) + (1-S_t) V_t^l(1,\lambda)}_{\text{Expected value of entering with } \varphi^l} \right) \leq \frac{1}{\psi_z} w_t, \tag{8}$$

which holds with equality if there is positive entry, i.e., $z_t > 0$. With high and low-process efficiency firms expanding at different rates, firm entry is necessary to generate a stable distribution of productivity types across product lines.

4.5 Distribution over quality and productivity gaps

In this section, I characterize the two-dimensional distribution of incumbents' quality and productivity gaps across all product lines, $\nu_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)$. This two-dimensional distribution characterizes the share of product lines operated by each productivity type, S, and labor demand for production and R&D. S and the labor demand, in return, enter the firms' optimization problem and the labor market clearing condition. From the firm's maximization problem, it will turn out that along the balanced growth path, the own-innovation and expansion rate are time-invariant and do not depend on the quality gap in a product line, i.e., they only vary across the firm's productivity type. Therefore, I characterize the distribution for the case where I and x only depend on the productivity type of the firm.

The distribution of quality and productivity gaps is characterized by a set of infinitely many differential equations. These differential equations capture the change in the mass of product lines that are of a specific quality gap, λ^Δ, and productivity gap, $\frac{\varphi_j}{\varphi_{j'}}$ as follows

$$\begin{aligned} \dot{\nu}_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right) &= I^l \nu_t\left(\Delta-1, \frac{\varphi_j}{\varphi_{j'}}\right) - \nu_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)(I^l + \tau_t) \quad \text{for} \quad \Delta \geq 2, \frac{\varphi_j}{\varphi_{j'}} \in \left\{\frac{\varphi^l}{\varphi^h}, \frac{\varphi^l}{\varphi^l}\right\} \\ \dot{\nu}_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right) &= I^h \nu_t\left(\Delta-1, \frac{\varphi_j}{\varphi_{j'}}\right) - \nu_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)(I^h + \tau_t) \quad \text{for} \quad \Delta \geq 2, \frac{\varphi_j}{\varphi_{j'}} \in \left\{\frac{\varphi^h}{\varphi^h}, \frac{\varphi^h}{\varphi^l}\right\}. \end{aligned} \tag{9}$$

For product lines with a unitary quality gap, $\Delta = 1$, the differential equations read:

$$\begin{aligned} \dot{\nu}_t\left(1, \frac{\varphi^l}{\varphi^h}\right) &= (1-S_t)x^l S_t + z_t(1-p^h)S_t - \nu_t\left(1, \frac{\varphi^l}{\varphi^h}\right)(I^l + \tau_t) \\ \dot{\nu}_t\left(1, \frac{\varphi^l}{\varphi^l}\right) &= (1-S_t)x^l(1-S_t) + z_t(1-p^h)(1-S_t) - \nu_t\left(1, \frac{\varphi^l}{\varphi^l}\right)(I^l + \tau_t) \\ \dot{\nu}_t\left(1, \frac{\varphi^h}{\varphi^h}\right) &= S_t x^h S_t + z_t p^h S_t - \nu_t\left(1, \frac{\varphi^h}{\varphi^h}\right)(I^h + \tau_t) \\ \dot{\nu}_t\left(1, \frac{\varphi^h}{\varphi^l}\right) &= S_t x^h(1-S_t) + z_t p^h(1-S_t) - \nu_t\left(1, \frac{\varphi^h}{\varphi^l}\right)(I^h + \tau_t). \end{aligned} \tag{10}$$

The law of motion for the mass of product lines with quality gap Δ and a given productivity gap is characterized by the inflow minus the outflow of product mass. Outflows are due to own-innovation (quality gap increases from Δ to $\Delta+1$) and creative destruction (quality gap gets reset from Δ to unity). For $\Delta \geq 2$, inflows into state Δ are due to own-innovation in product lines that previously had a quality gap of $\Delta - 1$. For $\Delta = 1$, inflows come from product lines where creative destruction resets previously accumulated quality gaps back to unity.

Using the distribution of quality and productivity gaps, the aggregate share of products with a high-productivity incumbent is defined as

$$S_t = \sum_{i=1}^{\infty} \left[\nu_t \left(i, \frac{\varphi^h}{\varphi^h} \right) + \nu_t \left(i, \frac{\varphi^h}{\varphi^l} \right) \right]. \tag{11}$$

Inserting the differential equations in eqs. (9) and (10) into eq. (11) it follows

$$\dot{S}_t = S_t x^h (1 - S_t) - (1 - S_t) x^l S_t + z_t \left(p^h (1 - S_t) - (1 - p^h) S_t \right). \tag{12}$$

Changes in S_t are due to high-productivity firms expanding into markets with low-productivity incumbents (first term), low-productivity firms expanding into markets with high-productivity incumbents (second term), high-productivity entrants replacing low-productivity incumbents and low-productivity entrants replacing high-productivity incumbents (final term).

The rate of creative destruction is

$$\tau_t = z_t + x^h S_t + x^l (1 - S_t). \tag{13}$$

Note that τ_t is a function of S_t. The rate of creative destruction depends on the distribution of productivity types across product lines.

4.6 Balanced growth path (BGP) characterization

Labor market clearing requires that production labor L_{Pt} and research labor L_{Rt} add up to one, the aggregate labor endowment:

$$1 = L_{Pt} + L_{Rt} \tag{14}$$

A balanced growth path (BGP) is a set of allocations $[x_{it}, I_{it}, \ell_{it}, z_t, S_t, y_{it}, C_t]_{it}$ and prices $[r_t, w_t, p_{it}]_{it}$ such that firms choose $[x_{it}, I_{it}, p_{it}]$ optimally, the representative household maximizes utility choosing $[C_t, y_{it}]_{it}$, the growth rate of aggregate variables is constant, the free-entry condition holds, all markets clear and the cross-sectional joint distribution of quality and productivity gaps ν_t is stationary.

In Appendix B.1, I show that the value function of the high productivity type firm takes on the following expression (with h replaced by l for the low productivity type)

$$\begin{aligned} V_t^h(n, [\mu_i]) =& V_{t,P}^h(n) + \sum_{k=1}^{n} V_{t,M}^h(\mu_k) \\ =& n \frac{1}{(\rho+\tau)} \frac{\zeta-1}{\psi_x} (x^h)^\zeta w_t + \sum_{k=1}^{n} \frac{\pi^h(\mu_k) + \frac{\zeta-1}{\psi_I^h} (I^h)^\zeta w_t \mu_k^{-1}}{\rho+\tau}. \end{aligned} \tag{15}$$

The value function is additive across products. The first part of the value function that represents the option value of expanding into new markets scales linearly in the number of products. The second part consists of flow profits and the option value to increase markups further. Both terms are scaled by the sum of the discount factor and the rate of creative destruction, the rate at which products get replaced. The value function is productivity type specific. First, the option value to expand (first term) is productivity type specific since different firm types expand at different rates. Second, firm productivity enters flow profits and the option value to increase markups in the future (second term) through productivity-specific markups and own-innovation rates.

In the following, I characterize the two-dimensional distribution of quality and productivity gaps along the BGP as a function of firm policies. This allows for optimal policies and the distribution to be solved jointly. I solve for the steady state distribution over quality and productivity gaps by setting the differential equations characterizing the law-of-motion in eq. (9) and (10) equal to zero. The stationary mass of product lines with quality gap λ^{Δ} and productivity gap φ^i/φ^j is given by

$$\nu\left(\Delta, \frac{\varphi^l}{\varphi^h}\right) = \left(\frac{I^l}{I^l+\tau}\right)^{\Delta} \frac{(1-S)x^l S + z(1-p^h)S}{I^l}$$
$$\nu\left(\Delta, \frac{\varphi^l}{\varphi^l}\right) = \left(\frac{I^l}{I^l+\tau}\right)^{\Delta} \frac{(1-S)x^l(1-S) + z(1-p^h)(1-S)}{I^l}$$
$$\nu\left(\Delta, \frac{\varphi^h}{\varphi^h}\right) = \left(\frac{I^h}{I^h+\tau}\right)^{\Delta} \frac{Sx^h S + zp^h S}{I^h}$$
$$\nu\left(\Delta, \frac{\varphi^h}{\varphi^l}\right) = \left(\frac{I^h}{I^h+\tau}\right)^{\Delta} \frac{Sx^h(1-S) + zp^h(1-S)}{I^h}$$

Summing the measure over all quality gaps for a given productivity gap yields the share of products with a given productivity gap:

$$S_{\varphi^l,\varphi^h} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^l}{\varphi^h}\right) = \frac{(1-S)x^l S + z(1-p^h)S}{\tau}$$
$$S_{\varphi^l,\varphi^l} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^l}{\varphi^l}\right) = \frac{(1-S)x^l(1-S) + z(1-p^h)(1-S)}{\tau}$$
$$S_{\varphi^h,\varphi^h} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^h}{\varphi^h}\right) = \frac{Sx^h S + zp^h S}{\tau}$$
$$S_{\varphi^h,\varphi^l} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^h}{\varphi^l}\right) = \frac{Sx^h(1-S) + zp^h(1-S)}{\tau}$$

The share of products with a given productivity gap is along the BGP equal to the share of total creative destruction that starts with a quality gap of unity for that given productivity gap at each instant in time. From this, S is implicitly defined by

$$S = S_{\varphi^h,\varphi^h} + S_{\varphi^h,\varphi^l} = \frac{Sx^h + zp^h}{\tau}. \tag{16}$$

In Appendix B.3, I show that the share of product lines with a quality gap smaller than m conditional on the productivity gap follows

$$P\left(\lambda^{\Delta} \leq m, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - m^{-\theta_l}\right),$$

where I denote $\theta_l = \frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}$. Conditional on the productivity gap, quality gaps follow a Pareto distribution with parameter θ. The Pareto shape parameter is affected by the rate of own-innovation I. The more own-innovation, the more mass is in the tail of the quality gap distribution. In Peters (2020), markups follow a Pareto distribution. I introduce ex-ante differences in firm productivities, which leads to the result that markups *conditional* on the productivity gap between incumbents and second-best firms are Pareto distributed.

The stationary two-dimensional distribution of productivity and quality gaps across all product lines characterizes (1) the aggregate labor income share Λ that enters the labor market clearing condition, (2) the TFP misallocation measure $\mathcal{M}$ that captures the static loss in output that arises from markup dispersion and (3) the average markup in the economy, see Appendix B.3 for the derivation.

$$\Lambda = \sum_{k\in\{h,l\}} \frac{\theta_k}{\theta_k+1} \sum_{n\in\{h,l\}} \frac{1}{\varphi_k/\varphi_n} S_{\varphi_k,\varphi_n}$$

$$\mathcal{M} = \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\left[S_{\varphi_k,\varphi_n}\left(\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)-\frac{1}{\theta_k}\right)\right]}}{\Lambda}$$

$$E[\mu] = \sum_{k\in\{h,l\}} \frac{\theta_k}{\theta_k-1} \sum_{n\in\{h,l\}} \frac{\varphi_k}{\varphi_n} \times S_{\varphi_k,\varphi_n}$$

All three have in common that they depend on the speed of own-innovation relative to creative destruction, θ, the size of productivity gaps, φ_k/φ_l, and the distribution of productivity gaps across product lines, S_{φ_k,φ_n}. I provide further moments of the markup distribution in Appendix B.4.

Growth in this model is the result of R&D at the product level. This occurs through either own-innovation, firm expansion or firm entry. The steady-state growth rate of aggregate variables is derived in Appendix B.5.

$$g = \frac{\dot{Q}_t}{Q_t} = \Bigg(\underbrace{SI^h + (1-S)I^l}_{\text{Incumbent own-innovation}} + \underbrace{Sx^h + (1-S)x^l}_{\text{Incumbent product expansion}} + \underbrace{z}_{\text{Entry}} \Bigg) \times \ln(\lambda) \tag{17}$$

The growth rate is equal to the aggregate arrival rate of innovation times the step size of innovation, $\ln(\lambda)$. Since firms with different productivities innovate at different rates, the growth rate depends on the distribution of productivity types across product lines, S.

To find the BGP solution of the model, I solve the optimality conditions of the firm (derived in Appendix B.1) jointly with the aggregate rate of creative destruction, eq. (13), the share of productivity types across product lines, eq. (18), the free entry condition, eq. (8) and the labor market clearing condition, eq. (14), see Appendix B.2 for details.

4.6.1 Discussion of the stationary distribution

In equilibrium, high-productivity firms expand into new markets at higher rates than low-productivity firms. Firm entry prevents high-productivity firms from capturing all product lines. To see this note that in steady state $\dot{S} = 0$ such that eq. (12) turns into

$$z\left(S - p^h\right) = S(1-S)(x^h - x^l). \tag{18}$$

It is worthwhile to discuss eq. (18) since it provides intuition on the relationship between expansion rates and firm entry. If high-productivity incumbents expand at higher rates than low-productivity firms ($x^h > x^l$), for the share of high-productivity incumbents to be constant along the BGP, S needs to be greater than p^h, the share of entrants with the high productivity type. In other words, the share of high-productivity firms among entrants must be lower than that of high-productivity firms in the economy. In this case, "sufficient" entry by low-productivity firms balances the relatively higher expansion rate by existing high-productivity incumbents and the share of lines operated by high-productivity firms remains constant. Eq. (18) highlights the role of firm entry. Without entry ($z = 0$), higher expansion rates by high-productivity incumbents would result in those firms eventually overtaking all product lines. Given $x^h > x^l$ and $0 < S < 1$, for eq. (18) to hold firm entry, z, needs to be positive.

In the special case where all entrants are of the low productivity type ($p^h = 0$), eq. (18) can be written as

$$Sx^h(1-S) - (1-S)x^l S = zS.$$

Entry by low-productivity firms that replace high-productivity firms (zS) makes up precisely for the lost market share of incumbent low-productivity firms, $Sx^h(1-S) - (1-S)x^l S$, such that the aggregate share of high-productivity firms remains constant.

4.7 Firm dynamics

In this section, I derive how firm markups, sales and survival evolve with age and characterize the firm size distribution. The results of this section will again be used when taking the model to the data.

4.7.1 Markup dynamics

Firm markups or $TFPR$ are defined in eq. (6) as $\mu_f = \frac{py_f}{wl_f} = \left(\frac{1}{n_f}\sum_{i\in N_f}\mu_{if}^{-1}\right)^{-1}$. The firm markup is the harmonic mean of its product markups. In Appendix B.6, I show that for a high process efficiency firm, the expected log markup conditional on firm age a_f is

$$E\left[\ln \mu_f | \text{firm age} = a_f, \varphi^h\right] = \underbrace{\ln\lambda \times \left(1 + I^h \times E[a_P^h|a_f]\right)}_{\text{Quality improvements}} + \underbrace{(1-S)\times \ln\left(\frac{\varphi^h}{\varphi^l}\right)}_{\text{Productivity advantage}}, \tag{19}$$

where $E[a_P^h|a_f]$ denotes the average product age of a high process efficiency firm conditional on firm age, which is given by

$$\begin{aligned}
E[a_P^h|a_f] &= \frac{1}{x^h}\left(\frac{\frac{1}{\tau}\left(1-e^{-\tau a_f}\right)}{\frac{1}{x^h+\tau}\left(1-e^{-(x^h+\tau)a_f}\right)} - 1\right)\left(1-\phi^h(a_f)\right) + a_f\phi^h(a_f)\\
\phi^h(a) &= e^{-x^h a}\frac{1}{\gamma^h(a)}\ln\left(\frac{1}{1-\gamma^h(a)}\right)\\
\gamma^h(a) &= \frac{x^h\left(1-e^{-(\tau-x^h)a}\right)}{\tau - x^h e^{-(\tau-x^h)a}},
\end{aligned}$$

Expected markups conditional on age consist of two terms. The first term in eq. (19) reflects how average quality gaps across the firm's products evolve with firm age. It captures the effects that as the firm ages, it improves the quality of its continuing items, the firm loses product lines for which it has accumulated quality gaps and acquires new products with initially low-quality gaps. This term is akin to the markup expression in Peters (2020). In Peters (2020), this term holds for all firms, whereas in my model, this term is specific to the productivity type of the firm as own-innovation and expansion rates vary by firm type. Permanent differences in the process efficiency level across firms affect not only expected markup growth (captured by the first term) but also introduce a level effect captured by the second term in eq. (19). The intuition behind the second term is that if a high process efficiency firm faces a low process efficiency second best producer in a given line, it can charge a φ^h/φ^l higher markup, which occurs in expectation in $1 - S$ of the firm's product lines.

The expected markup conditional on firm age for a low process efficiency firm is

$$E\left[\ln \mu_f | \text{firm age} = a_f, \varphi^l\right] = \underbrace{\ln \lambda \times \left(1 + I^l \times E[a_P^l | a_f]\right)}_{\text{Quality improvements}} + \underbrace{S \times \ln\left(\frac{\varphi^l}{\varphi^h}\right)}_{\text{Productivity disadvantage}}. \tag{20}$$

The first term capturing the quality advantage is equivalent to the first term in eq. (19) for the high type. $E[a_P^l | a_f]$ follows the same expression as $E[a_P^h | a_f]$ with h replaced by l. The second term in eq. (20) differs from eq. (19) as low productivity firms have a process efficiency disadvantage in a share S of their product lines, where they face a high productivity second best producer. Since $\varphi^l < \varphi^h$, this term is negative.

4.7.2 Sales dynamics

Firms lose products according to the same stochastic process as in Klette and Kortum (2004) that, in their model, results in a skewed sales distribution, a decreasing variance of sales growth in size, a declining exit probability in age and size and firm size growth being independent of size. In my model, the rate at which firms add products is heterogeneous across firms. High-process-efficiency firms add new products faster than low-process efficiency firms, which in turn affects firm sales and survival. Therefore, the properties related to firm size and survival in Klette and Kortum (2004) hold conditional on the productivity type of the firm.

Firm sales are proportional to the number of products a firm produces. Since more productive firms add new products faster, sales growth is specific to the productivity type. Expected log sales growth for a firm with process efficiency $\varphi^j, j \in \{h, l\}$ between age zero and age a_f is $E\left[\ln nY | a_f, \varphi^j\right] - E\left[\ln nY | 0, \varphi^j\right]$, where n is the number of products a firm is producing. Firm sales growth stems from aggregate growth and from the firm gaining and losing products as it ages

$$E\left[\ln nY | a_f, \varphi^j\right] - E\left[\ln nY | 0, \varphi^j\right] = \underbrace{g \times a_f}_{\text{Aggregate growth}} + \underbrace{E\left[\ln n | a_f, \varphi^j\right]}_{\text{Firm's product growth}}.$$

To derive $E\left[\ln n | a_f, \varphi^j\right]$ note that the probability of a high process efficiency firm producing n products at age a conditional on survival is $\left(1 - \gamma^j(a)\right)\left(\gamma^j(a)\right)^{n-1}$, where $\gamma^j(a) = x^j \frac{1 - e^{-(\tau - x^j)a}}{\tau - x^j e^{-(\tau - x^j)a}}$.

Therefore sales growth is given by

$$E\left[\ln nY|a_f,\varphi^j\right] - E\left[\ln nY|0,\varphi^j\right] = \underbrace{g \times a_f}_{\text{Aggregate growth}} + \underbrace{\left(1-\gamma^j(a_f)\right)\sum_{n=1}^{\infty}\ln n \times \left(\gamma^j(a_f)\right)^{n-1}}_{\text{Firm's product growth}}. \quad (21)$$

4.7.3 Employment dynamics and firm survival

Employment of the firm can be decomposed into the number of products that the firm produces and its markup as in Peters (2020). In my model, product and markup dynamics depend on the process efficiency of the firm such that

$$E[\ln l_f|\text{firm age} = a_f,\varphi^j] = E\left[\ln\left(\frac{nY}{w\mu_f}\right)|a_f,\varphi^j\right] = \ln\left(\frac{Y}{w}\right) + E\left[\ln n|a_f,\varphi^j\right] - E\left[\ln \mu_f|a_f,\varphi^j\right],$$

where $\varphi^j \in \{\varphi^h,\varphi^l\}$. Employment growth then simply is

$$E[\ln l_f|a_f,\varphi^j] - E[\ln l_f|0,\varphi^j] = \underbrace{E\left[\ln n|a_f,\varphi^j\right]}_{\text{Firm's product growth}} - \underbrace{\left(E\left[\ln \mu_f|a_f,\varphi^j\right] - E\left[\ln \mu_f|0,\varphi^j\right]\right)}_{\text{Firm's markup growth}}. \quad (22)$$

$E\left[\ln \mu_f|a_f,\varphi^j\right] - E\left[\ln \mu_f|0,\varphi^j\right]$ and $E\left[\ln n|a_f,\varphi^j\right]$ are derived in eq. (19), (20) and (21).

