Recent Changes in Firm Dynamics and the Nature of Economic Growth*

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Abstract

This paper presents a novel observation on the dynamics of firm size. The average firm size *conditional on firm age* has systematically increased since the 1990s relative to the size of entrants. The finding suggests that firm growth or the selection of surviving firms changed over time. I study trends in firm growth, firm selection, and their implications for aggregate economic growth in a Schumpeterian growth model with rich firm dynamics. Innate productivity differences across firms generate heterogeneity in expected life cycle trajectories. Caused by rising entry costs, more productive firms start expanding into new product markets faster. Size growth conditional on survival accelerates for more productive firms, and their share among surviving firms at any age increases, raising the average firm size conditional on age. These changes in firm growth and selection affect aggregate economic growth positively. However, falling firm entry dominates both effects, lowering economic growth and welfare.

Keywords: Firm dynamics, Aggregate economic growth, Firm entry, Reallocation, Administrative data

JEL codes: D22, E24, O31, O47, O50

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1 Introduction

Firm entry, firm size growth, and aggregate productivity growth are directly linked. Starting small, entering firms replace competing firms through product improvements. The firms that improve their products over time attract new customers and increase their market shares. At the same time, firms' product improvements generate aggregate productivity growth. The link between firm entry, firm size growth, and aggregate productivity growth lies at the heart of Schumpeterian growth theory. This paper presents a novel finding on the dynamics of firm size. The average firm size conditional on firm age has systematically increased since the 1990s relative to the size of entrants. This suggests that recent macroeconomic trends, particularly the rise in the average firm size, are not only due to a shift in the firm-age distribution toward older firms (Hopenhayn, Neira and Singhania, 2022; Karahan, Pugsley and Şahin, 2022). Average firm size has increased, even conditional on firm age, pointing toward systematic changes in the growth of surviving firms or selection among them. According to Schumpeterian growth theory, changes in firm growth entail changes in aggregate productivity growth. Changes in the selection of firms also matter for aggregate growth if firms grow at heterogeneous rates. I study trends in firm growth, selection, and aggregate productivity growth in a Schumpeterian growth model, in which expected firm growth systematically differs across firms. The model disentangles changes in firm growth from changes in the share of firms that grow at different expected rates and quantifies the implications for the macroeconomy, particularly productivity growth and welfare.

The first contribution of this paper is empirical. I document a new stylized fact on the dynamics of firms size using high-quality Swedish administrative data from tax records: the average firm size conditional on age has increased relative to the average size at entry. For example, the average sales of firms aged eight are roughly 56 percent higher than the average sales at entry for the cohorts established in the late 1990s, compared to 67 percent for the cohorts of the early 2010s. For employment, these differences are even more pronounced. Average employment at age eight is 29 percent higher than at entry for the cohorts of the late 1990s, compared to 47 percent for the cohorts of the 2010s. These trends are statistically significant and robust. I find similar patterns in U.S. Census data. For almost all sectors in the U.S. economy, firm size conditional on age has increased over time relative to the size of entrants. Previous studies (Hopenhayn, Neira and Singhania, 2022; Karahan, Pugsley and Şahin, 2022) find stability in firm size conditional on age patterns because these studies pool firms across all sectors. Declining firm size in the U.S. manufacturing sector masks increasing firm size in almost all other sectors (conditional on age).

The increase in firm size conditional on age suggests that firm growth or the selection of surviving firms (or both) has systematically changed over time. I study the implications for aggregate growth in a structural model that includes the following three elements. First, the model features a link between firm dynamics and economic growth in the spirit of Schumpeterian growth models (Aghion and Howitt, 1992; Grossman and Helpman, 1991; Klette and Kortum, 2004): incumbent firms (and potential entrants) gain market shares by expanding horizontally into new product markets through creative destruction (expansion R&D).[1] Second, the model features ex-ante heterogeneity in firm fundamentals. Some firms are born more productive than others and endogenously (innovate and) grow at systematically faster rates over their life cycle. This allows the average firm size conditional on age to increase without changes in the growth of surviving firms if less-productive firms with slower expected growth exit faster. Third, in standard models of creative destruction with constant markups, firm sales and employment growth from successful expansion R&D are identical. I include a second type of product innovation that allows for differential firm sales and employment growth. This type of innovation (internal R&D) enables incumbent firms to distance their competitors vertically in the quality space and, in equilibrium, charge a higher markup. Markup growth separates firm sales from employment growth, necessary to explain the disproportionate increase in firm employment conditional on age (relative to sales).

I estimate the model on a balanced growth path matching firm sales and employment conditional on age of the cohorts of the late 1990s and other macroeconomic moments. As a comparative statics exercise, I re-estimate the model parameters to match the increase in sales and employment conditional on age of the latest cohorts in the data. The estimation highlights a rise in the cost of entry as the (exogenous) cause behind the increase in firm size conditional on age. In response to rising entry costs, more productive firms, which charge higher markups and enjoy greater expected profits, expand into new product markets faster. This increases their firm growth conditional on survival and their share among surviving firms at any age, raising the average firm size conditional on age. To rationalize the disproportionate increase in average employment conditional on age (relative to sales), the model further asks for an increase in the internal R&D costs. The increase in internal R&D costs slows markup growth, accelerating employment relative to sales growth for all firms.

What are the implications for the macroeconomy? Caused by the rise in the cost of entry and internal R&D, the firm entry rate falls by eight percentage points (pp), and the aggregate growth rate declines by 0.62pp in the long-run. The decline in aggregate growth might seem surprising at first. I address the decrease shortly. The implied decline in firm entry and long-run growth account for roughly 80% of the decline in the firm entry rate and 60% of the measured decline in TFP growth in Sweden over the last three decades (Engbom, 2023). Further, changes in the selection of surviving firms translate into selection in the cross-section of firms. More productive firms represent a larger share in the cross-section of firms and capture a larger share of total sales. The reallocation of market shares to more productive firms that, in the model, feature relatively low labor shares and high markups is qualitatively consistent with Kehrig and Vincent (2021), De Loecker, Eeckhout and Unger (2020), and Baqaee and Farhi (2020).

To shed light on the fall in the aggregate growth rate, I quantify the contributions by incumbent firms and entrants in a growth decomposition. Changes in the long-run growth rate are due to (i) changes in incumbents' innovation rates, holding sales shares constant, (ii) real-

[1] These models capture salient features of firm dynamics (Lentz and Mortensen, 2008; Akcigit and Kerr, 2018).

location of sales shares across incumbents that innovate at different rates, and (iii) changes in entrants' innovation rates (firm entry). First, the average innovation rate by incumbents has increased, raising the long-run aggregate growth rate by 0.22pp. This is reflected in the growth acceleration of productive incumbents. Second, as more productive incumbents innovate (and grow) at systematically higher rates in equilibrium, the reallocation of market shares to these firms in the cross-section further increases the aggregate growth rate by 0.27pp. Hence, incumbents have contributed positively to changes in long-run growth, mainly due to the effects of reallocation. These reallocation effects are absent in standard models of creative destruction with ex-ante homogeneous firms. Third, rising entry costs slow firm entry. The fall in firm entry lowers the long-run growth rate by 1.1pp. Net of the positive contribution by incumbents, the long-run growth rate declines by 0.62pp. The decomposition results are robust to an alternative estimation, in which the productivity dispersion increases as in Aghion, Bergeaud, Boppart, Klenow and Li (2023). Lastly, I extend the decomposition over the transition period, which further trades off the long-run fall in the aggregate growth rate with an increase in the productivity level due to the reallocation of sales shares. Growth effects dominate level effects, resulting in a welfare loss.

What drives the increase in entry costs? Firms strategically invest in fixed assets, e.g., intangible capital, to raise barriers to entry. I provide suggestive evidence that a rise in the stock of fixed assets at the sector level increased barriers to entry. Sectors that experienced the largest rise in the stock of fixed assets display the greatest increase in firm size conditional on age relative to the size of entrants. At the same time, these sectors experienced the largest decline in economic growth, as predicted by the model. I provide further evidence on the mechanism at the firm level. The data suggests that more productive firms grow faster than less productive ones over their life cycle, particularly in the more recent years.

Related Literature. The empirical results relate to Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022), who document that firm employment conditional on age has been relatively stable in U.S. Census data since the 1980s. Both studies show this stability pooling firms across sectors. I find enormous heterogeneity in firm size trends across sectors using the same data. Declining firm size conditional on age in the U.S. manufacturing sector masks the reverse trend in almost all other sectors when pooling firms across sectors.[2] Based on their findings, both studies explain the recent increase in the average firm size and other macroeconomic trends through falling labor force growth, which shifts the firm age distribution while keeping firm size conditional on age constant. Both rule out rising entry costs behind the increase in the average firm size, as firm size conditional on age increases. I show that firm size conditional on age has increased in the data, highlighting that other forces that affect firm growth or selection impacted the aggregate economy too. Such forces include increasing barriers to entry (Davis, 2017; Gutiérrez and Philippon, 2018) or rising productivity dispersion (Aghion, Bergeaud, Boppart, Klenow and Li, 2023).[3]

[2] Van Vlokhoven (2021) further documents that profits and sales of firms in Compustat data have become more back-loaded.

[3] Further explanations behind recent macroeconomic trends include increasing costs of R&D (Bloom, Jones, Van Reenen and Webb, 2020), the increasing importance of intangible capital and information and

Sterk, Sedláček and Pugsley (2021) document changes in life cycle growth for U.S. firms over time. For the cohorts 1979 to 1993, the authors show that employment growth over the firm's life cycle slowed. The results presented in this paper are complementary rather than contradictory to theirs as I document trends for the cohorts from 1997 to 2017. The rise in industry concentration, and the fall in firm entry accelerated strongly during the turn of the millennium, as shown by Autor, Dorn, Katz, Patterson and Van Reenen (2020), and Akcigit and Ates (2021). Firm-size changes during this period are particularly useful to understand the forces behind these macroeconomic trends. Further, I document trends in average firm size conditional on age that reflect changes in firm growth and selection.

The structural model in this paper relates to Peters (2020). Peters (2020) builds an endogenous growth model with ex-ante homogeneous firms that conduct expansion and internal R&D. The model in this paper features ex-ante heterogeneity in firm productivity types to study firm selection. This introduces a new state variable to the firm's value function, namely the distribution of firm types across product lines. Firms keep track of this distribution to build markup expectations for their decision to enter new product lines.[4] Apart from technical implications, firm selection matters for long-run productivity growth, as highlighted by the growth decomposition in this paper. Further, Peters (2020) analyzes the model's steady state implications, whereas this paper solves for the transition between steady states to study welfare. Relatedly, Aghion, Bergeaud, Boppart, Klenow and Li (2023) build a model of creative destruction with ex-ante heterogeneous firms. The model abstracts from internal R&D. Internal R&D is necessary to explain the disproportionate increase in average firm employment conditional on age (relative to sales). Further, there is no firm entry or exit. Firm entry introduces the life cycle dimension to firms and plays a key role for changes in long-run productivity growth in this paper.

A separate strand of literature emphasizes the effects of reallocation on economic growth. China and East Germany are examples where long-term sustained growth followed the reallocation of market shares from state-owned enterprises to privately held companies (Song, Storesletten and Zilibotti, 2011; Findeisen, Lee, Porzio and Dauth, 2021). This reallocation potentially affects GDP per capita in a static sense through two channels. First, more productive firms gain market shares, thereby raising average productivity, and second, by reducing the extent of misallocation of production factors in the spirit of Hsieh and Klenow (2009). The reallocation of market shares could also affect the economy's long-run growth rate if firms innovate (or imitate) at heterogeneous rates (Acemoglu, Akcigit, Alp, Bloom and Kerr, 2018). The model in this paper accounts for the effect of reallocation on economic growth through all three channels: over the transition to the new balanced growth path, the reallocation of sales shares across firms affects aggregate output growth through changes

communications technology (ICT) (Crouzet and Eberly, 2019; Chiavari and Goraya, 2020; De Ridder, 2024; Hsieh and Rossi-Hansberg, 2023; Weiss, 2019), falling population growth (Bornstein, 2018; Engbom, 2023; Peters and Walsh, 2021; Hopenhayn, Neira and Singhania, 2022; Karahan, Pugsley and Şahin, 2022), declining interest rates (Chatterjee and Eyigungor, 2019; Liu, Mian and Sufi, 2022), changes in the quality of ideas (Olmstead-Rumsey, 2019) or declining imitation rates (Akcigit and Ates, 2023).

[4] Hence, ex-ante heterogeneity in productivity conceptually differs from heterogeneity in, e.g., R&D costs.

in average productivity, misallocation, and innovation rates. I find that these reallocation effects matter, even for long-run economic growth.

Akcigit and Kerr (2018), Garcia-Macia, Hsieh and Klenow (2019), and Peters (2020) decompose economic growth into the contributions by entrants and incumbent firms. These studies conclude that economic growth is mainly due to incumbent firms rather than entrants. While this is also the case in the parametrized model of this paper, I show that entrants play a more prominent role in explaining changes in economic growth.[5]

The paper proceeds as follows. Section 2 documents the changes in firm-size dynamics, and Section 3 lays out the model. Section 4 explains the empirical findings across balanced growth paths and quantifies the aggregate implications. The transitional dynamics are computed in Section 6. Section 7 provides robustness, and Section 8 concludes.

2 Trends in firm size

The section reports salient changes in the dynamics of firm size. I outline the data first.

2.1 Data

Data is provided by Statistics Sweden (SCB), the official statistical agency in Sweden. The main data set is *Företagens Ekonomi* (FEK), which covers information from balance sheets and profit and loss statements for the universe of Swedish firms at an annual frequency covering the period 1997-2017. FEK contains the main variables of interest: sales and employment (in full-time units). Before 1997, FEK was a sample covering large Swedish firms. To ensure full representativeness, I focus on the years 1997 forward. The data further contains information on the firm's legal type and industry at the five-digit level. I restrict the data to firms in the private economy. If not mentioned otherwise, I focus on the unbalanced panel of firms. The birth year of the firm is defined as the year it hires its first employee. I obtain this information from the auxiliary data set *Registerbaserad Arbetsmarknadsstatistik* (RAMS), containing the universe of employer-employee matches. I further restrict myself to firms that employ at least one worker according to RAMS.[6] Throughout the paper, nominal variables are deflated to 2017 Swedish Krona (SEK) using the GDP deflator. For more details about the data, see Section A in the Appendix.

Table 1 reports distributional statistics of firm sales, value added, and production inputs for the pooled data (1997 to 2017). The median firm lists sales of roughly 2.7 million SEK (approx. 0.27 million US dollars), value added of 1.1 million SEK, and employs two workers. The distribution of sales, value added, and all production inputs is highly right-skewed, as

[5] Likewise, Bartelsman and Doms (2000), Haltiwanger, Foster and Krizan (2001), Lentz and Mortensen (2008) and Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) decompose productivity growth into within- and between firm effects. This paper studies how these channels affect changes in productivity growth.

[6] The empirical results of this section are very similar when measuring firm employment using RAMS.

Table 1: Summary statistics (1997-2017)

	25th Pct.	50th Pct.	75th Pct.	Mean	SD	Obs.
*Sales**	1.2	2.7	7.8	27.8	568.2	4,918,996
*Value added**	0.5	1.1	2.9	7.6	142.3	4,918,996
Employment	1	2	5	9.9	131.1	4,918,996
*Wage bill**	0.2	0.6	1.6	3.7	53.0	4,918,996
*Capital stock**	0.04	0.2	1.1	9.3	277.0	4,918,996
*Intermediate Inputs**	0.4	0.9	2.6	10.8	270.0	4,918,996

Note: variables marked with * are in units of million 2017-SEK (1 SEK $\approx$ 0.1 US dollars). The capital stock is defined as fixed assets minus depreciation.

indicated by the mean and the 25th, 50th, and 75th percentiles. Average firm sales are 27.8 million SEK, and average employment is 9.9. In total, the data includes about 4.9 million firm-year observations. For the age-specific empirical analysis, I focus on firms established in 1997 or later, which reduces the sample size to 2.2 million firm-year observations. For these firms, age is not truncated.

2.2 Trends in firm size in Sweden

This section documents systematic changes in firm-size dynamics. To start with, I characterize firm size as a function of firm age non-parametrically using a regression framework. More specifically, I measure firm size by firm age as follows

$$\ln \text{Size}_{f,t} = \gamma_0 + \sum_{a_f=1}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{f,t}=a_f} + \theta_c + \theta_k + \epsilon_{f,t}, \tag{1}$$

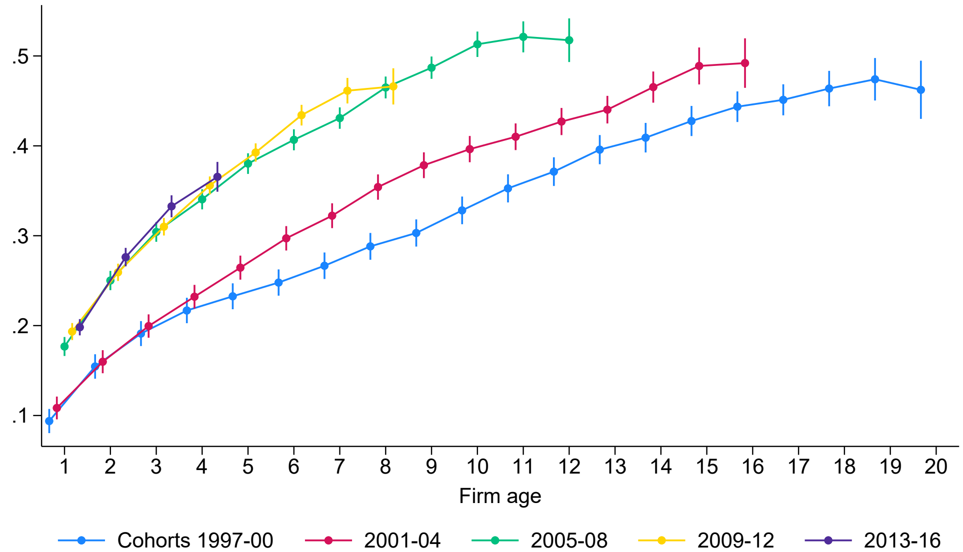
where f indexes firms, and $\mathbb{1}_{\text{Age}_{j,t}=a_f}$ denote firm-age dummies ranging from age one to twenty. θ_k and θ_c are industry (5-digit) and cohort fixed effects.[7] Within a given cohort and industry, γ_{a_f} captures the average firm size conditional on age a_f relative to the average size at entry (at age zero that is captured by the constant γ_0), i.e.,

$$\gamma_{a_f} = E\left[\ln \text{Size}_{f,t} | \text{Age}_{f,t} = a_f, c, k\right] - E\left[\ln \text{Size}_{f,t} | \text{Age}_{f,t} = 0, c, k\right]. \tag{2}$$

Firm employment and sales serve as the dependent variable in (1). Sedláček and Sterk (2017) argue that aggregate conditions at the year of entry have persistent effects on firm employment over the life cycle. Therefore, the baseline regression includes cohort fixed effects, but the results are robust to alternative specifications, as shown later.

[7]The cohort and industry dependence of the other variables is suppressed for clarity.

Figure 1: Average log employment relative to age zero (by cohorts)

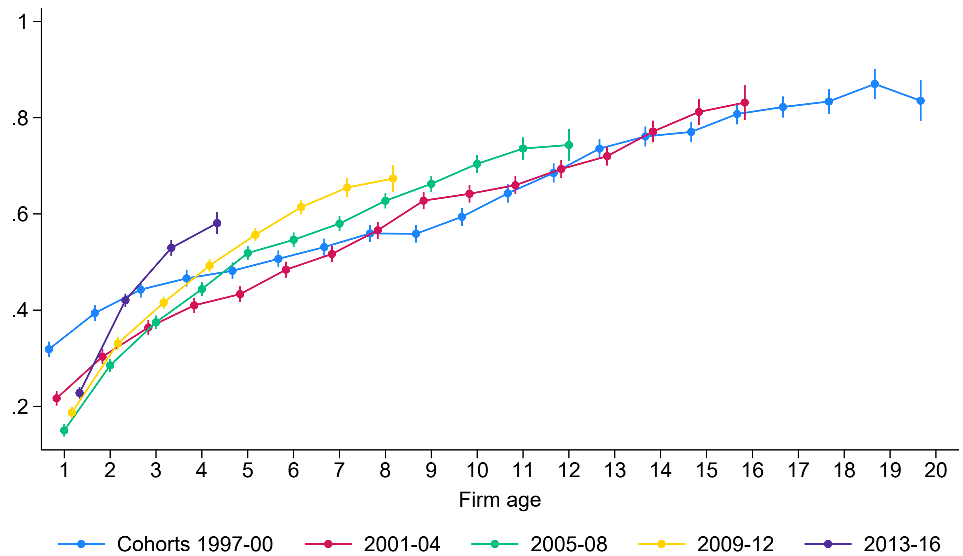


Notes: the figure shows average log employment for any firm age relative to average log employment at entry according to eq. (1) in Swedish registry data (unbalanced panel). Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

I group the cohorts 1997–2017 into five groups (each group includes four cohorts) and run regression (1) for each group to measure changes in firm size conditional on age across cohorts. Figure 1 plots the age coefficients, γ_1 to γ_{20}, for the different cohort groups with employment as the dependent variable. Figure 1 contains the main empirical result: the age coefficients γ_1 to γ_{20} gradually increase for more recent cohorts. In other words, firm size conditional on age has systematically increased since the late 1990s relative to the size at entry. For example, the average employment at age eight is 0.29 log points higher than the average employment at entry for the cohorts 1997 to 2000, compared to 0.47 log points for the cohorts 2009 to 2012. To be clear, Figure 1 documents an increase in average firm size conditional on age without taking a stance on the cause. In particular, the figure remains silent on whether firm growth accelerated or the selection of surviving firms changed.

I provide robustness checks for the above results, contained in Appendix B.1. The baseline regression in equation (1) controls for industry and cohort fixed effects. The results are virtually unchanged with interacted industry and cohort fixed effects. The cohort fixed effects control for shocks that are common to firms established in the same year. Alternatively, one could control for shocks that are common to firms in a given year, independent of their age. The increase in firm size conditional on age is even more pronounced when estimating (1) with interacted industry and year instead of cohort fixed effects. Further, I rule out selection effects due to the Great Recession as a cause behind the trends. I show the size conditional on age patterns of each cohort that followed the Great Recession, starting with the cohort in 2011. A steepening of the patterns is visible for every single cohort. Other structural forces might, of course, have affected the selection of firms. Selection effects between (ex-ante) heterogeneous firms will be highlighted in the model analysis later on. Lastly, I explore robustness concerning the definition of an entering firm. Firms enter the economy at different times during the year. If, for some reason, firms enter towards the end of the year for the more recent cohorts, employment during the first year (age zero) is lower, and employment growth between age zero and one is higher. First, firm employment at entry is very stable over time, indicating no systematic changes in the nature of entrants. Second, I estimate a variant of regression (1), regressing log size on age dummies ranging from two to twenty. This way, firm size is measured relative to the average firm size before age two. The alternative entrant classification somewhat reduces the jump at early ages for the cohorts following 2005 in Figure (1) while preserving the divergence at later ages.

