Recent Changes in Firm Dynamics and the Nature of Economic Growth*

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Abstract

This paper documents a novel observation on firm growth in high-quality administrative data: cumulative size growth over a firm's life cycle has systematically increased since the 1990s. I study these trends in an endogenous growth model of creative destruction that features ex-ante heterogeneity in firm productivity. In the model, the acceleration of firm size growth over the life cycle is the natural response to rising entry costs. Falling firm entry lowers the risk of replacement, which benefits particularly more productive incumbents that charge higher markups in equilibrium. As a result, more productive incumbents expand into new product markets faster, explaining the observed acceleration in firm size growth. The growth acceleration of more productive incumbents concentrates market shares among these firms, exerting positive reallocation effects on short and long-run productivity growth. Falling firm entry dominates the positive contribution of productive incumbents, lowering productivity growth and welfare.

Keywords: Firm dynamics, Aggregate productivity growth, Firm entry, Reallocation, Administrative data

JEL codes: D22, E24, O31, O47, O50

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1 Introduction

Firm growth and aggregate economic growth go hand in hand. Firms constantly compete for market shares in product markets. The firms that improve their products over time attract new customers and workers, increasing their sales and employment. Simultaneously, firms' product improvements generate aggregate economic growth. The link between firm growth and aggregate economic growth lies at the heart of Schumpeterian growth theory. This paper presents a novel finding on firm growth in high-quality administrative data. Sales and employment growth over firms' life cycles has systematically increased since the 1990s. I rationalize these changes in an endogenous growth model of creative destruction with rich firm dynamics. In the model, the acceleration of incumbent firm growth is the natural response to rising entry costs. Fewer firms enter the economy, but those that do, particularly the more productive ones among them, expand into new product markets faster due to increased expected profits. The acceleration of firm growth is, hence, consistent with many recent macroeconomic trends, in particular, falling firm entry, rising concentration, declining worker reallocation (creative destruction), and slowing aggregate productivity growth.

The first contribution of this paper is empirical. I document a new stylized fact about firm growth using Swedish administrative data from tax records: life cycle growth of firm sales and employment accelerated. Over the first eight years of the firm, sales increased by 55.9 percent for firms established in the late 1990s compared to 67.4 percent for the cohorts of the early 2010s. For employment growth, these differences are even more pronounced. Firm employment increased by 28.8 percent over the first eight years for the cohorts of the late 1990s compared to 46.6 percent for the cohorts of the 2010s. These trends are robust to various regression specifications, alternative entrant classifications, and can be observed for cohorts established after the Great Recession. I find similar patterns in U.S. Census data. For almost all sectors in the U.S. economy, firm size conditional on age has increased relative to the size of entrants over time, suggesting that growth over the firm's life cycle has accelerated. The reason why previous studies (Hopenhayn, Neira and Singhania, 2022; Karahan, Pugsley and Şahin, 2022) find stability in firm size conditional on age patterns is because these studies pool firms across all sectors. Declining firm size in the U.S. manufacturing sector masks increasing firm size in almost all other sectors (conditional on age).

I rationalize the changes in firm growth in a structural model. The model includes the following three elements. First, the model features a link between firm dynamics and economic growth in the spirit of Schumpeterian growth models (Aghion and Howitt, 1992; Grossman and Helpman, 1991; Klette and Kortum, 2004): incumbent firms (and potential entrants) gain market shares by expanding horizontally into new product markets through creative destruction (expansion R&D).[1] Second, in standard models of creative destruction with constant markups, firm sales and employment growth from horizontal expansion are identical. In line with the data, I include a second type of product innovation that permits differential

[1]These models are analytically tractable yet capture salient features of firm dynamics (Lentz and Mortensen, 2008; Akcigit and Kerr, 2018).

sales and employment growth (and changes therein). This type of innovation (internal R&D) allows incumbent firms to distance their competitors vertically in the quality space and, in equilibrium, charge a higher product markup. Markup growth drives a wedge between firm sales and employment growth. Third, the model includes innate (ex-ante) heterogeneity in firm fundamentals. Some firms in the economy are persistently more productive than others and, as an equilibrium outcome, (innovate and) grow at systematically faster rates. This allows for selection effects between firms that grow at different rates to explain the observed acceleration of firm growth.

I estimate the model on a balanced growth path matching firm sales and employment growth of the cohorts in the late 1990s and other macroeconomic moments. As a comparative statics exercise, I re-estimate two parameters of the model to match the acceleration of firm sales and employment life cycle growth of the latest cohorts in the data. The estimation highlights a rise in the cost of entry and an increase in incumbents' internal R&D costs as the cause behind the changes in firm growth. Rising firm entry costs increase the value of a product line, incentivizing incumbents to expand horizontally. The expansion into new product lines increases incumbents' sales and employment growth. In contrast, the increase in the internal R&D costs reduces incumbents' internal R&D efforts. Falling internal R&D slows incumbents' markup growth, accelerating their employment relative to sales growth. Hence, rising entry costs account for the *joint* acceleration of sales and employment growth, whereas increasing internal R&D costs explain the *relative* acceleration of employment growth.

The rise in the cost of entry raises incumbents' incentives to expand into new product markets, but, importantly, the strength of the effect varies across incumbents. In particular, declining firm entry increases the difference in the value of a product line between more and less productive incumbents. Similarly, rising internal R&D costs lower the value of a product line primarily for less productive incumbents as the continuation value of internal R&D is higher for less productive incumbents with initially lower markups. As a result, only the more productive incumbents increase their expansion R&D rates, accounting for the observed acceleration in firm life cycle growth. The increase in life cycle growth of the more productive firms is associated with a rise in their sales shares in the cross-section.

What are the long-run implications for the macroeconomy associated with the changes in firm growth? In response to the rise in the cost of entry and internal R&D, the firm entry rate falls by eight percentage points (pp), sales concentration rises, and the aggregate growth rate declines by 0.62pp. The implied decline in long-run growth and firm entry account for roughly 60% of the measured decline in TFP growth and 80% of the decline in the firm entry rate in Sweden over the last three decades (Engbom, 2023). The reallocation of market shares to more productive firms that, in the model, feature relatively low labor shares and high markups is further consistent with Kehrig and Vincent (2021), De Loecker, Eeckhout and Unger (2020), and Baqaee and Farhi (2020). I extend the analysis over the transition period, which trades off the long-run fall in the aggregate growth rate with the positive level effects due to the reallocation of sales shares to more productive incumbents. The growth effects dominate the level effects, resulting in a welfare loss.

I quantify the contributions by incumbent firms and entrants to the long-run fall in aggregate productivity growth in a growth decomposition. Changes in the long-run growth rate are due to (i) changes in incumbents' (internal and expansion) innovation rates, holding sales shares constant, (ii) reallocation of sales shares across incumbents that innovate at different rates, and (iii) changes in entrants' innovation rates (firm entry). First, incumbents' innovation rates have increased. Despite rising internal R&D costs, increasing entry costs incentivize incumbent innovation, raising the long-run growth rate by 0.22pp. Second, as more productive incumbents innovate at systematically higher rates in equilibrium, the reallocation of market shares to these firms increases the aggregate growth rate by 0.27pp. Hence, incumbents have contributed positively to changes in long-run growth since the 1990s, primarily due to the effects of reallocation. These reallocation effects are absent in standard models of creative destruction with ex-ante homogeneous firms. Third, rising entry costs slow firm entry. The fall in firm entry lowers the long-run growth rate by 1.1pp. Net of the positive contribution by incumbents, the long-run growth rate declines by 0.62pp. The results of the decomposition are robust to an alternative estimation, where an increase in the productivity dispersion across firms explains the magnitude of the decline in U.S. TFP growth as in Aghion, Bergeaud, Boppart, Klenow and Li (2023).

Lastly, the richness of the Swedish administrative data, particularly information on the capital stock and intermediate input usage for the universe of firms, allows me to test model predictions. I provide suggestive evidence that persistently more productive firms grow faster over their life cycles, a key implication of the ex-ante heterogeneity in firm productivity.

Related Literature. The empirical results relate to Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022), who document that firm employment conditional on age has been relatively stable in U.S. Census data since the 1980s. Both studies show this stability pooling firms across sectors. Using the same data, I find enormous heterogeneity in firm size trends across sectors. Declining incumbent firm size conditional on age in the U.S. manufacturing sector masks the reverse trends in almost all other sectors when pooling firms across sectors.[2]

Sterk, Sedláček and Pugsley (2021) document changes in life cycle growth for U.S. firms over time. For the cohorts 1979 to 1993, the authors show that employment growth over the firm's life cycle slowed. The results presented in this paper are complementary rather than contradictory to theirs as I document trends for the cohorts from 1997 to 2017. The rise in industry concentration, and the fall in firm entry accelerated strongly during the turn of the millennium, as shown by Autor, Dorn, Katz, Patterson and Van Reenen (2020), and Akcigit and Ates (2021). Firm-level changes during this period are particularly useful to understand the forces behind these macroeconomic trends.

The structural model in this paper relates to Peters (2020). Peters (2020) builds an endoge-

[2] Van Vlokhoven (2021) further documents that profits and sales of firms in Compustat data have become more back-loaded. While I share the observation that the sales growth over the firm's life cycle accelerated, I find that firm size at entry is relatively stable over time in U.S. Census and Swedish registry data.

nous growth model with ex-ante homogeneous firms that conduct expansion and internal R&D. The model in this paper features ex-ante heterogeneity in firm productivity types to study firm type selection. This introduces a new state variable to the firm's value function, namely the distribution of firm types across product lines. Firms keep track of this distribution to build markup expectations for their decision to enter new product lines.[3] Further, Peters (2020) analyzes the model's steady state implications, whereas this paper solves for the transition between steady states to quantify the implications of the changes in firm growth for welfare. Relatedly, Aghion, Bergeaud, Boppart, Klenow and Li (2023) build a model of creative destruction with ex-ante heterogeneous firms, abstracting from internal R&D and firm entry. Both elements are present in the model in this paper, which allows firm dynamics in sales, employment, and markups to be mapped to the data.

The comparative statics exercise is related to studies explaining recent macroeconomic trends in the U.S. economy. Proposed drivers for these trends are increasing costs of R&D (Bloom, Jones, Van Reenen and Webb, 2020) or increasing barriers to entry (Davis, 2017; Gutiérrez and Philippon, 2018).[4] I show that these mechanisms can explain a new stylized fact at the micro (firm) level, namely accelerating sales and employment life cycle growth.

A separate strand of literature emphasizes the effects of reallocation on economic growth. China and East Germany are examples where long-term sustained growth followed the reallocation of market shares from state-owned enterprises to privately held companies (Song, Storesletten and Zilibotti, 2011; Findeisen, Lee, Porzio and Dauth, 2021). This reallocation potentially affects GDP per capita in a static sense through two channels. First, more productive firms gain market shares, thereby raising average productivity, and second, by reducing the extent of misallocation of production factors in the spirit of Hsieh and Klenow (2009). However, the reallocation of market shares could also affect the economy's long-run growth rate if firms innovate (or imitate) at heterogeneous rates (Acemoglu, Akcigit, Alp, Bloom and Kerr, 2018). The model in this paper accounts for the effect of reallocation on economic growth through all three channels: over the transition to the new balanced growth path, the reallocation of sales shares across firms affects aggregate output growth through changes in average productivity, misallocation, and innovation rates. I find that these reallocation effects matter, even for long-run economic growth.

Akcigit and Kerr (2018), Garcia-Macia, Hsieh and Klenow (2019), and Peters (2020) decompose economic growth into the contributions by entrants and incumbent firms. These studies conclude that economic growth is mainly due to incumbent firms rather than entrants. While this is also the case in the parametrized model of this paper, I show that entrants rather

[3]Hence, ex-ante heterogeneity in productivity conceptually differs from heterogeneity in, e.g., R&D costs.

[4]Further explanations include the increasing importance of intangible capital and information and communications technology (ICT) (Crouzet and Eberly, 2019; Chiavari and Goraya, 2020; De Ridder, 2024; Hsieh and Rossi-Hansberg, 2023; Weiss, 2019), rising productivity dispersion (Aghion, Bergeaud, Boppart, Klenow and Li, 2023), falling population growth (Bornstein, 2018; Engbom, 2023; Peters and Walsh, 2021; Hopenhayn, Neira and Singhania, 2022; Karahan, Pugsley and Şahin, 2022), declining interest rates (Chatterjee and Eyigungor, 2019; Liu, Mian and Sufi, 2022), changes in the quality of ideas (Olmstead-Rumsey, 2019) or declining imitation rates (Akcigit and Ates, 2023).

than incumbents account for the *fall* in economic growth since the 1990s. This finding is consistent with the observation in Garcia-Macia, Hsieh and Klenow (2019) that the share of economic growth accounted for by entrants has declined in the U.S. since the 1990s.[5][6]

The paper proceeds as follows. Section 2 documents the acceleration of firm life cycle growth, and Section 3 lays out the model. Section 4 explains the empirical findings across balanced growth paths and quantifies the aggregate implications. The transitional dynamics are computed in Section 6. Section 7 provides robustness, and Section 8 concludes.

2 Trends in firm size growth

This section documents systematic changes in sales and employment growth over the firms' life cycle. I describe the data in a first step.

2.1 Data

Data is provided by Statistics Sweden (SCB), the official statistical agency in Sweden. The main data set is *Företagens Ekonomi* (FEK), which covers information from balance sheets and profit and loss statements for the universe of Swedish firms at an annual frequency covering the period 1997-2017. FEK contains the main variables of interest: sales and employment (in full-time units). Before 1997, FEK was a sample covering large Swedish firms. To ensure full representativeness, I focus on the years 1997 forward. The data further contains information on the firm's legal type and industry at the five-digit level. I restrict the data to firms in the private economy. I focus on the unbalanced panel of firms, where, due to the universal coverage of the data, the disappearance of a firm is interpreted as firm exit. Throughout the paper, nominal variables are deflated to 2017 Swedish Krona (SEK) using the GDP deflator. For more details about the data, see Section A in the Appendix.

I define the birth year of the firm as the year it hires its first employee. I obtain this information from the auxiliary data set *Registerbaserad Arbetsmarknadsstatistik* (RAMS), containing the universe of employer-employee matches. I further restrict myself to firms that employ at least one worker, according to RAMS.[7]

Table 1 reports distributional statistics of firm sales, value added, and production inputs for the pooled data (1997 to 2017). The median firm lists sales of roughly 2.7 million SEK

[5]Klenow and Li (2021) find that internal innovation by incumbents drove the speedup and slowdown in growth. Mapping products to establishments, their identification infers the contribution of creative destruction and variety creation from the entry and exit of establishments, while internal innovation accounts for incumbents' establishment growth. As most firms feature significant growth over their life cycle yet remain single establishment firms, these assumptions assign a large role to internal innovation.

[6]Bartelsman and Doms (2000), Haltiwanger, Foster and Krizan (2001), Lentz and Mortensen (2008) and Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) decompose productivity growth further into within- and between firm effects. This paper studies which of these channels explains the *changes* in productivity growth since the 1990s.

[7]The empirical results of this section are very similar when measuring firm employment using RAMS.

Table 1: Summary statistics (1997-2017)

	25th Pct.	50th Pct.	75th Pct.	Mean	SD	Obs.
*Sales**	1.2	2.7	7.8	27.8	568.2	4,918,996
*Value added**	0.5	1.1	2.9	7.6	142.3	4,918,996
Employment	1	2	5	9.9	131.1	4,918,996
*Wage bill**	0.2	0.6	1.6	3.7	53.0	4,918,996
*Capital stock**	0.04	0.2	1.1	9.3	277.0	4,918,996
*Intermediate Inputs**	0.4	0.9	2.6	10.8	270.0	4,918,996

Note: variables marked with * are in units of million 2017-SEK (1 SEK $\approx$ 0.1 US dollars). The capital stock is defined as fixed assets minus depreciation.

(approx. 0.27 million US dollars), value added of 1.1 million SEK, and employs two workers. The distribution of sales, value added, and all production inputs is highly right-skewed, as indicated by the mean and the 25th, 50th, and 75th percentiles. Average firm sales are 27.8 million SEK, and average employment is 9.9. In total, the data includes about 4.9 million firm-year observations. For the age-specific empirical analysis, I focus on firms established in 1997 or later, which reduces the sample size to 2.2 million firm-year observations. For these firms, age is not truncated.