Firm size dynamics determine firm survival since firms that lose their final product become inactive. Firms that grow faster in size have a higher probability of survival. Hence, firm survival is productivity-type specific. The share of high and low process efficiency firms surviving until age a is given by

$$s^h(a_f) = 1 - \tau\frac{1-e^{-(\tau-x^h)a_f}}{\tau - x^h e^{-(\tau-x^h)a_f}} \quad (23)$$

$$s^l(a_f) = 1 - \tau\frac{1-e^{-(\tau-x^l)a_f}}{\tau - x^l e^{-(\tau-x^l)a_f}}. \quad (24)$$

4.7.4 Firm size distribution

The mass of high and low process efficiency firms with $n \geq 2$ products follows the differential equations

$$\begin{aligned}\dot{M}_t^h(n) &= (n-1)x^h M_t^h(n-1) + (n+1)\tau M_t^h(n+1) - n(x^h+\tau)M_t^h(n)\\ \dot{M}_t^l(n) &= (n-1)x^l M_t^l(n-1) + (n+1)\tau M_t^l(n+1) - n(x^l+\tau)M_t^l(n),\end{aligned} \quad (25)$$

whereas the mass of firms with one product evolves according to

$$\begin{aligned}\dot{M}_t^h(1) &= zp^h + 2\tau M_t^h(2) - (x^h+\tau)M_t^h(1)\\ \dot{M}_t^l(1) &= z(1-p^h) + 2\tau M_t^l(2) - (x^l+\tau)M_t^l(1).\end{aligned} \quad (26)$$

The mass of firms with n products increases through firms with $n-1$ products expanding to size n at rate x^h or x^l per product or through firms with $n+1$ products losing one product at the rate of aggregate creative destruction. The mass of firms with n products decreases through firms with

n products either gaining or losing a product through expansion or creative destruction. The mass of firms with one product additionally increases through firm entry. The change in the total mass of firms at any point in time is $z - \tau(M_t^h(1) + M_t^l(1))$. To find the stationary firm size distribution, I set the time derivatives in equations (25) and (26) equal to zero. The steady-state mass of high and low process efficiency firms with n products is

$$M^h(n) = \frac{(x^h)^{n-1} z p^h}{n\tau^n} = \frac{zp^h}{x^h}\frac{1}{n}\left(\frac{x^h}{\tau}\right)^n$$
$$M^l(n) = \frac{(x^l)^{n-1} z(1-p^h)}{n\tau^n} = \frac{z(1-p^h)}{x^l}\frac{1}{n}\left(\frac{x^l}{\tau}\right)^n.$$

From this, one obtains (all in steady-state) the total mass of firms with n products

$$M(n) = M^h(n) + M^l(n) = \frac{(x^h)^{n-1} zp^h + (x^l)^{n-1} z(1-p^h)}{n\tau^n}, \tag{27}$$

the mass of firms of each productivity type

$$M^h = \sum_{n=1}^{\infty} M^h(n) = \frac{zp^h}{x^h}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{x^h}{\tau}\right)^n = \frac{zp^h}{x^h}\ln\left(\frac{\tau}{\tau - x^h}\right)$$
$$M^l = \sum_{n=1}^{\infty} M^l(n) = \frac{z(1-p^h)}{x^l}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{x^l}{\tau}\right)^n = \frac{z(1-p^h)}{x^l}\ln\left(\frac{\tau}{\tau - x^l}\right)$$

and the total mass of firms

$$M = M^h + M^l.$$

As in Lentz and Mortensen (2008), the share of firms with n products for each firm type, $M^h(n)/M^h$ and $M^l(n)/M^l$, follows the pdf of a logarithmic distribution with parameter x^h/τ and x^l/τ. The firm size distribution is highly skewed to the right.

Since there is a continuum of mass one of products and each product is mapped to one firm $\sum_{i=1}^{\infty} M(n) \times n = 1$. Further, the mass of high process efficiency firms producing n products is related to the previously defined share of product lines operated by high process efficiency firms, S, as follows

$$S = \sum_{n=1}^{\infty} M^h(n) \times n = \frac{zp^h}{\tau - x^h}.$$

From the firm size distribution, I obtain the share of high process efficiency firms

$$S_{M^h} = \frac{M^h}{M},$$

the entry rate

$$\text{Entry rate} = \frac{z}{M} \tag{28}$$

and the standard deviation of log firm sales

$$\sigma(\ln sales) \equiv \sigma(\ln pyn) = \sqrt{\sum_{n=1}^{\infty}\frac{M(n)}{M}(\ln n)^2 - \left(\sum_{n=1}^{\infty}\frac{M(n)}{M}\ln n\right)^2}. \tag{29}$$

4.8 Testing model predictions

In this section, I test predictions of the model about firms' price setting and determinants of revenue productivity.

4.8.1 Quality, price and revenue productivity growth

In the model, product innovations occur along the quality dimension, i.e., every product innovation leads to an improvement in product quality of size λ. A quality improvement through incumbent own-innovation increases the product price and markup according to eqs. (3) and (4). Linking micro data on firms' price setting with balance sheet data, I provide suggestive evidence that quality improvements are positively related to price growth at the product level and that firms with higher price growth display faster revenue productivity (or markup) growth. Taken together, the results suggest that innovations (quality improvements) are associated with faster revenue productivity (or markup) growth in the data.

To study firms' price setting, I exploit the micro data underlying the Swedish Producer Price Index (PPI) that contains reported item prices and information on quality-adjusted prices. If firms report to Statistics Sweden (SCB) that the nature of their good has changed, e.g., due to changes in quality, SCB assesses the change in quality and computes a quality-adjusted price for the new item. If the new item is an updated version of the old item, SCB keeps the item ID unchanged. I identify a quality change as an instance where SCB updates the quality-adjusted price while the item ID remains unchanged. Around the time of the quality improvement, the nominal price is expected to increase according to the theory.

To test whether changes in item quality are reflected in nominal price changes, I regress (annualized) monthly log price growth rates at the item level, i, on a dummy indicating whether a quality adjustment occurred in that month controlling for industry $\times$ year fixed effects, δ_{kt}:

$$\ln \frac{p_{ikt,month}}{p_{ikt,month-1}} = \beta_0 + \beta_1 \mathbb{1}\{Quality\ adjustment\}_{ikt,month} + \delta_{kt} + u_{ikt,month}.$$

Table 2: Change in nominal price after quality improvement

	$\ln \frac{p_t}{p_{t-1}}$
$\mathbb{1}\{Quality\ adjustment\}_t$	0.1833
	(5.193)
Constant	0.0179
	(73.802)
Industry $\times$ year FE	✓
N	971,467

Notes: t statistics in parentheses. Clustered standard errors at the industry $\times$ year level. $\frac{p_t}{p_{t-1}}$ denotes annualized month-to-month price growth. Number of quality adjustments for continued items in data: 6,676. Regression restricted to items with an annualized monthly price growth rate of $|\ln p_{ikt,month} - \ln p_{ikt,month-1}| < 0.5 \times 12$, which includes 99.5% of the observations.

The results are shown in Table 2. The coefficient on the quality adjustment dummy is positive and statistically significant. Items with changes in quality display an annualized 0.18 log point higher price increase than non-adjusted items. In line with the theory, quality adjustments are associated with price increases.

To show that firms with higher price growth display faster $TFPR$ growth, I regress log $TFPR$ changes at the firm level on price changes controlling for industry $\times$ year fixed effects.

$$\ln \frac{TFPR_{fkt}}{TFPR_{fkt-1}} = \beta_0 + \beta_1 \ln \frac{p_{fkt}}{p_{fkt-1}} + \delta_{kt} + u_{fkt}$$

Since prices are observed at the item level at a monthly frequency in the PPI microdata, I compute annual firm price growth rates as sales weighted December to December item price growth rates of the firm. This allows me to relate price changes to $TFPR$ changes that come with an annual frequency from the balance sheet data.

Table 3: Revenue productivity and output price growth

	$\ln \frac{TFPR_t}{TFPR_{t-1}}$
$\ln \frac{p_t}{p_{t-1}}$	0.2395
	(4.737)
Industry $\times$ year FE	✓
N	15,673

Notes: t statistics in parentheses. Clustered standard errors at the industry $\times$ year level. $\ln \frac{TFPR_{fkt}}{TFPR_{fkt-1}}$ denotes a firm's log TFPR growth with respect to the previous year. $\frac{p_t}{p_{t-1}}$ denotes a firm's December to December price growth averaged over the products of the firm (sales-weighted).

Table 3 shows the results. The coefficient on price changes is positive and significant. Larger price changes are associated with higher $TFPR$ growth. To be precise, firms with one log point higher price growth display on average 0.24 log points larger $TFPR$ growth. One comment is in order. The PPI micro data covers a sample and not the universe of item prices. The computed firm price growth rate, therefore, only reflects items included in the PPI and potentially does not include all of the firm's items. If the computed firm price growth rate is equal to the actual firm price growth rate plus a measurement error and actual price growth rates are uncorrelated with the measurement error, the regression coefficient is biased towards zero. Therefore, it should be viewed as a lower bound. Another reason why the estimate should be interpreted as a lower bound is that unobserved productivity changes of the firm are arguably negatively related to price changes and positively to $TFPR$ changes. Hence, price growth that originates from quality improvements potentially has larger effects on $TFPR$ growth than the one shown in Table 3.

The two regressions suggest that quality improvements lead to faster price growth and that firms with higher price growth increase their revenue productivity faster. The two facts link innovation with revenue productivity (or markup) growth through price growth.

4.8.2 Determinants of revenue productivity

In the model, revenue productivity (or markups) increases through own-innovation as the firm ages. This suggests that revenue productivity is positively related to firm age. Simultaneously, more productive firms (with higher revenue productivity) grow larger in size. This suggests that revenue productivity is positively related to firm size across firms. In Appendix C.1 I run a horse-race regression of revenue productivity on firm age and firm size. Consistent with the model, the regression identifies firm age and size as independent drivers of revenue productivity across firms. I further show that the revenue productivity-size relation within firms is smaller than across firms and that the revenue productivity-size relationship when size is measured by employment is smaller than when sales measure size. Both findings are consistent with the theory: first, the positive markup size correlation is driven by across-firm variation. More productive firms (with higher markups) end up large. Within firms, size growth (adding a new product) weakly lowers the firm markup, as products initially start with low markups. Second, as the firm increases its markup, it lowers its demand for labor. Therefore, the markup employment correlation is smaller than the markup sales correlation.

5 Model application

In this section, I study the drivers behind the trends in the Swedish economy. I estimate the model using Swedish administrative data in the first step.

5.1 Estimation

There are, in total, nine parameters in the model. The own-innovation efficiency for high and low productivity firms ψ_I^h, ψ_I^ℓ, the expansion efficiency ψ_x, the innovation cost curvature ζ, the entry efficiency ψ_z, the step size of innovation λ, the process efficiency differential $\frac{\varphi^h}{\varphi^\ell}$, the probability of being assigned the high process efficiency type conditional on entry p^h, and the discount factor ρ. I follow Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) and Peters (2020) that set ζ equal to two based on evidence from the microeconometric innovation literature (Blundell, Griffith and Windmeijer, 2002; Hall and Ziedonis, 2001). The discount factor ρ is set to 0.05. In the estimation, I target firm life cycle growth that, in the model, is productivity-type specific. Since firm productivity is unobserved in the data, I follow a model-based approach to classify firms into productivity types. The model predicts that firms with a high process efficiency expand faster in size. I identify high-productivity firms in the data as firms that are large on average conditional on firm age. In the first step, I regress firm sales on firm age, industry fixed effects and firm fixed effects. I then split firms into two groups according to their firm fixed effect and interpret firms with relatively high firm fixed effects as high process efficiency firms. Knowing the productivity type of a firm, I then compute p^h from the data as the share of firms at age zero that are of the high process efficiency type. I set the threshold for the firm fixed effect in the productivity classification such that 65% of the entrants are of the high type.[13] This leaves in total six parameters to estimate $(\psi_I^h, \psi_I^\ell, \psi_x, \psi_z, \lambda, \frac{\varphi^h}{\varphi^\ell})$.

I target six moments both at the firm and aggregate levels. At the firm level, I match the process efficiency-specific life cycle growth of $TFPR$ and employment (four moments in total).[14] I target

[13]In section 3 and the appendix, I show that the trends in the Swedish economy are not driven by a small share of firms, but the trends happen along the entire firm distribution. I, therefore, choose a more equal split of firms into productivity types than, e.g., focusing on the top 10% of firms. I check robustness concerning the threshold.

[14]Both employment and sales growth are natural candidates to target in the estimation. Using employment has the following advantages: sales levels of the firm are already used to classify firms into productivity types. Even though I target growth rates in the estimation, I expect fewer implications of the productivity classification for employment than for sales growth. Sales might be further related to value added in the numerator of $TFPR$. $TFPR$ growth is

employment growth between ages zero and 15, whereas, for $TFPR$, I target growth between ages zero and ten. $TFPR$ growth displays strong concavity. To match $TFPR$ growth in the firm's early years, I take age ten as the target. I further match the $TFPR$ difference between high and low process efficiency firms at age two to discipline the process efficiency differential.[15] The $TFPR$ and employment life cycles, as well as the difference in $TFPR$ between high and low process efficiency firms, are characterized analytically in eq. (19), (20) and (22). I obtain $TFPR$ and employment life cycle growth from the data by regressing log $TFPR$ and log employment (net of industry $\times$ year fixed effects) on firm age dummies (age zero left out) and a constant. For high process efficiency firms, $TFPR$ between age zero and ten grows by 0.114 log points and employment between age zero and 15 grows by 0.524 log points. For low process efficiency firms, I observe $TFPR$ growth of 0.098 log points and employment growth of 0.123 log points. The level difference of $TFPR$ between high and low process efficiency firms at age two is 0.046 log points. At the aggregate level, I target a TFP growth rate of 2.3%, which is taken from the Multifactor productivity (MFP) series by Statistics Sweden (SCB) measured in labor augmenting terms for 1996–2006. This growth rate is characterized analytically by eq. (17). Table 4 lists all targets. Since all moments are derived analytically as functions of the endogenous model outcomes, there is no need to simulate during the estimation. I estimate the model using the generalized method of moments (GMM).[16]

Table 4 shows the moment conditions and calibrated parameters. High process efficiency firms are roughly 40% more efficient in own-innovation than low process efficiency firms: $\psi_I^h/\psi_I^l = 1.39$, which is estimated from their steeper $TFPR$ growth profile. Firms with the high process efficiency type are roughly 4% more productive than low process efficiency firms: $\varphi^h/\varphi^l = 1.039$. Each product innovation improves an item's quality by roughly 6.3%: $\lambda = 1.063$.

5.2 Model fit of the Swedish economy

Table 4 shows that the model precisely hits all targeted moments. How does the model compare along untargeted moments of the Swedish economy? Figure 4 shows the life cycles of $TFPR$, employment and sales growth both for the data and the model. The displayed life cycle growth profiles are obtained from the data, as explained in the previous section. In the figure, only $TFPR$ growth with respect to age ten and employment growth with respect to age 15 are targeted; the remaining years are not. Sales life cycle growth is not targeted at all.[17]

Overall the model fits the growth profiles for $TFPR$, employment and sales well. For $TFPR$, the model overshoots the firm's later years slightly, particularly for low-process efficiency firms. However, at age 20, only 16% of the low and 27% of the high process efficiency firms are still active such that discrepancies between data and model for later years affect only a small share of firms. The employment fit is very good for both productivity types. Recall that the process efficiency type

already targeted in the calibration. The employment measure comes from a separate data source (RAMS).

[15]The reason I take age two to measure the gap is that this is the age when I observe most firms in the data. Age in my data is imputed based on the firm's first employee. This information comes from a data set (RAMS) independent of the balance sheet data (used to compute $TFPR$). Some firms do not report balance sheets the year they hire their first employee and hence show up in the balance sheet data after age zero.

[16]All targets receive equal weight. I use Julia's non-linear solver NLsolve with the trust-region method to minimize the distance between the model and data targets.

[17]In panel (a), I relate $TFPR$ growth from the data to the $TFPR$ level in the model by normalizing the growth profile to the model-implied $TFPR$ level for high and low process efficiency firms at age zero.

Table 4: Targeted moments and estimated parameters

		Data	Model	Source
Targets				
	$TFPR$ growth high prod.	0.114	0.114	Own calc.
	$TFPR$ growth low prod.	0.098	0.098	Own calc.
	Employment growth high prod.	0.524	0.524	Own calc.
	Employment growth low prod.	0.123	0.123	Own calc.
	$TFPR$ level difference	0.046	0.046	Own calc.
	Agg. growth rate g	2.3%	2.3%	SCB
Parameters				
ψ_I^h	*Inno. efficiency high prod.*		1.504	Estimated
ψ_I^l	*Inno. efficiency low prod.*		1.081	Estimated
ψ_x	*Expansion efficiency*		0.229	Estimated
ψ_z	*Entry efficiency*		1.336	Estimated
λ	*Step size of inno.*		1.063	Estimated
φ^h/φ^l	*High/low productivity gap*		1.039	Estimated
ρ	*Discount rate*		0.05	Set exog.
ζ	*Inno. cost curvature*		2	Set exog.
p^h	*Prob. high prod. type (entry)*		0.653	Set exog.

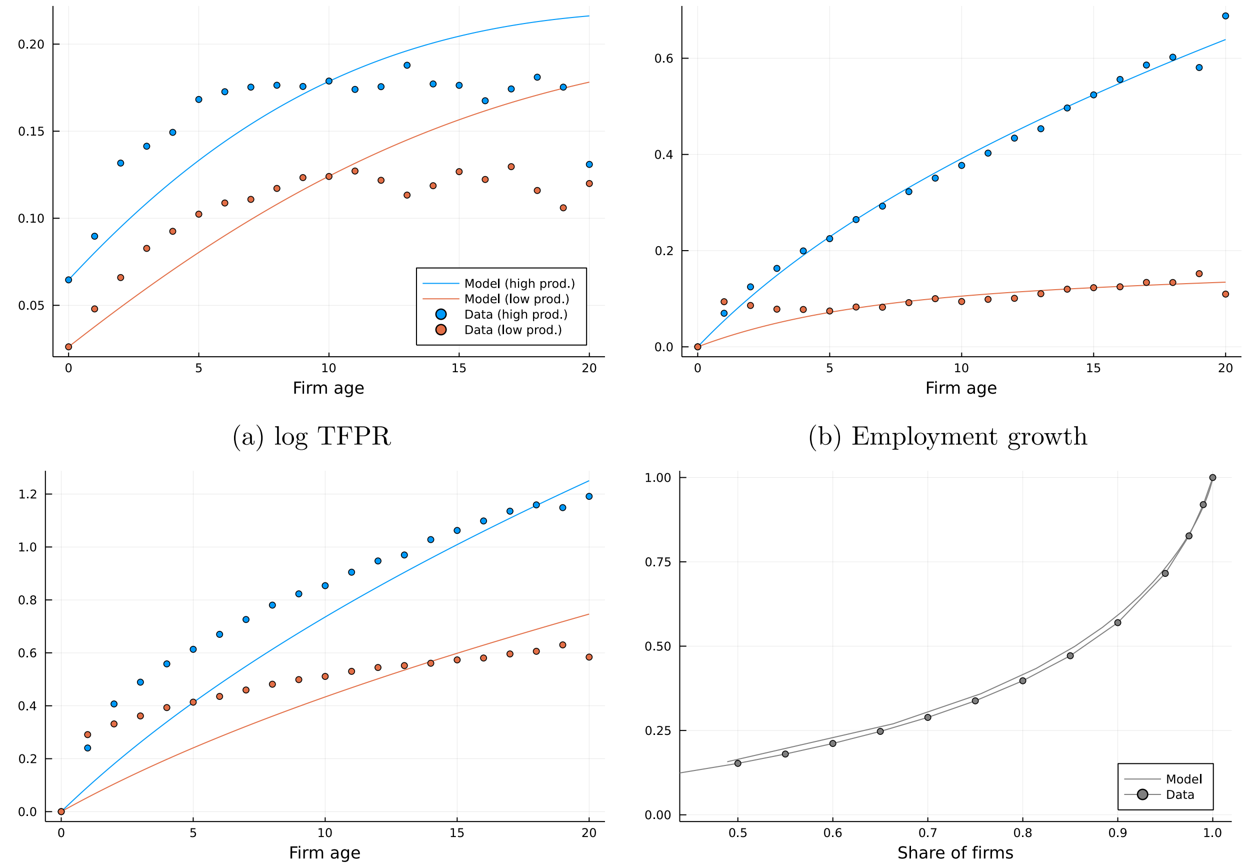
Notes: firm life cycle growth and p^h obtained from Swedish administrative data. Life cycle growth measured as log difference between age zero and ten ($TFPR$) or age zero and 15 (employment). The $TFPR$ difference between high and low productivity firms is measured at age two. The agg. growth target is obtained from Statistics Sweden (SCB), MFP series (1996–2006) in labor augmenting terms.

classification is based on sales, and the measure of employment comes from an independent data source (RAMS). Almost all data points align with the model-implied employment growth. Even though not targeted directly, the model fits the sales life cycle, particularly for the later years. I further compare the size distribution in the model and the data in panel (d). The size distribution is displayed in the form of the Lorenz curve, capturing the share of value added attributed to the smallest x% of firms. Both model and data show very similar concentration levels for output.