Figure 2: Average log sales relative to age zero (by cohorts)



Notes: the figure shows average log sales for any firm age relative to average log sales at entry according to eq. (1) in Swedish registry data (unbalanced panel). Cohorts are pooled as indicated in the legend. Firm sales are filtered at their 1% tails. The figure includes 95% confidence intervals.

The previous analysis focused on firm employment. Figure 2 reports the age coefficients of regression (1) with sales as the dependent variable. The figure confirms the same patterns: firm sales conditional on age increased relative to sales at entry, yet at a more muted rate. Average sales at age eight are roughly 0.56 log points higher than at entry for the cohorts 1997 to 2000, compared to 0.67 log points for the cohorts 2009 to 2012. I provide the same robustness checks in Appendix B.1. If anything, the alternative entrant classification strengthens the increase in sales conditional on age relative to entry, as it mutes the increase of the cohorts 1997-2000 during the early ages.

The evidence suggests that firm size conditional on age has increased relative to the size at entry, particularly for employment. What is driving the changes? I build a structural model to study the cause behind these trends. Before turning to the model, the following presents external validity to the documented trends.

2.3 Trends in firm size in the U.S.

Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022) document that firm size conditional on age has been relatively stable in the U.S. since the 1990s. Importantly, both studies pool firms across all sectors of the economy. I find enormous heterogeneity in the firm size-conditional on age patterns across sectors. In most sectors, firm size conditional on age has increased relative to the size of entrants. In some sectors, this increase is mild; in others, it is substantial. That the size patterns are stable when pooling firms across all sectors is due to firm size contracting in the manufacturing and accommodation sector, offsetting the increase in almost all other sectors.

I use data from the Business Dynamics Statistics (BDS) provided by the U.S. Census Bureau, which covers nearly all private-sector firms with paid employees.[8] The data covers the period 1978–2021. For completeness, Appendix B.2 shows the patterns of firm size conditional on age, pooling firms across sectors. These patterns are relatively stable, as noted by Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022). However, this result masks enormous heterogeneity across sectors. Figure 3 reports the size-conditional-on age patterns for selected economically relevant sectors, where sector classifications follow the two-digit NAICS codes.[9] Log employment of entrants (age zero) is normalized to zero to facilitate comparability across time and sectors.

Figure 3 illustrates substantial heterogeneity in the firm-size trends across sectors. For example, average employment of manufacturing firms aged 11-15 has shrunk relative to the average size of entrants. For firms in the retail trade sector, firm-size patterns have been stable. In stark contrast, average employment of firms aged 11-15 has increased substantially in the information sector, finance and insurance sector, professional, scientific and technical services sector, and the administrative and support services sector relative to that of entrants. In the information sector, employment of firms aged 11-15 increased by roughly 0.5 log points from the early 1990s to the late 2010s relative to employment of entrants. It is particularly noteworthy that the increase in the size gap is not only a feature of the high-growth period of the late 1990s but has occurred steadily over the last three decades.

Figure 4 reports the change in the log employment gap between firms aged 11-15 and entrants for all two-digit NAICS sectors in the private economy from 1992 to 2017; in essence, the time change of the blue line in Figure 3. The year 2017 is chosen for comparability with Section 2.2. This choice is conservative given that the size gaps in the service sectors further open up after 2017 in Figure 3.

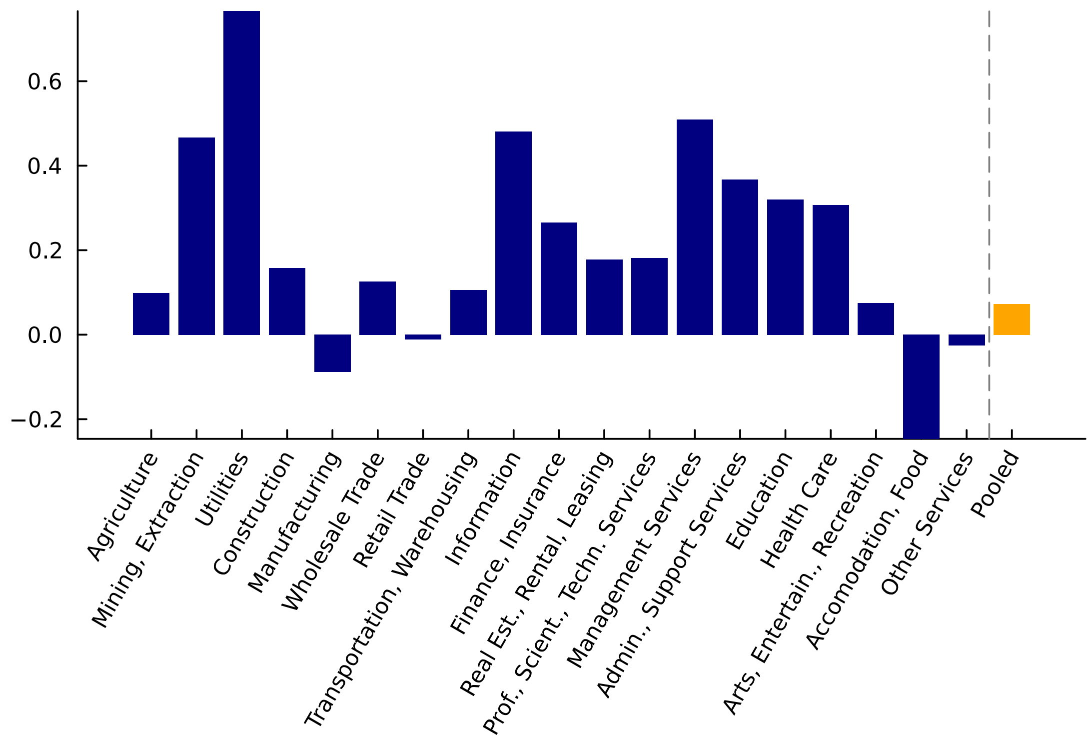
[8]Source: U.S. Census Bureau - Center for Economic Studies - Business Dynamics Statistics (2021), `https://bds.explorer.ces.census.gov`. Accessed July 13, 2024.

[9]For age groups 6-10 and 11-15, I skip the first four years to ensure a consistent age grouping over time.

Figure 3: Log employment by firm age and sector

Notes: the figure shows average log employment conditional on firm age in U.S. Census data (unbalanced panel). Log employment of entrants (age zero) is normalized to zero. Sector classifications correspond to two-digit NAICS codes.

Figure 4: Firm size (ages 11-15) relative to entrants, log change 1992-2017



Notes: the figure shows the change (1992–2017) in the gap between log employment of firms aged 11-15 and entrants in U.S. Census data. Sector classifications correspond to two-digit NAICS codes.

Several observations are noteworthy. First, Figure 4 confirms the enormous heterogeneity in firm-size trends across sectors. Second, almost all sectors display an increase in the size of firms aged 11-15 relative to entrants. Only two out of the nineteen sectors experienced a meaningful decline, namely the manufacturing sector and the accommodation and food sector.[10] Even though the decrease in firm size in the U.S. manufacturing sector is interesting in its own right, Figure 3 shows that the decline entirely occurred during the 2000s, arguably driven by forces outside the analysis of this paper, in particular the China shock. The decrease in the size of firms aged 11-15 relative to entrants in the accommodation and food sector is surprising. However, this decline occurred after 2009, before which firms of this age experienced an increase in the average size of roughly 0.2 log points (not shown). Figure 4 further highlights that the increase in firm size relative to entrants is particularly widespread across the services sectors. This finding is consistent with the fact that the number of markets served per firm has increased for services firms, as documented by Hsieh and Rossi-Hansberg (2023). Third, when pooling firms across sectors (last column), the decline in these two sectors mutes the increase in all other sectors. When pooling across all sectors, the change in the relative size of firms aged 11-15 is small yet positive at 0.07 log points.

[10]The nature of the decline is different in both sectors, however. Whereas in manufacturing, the decline in total employment outweighs the fall in the number of firms, in the accommodation and food sector, the rise in the number of firms exceeds the increase in total employment (all conditional on age).

One caveat is in order. The average age within the group of firms aged 11-15 has potentially increased itself. I argue that changes in the age distribution within the group are not the main driver behind the increase in their average size. First, Figure 3 shows that the size of three-year-old firms also increases over time in the highlighted sectors. Second, in sectors that experienced large inflows of new firms, potentially even lowering the average age within the 11-15 age bracket, the average size within the age bracket increased substantially. One example is the information sector during the 1990s and 2000s. Third, the rise in the average size of firms aged 6-10 in the service sectors in Figure 3 is so enormous that their size in 2021 exceeds that of firms aged 11-15 in 1992. Even if the average age increased from 6.0 to 10.0 within the group (the most extreme example), age selection cannot explain why these firms nowadays are so much larger than the ones aged 11-15 in 1992. The increase in firm size conditional on age is sheer too large.

Firm size conditional on age has increased relative to the size of entrants in almost all sectors of the U.S. economy, in line with the trends in the Swedish administrative data.

3 Model

This section outlines an endogenous growth model with firm dynamics that allows me to study trends in firm growth, firm selection, and their implications for aggregate growth.

3.1 Preferences and aggregate economy

Time is continuous. The economy consists of a representative household that chooses the path of consumption C_t and wealth A_t to maximize lifetime utility

$$U = \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

subject to the budget constraint $\dot{A}_t = r_t A_t + w_t L_t - C_t$. ρ denotes the discount rate, r_t the interest rate and w_t the real wage. The household supplies one unit of labor inelastically, i.e., $L_t = 1$. The optimality condition (Euler equation) for the household problem reads

$$\frac{\dot{C}_t}{C_t} = r_t - \rho.$$

Aggregate output is produced competitively using a Cobb-Douglas technology over a continuum of different products indexed by i (time subscripts suppressed)

$$Y = \exp\left(\int_0^1 \ln\left[q_i y_i\right] di\right),$$

where y_i and q_i denote the quantity and quality of product i. Output is consumed entirely such that $Y = C$. Expenditure minimization leads to the standard demand function

$$y_i = \frac{YP}{p_i}. \tag{3}$$

P is defined as the aggregate price index, which I normalize to 1.

3.2 Production

Firms potentially produce in a product market i with a linear technology

$$y_{if} = \varphi_f l_{if},$$

where y_{if} is the amount of product i produced by firm f, l_{if} is the amount of labor hired, and φ_f denotes the productivity of firm f. As in Aghion, Bergeaud, Boppart, Klenow and Li (2023), the firm's productivity is fixed over time t and markets i, which captures the notion that some firms are persistently more efficient at producing than others, e.g., due to a better business plan. For simplicity, firms are of a high or low productivity type, i.e., $\varphi_f \in \{\varphi^h, \varphi^l\}$ where $\varphi^h/\varphi^l > 1$, which I refer to as high- and low-type firms.

3.3 Static allocation

Taking the joint distribution of product qualities and firm productivity as exogenous in this section, I characterize the static allocations at the product, firm and aggregate levels.

3.3.1 Product level

Firms in product market i compete in prices (Bertrand competition). In equilibrium, only the firm with the highest quality-adjusted productivity $q_{if}\varphi_f$ produces product i (henceforth, incumbent). Under Bertrand competition, the incumbent firm engages in limit pricing and sets its price equal to the quality-adjusted marginal costs of the follower (the firm with the second highest quality-adjusted productivity)

$$p_{if} = \frac{q_{if}}{q_{if'}} \frac{w}{\varphi_{f'}}, \tag{4}$$

where f' indexes the follower in product market i. The price that the incumbent sets is increasing in the quality gap between the incumbent and the follower, as eq. (4) shows. Defining the product markup as the output price over marginal costs, it follows

$$\mu_{if} \equiv \frac{p_{if}}{w/\varphi_f} = \frac{q_{if}}{q_{if'}} \frac{\varphi_f}{\varphi_{f'}}. \tag{5}$$

The incumbent's markup for product i is increasing in the quality and productivity gap. The price setting of the incumbent gives rise to the following profits for product i

$$\pi_{if} = p_{if} y_{if} - w l_{if} = Y \left(1 - \frac{1}{\mu_{if}}\right),$$

with labor demand for product i

$$l_{if} = \frac{Y}{w} \mu_{if}^{-1}. \tag{6}$$

Employment in product line i is decreasing in the markup.

3.3.2 Firm level

Firm employment is the sum of employment across product lines

$$l_f = \sum_{i \in N_f} l_{if} = \frac{Y}{w} \left(\sum_{i \in N_f} \mu_{if}^{-1} \right),$$

where N_f denotes the set of product lines where firm f is the incumbent producer. Firm employment decreases in the markups within each product line but increases in the total number of lines. Hence, holding markups constant, firms that produce in more product lines feature higher employment. Vice versa, holding the number of product lines constant, firms with higher markups employ less labor. As sales are equalized across product lines, firm sales are given by $\left|N_f\right| Y \equiv n_f Y$, where n_f denotes the number of products firm f is producing. Hence, firms that produce in more product lines feature higher sales.

3.3.3 Aggregate level

Integrating employment across firms or products yields the total workforce in production:

$$L_P = \int_f l_f df = \frac{Y}{w} \int_0^1 \mu_{if}^{-1} di. \tag{7}$$

Taking logs and integrating eq. (5), one obtains an expression for the wage

$$w = \exp\left(\int_0^1 \ln q_{if} di \right) \times \exp\left(\int_0^1 \ln \varphi_{f(i)} di \right) \times \exp\left(\int_0^1 \ln \mu_{if}^{-1} di \right). \tag{8}$$

To find an expression for aggregate output, insert eq. (8) into eq. (7) to obtain

$$Y = Q \Phi \mathcal{M} L_P, \tag{9}$$

where

$$Q = \exp\left(\int_0^1 \ln q_{if} di\right), \quad \Phi = \exp\left(\int_0^1 \ln \varphi_{f(i)} di\right), \quad \mathcal{M} = \frac{\exp\left(\int_0^1 \ln \mu_{if}^{-1} di\right)}{\int_0^1 \mu_{if}^{-1} di}.$$

Aggregate output Y depends on geometric averages of quality Q and productivity Φ as well as on misallocation $\mathcal{M}$ and production labor L_P. As highlighted in Peters (2020), misallocation arises from markup dispersion ($\mathcal{M}$ is bounded by unity from above). In this model, markup dispersion is due to both quality and productivity heterogeneity. The product of Q, Φ and $\mathcal{M}$ captures aggregate Total Factor Productivity (TFP).

3.4 Dynamic firm problem

Incumbents continuously improve the quality of products, q_i, in the economy through two different types of R&D. Internal R&D increases the quality of incumbent firm f's own product, whereas, through expansion R&D, the incumbent f improves the quality of a random product of a competing incumbent. Item quality is improved step-wise such that every innovation (either internal or expansion R&D) increases q_i by a factor of λ. As Aghion, Bergeaud, Boppart, Klenow and Li (2023), I assume that the step size of quality improvements exceeds the productivity differential, $\lambda > \varphi^h/\varphi^l$. This assumption ensures that the firm with the highest quality version in a product line is the incumbent producer.[11] Denoting by λ^{Δ_i} the relative qualities of incumbent and second-best firms within a product line, i.e.,

$$\lambda^{\Delta_i} = \frac{q_{if}}{q_{if\prime}}$$

and by $[\mu_i]$ the set of markups, where firm f is producing, firm profits can be written as

$$\pi_{ft}(n, [\mu_i]) = \sum_{k=1}^{n} Y_t \left(1 - \frac{1}{\mu_k}\right) = \sum_{k=1}^{n} Y_t \left(1 - \frac{1}{\lambda^{\Delta_k}} \frac{1}{\frac{\varphi_{fk}}{\varphi_{f\prime k}}}\right) \equiv \sum_{k=1}^{n} \pi(\mu_k),$$

where $\pi(\mu_i)$ denote profits in product line i. Incumbent firms choose the rate of internal R&D, I_i, and the rate of expansion R&D, x_i, for each of their product lines, i. When choosing optimal internal and expansion R&D rates, firms take aggregate output Y_t, the real wage w_t, the share of lines operated by high-productivity firms S_t, the interest rate r_t and the rate of creative destruction τ_t as given. Denoting the time derivative by $\dot{V}_t^h()$, the value function of a high-productivity type firm (indexed by h) satisfies the following HJB equation:

$$r_t V_t^h(n, [\mu_i], S_t) - \dot{V}_t^h(n, [\mu_i], S_t) =$$

[11] Relaxing this assumption would give room for a race for incumbency between low-productivity entrants facing a high-productivity incumbent from which I abstract.

$$\sum_{k=1}^{n} \underbrace{\pi(\mu_k)}_{\text{Flow profits}} + \sum_{k=1}^{n} \underbrace{\tau_t \Big[V_t^h\left(n-1, [\mu_i]_{i\neq k}, S_t\right) - V_t^h(n, [\mu_i], S_t)\Big]}_{\text{Creative destruction}}$$

$$+ \max_{[x_k, I_k]} \left\{ \sum_{k=1}^{n} \underbrace{I_k \Big[V_t^h\left(n, \left[[\mu_i]_{i\neq k}, \mu_k \cdot \lambda\right], S_t\right) - V_t^h(n, [\mu_i], S_t)\Big]}_{\text{Internal R\&D}} \right.$$

$$+ \sum_{k=1}^{n} \underbrace{x_k \left[S_t V_t^h\left(n+1, \left[[\mu_i], \lambda\right], S_t\right) + (1-S_t) V_t^h\left(n+1, \left[[\mu_i], \lambda \cdot \frac{\varphi^h}{\varphi^l}\right], S_t\right) - V_t^h(n, [\mu_i], S_t)\right]}_{\text{Expansion R\&D}}$$

$$\left. - \underbrace{w_t \Gamma\left([x_i, I_i]; n, [\mu_i]\right)}_{\text{R\&D costs}} \right\}.$$

The value of a firm consists of flow profits, research costs, and three parts related to internal R&D, expansion R&D, and creative destruction. At the rate of creative destruction τ_t (determined in equilibrium), the firm loses one of its n products, in which case, it remains with $n-1$ products. At the optimally chosen rate I_k, internal R&D turns out successful (third row), and the firm charges a λ times higher markup on its product according to eq. (5). Alternatively, at the optimally chosen rate x_k, expansion R&D is successful (fourth row), and the firm acquires a new product (n increases by one).

Firm-type heterogeneity introduces novel elements to the value function. First, the value function is specific to the productivity type of the firm. Second, the share of product lines operated by each productivity type is a state variable (with two types, it is sufficient to keep track of S_t). When taking over a new product line through expansion R&D (fourth row), the probability of replacing a high-type incumbent is S_t, in which case the high-type entrant charges a markup of λ. With probability $1-S_t$, the replaced incumbent is of the low type, and the high-type entrant charges a markup of $\lambda \cdot \varphi^h/\varphi^l$. Firms take S_t as given; however, they affect it through their expansion R&D efforts x_k in equilibrium. The HJB equation for a low-productivity firm follows the same structure and is listed in the Appendix, Section C.1. The term related to expansion R&D (fourth row) varies since low-productivity firms build different markup expectations when entering a new product line.

$\Gamma\left([x_i, I_i]; n, [\mu_i]\right)$ denote the R&D costs. For their R&D activities, firms pay a cost of

$$\Gamma\left([x_i, I_i]; n, [\mu_i]\right) = \sum_{k=1}^{n} c\left(x_k, I_k; \mu_k\right) = \sum_{k=1}^{n} \left[\mu_k^{-1} \frac{1}{\psi_I} (I_k)^\zeta + \frac{1}{\psi_x} (x_k)^\zeta\right]$$

in terms of labor. $\zeta > 1$ ensures convexity of the cost function. R&D costs are additively separable to render a closed-form solution of the value function along the balanced growth path. ψ_I and ψ_x scale internal and external R&D costs and capture the R&D efficiency.[12]

[12] The incentives for internal R&D decrease with the quality gap that the firm has accumulated as profits within a product line are concave in the markup. I scale the internal R&D costs by the inverse markup to

Firm entry is determined as follows. Potential entrants produce a flow rate of entry z_t using a technology that is linear in labor: $z_t = \psi_z L_{Et}$, where ψ_z denotes the entry efficiency and L_{Et} research labor of entrants. Entrants improve the quality of a randomly selected product line. The productivity type is realized after entry and assigned with the exogenous probabilities p^h and $1-p^h$, respectively. Entrants start with a one-step quality gap. When $z_t > 0$, the free entry condition requires that the expected value of firm entry equals the entry costs

$$p^h E[V_t^h(1,\mu_i)] + (1-p^h)E[V_t^l(1,\mu_i)] = \frac{1}{\psi_z} w_t, \tag{10}$$

where the expected value of entering as a high- or low-type firm is

$$\begin{aligned} E[V_t^h(1,\mu_i)] =& S_t V_t^h(1,\lambda) + (1-S_t)V_t^h(1,\lambda \times \varphi^h/\varphi^l) \\ E[V_t^l(1,\mu_i)] =& S_t V_t^l(1,\lambda \times \varphi^l/\varphi^h) + (1-S_t)V_t^l(1,\lambda). \end{aligned}$$

Labor market clearing requires that production labor L_{Pt} and total research labor L_{Rt} add up to one, the aggregate labor endowment

$$1 = L_{Pt} + L_{Rt} = \int_0^1 \frac{Y_t}{w_t} \mu_{it}^{-1} di + \int_0^1 \left(\mu_{it}^{-1} \frac{I_{it}^\zeta}{\psi_I} + \frac{x_{it}^\zeta}{\psi_x} \right) di + \frac{z_t}{\psi_z}. \tag{11}$$

3.5 Cross-sectional distribution of quality and productivity gaps

The joint (cross-sectional) distribution of quality and productivity gaps is the key equilibrium object that characterizes aggregates in the model. On the one hand, quality and productivity gaps characterize the markup distribution that summarizes labor demand. On the other hand, the joint distribution characterizes the share of product lines operated by each productivity type, which is a state variable in the firm's optimization problem. This section characterizes the joint distribution of quality and productivity gaps as a function of firm policies, which allows the equilibrium distribution to be solved jointly with the policies.