2.2 Changes in firm life cycle growth

I use a regression framework to obtain a non-parametric estimate of size growth over the firm's life cycle. More specifically, I run the following regression

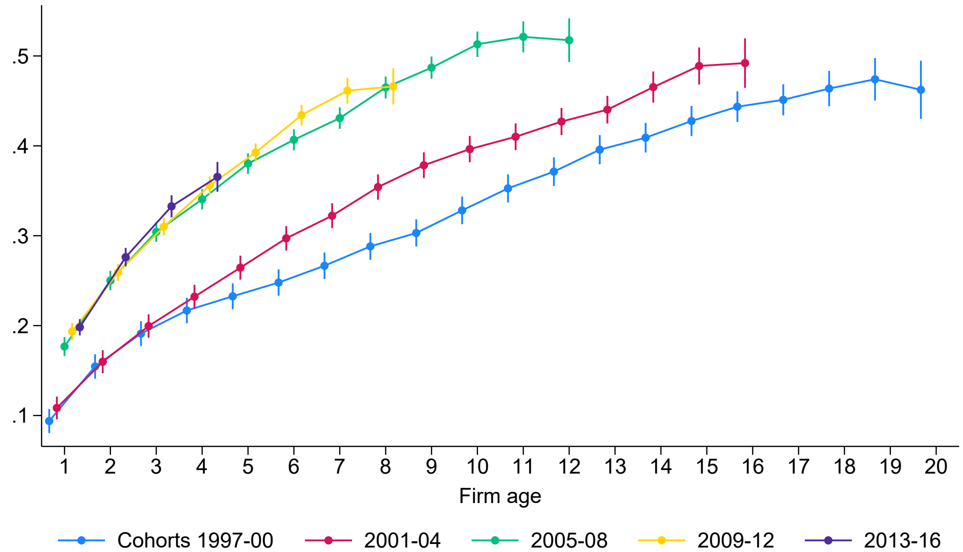
$$\ln \text{Size}_{j,t} = \gamma_0 + \sum_{a_f=1}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{j,t}=a_f} + \theta_c + \theta_k + \epsilon_{j,t}, \tag{1}$$

where $\mathbb{1}_{\text{Age}_{j,t}=a_f}$ is an indicator function for firms of age a_f. θ_k and θ_c denote 5-digit industry and cohort fixed effects.[8] Sedláček and Sterk (2017) argue that aggregate conditions at the year of entry have persistent effects on firm employment along the life cycle. Therefore, the baseline regression includes cohort fixed effects, but the results are robust to alternative specifications, as shown later. The constant γ_0 picks up the average log firm size at entry (age zero) and γ_{a_f} captures the log difference in average firm size between age a_f and zero, i.e., γ_1 to γ_{20} provide the non-parametric estimates of cumulative growth over the life cycle up to age 20. Firm employment and sales serve as the dependent variable in (1).

I run regression (1) for consecutive cohort groups (each group includes four cohorts) to capture changes in the life cycle profile over time. Figure 1 plots the age coefficients, γ_1 to

[8]The cohort and industry dependence of the other variables is suppressed for clarity.

Figure 1: Log employment relative to age zero (by cohort)

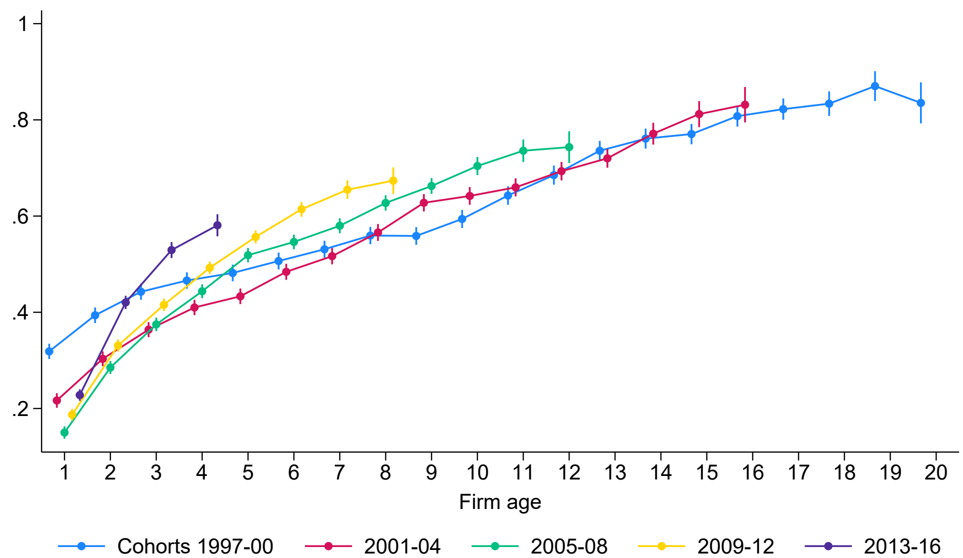


Notes: the figure shows cumulative employment growth over the firm's life cycle in Swedish registry data, measured as the difference between log employment at age a_f and age zero according to eq. (1). Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

γ_{20}, for the different cohort groups with employment as the dependent variable. Figure 1 contains the main empirical result: the age coefficients γ_1 to γ_{20} gradually increase for more recent cohorts. In other words, cumulative employment growth over the firm's life cycle has systematically increased since the late 1990s. For example, employment increased by about 29% over the first eight years for the cohorts 1997 to 2000, compared to 47% for the cohorts 2009 to 2012. About half of this increase is due to differences in growth over the first two years of the firm, with the remainder accruing over the later years.

I provide robustness checks for the above results; all contained in Appendix B.1. The baseline regression in equation (1) controls for industry and cohort fixed effects, as in Sterk, Sedláček and Pugsley (2021). The results are virtually unchanged with interacted industry and cohort fixed effects. One could alternatively argue that aggregate shocks that played out during the period under study affected firm employment independent of age. The acceleration of employment life cycle growth is even more pronounced with interacted industry and year fixed effects. Relatedly, selection effects among incumbent firms due to the Great Recession do not explain the acceleration of firm growth. I show the life cycle profiles of each cohort that followed the Great Recession, starting with the cohort in 2011. A steepening of the age gradient is visible for every single cohort. Other structural forces might, of course, have affected the selection of incumbent firms. Selection effects between (ex-ante) heterogeneous firms will be highlighted in the model analysis later on. Lastly, I explore robustness concerning the definition of an entering firm. Firms enter the economy at different times during the year. If, for some reason, firms enter towards the end of the year for the more recent cohorts, employment during the first year (age zero) is lower, and employment growth between age zero and one is higher. First, firm employment at entry is very stable over time, indicating no systematic changes in the nature of entrants. Second, I estimate a variant of regression (1), regressing log size on age dummies ranging from two to twenty. This way, firm life cycle growth is measured relative to the average firm size before age two. The results are similar to Figure (1), suggesting that the acceleration in size growth during the firms' early years is not affected by the classification of entrants.

Figure 2: Log sales relative to age zero (by cohort)



Notes: the figure shows cumulative sales growth over the firm's life cycle in Swedish registry data, measured as the difference between log sales at age a_f and age zero according to eq. (1). Cohorts are pooled as indicated in the legend. Firm sales are filtered at their 1% tails. The figure includes 95% confidence intervals.

The previous analysis focused on employment growth over the firm's life cycle. Firm sales growth also accelerated, yet at a muted rate. Figure 2 reports the age coefficients of regression (1) with sales as the dependent variable. The figure confirms the same patterns: sales growth over the firm's life cycle has accelerated. Over the first eight years of the firm, sales increased by about 56% for the cohorts 1997 to 2000, whereas sales increased by about 67% for the cohorts 2009 to 2012. The acceleration of sales life cycle growth is more suppressed than for employment, but a clear upward shift of the age coefficients is apparent. I provide the same robustness checks as for employment growth in Appendix B.1. If anything, the alternative entrant classification strengthens the acceleration of sales life cycle growth, as it mutes the employment growth during the early ages of the cohorts 1997-2000.

The evidence, hence, suggests that employment and sales growth over the life cycle accelerated, with a disproportionate increase in employment growth. What is driving the changes in firm growth? I build a structural model to study the cause behind these trends. Before turning to the model, the following presents external validity to the documented trends.

2.3 Trends in firm size in the U.S.

Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022) document that firm size conditional on age has been relatively stable in the U.S. since the 1990s, indicating no systematic changes in growth over the firm's life span. Importantly, both studies pool firms across all sectors of the economy when documenting firm size patterns. I find enormous heterogeneity in the firm size-conditional on age patterns across sectors. In most sectors, particularly the service sectors, firm size relative to entrants has increased. In some sectors, this increase is mild; in others, it is substantial. That the size patterns are stable when pooling firms across all sectors is due to firm size contracting in the manufacturing and accommodation sector, offsetting the increase in almost all other sectors.

I use data from the Business Dynamics Statistics (BDS) provided by the U.S. Census Bureau, covering nearly the universe of private sector firms with paid employees.[9] The data covers the period 1978–2021. For completeness, Appendix B.2 shows the patterns of firm size conditional on age, pooling firms across sectors. There are no systematic trends, as noted by Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022). However, this result masks enormous heterogeneity across sectors. Figure 3 reports the size-conditional-on age patterns for selected economically relevant sectors, where sector classifications follow the two-digit NAICS codes.[10] Log employment of entrants (age zero) is normalized to zero to facilitate comparability across time and sectors.

Figure 3 illustrates substantial heterogeneity in the firm-size trends across sectors. For example, employment of manufacturing firms aged 11-15 has shrunk relative to the average employment of entrants. For firms in the retail trade sector, firm-size patterns have been stable. In stark contrast, employment relative to that of entrants has increased substantially in the information sector, finance and insurance sector, professional, scientific and technical services sector, and the administrative and support services sector. In the information sector, employment of firms aged 11-15 increased by roughly 0.5 log points from the early 1990s to the late 2010s relative to employment of entrants. It is particularly noteworthy that the increase in the size gap in the information sector is not only a feature of the high-growth period of the late 1990s but has occurred steadily over the last three decades.

Figure 4 reports the change in the log employment gap between firms aged 11-15 and entrants for all two-digit NAICS sectors in the private economy from 1992 to 2017; in essence, the time change of the blue line in Figure 3 for all sectors of the U.S. economy. The year 2017 is chosen for comparability with Section 2.2. This choice is conservative given that the size gap in the service sectors further open up after 2017 in Figure 3.

Several observations are noteworthy. First, Figure 4 confirms the enormous heterogeneity in firm-size trends across sectors. Second, almost all sectors display an increase in the size of firms aged 11-15 relative to entrants. Only two out of the nineteen sectors experienced

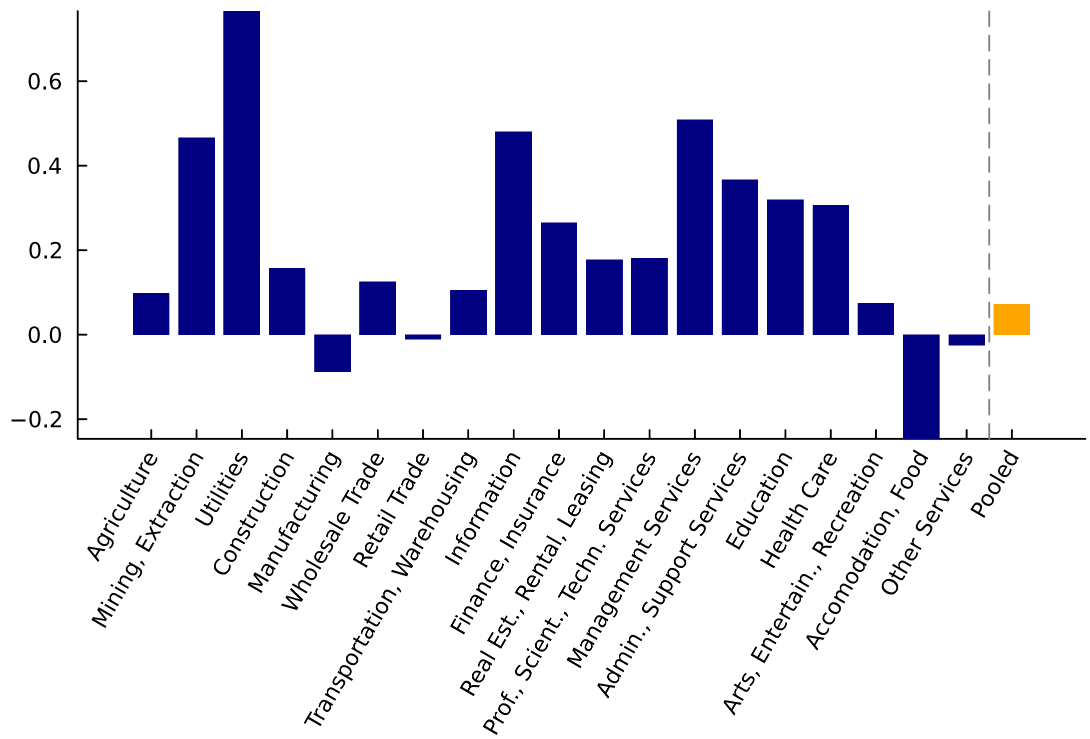
[9]Source: U.S. Census Bureau - Center for Economic Studies - Business Dynamics Statistics (2021), https://bds.explorer.ces.census.gov. Accessed July 13, 2024.

[10]For age groups 6-10 and 11-15, I skip the first four years to ensure a consistent age grouping over time.

Figure 3: Log employment by firm age and sector

Notes: the figure shows average log employment conditional on firm age in U.S. Census data. Log employment of entrants (age zero) is normalized to zero. Sector classifications correspond to two-digit NAICS codes.

Figure 4: Firm size (ages 11-15) relative to entrants, log change 1992-2017



Notes: the figure shows the change (1992–2017) in the gap between log employment of firms aged 11-15 and entrants in U.S. Census data. Sector classifications correspond to two-digit NAICS codes.

a meaningful decline, namely the manufacturing sector and the accommodation and food sector.[11] Even though the decrease in firm size in the U.S. manufacturing sector is interesting in its own right, Figure 3 shows that the decline entirely occurred during the 2000s, arguably driven by forces outside the analysis of this paper, in particular the China shock. The decrease in the size of firms aged 11-15 relative to entrants in the accommodation and food sector is surprising. However, this decline occurred after 2009, before which firms of this age experienced an increase in the average size of roughly 0.2 log points (not shown). Figure 4 further outlines that the increase in firm size relative to entrants is particularly widespread across the services sectors. This finding is consistent with the fact that the number of markets served per firm has increased for services firms, as documented by Hsieh and Rossi-Hansberg (2023). Third, when pooling firms across sectors (last column), the decline in these two sectors suffices to mute the increase in all other sectors. When pooling across all sectors, the change in the relative size of firms aged 11-15 is small yet positive.

One caveat is in order. The average age within the group of firms aged 11-15 has potentially increased itself. I argue that changes in the age distribution within the group are not the main driver behind the increase in their average size. First, Figure 3 shows that even the size

[11] The nature of the decline is different in both sectors, however. Whereas in manufacturing, the decline in total employment outweighs the fall in the number of firms (conditional on age), in the accommodation and food sector, the rise in the number of firms exceeds the increase in total employment.

of three-year-old firms is slightly increasing over time in the services sectors. Second, even in sectors that experienced large inflows of new firms, the size of firms aged 11-15 increased. One example is the information sector during the late 1990s and early 2000s. Third, the rise in the average size of firms aged 6-10 in the service sectors in Figure 3 is so enormous that their size in 2021 compares large to that of firms aged 11-15 in 1992. Even if the average age increased from 6.0 to 10.0 within the group (the most extreme example), age selection cannot explain why these firms nowadays are so much larger than the ones aged 11-15 in 1992. The increase in average size within the age groups is sheer too large.

To sum up, the size of mature firms has increased relative to entrants in almost all sectors of the U.S. economy. This suggests that firms, on average, grow faster over their life cycle, consistent with the findings in the Swedish administrative data.

3 Model

This section outlines an endogenous growth model with firm dynamics. I apply the model to study the cause behind the acceleration of firm life cycle growth in the later sections.

3.1 Preferences and aggregate economy

Time is continuous. The economy consists of a representative household that chooses the path of consumption C_t and wealth A_t to maximize lifetime utility

$$U = \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

subject to the budget constraint $\dot{A}_t = r_t A_t + w_t L_t - C_t$. ρ denotes the discount rate, r_t the interest rate and w_t the real wage. The household supplies one unit of labor inelastically, i.e., $L_t = 1$. The optimality condition (Euler equation) for the household problem reads

$$\frac{\dot{C}_t}{C_t} = r_t - \rho.$$

Aggregate output is produced competitively using a Cobb-Douglas technology over a continuum of different products indexed by i (time subscripts suppressed)

$$Y = \exp\left(\int_0^1 \ln\left[q_i y_i\right] di\right),$$

where y_i and q_i denote the quantity and quality of product i. Output is consumed entirely such that $Y = C$. Expenditure minimization leads to the standard demand function

$$y_i = \frac{YP}{p_i}. \tag{2}$$

P is defined as the aggregate price index, which I normalize to 1.

3.2 Production

Firms potentially produce in a product market i with a linear technology

$$y_{ij} = \varphi_j l_{ij},$$

where y_{ij} is the amount of product i produced by firm j, l_{ij} is the amount of labor hired, and φ_j denotes the productivity of firm j. Firm j produces different products with the same productivity, i.e., φ_j varies with j, but not with i. As in Aghion, Bergeaud, Boppart, Klenow and Li (2023), the firm's productivity is fixed over time, which captures the notion that some firms are persistently more efficient at producing than others, e.g., due to a better business plan. For simplicity, firms are of a high or low productivity type, i.e., $\varphi_j \in \{\varphi^h, \varphi^l\}$ where $\varphi^h/\varphi^l > 1$, which I refer to as high- and low-type firms.