$TFPR$ growth of high productivity firms is only slightly higher than $TFPR$ growth of low productivity firms both in the data and in the estimated model (11.4% vs. 9.8% over the first ten years). Note that this does not imply that both firm types own-innovate at a similar speed. More productive firms own-innovate and expand into new markets faster than less productive firms. The fact that more productive firms add new product lines with low markups to their product portfolio at a faster rate deflates their $TFPR$ growth. Their higher expansion rate, therefore, partly offsets the effect of faster own-innovation on firm $TFPR$ growth. Their stark expansion rate differences become apparent from the heterogeneous life cycle profiles for sales in Figure 4.

The model also compares well with other non-targeted dimensions. The model implies an equilib-

Figure 4: Model fit of firm growth and size distribution (untargeted)



(a) log TFPR

(b) Employment growth

(c) Sales growth

(d) Share of value added

Notes: figure shows the fit of life cycle growth for high and low process efficiency firms separately, as well as the fit of the firm size distribution. $TFPR$, employment and sales growth obtained from the data by regressing its log (net of industry × year fixed effects) on age dummies (age zero left out) and a constant. Coefficients indicate the log difference to age zero. $TFPR$ growth is normalized by the model implied log $TFPR$ at age zero for each productivity type in panel (a). In the above panels, only $TFPR$ growth at age ten and employment growth at age 15 are targeted in the calibration.

rium entry rate characterized by eq. (28), of 0.059. Using the same administrative records as I do in this paper, Engbom (2020) reports an entry rate for the Swedish economy of around 0.065 in the late 1980s, followed by a drop to 0.05 to 0.055 during 2010–2015. Regarding markups, the model generates a sales-weighted markup average of 1.19 across product lines. The cost-weighted markup is 1.16. Sandström (2020) documents the time series of sales-weighted average markups in Sweden using the same underlying data source as I do for firms with more than ten employees. Average markups range from 1.13 to 1.23 during 1998–2016. De Loecker and Eeckhout (2018) report a sales-weighted markup of 1.31 for large and mainly publicly traded firms in Sweden in 2016. The sales distribution across productivity types is also similar in the model and the data. The sales share of high process efficiency firms is 0.91 in the model (S) vs. 0.92 in the data measured as the sales share of high process efficiency firms in the cross-section of firms averaged across all years 1997–2017.

5.3 Explaining the recent economic trends

In this section, I apply the model to explain the 19.5% increase in the standard deviation of log $TFPR$, the 5% increase in the standard deviation of log sales shares and the 25% increase in $TFPR$ life cycle growth at age four. Starting from the calibrated balanced growth path (BGP), I re-estimate model parameters to replicate the trends along a new BGP. I focus on changes in the cost of own-innovation for the high productivity firms ψ_I^h, the cost of creative destruction ψ_x and the entry cost ψ_z. Changes in the R&D costs affect the firms' optimal innovation efforts directly through the optimality conditions. They are, therefore, particularly well-suited to match the changes in $TFPR$ dispersion, sales concentration and $TFPR$ life cycle growth quantitatively. A higher own-innovation rate increases $TFPR$ life cycle growth and $TFPR$ dispersion ceteris paribus. A faster rate of firm expansion leads to higher sales concentration. The rate of creative destruction limits the extent of life cycle growth and cross-sectional dispersion in $TFPR$ and sales. ψ_I^h, ψ_x and ψ_z therefore match changes in $TFPR$ life cycle growth as well as dispersion in $TFPR$ and sales quite flexibly. Substituting one of the parameters results in the estimation falling short of explaining the three trends quantitatively, as explained later on.

The 25% increase in $TFPR$ life cycle growth at age four in Figure 3b is obtained from the pooled sample of firms, i.e., firms of both productivity types. I, therefore, target a 25% increase in $TFPR$ life cycle growth for the pooled sample of firms in the model, weighting productivity type specific $TFPR$ growth obtained from eqs. (19) and (20) with the type specific entry probability (p^h and $1 - p^h$) and survival probability (eqs. (23) and (24) all evaluated at age four). The standard deviation of log sales is characterized by eq. (29). To compute the standard deviation of $TFPR$ across firms, I simulate the model.[18] I re-estimate $(\psi_I^h, \psi_x, \psi_z)$ using the simulated method of moments (SMM) matching the industry and firm level trends along a new BGP.[19]

Table 5: New balanced growth path

	Parameters			*Moments*		
	ψ_z	ψ_I^h	ψ_x	$\sigma(\ln TFPR)$	$\sigma(\ln sales)$	$TFPR$ life cycle
Data				+19.5%	+5.0%	+25.0%
Model	-11.1%	+30.8%	-12.8%	+19.7%	+5.0%	+25.0%

Notes: the table shows the estimated parameter changes. ψ_I^h denotes the own-innovation efficiency (high type), ψ_x the creative destruction efficiency and ψ_z the entry efficiency.

Table 5 shows the estimated changes for the three parameters. The estimation asks for an 11% increase in the entry costs, a 31% decrease in the cost of own-innovation for the high productivity type, and a 13% increase in the cost of creative destruction. The three parameter changes can quantitatively replicate the change in $TFPR$ life cycle growth, $TFPR$ dispersion and sales concentration vis-à-vis the initial BGP. The changes in own-innovation and creative destruction costs

[18] I simulate the economy for 200 years, and the last period is used to compute the standard deviation of log $TFPR$ across firms. Since time is continuous in the model, I discretize time into 50 intervals within a year and simulate an economy consisting of 15.000 products.

[19] All three targets receive equal weight. I use Julia's non-linear solver NLsolve with the trust-region method to minimize the distance between the three model and data targets.

capture the general notion that for more productive firms to improve their own-products in markets where they have already established themselves as the market leader has become easier. At the same time, it has become harder for firms (incumbents and entrants) to expand into new product markets where competing firms have already established themselves as market leaders.

The changes in the innovation costs replicate the rise in revenue productivity dispersion, market concentration and acceleration in firm markup growth as follows. The decrease in the own-innovation costs for the more productive firms incentivizes those firms to own-innovate faster. A rise in the own-innovation efforts accelerates markup accumulation of those firms, i.e., markup life cycle growth increases. The acceleration in markup life cycle growth is associated with an increase in cross-sectional markup (or revenue productivity) dispersion. At the same time, as more productive firms accumulate markups faster, they have a higher incentive to enter new product markets. Together with a fall in firm entry due to increased creative destruction costs for entrants, this increases market concentration. Somewhat surprising is the increase in the cost of firm expansion for incumbent firms ψ_x. To generate an increase in sales concentration, one might expect a decrease in the cost of firm expansion. The estimation asks for an increase in the cost of firm expansion because the increase in the entry cost more than explains the rise in sales concentration as illustrated in Table 6.

Table 6: Decomposing the rise in sales concentration

	Baseline	$(\psi_z \downarrow, \psi_I^h \uparrow, \psi_x \downarrow)$	$\psi_z \downarrow$	$\psi_I^h \uparrow$	$\psi_x^* \downarrow$
Sales concentration $\sigma(\ln sales)$	0.836	+5.0%	+44.8%	-15.3%	-45.5%

Notes: the table shows the effect of each parameter change on sales concentration in isolation. The Baseline column shows levels in the initial BGP; remaining columns show percentage changes. $\psi_z \downarrow$ denotes the 11.1% decline in the entry efficiency, $\psi_I^h \uparrow$ the 30.8% increase in the own-innovation efficiency (high type) and $\psi_x \downarrow$ the 12.8% decrease in the expansion efficiency. $\psi_x^* \downarrow$ shows the linearized response to the 12.8% decrease in ψ_x as $\psi_x \downarrow$ does not generate a BGP solution in isolation.

Table 6 shows the change in the standard deviation of log sales in response to the parameter changes (both jointly and in isolation). Starting from the new BGP, I obtain the effect of each parameter change in isolation by turning the parameter change off (and on). The increase in the entry cost leads to a 44.8% increase in the standard deviation of log sales. The decline in creative destruction that results from the increase in entry costs raises firms' expansion efforts by more than what is required to match the targeted 5% increase in sales concentration. As a result, the estimation asks for an increase in the firm expansion costs. This highlights the importance of replicating the empirical trends jointly, as a decrease in expansion costs generates a rise in concentration in isolation.

I have focused on changes in own-innovation, creative destruction and entry costs in the estimation. To test for alternative drivers of the empirical trends, I re-run the SMM estimation for all possible parameter triples in the model. It turns out that other parameter combinations, despite replicating the trends qualitatively, fail to explain them quantitatively. On the other hand, alternative parameter combinations that get close to replicating the trends quantitatively consistently feature a decrease in the own-innovation costs for the high type, an increase in entry costs, or an increase in the creative destruction costs, which is the reason why those three parameters in combination fit the trends particularly well. Firm-type heterogeneity also plays an important role. Own-innovation costs for the high-productivity firms decrease, while the costs for the low-productivity firms remain unchanged. Decreasing the own-innovation costs for the low-productivity firms by the same amount as for the high-type firms (alongside the other parameter changes) results in a decrease in $\sigma(\ln sales)$ by 9%. Sales concentration decreases because lowering the cost of own-innovation for the low-productivity type incentivizes those firms to expand in size, thereby stealing sales shares from the high-productivity firms.

5.4 Implications for the long-run economy

In response to the parameter changes, the model makes predictions about the entry rate, markups, misallocation and aggregate growth rate in the long run, see Table 7. First, the model predicts a decline in the entry rate, which is consistent with Engbom (2020), who, using the same data source as I do in this paper, documents a decline in the firm entry rate in Sweden that goes back to the 1980s. Second, the model predicts an aggregate cost-weighted markup increase from roughly 1.16 to 1.2. As explained in the previous section, the changes in innovation costs incentivize more productive firms to accumulate markups faster and to expand into new product markets. In other words, the firms that accumulate markups faster are the ones that increase their sales shares. This is in line with Kehrig and Vincent (2021), which shows that the fall in the aggregate labor income share in the US is generated by firms whose labor share fell as they grew in size. In my model, the aggregate labor income share is the inverse of the aggregate (cost-weighted) markup.

Static efficiency in the model is affected by markup dispersion and aggregate productivity changes. The output loss that arises from markup dispersion increases from roughly 1.2% to 1.9% ($\mathcal{M}$ falls by roughly 0.7%). The fall in the own-innovation costs is responsible for the rise in markup dispersion: as firms accumulate markups faster, misallocation through markup dispersion increases. At the same time, more productive firms take over product lines from less productive firms resulting in an increase in aggregate productivity by 0.06%. The increase in aggregate productivity is slight compared to the increase in markup dispersion, so overall static efficiency falls.

Table 7: Implications for the long-run economy

	Initial BGP	New BGP
Entry rate	0.059	-0.23pp
Agg. markup $E^{cost}[\mu] - 1$	0.159	+4.23pp
Agg. output		
Markup dispersion $\mathcal{M}$	0.988	-0.68%
Agg. productivity Φ		+0.06%
Growth rate g	0.023	+0.48pp

Notes: the table shows the effect of the estimated parameter changes on the long-run economy. The first column shows levels in the initial balanced growth path. The second column shows changes in percentage points (pp) or percent after the rise in entry costs, the fall in own-innovation costs and the rise in creative destruction costs.

Interestingly, the model predicts an increase in the aggregate growth rate from 2.3% to roughly 2.8%. Why is the aggregate growth rate increasing? To understand the increase in economic growth, I run counterfactuals changing each innovation cost in isolation. Table 8, last row, shows the change in the aggregate growth rate Δg in response to each parameter change. The change in own-innovation costs for the more productive firms accounts for the most significant fraction of the change in growth. This is because the more productive firms increase their own-innovation efforts in response to the changes in their own-innovation costs, thereby generating economic growth. The rise in own-innovation causes both a loss in aggregate output through markup dispersion and economic growth. The rise in the creative destruction costs for entrants, $\psi_z \downarrow$, also contributes a positive growth effect (second column, Table 8) by incentivizing incumbent firms to innovate faster. However, most of the increase in the aggregate growth rate is due to the decrease in own-innovation costs.

More productive firms (with higher innovation rates) increase their innovation efforts and expand into new product markets. I quantify how much of the increase in the aggregate growth rate comes from changes in economic growth generated in the average product line and the reallocation of product lines towards more innovative firms. The aggregate growth rate in eq. (17) can be written as

$$g = Sg^h + (1-S)g^l.$$

$g^h \equiv (I^h + x^h + z)\ln(\lambda)$ and $g^l \equiv (I^l + x^l + z)\ln(\lambda)$ denote the growth rate per product line, where a high or low productivity type firms is the incumbent. Using a standard shift-share decomposition, I decompose changes in g as follows

$$\Delta g = g_{new} - g_{old} = \underbrace{S_{old}\Delta g^h + (1-S_{old})\Delta g^\ell}_{\Delta\text{Within}} + \underbrace{g^h_{old}\Delta S - g^\ell_{old}\Delta S}_{\Delta\text{Between}} + \underbrace{\Delta g^h \Delta S - \Delta g^\ell \Delta S}_{\Delta\text{Cross term}}, \tag{30}$$

where *old* and *new* index steady-state variables before and after the parameter change. Changes in the aggregate growth rate are due to changes in innovation efforts of firms holding the composition of product lines across firm types constant (ΔWithin), due to changes in the composition of product lines across firms of different types holding innovation efforts of firms constant (ΔBetween) and due to changes in both innovation efforts and the composition of lines (ΔCross-term). The ΔCross-term turns out to be small such that I group ΔBetween and ΔCross-term into a common ΔReallocation term.

Table 8 shows the decomposition of changes in the aggregate growth rate between the initial and new BGP into the contributions from ΔWithin and ΔReallocation. The majority of the total change in aggregate growth (47.6bp) comes from the ΔWithin effect (45.6bp). The ΔReallocation effect is relatively small. The reason is that the decline in own-innovation costs for the high type leads to a strong ΔWithin response: as own-innovation becomes cheaper own-innovation efforts in the average product line increase. This does not imply that ΔReallocation effects are generally minor compared to ΔWithin effects in the estimated model. Column 2 shows the decomposition for the entry cost increase in isolation. The ΔReallocation effect is ten times the size of the ΔWithin effect: the increase in entry costs affects growth mainly through reallocation as more innovative firms capture more product lines. The increase in creative destruction costs (last column) also leads

Table 8: Decomposing the change in the agg. growth rate

	$(\psi_z \downarrow, \psi_I^h \uparrow, \psi_x \downarrow)$	$\psi_z \downarrow$	$\psi_I^h \uparrow$	$\psi_x^* \downarrow$
ΔWithin	45.6bp	1.6bp	44.6bp	4.1bp
ΔReallocation	2.0bp	11.6bp	-3.4bp	-11.2bp
Δg	47.6bp	13.3bp	41.2bp	-7.1bp

Notes: the table shows the effect of each estimated parameter change on the agg. growth rate and decomposes changes in growth into ΔWithin and ΔReallocation effects. ΔWithin and ΔReallocation follow the decomposition in eq. (30). $\psi_z \downarrow$ denotes the 11.1% decline in the entry efficiency, $\psi_I^h \uparrow$ the 30.8% increase in the own-innovation efficiency (high type) and $\psi_x \downarrow$ the 12.8% decrease in the expansion efficiency. $\psi_x^* \downarrow$ shows the linearized response to the 12.8% decrease in ψ_x as $\psi_x \downarrow$ does not generate a BGP solution in isolation.

to ΔReallocation effects that dominate the ΔWithin effect. In this case, the increase in creative destruction costs reallocates market shares to low productivity incumbents with lower innovation rates such that the ΔReallocation effect is negative. The ΔReallocation effects from changes in the entry and creative destruction costs offset each other, however, such that most of the total change in aggregate growth comes from the ΔWithin response.

The changes in innovation costs imply a fall in static efficiency and an increase in the aggregate growth rate of the economy. To trade off the changes in efficiency and growth, I compare the change in lifetime utility across the old and the new BGP. In particular, I compare lifetime utility of BGPs, starting from the same level of average quality $\mathcal{Q}$ (see Appendix E for a detailed description). Lifetime utility of a consumption path where C_t grows at rate g is

$$\mathcal{U}\left(\{C_t\}_{t=0}^{\infty}\right) = \int_0^{\infty} e^{-\rho t} \ln C_t dt = \frac{1}{\rho} \ln C_0 + \frac{g}{\rho^2} = \mathcal{U}\left(C_0, g\right).$$

Lifetime utility along the BGP depends on three variables: the fixed parameter ρ that captures the household's discount rate, detrended consumption C_0 and the economy's growth rate g. In the new BGP, detrended consumption is 2.6% lower than in the initial BGP. Both a fall in static efficiency and production labor reduce detrended consumption. The upside is that the economy's growth rate is 0.48pp higher in the new BGP. I measure the change in lifetime utility in permanent consumption-equivalent terms, ξ, as follows

$$\mathcal{U}\left((1+\xi)C_0^{old}, g^{old}\right) = \frac{\ln(1+\xi)}{\rho} + \mathcal{U}\left(C_0^{old}, g^{old}\right) = \mathcal{U}\left(C_0^{new}, g^{new}\right). \tag{31}$$

ξ measures the permanent increase in consumption in the *old* BGP that equates lifetime utility of the *old* and *new* BGP. The change in innovation costs is associated with a 7.1% permanent increase in consumption, $\xi = 0.071$. Note that this comparison does not consider the transition from the initial to the new BGP.

5.5 R&D policies

Creative destruction reduces markup dispersion, generating a static increase in aggregate output. Should a government, therefore, subsidize the creative destruction costs of firms? One example of a creative destruction subsidy is a patent waiver that has been discussed recently for Covid vaccines or green technologies. Lifting patent protection lowers firms' costs to adopt and improve the technology of incumbent patenting firms. What are the effects of such a policy on static efficiency and economic growth? Rising product market entry lowers the market power of incumbent patent holders. On the other hand, facing a higher risk of replacement, incumbent firms have a lower incentive to innovate their products with potentially damaging consequences for economic growth. Therefore, quantifying the effects of a patent waiver on economic growth and static efficiency requires a model that includes both product market entry and incumbent R&D.

The model presented in section 4, featuring creative destruction and incumbent within-product market R&D, is a suitable candidate for this exercise. I introduce a government to the model that taxes firm profits and subsidizes firms' R&D, either creative destruction or within-product market R&D expenditures, while running a balanced budget.[20] The profit tax and research subsidy affect the firm as follows. Following the notation in section 4, production profits in line i are given by

$$\pi_t(\mu_i) = (1-\chi_t)Y_t\left(1-\frac{1}{\mu_{it}}\right),$$

where χ_t denotes the government's tax rate. After-tax profits equal pre-tax profits minus the tax. The firm faces own-innovation and creative destruction research costs of

$$w_t\Gamma\left(x_{ti}, I_{ti}, \mu_{ti}\right) = w_t\left[(1-\upsilon_{It})\mu_{ti}^{-1}\frac{1}{\psi_{Ii}}I_{ti}^{\zeta} + (1-\upsilon_{xt})\frac{1}{\psi_x}x_{ti}^{\zeta}\right],$$

where υ_{It} and υ_{xt} denote the own-innovation and creative destruction specific subsidy rates. I allow for subsidies of both creative destruction and own-innovation costs. This is the same cost formulation as in section 4 with the addition that the firm gets rebated a fraction (υ_{It} or υ_{xt}) of its research costs. This formulation reduces to the model without the government with $\chi_t = \upsilon_{it} = \upsilon_{xt} = 0$. The government satisfies its budget constraint

$$\chi_t Y_t(1-\Lambda_t) = w_t\int_0^1\left[\upsilon_{It}\mu_i^{-1}\frac{1}{\psi_{Ii}}I_{ti}^{\zeta} + \upsilon_{xt}\frac{1}{\psi_x}x_{ti}^{\zeta}\right]di.$$

The government's budget constraint requires that total tax revenues ($1-\Lambda_t$ is the aggregate profit share) equal total research subsidies paid out (right-hand side). For a given tax rate χ_t, I solve for the creative destruction or own-innovation subsidy rate that clears the government's budget constraint jointly with the other equilibrium conditions.