The distribution of quality and productivity gaps ν is characterized by a set of infinitely many differential equations. For simplicity, I characterize the differential equations for firm-type specific expansion R&D rates, x_t^h and x_t^ℓ, and uniform internal R&D rates, I_t, as proven shortly in Proposition 1 for a balanced growth path.[13] For product lines where the incumbent is at least two quality steps ahead of the follower ($\Delta \geq 2$), the measure ν follows

$$\dot{\nu}_t \left(\Delta, \frac{\varphi_f}{\varphi_{f'}} \right) = I_t \nu_t \left(\Delta - 1, \frac{\varphi_f}{\varphi_{f'}} \right) - \nu_t \left(\Delta, \frac{\varphi_f}{\varphi_{f'}} \right) (I_t + \tau_t). \tag{12}$$

keep internal R&D incentives constant as in Peters (2020).

[13]The distribution along the transition path is characterized in the Appendix, Section D.

For product lines where the incumbent is one step ahead ($\Delta = 1$), the measure follows

$$\begin{aligned}
\dot{\nu}_t\left(1, \frac{\varphi^l}{\varphi^h}\right) &= (1-S_t)x_t^l S_t + z_t(1-p^h)S_t - \nu_t\left(1, \frac{\varphi^l}{\varphi^h}\right)(I_t+\tau_t) \\
\dot{\nu}_t\left(1, \frac{\varphi^l}{\varphi^l}\right) &= (1-S_t)x_t^l(1-S_t) + z_t(1-p^h)(1-S_t) - \nu_t\left(1, \frac{\varphi^l}{\varphi^l}\right)(I_t+\tau_t) \\
\dot{\nu}_t\left(1, \frac{\varphi^h}{\varphi^h}\right) &= S_t x_t^h S_t + z_t p^h S_t - \nu_t\left(1, \frac{\varphi^h}{\varphi^h}\right)(I_t+\tau_t) \\
\dot{\nu}_t\left(1, \frac{\varphi^h}{\varphi^l}\right) &= S_t x_t^h(1-S_t) + z_t p^h(1-S_t) - \nu_t\left(1, \frac{\varphi^h}{\varphi^l}\right)(I_t+\tau_t).
\end{aligned} \tag{13}$$

Changes in the measure $\dot{\nu}$ are due to inflows and outflows. Outflows arise from successful internal R&D (quality gap increases from Δ to $\Delta+1$) and creative destruction (quality gap is reset to unity). Inflows vary with the quality gap. For $\Delta \geq 2$, inflows into state Δ are due to successful internal R&D in product lines with quality gaps of $\Delta-1$. For $\Delta = 1$, inflows result from creative destruction. For example, the measure of products with a low-type incumbent and high-type second best firm $\nu_t\left(1, \frac{\varphi^l}{\varphi^h}\right)$ increases due to low-type incumbents and entrants replacing high-type incumbents, captured by $(1-S_t)x_t^l S_t + z_t(1-p^h)S_t$ in eq. (13). From the measure ν, one obtains the share of product lines operated by high-type firms

$$S_t = \sum_{i=1}^{\infty}\left[\nu_t\left(i, \frac{\varphi^h}{\varphi^h}\right) + \nu_t\left(i, \frac{\varphi^h}{\varphi^l}\right)\right]. \tag{14}$$

3.6 Balanced growth path characterization

I define a balanced growth path of the economy as follows.

Definition 1. *A balanced growth path (BGP) is a set of allocations $[x_{it}, I_{it}, \ell_{it}, z_t, S_t, y_{it}, C_t]_{it}$ and prices $[r_t, w_t, p_{it}]_{it}$ such that firms choose $[x_{it}, I_{it}, p_{it}]$ optimally, the representative household maximizes utility choosing $[C_t, y_{it}]_{it}$, the growth rate of aggregate variables is constant, the free-entry condition holds, all markets clear and the distribution of quality and productivity gaps is stationary.*

Along the balanced growth path, the economy can be characterized in closed form.

Proposition 1. *In the above setup, along a balanced growth path:*

1. *The value of a product line for a firm of productivity type $d \in \{h, l\}$ is given by*

$$V_t^d(1, \mu_i, S) = \frac{1}{\rho+\tau}\left[Y_t\left(1-\frac{1}{\mu_i}\right) + \frac{\zeta-1}{\psi_x}(x^d)^\zeta w_t + \frac{\zeta-1}{\psi_I} I^\zeta w_t \mu_i^{-1}\right] \tag{15}$$

   where $x^h > x^l$ and $I \equiv I^h = I^l$. The value of a firm is $V_t^d(n, [\mu_i], S) = \sum_{i=1}^n V_t^d(1, \mu_i, S)$.

2. *S_{φ^k,φ^p}, the constant share of product lines where the incumbent firm is of productivity*

type k and the second-best firm of type p is

$$
\begin{aligned}
S_{\varphi^l,\varphi^h} &\equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^l}{\varphi^h}\right) = \frac{(1-S)x^l S + z(1-p^h)S}{\tau} \\
S_{\varphi^l,\varphi^l} &\equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^l}{\varphi^l}\right) = \frac{(1-S)x^l(1-S) + z(1-p^h)(1-S)}{\tau} \\
S_{\varphi^h,\varphi^h} &\equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^h}{\varphi^h}\right) = \frac{Sx^h S + zp^h S}{\tau} \\
S_{\varphi^h,\varphi^l} &\equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^h}{\varphi^l}\right) = \frac{Sx^h(1-S) + zp^h(1-S)}{\tau},
\end{aligned}
$$

which defines the share of product lines operated by the high-productivity type

$$
S = S_{\varphi^h,\varphi^h} + S_{\varphi^h,\varphi^l} = \frac{Sx^h + zp^h}{\tau}. \tag{16}
$$

3. *The growth rate of aggregate variables is given by*

$$
g = \frac{\dot{Q}_t}{Q_t} = \Bigg(\underbrace{I}_{\textit{Incumbent internal R\&D}} + \underbrace{Sx^h + (1-S)x^l}_{\textit{Incumbent expansion R\&D}} + \underbrace{z}_{\textit{Entry}} \Bigg) \times \ln(\lambda). \tag{17}
$$

Proof. The Appendix, Sections C.1, C.2 and C.3, contains the proofs. □

The value of a product line in eq. (15) consists of three terms: profits for a given markup, the continuation value of expansion R&D, and the continuation value of internal R&D. The sum of the three terms is discounted by the discount rate and the rate of creative destruction. The more impatient the household or the higher the risk of replacement, the lower the value of a product line. Importantly, the value of a product line is productivity-type specific. More productive incumbents charge higher markups and enjoy greater profits in expectation. This affects their expansion R&D rates. The optimality condition for expansion R&D (Appendix, eq. (34)) equates the expected value of a product line to the marginal cost of expansion R&D. Hence, in equilibrium, more productive firms pay a higher marginal cost of expansion R&D, $x^h > x^l$. Lastly, the value of a firm equals the sum of the value of its product lines.

Proposition 1 further shows that S_{φ^k,φ^p}, the share of products lines where the incumbent firm is of type k and the second-best firm of type p, is constant along a balanced growth path. This share equals the fraction of creatively destroyed products at each instant of time, where the new incumbent is of type k and the replaced firm of type p. The share of product lines operated by high-productivity type firms S is equal to the sum of S_{φ^h,φ^h} and S_{φ^h,φ^l}. In particular, eq. (16) can be rearranged to

$$S = \frac{zp^h}{(1-S)(x^l - x^h) + z}, \tag{18}$$

which shows that S depends on the difference in the expansion R&D rates between firm types, $x^l - x^h$. Holding firm entry z fixed, an increase in the expansion rate of high-productivity incumbents must be matched by an equal rise (in absolute terms) in the expansion rate of less-productive firms for S to remain constant. Note the importance of firm entry. With z set to zero, for equation (18) to hold, either x^l equals x^h or there is an exterior solution for S. Without firm entry and $x^l \neq x^h$, the faster expanding, more productive firms eventually take over all product lines. Positive firm entry is necessary for both firm types to co-exist in equilibrium where $x^l \neq x^h$ and S is constant at its interior solution.

The aggregate rate of creative destruction is equal to the sum of firm-type specific expansion R&D rates weighted by their sales shares and the rate of entry

$$\tau = \int_0^1 x_i di + z = Sx^h + (1-S)x^l + z. \tag{19}$$

Long-run growth results from R&D at the product level. This occurs through successful internal R&D or (incumbent and entrant) creative destruction. The aggregate arrival rate of innovation is, hence, equal to the sum of the rates of creative destruction τ and internal R&D I. Multiplying the arrival rate by the step size of innovation delivers the aggregate growth rate g, as shown in eq. (17) of Proposition 1. Since expansion R&D rates are heterogeneous, changes in the share of product lines operated by each productivity type, S and $1-S$, affect the aggregate growth rate. Along the balanced growth path, both τ and g are constant.

The stationary distribution of productivity and quality gaps further characterizes the aggregate labor income share, the TFP misallocation measure $\mathcal{M}$, and the aggregate markup. I derive these objects analytically in the Appendix, Section C.2.

To find the balanced growth path, I jointly solve the optimality conditions of the firm (derived in Appendix C.1), the free entry condition, eq. (10), the labor market clearing condition, eq. (11), and the system of differential equations characterizing the distribution of productivity and quality gaps captured in eqs. (12) and (13). Appendix C.4 contains the details.

3.7 Firm dynamics

Firms lose products according to the same stochastic process as in Klette and Kortum (2004). However, in this model, firms add products at systematically different rates as optimally chosen expansion R&D rates vary with the firm's productivity type.[14] The following section

[14]Therefore, the properties related to firm size growth and survival in Klette and Kortum (2004) hold conditional on the firm type. In particular, conditional on the type, firm size, and growth are unrelated, as in Lentz and Mortensen (2008). For the unconditional firm size and growth correlation, two forces are at

derives productivity-type specific firm growth conditional on survival (life cycle growth) and survival probabilities.

3.7.1 Firm sales growth

Firm sales are proportional to the number of products a firm produces. As such, successful expansion R&D increases firm sales. Since optimal expansion R&D rates are productivity-type specific, so is expected sales growth. Conditional on survival, expected sales growth for a firm with productivity φ_f between age zero and age a_f is

$$E\left[\ln n_f Y|a_f,\varphi_f\right] - E\left[\ln n_f Y|0,\varphi_f\right] = \underbrace{g \times a_f}_{\text{Aggregate growth}} + \underbrace{E\left[\ln n_f|a_f,\varphi_f\right]}_{\text{Firm's product growth}},$$

where n_f is the number of products the firm is producing. Note that the probability of producing n products at age a conditional on survival is $(1-\gamma^j(a))\left(\gamma^j(a)\right)^{n-1}$, where $\gamma^j(a) = x^j \frac{1-e^{-(\tau-x^j)a}}{\tau - x^j e^{-(\tau-x^j)a}}$ and $j \in \{h, l\}$. Therefore expected sales growth conditional on survival is equal to

$$E\left[\ln n_f Y|a_f,\varphi_f\right] - E\left[\ln n_f Y|0,\varphi_f\right] = \underbrace{g \times a_f}_{\text{Aggregate growth}} + \underbrace{\left(1-\gamma^f(a_f)\right)\sum_{n=1}^{\infty} \ln n \times \left(\gamma^f(a_f)\right)^{n-1}}_{\text{Firm's product growth}}. \tag{20}$$

3.7.2 Firm markup growth

Firm markups are defined as $\mu_f = \frac{py_f}{wl_f}$. The Appendix, Section C.5 shows that for a high-productivity type firm, the expected log markup conditional on firm age a_f is

$$E\left[\ln \mu_f|a_f,\varphi^h\right] = \underbrace{\ln\lambda \times \left(1 + I \times E[a_P^h|a_f]\right)}_{\text{Quality improvements}} + \underbrace{(1-S)\times \ln\left(\frac{\varphi^h}{\varphi^l}\right)}_{\text{Productivity advantage}}, \tag{21}$$

where $E[a_P^h|a_f]$ is the average product age of a high-type firm conditional on firm age.

The expected firm markup conditional on age consists of two terms. The first term in eq. (21) is akin to Peters (2020) and reflects that internal R&D translates quality improvements within a firm's product line into markup growth at the firm level as it ages. In Peters (2020), this term holds for all firms, whereas in this model, this term is specific to the productivity type of the firm, as the average product age varies by firm type. The second term in eq. (21)

play. On the one hand, young (small) firms tend to grow quicker due to survival bias. On the other hand, more productive firms (with faster growth rates) are more likely to end up large. In the estimated (initial) balanced growth path, 74% of the firms are of the high productivity type. Hence, size is unrelated to growth for the vast majority of firms.

captures a new level effect that heterogeneity in productivity introduces. The intuition is that if a high-type incumbent faces a low-type second-best firm, it can charge a φ^h/φ^l higher markup, which occurs in expectation in $1-S$ of the incumbent's product lines. Expected markup growth conditional on survival then equals $E\left[\ln\mu_f|a_f,\varphi^h\right]-E\left[\ln\mu_f|0,\varphi^h\right]$.

The expected markup conditional on firm age for a low-productivity type firm follows

$$E\left[\ln\mu_f|a_f,\varphi^l\right]=\underbrace{\ln\lambda\times\left(1+I\times E[a_P^l|a_f]\right)}_{\text{Quality improvements}}+\underbrace{S\times\ln\left(\frac{\varphi^l}{\varphi^h}\right)}_{\text{Productivity disadvantage}}. \tag{22}$$

The first term captures quality improvements through internal R&D, equivalently to eq. (21). $E[a_P^l|a_f]$ follows the same expression as $E[a_P^h|a_f]$ with h replaced by l. The second term in eq. (22) differs from eq. (21). Low-productivity incumbents face a high-productivity second-best firm in a share S of their product lines. Since $\varphi^l<\varphi^h$, this term is negative.

3.7.3 Firm employment growth

Average employment conditional on age and productivity type is equal to

$$E[\ln l_f|=a_f,\varphi_f]=\ln\left(\frac{Y}{w}\right)+E\left[\ln n_f|a_f,\varphi_f\right]-E\left[\ln\mu_f|a_f,\varphi_f\right].$$

Since $\frac{Y}{w}$ is constant, employment growth conditional on survival is given by

$$E[\ln l_f|a_f,\varphi_f]-E[\ln l_f|0,\varphi_f]=\underbrace{E\left[\ln n_f|a_f,\varphi_f\right]}_{\text{Firm's product growth}}-\underbrace{\left(E\left[\ln\mu_f|a_f,\varphi_f\right]-E\left[\ln\mu_f|0,\varphi_f\right]\right)}_{\text{Firm's markup growth}}, \tag{23}$$

where $E\left[\ln n_f|a_f,\varphi_f\right]$ and $E\left[\ln\mu_f|a_f,\varphi_f\right]-E\left[\ln\mu_f|0,\varphi_f\right]$ are defined in eqs. (20)-(22). Employment growth equals firm sales growth minus markup growth.

3.7.4 Firm survival

Firm size dynamics determine firm survival. Since firm size growth is type-dependent, so is firm survival. The survival function in Klette and Kortum (2004) holds conditional on the firm type, i.e., the share of high and low type firms surviving until age a_f is

$$\chi^h(a_f)=1-\tau\frac{1-e^{-(\tau-x^h)a_f}}{\tau-x^he^{-(\tau-x^h)a_f}}\quad\text{and}\quad\chi^l(a_f)=1-\tau\frac{1-e^{-(\tau-x^l)a_f}}{\tau-x^le^{-(\tau-x^l)a_f}}. \tag{24}$$

The share of high-type firms among surviving firms at any age a_f is

$$s^h(a_f)=\frac{p^h\chi^h(a_f)}{p^h\chi^h(a_f)+(1-p^h)\chi^l(a_f)}, \tag{25}$$

which corresponds to the mass of high-type survivors relative to the total mass of survivors. Since $x^h > x^l$, one can show that size growth conditional on survival of high-productivity firms exceeds that of low-productivity ones and that the share of the former among surviving firms at age a_f increases in a_f.

3.7.5 Firm size conditional on age

Having defined firm growth conditional on survival and survival probabilities by productivity type allows us to characterize the average firm size conditional on age (relative to the size at entry), as measured in the data in Section 2.2. The average firm size conditional on age relative to the average size at entry can be decomposed as

$$
\begin{aligned}
&E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=a_f\right]-E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=0\right]=\\
&\qquad s^h(a_f)\times\underbrace{\left(E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=a_f,\varphi_f=\varphi^h\right]-E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=0,\varphi_f=\varphi^h\right]\right)}_{\text{Size growth cond. on survival (high-productivity type)}}\\
&\qquad+\left(1-s^h(a_f)\right)\times\underbrace{\left(E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=a_f,\varphi_f=\varphi^\ell\right]-E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=0,\varphi_f=\varphi^\ell\right]\right)}_{\text{Size growth cond. on survival (low-productivity type)}}\\
&\qquad+\left(s^h(a_f)-s^h(0)\right)\times\underbrace{\left(E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=0,\varphi_f=\varphi^h\right]-E\left[\ln \text{Size}_{f,t}|\text{Age}_{f,t}=0,\varphi_f=\varphi^\ell\right]\right)}_{\text{Firm exit correction term}}
\end{aligned}
\tag{26}
$$

Eq. (26) is the model-equivalent of eq. (2). In the model, firm size conditional on age a_f relative to size at entry is equal to the sum of firm growth conditional on survival of high- and low-productivity firms over the first a_f years, weighted by their share among surviving firms and a firm exit correction term. The correction term captures the fact that the share of each productivity type among firms aged a_f is not equal to their share at age zero. Note that $s^h(0) = p^h$. Eq. (26) illustrates that multiple sources give rise to an increase in firm size conditional on age relative to entry. One explanation is that firm growth conditional on survival has increased for either productivity type. Since firm growth conditional on survival is higher for more productive than less productive firms, an alternative explanation is that the share of high-productivity firms among surviving firms at age a_f has increased. This highlights changes in firm growth and selection among surviving firms as potential explanations behind the observed trends.

3.7.6 Firm size distribution

The model makes precise predictions about the firm size distribution. I derive the firm size distribution in Section C.6 in the Appendix. Denoting by M^h the mass of high-productivity type firms and by M the total mass of firms, I compute the share of high-productivity type firms in the cross-section as

$$S_{M^h} = \frac{M^h}{M}, \tag{27}$$

and the firm entry rate as

$$\text{Firm entry rate} = \frac{z}{M}. \tag{28}$$

4 Explaining the changes in firm life cycle growth

This section applies the model to explain the documented changes in firm size conditional on age. To this extent, I estimate the model along two balanced growth paths. The initial balanced growth path captures firm size patterns and aggregate economic conditions during the 1990s. I then re-estimate model parameters to explain the changes in firm size conditional on age of the latest cohorts in the data.

4.1 Initial balanced growth path

There are, in total, eight parameters in the model. The internal R&D efficiency ψ_I, the expansion R&D efficiency ψ_x, the innovation cost curvature ζ, the entry efficiency ψ_z, the step size of innovation λ, the productivity differential φ^h/φ^ℓ, the share of high-productivity type firms among entrants p^h, and the discount rate ρ. Two parameters are set exogenously, and the remaining parameters are estimated. I follow Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) and Peters (2020) that set ζ equal to two based on evidence from the microeconometric innovation literature (Blundell, Griffith and Windmeijer, 2002; Hall and Ziedonis, 2001). The discount rate ρ is set to 0.02, resulting in an annual discount factor of roughly 0.97%.

The remaining six parameters are estimated, targeting moments of firm size as well as cross-sectional firm heterogeneity and economic aggregates. In particular, I target firm size (sales and employment) conditional on age, dispersion in inverse labor shares across entrants, the firm entry rate, TFP growth, and the aggregate markup. Despite all parameters being identified jointly, there is a tight mapping between parameters and targets.

Matching sales and employment conditional on age (relative to entry) disciplines the firms' R&D efficiencies ψ_x and ψ_I. In the model, successful expansion R&D translates into sales growth. In the estimation, ψ_x adjusts expansion R&D costs such that sales conditional on age in the model matches that in the data. The internal R&D costs govern firms' markup growth. Since markup growth drives a wedge between sales and employment growth, targeting employment and sales jointly disciplines markups (all relative to entry) and, hence, the internal R&D efficiency ψ_I. The advantage of targeting employment instead of markups is that employment is directly observed in the data. I target average sales and employment at age eight relative to entry of the cohorts 1997 to 2000. According to Figures 1 and 2, average sales increased by 0.559 and employment by 0.288 log points for these cohorts. These numbers are matched with the model equivalent expression in eq. (26). Eight years are long enough to reflect firm size growth and still allows for estimating separate balanced growth paths (one for the early cohorts and one for the latest cohorts) over the data coverage period from 1997 to 2017. The model matches firm size conditional on age well, so the specific age targeted is not consequential.

The entry rate helps identify the entry efficiency of firms ψ_z. I compute the entry rate in the data as the share of firms equal to or less than one year of age. This results in an average entry rate over the period 1997-2005 of 14.3%, in line with Engbom (2023). I match this number with the model-implied entry rate in eq. (28).

Aggregate TFP growth disciplines the step-size improvement of innovation λ: the growth rate of TFP in eq. (17) directly depends on λ. I obtain TFP growth for the Swedish economy from Federal Reserve Economic Data (FRED) in labor augmenting terms.[15] After suffering a financial crisis in the early 90s, Sweden's economy featured strong growth towards the end of the century. During 1997–2005, TFP grew by 3.02% per year.

To pin down the productivity differential φ^h/φ^ℓ, I target the aggregate markup. The aggregate markup is a weighted average of product markups that, in return, depend on φ^h/φ^ℓ. Sandström (2020) and De Loecker and Eeckhout (2018) report sales-weighted markups for the Swedish economy. Sandström (2020) computes the markup in Swedish registry data focusing on firms with at least ten employees, whereas De Loecker and Eeckhout (2018) focus mainly on publicly listed firms. I target the average of both reported aggregate markups, resulting in a conservative estimate of 7.5%. Lastly, I target the standard deviation of log inverse labor shares across entering firms (sales relative to the wage bill). Given φ^h/φ^ℓ, the dispersion of labor shares at entry depends on the share of product lines operated by high-type firms (determined in equilibrium) and the share of high-type firms among entrants (the parameter p^h). The dispersion of inverse labor shares across entrants, hence, disciplines p^h. The standard deviation of log inverse labor shares of entering firms, averaged over 1997-2005, equals 0.053.[16] All targets are summarized in Table 2.

The estimation follows a two-step approach. In the first (global) step, the algorithm computes the sum of squared percentage deviations from the targeted moments for a large Sobol sequence of parameter vectors. All targets receive equal weights. In the second (local) step, I take the best candidates from the first step and perform a local search. The local search, again, minimizes the distance from the targets. The best parameter vectors from the second step converge to the same parameter values.