3.3 Static allocation

Taking the joint distribution of product qualities and firm productivity as exogenous in this section, I characterize the static allocations at the product, firm and aggregate levels.

3.3.1 Product level

Firms in product market i compete in prices (Bertrand competition). In equilibrium, only the firm with the highest quality-adjusted productivity $q_{ij}\varphi_j$ produces product i (henceforth, incumbent). Under Bertrand competition, the incumbent firm engages in limit pricing and sets its price equal to the quality-adjusted marginal costs of the follower (the firm with the second highest quality-adjusted productivity)

$$p_{ij} = \frac{q_{ij}}{q_{ij'}} \frac{w}{\varphi_{j'}}, \tag{3}$$

where j' indexes the follower in product market i. The price that the incumbent sets is increasing in the quality gap between the incumbent and the follower, as eq. (3) shows. The price-cost markup in market i for producer j is

$$\mu_{ij} \equiv \frac{p_{ij}}{w/\varphi_j} = \frac{q_{ij}}{q_{ij'}} \frac{\varphi_j}{\varphi_{j'}}. \tag{4}$$

The incumbent's markup for product i is increasing in the quality and productivity gap. The price setting of the incumbent gives rise to the following profits for product i

$$\pi_{ij} = p_{ij}y_{ij} - wl_{ij} = Y\left(1 - \frac{1}{\mu_{ij}}\right),$$

with labor demand for product i

$$l_{ij} = \frac{Y}{w}\mu_{ij}^{-1}. \tag{5}$$

Employment in product line i is decreasing in the markup.

3.3.2 Firm level

Summing employment over the set of product lines where firm j is the incumbent, N_j,

$$l_j = \sum_{i \in N_j} l_{ij} = \frac{Y}{w}\left(\sum_{i \in N_j} \mu_{ij}^{-1}\right).$$

Firm employment decreases in the markups within each product line but increases in the total number of lines. Hence, holding markups constant, firms that produce in more product lines feature higher employment. Vice versa, holding the number of product lines constant, firms with higher markups employ less labor. As sales are equalized across product lines, firm sales are given by $\left|N_j\right|Y \equiv n_jY$, where n_j denotes the number of products firm j is producing. Hence, firms that produce in more product lines feature higher sales.

3.3.3 Aggregate level

Integrating employment across firms or products yields the total workforce in production:

$$L_P = \int_j l_j dj = \frac{Y}{w}\int_0^1 \mu_{ij}^{-1} di. \tag{6}$$

Taking logs and integrating eq. (4), one obtains an expression for the wage

$$w = \exp\left(\int_0^1 \ln q_{ij} di\right) \times \exp\left(\int_0^1 \ln \varphi_{j(i)} di\right) \times \exp\left(\int_0^1 \ln \mu_{ij}^{-1} di\right). \tag{7}$$

To find an expression for aggregate output, insert eq. (7) into eq. (6) to obtain

$$Y = Q\Phi\mathcal{M}L_P, \tag{8}$$

where

$$Q = \exp\left(\int_0^1 \ln q_{ij} di\right), \quad \Phi = \exp\left(\int_0^1 \ln \varphi_{j(i)} di\right), \quad \mathcal{M} = \frac{\exp\left(\int_0^1 \ln \mu_{ij}^{-1} di\right)}{\int_0^1 \mu_{ij}^{-1} di}.$$

Aggregate output Y depends on geometric averages of quality Q and productivity Φ as well as on misallocation $\mathcal{M}$ and production labor L_P. As highlighted in Peters (2020), misallocation arises from markup dispersion ($\mathcal{M}$ is bounded by unity from above). In this model, markup dispersion is due to both quality and productivity heterogeneity. The product of Q, Φ and $\mathcal{M}$ captures aggregate Total Factor Productivity (TFP).

3.4 Dynamic firm problem

Incumbents continuously improve the quality of products, q_i, in the economy through two different types of R&D. Internal R&D increases the quality of incumbent firm j's own product, whereas, through expansion R&D, the incumbent j improves the quality of a random product of a competing incumbent. Item quality is improved step-wise such that every innovation (either internal or expansion R&D) increases q_i by a factor of λ. As Aghion, Bergeaud, Boppart, Klenow and Li (2023), I assume that the step size of quality improvements exceeds the productivity differential, $\lambda > \varphi^h/\varphi^l$. This assumption ensures that the firm with the highest quality version in a product line is the incumbent producer.[12] Denoting by λ^{Δ_i} the relative qualities of incumbent and second-best firms within a product line, i.e.,

$$\lambda^{\Delta_i} = \frac{q_{ij}}{q_{ij'}}$$

and by $[\mu_i]$ the set of markups, where firm j is producing, firm profits can be written as

$$\pi_{jt}(n, [\mu_i]) = \sum_{k=1}^{n} Y_t \left(1 - \frac{1}{\mu_k}\right) = \sum_{k=1}^{n} Y_t \left(1 - \frac{1}{\lambda^{\Delta_k}} \frac{1}{\frac{\varphi_{jk}}{\varphi_{j'k}}}\right) \equiv \sum_{k=1}^{n} \pi(\mu_k),$$

where $\pi(\mu_i)$ are profits in product line i. Incumbent firms choose the rate of internal R&D, I_i, and the rate of expansion R&D, x_i, for each of their product lines, i. When choosing optimal internal and expansion R&D rates, firms take aggregate output Y_t, the real wage w_t, the share of lines operated by high-productivity firms S_t, the interest rate r_t and the rate of creative destruction τ_t as given. Denoting the time derivative by $\dot{V}_t^h()$, the value function of

[12] Relaxing this assumption would give room for a race for incumbency between low-productivity entrants facing a high-productivity incumbent from which I abstract.

a high-productivity type firm (indexed by h) satisfies the following HJB equation:

$$
\begin{aligned}
&r_t V_t^h(n,[\mu_i],S_t) - \dot{V}_t^h(n,[\mu_i],S_t) = \\
&\quad \sum_{k=1}^{n} \underbrace{\pi(\mu_k)}_{\text{Flow profits}} + \sum_{k=1}^{n} \underbrace{\tau_t \Big[V_t^h\left(n-1,[\mu_i]_{i\neq k},S_t\right) - V_t^h(n,[\mu_i],S_t) \Big]}_{\text{Creative destruction}} \\
&\quad + \max_{[x_k,I_k]} \Bigg\{ \sum_{k=1}^{n} \underbrace{I_k \Big[V_t^h\left(n,\left[[\mu_i]_{i\neq k},\mu_k\cdot\lambda\right],S_t\right) - V_t^h(n,[\mu_i],S_t) \Big]}_{\text{Internal R\&D}} \\
&\quad + \sum_{k=1}^{n} \underbrace{x_k \Bigg[S_t V_t^h\left(n+1,\left[[\mu_i],\lambda\right],S_t\right) + (1-S_t) V_t^h\left(n+1,\left[[\mu_i],\lambda\cdot\frac{\varphi^h}{\varphi^l}\right],S_t\right) - V_t^h(n,[\mu_i],S_t) \Bigg]}_{\text{Expansion R\&D}} \\
&\quad - \underbrace{w_t \Gamma\left([x_i,I_i];n,[\mu_i]\right)}_{\text{R\&D costs}} \Bigg\}.
\end{aligned}
$$

The value of a firm consists of flow profits, research costs, and three parts related to internal R&D, expansion R&D, and creative destruction. At the rate of creative destruction τ_t (determined in equilibrium), the firm loses one of its n products, in which case, it remains with $n-1$ products. At the optimally chosen rate I_k, internal R&D turns out successful (third row), and the firm charges a λ times higher markup on its product according to eq. (4). Alternatively, at the optimally chosen rate x_k, expansion R&D is successful (fourth row), and the firm acquires a new product (n increases by one).

Firm-type heterogeneity introduces new elements to the value function. First, the value function is specific to the productivity type of the firm. Second, the share of product lines operated by each productivity type is a state variable (with two types, it is sufficient to keep track of S_t). When taking over a new product line through expansion R&D (fourth row), the probability of replacing a high-type incumbent is S_t, in which case the high-type entrant charges a markup of λ. With probability $1-S_t$, the replaced incumbent is of the low type, and the high-type entrant charges a markup of $\lambda\cdot\varphi^h/\varphi^l$. Firms take S_t as given; however, they affect it through their expansion R&D efforts x_k in equilibrium. The HJB equation for a low-productivity firm follows the same structure and is listed in the Appendix, Section C.1. The term related to expansion R&D (fourth row) varies since low-productivity firms build different markup expectations when entering a new product line.

$\Gamma\left([x_i,I_i];n,[\mu_i]\right)$ denote the R&D costs. For their R&D activities, firms pay a cost of

$$
\Gamma\left([x_i,I_i];n,[\mu_i]\right) = \sum_{k=1}^{n} c\left(x_k,I_k;\mu_k\right) = \sum_{k=1}^{n} \left[\mu_k^{-1}\frac{1}{\psi_I}(I_k)^{\zeta} + \frac{1}{\psi_x}(x_k)^{\zeta} \right]
$$

in terms of labor. $\zeta>1$ ensures convexity of the cost function. R&D costs are additively separable to render a closed-form solution of the value function along the balanced growth path.

ψ_I and ψ_x scale internal and external R&D costs and capture the R&D efficiency.[13]

Firm entry is determined as follows. Potential entrants produce a flow rate of entry z_t using a technology that is linear in labor: $z_t = \psi_z L_{Et}$, where ψ_z denotes the entry efficiency and L_{Et} research labor by entrants. Entrants improve the quality of a randomly selected product line. The productivity type is realized after entry and assigned with the exogenous probabilities p^h and $1-p^h$, respectively. Entrants start with a one-step quality gap. When $z_t > 0$, the free entry condition requires that the expected value of firm entry equals the entry costs

$$p^h E[V_t^h(1,\mu_i)] + (1-p^h)E[V_t^l(1,\mu_i)] = \frac{1}{\psi_z} w_t, \tag{9}$$

where the expected value of entering as a high- or low-type firm is

$$\begin{aligned} E[V_t^h(1,\mu_i)] =& S_t V_t^h(1,\lambda) + (1-S_t)V_t^h(1,\lambda \times \varphi^h/\varphi^l) \\ E[V_t^l(1,\mu_i)] =& S_t V_t^l(1,\lambda \times \varphi^l/\varphi^h) + (1-S_t)V_t^l(1,\lambda). \end{aligned}$$

Labor market clearing requires that production labor L_{Pt} and total research labor L_{Rt} add up to one, the aggregate labor endowment

$$1 = L_{Pt} + L_{Rt} = \int_0^1 \frac{Y_t}{w_t}\mu_{it}^{-1} di + \int_0^1 \left(\mu_{it}^{-1}\frac{I_{it}^{\zeta}}{\psi_I} + \frac{x_{it}^{\zeta}}{\psi_x} \right) di + \frac{z_t}{\psi_z}. \tag{10}$$

3.5 Cross-sectional distribution of quality and productivity gaps

The joint (cross-sectional) distribution of quality and productivity gaps is the key equilibrium object that characterizes aggregates in the model. On the one hand, quality and productivity gaps characterize the markup distribution that determines labor demand. On the other hand, the distribution keeps track of the share of product lines operated by each productivity type, which is a state variable in the firm's optimization problem. This section characterizes the joint distribution of quality and productivity gaps as a function of firm policies, which allows the equilibrium distribution to be solved jointly with the policies.

The distribution of quality and productivity gaps ν is characterized by a set of infinitely many differential equations. For simplicity, I characterize the differential equations for firm-type specific expansion R&D rates, x_t^h and x_t^ℓ, and uniform internal R&D rates, I_t, as proven shortly in Proposition 1 for a balanced growth path.[14] For product lines where the incumbent

[13] The incentives for internal R&D decrease with the quality gap that the firm has accumulated as profits within a product line are concave in the markup. I scale the internal R&D costs by the inverse markup to keep internal R&D incentives constant as in Peters (2020).

[14] The distribution along the transition path is characterized in the Appendix, Section D.

is at least two quality steps ahead of the follower ($\Delta \geq 2$), the measure ν follows

$$\dot{\nu}_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right) = I_t \nu_t \left(\Delta - 1, \frac{\varphi_j}{\varphi_{j'}}\right) - \nu_t\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)(I_t + \tau_t). \tag{11}$$

For product lines where the incumbent is one step ahead ($\Delta = 1$), the measure follows

$$\begin{aligned}
\dot{\nu}_t\left(1, \frac{\varphi^l}{\varphi^h}\right) &= (1 - S_t)x_t^l S_t + z_t(1 - p^h)S_t - \nu_t\left(1, \frac{\varphi^l}{\varphi^h}\right)(I_t + \tau_t) \\
\dot{\nu}_t\left(1, \frac{\varphi^l}{\varphi^l}\right) &= (1 - S_t)x_t^l(1 - S_t) + z_t(1 - p^h)(1 - S_t) - \nu_t\left(1, \frac{\varphi^l}{\varphi^l}\right)(I_t + \tau_t) \\
\dot{\nu}_t\left(1, \frac{\varphi^h}{\varphi^h}\right) &= S_t x_t^h S_t + z_t p^h S_t - \nu_t\left(1, \frac{\varphi^h}{\varphi^h}\right)(I_t + \tau_t) \\
\dot{\nu}_t\left(1, \frac{\varphi^h}{\varphi^l}\right) &= S_t x_t^h(1 - S_t) + z_t p^h(1 - S_t) - \nu_t\left(1, \frac{\varphi^h}{\varphi^l}\right)(I_t + \tau_t).
\end{aligned} \tag{12}$$

Changes in the measure $\dot{\nu}$ are due to inflows and outflows. Outflows arise from successful internal R&D (quality gap increases from Δ to $\Delta+1$) and creative destruction (quality gap is reset to unity). Inflows vary with the quality gap. For $\Delta \geq 2$, inflows into state Δ are due to successful internal R&D in product lines with quality gaps of $\Delta-1$. For $\Delta = 1$, inflows result from creative destruction. For example, the measure of products with a low-type incumbent and high-type second best firm $\nu_t\left(1, \frac{\varphi^l}{\varphi^h}\right)$ increases due to low-type incumbents and entrants replacing high-type incumbents, captured by $(1 - S_t)x_t^l S_t + z_t(1 - p^h)S_t$ in eq. (12). From the measure ν, one obtains the share of product lines operated by high-type firms

$$S_t = \sum_{i=1}^{\infty}\left[\nu_t\left(i, \frac{\varphi^h}{\varphi^h}\right) + \nu_t\left(i, \frac{\varphi^h}{\varphi^l}\right)\right]. \tag{13}$$

3.6 Balanced growth path characterization

I define a balanced growth path of the economy as follows.

Definition 1. *A balanced growth path (BGP) is a set of allocations $[x_{it}, I_{it}, \ell_{it}, z_t, S_t, y_{it}, C_t]_{it}$ and prices $[r_t, w_t, p_{it}]_{it}$ such that firms choose $[x_{it}, I_{it}, p_{it}]$ optimally, the representative household maximizes utility choosing $[C_t, y_{it}]_{it}$, the growth rate of aggregate variables is constant, the free-entry condition holds, all markets clear and the distribution of quality and productivity gaps is stationary.*

Along the balanced growth path, the economy can be characterized in closed form.

Proposition 1. *In the above setup, along a balanced growth path:*

1. *The value of a product line for a firm of productivity type $d \in \{h, l\}$ is given by*

$$V_t^d(1, \mu_i, S) = \frac{1}{\rho + \tau}\left[Y_t\left(1 - \frac{1}{\mu_i}\right) + \frac{\zeta - 1}{\psi_x}(x^d)^\zeta w_t + \frac{\zeta - 1}{\psi_I} I^\zeta w_t \mu_i^{-1}\right] \tag{14}$$

where $x^h > x^l$ and $I \equiv I^h = I^l$. The value of a firm is $V_t^d(n, [\mu_i], S) = \sum_{i=1}^n V_t^d(1, \mu_i, S)$.