Table 9 shows the effect of a 10% profit tax on the aggregate growth rate and static efficiency under different R&D policies. I consider a subsidy on creative destruction costs, a subsidy on own-innovation costs and a subsidy on both costs (all financed by a 10% profit tax). The first row shows the subsidy on creative destruction. The subsidy that clears the government's budget constraint is

[20] The government could alternatively subsidize entry costs. Table 15 shows that firm entry contributes very little to aggregate growth. The effects of such a policy are, therefore, small. Instead of taxing profits, the government could also tax the firm's revenues. Taxing revenues instead of profits affects firms' incentives for own-innovation and creative destruction differently. The results with a revenue tax are quantitatively different but qualitatively identical to those with a profit tax.

a 26% subsidy on the firms' creative destruction costs. The creative destruction subsidy results in a decrease in the aggregate growth rate of 0.2pp. Firms react to the increase in creative destruction by lowering their own-innovation efforts. The increased risk of replacement lowers the incentives for firms to accumulate markups. The decrease in own-innovation weighs large on the aggregate growth rate. Own-innovation is the driver of economic growth in the estimated model. In Appendix D, I decompose economic growth into its origins. I show that most economic growth is generated by own-innovation (68%). Creative destruction only accounts for 32%. The fall in own-innovation, therefore, has a significant effect on the aggregate growth rate. The upside of such a policy is that it improves static efficiency. The reduction in markup dispersion is equivalent to a permanent 0.41% gain in output (captured by $\Delta\mathcal{M}$). However, the increase in creative destruction leads to a reallocation of sales shares from the high to the low-productivity firms, which decreases aggregate productivity and output permanently by 0.17% (captured by $\Delta\Phi$). The creative destruction policy, therefore, has contrasting effects on static efficiency. As the effect of reduced markup dispersion outweighs the effect of reduced aggregate productivity, the creative destruction subsidy improves static efficiency. In sum, a subsidy on creative destruction costs decreases the aggregate growth rate but improves static efficiency. Trading off static efficiency and economic growth, lifetime utility is 3.7% lower (in permanent consumption equivalents) in the BGP with the creative destruction subsidy than without the subsidy (last column).

Table 9: Effect of innovation subsidies on growth and static efficiency

	Subsidy rates		*Growth rate*	*Static efficiency*		*Utility*
	Creative destr.	Own-inno.	Δg	$\Delta\mathcal{M}$	$\Delta\Phi$	$\xi\%$
Profit tax Tax rate = 0.1	0.26		-0.2pp	+0.41%	-0.17%	-3.7%
Profit tax Tax rate = 0.1		0.29	+0.4pp	-0.51%	-0.03%	+5.3%
Profit tax Tax rate = 0.1	0.16	0.16	+0.1pp	+0.09%	-0.14%	+0.3%

Notes: the table shows the effect of different innovation subsidies financed by a 10% profit tax on the agg. growth rate and static efficiency. $\Delta\mathcal{M}$ and $\Delta\Phi$ capture the permanent change in output due to changes in markup dispersion and agg. productivity. ξ denotes the permanent increase in consumption in the BGP without the subsidy that equates lifetime utility of both BGPs.

The second row of Table 9 shows the effects of an own-innovation cost subsidy. A 29% subsidy rate clears the government's budget constraint in equilibrium. The effect on economic growth is large and positive, raising the aggregate growth rate by 0.4pp. The own-innovation subsidy strongly affects the aggregate growth rate because own-innovation is the primary driver of economic growth. The increase in growth does not come without a cost, however. Own-innovation generates economic growth but introduces static efficiency losses through markup dispersion. Relative to the BGP without the own-innovation subsidy, detrended (or permanent) output in the new BGP is 0.51% lower as a result of the increase in markup dispersion (captured by $\Delta\mathcal{M}$). Relative to the creative destruction subsidy, detrended output is almost 1% lower. Overall the positive effect on economic growth outweighs the negative static efficiency effect of the own-innovation subsidy. A permanent increase in consumption of 5.3% in the BGP without the subsidy equates lifetime utility of the

BGPs with and without the own-innovation subsidy (last column).

In practice, differentiating between own-innovation and creative destruction costs might be infeasible for a policymaker. As a last counterfactual, I study the effects of a subsidy on the total innovation costs of the firm (own-innovation and creative destruction). Both subsidies, in isolation, have contrasting effects on economic growth and static efficiency. The effect of a general research subsidy on growth and static efficiency, therefore, depends on the relative strength of each effect. A general research subsidy that subsidizes creative destruction and own-innovation at an identical rate (last row) has a muted yet overall positive effect on the aggregate growth rate. The positive growth effect of the own-innovation subsidy outweighs the negative effect of the creative destruction subsidy. The subsidy leads to a 0.1pp increase in the aggregate growth rate. The effect on static efficiency is negative but small. A reduction in markup dispersion leads to a permanent 0.09% gain in output; however, reallocation of market shares towards less productive firms lowers aggregate productivity by 0.14%. Overall, the difference between lifetime utility of the BGP with and without the general research subsidy is equivalent to a permanent 0.3% increase in consumption (last column).

In the counterfactuals, the own-innovation cost subsidy generates economic growth but lowers static efficiency. On the other hand, the creative destruction cost subsidy improves static efficiency but harms economic growth, posing a tradeoff for policymakers to address economic growth and static efficiency.

5.6 Discussion

In this section, I discuss alternative mechanisms proposed in the literature, the sensitivity of my findings to the model specifics, a plausible root behind the identified changes in R&D costs and the role of firm productivity heterogeneity.

5.6.1 Alternative mechanisms proposed in the literature

To explain the recent trends in the US economy at the industry level, several mechanisms have been put forth. I discuss why, through the lens of the model outlined in this paper, these mechanisms are inconsistent with the acceleration in revenue productivity (markup) growth at the firm level that I document in section 3. I study alternative mechanisms in a comparative statics exercise by changing parameters as described in the following. Aghion, Bergeaud, Boppart, Klenow and Li (2019) propose a rising efficiency advantage of large firms or a fall in the cost of spanning multiple product markets. A rising efficiency advantage in this model, $\varphi^h/\varphi^l \uparrow$, allows the more productive firms to charge higher markups. In response, more productive firms increase their expansion R&D efforts and market concentration rises. At the same time, as more productive firms charge higher markups, cross-sectional $TFPR$ dispersion increases. A rise in the efficiency advantage is hence consistent with the observed industry trends. Changes in φ^h/φ^l do not affect incentives to own-innovate directly though, such that firm markup growth hardly changes. As an alternative Aghion, Bergeaud, Boppart, Klenow and Li (2019) suggest that the overhead costs of spanning multiple product lines have fallen. In my model, the closest equivalent to this mechanism is a fall in the expansion costs, $\psi_x \uparrow$. Such fall incentivizes firms to expand into new markets with rising market concentration as a consequence. As firms add new products with low markups to their existing portfolio, firm markup growth, however, slows down and dispersion of markups (or $TFPR$) across firms falls. Olmstead-Rumsey (2019) argues that patent quality has fallen over time. Interpreting a fall in patent quality as a decrease in the step-size of innovation, $\lambda \downarrow$, more incremental quality improvements lower the incentives for firms to own-innovate. As a result, markup growth slows down and markup ($TFPR$) dispersion across firms decreases. Lastly, Liu, Mian and Sufi (2022) connect a fall in the real interest rate to the recent trends in the US economy. I study the effect of a fall in the real interest rate in my model by lowering the discount rate, ρ. In response to a fall in the discount rate, firm entry increases. As firms enter with initially low markups and steal sales shares from incumbents, sales concentration and markup ($TFPR$) dispersion in the cross-section fall. The failure of the above mechanisms to account for the acceleration in markup growth emphasizes the importance of falling own-innovation costs that I highlight in this paper. The fall in own-innovation costs incentivizes firms to increase their R&D efforts within their product markets, generating faster firm markup growth.

5.6.2 Sensitivity of the findings to model specifics

Changes in the R&D costs, particularly a fall in the within-product market R&D costs and a rise in the R&D costs to enter new markets, are consistent with the empirical trends. Would alternative models of firm dynamics that abstract from firm R&D highlight different channels? I argue that very similar mechanisms explain the empirical trends in alternative models of firm dynamics. Naturally, models that abstract from firm R&D do not allow changes in R&D costs to play a role. However, the outlined changes in R&D costs affect firms more generally beyond the direct cost of R&D. The fall in within-product market R&D costs allows incumbent firms to accumulate markups faster, while the rise in product market entry costs deters potential entrants. Mechanisms that ease incumbents' markup growth while hindering market entry by new firms are promising mechanisms to explain the empirical trends in models that abstract from firm R&D. Concrete examples are rising fixed costs of production within product markets or a fall in the marginal costs for incumbent producers. Both changes in (production) costs favor incumbent producers over new market entrants. De Ridder (2019) highlights the rising importance of intangible inputs in production to explain the recent trends in the US economy. De Ridder argues that intangible inputs reduce marginal costs and raise fixed costs of production. Hence, the rising importance of intangibles affects the power structure between incumbents and entrants, similar to the proposed changes in R&D costs. I argue in the next section that the rising importance of intangible inputs is a driver behind the proposed changes in R&D costs.

Although similar *mechanisms* explain the empirical trends in alternative models that abstract from R&D, the *implications* about economic growth are different. In the model proposed in this paper, firms grow in size and increase their markups through R&D. Rising within-product market R&D efforts generate a positive aggregate output growth effect in the long run alongside the acceleration in firm markup growth. In section 4.8, I show that firms that improve the quality of their products raise their product prices and that firms with faster price growth display faster revenue productivity (markup) growth providing suggestive evidence that links markup growth to economic growth. I further observe that firms' sales and revenue productivity growth accelerated the fastest during 1997–2007, a period of high aggregate TFP growth. Autor, Dorn, Katz, Patterson and Van Reenen (2020) and Crouzet and Eberly (2018) study the relationship between industry concentration and TFP more systematically for the US and conclude that industries where concentration has been rising the strongest experienced the fastest increase in TFP.

5.6.3 Plausible root behind the identified changes in R&D costs

Chiavari and Goraya (2020) and De Ridder (2019) explain the recent trends in the US economy through the rising importance of intangible inputs of production (software, networks, brand value, etc.). I argue that the identified fall in the within-product market R&D costs and increase in the product market entry costs likely result from the rise in intangibles. For example, a firm uses software as an intangible input to develop, produce and distribute its products. Such software facilitates firms to optimize and improve their products, while its costly acquisition poses a barrier for new firms to enter a product market. Similar arguments can be made for networks (of production and distribution) or a firm's brand value. I find evidence for the rising importance of intangible inputs in the Swedish economy. With the caveat that balance sheets potentially report firms' intangible capital with measurement error, the share of aggregate intangible capital in total capital rose from 1.5% in 1997 to 4.5% in 2017. In the cross-section of firms in 2017 that reported a positive intangible capital stock, I further find that firms with an above-average intangible capital share have 5% higher $TFPR$ than those below the average. Future research could try to identify to which extent intangible capital facilitates within-product market R&D and revenue productivity (or markup) growth of firms.

5.6.4 The role of firm productivity heterogeneity

One of the contributions of this paper is to add ex-ante heterogeneity across firms to an endogenous growth model that features firm markup and sales growth. This heterogeneity in firm productivity types plays a role in explaining the recent economic trends at the industry and firm level and is necessary to match several stylized facts on firm dynamics. First, the identified changes in R&D costs are productivity-type specific. In particular, only the within-product market R&D costs of the more productive firms have fallen, while the R&D costs of the less productive firms remained unchanged. The fall in the R&D costs incentivizes the high-type firms to expand into new product markets and market concentration rises as a result. Lowering the within-product market R&D costs for the low type by the same amount as for the high type would, in fact, lower sales concentration, $\sigma(\ln sales)$, by 9% in the long run relative to the initial BGP. In the data, sales concentration instead rose by 5%. Lowering the R&D costs for the low type incentivizes those firms to expand in size at the expense of the high type firms.

Second, permanent productivity heterogeneity gives rise to systematic differences in growth rates across firms. In the model, more productive firms choose to grow faster as those firms can charge higher markups upon market entry. This result is in line with Sterk, Sedláček and Pugsley (2021), which shows that differences in size growth across firms result from ex-ante heterogeneity rather than ex-post realized shocks. Systematic differences in size growth across firms further help to match the observed concentration in the cross-sectional firm size distribution. In Figure 12 in the Appendix, I show the fit of the firm size distribution for two different specifications of the model: with and without productivity differences across firms.[21] In the version without productivity differences, $\varphi^h/\varphi^l = 1$, all firms choose identical expansion R&D rates and realized differences in growth rates are entirely due to ex-post shocks. The model fit under this constrained version worsens considerably.

Third, markups are *separately* increasing in firm age and size in the model. As firms age, within-

[21] I re-solve the model for the case $\varphi^h/\varphi^l = 1$.

product market R&D increases markups over time. On the other hand, more productive firms with higher markups choose to grow faster. Firm age and size are, therefore, separately related to markups. I provided suggestive evidence for those separate correlations in section 4.8 regressing revenue productivity on age and sales of the firm. Both coefficients are positive and significant. Although a large class of models features positive markup age and markup size correlations (e.g., models of oligopolistic competition, models with a decreasing price elasticity of demand or models of customer capital acquisition), the same force drives both the markup age and markup size relationship in those models. The markup age correlation disappears conditional on firm size. Adding a separate mechanism would be required in those models to generate a positive markup age correlation conditional on firm size. A related point is that I find suggestive evidence that the markup size correlation is large across firms but small within firms; see section C.1 in the Appendix. For the class of models listed above, markups increase as the firm grows in size (positive within-firm correlation). In contrast, in my model, due to the productivity heterogeneity, the more productive firms (with higher markups) end up large (positive across-firm correlation).

Lastly, firms of different productivity types produce with distinct efficiencies, charge heterogeneous markups and innovate at contrasting speeds. Any form of reallocation of market shares across firms of different productivity types affects aggregate output through changes in the growth rate of product quality, average productivity and markup dispersion. Those reallocation effects were absent if abstracted from productivity-type heterogeneity.

6 Conclusion

I show in this paper that sales concentration and dispersion of revenue productivity within industries increased in Sweden during 1997-2017. Those trends are in line with recent developments in the US economy. While the trends at the industry level raise cause for concern, I find that firm life cycle growth of sales, revenue productivity, and employment accelerated for firms established after 1997. One exciting avenue for future research is to document whether a similar acceleration of firm life cycle growth can be observed for other countries. Van Vlokhoven (2021) provides suggestive evidence of accelerating sales life cycle growth for US firms in Compustat data and documents a steepening of the profit-age gradient.

To explain the empirical trends, I build a dynamic model of heterogeneous firms with endogenous markups. Firms grow in size by entering their competitors' product markets through R&D (creative destruction), thereby stealing sales shares. On the other hand, firms increase their markups by distancing themselves within their product markets from their competitors through R&D. Sales and markup growth vary systematically across firms. Firms differ in their permanent productivity level, which introduces heterogeneity in the optimal R&D efforts across firms.

To rationalize the empirical trends, the model asks for a reduction in the R&D costs for the more productive firms to distance their competitors, whereas barriers to entering new product markets have increased. One plausible driver that can simultaneously account for the changes in own-innovation and creative destruction costs is the rising importance of software and networks. Globalization and digitalization gave prominence to such intangible inputs for product planning, development, production, and distribution. The costly acquisition of software and networks deters potential entrants while lowering the cost for product improvements of incumbent firms that have already acquired such infrastructure. Chiavari and Goraya (2020) argue that the rising importance of intangible capital can explain the increase in revenue productivity dispersion and sales concentration in the US. De Ridder (2019) also links the rise in intangible inputs to the recent trends in the US economy. Future work could try to document to what extent the rise in intangible capital facilitates incumbent own-innovation and deters potential entrants.

In response to the changes in the R&D costs, the entry rate declines, the aggregate markup increases, and the aggregate output loss due to markup dispersion rises from 1.2% to 1.9% in the long run. There is a positive side effect. The rise in within-product market R&D, despite worsening static efficiency, increases the long-run aggregate growth rate by almost half a percentage point. Aggregate TFP growth and revenue productivity dispersion started to accelerate in the US and the Swedish economy in the mid-1990s, indicating a simultaneous rise in economic growth and fall in static efficiency. In fact, I observe that during 1997–2007, a period of high aggregate TFP growth, revenue productivity growth of firms accelerated the fastest.

I study the effect of different R&D policies on static efficiency and economic growth. A subsidy on the firms' creative destruction costs improves static efficiency at the expense of economic growth as firms lower their within-product market R&D efforts. This policy counterfactual relates to the recent debate on lifting patent protection (e.g., for Covid vaccines or green technologies) to facilitate new firms adapting and improving the technology of incumbent, patenting firms akin to a creative destruction subsidy. The model highlights the adverse effects on economic growth arising from such a policy. A subsidy on the R&D costs of incumbents to distance their competitors instead generates economic growth. Within-product market R&D by incumbents is the primary source of economic growth in the estimated model. The effect of R&D policies on economic growth depends on the response of incumbents' within-product market R&D efforts to such policies.

One key aspect of this paper is the distinction between static efficiency and economic growth. To the extent that product quality growth (as in this model) or physical productivity growth arises through R&D that allows firms to increase markups, misallocation of production factors increases with economic growth. In this case, measures indicating worsening static efficiency, e.g., rising market concentration or revenue productivity dispersion, overlook the dynamic growth effects.

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Appendices

A Trends in the Swedish economy

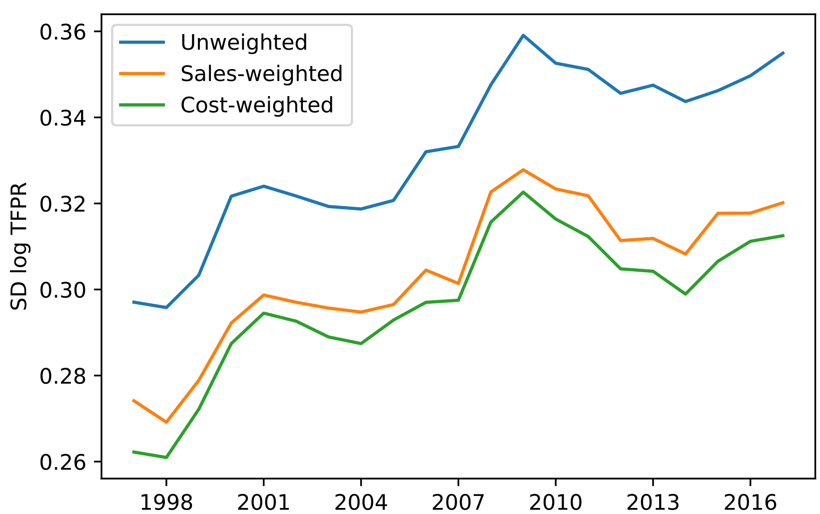
A.1 Rise in $TFPR$ dispersion

In this section, I document robustness of the increase in industry $TFPR$ dispersion.

A.1.1 Sales- and cost-weighted standard deviation

Figure 5 shows the sales- and (labor) cost-weighted standard deviation of log $TFPR$ averaged across all 5-digit industries. The unweighted standard deviation is the one shown in the main text.

Figure 5: Weighted standard deviation of log $TFPR$

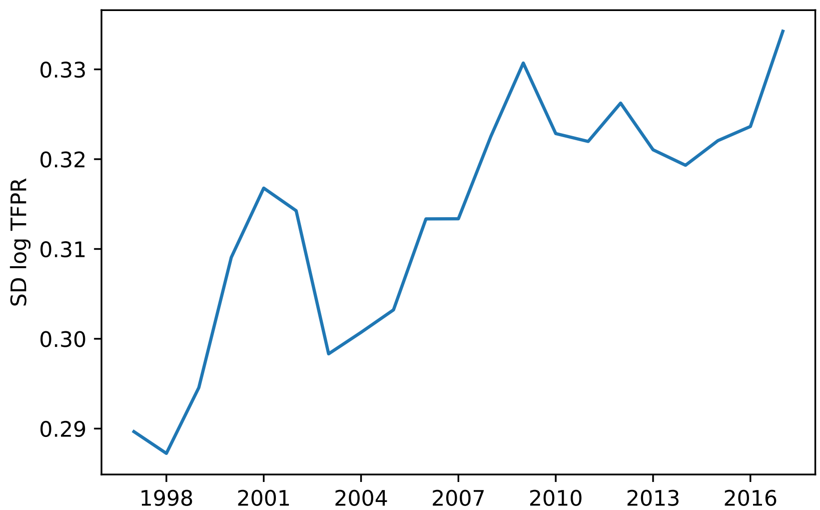


Notes: standard deviation of log $TFPR$ averaged across all (5-digit) industries. Orange and green show the sales- and (labor) cost-weighted standard deviation. $TFPR = VA/(K^{\alpha}(wL)^{1-\alpha})$.