[15] FRED series RTFPNASEA632NRUG. The labor share is obtained from FRED, series LABSHPSEA156NRUG, averaged over 1997–2005.

[16] For firms with a low wage bill, inverse labor shares explode. Therefore, I focus on firms with a sales-to-wage bill ratio between one and three (model implied markups between 0% and 200%). Further, sales relative to the wage bill in the data may vary for reasons outside the model. I bin firms into equally sized groups based on their capital and intermediate inputs and compute the dispersion of log inverse labor shares across firms within these groups.

Table 2: Initial balanced growth path. Moments and parameters

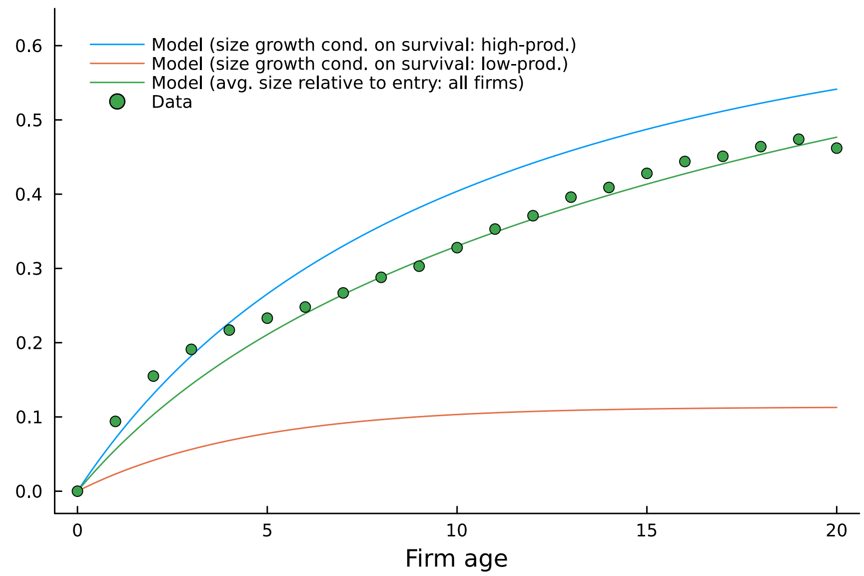
	Data	Model
Moments		
Avg. sales age 8 relative to entry in logs (cohorts 1997–2000)	0.559	0.558
Avg. employment age 8 relative to entry in logs (cohorts 1997–2000)	0.288	0.288
Cross-sectional SD of log labor shares across entrants (1997–2005)	0.053	0.053
TFP growth g in % (1997–2005; FRED)	3.02	3.02
Entry rate in % (1997–2005)	14.3	14.3
Agg. markup μ in % (Sandström, 2020; De Loecker and Eeckhout, 2018)	7.5	7.5
Parameters		
ψ_I *Internal R&D efficiency*		0.144
ψ_x *Expansion R&D efficiency*		0.282
ψ_z *Entry R&D efficiency*		1.483
λ *Step size of innovation*		1.136
φ^h/φ^ℓ *Productivity gap*		1.091
p^h *Share of high type among entrants*		0.683
Set exogenously		
ρ *Discount rate*		0.02
ζ *R&D cost curvature*		2

Notes: except for aggregate productivity (TFP) growth and μ, the moments are computed using Swedish registry data. TFP growth is obtained from Federal Reserve Economic Data (FRED), series RTFPNASEA632NRUG, in labor augmenting terms (the labor share is obtained from FRED, series LABSHPSEA156NRUG, averaged over the same period 1997–2005).

Table 2 shows the estimation results. The model replicates all targeted moments well. The estimated parameters can be interpreted as follows: successful innovation increases product quality by 13.6%. High and low-type firms' productivity differs by 9.1%, and 68.3% of firms enter the economy as high-type firms. The share of high-type firms at entry is relatively large, consistent with the relatively moderate productivity advantage. Hence, for the interpretation of the results, it is more suitable to think of high- and low-type firms as above- and below-median firms (at entry) rather than superstar vs. the rest of firms.

Along the balanced growth path, the constant share of high-productivity type firms in the cross-section, S_{M^h} in eq. (27), equals 74%. The number is larger than their share at entry ($p^h = 0.683$) due to high-type firms choosing higher expansion R&D rates than low-type firms: $x^h - x^\ell = 0.075$. This is reflected in their size growth. Conditional on survival, employment grows on average by 0.36 log points for high-type firms but only by 0.1 log points for low-type firms over the first eight years. Weighting by the share of each type among surviving firms at age eight and correcting for firm exit according to eq. (26), this results in an average employment at age eight relative to entry of 0.288 log points, as targeted in Table 2. Figure 5 shows the employment trajectories for each productivity type over the entire life cycle. The figure clearly illustrates the heterogeneity in employment growth profiles. Conditional on survival, the difference in employment growth over the first 20 years between both productivity types amounts to roughly 0.4 log points. Figure 5 includes the average employment conditional on age relative to entry (model and data). Despite being untargeted, except at age eight, the model provides a good fit of the entire age path.

Figure 5: Firm employment dynamics



Notes: the figure shows firm log employment dynamics in the model (initial balanced growth path) and the data (cohorts 1997–2000 in Swedish registry data). Size growth conditional on survival of high- and low-productivity firms and the average size relative to entry are characterized in eq. (26). The average size at age eight relative to entry has been targeted.

Sterk, Sedláček and Pugsley (2021) emphasize the importance of ex-ante heterogeneity in firm life cycle trajectories. In this model, heterogeneity in expected firm growth arises from heterogeneous expansion R&D rates (x^h and x^l) specific to the firm's productivity type. I provide suggestive evidence that firms with permanently higher productivity are associated with faster size growth conditional on survival in the data, see Section 7.3.

4.2 New balanced growth path

This section estimates the model on a new balanced growth path that replicates the changes in firm size patterns vis-a-vis the initial balanced growth path. To replicate the changes in firm sales and employment conditional on age (two moments), I re-estimate two parameters, particularly the internal R&D efficiency ψ_I and the entry efficiency ψ_z. These two parameters are promising candidates because one affects sales and employment growth jointly, whereas the other moves employment relative to sales growth, as explained shortly. I test alternative parameter changes as a robustness check.

Table 3 shows the targeted moments and estimated parameters. For the cohorts 2009 to 2012, average sales at age eight exceeded sales at entry by 0.674 log points (compared to 0.559 for the cohorts 1997 to 2000). Average employment at age eight exceeded employment

Table 3: New balanced growth path. Moments and parameters

	Data	Model
Moments		
Avg. sales age 8 relative to entry in logs (cohorts 2009–2012)	0.674	0.674
Avg. employment age 8 relative to entry in logs (cohorts 2009–2012)	0.466	0.466
Parameters		
ψ_I *Internal R&D efficiency* (Δ in %)		-51.0
ψ_z *Entry R&D efficiency* (Δ in %)		-22.0

Notes: the table reports targeted moments in the new balanced growth path in logs and changes in the estimated parameters vis-a-vis the initial balanced growth path in percent.

at entry by 0.466 log points for the cohorts 2009 to 2012 (compared to 0.228 for the cohorts 1997 to 2000). The model matches these changes by lowering the entry efficiency by 22% and the internal R&D efficiency by 51%, i.e., by raising the cost of firm entry and internal R&D. Section 7.2 provides suggestive evidence that an increase in the stock of fixed assets at the sector level, such as intellectual property products or structures, acts like an increase in entry costs. Likewise, goods-producing firms increasingly offer (less patentable) services, arguably increasing the cost of distancing competitors in the quality space (internal R&D).

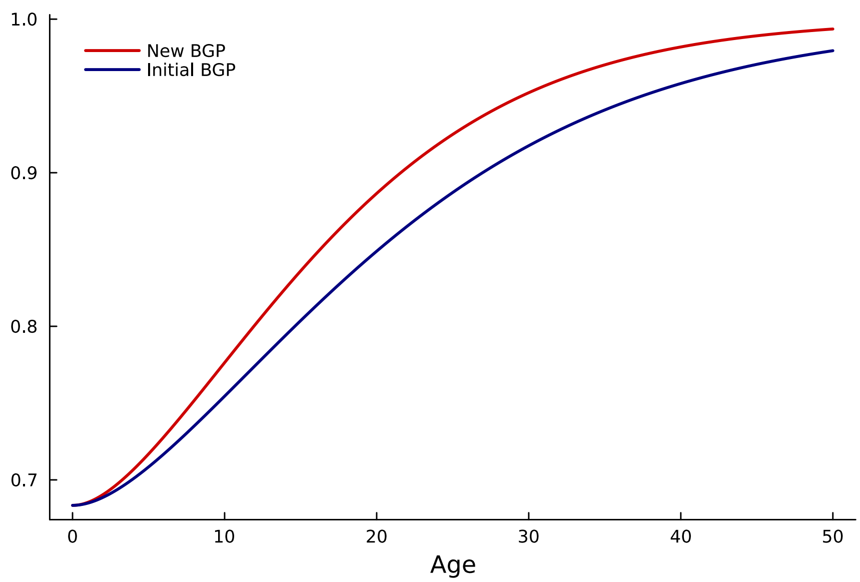
Table 4: Firm size growth conditional on survival over the first eight years

	Initial BGP	New BGP	$\psi_I \downarrow$ only	$\psi_z \downarrow$ only
Sales (high productivity)	0.625	0.792	0.612	0.796
Sales (low productivity)	0.370	0.317	0.339	0.362
Employment (high productivity)	0.357	0.585	0.383	0.547
Employment (low productivity)	0.096	0.106	0.107	0.104

Notes: the table shows firm size growth conditional on survival over the first eight years for sales and employment in logs by firm productivity type, as defined in eqs. (20)-(23) . $\psi_I \downarrow$ denotes the 51% fall in the internal R&D efficiency and $\psi_z \downarrow$ the 22% fall in the entry efficiency across balanced growth paths (BGPs).

The decomposition in eq. (26) illustrated how changes in within-firm growth and firm selection conditional on age affect firm size conditional on age. How does the rise in entry and internal R&D cost affect either channel? Table 4 quantifies within-firm growth in sales and employment for high- and low-productivity firms along the initial and new balanced growth path. Sales growth over the first eight years of high-productivity firms increases from 0.625 to 0.792 log points. Employment growth increases from 0.357 to 0.585 log points. Hence, within-firm growth of high-productivity firms accelerates. Within-firm growth of low-productivity firms decelerates for sales and remains roughly constant for employment. The last two columns of the table report within-firm growth for each parameter change in isolation. Comparing both columns shows that the increase in within-firm sales and employment growth of the high-productivity firms is mostly due to the rise in the entry costs. The rise in internal R&D costs slows firm markup growth and increases employment relative to sales growth, helping to explain the disproportionate increase in employment conditional on age. To see why the size growth of high-productivity firms increases relative to low-productivity firms, note that both cost changes increase the value of a product line of high-productivity firms relative to low-productivity ones. Falling firm entry due to increasing entry costs increases the difference in discounted profit streams for both firm types in eq. (15). Likewise, the continuation value of internal R&D in eq. (15) is higher for less productive firms that, so far, have accumulated fewer markups. The rise in the internal R&D costs disproportionately lowers the value of a product line for less productive firms. The expected value of a product line determines the expansion R&D rate according to the optimality condition in eq. (34). I provide suggestive evidence that firm life cycle growth of more productive firms accelerated in Section 7.3.

Figure 6: Share of high-productivity type firms among surviving firms



Notes: the figure shows the share of high-productivity type firms among firms of age a_f, $s^h(a_f)$ in eq. (25), for the initial and new balanced growth path (BGP).

How do the cost changes affect firm selection? Figure 6 shows the share of high-productivity type firms among firms of age a_f, defined by eq. (25), for the initial and new balanced growth path. The share of high-type firms increases relative to the initial balanced growth path for all ages. Since firm survival probabilities in eq. (24) depend on the productivity-type specific expansion R&D rate, the increase in within-firm growth of high-productivity firms increases their share among surviving firms at any age. Note that this share equals p^h at age zero and converges to one with firm age for any balanced growth path. Only high-type firms are present among older firms as their expansion R&D rates exceed the ones of low-type firms. Firm selection conditional on age translates into firm selection in the cross-section of firms. Integrating the share of high-type firms conditional on age in Figure 6 over the firm-age distribution yields the share of high-type firms in the cross-section, defined by S_{M^h} in eq.

(27). The cross-sectional share of high-type firms increases by 12pp across the balanced growth paths. Selection effects at the product level are even larger than at the firm level. The cross-sectional sales share of high-type firms, S, increases by 17pp. The sales share of high-type firms increases by more than their share in the cross-section of firms, as low-type firms with more than one product lose sales shares without exiting the economy.

In sum, the model explains the increase in firm size conditional on age as follows. First, the life cycle growth of productive firms accelerates. Second, holding firm growth constant, firms with faster life cycle growth represent a larger share among surviving firms at any age.

5 Long-run macroeconomic implications

What are the implications for the aggregate economy associated with the changes in within-firm growth and selection? This section quantifies the response of economic aggregates.

The rise in the cost of entry and internal R&D cause a long-run fall in the growth rate g and firm entry: the aggregate growth rate declines by 0.62pp, and the firm entry rate drops by 8pp. In Sweden, average TFP growth between 2010 and 2015 declined by about 1pp relative to 1997–2005. Further, Engbom (2023) documents a fall in the entry rate by about 10pp from the early 1990s to the mid-2010s in the Swedish economy. The comparative statics, therefore, account for roughly 60 percent of the fall in economic growth and 80 percent of the decline in firm entry since the 1990s. The reallocation of market shares to more productive incumbents further increases the average productivity, Φ in eq. (9), across balanced growth paths by 1.5%. The rise in average productivity and fall in the long-run growth rate g pose contrasting level and growth effects on aggregate output that leave the implications for welfare ambiguous. The next section examines the effect on welfare. Further, the reallocation of sales shares to more productive firms that, in the model, feature relatively low labor shares is qualitatively consistent with Kehrig and Vincent (2021). Similarly, De Loecker, Eeckhout and Unger (2020) and Baqaee and Farhi (2020) document a reallocation of sales shares to firms with relatively high markups in Compustat data.[17] In sum, the observed increase in firm size conditional on age is associated with falling entry, rising concentration, a slowdown in productivity growth, and a reallocation of market shares to low-labor share firms.

Incumbent innovation, reallocation, entry and growth

To shed light on the long-run fall in the aggregate growth rate, this section decomposes the fall into the contributions by incumbent firms and entrants. The aggregate growth rate g naturally lends itself to such decomposition. Along a balanced growth path, the aggregate

[17] In the estimated model, expected differences in markup growth are small compared to the difference in markup levels at birth between high- and low-productivity firms, so most high-productivity firms remain high-markup (low-labor share) firms throughout.

growth rate defined in eq. (17), can be written as

$$g = Sg^h + (1 - S)g^\ell + g^z,$$

where $g^h \equiv (I + x^h)\ln(\lambda)$, $g^\ell \equiv (I + x^\ell)\ln(\lambda)$ and $g^z \equiv z\ln(\lambda)$ capture the contributions by high-type incumbents, low-type incumbents, and entrants to economic growth. Note that for the total contribution by incumbents, their innovation rates and the share of product lines operated by each type matter. Using a shift-share decomposition, I decompose changes in the growth rate across balanced growth paths, $\Delta g \equiv g_{new} - g_{old}$, as follows

$$\Delta g = \underbrace{S_{old}\Delta g^h + (1 - S_{old})\Delta g^\ell}_{\Delta\text{Within}} + \underbrace{g^h_{old}\Delta S - g^\ell_{old}\Delta S}_{\Delta\text{Between}} + \underbrace{\Delta g^h\Delta S - \Delta g^\ell\Delta S}_{\Delta\text{Cross}} + \underbrace{\Delta g^z}_{\Delta\text{Entry}}, \tag{29}$$

where *old* and *new* index balanced growth path variables before and after the parameter change. Changes in the aggregate growth rate are due to changes in innovation rates holding the distribution of sales shares constant (ΔWithin), due to changes in the distribution of sales shares holding innovation rates constant (ΔBetween), due to changes in both innovation rates and sales shares (ΔCross) as well as due to changes in firm entry (Δg^z). That high- and low-productivity firms innovate (and grow) at systematically different rates allows firm selection to affect long-run growth through changes in their sales shares. The ΔWithin, ΔBetween, and ΔCross terms capture changes due to incumbents, whereas Δg^z captures changes due to entrants. Because the ΔCross term is absent without firm type heterogeneity, I group the ΔBetween and ΔCross-term into a common ΔReallocation term.

Table 5: Decomposing the fall in the aggregate growth rate

	Δg ($\psi_I \downarrow, \psi_z \downarrow$)	Δg ($\psi_I \downarrow$)	Δg ($\psi_z \downarrow$)
ΔWithin	+0.22	-0.23	+0.47
ΔReallocation	+0.27	+0.01	+0.20
ΔEntry	-1.10	-0.11	-0.93
Total	-0.62	-0.33	-0.26

Notes: the table shows the contributions to the change in the aggregate growth rate g across the balanced growth paths according to the decomposition in eq. (29) in percentage points. ΔReallocation is the sum of the ΔBetween and ΔCross terms. g in the initial balanced growth path is equal to 3.02%. $\psi_I \downarrow$ denotes the 51% fall in the internal R&D efficiency and $\psi_z \downarrow$ the 22% fall in the entry efficiency.

Table 5 quantifies the different contributions to the fall in the aggregate growth rate. First, the ΔWithin term is positive at 0.22pp, indicating that incumbents' innovation rates increased on average. This is reflected in within-firm growth. Growth of high-productivity firms increases by more than it decreases for low-productivity firms in Table 4. Second, the reallocation of sales shares to more productive firms that endogenously feature higher innovation rates contributed positively to economic growth. The ΔReallocation term is positive at 0.27pp. Changes in incumbent innovation (ΔWithin + ΔReallocation) raised the aggregate growth rate by a total of 0.49pp. ΔReallocation accounts for 55% (0.27/0.49) of the total contribution by incumbent firms. Thus, incumbents mainly contributed to changes in long-run growth through the reallocation of sales shares to more innovative firms. This channel is absent in standard models of creative destruction where firms innovate at identical rates. Lastly, falling firm entry lowers the aggregate growth rate substantially by 1.1pp. The fall in firm entry dominates the positive contribution by incumbents, resulting in a total decline of the growth rate of 0.62pp.

That the ΔWithin term is positive may be surprising given that R&D costs of incumbents have increased. Columns 3 and 4 of Table 5 repeat the decomposition for each parameter change in isolation. The ΔWithin effect of a rise in the internal R&D costs is negative (-0.23pp). At the same time, the rise in the entry costs generates a positive ΔWithin effect. Rising barriers to entry incentivize more productive firms to expand into new product markets. Overall, the positive ΔWithin effect following the rise in the entry costs outweighs the negative ΔWithin effect of the rising internal R&D costs. Note also that the positive ΔReallocation effect is mainly due to the rise in the entry costs.

The results of the decomposition complement the findings in Akcigit and Kerr (2018), Garcia-Macia, Hsieh and Klenow (2019), and Peters (2020). These studies show that economic growth is mainly due to incumbent firms.[18] The decomposition in this paper suggests that entrants play a more prominent role when explaining *changes* in economic growth. I show in Section 7.1.1 that a rise in productivity dispersion, recently entertained in Aghion, Bergeaud, Boppart, Klenow and Li (2023) as the cause behind rising concentration and falling growth, implies very similar ΔWithin, ΔReallocation and ΔEntry contributions.

6 Transition dynamics

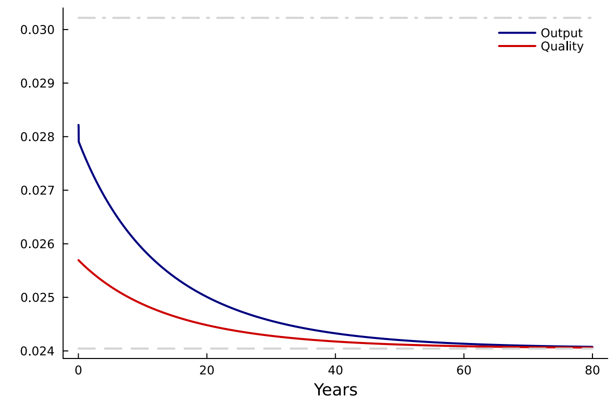
The previous section analyzed the long-run effects associated with the changes in within-firm growth and selection. The reallocation of sales shares to more productive incumbents introduces an interesting tradeoff between rising average productivity, Φ in eq. (9), and the long-run fall in the aggregate growth rate, which leaves the effect on welfare unclear. I solve the model numerically over the transition period to study the implications for welfare in this section. The solution algorithm, outlined in detail in the Appendix, Section D, works as follows. I solve for policy and value functions from the ending balanced growth path backward for a guessed sequence of wage growth, interest rates, and distribution of firm types over the product space (S_t). I then use the obtained policy functions over the transition period to simulate the two-dimensional distribution of quality and productivity gaps forward, starting from the initial balanced growth path. Using the evolution of this distribution over the transition, I back out the implied sequences of wage growth, interest rates, and S_t. The

[18] Decomposing growth levels shows that this is also the case in this model in both balanced growth paths.

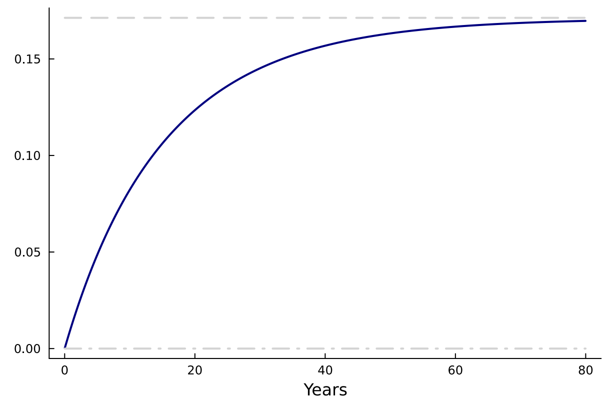
transition path is the fixed point between the guessed and implied sequences.