2. *S_{φ^k,φ^p}, the constant share of product lines where the incumbent firm is of productivity type k and the second-best firm of type p is*

$$\begin{aligned}
S_{\varphi^l,\varphi^h} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^l}{\varphi^h}\right) =& \frac{(1-S)x^l S + z(1-p^h)S}{\tau} \\
S_{\varphi^l,\varphi^l} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^l}{\varphi^l}\right) =& \frac{(1-S)x^l(1-S) + z(1-p^h)(1-S)}{\tau} \\
S_{\varphi^h,\varphi^h} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^h}{\varphi^h}\right) =& \frac{Sx^h S + zp^h S}{\tau} \\
S_{\varphi^h,\varphi^l} \equiv \sum_{i=1}^{\infty} \nu\left(i, \frac{\varphi^h}{\varphi^l}\right) =& \frac{Sx^h(1-S) + zp^h(1-S)}{\tau},
\end{aligned}$$

which defines the share of product lines operated by the high-productivity type

$$S = S_{\varphi^h,\varphi^h} + S_{\varphi^h,\varphi^l} = \frac{Sx^h + zp^h}{\tau}. \tag{15}$$

3. *The growth rate of aggregate variables is given by*

$$g = \frac{\dot{Q}_t}{Q_t} = \left(\underbrace{I}_{\textit{Incumbent internal R\&D}} + \underbrace{Sx^h + (1-S)x^l}_{\textit{Incumbent expansion R\&D}} + \underbrace{z}_{\textit{Entry}}\right) \times \ln(\lambda). \tag{16}$$

Proof. The Appendix, Sections C.1, C.2 and C.3, contains the proofs. □

The value of a product line in eq. (14) consists of three terms: profits for a given markup, the continuation value of expansion R&D, and the continuation value of internal R&D. The sum of the three terms is discounted by the discount rate and the rate of creative destruction. The more impatient the household or the higher the risk of replacement, the lower the value of a product line. Importantly, the value of a product line is productivity-type specific. More productive incumbents charge higher markups and enjoy greater profits in expectation. The optimality condition for expansion R&D (Appendix, eq. (35)) relates the expected value of a product line to the marginal cost of expansion R&D. Hence, in equilibrium, more productive firms pay a higher marginal cost of expansion R&D, i.e., $x^h > x^l$. Lastly, the value of a firm is equal to the sum of the value of its product lines.

Proposition 1 further shows that S_{φ^k,φ^p}, the share of products lines where the incumbent firm is of type k and the second-best firm of type p, is constant along a balanced growth path. This share equals the fraction of creatively destroyed products at each instant of time, where the new incumbent is of type k and the replaced firm of type p. The share of product lines operated by high-productivity type firms S is equal to the sum of S_{φ^h,φ^h} and S_{φ^h,φ^l}. In particular, eq. (15) can be rearranged to

$$S = \frac{zp^h}{(1-S)(x^l - x^h) + z}, \tag{17}$$

which shows that S depends on the difference in the expansion R&D rates between firm types, $x^l - x^h$. Holding firm entry z fixed, an increase in the expansion rate of high-productivity incumbents must be matched by an equal rise (in absolute terms) in the expansion rate of less-productive firms for S to remain constant. Note the importance of firm entry. With z set to zero, for equation (17) to hold, either x^l equals x^h or there is an exterior solution for S. Without firm entry and $x^l \neq x^h$, the faster expanding, more productive firms eventually take over all product lines. Positive firm entry is necessary for both firm types to co-exist in equilibrium where $x^l \neq x^h$ and S is constant at its interior solution.

The aggregate rate of creative destruction is equal to the sum of firm-type specific expansion R&D rates weighted by their sales shares and the rate of entry

$$\tau = Sx^h + (1-S)x^l + z. \tag{18}$$

Long-run growth results from R&D at the product level. This occurs through successful internal R&D or (incumbent and entrant) creative destruction. The aggregate arrival rate of innovation is, hence, equal to the sum of the rates of creative destruction τ and internal R&D I. Multiplying the arrival rate by the step size of innovation delivers the aggregate growth rate g, as shown in eq. (16) of Proposition 1. Since expansion R&D rates are heterogeneous, changes in the share of product lines operated by each productivity type, S and $1-S$, affect the aggregate growth rate. Along the balanced growth path, both τ and g are constant.

The stationary distribution of productivity and quality gaps further characterizes the aggregate labor income share, the TFP misallocation measure $\mathcal{M}$, and the aggregate markup. I derive these objects analytically in the Appendix, Section C.2.

To find the balanced growth path, I jointly solve the optimality conditions of the firm (derived in Appendix C.1), the free entry condition, eq. (9), the labor market clearing condition, eq. (10), and the system of differential equations characterizing the distribution of productivity and quality gaps captured in eqs. (11) and (12). Appendix C.4 contains the details.

3.7 Firm dynamics

Firms lose products according to the same stochastic process as in Klette and Kortum (2004). However, in this model, firms add products at systematically different rates as optimally chosen expansion R&D rates vary with the firm's productivity type.[15] As a result, expected firm life cycle trajectories differ across firms (Sterk, Sedláček and Pugsley, 2021). The following section derives firm-type specific sales and employment life cycle growth that will be mapped to the life cycle profiles obtained in the data.

3.7.1 Firm sales growth

Firm sales are proportional to the number of products a firm produces. As such, successful expansion R&D increases firm sales. Since optimal expansion R&D rates are productivity-type specific, so is expected sales growth. Conditional on survival, expected sales growth for a firm with productivity $\varphi^j, j \in \{h, l\}$ between age zero and age a_f is

$$E\left[\ln n_f Y | a_f, \varphi^j\right] - E\left[\ln n_f Y | 0, \varphi^j\right] = \underbrace{g \times a_f}_{\text{Aggregate growth}} + \underbrace{E\left[\ln n_f | a_f, \varphi^j\right]}_{\text{Firm's product growth}},$$

where n_f is the number of products the firm is producing. To derive $E\left[\ln n_f | a_f, \varphi^j\right]$ note that the probability of a high-productivity type firm producing n products at age a conditional on survival is $(1-\gamma^j(a))\left(\gamma^j(a)\right)^{n-1}$, where $\gamma^j(a) = x^j \frac{1-e^{-(\tau - x^j)a}}{\tau - x^j e^{-(\tau - x^j)a}}$ and $x^j \in \{x^h, x^l\}$. Therefore expected sales growth is given by

$$E\left[\ln n_f Y | a_f, \varphi^j\right] - E\left[\ln n_f Y | 0, \varphi^j\right] = \underbrace{g \times a_f}_{\text{Aggregate growth}} + \underbrace{\left(1 - \gamma^j(a_f)\right) \sum_{n=1}^{\infty} \ln n \times \left(\gamma^j(a_f)\right)^{n-1}}_{\text{Firm's product growth}}. \tag{19}$$

Relative sales growth of the firm is equal to the firm's product growth.

3.7.2 Firm markup growth

Firm markups are defined as $\mu_f = \frac{p y_f}{w l_f}$. The Appendix, Section C.5 shows that for a high-productivity type firm, the expected log markup conditional on firm age a_f is

$$E\left[\ln \mu_f | a_f, \varphi^h\right] = \underbrace{\ln \lambda \times \left(1 + I \times E[a_P^h | a_f]\right)}_{\text{Quality improvements}} + \underbrace{(1 - S) \times \ln\left(\frac{\varphi^h}{\varphi^l}\right)}_{\text{Productivity advantage}}, \tag{20}$$

[15] Therefore, the properties related to firm size growth and survival in Klette and Kortum (2004) hold conditional on the firm type. In particular, conditional on the type, firm size, and (expected) growth are unrelated, i.e., Gibrat's law holds conditionally as in Lentz and Mortensen (2008). For the unconditional firm size and growth correlation, two forces are at play. On the one hand, young (small) firms tend to grow quicker due to survival bias. On the other hand, more productive firms (with faster growth rates) are more likely to end up large. In the estimated (initial) balanced growth path, 74% of the firms are of the high productivity type. Hence, size is unrelated to growth (in expectation) for the vast majority of firms.

where $E[a_P^h|a_f]$, the average product age of a high-type firm conditional on firm age, is

$$E[a_P^h|a_f] = \frac{1}{x^h}\left(\frac{\frac{1}{\tau}\left(1-e^{-\tau a_f}\right)}{\frac{1}{x^h+\tau}\left(1-e^{-(x^h+\tau)a_f}\right)} - 1\right)\left(1-\phi^h(a_f)\right) + a_f\phi^h(a_f)$$
$$\phi^h(a) = e^{-x^h a}\frac{1}{\gamma^h(a)}\ln\left(\frac{1}{1-\gamma^h(a)}\right)$$
$$\gamma^h(a) = \frac{x^h\left(1-e^{-(\tau-x^h)a}\right)}{\tau - x^h e^{-(\tau-x^h)a}}.$$

The expected firm markup conditional on age consists of two terms. The first term in eq. (20) is akin to Peters (2020) and, reflects that internal R&D translates quality improvements within a firm's product line into markup growth at the firm level as it ages. In Peters (2020), this term holds for all firms, whereas in this model, this term is specific to the productivity type of the firm, as the average product age varies by firm type. The second term in eq. (20) captures a new level effect that heterogeneity in productivity introduces. The intuition is that if a high-productivity type incumbent faces a low-type second-best firm in a given line, it can charge a φ^h/φ^l higher markup, which occurs in expectation in $1-S$ of the incumbent's product lines.

The expected markup conditional on firm age for a low-productivity type firm follows

$$E\left[\ln\mu_f|a_f,\varphi^l\right] = \underbrace{\ln\lambda\times\left(1+I\times E[a_P^l|a_f]\right)}_{\text{Quality improvements}} + \underbrace{S\times\ln\left(\frac{\varphi^l}{\varphi^h}\right)}_{\text{Productivity disadvantage}}. \tag{21}$$

The first term captures quality improvements through internal R&D, equivalently to eq. (20). $E[a_P^l|a_f]$ follows the same expression as $E[a_P^h|a_f]$ with h replaced by l. The second term in eq. (21) differs from eq. (20). Low-productivity incumbents face a high-productivity second-best firm in a share S of their product lines. Since $\varphi^l < \varphi^h$, this term is negative.

3.7.3 Firm employment growth

Average employment conditional on age and firm type is equal to

$$E[\ln l_f| = a_f,\varphi^j] = \ln\left(\frac{Y}{w}\right) + E\left[\ln n_f|a_f,\varphi^j\right] - E\left[\ln\mu_f|a_f,\varphi^j\right].$$

Since $\frac{Y}{w}$ is constant along the balanced growth path, employment growth is given by

$$E[\ln l_f|a_f,\varphi^j] - E[\ln l_f|0,\varphi^j] = \underbrace{E\left[\ln n_f|a_f,\varphi^j\right]}_{\text{Firm's product growth}} - \underbrace{\left(E\left[\ln\mu_f|a_f,\varphi^j\right] - E\left[\ln\mu_f|0,\varphi^j\right]\right)}_{\text{Firm's markup growth}}, \tag{22}$$

where $E[\ln n_f|a_f, \varphi^j]$ and $E[\ln \mu_f|a_f, \varphi^j] - E[\ln \mu_f|0, \varphi^j]$ are defined in eqs. (19)-(21). Employment growth equals sales growth minus markup growth.

3.7.4 Firm survival and unconditional life cycle growth

Firm size dynamics determine firm survival. Since firm size growth is type-dependent, so is firm survival. The survival function in Klette and Kortum (2004) holds conditional on the firm type, i.e., the share of high and low type firms surviving until age a_f is

$$\chi^h(a_f) = 1 - \tau \frac{1 - e^{-(\tau - x^h)a_f}}{\tau - x^h e^{-(\tau - x^h)a_f}} \tag{23}$$

$$\chi^l(a_f) = 1 - \tau \frac{1 - e^{-(\tau - x^l)a_f}}{\tau - x^l e^{-(\tau - x^l)a_f}}. \tag{24}$$

The firm survival function can be used to compute firm sales, and employment growth unconditionally of the firm type. The share of high-type firms among firms at age a_f is

$$s^h(a_f) = \frac{p^h \chi^h(a_f)}{p^h \chi^h(a_f) + (1 - p^h)\chi^l(a_f)}. \tag{25}$$

The share corresponds to the mass of high-type survivors relative to the total mass of survivors. Unconditional employment growth between age zero and a_f is then given by

$$s^h(a_f)\left(E[\ln l_f|a_f, \varphi^h] - E[\ln l_f|0, \varphi^h]\right) + \left(1 - s^h(a_f)\right)\left(E[\ln l_f|a_f, \varphi^l] - E[\ln l_f|0, \varphi^l]\right). \tag{26}$$

Unconditional sales growth is defined similarly. When estimating the model, I match eq. (26) to observed employment growth in the data.

3.7.5 Firm size distribution

The model makes precise predictions about the firm size distribution. I derive the firm size distribution in Section C.6 in the Appendix. Denoting by M^h the mass of high-productivity type firms and by M the total mass of firms, I compute the share of high-productivity type firms in the cross-section as

$$S_{M^h} = \frac{M^h}{M}, \tag{27}$$

and the firm entry rate as

$$\text{Firm entry rate} = \frac{z}{M}. \tag{28}$$

4 Explaining the changes in firm life cycle growth

This section applies the model to explain the documented changes in firm sales and employment life cycle growth. To this extent, I estimate the model along two balanced growth paths. The initial balanced growth path captures firm life cycle growth and aggregate economic conditions during the 1990s. I then re-estimate model parameters to explain the changes in firm life cycle growth of the latest cohorts in the data.

4.1 Initial balanced growth path

There are, in total, eight parameters in the model. The internal R&D efficiency ψ_I, the expansion R&D efficiency ψ_x, the innovation cost curvature ζ, the entry efficiency ψ_z, the step size of innovation λ, the productivity differential φ^h/φ^ℓ, the share of high-productivity type firms among entrants p^h, and the discount rate ρ. Two parameters are set exogenously, and the remaining parameters are estimated. I follow Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) and Peters (2020) that set ζ equal to two based on evidence from the microeconometric innovation literature (Blundell, Griffith and Windmeijer, 2002; Hall and Ziedonis, 2001). The discount rate ρ is set to 0.02, resulting in an annual discount factor of roughly 0.97%.

The remaining six parameters are estimated, targeting moments of firm life cycle growth as well as cross-sectional firm heterogeneity and economic aggregates. In particular, I target firms' sales and employment life cycle growth, dispersion in inverse labor shares across entrants, the firm entry rate, TFP growth, and the aggregate markup. Despite all parameters being identified jointly, there is a tight mapping between parameters and targets.

Matching sales and employment growth disciplines the firms' R&D efficiencies ψ_x and ψ_I. In the model, successful expansion R&D translates into sales growth. In the estimation, ψ_x adjusts expansion R&D costs such that sales life cycle growth in the model matches that in the data. The internal R&D costs govern firms' markup growth. Since markup growth drives a wedge between sales and employment growth, targeting employment and sales growth jointly disciplines markup growth and, hence, the internal R&D efficiency ψ_I. The advantage of targeting employment instead of markup growth is that employment is directly observed in the data. I target sales and employment growth over the first eight years of the firm. This period is long enough to capture firms' life cycle growth and still allows for estimating separate balanced growth paths (one for the early cohorts and one for the latest cohorts) over the data coverage period from 1997 to 2017. The model matches growth over the firm's life cycle well, so the specific age targeted is not consequential. In the model, sales and employment growth are specific to the productivity type of the firm. In the data, the productivity type is unobserved. I match observed sales and employment growth in the data with unconditional (of the productivity type) firm growth, defined in eq. (26).[16]

[16]The alternative is to impute the productivity type of a firm in the data and to measure type-specific sales and employment growth. I match firm life cycle growth, unconditional of the productivity type, to avoid classifying firms incorrectly into types that would affect the parameter estimates and the firm composition. Matching type-specific growth would further require setting the productivity threshold exogenously.

Therefore, the composition of firm types conditional on firm age (an equilibrium outcome) is such that type specific life cycle growth weighted by the (age-conditional) type composition matches observed growth in the data. For the initial balanced growth path, I target sales and employment growth of the cohorts 1997 to 2000. For these cohorts, sales grew by 55.9% and employment by 28.8% over the first eight years of the firm.

The entry rate helps identify the entry efficiency of firms ψ_z. I compute the entry rate in the data as the share of firms equal to or less than one year of age. This results in an average entry rate over the period 1997-2005 of 14.3%, in line with Engbom (2023). I match this number with the model-implied entry rate in eq. (28).

Aggregate TFP growth disciplines the step-size improvement of innovation λ: the growth rate of TFP in eq. (16) directly depends on λ. I obtain TFP growth for the Swedish economy from Federal Reserve Economic Data (FRED) in labor augmenting terms.[17] After suffering a financial crisis in the early 90s, Sweden's economy featured strong growth towards the end of the century. During 1997–2005, TFP grew by 3.02% per year.

To pin down the productivity differential φ^h/φ^ℓ, I target the aggregate markup. The aggregate markup is a weighted average of product markups that, in return, depend on φ^h/φ^ℓ. Sandström (2020) and De Loecker and Eeckhout (2018) report sales-weighted markups for the Swedish economy. Sandström (2020) computes the markup in Swedish registry data focusing on firms with at least ten employees, whereas De Loecker and Eeckhout (2018) focus mainly on publicly listed firms. I target the average of both reported aggregate markups, resulting in a conservative estimate of 7.5%. Lastly, I target the standard deviation of log inverse labor shares across entering firms (sales relative to the wage bill). Given φ^h/φ^ℓ, the dispersion of labor shares at entry depends on the share of product lines operated by high-type firms (determined in equilibrium) and the share of high-type firms among entrants (the parameter p^h). The dispersion of inverse labor shares across entrants, hence, disciplines p^h. The standard deviation of log inverse labor shares of entering firms, averaged over 1997-2005, equals 0.053.[18] All targets are summarized in Table 2.