A.1.2 Standard deviation for large industries

Figure 6 shows the standard deviation of log $TFPR$ averaged across industries with at least 50 firms.

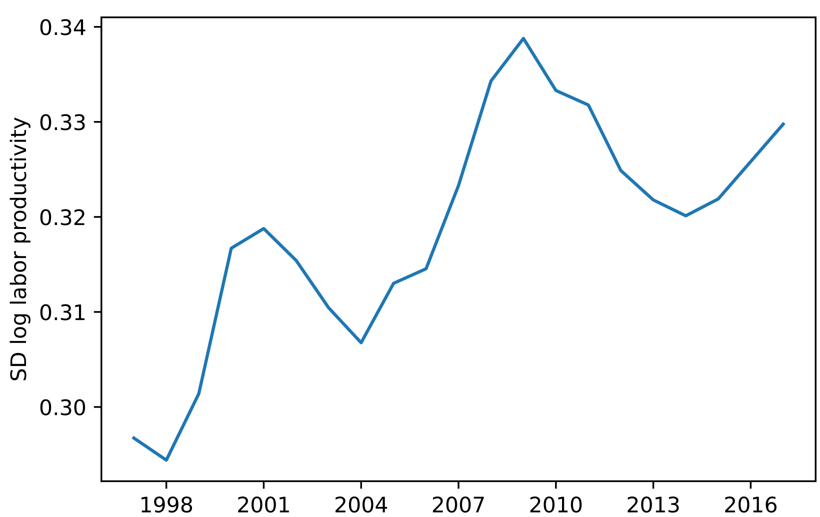
Figure 6: log $TFPR$ standard deviation for large industries



Notes: standard deviation of log $TFPR$ averaged across all (5-digit) industries. Only industries with at least 50 firms included. $TFPR = VA/(K^{\alpha}(wL)^{1-\alpha})$.

A.1.3 Dispersion in labor productivity

Figure 7: Industry labor productivity distribution over time

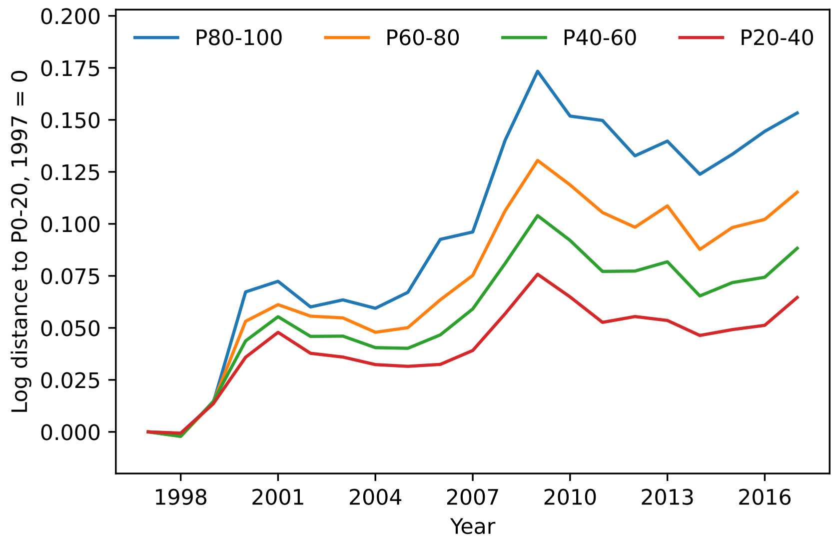


Notes: labor productivity computed as value added over the wage bill of the firm. Standard deviation of log labor productivity averaged across all (5-digit) industries.

A.1.4 Distribution of $TFPR$

Figure 8 shows the average (unweighted) industry log $TFPR$ gap between a given quantile (P80-100, P60-80, P40-60, P20-40) and the bottom quantile (P0-20) of the industry $TFPR$ distribution. The distance to the bottom quantile increased over time for all quantiles. The higher the quantile, the further the gap widened.

Figure 8: Industry $TFPR$ distribution over time



Notes: $TFPR = VA/(K^{\alpha}(wL)^{1-\alpha})$. Figure shows the within-industry $\ln TFPR$ gap between a given quantile and the bottom quantile of the $TFPR$ distribution averaged across industries (unweighted). Gap normalized to zero in 1997.

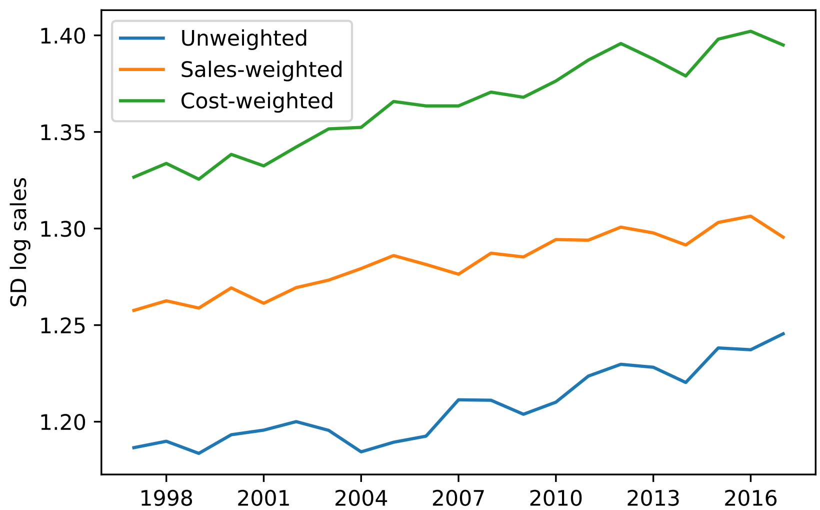
A.2 Rise in sales concentration

This section provides robustness to the reported increase in sales concentration in the main text.

A.2.1 Weighted standard deviations

Figure 9 shows the sales- and (labor) cost-weighted standard deviation of log sales over time.

Figure 9: Weighted standard deviation log sales

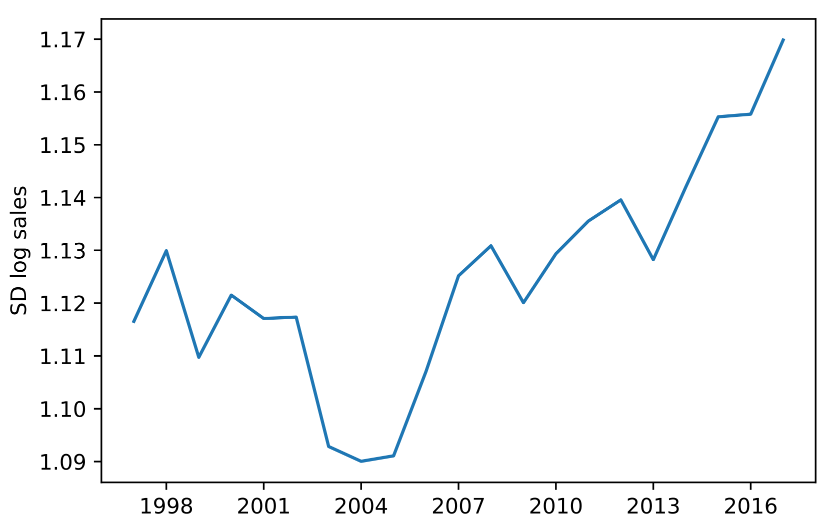


Notes: standard deviation of log sales averaged across all (5-digit) industries. Orange and green show the sales- and (labor) cost-weighted standard deviation.

A.2.2 Standard deviation log sales for large industries

Figure 10 shows the standard deviation of log sales for industries with at least 50 firms over time.

Figure 10: Standard deviation of log sales for large industries

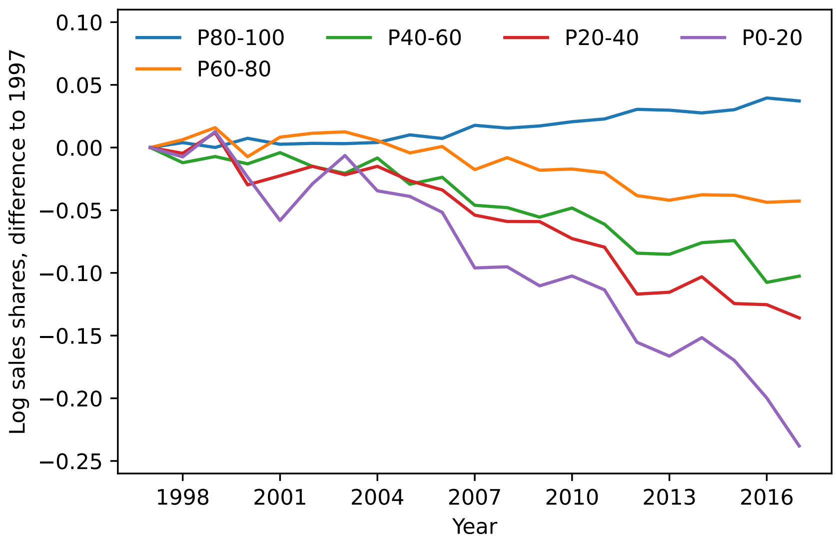


Notes: standard deviation of log sales averaged across all (5-digit) industries. Only industries with at least 50 firms included.

A.2.3 Sales shares by quantile

Figure 11 shows the percentage change in average (unweighted) industry sales shares for different quantiles of the industry sales distribution. Higher quantiles of the sales distribution saw an increase in their average sales share, whereas lower quantiles had their sales shares decline over time, i.e., the industry sales distribution widened.

Figure 11: Changes in sales share by quantile over time

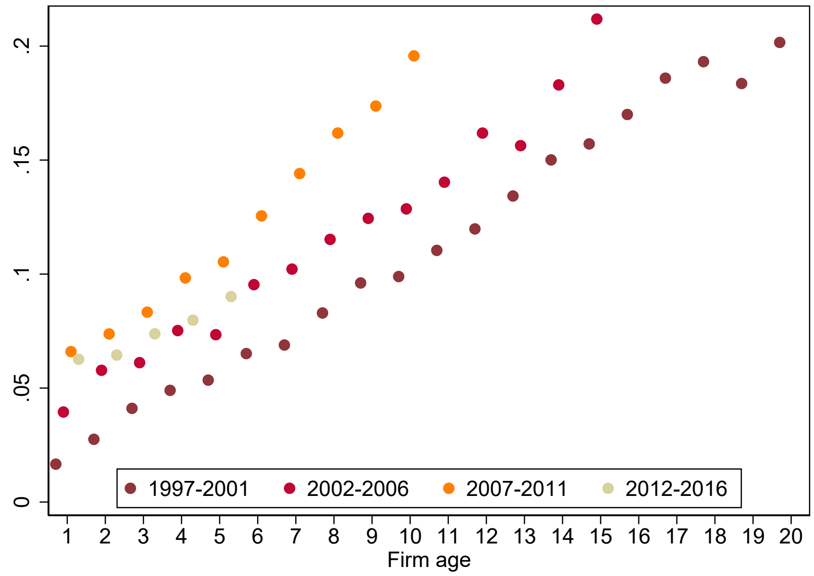


Notes: figure shows the change in a quantile's average (unweighted) industry log sales share. Quantiles refer to quantiles of the within-industry sales distribution.

A.3 Acceleration in firm life cycle growth

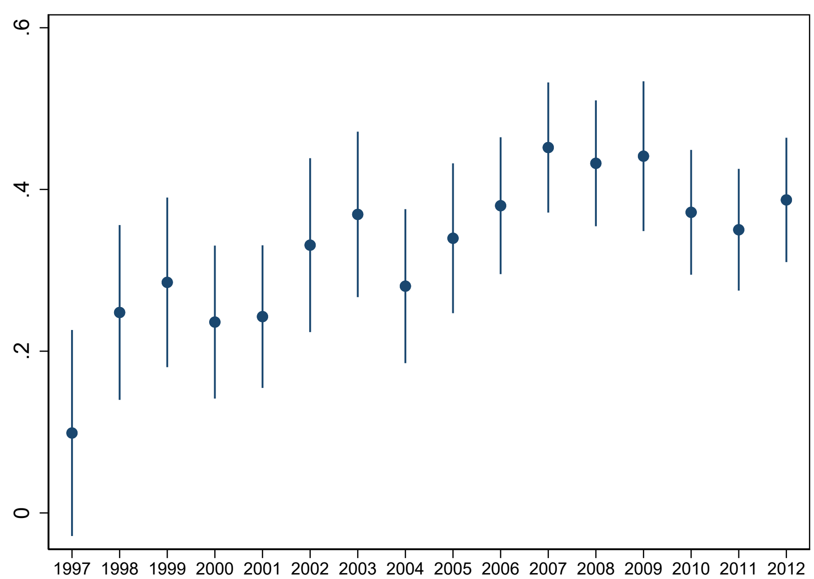
This section provides robustness to the reported increase in firm life cycle growth.

A.3.1 Employment life cycle growth



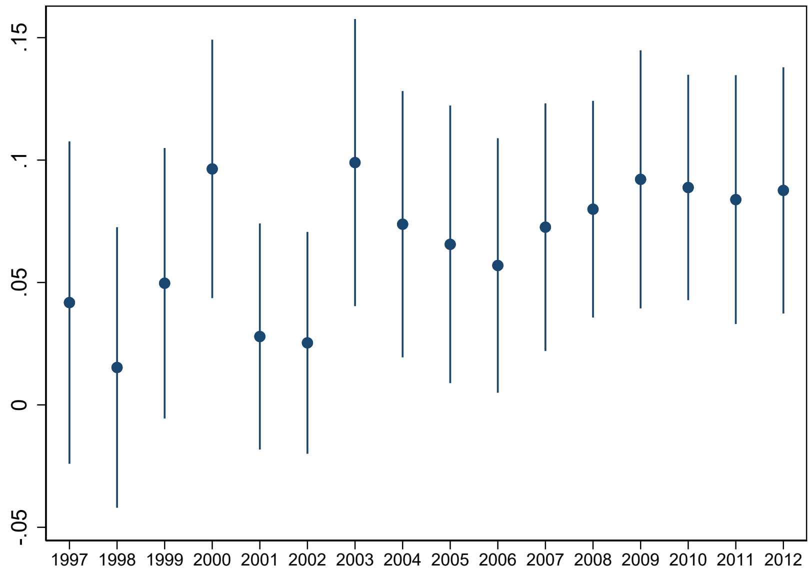
Notes: dots reflect the difference in log employment to age zero (relative to the industry mean). Cohorts pooled as indicated in the legend.

A.3.2 Four year sales growth over time



Notes: coefficients obtained by regressing log sales (net of industry × year FE) on age dummies and a constant (age zero left out) for each cohort separately. Coefficients on the age four dummy shown (with 95% confidence bands), which reflect the change in log sales between age zero and age four relative to the industry mean. The x-axis indicates the cohort.

A.3.3 Four year revenue productivity growth over time



Notes: coefficients obtained by regressing log $TFPR$ (net of industry $\times$ year FE) on age dummies and a constant (age zero left out) for each cohort separately. Coefficients on the age four dummy shown (with 95% confidence bands), which reflect the change in log $TFPR$ between age zero and age four relative to the industry mean. The x-axis indicates the cohort.

B Model

B.1 Solving the dynamic firm problem

The HJB for a high process efficiency firm h reads

$$\begin{aligned}
& r_t V_t^h(n,[\mu_i]) - \dot{V}_t^h(n,[\mu_i]) = \\
& \qquad \sum_{k=1}^{n} \pi(\mu_k) + \sum_{k=1}^{n} \tau_t \Big[V_t^h\left(n-1,[\mu_i]_{i\neq k}\right) - V_t^h(n,[\mu_i]) \Big] \\
& \qquad + \max_{[x_k,I_k]} \Bigg\{ \sum_{k=1}^{n} I_k \Big[V_t^h\left(n,[[\mu_i]_{i\neq k},\mu_k\times\lambda]\right) - V_t^h(n,[\mu_i]) \Big] \\
& \qquad + \sum_{k=1}^{n} x_k \Big[S_t V_t^h\left(n+1,[[\mu_i],\lambda]\right) + (1-S_t) V_t^h\left(n+1,\Big[[\mu_i],\lambda\times\varphi^h/\varphi^l\Big]\right) - V_t^h(n,[\mu_i]) \Big] \\
& \qquad - w_t \Bigg[\mu_k^{-1}\frac{1}{\psi_I^h}(I_k)^\zeta + \frac{1}{\psi_x}(x_k)^\zeta \Bigg] \Bigg\}
\end{aligned}$$

As in Peters (2020), guess that the value function of the firm consists of a component that is common to all lines and a line-specific component[22]

$$V_t^h(n,[\mu_i]) = V_{t,P}^h(n) + \sum_{k=1}^n V_{t,M}^h(\mu_k)$$

so that

$$\begin{aligned}
\dot{V}_t^h(n,[\mu_i]) &= \dot{V}_{t,P}^h(n) + \sum_{k=1}^n \dot{V}_{t,M}^h(\mu_k) \\
V_t^h\left(n-1,[\mu_i]_{i\neq k}\right) - V_t^h(n,[\mu_i]) &= V_{t,P}^h(n-1) - V_{t,P}^h(n) - V_{t,M}^h(\mu_k) \\
V_t^h\left(n,[[\mu_i]_{i\neq k},\mu_k\times\lambda]\right) - V_t^h(n,[\mu_i]) &= V_{t,M}^h(\mu_k\times\lambda) - V_{t,M}^h(\mu_k)
\end{aligned}$$

and

$$\begin{aligned}
&S_t V_t^h\left(n+1,[[\mu_i],\lambda]\right) + (1-S_t)V_t^h\left(n+1,\left[[\mu_i],\lambda\times\varphi^h/\varphi^l\right]\right) - V_t^h(n,[\mu_i] = \\
&S_t\left(V_{t,P}^h(n+1) + \sum_{k=1}^n V_{t,M}^h(\mu_k) + V_{t,M}^h(\lambda)\right) + (1-S_t)\left(V_{t,P}^h(n+1) + \sum_{k=1}^n V_{t,M}^h(\mu_k) + V_{t,M}^h(\lambda\times\varphi^h/\varphi^l)\right) \\
&- V_{t,P}^h(n) - \sum_{k=1}^n V_{t,M}^h(\mu_k) = \\
&V_{t,P}^h(n+1) - V_{t,P}^h(n) + S_t V_{t,M}^h(\lambda) + (1-S_t)V_{t,M}^h(\lambda\times\varphi^h/\varphi^l).
\end{aligned}$$

Substituting the guess into the HJB

$$\begin{aligned}
r_t\Big[V_{t,P}^h(n) + \sum_{k=1}^n V_{t,M}^h(\mu_k)\Big] - \dot{V}_{t,P}^h(n) - \sum_{k=1}^n \dot{V}_{t,M}^h(\mu_k) = \\
&\sum_{k=1}^n \pi(\mu_k) + \sum_{k=1}^n \tau_t\Big[V_{t,P}^h(n-1) - V_{t,P}^h(n) - V_{t,M}^h(\mu_k)\Big] \\
&+ \max_{[x_k,I_k]}\Bigg\{\sum_{k=1}^n I_k\Big[V_{t,M}^h(\mu_k\times\lambda) - V_{t,M}^h(\mu_k)\Big] \\
&+ \sum_{k=1}^n x_k\Big[V_{t,P}^h(n+1) - V_{t,P}^h(n) + S_t V_{t,M}^h(\lambda) + (1-S_t)V_{t,M}^h(\lambda\times\varphi^h/\varphi^l)\Big] \\
&- w_t\left[\sum_{k=1}^n \mu_k^{-1}\frac{1}{\psi_I^h}(I_k)^\zeta + \frac{1}{\psi_x}(x_k)^\zeta\right]\Bigg\}
\end{aligned}$$

and rearranging terms yields

$$\begin{aligned}
&r_t V_{t,P}^h(n) - \dot{V}_{t,P}^h(n) + \sum_{k=1}^n\Big[r_t V_{t,M}^h(\mu_k) - \dot{V}_{t,M}^h(\mu_k)\Big] = \\
&\sum_{k=1}^n\left\{\pi(\mu_k) - \tau_t V_{t,M}^h(\mu_k) + \max_{I_k}\left\{I_k\Big[V_{t,M}^h(\mu_k\times\lambda) - V_{t,M}^h(\mu_k)\Big] - w_t\mu_k^{-1}\frac{1}{\psi_I^h}(I_k)^\zeta\right\}\right\} \\
&+ \sum_{k=1}^n \tau_t\Big[V_{t,P}^h(n-1) - V_{t,P}^h(n)\Big] \\
&+ \max_{[x_k]}\left\{\sum_{k=1}^n x_k\Big[V_{t,P}^h(n+1) - V_{t,P}^h(n) + S_t V_{t,M}^h(\lambda) + (1-S_t)V_{t,M}^h(\lambda\times\varphi^h/\varphi^l)\Big] - w_t\frac{1}{\psi_x}(x_k)^\zeta\right\}.
\end{aligned}$$

[22] The notation follows Peters (2020) where possible.