Figure 7: Transition dynamics

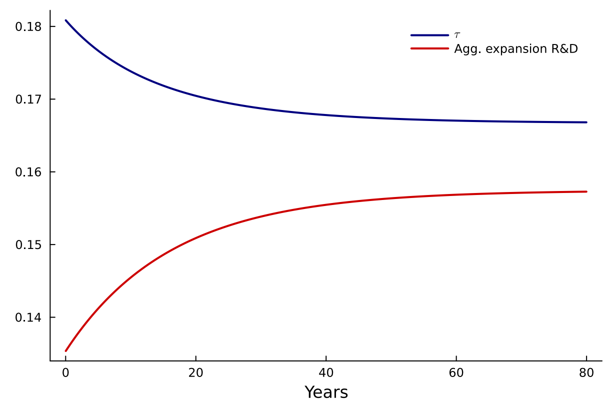
(a) Output and quality growth



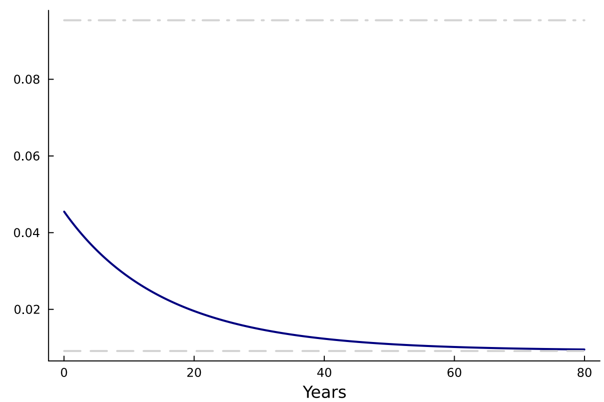
(b) Change in concentration, $S_t - S_{\text{initial}}$



(c) Rate of creative destruction, τ_t



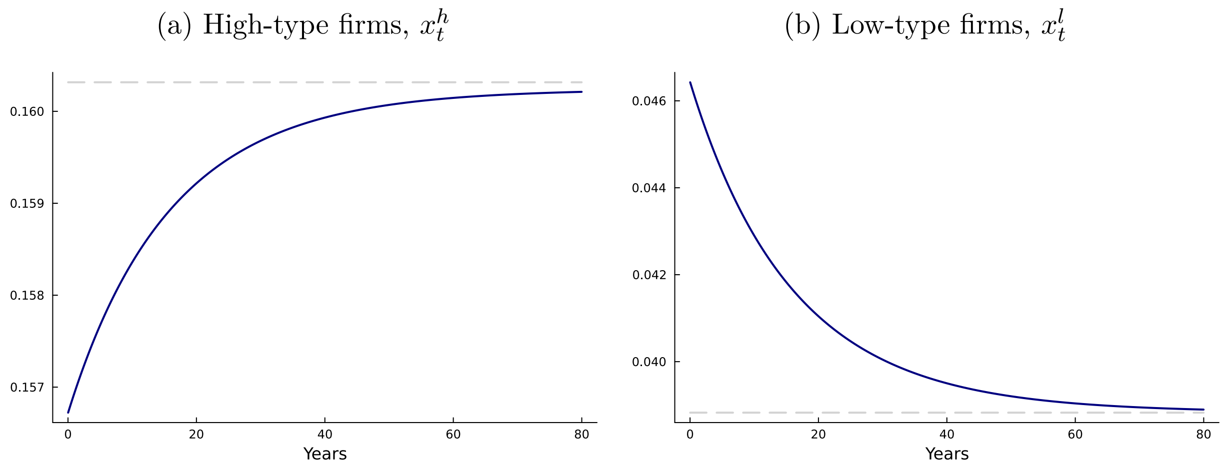
(d) Rate of entry, z_t



Notes: the figure shows the response in equilibrium outcomes following the increase in the cost of entry and internal R&D as in Table 3 in period zero. Output and quality growth (Panel a) refer to the growth rate of Y_t and Q_t in percent. The change in concentration refers to the change in the sales share of high-productivity type firms relative to the initial balanced growth path in percentage points. The gray dashed and dash-dotted lines indicate the ending and initial balanced growth paths, respectively. Aggregate expansion R&D in panel (c) is computed as $S_t \times x_t^h + (1 - S_t) \times x_t^l$.

Starting from the initial balanced growth path, I introduce the estimated rise in entry and internal R&D costs (Table 3) as shocks, after which no further parameter changes occur. Figure 7 shows the paths of output (Y_t) growth (in %), quality (Q_t) growth (in %), changes in the sales share of high productivity type firms S_t with respect to the initial balanced growth path (in pp), the rate of creative destruction (τ_t), and the rate of entry (z_t) over the transition period. Convergence is relatively quick. Most changes in equilibrium outcomes occur over the first 20 years of the transition. Both output and quality growth decline on impact and converge quickly after to their new long-run values, as shown in Panel (a). Along a balanced growth path, quality and output grow at the same rate. Over the transition, aggregate quality growth differs from output growth with growth in average productivity, markup dispersion, and production labor, explaining the residual according to eq. (9). Output growth declines by less than quality growth on impact as the rising sales share by high productivity firms, S_t, shown in Panel (b), contributes positively to growth in average productivity and hence aggregate output. Over the entire transition period, S_t increases by 17pp. The rise in average productivity does not suffice to counteract the fall in quality growth. Panel (a) shows that output growth follows the declining pattern of quality growth.[19]

Figure 8: Expansion R&D rates over the transition



Notes: the figure shows the evolution of the optimal expansion R&D rates by high- and low-type firms following the increase in the cost of entry and internal R&D as in Table 3 in period zero.

That quality growth steadily declines over the transition period is not self-evident as contrasting forces are at play. On the one hand, firm entry declines over the transition, as shown in Panel (d), which lowers quality growth. On the other hand, external and internal R&D efforts by incumbents are also subject to change over the transition. Figure 8 shows the evolution of expansion R&D rates by high- and low-type firms. Consistent with the rise in concentration, expansion rates of high-type firms increase while the ones of low-type firms decline over the transition. Aggregate expansion R&D rates (productivity-type specific R&D rates weighted by their respective sales shares) are, in fact, increasing over the transition as shown in Panel (c) of Figure 7. That falling entry outweighs the rise in aggregate expansion R&D becomes evident after looking at the path of the rate of creative destruction τ_t, also shown in Panel (c). The rate of creative destruction is the sum of the aggregate expansion R&D rate and the firm entry rate z_t. The rate of creative destruction is strictly falling over the transition, highlighting that falling firm entry dominates rising aggregate expansion R&D. Falling firm entry drives the decline in quality and output growth over the transition, dominating the positive reallocation effects on average productivity.[20]

As output growth gradually declines right from the shock period in Figure 7, the net effect on welfare is negative. To quantify the change in welfare, I compute the permanent consumption change (in percent) along the initial balanced growth path that makes the consumer

[19]Changes in misallocation, $\mathcal{M}_t$, have a negligible effect on output growth during the transition.

[20]Internal R&D also declines over the transition period (not shown). However, this effect is small.

as well off as with the obtained consumption stream over the transition towards the new balanced growth path. I find that welfare decreases by 23.3%. This number is sizable and should be interpreted with substantial caution. The initial balanced growth path matches macroeconomic conditions (and firm growth) during the late 1990s. Aggregate productivity growth averaged about 3% during this period in Sweden. Therefore, the transition path is compared to a scenario in which the high growth period of the late 1990s would have continued forever. Targeting a lower aggregate growth rate in the initial balanced growth path that reflects average growth before the 1990s boom, as in Aghion, Bergeaud, Boppart, Klenow and Li (2023) or De Ridder (2024), would result in a lower welfare loss. However, this would introduce an inconsistency in targeted moments: targeted firm growth reflects conditions during the late 1990s, while aggregate growth refers to an earlier period. Note also that the decline in output growth is monotone, i.e., there is no initial burst in output growth as declining firm entry outweighs rising expansion R&D and average productivity over the entire transition. Given that the initial balanced growth path reflects the high growth period of the late 1990s, it is consistent with the data that the transition does not feature a further burst in growth. This does, however, translate into a larger welfare loss.

If one were to compare welfare of two different balanced growth paths that grow at the rates of the estimated initial and ending balanced growth paths (without taking the transition nor any level effects into account) the consumption equivalent change (in percent) ξ is determined by $\ln(1+\xi) = (g^{\text{ending}} - g^{\text{initial}})/\rho$, where g^{ending} and g^{initial} refer to the growth rates of the initial and ending balanced growth paths. Given that the growth rate declines by roughly six percentage points across the balanced growth paths and ρ equals 0.02, the welfare loss amounts to 26.6% ($\xi = -0.266$). Comparing this number to the 23.3% welfare loss above shows again that the fall in output growth during the transition is mainly driven by declining quality growth and that the transition to the new balanced growth path is fast.

7 Robustness checks

7.1 Alternative explanations for the life cycle trends

The main estimation explains the changes in two moments, firm sales and employment conditional on age, through changes in two parameters: rising entry and internal R&D costs. This section discusses alternative explanations.

7.1.1 Rising productivity dispersion

Aghion, Bergeaud, Boppart, Klenow and Li (2023) explain the fall in economic growth and the rise in concentration in the U.S. economy through rising productivity dispersion of incumbents (as well as changes in the R&D efficiency). In line with their story, I estimate an alternative ending balanced growth path where the parameters subject to change are the productivity gap φ^h/φ^ℓ (instead of the entry efficiency) and the internal R&D efficiency ψ_I

(as in the previous estimation).

Table 6: Alternative new balanced growth path. Moments and parameters

	Data	Model
Moments		
Avg. sales age 8 relative to entry in logs (cohorts 2009–2012)	0.674	0.579
Avg. employment age 8 relative to entry in logs (cohorts 2009–2012)	0.466	0.362
Parameters		
ψ_I *Internal R&D efficiency* (Δ in %)		-54
φ^h/φ^ℓ *Productivity gap* (Δ in %)		+6

Notes: the table shows changes in moments (in percentage points) and parameters (in percent) with respect to the initial balanced growth path.

Table 6 shows the estimation results. The internal R&D efficiency falls by 54% (compared to 51% in the previous estimation), and the productivity gap increases by 6%.[21] The implied changes in firm sales and employment conditional on age are qualitatively in line with the data, yet fall short in explaining them quantitatively.[22] Nevertheless, the implied changes for the aggregate economy are consistent with recent macroeconomic trends: the long-run aggregate growth rate falls by 0.49pp, the firm entry rate declines by 3pp, and concentration, S, rises. Hence, the increase in the productivity gap and internal R&D costs give rise to a similar fall in the aggregate growth rate as the one targeted in Aghion, Bergeaud, Boppart, Klenow and Li (2023) (-0.42pp).

Table 7: Decomposing the fall in the aggregate growth rate revisited

	Δg ($\psi_I\downarrow, \varphi^h/\varphi^\ell\uparrow$)	Δg ($\psi_I\downarrow$)	Δg ($\varphi^h/\varphi^\ell\uparrow$)
ΔWithin	-0.13	-0.24	+0.11
ΔReallocation	+0.18	+0.01	+0.13
ΔEntry	-0.53	-0.12	-0.35
Total	-0.49	-0.35	-0.11

Notes: the table shows the contributions to the change in the aggregate growth rate g across the balanced growth paths according to the decomposition in eq. (29) in percentage points. ΔReallocation is the sum of the ΔBetween and ΔCross terms. g in the initial balanced growth path is equal to 3.02%. $\psi_I\downarrow$ denotes the 54% fall in the internal R&D efficiency and $\varphi^h/\varphi^\ell\uparrow$ the 6% rise in the productivity gap.

I decompose the implied fall in the aggregate growth rate according to eq. (29.) as before. First, changes in incumbent innovation rates, ΔWithin, lower the growth rate slightly

[21] For this estimation, I assume that entrants always replace incumbents after a successful innovation as the estimated productivity gap exceeds the step size of innovation λ. Estimating the parameters with the constraint $\varphi^h/\varphi^\ell < \lambda$ results in the constraint binding at $\varphi^h/\varphi^\ell = 1.136$, which is the value of λ.

[22] For a large enough productivity disadvantage, low-type firms stop expanding into new product markets and remain one-product firms, which reduces the degrees of freedom of the model to match the trends.

(-0.13pp), whereas the reallocation of sales shares, ΔReallocation, towards the more productive firms with higher innovation rates generates a positive growth effect (+0.18pp), shown in Table 7. ΔReallocation outweighs ΔWithin, as in the previous comparative statics estimation. Second, the fall in firm entry more than explains the fall in the aggregate growth rate: -0.53pp compared to -0.49pp. Therefore, the two findings that incumbent firms have mainly contributed to changes in long-run growth through reallocation effects and that the decline in the aggregate growth rate is driven by a fall in firm entry even hold for an alternative estimation, in which the entry costs remain unchanged. Comparing the last column of Table 5 and Table 7 shows that the rising productivity gap works similarly as rising entry costs on growth: both generate positive ΔWithin and ΔReallocation effects that are dominated by a negative ΔEntry effect. As for the rise in entry costs, an increase in the productivity gap widens the gap in expected profits per product line across incumbents, incentivizing the more productive firms to expand faster. The ΔWithin, ΔReallocation, and ΔEntry contributions resulting from the rise in the internal R&D costs are quantitatively almost identical to the previous estimation.

In Aghion, Bergeaud, Boppart, Klenow and Li (2023), all firms innovate at the same rate, and there is no firm entry such that changes in within-firm innovation rates, ΔWithin, fully explain the decline in the aggregate growth rate. Table 7 suggests that reallocation effects and firm entry matter for changes in long-run growth. The ΔReallocation effect outweighs the ΔWithin effect, and ΔEntry dominates both.

Would the role of entry change when relaxing the assumption of a unitary demand elasticity? With a demand elasticity greater than one, firms also gain market shares through successful internal R&D. This suggests that, ceteris paribus, an even larger rise in firm entry costs would be required to offset the negative size-growth effect from rising internal R&D costs when matching the increase in firm life cycle growth.

7.1.2 Firm type selection on entry

Rising entry costs incentivize more productive firms to expand, driving less productive ones out of the economy. Rising entry costs, hence, induce selection effects among incumbents. However, the distribution of productivity types among entrants is unaffected, as this is governed exogeneously by the model parameter p^h. Potentially, the selection of entrants has changed over time.

Eq. (6) characterizes the employment of entrants. Employment at entry is a function of the markup. Hence, systematic changes in the productivity types of entrants should be reflected in average employment. Figure 17 in the Appendix displays the average employment of entrants by sector over time in the U.S. Census data. The size of entrants shows little variation over time, indicating no systematic changes in entrants' productivity.

7.1.3 Other explanations

Two of the six parameters estimated along the initial balanced growth path have not been discussed thus far. The step size improvement of innovations λ and the expansion R&D efficiency ψ_x. A fall in λ could be interpreted as falling research productivity or innovations becoming more incremental (Bloom, Jones, Van Reenen and Webb, 2020; Olmstead-Rumsey, 2019). As λ falls, markup levels and growth decrease, reducing incumbents' incentives to enter new product markets. Hence, a fall in λ *reduces* within-firm sales growth. Lastly, to match the increase in firm size conditional on age relative to entry, the model asks for an increase in the expansion R&D efficiency ψ_x. Increasing R&D efficiency contrasts the fall in research productivity documented by Bloom, Jones, Van Reenen and Webb (2020).

7.2 External evidence at the sector level

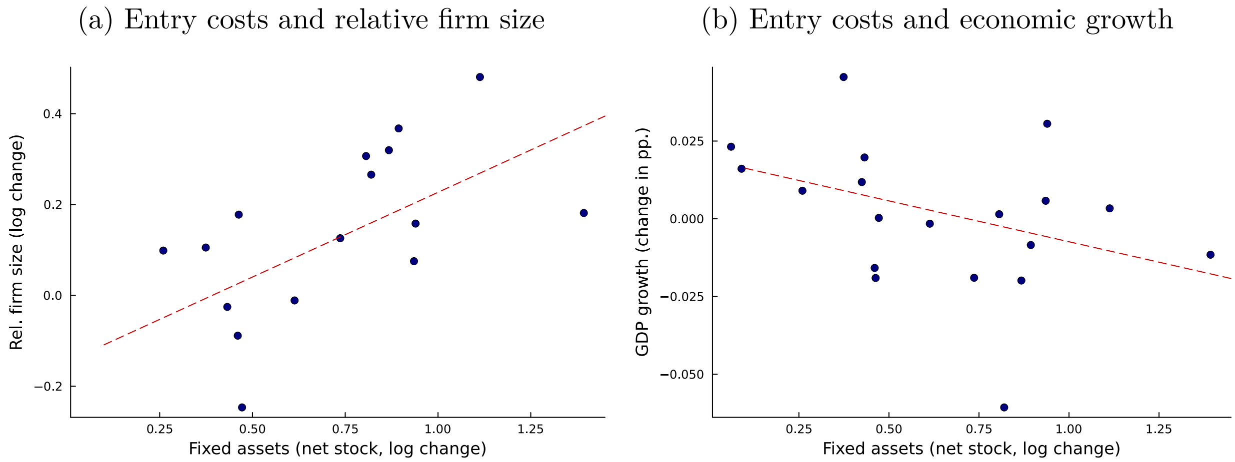
This paper argues that rising entry costs cause the increase in firm sales and employment, conditional on age. Rising internal R&D costs further explain the disproportionate increase in employment. I provide suggestive evidence of the mechanism at the sector level in this section. For the sector-level analysis, I use publicly available U.S. data.

What drives the increase in entry costs? Firms strategically invest in fixed assets like intellectual property products or structures to shield themselves from entrants. As one example, (De Ridder, 2024) links rising market power to the introduction of intangible assets. I provide evidence that changes in the entry costs, proxied by changes in the U.S. Bureau of Economic Analysis quantity index for the net stock of private fixed assets at the sector level, align with the model's predictions.[23] Panel (a) in Figure 9 suggests that in U.S. sectors where entry costs rose the most from 1992 to 2017, the employment gap between firms aged 11-15 and entrants increased the fastest. A linear fit shows that a 0.1 log point increase in the sector-level stock of fixed assets is associated with a 0.035 log point increase in the employment gap between firms aged 11-15 and entrants (correlation is 0.58). The sectors mining, utilities, and management of companies are excluded in Panel (a) due to the small number of firms. Panel (b) in Figure 9 provides further evidence of the aggregate predictions of the rise in entry costs. The growth rate of real value added declined strongest in U.S. sectors that experienced the greatest increase in fixed assets. Despite the increase in firm size conditional on age, as shown in Panel (a), economic growth declined in these sectors. In the model, falling firm entry outweighs the positive contribution by incumbents in response to rising entry costs, slowing economic growth. Real value added by sector is obtained from the U.S. Bureau of Economic Analysis and Panel (b) shows the difference in the annual growth rate between 2012-2017 and 1997-2002 in percentage points (series starts in 1997).[24]

[23]Source: U.S. Bureau of Economic Analysis, "Table 3.2ESI. Chain-Type Quantity Indexes for Net Stock of Private Fixed Assets by Industry" (accessed Sunday, August 11, 2024). Fixed assets include intellectual property products, structures, and equipment.

[24]Source: U.S. Bureau of Economic Analysis, "Real Value Added by Industry" (accessed Wednesday, August 14, 2024).

Figure 9: Changes in entry costs across sectors



Notes: the x-axis shows the log change (1992–2017) in the net stock of private fixed assets at the sector level obtained from the U.S. Bureau of Economic Analysis, the proxy of firm entry costs. Panel (i): the y-axis contains the log change (1992–2017) in the employment of firms aged 11-15 relative to entrants by sector obtained from the U.S. Census Bureau's Business Dynamics Statistics. Three sectors are excluded due to the small number of firms (mining, utilities, management of companies). Panel (b): the y-axis shows the difference in the annual growth rate of real value added between 2012–2017 and 1997–2002 in percentage points (pp.) for all sectors obtained from the U.S. Bureau of Economic Analysis.

Davis (2017) and Gutiérrez and Philippon (2018) argue that the increasing complexity of regulatory requirements and tax systems, as well as rising lobbying expenditures disproportionately hurt entrants, providing an additional argument of rising entry costs.

One force that potentially contributed to increasing internal R&D costs is related to the rising importance of the service sector. The composition of industries in which firms operate has changed over time. Consider, for example, the car manufacturer Volvo. Volvo recently offers the following services: car maintenance, insurance, leasing, and car sharing. Similarly, the clothing manufacturer H&M now provides repair and recycling services or even clothing rentals. Arguably, services are generally more difficult to patent than manufactured products, i.e., it is harder to distance competitors in the quality space for services than for goods. To the extent that manufacturing firms offer more and more of such services (or service firms that manufacture a product reduce their manufacturing activities), this implies that the average internal R&D efficiency of a firm (the internal R&D efficiency in a product or service line averaged over the firm's products and services) has declined. The aggregate-level evidence of a rise in the share of the workforce employed in the service sector is in line with the above examples.[25] A rising share of services in a firm's portfolio could also explain why, despite the convincing evidence in Akcigit and Ates (2023) of incumbents using patents more strategically (for the set of patentable products), for the firm's average line, it has become harder to prevent competitors from catching up.

Bloom, Jones, Van Reenen and Webb (2020) document that research productivity has de-

[25] 1997 to 2012: 75 to 81% (US), 72 to 79% (Sweden). FRED data (USAPESANA and SWEPESANA).

clined across U.S. sectors. The notion of ideas getting harder to find is consistent with rising internal R&D costs. Their main example of rising research labor required by incumbent firms to keep growth in the number of transistors on microchips constant can be interpreted as evidence of rising internal R&D costs.[26]

7.3 External evidence at the firm level

This section provides suggestive evidence on the mechanism at the firm level, documenting that more productive firms grow faster in size and that this relationship has strengthened over time. For the firm-level analysis, I turn back to the Swedish administrative data. A strength of the data is that it contains information on the capital stock and intermediate input usage for the universe of firms when measuring firm productivity.

Firm productivity is generally unobserved in the data. I use a model-based approach to infer the firms' productivity. As firms enter the model economy with one product, eq. (5) captures firm markups upon entry. Eq. (4) implies that their productivity advantage allows more productive firms to charge higher markups in equilibrium. Guided by the theory, I proxy firm productivity by its markup (sales relative to wage bill) at age zero, and regress observed firm life cycle growth on the productivity proxy including cohort and industry controls

$$\Delta \ln \text{Size}_{\text{Age}_{f,t}=8} = \beta_0 + \beta_1 \log\left(\frac{py}{wl}\right)_{\text{Age}_{f,t}=0} + \beta_2 \mathbb{1}_{c>2003} \log\left(\frac{py}{wl}\right)_{\text{Age}_{f,t}=0} + \theta_c + \theta_k + \epsilon_{f,t}.$$

$\Delta \ln \text{Size}_{\text{Age}_{f,t}=8}$ denotes size growth of firms surviving up to age eight and py/wl sales relative to the wage bill. To avoid a spurious relationship between sales or the wage bill at age zero on the right-hand side and size growth on the left-hand side, I measure firm size growth from age one to eight and use employment as the measure of firm size. Since the regression restricts to firms surviving until age eight, β_1 captures the effect of firm productivity on within-firm growth. $\mathbb{1}_{c>2003}$ is a dummy for cohorts established after 2003 that captures how the relationship between firm productivity and size growth changed over time.[27]

Table 8 reports the results for different specifications. Column one shows the baseline regression, column two focuses on firms with sales larger than the wage bill at entry, and columns three and four further control for the firm's capital and intermediate inputs. Across all specifications, β_1 and β_2 are positive, i.e., firms with higher inverse labor income shares at entry, perhaps due to higher productivity, feature faster life cycle growth. This relationship has strengthened over time. More productive firms have grown faster in recent cohorts. In

[26] While rising expansion R&D costs are also consistent with declining research productivity in theory, the data speaks against rising expansion R&D costs and in favor of increasing internal R&D costs: rising expansion R&D costs slow down both firm sales and employment growth. In contrast, rising internal R&D costs increase firm employment relative to sales conditional on age, as I document.