The estimation follows a two-step approach. In the first (global) step, the algorithm computes the sum of squared percentage deviations from the targeted moments for a large Sobol sequence of parameter vectors. All targets receive equal weights. In the second (local) step, I take the best candidates from the first step and perform a local search. The local search, again, minimizes the distance from the targets. The best parameter vectors from the second step converge to the same parameter values.

Table 2 shows the estimation results. The model replicates all targeted moments well. The

[17] FRED series RTFPNASEA632NRUG. The labor share is obtained from FRED, series LABSHPSEA156NRUG, averaged over 1997–2005.

[18] For firms with a low wage bill, inverse labor shares explode. Therefore, I focus on firms with a sales-to-wage bill ratio between one and three (model implied markups between 0% and 200%). Further, sales relative to the wage bill in the data may vary for reasons outside the model. I bin firms into equally sized groups based on their capital and intermediate inputs and compute the dispersion of log inverse labor shares across firms within these groups.

Table 2: Initial balanced growth path. Moments and parameters

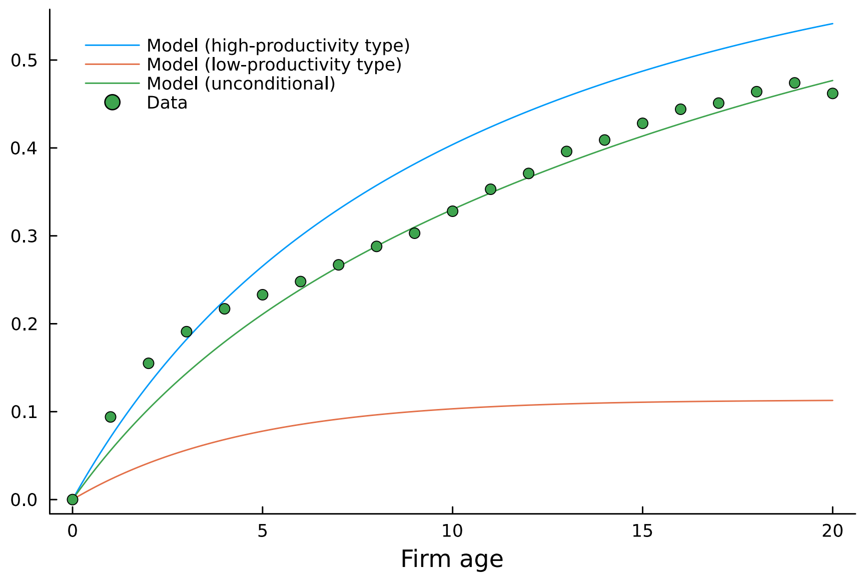
	Data	Model
Moments		
Sales growth by age 8 in % (cohorts 1997–2000)	55.9	55.8
Employment growth by age 8 in % (cohorts 1997–2000)	28.8	28.8
Cross-sectional SD of log labor shares across entrants (1997–2005)	0.053	0.053
TFP growth g in % (1997–2005; FRED)	3.02	3.02
Entry rate in % (1997–2005)	14.3	14.3
Agg. markup μ in % (Sandström, 2020; De Loecker and Eeckhout, 2018)	7.5	7.5
Parameters		
ψ_I *Internal R&D efficiency*		0.144
ψ_x *Expansion R&D efficiency*		0.282
ψ_z *Entry R&D efficiency*		1.483
λ *Step size of innovation*		1.136
φ^h/φ^ℓ *Productivity gap*		1.091
p^h *Share of high type among entrants*		0.683
Set exogenously		
ρ *Discount rate*		0.02
ζ *R&D cost curvature*		2

Notes: except for aggregate productivity (TFP) growth and μ, the moments are computed using Swedish registry data. TFP growth is obtained from Federal Reserve Economic Data (FRED), series RTFPNASEA632NRUG, in labor augmenting terms (the labor share is obtained from FRED, series LABSHPSEA156NRUG, averaged over the same period 1997–2005).

estimated parameters can be interpreted as follows: successful innovation increases product quality by 13.6%. High and low-type firms' productivity differs by 9.1%, and 68.3% of firms enter the economy as high-type firms. The share of high-type firms at entry is relatively high, consistent with the relatively moderate productivity advantage. For the interpretation of the results, it is more suitable to think of high- and low-type firms as above- and below-median firms (at entry) rather than superstar firms vs. the rest.

Along the balanced growth path, the constant share of high-productivity type firms in the cross-section, S_{M^h} in eq. (27), equals 74%. This number is larger than their share at entry ($p^h = 0.683$) due to high-type firms choosing higher expansion R&D rates than low-type firms: $x^h - x^\ell = 0.075$. This is reflected in their life cycle growth. Conditional on survival, employment grows on average by 36% for high-type firms but only by 10% for low-type firms over the first eight years. Weighted by the share of each firm type among surviving firms at age eight as in eq. (26), this results in employment growth of 28.8%, as reported in Table 2. Figure 5 shows the employment growth trajectory for each productivity type over the entire life cycle. The figure clearly illustrates the heterogeneity in employment growth profiles. Over the first 20 years, the difference in employment growth between both productivity types amounts to roughly 0.4 log points. The figure further reports the employment growth trajectory unconditional of the firm type, as well as the observed employment growth in the data. Despite being untargeted (except for age eight), the model provides an exceptional fit of the entire employment growth trajectory.

Figure 5: Employment life cycle growth



Notes: the figure shows log employment growth over the firm's life cycle in the model (initial balanced growth path) and the data (cohorts 1997–2000 in Swedish registry data). Employment growth at age eight has been targeted.

Sterk, Sedláček and Pugsley (2021) emphasize the importance of ex-ante heterogeneity in firm life cycle trajectories. In this model, heterogeneity in expected life cycle trajectories arises from heterogeneous expansion R&D rates (x^h and x^l) specific to the firm's productivity type. I provide suggestive evidence that firms with permanently higher productivity are associated with faster life cycle growth in the data, see Section 7.2.

4.2 New balanced growth path

This section estimates the model on a new balanced growth path that replicates the changes in firm life cycle growth vis-a-vis the initial balanced growth path. To replicate the changes in firm sales and employment growth (two moments), I re-estimate two parameters, particularly the internal R&D efficiency ψ_I and the entry efficiency ψ_z. These two parameters are promising candidates because one affects sales and employment growth jointly, whereas the other moves employment relative to sales growth, as explained shortly. I test alternative parameter changes as a robustness check.

Table 3 shows the changes in the targeted moments and estimated parameters. For the cohorts 2009 to 2012, sales growth over the first eight years averaged 67.4% (an increase of 11.5pp relative to the cohorts 1997 to 2000) and employment growth 46.6% (an increase of 17.8pp). The model matches these changes by lowering the internal R&D efficiency by 51%

Table 3: New balanced growth path. Moments and parameters

	Data (%)	Model (%)	ΔBGPs (pp)
Moments			
Sales growth by age 8 (cohorts 2009–2012)	67.4	67.4	+11.5
Employment growth by age 8 (cohorts 2009–2012)	46.6	46.6	+17.8
Parameters			
ψ_I *Internal R&D efficiency* (Δ in %)			-51.0
ψ_z *Entry R&D efficiency* (Δ in %)			-22.0

Notes: the column ΔBGPs reports the difference between ending and initial balanced growth path moments (in percentage points) and parameters (in percent).

and the entry efficiency by 22%, i.e., by raising the cost of internal R&D and firm entry. Rising entry costs are consistent with Davis (2017) and Gutiérrez and Philippon (2018), who argue that the increasing complexity of regulatory requirements and lobbying expenditures disadvantage entrants. The increase in internal R&D costs is reminiscent of the observation in Bloom, Jones, Van Reenen and Webb (2020) that research productivity has fallen in the U.S. Section 7.3 discusses potential forces behind the rise in the cost of entry and internal R&D in more detail.

How does the estimated rise in internal R&D and entry costs affect employment and sales life cycle growth? Table 4 shows the effect of each parameter change on incumbents' innovation rates. The rise in the internal R&D costs lowers internal R&D rates by 49.4% relative to the initial balanced growth path for both productivity types (second column). The fall in internal R&D rates slows markup growth and accelerates employment life cycle growth according to eqs. (20) and (22). In contrast, the rise in the entry costs increases expansion R&D rates (+32.8% for the high-productivity type and +1.3% for the low type). To see why expansion R&D rates increase, note that the optimality condition for the expansion R&D rate equates the expected value of a product line with the marginal cost of expansion R&D. Firm entry, as part of the rate of creative destruction, deflates the value of a product line in eq. (14). A fall in firm entry raises the value of a product line such that firms optimally choose to pay a higher marginal cost of expansion R&D. As expansion R&D results in firm sales and employment growth according to eqs. (19) and (22), the increase in expansion R&D rates accelerates both sales and employment life cycle growth. Hence, rising entry costs increase sales and employment life cycle growth while rising internal R&D costs increase employment *relative* to sales growth.

Importantly, Table 4 shows that the rise in the entry costs increases the expansion rates for incumbents of both productivity types, but disproportionately so for the more productive ones. This asymmetry arises as follows. The entry rate z deflates the value of a product line in eq. (14). In the limit where firm entry becomes costless, the value of a product line converges to zero for both high- and low-productivity incumbents, and productivity

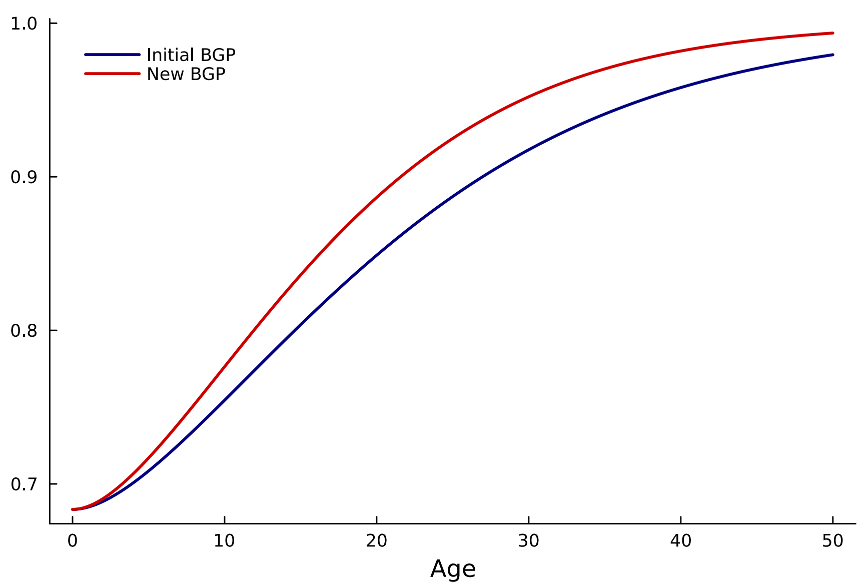
Table 4: Changes in innovation rates

	Initial BGP	$\psi_I \downarrow$ (%)	$\psi_z \downarrow$ (%)	$\psi_I \downarrow, \psi_z \downarrow$ (%)
x^h	0.1187	+1.0	+32.8	+35.0
x^ℓ	0.0439	-5.9	+1.3	-11.5
I^h, I^ℓ	0.0367	-49.4	+13.8	-41.9

Notes: the table shows the change in firm expansion R&D rates (by productivity type, x^h and x^ℓ) and internal R&D rates in percent. $\psi_I \downarrow$ denotes the 51% fall in the internal R&D efficiency and $\psi_z \downarrow$ the 22% fall in the entry efficiency.

differences become irrelevant. Intuitively, the stream of profits in a product line is higher for more productive firms with larger markups, which becomes inconsequential if incumbents get replaced instantly by new entrants. Hence, a fall in firm entry increases the difference in the value of a product line between more and less productive firms, which widens the gap in their optimal expansion R&D rates. Table 4 shows that the rise in internal R&D costs also has heterogeneous effects on the expansion R&D rates. To see why this is the case, note that the continuation value of internal R&D in eq. (14) is higher for less productive firms that, so far, have accumulated fewer markups. Hence, the rise in the internal R&D costs disproportionately lowers the value of a product line for less productive firms, which decrease their expansion R&D rates. The last column of Table 4 shows that the rise in entry and internal R&D costs together increase the expansion R&D rates of high-type firms by 35% while lowering the low-type expansion R&D rates by 11.5%. Hence, the observed acceleration of firm life cycle growth is entirely due to accelerating life cycle growth of the more productive incumbents.

Figure 6: Share of high-productivity type firms



Notes: the figure shows the share of high-productivity type firms among firms of age a_f, $s^h(a_f)$ in eq. (25), for the initial and new balanced growth path.

The changes in expansion R&D rates by high- and low-productivity firms imply changes in firm selection. Figure 6 shows the share of high-productivity type firms among firms of age a_f, defined by $s^h(a_f)$ in eq. (25), for the initial and new balanced growth path. For any balanced growth path, this share equals p^h at age zero and converges to one with firm age. Among older firms, only high-type firms are represented as their expansion R&D rates exceed the ones of low-type firms. Figure 6 shows that the share of high-type firms increases relative to the initial balanced growth path for all ages. Since product market incumbency is tied to firm survival, the increase in expansion rates of more productive relative to less productive firms increases their share among surviving firms at any age.

Firm selection conditional on age translates into firm selection in the cross-section of firms. Integrating the share of high-type firms conditional on age in Figure 6 over the firm-age distribution yields the share of high-type firms in the cross-section, defined by S_{M^h} in eq. (27). The cross-sectional share of high-type firms increases by 12pp across the balanced growth paths. Selection effects at the product level are even larger than at the firm level. The cross-sectional sales share of high-type firms, S, increases by 17pp. The sales share of high-type firms increases by more than their share in the cross-section of firms, as low-type firms with more than one product lose sales shares without exiting the economy.

5 Long-run macroeconomic implications

What are the implications for the aggregate economy? The rise in the cost of entry and internal R&D cause a long-run fall in the growth rate g and firm entry: the aggregate growth rate declines by 0.62pp, and the firm entry rate drops by 8pp. In Sweden, average TFP growth between 2010 and 2015 declined by about 1pp relative to 1997–2005. Further, Engbom (2023) documents a fall in the entry rate by about 10pp from the early 1990s to the mid-2010s in the Swedish economy. The comparative statics, therefore, account for roughly 60 percent of the fall in economic growth and 80 percent of the decline in firm entry since the 1990s. The reallocation of market shares to more productive incumbents further increases aggregate productivity, Φ in eq. (8), by 1.5%. The rise in aggregate productivity and fall in the long-run growth rate g pose contrasting level and growth effects on aggregate output that leave the implications for welfare ambiguous. The next section examines the effect on welfare. Further, the reallocation of sales shares to more productive firms that, in the model, feature relatively low labor shares is qualitatively consistent with Kehrig and Vincent (2021). Similarly, De Loecker, Eeckhout and Unger (2020) and Baqaee and Farhi (2020) document a reallocation of sales shares to firms with relatively high markups in Compustat data.[19] In sum, the observed acceleration in firm sales and employment life cycle growth is consistent with falling firm entry, rising concentration, a slowdown in productivity growth, and a reallocation of market shares to low-labor share firms.

[19] In the estimated model, expected differences in markup growth are small compared to the difference in markup levels at birth between high- and low-productivity firms, so most high-productivity firms remain high-markup (low-labor share) firms throughout.

Incumbent innovation, reallocation, entry and growth

What drives the long-run fall in the aggregate growth rate? This section quantifies the different contributions by incumbents and entrants to the fall in long-run growth. The aggregate growth rate g naturally lends itself to such decomposition. Along a balanced growth path, the aggregate growth rate defined in eq. (16), can be written as

$$g = Sg^h + (1 - S)g^\ell + g^z,$$

where $g^h \equiv (I + x^h)\ln(\lambda)$, $g^\ell \equiv (I + x^\ell)\ln(\lambda)$ and $g^z \equiv z\ln(\lambda)$ capture contributions by high-type incumbents, low-type incumbents, and entrants to economic growth. Note that for the total contribution by incumbents, their innovation rates and the share of product lines operated by each type matter. Using a shift-share decomposition, I decompose changes in the growth rate across balanced growth paths, $\Delta g \equiv g_{new} - g_{old}$, as follows

$$\Delta g = \underbrace{S_{old}\Delta g^h + (1 - S_{old})\Delta g^\ell}_{\Delta\text{Within}} + \underbrace{g^h_{old}\Delta S - g^\ell_{old}\Delta S}_{\Delta\text{Between}} + \underbrace{\Delta g^h\Delta S - \Delta g^\ell\Delta S}_{\Delta\text{Cross}} + \underbrace{\Delta g^z}_{\Delta\text{Entry}}, \tag{29}$$

where *old* and *new* index balanced growth path variables before and after the parameter change. Changes in the aggregate growth rate are due to changes in innovation rates holding the distribution of sales shares constant (ΔWithin), due to changes in the distribution of sales shares holding innovation rates constant (ΔBetween), due to changes in both innovation rates and sales shares (ΔCross) as well as due to changes in firm entry (Δg^z). The ΔWithin, ΔBetween, and ΔCross terms capture changes due to incumbents, whereas Δg^z captures changes due to entrants. Because the ΔCross term is absent without firm type heterogeneity, I group the ΔBetween and ΔCross-term into a common ΔReallocation term.