First solve

$$
\begin{aligned}
r_t V^h_{t,M}(\mu_i) - \dot{V}^h_{t,M}(\mu_i) =& \pi(\mu_i) - \tau_t V^h_{t,M}(\mu_i) \\
&+ \max_{I_i} \Big\{ I_i \Big[V^h_{t,M}(\mu_i \times \lambda) - V^h_{t,M}(\mu_i) \Big] - w_t \mu_i^{-1} \frac{1}{\psi^h_I} (I_i)^\zeta \Big\}.
\end{aligned} \tag{32}
$$

Once we know $V^h_{t,M}$, we can solve for $V^h_{t,P}$ in

$$
\begin{aligned}
r_t V^h_{t,P}(n) - \dot{V}^h_{t,P}(n) =& \sum_{k=1}^{n} \tau_t \Big[V^h_{t,P}(n-1) - V^h_{t,P}(n) \Big] \\
&+ \max_{[x_k]} \Big\{ \sum_{k=1}^{n} x_k \Big[V^h_{t,P}(n+1) - V^h_{t,P}(n) + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) \Big] - w_t \frac{1}{\psi_x} (x_k)^\zeta \Big\}
\end{aligned} \tag{33}
$$

Assume (and verified below) that in steady-state $V^h_{t,P}$ and $V^h_{t,M}$ grow at the constant rate g such that

$$
\begin{aligned}
\dot{V}^h_{t,P}(n) =& g V^h_{t,P}(n) \\
\dot{V}^h_{t,M}(\mu_i) =& g V^h_{t,M}(\mu_i)
\end{aligned}
$$

In steady-state we can then write eq. (32) as

$$
(r - g + \tau) V^h_{t,M}(\mu_i) = \pi(\mu_i) + \max_{I_i} \Big\{ I_i \Big[V^h_{t,M}(\mu_i \times \lambda) - V^h_{t,M}(\mu_i) \Big] - w_t \mu_i^{-1} \frac{1}{\psi^h_I} (I_i)^\zeta \Big\}. \tag{34}
$$

Guess that[23]

$$
V^h_{t,M}(\mu_i) = \kappa_t - \alpha_t \mu_i^{-1}
$$

so that

$$
V^h_{t,M}(\mu_i \times \lambda) - V^h_{t,M}(\mu_i) = \alpha_t \mu_i^{-1} \left(1 - \frac{1}{\lambda}\right).
$$

The FOC for I_i then reads

$$
\alpha_t \mu_i^{-1} \left(1 - \frac{1}{\lambda}\right) = w_t \mu^{-1} \frac{1}{\psi^h_I} \zeta \, (I_i)^{\zeta - 1}.
$$

Rearranging yields

$$
\left(\frac{\alpha_t}{w_t} \left(1 - \frac{1}{\lambda}\right) \frac{\psi^h_I}{\zeta} \right)^{\frac{1}{\zeta - 1}} = I_i. \tag{35}
$$

It will turn out that α_t / w_t is constant such that I_i is time independent. Using the guess for the value function, the FOC, and the Euler equation $\rho = r - g$ in eq. (34) we get

$$
\begin{aligned}
(\rho + \tau) \left[\kappa_t - \alpha_t \mu_i^{-1} \right] =& Y_t \left(1 - \frac{1}{\mu_i}\right) + w_t \mu^{-1} \frac{1}{\psi^h_I} \zeta \, (I_i)^\zeta - w_t \mu_i^{-1} \frac{1}{\psi^h_I} (I_i)^\zeta \\
=& Y_t \left(1 - \frac{1}{\mu_i}\right) + \frac{\zeta - 1}{\psi^h_I} w_t \mu_i^{-1} (I_i)^\zeta.
\end{aligned}
$$

[23] It will turn out from eq. (35) that $\alpha_t > 0$, otherwise I_i would not be positive, such that $V^h_{t,M}(\mu_i)$ is increasing in μ_i.

Matching coefficients we obtain

$$\begin{aligned}(\rho+\tau)\kappa_t =& Y_t \\ \Leftrightarrow \kappa_t =& \frac{Y_t}{\rho+\tau}\end{aligned}$$

and

$$\begin{aligned}-\alpha_t\mu_i^{-1} =& \frac{-Y_t\mu_i^{-1} + \frac{\zeta-1}{\psi_I^h} w_t \mu_i^{-1}(I_i)^\zeta}{\rho+\tau} \\ \Leftrightarrow \alpha_t =& \frac{Y_t - \frac{\zeta-1}{\psi_I^h}(I_i)^\zeta w_t}{\rho+\tau}.\end{aligned}$$

This confirms that α_t/w_t is indeed time independent. The value function is

$$\begin{aligned}V_{t,M}^h(\mu_i) =& \kappa_t - \alpha_t\mu_i^{-1} \\ =& \frac{Y_t}{\rho+\tau} - \frac{Y_t\mu_i^{-1} - \frac{\zeta-1}{\psi_I^h}(I_i)^\zeta w_t\mu_i^{-1}}{\rho+\tau} \\ =& \frac{\pi(\mu_i) + \frac{\zeta-1}{\psi_I^h}(I_i)^\zeta w_t\mu_i^{-1}}{\rho+\tau}.\end{aligned}$$

Inserting α into the optimality condition, I_i solves

$$I_i = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta-1}{\psi_I^h}(I_i)^\zeta\right)\left(1-\frac{1}{\lambda}\right)\frac{\psi_I^h}{\zeta(\rho+\tau)}\right)^{\frac{1}{\zeta-1}}.$$

Own-innovation rates are not product line specific but just specific to the productivity type of the producer: $I_i = I^h$. They are further time-invariant. The optimality condition for the high productivity type own-innovation rate reads

$$I^h = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta-1}{\psi_I^h}(I^h)^\zeta\right)\left(1-\frac{1}{\lambda}\right)\frac{\psi_I^h}{\zeta(\rho+\tau)}\right)^{\frac{1}{\zeta-1}}$$

Equivalently the optimality condition for the own-innovation rate of low productivity firms reads

$$I^l = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta-1}{\psi_I^l}(I^l)^\zeta\right)\left(1-\frac{1}{\lambda}\right)\frac{\psi_I^l}{\zeta(\rho+\tau)}\right)^{\frac{1}{\zeta-1}}.$$

With this at hand, we can turn back to the differential equation for $V_{t,P}^h(n)$ in eq. (33).

$$\begin{aligned}r_tV_{t,P}^h(n) - \dot{V}_{t,P}^h(n) = \sum_{k=1}^n \tau_t\Big[V_{t,P}^h(n-1) - V_{t,P}^h(n)\Big] \\ + \max_{[x_k]}\Big\{\sum_{k=1}^n x_k\Big[V_{t,P}^h(n+1) - V_{t,P}^h(n) + S_tV_{t,M}^h(\lambda) + (1-S_t)V_{t,M}^h(\lambda\times\varphi^h/\varphi^l)\Big] - w_t\frac{1}{\psi_x}(x_k)^\zeta\Big\}\end{aligned}$$

In addition to the guess that $V^h_{t,P}(n)$ grows at rate g, conjecture that $V^h_{t,P}(n) = n \times v^h_t$. Combined with the Euler we get

$$(\rho+\tau) n v^h_t = \max_{[x_k]} \Big\{ \sum_{k=1}^{n} x_k \Big[v^h_t + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) \Big] - w_t \frac{1}{\psi_x} (x_k)^\zeta \Big\}. \tag{36}$$

The optimality condition for x_k is given by

$$v^h_t + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) = w_t \frac{\zeta}{\psi_x} (x_k)^{\zeta-1}. \tag{37}$$

Several observations are noteworthy. First, the FOC shows that optimal expansion rates are independent of quality and productivity gaps in line k. Second, the optimal expansion rate x_k depends on the own productivity type through the value function and the expansion cost. We can hence drop the item indexation: $x_k = x^h$. This is intuitive since, for expansion, a random new line will be drawn. The expected value of acquiring a new line depends on the own productivity. Since productivity is firm-specific, expansion rates across lines within the same firm are identical. Third, $v_t, V^h_{t,M}, w_t$ all grow at the same rate g, which implies that expansion rates are constant over time. We can hence write eq. (36) as

$$(\rho+\tau) n v^h_t = n w_t \frac{\zeta}{\psi_x} (x^h)^\zeta - n w_t \frac{1}{\psi_x} (x^h)^\zeta.$$

or

$$v^h_t = \frac{1}{(\rho+\tau)} \frac{\zeta-1}{\psi_x} (x^h)^\zeta w_t.$$

Gathering all terms, the value function is given by

$$\begin{aligned} V^h_t(n, [\mu_i]) =& V^h_{t,P}(n) + \sum_{k=1}^{n} V^h_{t,M}(\mu_k) \\ =& n v^h_t + \sum_{k=1}^{n} V^h_{t,M}(\mu_k) \\ =& n \frac{1}{(\rho+\tau)} \frac{\zeta-1}{\psi_x} (x^h)^\zeta w_t + \sum_{k=1}^{n} \frac{\pi(\mu_k) + \frac{\zeta-1}{\psi^h_I} (I^h)^\zeta w_t \mu_k^{-1}}{\rho+\tau}. \end{aligned}$$

Using the expression for v^h_t in the optimality condition for the expansion rate, write eq. (37) as

$$\begin{aligned} \frac{1}{(\rho+\tau)} \frac{\zeta-1}{\psi_x} (x^h)^\zeta w_t + S_t \frac{\pi(\lambda) + \frac{\zeta-1}{\psi^h_I} (I^h)^\zeta w_t \lambda^{-1}}{\rho+\tau} + (1-S_t) \frac{\pi(\lambda \times \varphi^h/\varphi^l) + \frac{\zeta-1}{\psi^h_I} (I^h)^\zeta w_t \lambda^{-1} \frac{\varphi^l}{\varphi^h}}{\rho+\tau} \\ = w_t \frac{\zeta}{\psi_x} (x^h)^{\zeta-1}. \end{aligned}$$

Simplifying gives

$$\begin{aligned} \frac{\zeta-1}{\psi_x} (x^h)^\zeta + S_t \left(\pi(\lambda)/w_t + \frac{\zeta-1}{\psi^h_I} (I^h)^\zeta \lambda^{-1} \right) + (1-S_t) \left(\pi(\lambda \times \varphi^h/\varphi^l)/w_t + \frac{\zeta-1}{\psi^h_I} (I^h)^\zeta \lambda^{-1} \frac{\varphi^l}{\varphi^h} \right) \\ = (\rho+\tau) \frac{\zeta}{\psi_x} (x^h)^{\zeta-1}. \end{aligned}$$

Inserting the profit function

$$\frac{\zeta-1}{\psi_x}(x^h)^\zeta + S_t\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I^h}(I^h)^\zeta\lambda^{-1}\right)+(1-S_t)\left(\frac{Y_t}{w_t}\left(1-\frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I^h}(I^h)^\zeta\lambda^{-1}\frac{\varphi^l}{\varphi^h}\right)$$
$$=(\rho+\tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta-1}.$$

The optimality condition for the expansion rate of the low productivity type reads

$$\frac{\zeta-1}{\psi_x}(x^l)^\zeta + S_t\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right)+\frac{\zeta-1}{\psi_I^l}(I^l)^\zeta\lambda^{-1}\frac{\varphi^h}{\varphi^l}\right)+(1-S_t)\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I^l}(I^l)^\zeta\lambda^{-1}\right)$$
$$=(\rho+\tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta-1}.$$

B.2 Solving for the steady state equilibrium

In the model there are the eight unknown variables $x^h, x^l, I^h, I^l, z, \tau, \frac{Y_t}{w_t}, S$ and the markup distribution $\nu()$ in eight equations plus the system of differential equations characterizing $\nu()$.

Optimality condition for high productivity own-innovation rate

$$I^h=\left(\left(\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I^h}(I^h)^\zeta\right)\left(1-\frac{1}{\lambda}\right)\frac{\psi_I^h}{\zeta(\rho+\tau)}\right)^{\frac{1}{\zeta-1}}$$

Optimality condition for low productivity own-innovation rate

$$I^l=\left(\left(\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I^l}(I^l)^\zeta\right)\left(1-\frac{1}{\lambda}\right)\frac{\psi_I^l}{\zeta(\rho+\tau)}\right)^{\frac{1}{\zeta-1}}$$

Optimality condition for high productivity expansion rate

$$\frac{\zeta-1}{\psi_x}(x^h)^\zeta + S\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I^h}(I^h)^\zeta\lambda^{-1}\right)+(1-S)\left(\frac{Y_t}{w_t}\left(1-\frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I^h}(I^h)^\zeta\lambda^{-1}\frac{\varphi^l}{\varphi^h}\right)$$
$$=(\rho+\tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta-1}$$

Optimality condition for low productivity expansion rate

$$\frac{\zeta-1}{\psi_x}(x^l)^\zeta + S\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right)+\frac{\zeta-1}{\psi_I^l}(I^l)^\zeta\lambda^{-1}\frac{\varphi^h}{\varphi^l}\right)+(1-S)\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I^l}(I^l)^\zeta\lambda^{-1}\right)$$
$$=(\rho+\tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta-1}$$

Free entry condition

$$p^h\Big(SV_t^h(1,\lambda)+(1-S)V_t^h(1,\lambda\times\varphi^h/\varphi^l)\Big)+(1-p^h)\Big(SV_t^l(1,\lambda\times\varphi^l/\varphi^h)+(1-S)V_t^l(1,\lambda)\Big)=\frac{1}{\psi_z}w_t,$$

where

$$V_t^d(1,\mu)=\frac{1}{(\rho+\tau)}\frac{\zeta-1}{\psi_x}(x^d)^\zeta w_t+\frac{Y_t\left(1-\frac{1}{\mu}\right)+\frac{\zeta-1}{\psi_I^d}(I^d)^\zeta w_t\mu^{-1}}{\rho+\tau}$$

Labor market clearing condition

$$1 = \frac{Y_t}{w_t}\sum_{\frac{\varphi_j}{\varphi_{j'}}}\sum_i \frac{1}{\lambda^i \frac{\varphi_j}{\varphi_{j'}}}\nu\left(i, \frac{\varphi_j}{\varphi_{j'}}\right) + \sum_{\frac{\varphi_j}{\varphi_{j'}}} \frac{1}{\psi_I^j}(I^j)^\zeta \sum_i \frac{1}{\lambda^i \frac{\varphi_j}{\varphi_{j'}}}\nu\left(i, \frac{\varphi_j}{\varphi_{j'}}\right) + S\frac{1}{\psi_x}(x^h)^\zeta + (1-S)\frac{1}{\psi_x}(x^l)^\zeta + \frac{z}{\psi_z}$$

Creative destruction

$$\tau = z + Sx^h + (1-S)x^l$$

Share of high productivity type

$$S = \sum_{i=1}^{\infty}\left[\nu\left(i, \frac{\varphi^h}{\varphi^h}\right) + \nu\left(i, \frac{\varphi^h}{\varphi^l}\right)\right],$$

where ν, the stationary distribution of quality and productivity gaps, is characterized by

$$0 = \dot{\nu}\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right) = I^l \nu\left(\Delta - 1, \frac{\varphi_j}{\varphi_{j'}}\right) - \nu\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)(I^l + \tau) \quad \text{for} \quad \Delta \geq 2, \frac{\varphi_j}{\varphi_{j'}} \in \left\{\frac{\varphi^l}{\varphi^h}, \frac{\varphi^l}{\varphi^l}\right\}$$
$$0 = \dot{\nu}\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right) = I^h \nu\left(\Delta - 1, \frac{\varphi_j}{\varphi_{j'}}\right) - \nu\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)(I^h + \tau) \quad \text{for} \quad \Delta \geq 2, \frac{\varphi_j}{\varphi_{j'}} \in \left\{\frac{\varphi^h}{\varphi^h}, \frac{\varphi^h}{\varphi^l}\right\}$$

and for the case of a unitary quality gap

$$0 = \dot{\nu}\left(1, \frac{\varphi^l}{\varphi^h}\right) = (1-S)x^l S + z_t(1-p^h)S - \nu\left(1, \frac{\varphi^l}{\varphi^h}\right)(I^l + \tau)$$
$$0 = \dot{\nu}\left(1, \frac{\varphi^l}{\varphi^l}\right) = (1-S)x^l(1-S) + z_t(1-p^h)(1-S) - \nu\left(1, \frac{\varphi^l}{\varphi^l}\right)(I^l + \tau)$$
$$0 = \dot{\nu}\left(1, \frac{\varphi^h}{\varphi^h}\right) = Sx^h S + z_t p^h S - \nu\left(1, \frac{\varphi^h}{\varphi^h}\right)(I^h + \tau)$$
$$0 = \dot{\nu}\left(1, \frac{\varphi^h}{\varphi^l}\right) = Sx^h(1-S) + z_t p^h(1-S) - \nu\left(1, \frac{\varphi^h}{\varphi^l}\right)(I^h + \tau)$$

To simplify the system of equations, first rewrite the creative destruction equation

$$z = (\tau - Sx^h - (1-S)x^l)$$

such that z can be substituted out from the remaining equations. Second, as derived in the main text, from the differential equations characterizing the distribution of quality and productivity gaps in steady-state, we obtain the share of high-productivity incumbents in the economy

$$\begin{aligned} S =& S_{\varphi^h,\varphi^h} + S_{\varphi^h,\varphi^l} \\ =& \frac{Sx^h + zp^h}{\tau}. \end{aligned}$$

Third, the optimality conditions for expansion rates (multiplied by p^h and $(1-p^h)$) and the free entry condition together imply

$$\frac{1}{\psi_x}p^h(x^h)^{\zeta-1} + \frac{1}{\psi_x}(1-p^h)(x^l)^{\zeta-1} = \frac{1}{\psi_z \zeta}.$$

The system of equilibrium conditions can hence be reduced to:

Optimality condition for high productivity own-innovation rate

$$I^h = \left(\left(\frac{Y_t}{w_t} \psi_I^h - (\zeta - 1)(I^h)^\zeta \right) \frac{\left(1 - \frac{1}{\lambda}\right)}{\zeta(\rho + \tau)} \right)^{\frac{1}{\zeta - 1}}$$

Optimality condition for low productivity own-innovation rate

$$I^l = \left(\left(\frac{Y_t}{w_t} \psi_I^l - (\zeta - 1)(I^l)^\zeta \right) \frac{\left(1 - \frac{1}{\lambda}\right)}{\zeta(\rho + \tau)} \right)^{\frac{1}{\zeta - 1}}$$

Optimality condition for high productivity expansion rate

$$\frac{\zeta - 1}{\psi_x}(x^h)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + (\zeta - 1)(I^h)^\zeta \lambda^{-1} \frac{1}{\psi_I^h}\right) + (1 - S)\left(\frac{Y_t}{w_t}\left(1 - \frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right) + (\zeta - 1)(I^h)^\zeta \lambda^{-1} \frac{\varphi^l}{\psi_I^h \varphi^h}\right)$$
$$= (\rho + \tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta - 1}$$

Optimality condition for low productivity expansion rate

$$\frac{\zeta - 1}{\psi_x}(x^l)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right) + (\zeta - 1)(I^l)^\zeta \lambda^{-1} \frac{\varphi^h}{\psi_I^l \varphi^l}\right) + (1 - S)\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + (\zeta - 1)(I^l)^\zeta \lambda^{-1} \frac{1}{\psi_I^l}\right)$$
$$= (\rho + \tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta - 1}$$

Free entry

$$p^h \frac{(x^h)^{\zeta - 1}}{\psi_x} + (1 - p^h)\frac{(x^l)^{\zeta - 1}}{\psi_x} = \frac{1}{\psi_z \zeta}$$

Labor market clearing condition

$$1 = \frac{Y_t}{w_t}\Lambda + \Lambda_I + S\frac{1}{\psi_x}(x^h)^\zeta + (1 - S)\frac{1}{\psi_x}(x^l)^\zeta + \frac{\tau - Sx^h - (1 - S)x^l}{\psi_z},$$

where[24]

$$\Lambda = \sum_{k \in \{h,l\}} \frac{\theta_k}{\theta_k + 1} \sum_{n \in \{h,l\}} \frac{1}{\varphi_k/\varphi_n} S_{\varphi_k,\varphi_n}$$
$$\Lambda_I = \sum_{k \in \{h,l\}} \frac{1}{\psi_I^k}(I^k)^\zeta \frac{\theta_k}{\theta_k + 1} \sum_{n \in \{h,l\}} \frac{1}{\varphi_k/\varphi_n} S_{\varphi_k,\varphi_n}$$
$$\theta_k = \frac{\ln(I^k + \tau) - \ln(I^k)}{\ln(\lambda)}$$

Share of high productivity type

$$S = \frac{Sx^h + (\tau - Sx^h - (1 - S)x^l)p^h}{\tau}$$

This constitutes a system of seven equations in seven unknowns $(x^h, x^l, I^h, I^l, \tau, \frac{Y_t}{w_t}, S)$, which I solve using a root finder.