[27] Since the data spans 1997 to 2017, I observe the cohorts from 1997 to 2009 at age eight.

Table 8: Firm productivity and size growth

	$\Delta \ln \text{Size}_{\text{Age}=8}$	$\Delta \ln \text{Size}_{\text{Age}=8}$	$\Delta \ln \text{Size}_{\text{Age}=8}$	$\Delta \ln \text{Size}_{\text{Age}=8}$
$\log\left(\frac{py}{wl}\right)_{\text{Age}=0}$	0.066	0.095	0.104	0.113
	(0.006)	(0.006)	(0.006)	(0.006)
$\mathbb{1}_{c>2003} \log\left(\frac{py}{wl}\right)_{\text{Age}=0}$	0.011	0.015	0.017	0.017
	(0.008)	(0.008)	(0.008)	(0.008)
$\log K_{\text{Age}=0}$			-0.031	-0.009
			(0.002)	(0.003)
$\log M_{\text{Age}=0}$				-0.053
				(0.003)
Cohort fixed effects	✓	✓	✓	✓
Industry fixed effects	✓	✓	✓	✓
$\log\left(\frac{py}{wl}\right)_{\text{Age}=0} > 0$		✓	✓	✓
N	63,521	62,692	58,304	58,192
R^2	0.04	0.05	0.05	0.05

Notes: the regression coefficients are obtained in Swedish administrative data restricted to firms surviving up to age eight. Firm size growth is measured from age one to eight, $\Delta \ln \text{Size}_{\text{Age}=8} \equiv \ln \text{Size}_{\text{Age}_{j,t}=8} - \ln \text{Size}_{\text{Age}_{j,t}=1}$, using firm employment. $\log\left(py/wl\right)_{\text{Age}=0}$ denotes the log inverse labor share at age zero, the proxy of firm productivity, as explained in the main text. $\log K$ and $\log M$ denote the firm's capital and intermediate inputs, respectively. Robust standard errors are in parentheses.

the preferred specification that controls for the firm's capital and intermediate inputs, for firms established in 2003 or before, a 1% higher inverse labor shares at entry is associated with approximately 0.113pp faster employment growth up to age eight. For the cohorts of 2004 and after, this number increases to 0.13pp ($\beta_1 + \beta_2$). The coefficients are significant at common significance levels, and the acceleration of life cycle growth of productive firms for the more recent cohorts is robust across all specifications.[28]

8 Conclusion

The average firm size conditional on firm age has steadily increased over time. This suggests systematic changes in firm growth or the selection of surviving firms. I build a structural model to study changes in firm growth, selection, and aggregate growth. The model identifies rising entry costs as the cause behind the increase in the average firm size conditional on age. In response to rising entry costs, more productive firms grow faster conditional on survival, and represent a larger share among surviving firms at any age. Both increase the average firm size conditional on age.

Changes in the selection of incumbent firms have implications for long-run aggregate growth. As more productive firms innovate and grow at higher rates, their rising market shares in the cross-section increase the long-run aggregate growth rate. These effects matter quantitatively: incumbents have mainly contributed to changes in the long-run aggregate growth rate

[28] I obtain similar results when using $TFPR$ at age zero instead of labor productivity as the revenue productivity measure, where $TFPR \equiv \frac{py}{K^{\alpha}(wl)^{1-\alpha}}$ with α estimated at the industry level using cost shares.

since the 1990s through these reallocation effects, highlighting the importance of changes in industry concentration for long-run growth. Policymakers should trade off the dynamic effects of reallocation with the usual static efficiency losses when evaluating antitrust policies. However, falling firm entry, caused by the rise in entry costs, outweighs the positive contribution by incumbents, slowing long-run productivity growth. This suggests a promising role for policies that support firm startups to reverse the decline in productivity growth.

How does the reallocation of market shares to more productive incumbents compare to other, more severe, episodes of reallocation? Over the last decades, many Western economies privatized their education, health care, transportation, or communication sectors. It would be interesting to decompose changes in long-run growth following these events into changes in innovation rates, reallocation, and firm entry, as in this paper. To disentangle how reallocation ultimately affects short and long-run economic growth following privatization, one could further compare the effect of reallocation on innovation to the effects of reallocation on average productivity and misallocation. The quantitative framework in this paper, disciplined by changes in firm dynamics, could separate these forces.

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Appendices

A Data

The main data set, *Företagens Ekonomi* (FEK), covers information from balance sheets and profit and loss statements for the universe of Swedish firms. From this data, I obtain the main variables of interest, namely sales (*Nettoomsättning*, variable name: *Nettoomsattning*) and employment (*Antal anställda*, variable name: *MedelantalAnstallda*). In the FEK codebook by Statistics Sweden, these variables are defined as follows.[29] Sales refer to income from the companies' main business for goods sold and provided services. Employment refers to the average number of employees in full-time units in accordance with the company's annual report. As described in the main text, I focus on firms in the private sector. These firms have a legal type (variable name: *JurForm*) less than 50 or equal to 96.

The 5-digit industry classification (SNI codes) changed twice between 1997 and 2017, once in 2002 and once in 2007. I ensure a consistent industry classification using the following steps. During the year of the change, I observe both the old and the new industry classifications. For the firms present in the data in the year of the classification change, extending the new industry classification further back in time before the change is straightforward. This way, the industry codes of almost all firms are updated. A firm might be in the data before and after the classification change but not for the year of the change. For these firms, the above method does not work. If the firm appears in the data one year after the classification change, I use the observed classification after the change to update the classification before the change. For firms that are absent for several years around the year of change, I use industry mappings provided by Statistics Sweden. These mappings do not always provide a 1:1 mapping between industries before and after the classification change, so I use the most common transitions for the m:m mappings.

One concern is that changes in the firm structure, e.g., when firms merge with other firms, change the firm ID. To address this concern, I impute changes in firm IDs using worker flows between firms. The auxiliary data set *Registerbaserad Arbetsmarknadsstatistik* (RAMS) contains the universe of employer-employee matches. I impute changes in the firm ID of firms with at least five employees as follows: if more than 50% of the workforce of firm A in year t makes up for more than 50% of the workforce of firm B in year $t+1$, I substitute firm B's firm ID by firm A's firm ID following $t+1$. The empirical results remain virtually unchanged

[29] `https://www.scb.se/contentassets/9dd20ce462644cc19f6f04eb2edbbe28/nv0109_kd_2017_bv_190508_v2.pdf`, accessed 07.02.2024.

when excluding firms for whom the imputed firm ID differs from the observed firm ID.

B Trends in firm size

B.1 Firm-size trends in Sweden

This section provides robustness checks to the documented increase in average firm sales and employment conditional on age relative to entrants. I document robustness with respect to alternative fixed effects specifications, firm selection due to the Great Recession and the classification of entrants.

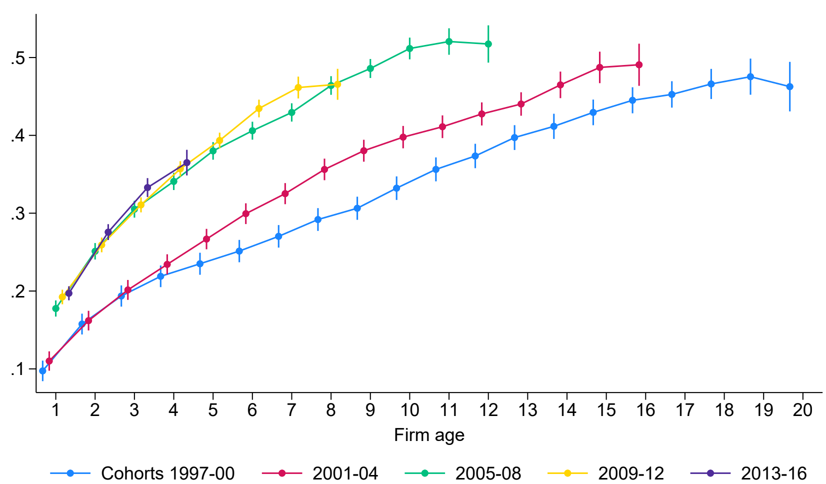
B.1.1 Employment

The baseline regression in (1) controls for cohort and 5-digit industry fixed effects. The results of the regression are virtually unchanged with *interacted* cohort and industry fixed effects as in

$$\ln \text{Employment}_{j,t} = \gamma_0 + \sum_{a_f=1}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{j,t}=a_f} + \theta_{c,k} + \epsilon_{j,t},$$

where, as before, c denotes cohorts and k industries. The estimated coefficients $\gamma_1 - \gamma_{20}$ are shown in Figure 10.

Figure 10: Cohort × industry fixed effects



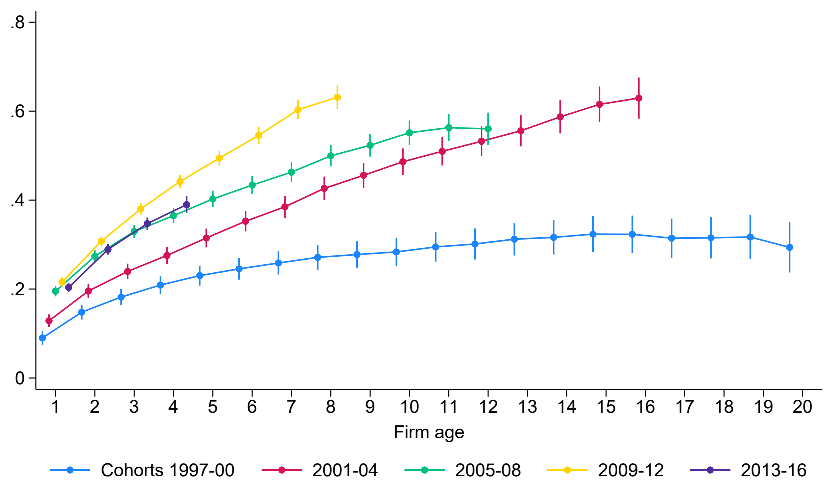
Notes: the figure shows average log employment for any firm age relative to average log employment at entry in Swedish registry data (unbalanced panel). Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

Alternatively, one could control for shocks that equally affect firms in a given year, independent of their age. The following regression includes year × industry fixed effects as in

$$\ln \text{Employment}_{j,t} = \gamma_0 + \sum_{a_f=1}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{j,t}=a_f} + \theta_{t,k} + \epsilon_{j,t}.$$

Figure 11 displays the age coefficients. If anything, the increase in employment conditional on age is even stronger.

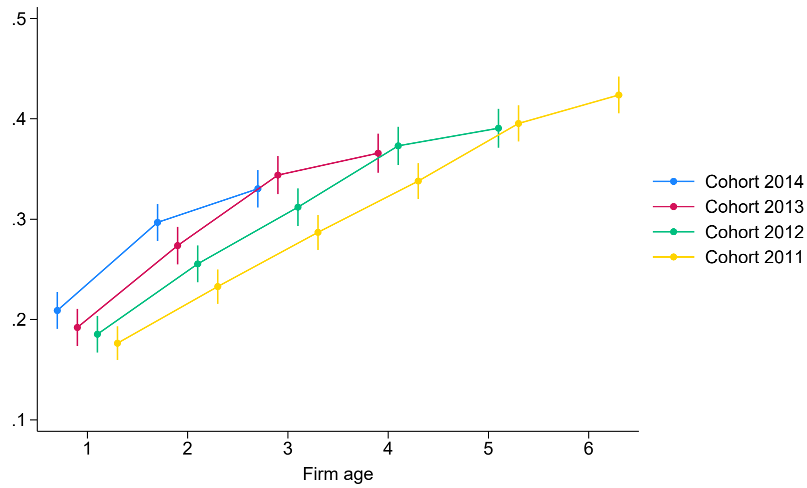
Figure 11: Year × industry fixed effects



Notes: the figure shows average log employment for any firm age relative to average log employment at entry in Swedish registry data (unbalanced panel). Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

Potentially, the Great Recession induced less productive firms to exit, driving up average firm size conditional on age. I provide evidence that selection effects due to the Great Recession are not behind the increase in firm employment conditional on age.

Figure 12: Post Great Recession



Notes: the figure shows average log employment for any firm age relative to average log employment at entry in Swedish registry data (unbalanced panel). Cohorts are indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

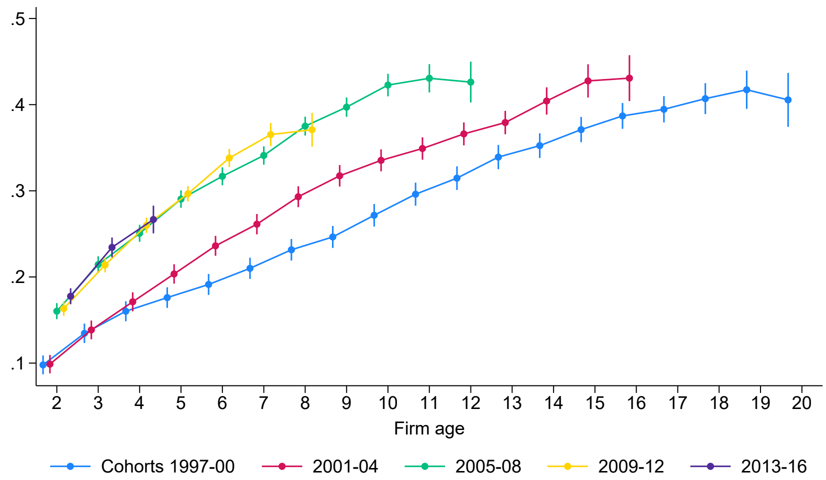
Figure 12 shows employment conditional on age relative to entry (regression (1) with industry fixed effects) for each cohort following the Great Recession. As shown in the figure, every cohort displays higher firm employment conditional on age than the cohort just before. That these patterns hold as clearly for the cohorts after the Great Recession suggests that the main results in Figure 1 are not driven by selection effects among incumbent firms due to the Great Recession.

Lastly, I show that the classification of an entering firm does not affect the documented patterns. In the following, I label firms of age zero and one as entrants and measure employment conditional on age relative to average employment of firms below age two. In particular, I measure relative firm size as follows

$$\ln \mathrm{Size}_{j,t} = \gamma_0 + \sum_{a_f=2}^{20} \gamma_{a_f} \mathbb{1}_{\mathrm{Age}_{j,t}=a_f} + \theta_c + \theta_k + \epsilon_{j,t},$$

where in comparison with regression (1), the firm age one dummy has been dropped. Average firm size of firms below age two is now captured by γ_0. The age coefficients are plotted in Figure 13. Employment conditional on age relative to entry looks comparable to Figure 1 in the main text. If anything, the jump at early firm ages is more muted.

Figure 13: Alternative entrant classification

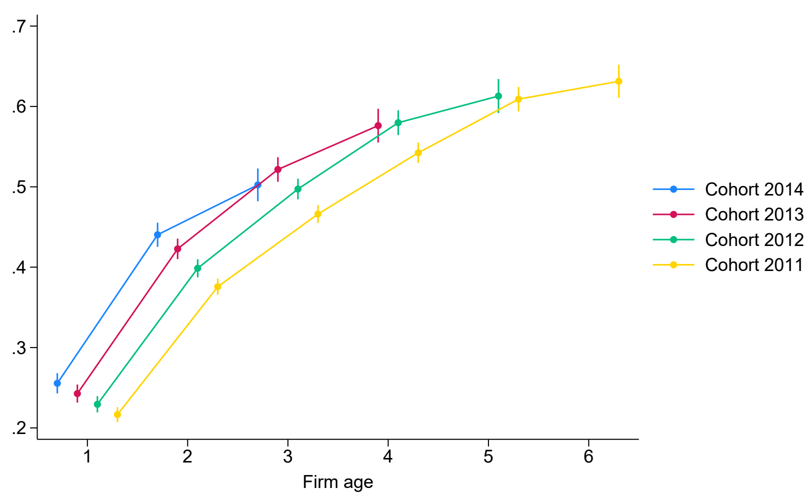


Notes: the figure shows average log employment for any firm age relative to average log employment at entry in Swedish registry data (unbalanced panel). Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

B.1.2 Sales

I repeat the above robustness exercises for sales. Figure 14 shows sales conditional on age relative to entry for cohorts following the Great Recession. The increase is apparent for each cohort, suggesting that structural forces other than the Great Recession drive it.

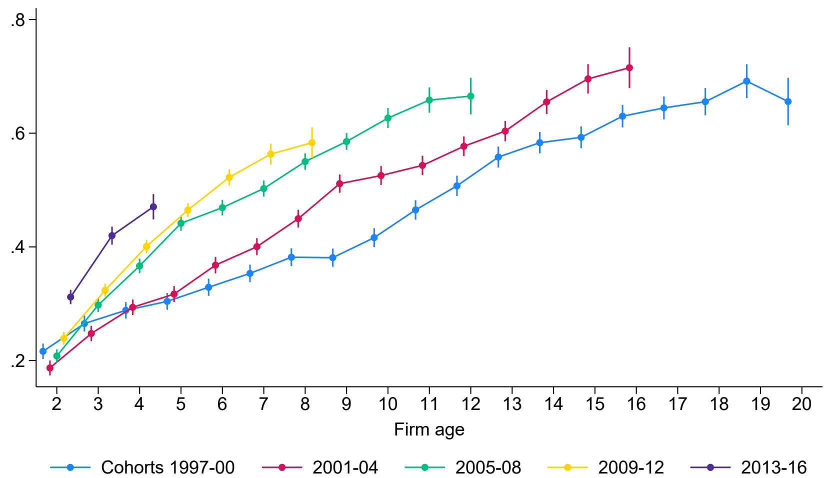
Figure 14: Post Great Recession



Notes: the figure shows average log sales for any firm age relative to average log sales at entry in Swedish registry data (unbalanced panel). Cohorts are indicated in the legend. Firm sales are filtered at their 1% tails. The figure includes 95% confidence intervals.

The cohorts 1997-2000 display a particularly steep increase in sales for early ages in Figure (2) in the main text. I show that this increase looks more muted when labelling firms less than age two as entrants, exactly as in the robustness check for employment.

Figure 15: Alternative entrant classification



Notes: the figure shows average log sales for any firm age relative to average log sales at entry in Swedish registry data (unbalanced panel). Cohorts are indicated in the legend. Cohorts are indicated in the legend. Firm sales are filtered at their 1% tails. The figure includes 95% confidence intervals.

Figure 15 shows the age coefficients for the alternative entrant classification. The steep rise in average sales during the early ages of the cohorts 1997-2000 disappears and the overall increase of firm size conditional on age of the later cohorts becomes more apparent.

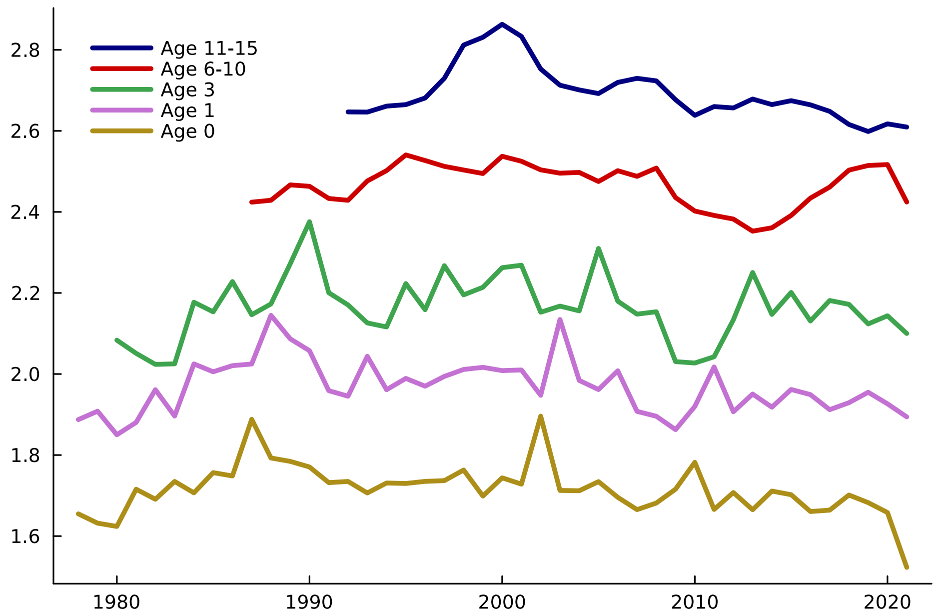
B.2 Firm-size trends in the U.S.

This section documents additional trends in firm size in the U.S. using the Business Dynamics Statistics (BDS) produced by the U.S. Census Bureau.

B.2.1 Replication of previous studies

Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022) establish that average employment conditional on firm age has been relatively stable over time. Figure 16 replicates their findings, showing no systematic trends in log employment conditional on firm age over time. Firms are pooled across all sectors when computing averages, as done in both studies.

Figure 16: Log employment by firm age, firms pooled across sectors



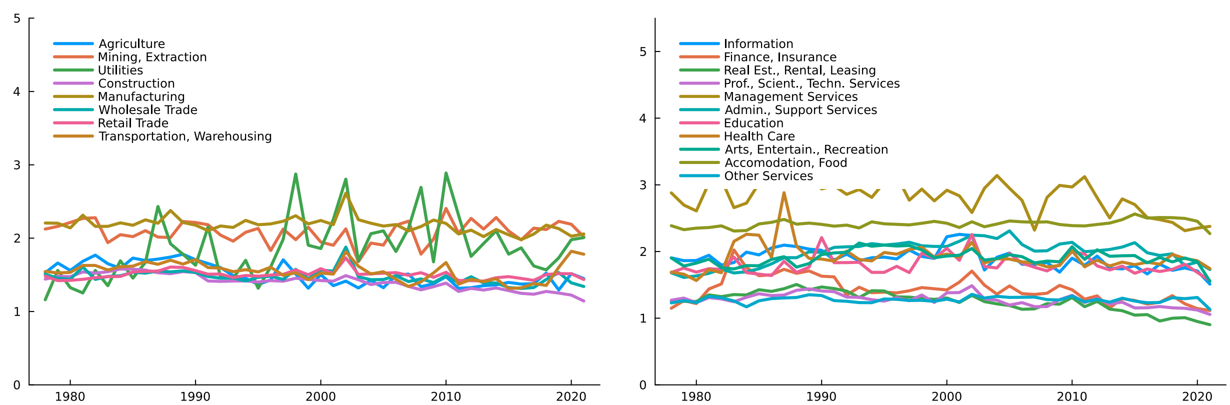
Notes: the figure shows average log employment conditional on firm age in U.S. Census data. Firms are pooled across all sectors.