Table 5: Decomposing the fall in the aggregate growth rate

	$\psi_I \downarrow, \psi_z \downarrow$	$\psi_I \downarrow$	$\psi_z \downarrow$
ΔWithin	+0.22	-0.23	+0.47
ΔReallocation	+0.27	+0.01	+0.20
ΔEntry	-1.10	-0.11	-0.93
Δg	-0.62	-0.33	-0.26

Notes: the table shows the contributions to the change in the aggregate growth rate g across the balanced growth paths according to the decomposition in eq. (29) in percentage points. ΔReallocation is the sum of the ΔBetween and ΔCross terms. g in the initial balanced growth path is equal to 3.02%. $\psi_I \downarrow$ denotes the 51% fall in the internal R&D efficiency and $\psi_z \downarrow$ the 22% fall in the entry efficiency.

Table 5 quantifies the different contributions to the fall in the aggregate growth rate. First, the ΔWithin term is positive at 0.22pp, indicating that incumbents' innovation rates increased. Second, the reallocation of sales shares to more productive firms that endogenously feature higher innovation rates contributed positively to economic growth. The ΔReallocation term is positive at 0.27pp. Changes in incumbent innovation (ΔWithin + ΔReallocation) raised the aggregate growth rate by a total of 0.49pp. ΔReallocation accounts for 55% (0.27/0.49) of the total contribution by incumbent firms. Thus, incumbents mainly contributed to changes in long-run growth through the reallocation of sales shares to more innovative firms. This channel is absent in standard models of creative destruction where firms innovate at identical rates. Lastly, falling firm entry lowers the aggregate growth rate substantially by 1.1pp. The fall in firm entry dominates the positive contribution by incumbents, resulting in a total decline of the growth rate of 0.62pp. Falling firm entry squares the rise in incumbent innovation with a fall in aggregate economic growth.

That the ΔWithin term is positive may be surprising given that R&D costs of incumbents have increased. Columns 3 and 4 of Table 5 repeat the decomposition for each parameter change in isolation. The ΔWithin effect of a rise in the internal R&D costs is negative (-0.23pp). At the same time, the rise in the entry costs generates a positive ΔWithin effect. Rising barriers to entry incentivize incumbent firms to innovate faster. Overall, the positive ΔWithin effect following the rise in the entry costs outweighs the negative ΔWithin effect of the rising internal R&D. Note also that the positive ΔReallocation effect is mainly due to the rise in the entry costs.

The results of the decomposition complement the findings in Akcigit and Kerr (2018), Garcia-Macia, Hsieh and Klenow (2019), and Peters (2020). These studies show that economic growth is mainly due to incumbent firms.[20] The decomposition in this paper suggests that entrants play a more prominent role when explaining *changes* in economic growth. That falling firm entry drives the decline in the aggregate growth rate is consistent with the observation in Garcia-Macia, Hsieh and Klenow (2019) that the relative contribution to economic growth by entrants has declined over time.

As a robustness exercise, I show in Section 7.1.1 that a rise in the productivity differential φ^h/φ^l, recently entertained in Aghion, Bergeaud, Boppart, Klenow and Li (2023) as the cause behind rising concentration and falling growth, implies very similar ΔWithin, ΔReallocation and ΔEntry contributions.

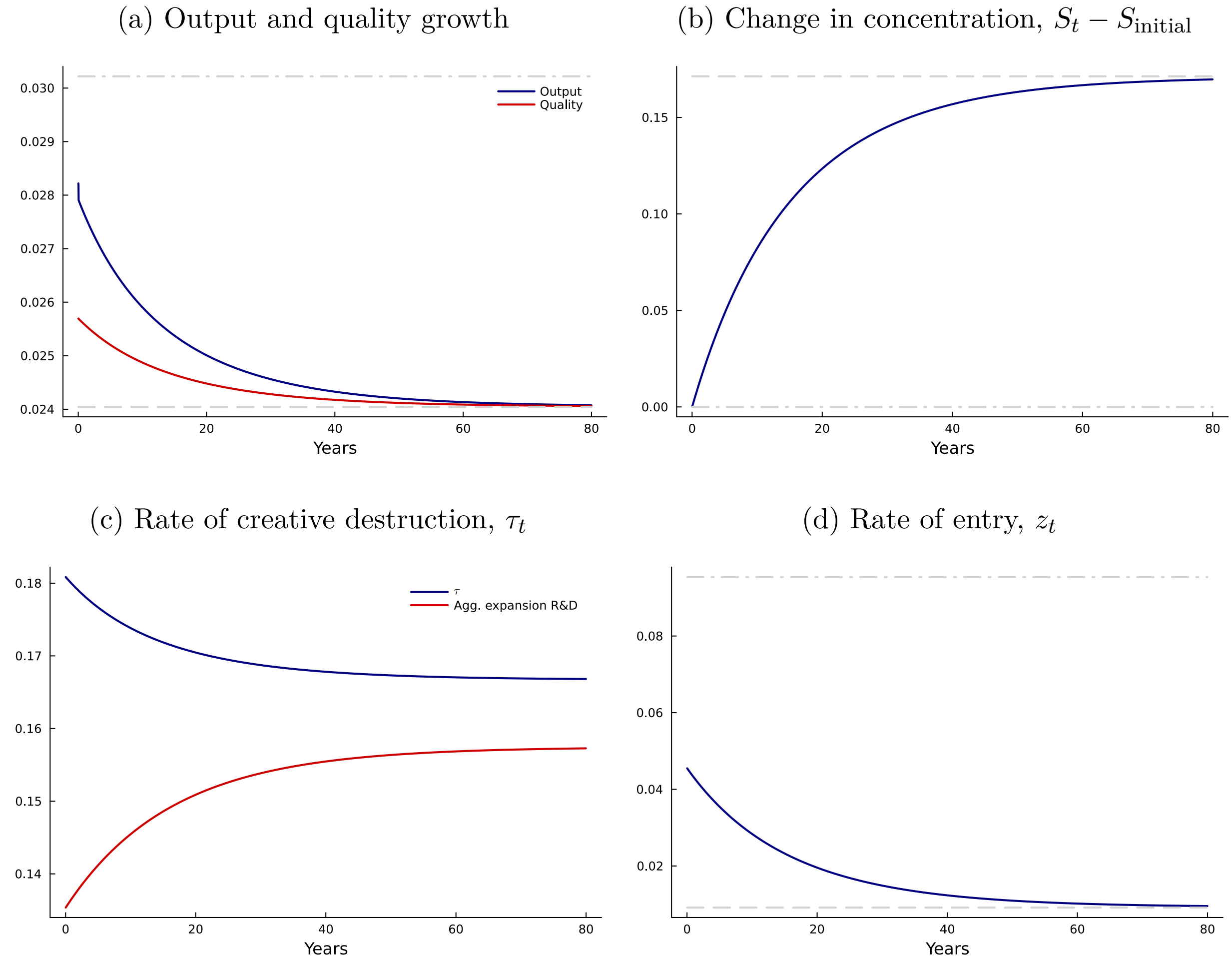
6 Transition dynamics

The previous section analyzed the long-run effects associated with the acceleration in firm life cycle growth. The reallocation of sales shares to more productive incumbents introduces an interesting tradeoff between rising average productivity, Φ in eq. (8), and the long-run fall in the aggregate growth rate, which leaves the effect on welfare unclear. I solve the model numerically over the transition period to study the implications for welfare in this section.

[20]Decomposing growth levels shows that this is also the case in this model in both balanced growth paths.

The solution algorithm, outlined in detail in the Appendix, Section D, works as follows. I solve for policy and value functions from the ending balanced growth path backward for a guessed sequence of wage growth, interest rates, and distribution of firm types over the product space (S_t). I then use the obtained policy functions over the transition period to simulate the two-dimensional distribution of quality and productivity gaps forward, starting from the initial balanced growth path. Using the evolution of this distribution over the transition, I back out the implied sequences of wage growth, interest rates, and S_t. The transition path is the fixed point between the guessed and implied sequences.

Figure 7: Transition dynamics



(a) Output and quality growth

(b) Change in concentration, $S_t - S_{\text{initial}}$

(c) Rate of creative destruction, τ_t

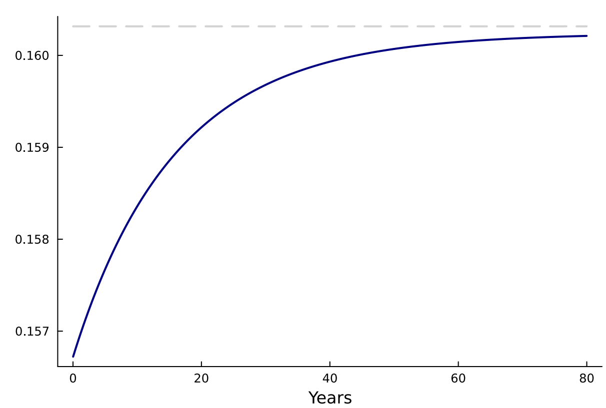
(d) Rate of entry, z_t

Notes: the figure shows the response in equilibrium outcomes following the increase in the cost of entry and internal R&D as in Table 3 in period zero. Output and quality growth (Panel a) refer to the growth rate of Y_t and Q_t in percent. The change in concentration refers to the change in the sales share of high-productivity type firms relative to the initial balanced growth path in percentage points. The gray dashed and dash-dotted lines indicate the ending and initial balanced growth paths, respectively. Aggregate expansion R&D in panel (c) is computed as $S_t \times x_t^h + (1 - S_t) \times x_t^l$.

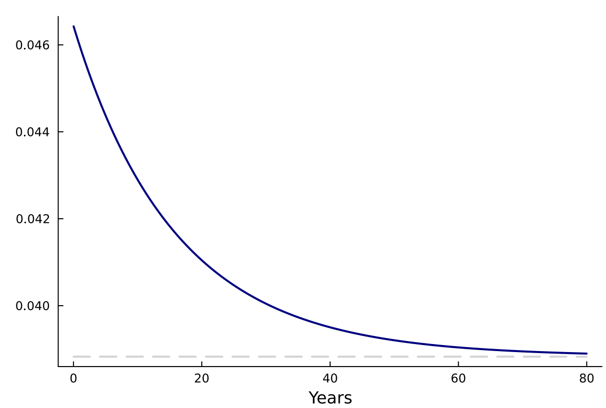
Starting from the initial balanced growth path, I introduce the estimated rise in entry and internal R&D costs (Table 3) as shocks, after which no further parameter changes occur. Figure 7 shows the paths of output (Y_t) growth (in %), quality (Q_t) growth (in %), changes in the sales share of high productivity type firms S_t with respect to the initial balanced growth path (in pp), the rate of creative destruction (τ_t), and the rate of entry (z_t) over the transition period. Convergence is relatively quick. Most changes in equilibrium outcomes occur over the first 20 years of the transition. Both output and quality growth decline on impact and converge quickly after to their new long-run values, as shown in Panel (a). Along a balanced growth path, quality and output grow at the same rate. Over the transition, aggregate quality growth differs from output growth with growth in average productivity, markup dispersion, and production labor, explaining the residual according to eq. (8). Output growth declines by less than quality growth on impact as the rising sales share by high productivity firms, S_t, shown in Panel (b), contributes positively to growth in average productivity and hence aggregate output. Over the entire transition period, S_t increases by 17pp. The rise in average productivity does not suffice to counteract the fall in quality growth. Panel (a) shows that output growth follows the declining pattern of quality growth.[21]

Figure 8: Expansion R&D rates over the transition

(a) High-type firms, x_t^h



(b) Low-type firms, x_t^l



Notes: the figure shows the evolution of the optimal expansion R&D rates by high- and low-type firms following the increase in the cost of entry and internal R&D as in Table 3 in period zero.

That quality growth steadily declines over the transition period is not self-evident as contrasting forces are at play. On the one hand, firm entry declines over the transition, as shown in Panel (d), which lowers quality growth. On the other hand, external and internal R&D efforts by incumbents are also subject to change over the transition. Figure 8 shows the evolution of expansion R&D rates by high- and low-type firms. Consistent with the rise in concentration, expansion rates of high-type firms increase while the ones of low-type firms decline over the transition. Aggregate expansion R&D rates (productivity-type specific R&D rates weighted by their respective sales shares) are, in fact, increasing over the transition as shown in Panel (c) of Figure 7. That falling entry outweighs the rise in aggregate expansion R&D becomes evident after looking at the path of the rate of creative destruction τ_t, also shown in Panel (c). The rate of creative destruction is the sum of the aggregate expansion R&D rate and the firm entry rate z_t. The rate of creative destruction is strictly falling

[21] Changes in misallocation, $\mathcal{M}_t$, have a negligible effect on output growth during the transition.

over the transition, highlighting that falling firm entry dominates rising aggregate expansion R&D. Falling firm entry drives the decline in quality and output growth over the transition, dominating the positive reallocation effects on average productivity.[22]

What is the effect on welfare? As output growth gradually declines right from the shock period in Figure 7, the net effect on welfare is negative. To quantify the change in welfare, I compute the permanent consumption change (in percent) along the initial balanced growth path that makes the consumer as well off as with the obtained consumption stream over the transition towards the new balanced growth path. I find that welfare decreases by 23.3%. This number is sizable and should be interpreted with substantial caution. The initial balanced growth path matches macroeconomic conditions (and firm growth) during the late 1990s. Aggregate productivity growth averaged about 3% during this period in Sweden. Therefore, the transition path is compared to a scenario in which the high growth period of the late 1990s would have continued forever. Targeting a lower aggregate growth rate in the initial balanced growth path that reflects average growth before the 1990s boom, as in Aghion, Bergeaud, Boppart, Klenow and Li (2023) or De Ridder (2024), would result in a lower welfare loss. However, this would introduce an inconsistency in targeted moments: targeted firm growth reflects conditions during the late 1990s, while aggregate growth refers to an earlier period. Note also that the decline in output growth is monotone, i.e., there is no initial burst in output growth as declining firm entry outweighs rising expansion R&D and average productivity over the entire transition. Given that the initial balanced growth path reflects the high growth period of the late 1990s, it is consistent with the data that the transition does not feature a further burst in growth. This does, however, translate into a larger welfare loss.

If one were to compare welfare of two different balanced growth paths that grow at the rates of the estimated initial and ending balanced growth paths (without taking the transition nor any level effects into account) the consumption equivalent change (in percent) ξ is determined by $\ln(1+\xi) = (g^{\text{ending}} - g^{\text{initial}})/\rho$, where g^{ending} and g^{initial} refer to the growth rates of the initial and ending balanced growth paths. Given that the growth rate declines by roughly six percentage points across the balanced growth paths and ρ equals 0.02, the welfare loss amounts to 26.6% ($\xi = -0.266$). Comparing this number to the 23.3% welfare loss above shows again that the fall in output growth during the transition is mainly driven by declining quality growth and that the transition to the new balanced growth path is fast.

7 Robustness checks

This section provides robustness to the main findings. The section concludes with a discussion of the broader implications of the changes in firm life cycle growth.

[22] Internal R&D also declines over the transition period (not shown). However, this effect is small.

7.1 Alternative explanations

The main comparative statics estimation explains the changes in two moments, sales and employment growth, through changes in two parameters: rising entry and internal R&D costs. This section discusses alternative explanations for the observed trends in firm growth.

7.1.1 Rising productivity dispersion

Aghion, Bergeaud, Boppart, Klenow and Li (2023) explain the fall in economic growth and the rise in concentration in the U.S. economy through rising productivity dispersion of incumbents (as well as changes in the R&D efficiency). In line with their story, I estimate an alternative ending balanced growth path where the parameters subject to change are the productivity gap φ^h/φ^ℓ (instead of the entry efficiency) and the internal R&D efficiency ψ_I (as in the previous estimation).

Table 6: Alternative new balanced growth path. Moments and parameters

	ΔData (pp)	ΔModel (pp)
Moments		
Sales growth by age 8 (cohorts 2009–2012)	+11.5	+2.1
Employment growth by age 8 (cohorts 2009–2012)	+17.8	+7.4
Parameters		
ψ_I *Internal R&D efficiency* (Δ in %)		-54
φ^h/φ^ℓ *Productivity gap* (Δ in %)		+6

Notes: the table shows changes in moments (in percentage points) and parameters (in percent) with respect to the initial balanced growth path.

Table 6 shows the estimation results. The internal R&D efficiency falls by 54% (compared to 51% in the previous estimation), and the productivity gap increases by 6%.[23] The implied changes in firm sales and employment growth are qualitatively in line with the data, yet fall short in explaining them quantitatively.[24] Therefore, changes in the productivity gap cannot fully account for the changes in firm growth. Nevertheless, changes in the aggregate economy are consistent with recent macroeconomic trends: the long-run aggregate growth rate falls by 0.49pp, the firm entry rate declines by 3pp, and concentration, S, rises. Therefore, the increase in the productivity gap and internal R&D costs give rise to a similar fall in the aggregate growth rate as the one targeted in Aghion, Bergeaud, Boppart, Klenow and Li (2023) (-0.42pp).