[24] For the derivation of Λ I assume a continuous distribution of quality gaps.

B.3 Deriving Λ and $\mathcal{M}$

From the two-dimensional distribution of quality and productivity gaps along the balanced growth path it follows that

$$\Pr\left(\Delta \leq d, \frac{\varphi^l}{\varphi^h}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - \left(\frac{I^l}{I^l+\tau}\right)^d\right)$$
$$\Pr\left(\Delta \leq d, \frac{\varphi^l}{\varphi^l}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^l}{\varphi^l}\right) = S_{\varphi^l,\varphi^l}\left(1 - \left(\frac{I^l}{I^l+\tau}\right)^d\right)$$
$$\Pr\left(\Delta \leq d, \frac{\varphi^h}{\varphi^h}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^h}{\varphi^h}\right) = S_{\varphi^h,\varphi^h}\left(1 - \left(\frac{I^h}{I^h+\tau}\right)^d\right)$$
$$\Pr\left(\Delta \leq d, \frac{\varphi^h}{\varphi^l}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^h}{\varphi^l}\right) = S_{\varphi^h,\varphi^l}\left(1 - \left(\frac{I^h}{I^h+\tau}\right)^d\right).$$

Focusing on product lines where a low-productivity incumbent faces a high-productivity second-best firm:

$$\begin{aligned}
P\left(\frac{\varphi^l}{\varphi^h}, \Delta \leq d\right) =& S_{\varphi^l,\varphi^h}\left(1 - \left(\frac{I^l}{I^l+\tau}\right)^d\right) \\
=& S_{\varphi^l,\varphi^h}\left(1 - e^{\ln\left(\left(\frac{I^l}{I^l+\tau}\right)^d\right)}\right) \\
=& S_{\varphi^l,\varphi^h}\left(1 - e^{-d[\ln(I^l+\tau)-\ln(I^l)]}\right)
\end{aligned}$$

and

$$\begin{aligned}
P\left(\frac{\varphi^l}{\varphi^h}, \ln\left(\lambda^\Delta\right) \leq d\right) =& P\left(\frac{\varphi^l}{\varphi^h}, \Delta \ln(\lambda) \leq d\right) \\
=& P\left(\frac{\varphi^l}{\varphi^h}, \Delta \leq \frac{d}{\ln(\lambda)}\right) \\
=& S_{\varphi^l,\varphi^h}\left(1 - e^{-\frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}d}\right)
\end{aligned}$$

Conditional on the productivity gap, $\ln\left(\lambda^\Delta\right)$ is exponentially distributed with parameter $\frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}$. Further

$$\begin{aligned}
P\left(\frac{\varphi^l}{\varphi^h}, \lambda^\Delta \leq d\right) =& P\left(\frac{\varphi^l}{\varphi^h}, \Delta \leq \frac{\ln(d)}{\ln(\lambda)}\right) \\
=& S_{\varphi^l,\varphi^h}\left(1 - e^{-\frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}\ln(d)}\right) \\
=& S_{\varphi^l,\varphi^h}\left(1 - d^{-\frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}}\right)
\end{aligned}$$

Conditional on the productivity gap, quality gaps follow a Pareto distribution with parameter $\frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}$. Denote $\theta_l = \frac{\ln(I^l+\tau)-\ln(I^l)}{\ln(\lambda)}$. We then have

$$P\left(\frac{\varphi^l}{\varphi^h}, \lambda^\Delta \leq m\right) = S_{\varphi^l,\varphi^h}\left(1 - m^{-\theta_l}\right).$$

Doing the same steps for lines with different productivity gaps, the aggregate labor share can be computed as

$$\begin{aligned}
\Lambda &= \sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int_1^\infty \frac{1}{\varphi_k/\varphi_n}\frac{1}{m}S_{\varphi_k,\varphi_n}\theta_k m^{-(\theta_k+1)}dm \\
&= \sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\frac{1}{\varphi_k/\varphi_n}S_{\varphi_k,\varphi_n}\theta_k\int_1^\infty m^{-(\theta_k+2)}dm \\
&= \sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\frac{1}{\varphi_k/\varphi_n}S_{\varphi_k,\varphi_n}\theta_k\left[-\frac{1}{\theta_k+1}m^{-(\theta_k+1)}\right]_1^\infty \\
&= \sum_{k\in\{h,l\}}\frac{\theta_k}{\theta_k+1}\sum_{n\in\{h,l\}}\frac{1}{\varphi_k/\varphi_n}S_{\varphi_k,\varphi_n}.
\end{aligned}$$

The TFP misallocation statistic $\mathcal{M}$ is then

$$\begin{aligned}
\mathcal{M} &= \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int\left[\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta_k m^{-(\theta_k+1)}\right]dm}}{\Lambda} \\
&= \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int\left[\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)S_{\varphi_k,\varphi_n}\theta_k m^{-(\theta_k+1)}+\ln\left(\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta_k m^{-(\theta_k+1)}\right]dm}}{\Lambda} \\
&= \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\left[\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)S_{\varphi_k,\varphi_n}+\int\ln\left(\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta_k m^{-(\theta_k+1)}dm\right]}}{\Lambda} \\
&= \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\left[S_{\varphi_k,\varphi_n}\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)-S_{\varphi_k,\varphi_n}\frac{1}{\theta_k}\right]}}{\Lambda} \\
&= \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\left[S_{\varphi_k,\varphi_n}\left(\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)-\frac{1}{\theta_k}\right)\right]}}{\Lambda}
\end{aligned}$$

where I have made use of

$$\int_1^\infty \ln\left(\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta_k m^{-(\theta_k+1)}dm = \left[\frac{\theta_k\ln(m)+1}{\theta_k m^{\theta_k}}+C\right]_1^\infty = -\frac{1}{\theta_k}.$$

Alternatively note that this expression is equal to $-S_{\varphi_k,\varphi_n}E[\ln(\lambda^\Delta)|\varphi_k,\varphi_n]$. I have shown above that $\ln(\lambda^\Delta)$ conditional on the productivity gap is exponentially distributed with parameter θ_k. From the characteristics of an exponential distribution, its expected value is $1/\theta_k$.

B.4 Moments of the markup distribution

Mean of markups (unweighted or sales weighted with Cobb-Douglas aggregator)

$$
\begin{aligned}
E[\mu] &= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \int_1^\infty \frac{\varphi_k}{\varphi_n} m S_{\varphi_k,\varphi_n} \theta_k m^{-(\theta_k+1)} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \frac{\varphi_k}{\varphi_n} S_{\varphi_k,\varphi_n} \theta_k \int_1^\infty m^{-\theta_k} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \frac{\varphi_k}{\varphi_n} S_{\varphi_k,\varphi_n} \theta_k \left[\frac{1}{1-\theta_k} m^{1-\theta_k}\right]_1^\infty \\
&= \sum_{k\in\{h,l\}} \frac{\theta_k}{\theta_k - 1} \sum_{n\in\{h,l\}} \frac{\varphi_k}{\varphi_n} S_{\varphi_k,\varphi_n},
\end{aligned}
$$

where in the last equation it is assumed that $\theta > 1$, which is true if $\frac{\tau}{I} > \lambda - 1$. Otherwise, the mean is ∞. Note that this is simply the mean of a Pareto distribution (once $\frac{\varphi_k}{\varphi_l} S_{\varphi_k,\varphi_l}$ is taken out of the integral). The geometric mean is computed from previously derived expressions:

$$
E[\mu^{geo}] = e^{-\ln(\mathcal{M}\times\Lambda)}.
$$

2nd moment of markups

$$
\begin{aligned}
E[\mu^2] &= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \int_1^\infty \left(\frac{\varphi_k}{\varphi_n} m\right)^2 S_{\varphi_k,\varphi_n} \theta_k m^{-(\theta_k+1)} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \left(\frac{\varphi_k}{\varphi_n}\right)^2 S_{\varphi_k,\varphi_n} \theta_k \int_1^\infty m^{1-\theta_k} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \left(\frac{\varphi_k}{\varphi_n}\right)^2 S_{\varphi_k,\varphi_n} \theta_k \left[\frac{1}{2-\theta_k} m^{2-\theta_k}\right]_1^\infty \\
&= \sum_{k\in\{h,l\}} \frac{\theta_k}{\theta_k - 2} \sum_{n\in\{h,l\}} \left(\frac{\varphi_k}{\varphi_n}\right)^2 S_{\varphi_k,\varphi_n}.
\end{aligned}
$$

Variance of markups

$$
\begin{aligned}
Var(\mu) = E[\mu^2] - E[\mu]^2 = &\sum_{k\in\{h,l\}} \frac{\theta_k}{\theta_k - 2} \sum_{n\in\{h,l\}} \left(\frac{\varphi_k}{\varphi_n}\right)^2 S_{\varphi_k,\varphi_n} \\
&- \left(\sum_{k\in\{h,l\}} \frac{\theta_k}{\theta_k - 1} \sum_{n\in\{h,l\}} \frac{\varphi_k}{\varphi_n} S_{\varphi_k,\varphi_n}\right)^2
\end{aligned}
$$

Without any differences in productivity ($\varphi_h = \varphi_l$) and own-innovation rates $\theta_h = \theta_l$, the variance collapses to $\frac{\theta}{\theta-2} - \left(\frac{\theta}{\theta-1}\right)^2 = \frac{\theta}{(\theta-2)(\theta-1)^2}$, which is just the variance of the Pareto distribution μ collapses to.

Mean of log markups

$$
\begin{aligned}
E[\ln \mu] &= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \int_1^\infty \ln\left(\frac{\varphi_k}{\varphi_n} m\right) S_{\varphi_k,\varphi_n} \theta_k m^{-(\theta_k+1)} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \int_1^\infty \left(\ln\left(\frac{\varphi_k}{\varphi_n}\right) + \ln m\right) S_{\varphi_k,\varphi_n} \theta_k m^{-(\theta_k+1)} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \left(\ln\left(\frac{\varphi_k}{\varphi_n}\right) S_{\varphi_k,\varphi_n} + S_{\varphi_k,\varphi_n} \theta_k \int_1^\infty \ln(m) m^{-(\theta_k+1)} dm\right) \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \left(S_{\varphi_k,\varphi_n} \left(\ln\left(\frac{\varphi_k}{\varphi_n}\right) + \frac{1}{\theta_k}\right)\right)
\end{aligned}
$$

2nd moment of log markups

$$
\begin{aligned}
E[(\ln \mu)^2] &= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \int_1^\infty \left[\ln\left(\frac{\varphi_k}{\varphi_n} m\right)\right]^2 S_{\varphi_k,\varphi_n} \theta_k m^{-(\theta_k+1)} dm \\
&= \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} S_{\varphi_k,\varphi_n} \frac{\theta_k \ln\left(\frac{\varphi_k}{\varphi_n}\right)\left(\theta_k \ln\left(\frac{\varphi_k}{\varphi_n}\right) + 2\right) + 2}{\theta_k^2}
\end{aligned}
$$

The variance of log markups is then

$$
Var(\ln \mu) = E[(\ln \mu)^2] - E[(\ln \mu)]^2,
$$

which is computed using the above expressions. Without any differences in productivity ($\varphi_h = \varphi_l$) and own-innovation rates ($\theta_h = \theta_l$), the variance collapses to $1/\theta^2$, which is just the variance of the exponential distribution that $\ln \mu$ collapses to.

B.5 Deriving the steady-state growth rate of aggregate variables

The growth rate of Q_t determines the growth rate of aggregate variables.

$$
g = \frac{\dot{Q}_t}{Q_t} = \frac{\partial \ln(Q_t)}{\partial t}
$$

Quality of a product in a given product line increases through own-innovation, firm expansion or firm entry. For the growth rate of Q_t we have

$$
\begin{aligned}
\ln(Q_{t+\Delta}) &= \int_0^1 \ln(q_{t+\Delta,i}) di \\
&= \int_0^1 \Big[(\Delta S I^h + \Delta(1-S)I^l + \Delta S x^h + \Delta(1-S)x^l + \Delta z) \ln(q_{t,i}\lambda) \\
&\quad + (1 - \Delta S I^h - \Delta(1-S)I^l + \Delta S x^h + \Delta(1-S)x^l + \Delta z) \ln(q_{t,i})\Big] di \\
&= \int_0^1 \Big[(\Delta S I^h + \Delta(1-S)I^l + \Delta S x^h + \Delta(1-S)x^l + \Delta z) \ln(\lambda) + \ln(q_{t,i})\Big] di
\end{aligned}
$$

so that

$$\frac{\ln(Q_{t+\Delta})-\ln(Q_t)}{\Delta}=\Big(SI^h+(1-S)I^l+Sx^h+(1-S)x^l+z\Big)\ln(\lambda)$$
$$=\Big(SI^h+(1-S)I^l+\tau\Big)\ln(\lambda).$$

For $\Delta\to 0$, $g=\Big(SI^h+(1-S)I^l+\tau\Big)\ln(\lambda)$.

B.6 Markup dynamics

Firm markups are defined by $\mu_f=\frac{py_f}{wl_f}=\left(\frac{1}{n}\sum_{k=1}^{n}\mu_{kf}^{-1}\right)^{-1}$. Therefore

$$\ln\mu_f=-\ln\left(\frac{1}{n}\sum_{k=1}^{n}\mu_k^{-1}\right).$$

Rewrite the term in brackets (for a high-productivity firm) as

$$\frac{1}{n}\sum_{k=1}^{n}\mu_k^{-1}=\frac{1}{n}\sum_{k=1}^{n}e^{-\ln\mu_k}=\frac{1}{n}\left(\sum_{i=1}^{n_i}e^{-\ln\frac{\varphi^h}{\varphi^l}-\ln\Delta_i\ln\lambda}+\sum_{j=1}^{n_j}e^{-\ln\Delta_j\ln\lambda}\right),\tag{38}$$

where i indexes the product lines where the high productivity firm faces a low productivity second best producer, j the lines where it faces a high productivity second best producer and $n_i+n_j=n$. A two-dimensional linear Taylor expansion around $\ln\lambda=0$ and $\ln\frac{\varphi^h}{\varphi^l}=0$ gives

$$\frac{1}{n}\left(\sum_{i=1}^{n_i}e^{-\ln\frac{\varphi^h}{\varphi^l}-\ln\Delta_i\ln\lambda}+\sum_{j=1}^{n_j}e^{-\ln\Delta_j\ln\lambda}\right)\approx 1-\left(\frac{1}{n}\sum_{k=1}^{n}\Delta_k\right)\ln\lambda-\frac{n_i}{n}\ln\left(\frac{\varphi^h}{\varphi^l}\right)$$

such that

$$E\Big[\ln\mu_f|\text{firm age}=a_f,\varphi^h\Big]\approx E\left[\frac{1}{n}\sum_{k=1}^{n}\Delta_k|\text{firm age}=a_f,\varphi^h\right]\ln\lambda+(1-S)\ln\left(\frac{\varphi^h}{\varphi^l}\right),$$

where I have used the fact that the expected share of the firm's product lines with a low productivity second best producer is equal to the aggregate share of product lines where the active producer is of the low productivity type. From Peters (2020) I know that

$$E\left[\frac{1}{n}\sum_{k=1}^{n}\Delta_k|\text{firm age}=a_f,\varphi^h\right]\ln\lambda=\Big(1+I^h\times E[a_P^h|a_f]\Big)\ln\lambda,$$

where $E[a_P^h|a_f]$ denotes the average product age of a high process efficiency firm conditional on firm age a_f and

$$E[a_P^h|a_f]=\frac{1}{x^h}\left(\frac{\frac{1}{\tau}\left(1-e^{-\tau a_f}\right)}{\frac{1}{x^h+\tau}\left(1-e^{-(x^h+\tau)a_f}\right)}-1\right)\Big(1-\phi^h(a_f)\Big)+a_f\phi^h(a_f)$$
$$\phi^h(a)=e^{-x^h a}\frac{1}{\gamma^h(a)}\ln\left(\frac{1}{1-\gamma^h(a)}\right)$$
$$\gamma^h(a)=\frac{x^h\left(1-e^{-(\tau-x^h)a}\right)}{\tau-x^h e^{-(\tau-x^h)a}},$$

which gives the expression in the main text.

For a firm of the low process efficiency type, the last term in eq. (38) reads

$$\frac{1}{n}\left(\sum_{i=1}^{n_i} e^{-\ln\Delta_i \ln\lambda} + \sum_{j=1}^{n_j} e^{-\ln\frac{\varphi^l}{\varphi^h} - \ln\Delta_j \ln\lambda}\right),$$

where i indexes the product lines where the low productivity producer faces a low productivity second best producer, j the lines where it faces a high productivity second best producer and $n_i + n_j = n$. Following the same steps as for a high productivity firm, this time linearizing around $\ln\frac{\varphi^l}{\varphi^h} = 0$ (and $\ln\lambda = 0$) gives

$$E\left[\ln\mu_f|\text{firm age} = a_f, \varphi^l\right] \approx \left(1 + I^l \times E[a^l_P|a_f]\right)\ln\lambda + S\ln\left(\frac{\varphi^l}{\varphi^h}\right),$$

where again I have made use of the fact that the share of the firm's product lines with a high productivity second best producer is equal to the aggregate share of product lines where the active producer is of the high productivity type. $E[a^l_P|a_f]$ is exactly defined as $E[a^h_P|a_f]$ with x^h replaced by x^l in the above expressions.

C Testing model predictions

C.1 Determinants of revenue productivity

Fact 1: $TFPR$ increases independently in firm age and firm sales

To show that both age and size drive $TFPR$, I run a horse-race regression of $TFPR$ on firm age and size. I use lagged firm sales as the baseline measure for firm size. In the model I present in section 4, firm sales are proportional to the number of products a firm produces. I use lagged instead of contemporaneous sales to account for a potential positive correlation between measurement errors in sales and value added entering the numerator of $TFPR$ that could lead to a spurious correlation between $TFPR$ and size. The main specification of the horse-race regression is

$$\ln TFPR_{fkt} = \beta_0 + \beta_1 Age_{fkt} + \beta_2 \ln Sales_{fkt-1} + \delta_{kt} + u_{fkt},$$

where $Sales_{fkt-1}$ denotes lagged firm sales and δ_{kt} captures industry $\times$ year fixed effects.

Table 10: Horse-race regression: revenue productivity, age and size

	(1)	(2)	(3)	(4)	(5)	(6)
	$\ln TFPR_t$	$\ln TFPR_t$	$\ln \widehat{TFPR}_t$	$\ln TFPR_t$	$\ln TFPR_t$	$\ln TFPR_t$
Age_t	0.0090	0.0055	0.0065	0.0023	0.0132	0.0072
	(14.111)	(10.666)	(14.052)	(4.052)	(21.294)	(15.029)
$\ln Sales_t$	0.0607		0.0432			
	(26.554)		(18.923)			
$\ln Sales_{t-1}$		0.0371		0.0040		
		(15.563)		(1.234)		
$\ln Employment_t$					-0.0266	-0.0154
					(-15.632)	(-4.578)
Industry × year FE	✓	✓	✓		✓	
Firm FE				✓		✓
$Age_t < 10$	✓	✓	✓	✓	✓	✓
N	300,518	213,864	214,270	213,864	300,518	300,518

Notes: t statistics in parentheses. Clustered standard errors at the fixed effects level. $\ln \widehat{TFPR}$ denotes $\ln TFPR$ predicted by its lagged value (run for each firm). $TFPR$ is computed from balance sheet data, whereas employment is obtained from separate matched employer-employee data.