B.2.2 Firm size at entry

The main empirical finding of the paper is that the firm size of incumbent firms has increased relative to the size of entrants. The size of entrants has remained relatively stable over time as shown in Figure 17 for each sector separately.

Figure 17: Log employment of entrants, by sector

(a) Goods (b) Services



Notes: the figure shows average log employment of entrants in U.S. Census data by sector. Sector classifications correspond to two-digit NAICS codes.

C Model

C.1 Solving the dynamic firm problem

The HJB for a high productivity-type firm h reads[30]

$$
\begin{aligned}
&r_t V_t^h(n,[\mu_i],S_t) - \dot{V}_t^h(n,[\mu_i],S_t) = \\
&\quad \sum_{k=1}^{n} \pi(\mu_k) + \sum_{k=1}^{n} \tau_t \Big[V_t^h\left(n-1,[\mu_i]_{i\neq k},S_t\right) - V_t^h(n,[\mu_i],S_t) \Big] \\
&\quad + \max_{[x_k,I_k]} \Bigg\{ \sum_{k=1}^{n} I_k \Big[V_t^h\left(n,\left[[\mu_i]_{i\neq k},\mu_k\times\lambda\right],S_t\right) - V_t^h(n,[\mu_i],S_t) \Big] \\
&\quad + \sum_{k=1}^{n} x_k \Big[S_t V_t^h\left(n+1,\left[[\mu_i],\lambda\right],S_t\right) + (1-S_t) V_t^h\left(n+1,\left[[\mu_i],\lambda\times\varphi^h/\varphi^l\right],S_t\right) - V_t^h(n,[\mu_i],S_t) \Big] \\
&\quad - w_t \left[\mu_k^{-1} \frac{1}{\psi_I}(I_k)^{\zeta} + \frac{1}{\psi_x}(x_k)^{\zeta} \right] \Bigg\}
\end{aligned}
$$

The HBJ for a low productivity-type firm l reads

$$
\begin{aligned}
&r_t V_t^l(n,[\mu_i],S_t) - \dot{V}_t^l(n,[\mu_i],S_t) = \\
&\quad \sum_{k=1}^{n} \pi(\mu_k) + \sum_{k=1}^{n} \tau_t \Big[V_t^l\left(n-1,[\mu_i]_{i\neq k},S_t\right) - V_t^l(n,[\mu_i],S_t) \Big] \\
&\quad + \max_{[x_k,I_k]} \Bigg\{ \sum_{k=1}^{n} I_k \Big[V_t^l\left(n,\left[[\mu_i]_{i\neq k},\mu_k\times\lambda\right],S_t\right) - V_t^l(n,[\mu_i],S_t) \Big] \\
&\quad + \sum_{k=1}^{n} x_k \Big[S_t V_t^l\left(n+1,\left[[\mu_i],\lambda\times\varphi^l/\varphi^h\right],S_t\right) + (1-S_t) V_t^l\left(n+1,\left[[\mu_i],\lambda\right],S_t\right) - V_t^l(n,[\mu_i],S_t) \Big] \\
&\quad - w_t \left[\mu_k^{-1} \frac{1}{\psi_I}(I_k)^{\zeta} + \frac{1}{\psi_x}(x_k)^{\zeta} \right] \Bigg\}.
\end{aligned}
$$

I solve for the value function of a high-type firm, however the steps for the low-type firm are equivalent. For clarity, I suppress the dependence of the value function on S_t in the following. Guess that the value function of the firm consists of a component that is common to all lines and a line-specific component

$$V_t^h(n,[\mu_i]) = V_{t,P}^h(n) + \sum_{k=1}^{n} V_{t,M}^h(\mu_k).$$

[30] The notation follows Peters (2020).

Substituting the guess into the HJB, $V^h_{t,P}(n)$ and $V^h_{t,M}(\mu_k)$ solve the following differential equations

$$\begin{aligned} r_t V^h_{t,M}(\mu_i) - \dot{V}^h_{t,M}(\mu_i) =& \pi(\mu_i) - \tau_t V^h_{t,M}(\mu_i) \\ &+ \max_{I_i}\left\{ I_i \Big[V^h_{t,M}(\mu_i \times \lambda) - V^h_{t,M}(\mu_i) \Big] - w_t \mu_i^{-1} \frac{1}{\psi_I}(I_i)^{\zeta} \right\} \end{aligned} \tag{30}$$

and

$$\begin{aligned} r_t V^h_{t,P}(n) - \dot{V}^h_{t,P}(n) &= \sum_{k=1}^{n} \tau_t \Big[V^h_{t,P}(n-1) - V^h_{t,P}(n) \Big] \\ + \max_{[x_k]} &\left\{ \sum_{k=1}^{n} x_k \Big[V^h_{t,P}(n+1) - V^h_{t,P}(n) + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) \Big] - w_t \frac{1}{\psi_x}(x_k)^{\zeta} \right\}. \end{aligned} \tag{31}$$

Assume that in steady-state $V^h_{t,P}$ and $V^h_{t,M}$ grow at the constant rate g. Using this guess in eq. (30) and following Peters (2020), we obtain for $V^h_{t,M}(\mu_i)$

$$V^h_{t,M}(\mu_i) = \frac{\pi(\mu_i) + \frac{\zeta-1}{\psi_I}(I_i)^{\zeta} w_t \mu_i^{-1}}{\rho + \tau},$$

where I_i solves

$$I_i = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta - 1}{\psi_I}(I_i)^{\zeta} \right) \left(1 - \frac{1}{\lambda} \right) \frac{\psi_I}{\zeta(\rho+\tau)} \right)^{\frac{1}{\zeta-1}}. \tag{32}$$

Eq. (32) shows that internal innovation rates I_i are time invariant, and independent of the product line and the productivity type of the firm, $I \equiv I^h = I^l$.

With this at hand, we can turn back to the differential equation for $V^h_{t,P}(n)$ in eq. (31). In addition to the guess that $V^h_{t,P}(n)$ grows at rate g, conjecture that $V^h_{t,P}(n) = n \times v^h_t$. Combined with the Euler we get

$$(\rho+\tau) n v^h_t = \max_{[x_k]} \left\{ \sum_{k=1}^{n} x_k \Big[v^h_t + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) \Big] - w_t \frac{1}{\psi_x}(x_k)^{\zeta} \right\}. \tag{33}$$

The optimality condition for x_k is given by

$$v^h_t + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) = w_t \frac{\zeta}{\psi_x}(x_k)^{\zeta-1}. \tag{34}$$

Several observations are noteworthy. First, eq. (34) shows that optimal expansion rates are independent of quality and productivity gaps in line k. We can hence drop the item indexation: $x_k = x^d$, where $d \in \{h, \ell\}$. Second, $v_t, V^h_{t,M}, w_t$ all grow at the same rate g, which implies that expansion rates are constant over time. We can hence write eq. (33) as

$$v_t^h = \frac{1}{(\rho+\tau)} \frac{\zeta - 1}{\psi_x} (x^h)^\zeta w_t.$$

Gathering all terms, the value function is given by

$$\begin{aligned} V_t^h(n, [\mu_i]) =& V^h_{t,P}(n) + \sum_{k=1}^{n} V_{t,M}(\mu_k) \\ =& n v_t^h + \sum_{k=1}^{n} V_{t,M}(\mu_k) \\ =& n \frac{1}{(\rho+\tau)} \frac{\zeta-1}{\psi_x} (x^h)^\zeta w_t + \sum_{k=1}^{n} \frac{\pi(\mu_k) + \frac{\zeta-1}{\psi_I} I^\zeta w_t \mu_k^{-1}}{\rho+\tau}, \end{aligned} \tag{35}$$

which is the expression for the value function stated in the main text, Proposition 1.

Using the expression for v_t^h, write the optimality condition in eq. (34) as

$$\begin{aligned} \frac{\zeta-1}{\psi_x}(x^h)^\zeta + S_t \left(\frac{Y_t}{w_t} \left(1 - \frac{1}{\lambda}\right) + \frac{\zeta-1}{\psi_I} I^\zeta \lambda^{-1} \right) + (1 - S_t) \left(\frac{Y_t}{w_t} \left(1 - \frac{\varphi^l}{\varphi^h} \frac{1}{\lambda}\right) + \frac{\zeta-1}{\psi_I} I^\zeta \lambda^{-1} \frac{\varphi^l}{\varphi^h} \right) \\ = (\rho+\tau) \frac{\zeta}{\psi_x} (x^h)^{\zeta-1}. \end{aligned}$$

Following the same steps for low-productivity firms, we obtain the optimality condition

$$\begin{aligned} \frac{\zeta-1}{\psi_x}(x^l)^\zeta + S_t \left(\frac{Y_t}{w_t} \left(1 - \frac{1}{\lambda} \frac{\varphi^h}{\varphi^l}\right) + \frac{\zeta-1}{\psi_I} I^\zeta \lambda^{-1} \frac{\varphi^h}{\varphi^l} \right) + (1 - S_t) \left(\frac{Y_t}{w_t} \left(1 - \frac{1}{\lambda}\right) + \frac{\zeta-1}{\psi_I} I^\zeta \lambda^{-1} \right) \\ = (\rho+\tau) \frac{\zeta}{\psi_x} (x^l)^{\zeta-1}. \end{aligned}$$

Lastly, I prove that more productive firms choose higher expansion R&D rates, i.e., $x^h > x^l$. Intuitively, the proof shows that an increase in productivity raises the stream of profits in a product line. The continuation value and the marginal cost of expansion R&D have to rise for the optimality condition of the expansion R&D rate to hold, implying that the expansion R&D rate increases. First note that product markups are increasing in firm productivity, as shown in eq. (5). Next, I totally differentiate the optimality condition for expansion R&D rate and show that the expansion R&D rate is increasing in the markup.

Write the optimality condition for expansion R&D as[31]

$$\frac{\zeta-1}{\psi_x}(x^h)^\zeta + S_t\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\mu^h}\right)+\frac{\zeta-1}{\psi_I}I^\zeta\frac{1}{\mu^h}\right)+(1-S_t)\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\mu^l}\right)+\frac{\zeta-1}{\psi_I}I^\zeta\frac{1}{\mu^l}\right)$$
$$=(\rho+\tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta-1},$$

where μ^h and μ^l denote the initial markup charged when facing a high- and low-productivity firm. Totally differentiate with respect to markups and the expansion R&D rate

$$S\frac{1}{(\mu^h)^2}\left(\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I}I^\zeta\right)d\mu+(1-S)\frac{1}{(\mu^l)^2}\left(\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I}I^\zeta\right)d\mu$$
$$=\frac{\zeta(\zeta-1)}{\psi_x}\left((\rho+\tau)(x^h)^{\zeta-2}-(x^h)^{\zeta-1}\right)dx^h,$$

where $d\mu^h=d\mu^l\equiv d\mu$. Since $x^h>0$, the above can be rearranged to

$$\frac{1}{(x^h)^{\zeta-2}}\left[S\frac{1}{(\mu^h)^2}\left(\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I}I^\zeta\right)+(1-S)\frac{1}{(\mu^l)^2}\left(\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I}I^\zeta\right)\right]d\mu$$
$$=\frac{\zeta(\zeta-1)}{\psi_x}\left(\rho+\tau-x^h\right)dx^h.$$

The left hand side captures the effect of changes in markups on profits. The right hand side captures the effect of changes in the expansion R&D rate on the continuation value and marginal costs of expansion R&D. From eq. (32) we know that $\frac{Y_t}{w_t}-\frac{\zeta-1}{\psi_I}I^\zeta>0$, otherwise the optimal internal R&D rate is negative. For a stationary firm size distribution, we must further have $\tau>x^h$. From this, it follows that $dx^h/d\mu>0$, which concludes the proof.

C.2 Joint distribution of quality and productivity gaps

I characterize the two-dimensional distribution of quality and productivity gaps along the BGP as a function of firm policies. This allows for optimal policies and the distribution to be solved jointly. I solve for the steady state distribution over quality and productivity gaps by setting the differential equations characterizing the law-of-motion in eq. (12) and (13) equal to zero. From this, one obtains the stationary mass of product lines with quality gap

[31] This uses the optimality condition of the high-type firm but the one of the low type works equivalently.

λ^{Δ} and productivity gap φ^i/φ^j

$$\nu\left(\Delta, \frac{\varphi^l}{\varphi^h}\right) = \left(\frac{I}{I+\tau}\right)^{\Delta} \frac{(1-S)x^l S + z(1-p^h)S}{I}$$
$$\nu\left(\Delta, \frac{\varphi^l}{\varphi^l}\right) = \left(\frac{I}{I+\tau}\right)^{\Delta} \frac{(1-S)x^l(1-S) + z(1-p^h)(1-S)}{I}$$
$$\nu\left(\Delta, \frac{\varphi^h}{\varphi^h}\right) = \left(\frac{I}{I+\tau}\right)^{\Delta} \frac{Sx^h S + zp^h S}{I}$$
$$\nu\left(\Delta, \frac{\varphi^h}{\varphi^l}\right) = \left(\frac{I}{I+\tau}\right)^{\Delta} \frac{Sx^h(1-S) + zp^h(1-S)}{I}.$$

Summing over all Δ for a given productivity gap gives $S_{\varphi^l,\varphi^h}, S_{\varphi^l,\varphi^l}, S_{\varphi^h,\varphi^h}, S_{\varphi^h,\varphi^l}$ as stated in Proposition 1 in main text. It follows that

$$\Pr\left(\Delta \leq d, \frac{\varphi^l}{\varphi^h}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - \left(\frac{I}{I+\tau}\right)^{d}\right)$$
$$\Pr\left(\Delta \leq d, \frac{\varphi^l}{\varphi^l}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^l}{\varphi^l}\right) = S_{\varphi^l,\varphi^l}\left(1 - \left(\frac{I}{I+\tau}\right)^{d}\right)$$
$$\Pr\left(\Delta \leq d, \frac{\varphi^h}{\varphi^h}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^h}{\varphi^h}\right) = S_{\varphi^h,\varphi^h}\left(1 - \left(\frac{I}{I+\tau}\right)^{d}\right)$$
$$\Pr\left(\Delta \leq d, \frac{\varphi^h}{\varphi^l}\right) = \sum_{i=1}^{d} \nu\left(i, \frac{\varphi^h}{\varphi^l}\right) = S_{\varphi^h,\varphi^l}\left(1 - \left(\frac{I}{I+\tau}\right)^{d}\right).$$

Focusing on product lines where a low-productivity incumbent faces a high-productivity second-best firm:

$$P\left(\Delta \leq d, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - e^{-d[\ln(I+\tau) - \ln I]}\right)$$

or

$$P\left(\ln\left(\lambda^{\Delta}\right) \leq d, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - e^{-\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}d}\right).$$

Conditional on the productivity gap, $\ln\left(\lambda^{\Delta}\right)$ is exponentially distributed with parameter

$\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}$. Further

$$P\left(\lambda^{\Delta}\leq d,\frac{\varphi^{l}}{\varphi^{h}}\right)=S_{\varphi^{l},\varphi^{h}}\left(1-d^{-\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}}\right).$$

Conditional on the productivity gap, quality gaps follow a Pareto distribution with parameter $\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}$. Denote $\theta=\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}$. We then have

$$P\left(\lambda^{\Delta}\leq m,\frac{\varphi^{l}}{\varphi^{h}}\right)=S_{\varphi^{l},\varphi^{h}}\left(1-m^{-\theta}\right).$$

Conditional on the productivity gap, quality gaps follow a Pareto distribution with parameter θ. As in Peters (2020), θ is affected by the rate of internal R&D I relative to creative destruction τ. The higher the rate of internal R&D, the more mass is in the tail of the quality gap distribution. The difference to Peters (2020) is that, in this model, quality gaps *conditional* on the productivity gap are Pareto distributed.

After repeating the same steps for lines with different productivity gaps, we obtain the aggregate labor income share as follows[32]

$$\begin{aligned}\Lambda=&\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int_{1}^{\infty}\frac{1}{\varphi_k/\varphi_n}\frac{1}{m}S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}dm\\=&\frac{\theta}{\theta+1}\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\frac{1}{\varphi_k/\varphi_n}S_{\varphi_k,\varphi_n}.\end{aligned}$$

The TFP misallocation statistic $\mathcal{M}$ is then given by

$$\begin{aligned}\mathcal{M}=&\frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int\left[\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}\right]dm}}{\Lambda}\\=&\frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\left[S_{\varphi_k,\varphi_n}\left(\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)-\frac{1}{\theta}\right)\right]}}{\Lambda},\end{aligned}$$

where I have made use of

$$\int_{1}^{\infty}\ln\left(\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}dm=\left[\frac{\theta\ln(m)+1}{\theta m^{\theta}}+C\right]_{1}^{\infty}=-\frac{1}{\theta}.$$

Alternatively note that this expression is equal to $-S_{\varphi_k,\varphi_n}E[\ln(\lambda^{\Delta})|\varphi_k,\varphi_n]$. I have shown

[32] For the derivation, I assume a continuous distribution of quality gaps.

above that $\ln(\lambda^{\Delta})$ conditional on the productivity gap is exponentially distributed with parameter θ. From the characteristics of an exponential distribution, its expected value is $1/\theta$.

The aggregate markup is then given by

$$\begin{aligned}E[\mu] =& \sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int_1^{\infty}\frac{\varphi_k}{\varphi_n}mS_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}dm \\ =&\frac{\theta}{\theta-1}\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\frac{\varphi_k}{\varphi_n}S_{\varphi_k,\varphi_n}.\end{aligned}$$

This concludes the derivations of the aggregate labor income share, TFP misallocation statistic and aggregate markup. In Peters (2020), the step size of innovation and the rate of creative destruction relative to internal R&D, captured by θ, fully characterize the aggregate labor income share, the misallocation measure, and the aggregate markup. In this model, these statistics further depend on the size and distribution of the productivity gaps. For example, a rise in the productivity gap or a reallocation of sales shares towards high-productivity firms lowers the aggregate labor income share and raises the markup, ceteris paribus.

C.3 Deriving the steady-state growth rate of aggregate variables

The growth rate of Q_t determines the growth rate of aggregate variables

$$g = \frac{\dot{Q}_t}{Q_t} = \frac{\partial \ln(Q_t)}{\partial t}.$$

Quality of a product in a given product line increases through internal R&D, expansion R&D or firm entry. For the growth rate of Q_t over a discrete time interval Δ, we have

$$\ln(Q_{t+\Delta}) = \int_0^1 \Big[(\Delta I + \Delta S x^h + \Delta(1-S)x^l + \Delta z)\ln(\lambda) + \ln(q_{t,i})\Big]\, di$$

so that

$$\frac{\ln(Q_{t+\Delta}) - \ln(Q_t)}{\Delta} = \Big(I + Sx^h + (1-S)x^l + z\Big)\ln(\lambda).$$

For $\Delta \to 0$, $g = \Big(I + Sx^h + (1-S)x^l + z\Big)\ln(\lambda)$ as stated in Proposition 1.

C.4 Solving for the steady state equilibrium

In the model, there are the seven unknown variables $x^h, x^l, I, z, \tau, \frac{Y_t}{w_t}, S$ and the markup distribution $\nu()$ in seven equations plus the system of differential equations characterizing $\nu()$.