[23] For this estimation, I assume that entrants always replace incumbents after a successful innovation as the estimated productivity gap exceeds the step size of innovation λ. Estimating the parameters with the constraint $\varphi^h/\varphi^\ell < \lambda$ results in the constraint binding at $\varphi^h/\varphi^\ell = 1.136$, which is the value of λ.

[24] For a large enough productivity disadvantage, low-type firms stop expanding into new product markets and remain one-product firms, which reduces the degrees of freedom in the model to match the increase in sales and employment life cycle growth.

Table 7: Decomposing the fall in the aggregate growth rate revisited

	$\psi_I \downarrow, \varphi^h/\varphi^\ell \uparrow$	$\psi_I \downarrow$	$\varphi^h/\varphi^\ell \uparrow$
ΔWithin	-0.13	-0.24	+0.11
ΔReallocation	+0.18	+0.01	+0.13
ΔEntry	-0.53	-0.12	-0.35
Δg	-0.49	-0.35	-0.11

Notes: the table shows the contributions to the change in the aggregate growth rate g across the balanced growth paths according to the decomposition in eq. (29) in percentage points. ΔReallocation is the sum of the ΔBetween and ΔCross terms. g in the initial balanced growth path is equal to 3.02%. $\psi_I \downarrow$ denotes the 54% fall in the internal R&D efficiency and $\varphi^h/\varphi^\ell \uparrow$ the 6% rise in the productivity gap.

I decompose the implied fall in the aggregate growth rate according to eq. (29.) as before. First, changes in incumbent innovation rates, ΔWithin, lower the growth rate slightly (-0.13pp), whereas the reallocation of sales shares, ΔReallocation, towards the more productive firms with higher innovation rates generates a positive growth effect (+0.18pp), shown in Table 7. ΔReallocation outweighs ΔWithin, as in the previous comparative statics estimation. Second, the fall in firm entry more than explains the fall in the aggregate growth rate: -0.53pp compared to -0.49pp. Therefore, the two findings that incumbent firms have mainly contributed to changes in long-run growth through reallocation effects and that the decline in the aggregate growth rate is driven by a fall in firm entry even hold for an alternative estimation, in which the entry costs remain unchanged. Comparing the last column of Table 5 and Table 7 shows that the rising productivity gap works similarly as rising entry costs on growth: both generate positive ΔWithin and ΔReallocation effects that are dominated by a negative ΔEntry effect. As for the rise in entry costs, an increase in the productivity gap widens the gap in expected profits per product line across incumbents, incentivizing the more productive firms to expand faster. The faster expansion of the more productive firms generates the ΔReallocation effect. The ΔWithin, ΔReallocation, and ΔEntry contributions resulting from the rise in the internal R&D costs are quantitatively almost identical to the previous estimation.

In Aghion, Bergeaud, Boppart, Klenow and Li (2023), all firms innovate at the same rate, and there is no firm entry such that changes in within-firm innovation rates, ΔWithin, fully explain the decline in the aggregate growth rate. Table 7 suggests that reallocation effects and firm entry matter for changes in long-run growth. The ΔReallocation effect outweighs the ΔWithin effect, and ΔEntry dominates both.

Would the role of entry change when relaxing the assumption of a unitary demand elasticity? With a demand elasticity greater than one, firms also gain market shares through successful internal R&D. This suggests that, ceteris paribus, an even larger rise in firm entry costs would be required to offset the negative size-growth effect from rising internal R&D costs when matching the increase in firm life cycle growth.

7.1.2 Firm type selection on entry

Rising entry costs incentivize more productive firms to expand, driving less productive ones out of the economy. Rising entry costs, hence, induce selection effects among incumbents. However, the distribution of productivity types among entrants is unaffected, as this is governed exogeneously by the model parameter p^h. Potentially, the selection of entrants has changed over time. In particular, the observed acceleration of firm growth could, in theory, be due to more productive firms entering the economy (an increase in p^h).

Eq. (5) characterizes the employment of entrants. Employment at entry is a function of the markup. Hence, systematic changes in the productivity types of entrants should be reflected in average employment. Figure 16 in the Appendix displays the average employment of entrants by sector over time in the U.S. Census data. The size of entrants shows little variation over time, indicating no systematic changes in the types of entrants.

7.1.3 Other explanations

Two of the six parameters estimated along the initial balanced growth path have not been discussed thus far. The step size improvement of innovations λ and the expansion R&D efficiency ψ_x. A fall in λ could be interpreted as falling research productivity or innovations becoming more incremental (Bloom, Jones, Van Reenen and Webb, 2020; Olmstead-Rumsey, 2019). As λ falls, markup levels and growth decrease, reducing incumbents' incentives to enter new product markets. Hence, a fall in λ reduces firm sales growth.

Lastly, the acceleration in firm sales and employment growth would require an increase in the expansion R&D efficiency ψ_x. Increasing R&D efficiency contrasts the fall in research productivity documented by Bloom, Jones, Van Reenen and Webb (2020).

7.2 Firm productivity and life cycle growth

Sterk, Sedláček and Pugsley (2021) highlight the importance of ex-ante heterogeneity in firm life cycle trajectories. In this model, heterogeneity in expected life cycle profiles arises endogenously through heterogeneous innovation rates related to ex-ante heterogeneity in firm productivity. This section provides suggestive evidence that firms with relatively higher innate productivity are associated with faster life cycle growth in the data.

Firm productivity is generally unobserved in the data. I use a model-based approach to infer the firms' productivity. As firms enter the model economy with one product, eq. (4) captures firm markups upon entry. Eq. (4) implies that their productivity advantage allows more productive firms to charge higher markups in equilibrium. Guided by the theory, I proxy firm productivity by its markup (sales relative to wage bill) at age zero, and regress observed firm life cycle growth on the productivity proxy

Table 8: Firm productivity and size growth

	$\Delta \ln \text{Size}_{\text{Age}=8}$	$\Delta \ln \text{Size}_{\text{Age}=8}$	$\Delta \ln \text{Size}_{\text{Age}=8}$	$\Delta \ln \text{Size}_{\text{Age}=8}$
$\log\left(\frac{py}{wl}\right)_{\text{Age}=0}$	0.130	0.198	0.222	0.237
	(0.006)	(0.005)	(0.005)	(0.006)
$\log K_{\text{Age}=0}$			-0.041	0.003
			(0.003)	(0.003)
$\log M_{\text{Age}=0}$				-0.107
				(0.004)
Cohort fixed effects	✓	✓	✓	✓
Industry fixed effects	✓	✓	✓	✓
$\log\left(\frac{py}{wl}\right)_{\text{Age}=0} > 0$		✓	✓	✓
N	66,817	65,875	60,950	60,832
R^2	0.06	0.08	0.08	0.10

Notes: the table reports the regression coefficient β_1 of eq. (30). Firm size growth over the first eight years, $\Delta \ln \text{Size}_{\text{Age}_{j,t}=8} \equiv \ln \text{Size}_{\text{Age}_{j,t}=8} - \ln \text{Size}_{\text{Age}_{j,t}=0}$, is measured using firm employment. $\log (py/wl)_{\text{Age}_{j,t}=0}$ denotes the log inverse labor share at age zero, the proxy of firm productivity, as explained in the main text. $\log K$ and $\log M$ denote the firm's capital stock and intermediate inputs, respectively. Robust standard errors are in parentheses.

$$\ln \text{Size}_{\text{Age}_{j,t}=a_f} - \ln \text{Size}_{\text{Age}_{j,t}=0} = \beta_0 + \beta_1 \log \left(\frac{py}{wl}\right)_{\text{Age}_{j,t}=0} + \theta_c + \theta_k + \epsilon_{j,t}. \tag{30}$$

py/wl denotes sales relative to the wage bill of the firm (inverse labor share). Otherwise, the notation follows eq. (1). As in the model estimation, I focus on firm size growth over the first eight years, i.e., $a_f = 8$. I use employment as the measure of firm size to avoid sales at age zero on both sides of eq. (30).

Table 8 shows the results. The regression coefficient of interest, β_1, stands at 0.13, i.e., within the same industry and cohort, firms with 1% higher inverse labor shares at entry are associated with approximately 0.13pp faster employment growth over the first eight years. For the model-relevant subsample of firms with positive markups (firms with inverse labor shares larger than one), the regression coefficient increases to 0.198 (column two). One strength of the Swedish data is that it contains information on the capital stock and intermediate input usage. Higher inverse labor shares at entry are positively related to firm life cycle growth, even when controlling for capital and intermediate inputs. Including capital or intermediate inputs at age zero in the regression increases β_1 to 0.222 and 0.237, respectively (third and fourth column).[25] Across all specifications, β_1 remains highly significant, with an almost constant (robust) standard error of 0.005. The data confirms that firms with relatively higher inverse labor shares at entry, perhaps due to systematically higher productivity as suggested by the model, display faster life cycle growth.

[25] I obtain similar results when using $TFPR$ at age zero instead of labor productivity as the markup measure, where $TFPR \equiv \frac{py}{K^{\alpha}(wl)^{1-\alpha}}$ with α estimated at the industry level using cost shares.

7.3 Discussion

This paper argues that rising entry and internal R&D costs cause the acceleration of employment and sales growth. A thorough analysis of the cause is outside the scope of the paper. However, the estimated cost changes relate to evidence in other studies.

What is driving the increase in entry costs? Davis (2017) and Gutiérrez and Philippon (2018) argue that the increasing complexity of regulatory requirements and the tax system, as well as rising lobbying expenditures disproportionately affect entrants. De Ridder (2024) documents the rising importance of intangible capital in production. Competing with incumbents becomes increasingly difficult for entrants or young firms with little or no stock of intangible capital. Interestingly, the firm size of incumbents relative to entrants has increased strongly in service sectors, where intangible capital is particularly relevant in production.

One force that potentially contributed to increasing internal R&D costs is related to the rising importance of the service sector. Firms commonly operate in multiple industries simultaneously, and the composition of industries in which they operate has changed over time. Consider, for example, the car manufacturer Volvo. Over time, Volvo has added the following services to its portfolio: car maintenance, insurance, leasing, and, most recently, car sharing. Similarly, the clothing manufacturer H&M now offers repair and recycling services or even clothing rentals. Arguably, services are generally more difficult to patent than manufactured products, i.e., it is harder to distance competitors in the quality space for services than for goods. To the extent that manufacturing firms offer more and more of such services (or service firms that manufacture a product reduce their manufacturing activities), this implies that the average internal R&D efficiency of a firm (the internal R&D efficiency in a product or service line averaged over the firm's products and services) has declined. The aggregate-level evidence of a rise in the share of the workforce employed in the service sector (72% in 1997 to 79% in 2012 in Sweden) is in line with the above examples.[26] A rising share of services in a firm's portfolio could also explain why, despite the convincing evidence in Akcigit and Ates (2023) of incumbents using patents more strategically (for the set of patentable products), for the firm's average line, it has become harder to prevent competitors from catching up.

Bloom, Jones, Van Reenen and Webb (2020) document that research productivity has declined in the U.S. The notion of ideas getting harder to find is consistent with rising internal R&D costs, as estimated in section 4.2.[27] While rising expansion R&D costs are also consistent with declining research productivity in theory, the data speaks against rising expansion R&D costs and in favor of increasing internal R&D costs: rising expansion R&D costs slow down both sales and employment life cycle growth. In contrast, rising internal R&D costs accelerate employment relative to sales life cycle growth, as I document.

Whether rising internal or expansion R&D costs contribute to falling research productivity

[26]Data from FRED, series SWEPESANA.

[27]Their main example of rising research labor required by incumbent firms to keep growth in the number of transistors on microchips constant can be interpreted as evidence of rising internal R&D costs.

has broader implications for the observed fall in TFP growth: the comparative statics in Section 4.2 highlight the reallocation of market shares to more productive incumbents that innovative at higher rates in equilibrium. The reallocation increases average productivity and long-run growth. This suggests that the observed fall in TFP growth is instead due to falling innovation rates (e.g., by entrants as the decomposition in Section 5 revealed). Rising expansion R&D costs, on the other hand, would lower the market shares by the more productive and innovative firms.[28] In this case, reallocation of market shares decreases average productivity and long-run growth, which would be part of the story behind the observed decline in TFP growth. To which extent reallocation contributed to or mitigated the fall in TFP growth depends on the nature of the decline in research productivity. The observed changes in sales and employment life cycle growth point to rising internal R&D costs as the more likely cause behind falling research productivity. Future research could study trends in research output relative to research inputs as in Bloom, Jones, Van Reenen and Webb (2020) by the type of innovation.

The acceleration in employment relative to sales growth has further implications for aggregate markups. Rising employment relative to sales growth increases a firm's labor income share. Autor, Dorn, Katz, Patterson and Van Reenen (2020) find increasing within-firm labor income shares in most U.S. sectors. This suggests that explanations for falling aggregate labor income shares should feature reallocation effects between firms with high and low labor income shares as in Aghion, Bergeaud, Boppart, Klenow and Li (2023) or De Ridder (2024) rather than falling labor income shares within firms.

8 Conclusion

Sales and employment growth over the firm's life cycle have accelerated. For firms established in the late 1990s, sales grew by 55.9 percent over the first eight years compared to 67.4 percent for firms established in the early 2010s. Similarly, employment growth increased from 28.8 percent to 46.6 percent. I study the cause behind these trends in a model of creative destruction with ex-ante heterogeneous firms. The model shows that accelerating sales and employment growth are the natural response to rising entry costs. Falling firm entry reduces the risk of replacement, which incentivizes, in particular, ex-ante more productive firms to expand into new product markets. Hence, the model suggests that the observed acceleration of firm life cycle growth is due to the accelerated growth of more productive incumbents. To explain the acceleration of employment growth *relative* to sales growth, the model highlights rising costs of distancing competitors within product markets.

More productive incumbents expanding into new product markets results in an increase in their share in the cross-section of firms and a reallocation of sales shares. As more productive incumbents endogenously innovate at systematically higher rates, rising sales shares of these firms increase the long-run aggregate growth rate. These effects matter

[28] In the limit where expansion R&D costs go to infinity, their sales share converges to p^h from above.

quantitatively: incumbents have mainly contributed to changes in long-run economic growth since the 1990s through these reallocation effects, highlighting the importance of changes in industry concentration for long-run growth. Policymakers should trade off the dynamic effects of reallocation with the usual static efficiency losses when evaluating antitrust policies. The model highlights falling firm entry, caused by the rise in entry costs, as the driver behind the recent slowdown in productivity growth. This suggests a promising role for policies that support new firm formation to reverse the decline in productivity growth.

How does the reallocation of market shares to more productive incumbents compare to other, more severe, episodes of reallocation? Over the last decades, many Western economies privatized their education, health care, transportation, or communication sectors. It would be interesting to decompose changes in long-run growth following these events into changes in innovation rates, reallocation, and firm entry, as in this paper. To disentangle how reallocation ultimately affects short and long-run economic growth following privatization, one could further compare the effect of reallocation on innovation to the effects of reallocation on average productivity and misallocation. The quantitative framework in this paper, disciplined by changes in firm dynamics, could separate these forces.

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Appendices

A Data

The main data set, *Företagens Ekonomi* (FEK), covers information from balance sheets and profit and loss statements for the universe of Swedish firms. From this data, I obtain the main variables of interest, namely sales (*Nettoomsättning*, variable name: *Nettoomsattning*) and employment (*Antal anställda*, variable name: *MedelantalAnstallda*). In the FEK codebook by Statistics Sweden, these variables are defined as follows.[29] Sales refer to income from the companies' main business for goods sold and provided services. Employment refers to the average number of employees in full-time units in accordance with the company's annual report. As described in the main text, I focus on firms in the private sector. These firms have a legal type (variable name: *JurForm*) less than 50 or equal to 96.

The 5-digit industry classification (SNI codes) changed twice between 1997 and 2017, once in 2002 and once in 2007. I ensure a consistent industry classification using the following steps. During the year of the change, I observe both the old and the new industry classifications. For the firms present in the data in the year of the classification change, extending the new industry classification further back in time before the change is straightforward. This way, the industry codes of almost all firms are updated. A firm might be in the data before and after the classification change but not for the year of the change. For these firms, the above method does not work. If the firm appears in the data one year after the classification change, I use the observed classification after the change to update the classification before the change. For firms that are absent for several years around the year of change, I use industry mappings provided by Statistics Sweden. These mappings do not always provide a 1:1 mapping between industries before and after the classification change, so I use the most common transitions for the m:m mappings.

One concern is that changes in the firm structure, e.g., when firms merge with other firms, change the firm ID. To address this concern, I impute changes in firm IDs using worker flows between firms. The auxiliary data set *Registerbaserad Arbetsmarknadsstatistik* (RAMS) contains the universe of employer-employee matches. I impute changes in the firm ID of firms with at least five employees as follows: if more than 50% of the workforce of firm A in year t makes up for more than 50% of the workforce of firm B in year $t+1$, I substitute firm B's firm ID by firm A's firm ID following $t+1$. The empirical results remain virtually unchanged

[29] https://www.scb.se/contentassets/9dd20ce462644cc19f6f04eb2edbbe28/nv0109_kd_2017_bv_190508_v2.pdf, accessed 07.02.2024.

when excluding firms for whom the imputed firm ID differs from the observed firm ID.