The regression output is shown in Table 10. Columns (1)-(3) show the $TFPR$, age and size relationship controlling for 5-digit industry × year fixed effects. Throughout the three columns, the coefficients on age and size are positive and significant. Column (1) shows the relationship between $TFPR$, age and contemporaneous sales. The coefficient on age suggests that conditional on firm size, firms one year older than the average firm in an industry-year display 0.009 log points higher $TFPR$. At the same time, conditional on firm age, firms with one log point larger sales than the industry-year average display 0.061 log points higher $TFPR$. The main specification is shown in column (2) with lagged sales as the size measure. Increasing age by one year increases $TFPR$ by 0.006 log points conditional on size and increasing size by one log point increases $TFPR$ by 0.037 log points conditional on age. The decline in the size coefficient from column (1) to (2) indicates that the size coefficient in column (1) is partly driven by the contemporaneous link between sales and value added (entering $TFPR$). However, the age and size coefficients remain large and significant in column (2). To provide further robustness that the size coefficient is not driven by random fluctuations in $TFPR$ that correlate with sales, I predict firm $\ln TFPR$ by its lagged value for each firm separately. Apart from breaking the link between contemporaneous value added and sales, predicting $TFPR$ by its lag smoothes the firm $TFPR$ profile. Column (3) shows the regression of predicted $TFPR$ on age and sales. The coefficients are comparable in size to column (2). Hence, in all specifications, $TFPR$ is increasing in firm age and size independently across firms.

All specifications restrict to firms younger than ten years since the $TFPR$-age profile flattens out at age ten. See Table 11 for results of the horse-race regression, including older firms. The size coefficient is close to the one reported for the main specification. In the same table, I further report results of separate $TFPR$-age and $TFPR$-size regressions and show that the above findings extend to using inverse labor shares instead of $TFPR$ as a measure of a firm's revenue productivity.

Fact 2: Controlling for firm fixed effects the $TFPR$-size relationship is small

The previous fact showed that firm age and size are important independent drivers of $TFPR$ across firms. I show next that an increase in size within the firm has a negligible effect on $TFPR$.

Column (4) of Table 10 shows the main specification of the horse-race regression with firm instead of industry $\times$ year fixed effects. Whereas the age coefficient remains statistically significant, the size coefficient declines by an order of magnitude and turns insignificant. Within the firm, a one log point increase in size is associated with a 0.004 log point increase in $TFPR$. The size effect is not only statistically but also economically minuscule.

This result relates to Gamber (2021), who using Compustat data, provides indirect evidence of markups increasing in sales. Regressing log variable input expenditures (e.g., wage bill) on log sales, he finds coefficients smaller than one, indicating that markups increase in sales. The rate at which variable input expenditures increase with sales is lower when controlling for firm than for industry $\times$ year fixed effects suggesting that markups increase faster in size within firms than across firms. This contrasts my findings. To relate my analysis to his, I regress $\ln TFPR$ on lagged sales without controlling for age in columns (1)-(2) of Table 12 in the Appendix. The size effect on $TFPR$ within firms remains small relative to the across-firm size effect. I further show in columns (3)-(4) that the effect of size on inverse labor shares declines by an order of magnitude when controlling for firm relative to industry $\times$ year fixed effects. To speak directly to his results, I run his regression with my data regressing log wage bill on log sales and find a lower coefficient with industry $\times$ year fixed effects than with firm fixed effects, which is consistent with size having a larger effect on markups across firms than within. The results of this regression are shown in Table 13 columns (1)-(2). One potential explanation for the difference in our findings is the coverage of firms. Using registry data my analysis covers the universe of firms in Sweden. Gamber (2021) relies on Compustat data, which is restricted to publicly listed firms. In fact, if I restrict my data to firms with sales exceeding 100 Mio. 2017 SEK (4.7% of all firms), I do find evidence that the size effect on markups is larger within firms than across (Table 13 columns (3)-(4)). That the size effect on the markup is larger across than within firms in the unrestricted sample but not among the subset of very large firms is potentially driven by the fact that when looking at the complete cross-section of firms in an industry, variation in firm size is mainly driven by heterogeneity in firm fundamentals (e.g. process efficiency) that in return affects the markup. When narrowing in on large, successful firms in an industry, those firms potentially look more alike in terms of firm fundamentals since, e.g., a high process efficiency level is required to reach that size. This would suggest that across-firm differences in size have larger effects on markups in the complete cross-section than when looking at a subset of either small or large firms. I repeat my regression of $TFPR$ on lagged sales with industry $\times$ year fixed effects for a subsample of small or large firms. Column (5) of Table 12 shows that for the subsample of large firms, the size coefficient drops by half compared to the size coefficient for the complete cross-section in column (1) and for the sample of small firms the coefficient is indistinguishable from zero, column (6). Hence, the large size coefficient in the complete cross-section is mainly driven by variation between large and small firms rather than by variation among small or large firms.

Fact 3: $TFPR$ increases faster in sales than employment

Columns (5) and (6) of Table 10 show results of the horse-race regression of $TFPR$ on age and employment. The measure for employment is obtained from a separate matched employer-employee data set ($RAMS$). While the age coefficient remains positive and significant, the coefficient on employment is negative. Conditional on firm age firms with one log point larger employment relative to average employment in the industry-year display 0.027 log points lower $TFPR$. Within firms, a one log point increase in employment is associated with 0.015 log points lower $TFPR$. I provide further robustness for the $TFPR$-employment relationship in Table 14 using both the wage bill to measure employment or using predicted $TFPR$. The sign and statistical significance of the coefficient on employment or the wage bill vary across specifications; however, in all specifications is the coefficient on employment or the wage bill lower than the one reported for sales in the previous tables.

Stylized facts 1-3 suggest that first, firm age and size matter independently for markups, particularly for explaining across-firm differences. Second, conditional on firm fixed effects, the increase in markups associated with an increase in firm size is small. That size differences across firms matter for explaining across-firm differences in markups, but within-firm sales growth has a small effect on the markup suggests that across-firm differences in size are driven by firm fundamentals that affect the markup (e.g., differences in productivity). In contrast, within-firm sales growth arises from factors less related to markup growth. Third, markups increase less strongly in employment than in sales. These findings have precise predictions for modeling firm dynamics and, in fact, oppose some commonly used theories. First, markups varying with age and size contradict models where firms charge constant markups due to, e.g., constant price elasticity of demand as in Melitz (2003). Second, stylized fact 1 highlights both firm age and sales as determinants of markups. Models that feature either a positive markup-age or markup-size relationship miss one dimension of markup heterogeneity. Further, the horse-race regression emphasizes age and size as *independent* determinants of markups. This opposes theories where the same mechanism drives a positive markup-age and markup-size relationship (e.g., through demand accumulation as in Foster, Haltiwanger and Syverson (2008)) or where the markup-age correlation simply arises through a positive age-size correlation. Holding size constant, the horse-race regression predicts a positive relationship between markups and age. Conditioning on size holds the driver behind size differences constant (e.g., productivity), which implies that the markup-age relationship must arise from a separate mechanism. Third, fact 2 shows that increasing sales within the firm has little effect on the markup. This contrasts models where firms increase their markups as they grow in size due to, e.g., the price elasticity of demand decreasing in size either by construction or where this arises endogenously as in models of oligopolistic competition à la Atkeson and Burstein (2008). These models feature a positive within-firm markup-size correlation, whereas I find that this relationship is insignificant in the data. Fact 2 also relates to the story of demand accumulation. If accumulation of customer capital were behind the positive $TFPR$-size relationship, then within-firm size growth should be positively related to $TFPR$, which is rejected by fact 2.

C.2 Auxiliary regressions

Table 11: Revenue productivity, age and size

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\ln TFPR_t$	$\ln TFPR_t$	$\ln TFPR_t$	$\ln TFPR_t$	$\ln \frac{VA}{wL}$	$\ln \frac{VA}{wL}$	$\ln \frac{VA}{wL}$	$\ln \frac{VA}{wL}$
Age_t	0.0127		0.0055	0.0026	0.0131		0.0044	0.0030
	(20.762)		(10.666)	(10.010)	(31.036)		(11.359)	(14.471)
$\ln Sales_{t-1}$		0.0390	0.0371	0.0337		0.0682	0.0668	0.0640
		(16.635)	(15.563)	(15.652)		(33.179)	(31.939)	(30.956)
Industry × year FE	✓	✓	✓	✓	✓	✓	✓	✓
$Age_t < 10$	✓	✓	✓		✓	✓	✓	
N	300,518	213,864	213,864	302,038	300,518	213,864	213,864	302,038

Notes: t statistics in parentheses. Clustered standard errors at the industry × year level. VA denotes value added and wL the wage bill.

Table 12: Revenue productivity and size, across and within firms

	(1)	(2)	(3)	(4)	(5)	(6)
	$\ln TFPR_t$	$\ln TFPR_t$	$\ln \frac{VA}{wL}$	$\ln \frac{VA}{wL}$	$\ln TFPR_t$	$\ln TFPR_t$
$\ln Sales_{t-1}$	0.0390	0.0093	0.0682	0.0068	0.0204	0.0011
	(16.635)	(3.411)	(33.179)	(2.651)	(3.592)	(0.108)
Industry × year FE	✓		✓		✓	✓
Firm FE		✓		✓		
$Sales > 50$ Mio. 2017 SEK					✓	
$Sales < 5$ Mio. 2017 SEK						✓
N	213,864	213,864	213,864	213,864	23,998	29,273

Notes: t statistics in parentheses. Clustered standard errors at the fixed effects level. VA denotes value added and wL the wage bill.

Table 13: Wage bill and sales, across and within firms

	(1)	(2)	(3)	(4)
	$\ln wL_t$	$\ln wL_t$	$\ln wL_t$	$\ln wL_t$
$\ln Sales_t$	0.6849	0.7082	0.7390	0.7271
	(115.879)	(52.659)	(55.572)	(30.175)
Industry × year FE	✓		✓	
Firm FE		✓		✓
$Sales > 100$ Mio. 2017 SEK			✓	✓
N	300,518	300,518	13,479	13,479

Notes: t statistics in parentheses. Clustered standard errors at the fixed effects level. wL denotes the wage bill.

Table 14: Horse-race regression with employment or wage bill

	(1) $\ln \widehat{TFPR}_t$	(2) $\ln \widehat{TFPR}_t$	(3) $\ln \widehat{TFPR}_t$	(4) $\ln \widehat{TFPR}_t$	(5) ln TFPR	(6) ln TFPR
Age_t	0.0081	0.0079	0.0031	0.0039	0.0080	0.0066
	(17.740)	(16.949)	(10.023)	(10.887)	(15.154)	(11.066)
$\ln Employment_t$	-0.0182		-0.0012			
	(-12.254)		(-0.576)			
$\ln wL_t$		0.0033		-0.0134		
		(1.956)		(-4.466)		
$\ln wL_{t-1}$					-0.0020	-0.0372
					(-1.197)	(-11.569)
Industry × year FE	✓	✓			✓	
Firm FE			✓	✓		✓
$Age_t < 10$	✓	✓	✓	✓	✓	✓
N	214,270	214,270	214,270	214,270	213,864	213,864

Notes: t statistics in parentheses. Clustered standard errors at the fixed effects level. $\ln \widehat{TFPR}$ denotes $\ln TFPR$ predicted by its lagged value (run for each firm).

D Decomposing the sources of economic growth

Incumbent creative destruction creates growth yet lowers incentives for firms to engage in own-innovation. Is more creative destruction by incumbents beneficial or harmful to economic growth? The answer to this question depends on the relative contributions of creative destruction and own-innovation to aggregate growth. Similarly, firm entry generates growth; however, it harms incumbent firms. Does more firm entry lead to more economic growth? The answer depends on the relative contributions by entrants and incumbents to aggregate growth.

In this section, I decompose the aggregate growth rate into its contributions by incumbent own-innovation and expansion (differentiating by productivity type) and the contribution by firm entry. The expression for the growth rate in eq. (17), repeated below, naturally lends itself to such decomposition

$$g = \Bigg(\underbrace{\underbrace{SI^h + (1-S)I^l}_{\text{Incumbent own-innovation}} + \underbrace{Sx^h + (1-S)x^l}_{\text{Incumbent product expansion}}}_{\text{Incumbents}} + \underbrace{z}_{\text{Entrants}} \Bigg) \times \ln(\lambda).$$

The curly brackets highlight the separate contributions to the growth rate. The exponents indicate the contributions by productivity type. Table 15 quantifies each component of the growth rate. Columns differentiate between own-innovation and creative destruction, whereas rows differentiate between incumbents (by productivity type) and entrants. Differentiating between innovation types, own-innovation makes up for the majority of aggregate growth (68.1%). Creative destruction accounts for the remaining 31.9%. Between incumbents and entrants, incumbents account for almost all growth (95%). Entry generates relatively little growth (5%). Among incumbents, own-innovation accounts for 71.7% of growth (creative destruction for 28.3%). Those numbers are similar to Peters (2020), who reports a share of aggregate growth accounted for by entrants of 4% and to Garcia-Macia, Hsieh and Klenow (2019), who find that among incumbents, own-innovation accounts for 75%-80% of growth.[25] Differentiating further by productivity type, firms of the high productivity type account for 94% of economic growth generated by incumbents, while the low type accounts for 6%. High productivity type firms, making up 65% of the entering firms, but 94% of aggregate growth among incumbents hence over-proportionally contribute to aggregate growth. Their larger contribution to aggregate growth is due to those firms endogenously innovating at faster rates than low-productivity firms, which is separate from their contribution to aggregate productivity.

Table 15: Decomposing the sources of economic growth (initial BGP)

	Own-innovation	Creative destruction	
Incumbents φ^h	0.0146 (63.6%)	0.0059 (25.7%)	0.0205 (89.3%)
Incumbents φ^ℓ	0.0011 (4.6%)	0.0003 (1.1%)	0.0013 (5.7%)
Entrants		0.0012 (5%)	0.0012 (5%)
	0.0157 (68.1%)	0.0073 (31.9%)	0.0230 (100%)

Notes: table shows the contribution to growth by incumbents vs. entrants, by own-innovation vs. expansion and high (φ^h) vs. low (φ^h) productivity type for the initial balanced growth path (BGP). Percentages in brackets refer to the share of total growth (0.023).

The contributions to aggregate growth derive from firms' innovation efforts that give rise to revenue productivity (markup) and sales life cycle growth. Firm life cycle growth has been targeted explicitly in the model estimation. The reason why not more aggregate growth is generated through creative destruction is that this would imply faster employment life cycle growth that, in return, is inconsistent with the life cycle growth observed in the data. Similarly, for low-productivity firms to contribute more to aggregate growth would require those firms to expand faster in employment, which would be inconsistent with the data.

I perform the same decomposition for the new BGP following the fall in own-innovation costs (high type) and the rise in entry and creative destruction costs. The results are shown in Table (16). In line with the intuition, the share of aggregate growth accounted for by entrants declines along with the share accounted for by creative destruction. On the other hand, the contribution of own-innovation by the high-productivity firms increases.

[25] Garcia-Macia, Hsieh and Klenow (2019) infer the contributions by studying employment fluctuations, while in Peters (2020), contributions to aggregate growth are deduced from firm life cycle growth as in this paper.

Table 16: Decomposing the sources of economic growth (new BGP)

	Own-innovation	Creative destruction	
Incumbents φ^h	0.0196 (70.7%)	0.0061 (21.8%)	0.0257 (92.5%)
Incumbents φ^ℓ	0.0009 (3.2%)	0.0002 (0.7%)	0.0011 (3.9%)
Entrants		0.0010 (3.7%)	0.0010 (3.7%)
	0.0205 (73.9%)	0.0073 (26.1%)	0.0278 (100%)

Notes: table shows the contribution to growth by incumbents vs. entrants, by own-innovation vs. expansion and high (φ^h) vs. low (φ^h) productivity type for the new balanced growth path. Percentages in brackets refer to the share of total growth (0.0278).

E Trading off static efficiency and growth

Utility from a consumption path where C_t grows at rate g is

$$\mathcal{U}\left(\{C_t\}_{t=0}^{\infty}\right) = \int_0^\infty e^{-\rho t}\ln C_t dt = \frac{1}{\rho}\ln C_0 + \frac{g}{\rho^2} = \mathcal{U}\left(C_0, g\right).$$

Utility of the consumption stream depends on the detrended consumption level C_0 and the growth rate. I evaluate the change in utility streams across two BGPs in permanent consumption-equivalent terms ξ as follows

$$\mathcal{U}\left((1+\xi)C_0^{old}, g^{old}\right) = \frac{\ln(1+\xi)}{\rho} + \mathcal{U}\left(C_0^{old}, g^{old}\right) = \mathcal{U}\left(C_0^{new}, g^{new}\right).$$

ξ measures the change in permanent consumption along the old BGP that equates utility of the old and new BGP. ξ then solves

$$\frac{\ln(1+\xi)}{\rho} = \frac{1}{\rho}\ln C_0^{new} + \frac{g^{new}}{\rho^2} - \frac{1}{\rho}\ln C_0^{old} - \frac{g^{old}}{\rho^2}.$$

Rearranging gives

$$\xi = exp\left(\frac{\ln\left(\frac{C_0^{new}}{C_0^{old}}\right)\rho + g^{new} - g^{old}}{\rho}\right) - 1.$$

The change in utility streams across BGPs depends on the discount rate, the relative detrended consumption levels and the difference in growth rates. To compute the relative detrended consumption levels, I assume that both BGPs start off from the same average quality level. Differences in C_0 across BGPs then arise from differences in average productivity Φ, markup dispersion $\mathcal{M}$ and production labor L_P (all constant along each BGP)

$$\frac{C_0^{new}}{C_0^{old}} = \frac{\Phi^{new}}{\Phi^{old}} \times \frac{\mathcal{M}^{new}}{\mathcal{M}^{old}} \times \frac{L_P^{new}}{L_P^{old}}.$$

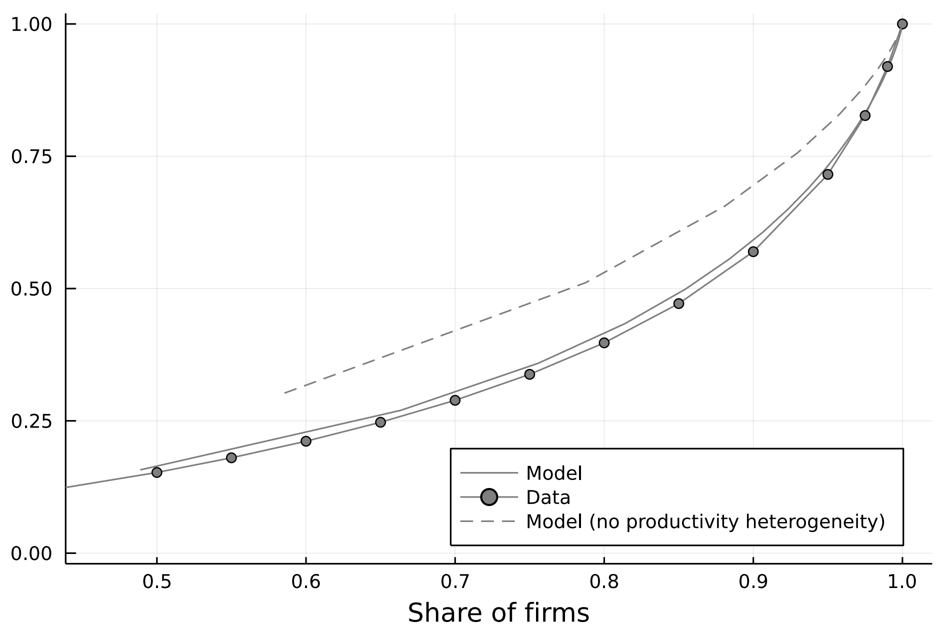
$\mathcal{M}$ and L_P are derived in the main text. The change in average productivity is given by

$$\frac{\Phi^{new}}{\Phi^{old}} = \left(\frac{\varphi^h}{\varphi^l}\right)^{S^{new} - S^{old}}.$$

F Firm type heterogeneity and the size distribution

To illustrate the effect of systematic differences in size growth across firms on the cross-sectional firm size distribution, I compare the model-implied Lorenz curve of value added for two different specifications, see Figure 12. The Lorenz curve captures the share of value added (y-axis) that is accounted for by the smallest x-percent of firms (x-axis). I allow for productivity differences across firms in the baseline version (solid line). In this version, firms of different productivity types choose systematically different expansion R&D rates. The solid line aligns well with the empirical Lorenz curve, i.e., the baseline model matches the firm size distribution in the data well. Recall that the firm size distribution was not targeted in the parameter estimation. The dashed line shows the fit of the firm size distribution when eliminating productivity differences across firms, $\varphi^h/\varphi^\ell = 1$, and re-solving the model. In this specification, all firms choose identical expansion R&D rates and differences in realized growth rates across firms are entirely due to ex-post shocks. As a result, the fit of the firm size distribution is worsened considerably.

Figure 12: Share of value added (Lorenz curve)



Notes: solid line shows model fit of the value-added distribution with baseline parameter values. Dashed line shows the model fit with baseline parameters and $\varphi^h/\varphi^\ell = 1$.