Optimality condition for the internal innovation rate

$$I = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta - 1}{\psi_I} I^\zeta \right) \left(1 - \frac{1}{\lambda}\right) \frac{\psi_I}{\zeta(\rho + \tau)} \right)^{\frac{1}{\zeta - 1}}$$

Optimality condition for high-productivity type expansion rate

$$\frac{\zeta - 1}{\psi_x}(x^h)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + \frac{\zeta - 1}{\psi_I} I^\zeta \lambda^{-1}\right) + (1 - S)\left(\frac{Y_t}{w_t}\left(1 - \frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right) + \frac{\zeta - 1}{\psi_I} I^\zeta \lambda^{-1}\frac{\varphi^l}{\varphi^h}\right)$$
$$= (\rho + \tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta - 1}$$

Optimality condition for low-productivity type expansion rate

$$\frac{\zeta - 1}{\psi_x}(x^l)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right) + \frac{\zeta - 1}{\psi_I l} I^\zeta \lambda^{-1}\frac{\varphi^h}{\varphi^l}\right) + (1 - S)\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + \frac{\zeta - 1}{\psi_I l} I^\zeta \lambda^{-1}\right)$$
$$= (\rho + \tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta - 1}$$

Free entry condition

$$p^h\Big(SV_t^h(1, \lambda) + (1 - S)V_t^h(1, \lambda \times \varphi^h/\varphi^l)\Big) + (1 - p^h)\Big(SV_t^l(1, \lambda \times \varphi^l/\varphi^h) + (1 - S)V_t^l(1, \lambda)\Big) = \frac{1}{\psi_z} w_t,$$

where

$$V_t^d(1, \mu) = \frac{1}{(\rho + \tau)}\frac{\zeta - 1}{\psi_x}(x^d)^\zeta w_t + \frac{Y_t\left(1 - \frac{1}{\mu}\right) + \frac{\zeta - 1}{\psi_I} I^\zeta w_t \mu^{-1}}{\rho + \tau}$$

Labor market clearing condition

$$1 = \frac{Y_t}{w_t}\sum_{\frac{\varphi_j}{\varphi_{j'}}}\sum_i \frac{1}{\lambda^i \frac{\varphi_j}{\varphi_{j'}}}\nu\left(i, \frac{\varphi_j}{\varphi_{j'}}\right) + \frac{1}{\psi_I} I^\zeta \sum_{\frac{\varphi_j}{\varphi_{j'}}}\sum_i \frac{1}{\lambda^i \frac{\varphi_j}{\varphi_{j'}}}\nu\left(i, \frac{\varphi_j}{\varphi_{j'}}\right) + S\frac{1}{\psi_x}(x^h)^\zeta + (1 - S)\frac{1}{\psi_x}(x^l)^\zeta + \frac{z}{\psi_z}$$

Creative destruction

$$\tau = z + Sx^h + (1 - S)x^l$$

Share of high productivity type

$$S = \sum_{i=1}^{\infty} \left[\nu \left(i, \frac{\varphi^h}{\varphi^h} \right) + \nu \left(i, \frac{\varphi^h}{\varphi^l} \right) \right],$$

where ν, the stationary distribution of quality and productivity gaps, is characterized by

$$0 = \dot{\nu} \left(\Delta, \frac{\varphi_j}{\varphi_{j'}} \right) = I\nu \left(\Delta - 1, \frac{\varphi_j}{\varphi_{j'}} \right) - \nu \left(\Delta, \frac{\varphi_j}{\varphi_{j'}} \right) (I + \tau) \quad \text{for} \quad \Delta \geq 2$$

and for the case of a unitary quality gap

$$\begin{aligned}
0 = \dot{\nu} \left(1, \frac{\varphi^l}{\varphi^h} \right) &= (1 - S)x^l S + z_t(1 - p^h)S - \nu \left(1, \frac{\varphi^l}{\varphi^h} \right) (I + \tau) \\
0 = \dot{\nu} \left(1, \frac{\varphi^l}{\varphi^l} \right) &= (1 - S)x^l(1 - S) + z_t(1 - p^h)(1 - S) - \nu \left(1, \frac{\varphi^l}{\varphi^l} \right) (I + \tau) \\
0 = \dot{\nu} \left(1, \frac{\varphi^h}{\varphi^h} \right) &= Sx^h S + z_t p^h S - \nu \left(1, \frac{\varphi^h}{\varphi^h} \right) (I + \tau) \\
0 = \dot{\nu} \left(1, \frac{\varphi^h}{\varphi^l} \right) &= Sx^h(1 - S) + z_t p^h(1 - S) - \nu \left(1, \frac{\varphi^h}{\varphi^l} \right) (I + \tau).
\end{aligned}$$

To simplify the system of equations, first rewrite the rate of creative destruction

$$z = (\tau - Sx^h - (1 - S)x^l)$$

such that z can be substituted out from the remaining equations. Second, based on Proposition 1, we know

$$S = S_{\varphi^h, \varphi^h} + S_{\varphi^h, \varphi^l} = \frac{Sx^h + zp^h}{\tau}.$$

Third, the optimality conditions for expansion rates (multiplied by p^h and $(1 - p^h)$) and the free entry condition together imply

$$\frac{1}{\psi_x} p^h (x^h)^{\zeta - 1} + \frac{1}{\psi_x} (1 - p^h)(x^l)^{\zeta - 1} = \frac{1}{\psi_z \zeta}.$$

The system of equilibrium conditions can hence be reduced to:

Optimality condition for the internal innovation rate

$$I = \left(\left(\frac{Y_t}{w_t} \psi_I - (\zeta - 1) I^\zeta \right) \frac{\left(1 - \frac{1}{\lambda}\right)}{\zeta(\rho + \tau)} \right)^{\frac{1}{\zeta - 1}}$$

Optimality condition for high-productivity type expansion rate

$$\begin{aligned}\frac{\zeta - 1}{\psi_x}(x^h)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + (\zeta - 1) I^\zeta \lambda^{-1} \frac{1}{\psi_I}\right) + (1 - S)\left(\frac{Y_t}{w_t}\left(1 - \frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right) + (\zeta - 1) I^\zeta \lambda^{-1} \frac{\varphi^l}{\psi_I \varphi^h}\right) \\ = (\rho + \tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta - 1}\end{aligned}$$

Optimality condition for low-productivity type expansion rate

$$\begin{aligned}\frac{\zeta - 1}{\psi_x}(x^l)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right) + (\zeta - 1) I^\zeta \lambda^{-1} \frac{\varphi^h}{\psi_I l \varphi^l}\right) + (1 - S)\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + (\zeta - 1) I^\zeta \lambda^{-1} \frac{1}{\psi_I l}\right) \\ = (\rho + \tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta - 1}\end{aligned}$$

Free entry

$$p^h \frac{(x^h)^{\zeta - 1}}{\psi_x} + (1 - p^h)\frac{(x^l)^{\zeta - 1}}{\psi_x} = \frac{1}{\psi_z \zeta}$$

Labor market clearing condition

$$1 = \frac{Y_t}{w_t}\Lambda + \Lambda_I + S\frac{1}{\psi_x}(x^h)^\zeta + (1 - S)\frac{1}{\psi_x}(x^l)^\zeta + \frac{\tau - Sx^h - (1 - S)x^l}{\psi_z},$$

where

$$\begin{aligned}\Lambda =& \frac{\theta}{\theta + 1}\sum_{k \in \{h,l\}}\sum_{n \in \{h,l\}}\frac{1}{\varphi_k / \varphi_n} S_{\varphi_k, \varphi_n} \\ \Lambda_I =& \frac{1}{\psi_I} I^\zeta \frac{\theta}{\theta + 1}\sum_{k \in \{h,l\}}\sum_{n \in \{h,l\}}\frac{1}{\varphi_k / \varphi_n} S_{\varphi_k, \varphi_n} \\ \theta =& \frac{\ln(I + \tau) - \ln(I)}{\ln(\lambda)}\end{aligned}$$

Share of high productivity type

$$S = \frac{Sx^h + (\tau - Sx^h - (1 - S)x^l)p^h}{\tau}$$

The expressions related to the labor market clearing condition are derived in Section C.2. This constitutes a system of seven equations in seven unknowns $(x^h, x^l, I, \tau, \frac{Y_t}{w_t}, S)$, which I solve using a root finder.

C.5 Firm markups

Firm markups are defined by $\mu_f = \frac{py_f}{wl_f} = \left(\frac{1}{n}\sum_{k=1}^{n}\mu_{kf}^{-1}\right)^{-1}$. Therefore

$$\ln \mu_f = -\ln\left(\frac{1}{n}\sum_{k=1}^{n}\mu_k^{-1}\right).$$

Rewrite the term in brackets (for a high-productivity firm) as

$$\frac{1}{n}\sum_{k=1}^{n}\mu_k^{-1} = \frac{1}{n}\sum_{k=1}^{n}e^{-\ln\mu_k} = \frac{1}{n}\left(\sum_{i=1}^{n_i}e^{-\ln\frac{\varphi^h}{\varphi^l}-\Delta_i\ln\lambda} + \sum_{j=1}^{n_j}e^{-\Delta_j\ln\lambda}\right), \tag{36}$$

where i indexes the product lines where the high productivity firm faces a low productivity second best producer, j the lines where it faces a high productivity second best producer and $n_i + n_j = n$. A two-dimensional linear Taylor expansion around $\ln\lambda = 0$ and $\ln\frac{\varphi^h}{\varphi^l} = 0$ gives

$$\frac{1}{n}\left(\sum_{i=1}^{n_i}e^{-\ln\frac{\varphi^h}{\varphi^l}-\Delta_i\ln\lambda} + \sum_{j=1}^{n_j}e^{-\Delta_j\ln\lambda}\right) \approx 1 - \left(\frac{1}{n}\sum_{k=1}^{n}\Delta_k\right)\ln\lambda - \frac{n_i}{n}\ln\left(\frac{\varphi^h}{\varphi^l}\right)$$

such that

$$E\left[\ln\mu_f|\text{firm age} = a_f, \varphi^h\right] \approx E\left[\frac{1}{n}\sum_{k=1}^{n}\Delta_k|\text{firm age} = a_f, \varphi^h\right]\ln\lambda + (1-S)\ln\left(\frac{\varphi^h}{\varphi^l}\right),$$

where I have used the fact that (in expectation) the share of the firm's product lines with a low productivity second best producer is equal to the aggregate share of product lines where the incumbent is of the low productivity type. From Peters (2020), we know that

$$E\left[\frac{1}{n}\sum_{k=1}^{n}\Delta_k|\text{firm age} = a_f, \varphi^h\right]\ln\lambda = \left(1 + I \times E[a_P^h|a_f]\right)\ln\lambda,$$

where $E[a_P^h|a_f]$ denotes the average product age of a high-productivity type firm conditional on firm age a_f and

$$E[a_P^h|a_f] = \frac{1}{x^h}\left(\frac{\frac{1}{\tau}\left(1-e^{-\tau a_f}\right)}{\frac{1}{x^h+\tau}\left(1-e^{-(x^h+\tau)a_f}\right)} - 1\right)\left(1-\phi^h(a_f)\right) + a_f\phi^h(a_f)$$
$$\phi^h(a) = e^{-x^h a}\frac{1}{\gamma^h(a)}\ln\left(\frac{1}{1-\gamma^h(a)}\right)$$
$$\gamma^h(a) = \frac{x^h\left(1-e^{-(\tau-x^h)a}\right)}{\tau - x^h e^{-(\tau-x^h)a}}.$$

For a firm of the low-productivity type, the last term in eq. (36) reads

$$\frac{1}{n}\left(\sum_{i=1}^{n_i} e^{-\ln\Delta_i \ln\lambda} + \sum_{j=1}^{n_j} e^{-\ln\frac{\varphi^l}{\varphi^h} - \ln\Delta_j\ln\lambda}\right),$$

where i indexes the product lines where the low-productivity producer faces a low-productivity second best producer, j the lines where it faces a high-productivity second best producer and $n_i + n_j = n$. Following the same steps as for a high-productivity firm, this time linearizing around $\ln\frac{\varphi^l}{\varphi^h} = 0$ (and $\ln\lambda = 0$) gives

$$E\left[\ln\mu_f|\text{firm age} = a_f, \varphi^l\right] \approx \left(1 + I \times E[a_P^l|a_f]\right)\ln\lambda + S\ln\left(\frac{\varphi^l}{\varphi^h}\right),$$

where again I have made use of the fact that (in expectation) the share of the firm's product lines with a high-productivity second best producer is equal to the aggregate share of product lines where the incumbent is of the high-productivity type. $E[a_P^l|a_f]$ is exactly defined as $E[a_P^h|a_f]$ with x^h replaced by x^l in the above expressions.

C.6 Firm size distribution

The mass of high- and low-productivity type firms with $n \geq 2$ products follows the differential equations

$$\dot{M}_t^h(n) = (n-1)x_t^h M_t^h(n-1) + (n+1)\tau_t M_t^h(n+1) - n(x_t^h + \tau_t)M_t^h(n)$$
$$\dot{M}_t^l(n) = (n-1)x_t^l M_t^l(n-1) + (n+1)\tau_t M_t^l(n+1) - n(x_t^l + \tau_t)M_t^l(n), \qquad (37)$$

whereas the mass of firms with one product evolves according to

$$\begin{aligned}\dot{M}_t^h(1) &= z_t p^h + 2\tau_t M_t^h(2) - (x_t^h + \tau_t) M_t^h(1) \\ \dot{M}_t^l(1) &= z_t(1-p^h) + 2\tau_t M_t^l(2) - (x_t^l + \tau_t) M_t^l(1).\end{aligned} \tag{38}$$

The mass of firms with n products increases through firms with $n-1$ products expanding to size n at rate x_t^h or x_t^l per product or through firms with $n+1$ products losing a product at the rate of aggregate creative destruction τ_t. The mass of firms with n products decreases through firms with n products either gaining or losing a product through expansion or creative destruction. The mass of firms with one product additionally increases through firm entry.

Proposition 2. *The stationary firm size distribution along the balanced growth path is characterized as follows.*

1. *The mass of high and low productivity firms with n products is*

$$\begin{aligned}M^h(n) &= \frac{(x^h)^{n-1} z p^h}{n\tau^n} = \frac{zp^h}{x^h}\frac{1}{n}\left(\frac{x^h}{\tau}\right)^n \\ M^l(n) &= \frac{(x^l)^{n-1} z(1-p^h)}{n\tau^n} = \frac{z(1-p^h)}{x^l}\frac{1}{n}\left(\frac{x^l}{\tau}\right)^n.\end{aligned}$$

2. *The total mass of firms with n products is*

$$M(n) = M^h(n) + M^l(n) = \frac{(x^h)^{n-1} z p^h + (x^l)^{n-1} z(1-p^h)}{n\tau^n}.$$

3. *The mass of firms of each productivity type is*

$$\begin{aligned}M^h &= \sum_{n=1}^{\infty} M^h(n) = \frac{zp^h}{x^h}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{x^h}{\tau}\right)^n = \frac{zp^h}{x^h}\ln\left(\frac{\tau}{\tau - x^h}\right) \\ M^l &= \sum_{n=1}^{\infty} M^l(n) = \frac{z(1-p^h)}{x^l}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{x^l}{\tau}\right)^n = \frac{z(1-p^h)}{x^l}\ln\left(\frac{\tau}{\tau - x^l}\right)\end{aligned}$$

4. *The total mass of firms is*

$$M = M^h + M^l.$$

Proof. These results follow from setting the time derivatives in equations (37) and (38) equal to zero and solving the system of equations. □

For each firm type, the share of firms with n products, $M^h(n)/M^h$ and $M^l(n)/M^l$, follows the PDF of a logarithmic distribution with parameter x^h/τ and x^l/τ as in Lentz and Mortensen (2008). The firm size distribution is highly skewed to the right.

Since there is a continuum of mass one of products and each product is mapped to one firm $\sum_{i=1}^{\infty} M(n) \times n = 1$. Further, the mass of high-productivity type firms producing n products is related to the share of product lines operated by high-type firms, S, as follows

$$S = \sum_{n=1}^{\infty} M^h(n) \times n = \frac{zp^h}{\tau - x^h}.$$

D Computation of transition dynamics

In this section, I lay out the numerical procedure to solve for the transition path. Since time is continuous, I solve a discretized version of the model where the solution converges to the one in continuous time for small enough time intervals. As shown in Appendix C, value functions are additive across product lines. Therefore, I solve the problem of two representative one-product firms: one of the high productivity type and one of the low productivity type.

I normalize the value function by the wage w_t to obtain a stationary problem. The value function for the high-type firm (in discrete time) reads

$$\begin{aligned}
&\frac{V_t^h(1,\mu_i,S_t)}{w_t} = \frac{Y_t}{w_t}\left(1-\frac{1}{\mu_i}\right)dt \\
&- \tau_t \exp(-r_t dt)\frac{V_{t+dt}^h(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}dt \\
&+ \max_{x_t^h}\left\{x_t^h \exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^h(1,\lambda,S_{t+dt})}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^h(1,\lambda\frac{\varphi^h}{\varphi^l},S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_x}(x_t^h)^\zeta dt\right\} \\
&+ \max_{I_t^h}\left\{I_t^h \exp(-r_t dt)\left(\frac{V_{t+dt}^h(1,\mu_i\lambda,S_{t+dt})}{w_{t+dt}} - \frac{V_{t+dt}^h(1,\mu_i,S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_I}\mu_i^{-1}(I_t^h)^\zeta dt\right\} \\
&+ \exp(-r_t dt)\frac{V_{t+dt}^h(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}.
\end{aligned} \tag{39}$$

The value function for the low-type firm reads

$$\begin{aligned}
&\frac{V_t^l(1,\mu_i,S_t)}{w_t} = \frac{Y_t}{w_t}\left(1-\frac{1}{\mu_i}\right)dt \\
&-\tau_t \exp(-r_t dt)\frac{V_{t+dt}^l(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}dt \\
&+\max_{x_t^l}\left\{x_t^l \exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^l(1,\lambda\frac{\varphi^l}{\varphi^h},S_{t+dt})}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^l(1,\lambda,S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_x}(x_t^l)^\zeta dt\right\} \\
&+\max_{I_t^l}\left\{I_t^l \exp(-r_t dt)\left(\frac{V_{t+dt}^l(1,\mu_i\lambda,S_{t+dt})}{w_{t+dt}} - \frac{V_{t+dt}^l(1,\mu_i,S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_I}\mu_i^{-1}(I_t^h)^\zeta dt\right\} \\
&+\exp(-r_t dt)\frac{V_{t+dt}^l(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}.
\end{aligned} \tag{40}$$

From this, one obtains the first order conditions for the policy functions. For the optimal expansion R&D rate of the high type firm x_t^h (again suppressing the dependence of the value function on S_t):

$$\exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^h(1,\lambda)}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^h(1,\lambda\frac{\varphi^h}{\varphi^l})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_x}(x_t^h)^{\zeta-1} \tag{41}$$

and for the low type firm x_t^l:

$$\exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^l(1,\lambda\frac{\varphi^l}{\varphi^h})}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^l(1,\lambda)}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_x}(x_t^l)^{\zeta-1}. \tag{42}$$

Both are independent of the markup μ_i. For the optimal internal R&D rates of the high type, I_t^h, one obtains

$$\exp(-r_t dt)\left(\frac{V_{t+dt}^h(1,\mu_i\lambda)}{w_{t+dt}} - \frac{V_{t+dt}^h(1,\mu_i)}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_I}\mu_i^{-1}(I_t^h)^{\zeta-1} \tag{43}$$

and similarly for I_t^l

$$\exp(-r_t dt)\left(\frac{V_{t+dt}^l(1,\mu_i\lambda)}{w_{t+dt}} - \frac{V_{t+dt}^l(1,\mu_i)}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_I}\mu_i^{-1}(I_t^l)^{\zeta-1}. \tag{44}$$

Equations (39) to (44) characterize the firm problem in discrete time. These equations are supplemented by the law of motion for the two dimensional distribution of quality and productivity gaps

$$\nu_{t+dt}\left(\Delta,\frac{\varphi_j}{\varphi_{j'}}\right)-\nu_t\left(\Delta,\frac{\varphi_j}{\varphi_{j'}}\right)=dt\left[I_{\mu_i,t}\nu_t\left(\Delta-1,\frac{\varphi_j}{\varphi_{j'}}\right)-\nu_t\left(\Delta,\frac{\varphi_j}{\varphi_{j'}}\right)(I_{\mu_i,t}+\tau_t)\right]\quad\text{for}\quad\Delta\geq 2 \tag{45}$$

and for product lines with a unitary quality gap, $\Delta = 1$,

$$\begin{aligned}
\nu_{t+dt}\left(1,\frac{\varphi^l}{\varphi^h}\right)-\nu_t\left(1,\frac{\varphi^l}{\varphi^h}\right)&=dt\left[(1-S_t)x_t^lS_t+z_t(1-p^h)S_t-\nu_t\left(1,\frac{\varphi^l}{\varphi^h}\right)(I_{\mu_i,t}+\tau_t)\right]\\
\nu_{t+dt}\left(1,\frac{\varphi^l}{\varphi^l}\right)-\nu_t\left(1,\frac{\varphi^l}{\varphi^l}\right)&=dt\left[(1-S_t)x_t^l(1-S_t)+z_t(1-p^h)(1-S_t)-\nu_t\left(1,\frac{\varphi^l}{\varphi^l}\right)(I_{\mu_i,t}+\tau_t)\right]\\
\nu_{t+dt}\left(1,\frac{\varphi^h}{\varphi^h}\right)-\nu_t\left(1,\frac{\varphi^h}{\varphi^h}\right)&=dt\left[S_tx_t^hS_t+z_tp^hS_t-\nu_t\left(1,\frac{\varphi^h}{\varphi^h}\right)(I_{\mu_i,t}+\tau_t)\right]\\
\nu_{t+dt}\left(1,\frac{\varphi^h}{\varphi^l}\right)-\nu_t\left(1,\frac{\varphi^h}{\varphi^l}\right)&=dt\left[S_tx_t^h(1-S_t)+z_tp^h(1-S_t)-\nu_t\left(1,\frac{\varphi^h}{\varphi^l}\right)(I_{\mu_i,t}+\tau_t)\right]
\end{aligned} \tag{46}$$

and a standard Euler equation

$$\frac{C_{t+dt}}{C_t}=\exp(-\rho dt)(1+r_{t+dt}dt). \tag{47}$$

Further, the (static) free entry and labor market clearing conditions remain unchanged and are characterized in the main text by equations (10) and (11).

The algorithm to compute the transition path assumes that an initial and ending balanced growth path has been solved for including the (stationary) two-dimensional distribution of quality and productivity gaps. I choose $dt = 0.02$ and set the transition period to 100 years (T), after which I assume the economy has reached its new balanced growth path. I further truncate the two dimensional distribution of quality and productivity gaps along the quality dimension at $\Delta = 30$, implying a maximum quality gap of λ^{30}. No mass reaches this state during the transition such that this assumption is satisfied. I then compute the transition path as follows:

1. Guess a path of interest rates r_t and wage growth $\frac{w_{d+dt}}{w_t}$ over the transition (equal to their values in the final balanced growth path)

   (a) Guess a path for S_t over the transition (equal to its value in the final balanced growth path).

i. Starting backwards in period T, solve for optimal policy functions in $T - dt$ using equations (41)-(44).[33]

ii. Solve for τ_{T-dt} that ensures that the free entry condition (10) holds.

iii. Compute the value function in $T - dt$ using equations (39) and (40).

iv. Iterate backwards until the first time period.

v. Starting from the initial balanced growth path, simulate S_t forward using[34]

$$S_{t+dt} = S_t + dt\left[S_t x_t^h (1 - S_t) - (1 - S_t) x_t^l S_t + z_t (p^h (1 - S_t) - (1 - p^h) S_t)\right],$$

where z_t can be substituted out by equation (19).

(b) Update the guess for S_t from step v and go back to step i. Iterate until the guessed path for S_t converges to the implied one.

2. Starting from the initial balanced growth path, simulate the two dimensional distribution of quality and productivity gaps forward using equations 45 and 46.

3. Solve for the sequence of $\frac{Y_t}{w_t}$ from the labor market clearing condition.

4. Compute the sequence of quality growth using

$$\frac{Q_{t+dt}}{Q_t} = \exp\left(\left[\int_0^1 I_{\mu_i,t} di + S_t x_t^h + (1 - S_t) x_t^l + z_t\right] dt \ln(\lambda)\right).$$

5. Compute the sequence of aggregate productivity growth using

$$\frac{\Phi_{t+dt}}{\Phi_t} = \left(\frac{\varphi^h}{\varphi^l}\right)^{S_{t+dt} - S_t}.$$

6. Using the two dimensional distribution of quality and productivity gaps, compute the sequence of $\mathcal{M}_t$ defined in equation (9).

7. Compute the sequence of production labor L_{Pt} using equation (7).

8. Compute the sequence of aggregate output growth $\frac{Y_{t+dt}}{Y_t}$ using equation (9).

[33] I solve for the optimal I_i (at each point in time) over a two-dimensional grid of quality and productivity gaps.

[34] One could already simulate the entire two-dimensional distribution of quality and productivity gaps forward here. However, for the inner loop, it is sufficient to iterate on S_t.

9. With the path of aggregate output growth, obtain the implied path of interest rates from the Euler equation (47).
10. With the paths of aggregate output growth and Y_t/w_t, obtain the implied path of wage growth $\frac{w_{d+dt}}{w_t}$.
11. Update the guesses for the interest rate and wage growth and go back to step (a). Iterate until the guessed and implied paths converge.