B Trends in firm dynamics

B.1 Changes in firm life cycle growth

This section provides robustness checks to the documented acceleration in employment and sales life cycle growth. I document robustness with respect to alternative fixed effects specifications, firm selection due to the Great Recession and the classification of entrants.

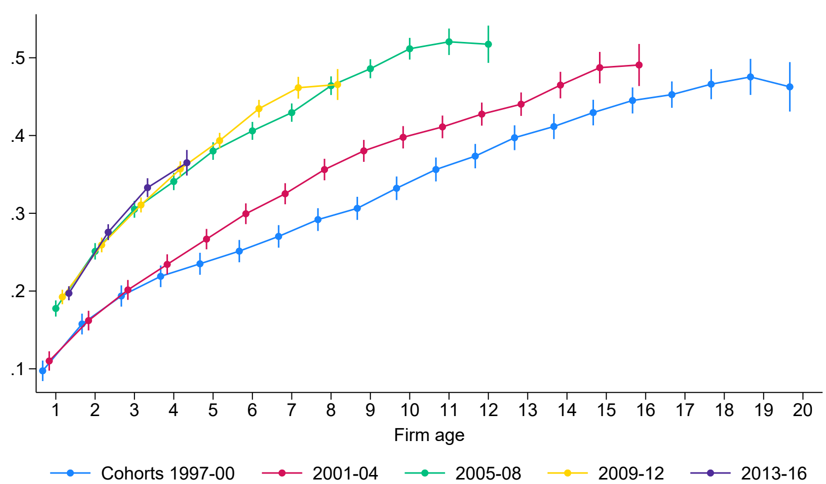
B.1.1 Employment

The baseline regression in (1) controls for cohort and 5-digit industry fixed effects. The results of the regression are virtually unchanged with *interacted* cohort and industry fixed effects as in

$$\ln \text{Employment}_{j,t} = \gamma_0 + \sum_{a_f=1}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{j,t}=a_f} + \theta_{c,k} + \epsilon_{j,t},$$

where, as before, c denotes cohorts and k industries. The estimated coefficients $\gamma_1 - \gamma_{20}$ are shown in Figure 9.

Figure 9: Employment life cycle growth with cohort × industry fixed effects



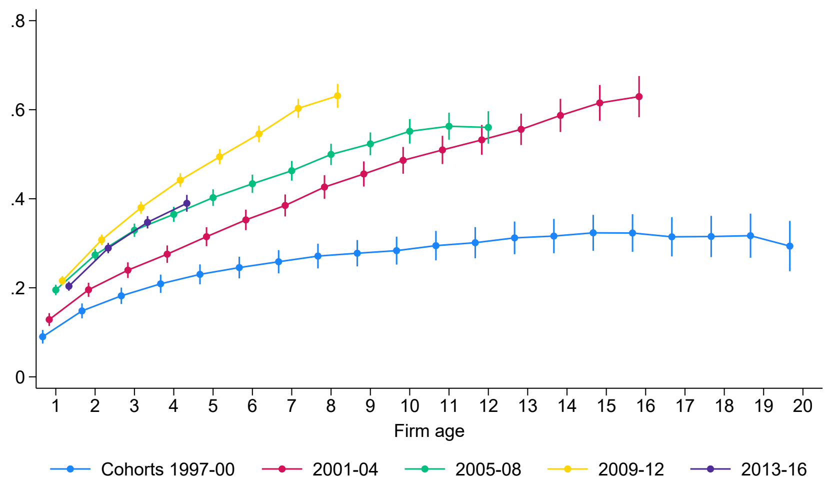
Notes: the figure shows cumulative employment growth over the firm's life cycle in Swedish registry data, measured as the difference between log employment at age a_f and age zero. Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

Alternatively one could control for industry specific, time-varying, shocks to employment by including year × industry fixed effects in the life cycle regression as in

$$\ln \text{Employment}_{j,t} = \gamma_0 + \sum_{a_f=1}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{j,t}=a_f} + \theta_{t,k} + \epsilon_{j,t}.$$

Figure 10 displays the age coefficients. If anything, the acceleration of employment growth is even stronger.

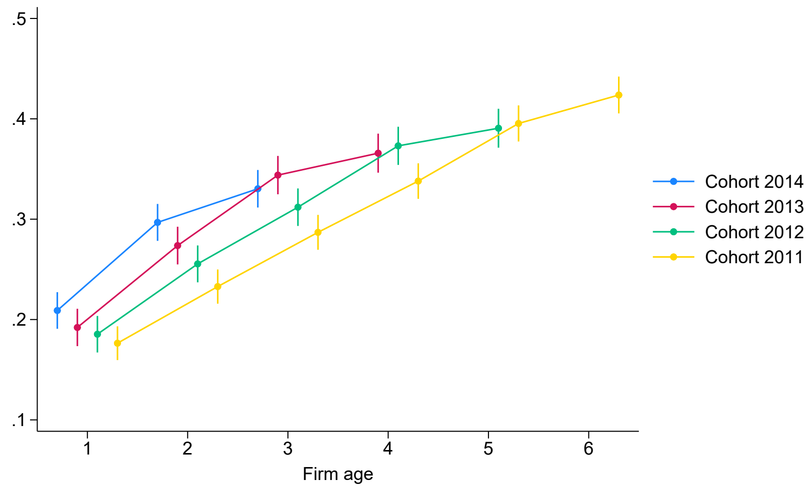
Figure 10: Employment life cycle growth with year × industry fixed effects



Notes: the figure shows cumulative employment growth over the firm's life cycle in Swedish registry data, measured as the difference between log employment at age a_f and age zero. Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

Potentially, the Great Recession induced less productive firms to exit, driving up average firm size following 2009. I provide evidence that selection effects due to the Great Recession are not behind the acceleration of employment growth.

Figure 11: Employment life cycle growth after the Great Recession



Notes: the figure shows cumulative employment growth over the firm's life cycle in Swedish registry data, measured as the difference between log employment at age a_f and age zero. Cohorts are indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

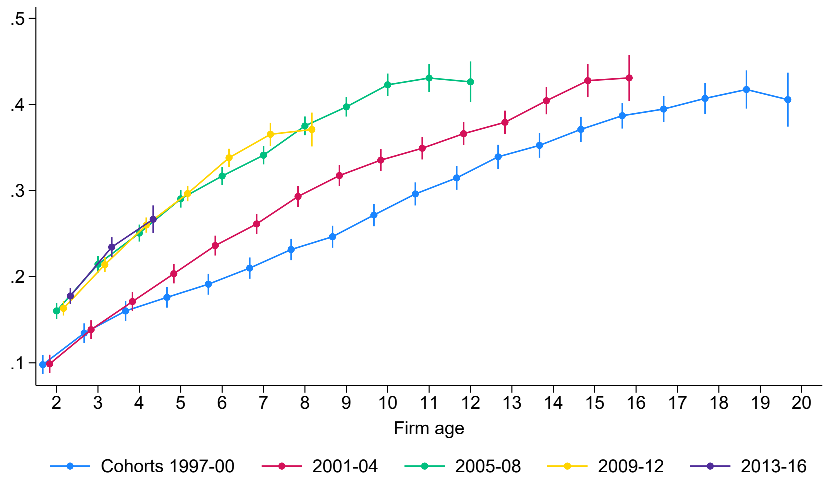
Figure 11 shows employment life cycle growth (regression (1) with industry fixed effects) for each cohort following the Great Recession. As shown in the figure, every cohort experiences faster life cycle growth than the cohort just before. That these patterns hold as clearly for the cohorts after the Great Recession suggests that the main results in Figure 1 are not driven by selection effects among incumbent firms due to the Great Recession.

Lastly, I show that the classification of an entering firm does not affect the documented patterns. In the following, I label firms of age zero and one as entrants and measure employment life cycle growth relative to average employment of firms below age two. In particular, I measure firm life cycle growth as follows

$$\ln \text{Size}_{j,t} = \gamma_0 + \sum_{a_f=2}^{20} \gamma_{a_f} \mathbb{1}_{\text{Age}_{j,t}=a_f} + \theta_c + \theta_k + \epsilon_{j,t},$$

where in comparison with regression (1), the firm age one dummy has been dropped. Average firm size of firms below age two is now captured by γ_0. The age coefficients are plotted in Figure 12. Employment growth, particularly during the early years of the firm, looks comparable to Figure 1 in the main text.

Figure 12: Employment life cycle growth with alternative entrant classification

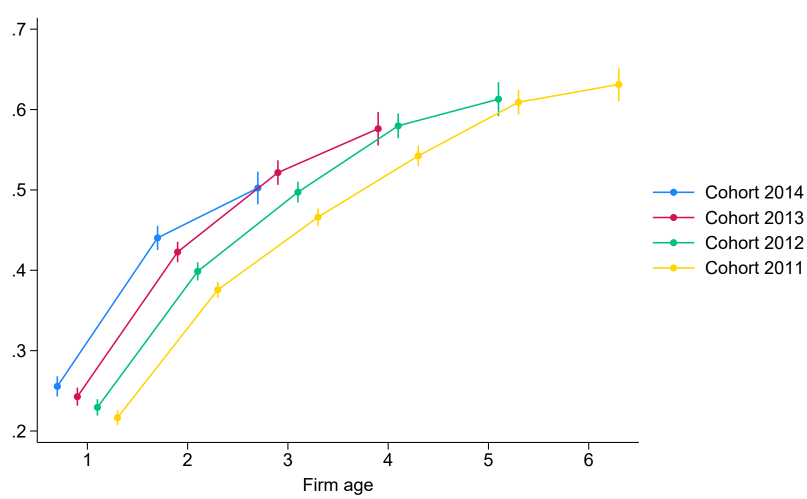


Notes: the figure shows cumulative employment growth over the firm's life cycle in Swedish registry data, measured as the difference between log employment at age a_f and age zero. Cohorts are pooled as indicated in the legend. Firm employment is filtered at its 1% tails. The figure includes 95% confidence intervals.

B.1.2 Sales

I repeat the above robustness exercises for sales growth. Figure 13 shows sales growth over the firm's life cycle for cohorts following the Great Recession. The acceleration of sales growth is apparent for each cohort, suggesting that structural forces other than the Great Recession drive the acceleration.

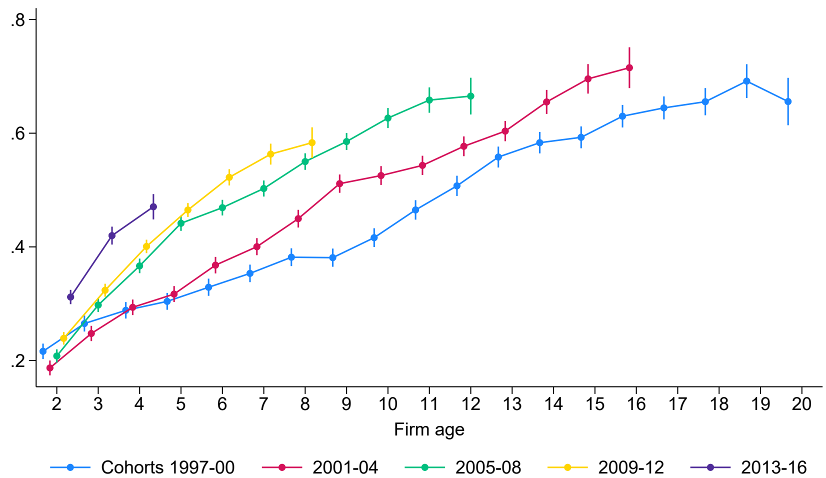
Figure 13: Sales life cycle growth after the Great Recession



Notes: the figure shows cumulative sales growth over the firm's life cycle in Swedish registry data, measured as the difference between log sales at age a_f and age zero. Cohorts are indicated in the legend. Firm sales are filtered at their 1% tails. The figure includes 95% confidence intervals.

The cohorts 1997-2000 display a particularly steep sales growth during the early ages in Figure (2) in the main text. I show that this increase looks more muted when labelling firms less than age two as entrants, exactly as in the robustness check for employment growth.

Figure 14: Sales life cycle growth with alternative entrant classification



Notes: the figure shows cumulative sales growth over the firm's life cycle in Swedish registry data, measured as the difference between log sales at age a_f and age zero. Cohorts are indicated in the legend. Firm sales are filtered at their 1% tails. The figure includes 95% confidence intervals.

Figure 14 shows the age coefficients for the alternative entrant classification. The steep employment growth during the early ages of the cohorts 1997-2000 disappears and the overall acceleration of sales growth over the life cycle of the later cohorts becomes more apparent.

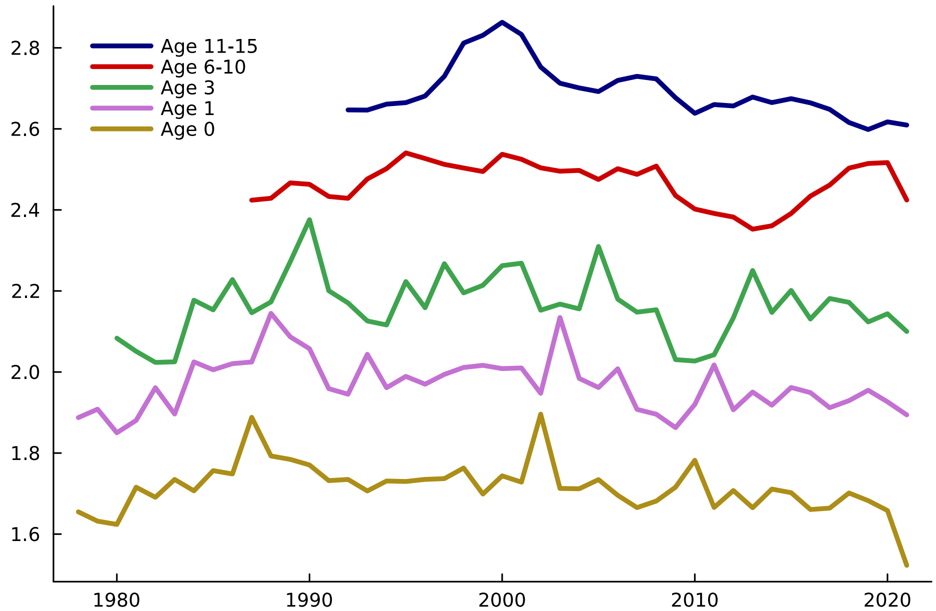
B.2 Firm-size trends in the U.S.

This section documents additional trends in firm size in the U.S. using the Business Dynamics Statistics (BDS) produced by the U.S. Census Bureau.

B.2.1 Replication of previous studies

Karahan, Pugsley and Şahin (2022) and Hopenhayn, Neira and Singhania (2022) establish that average employment conditional on firm age has been relatively stable over time. Figure 15 replicates their findings, showing no systematic trends in log employment conditional on firm age over time. Firms are pooled across all sectors when computing averages, as done in both studies.

Figure 15: Log employment by firm age, firms pooled across sectors



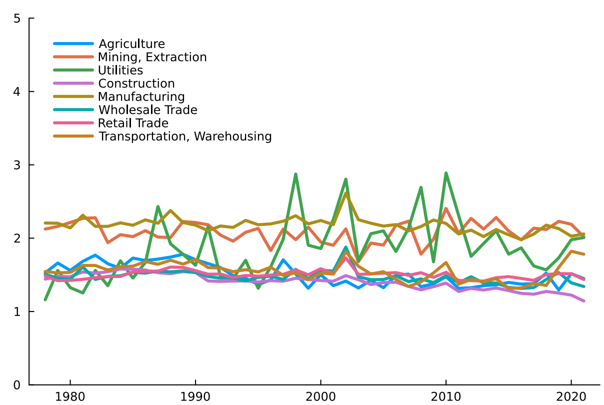
Notes: the figure shows average log employment conditional on firm age in U.S. Census data. Firms are pooled across all sectors.

B.2.2 Firm size at entry

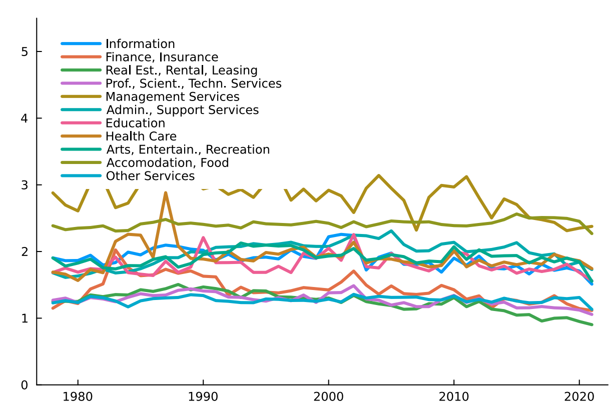
The main empirical finding of the paper is that the firm size of incumbent firms has increased relative to the size of entrants. The size of entrants has remained relatively stable over time as shown in Figure 16 for each sector separately.

Figure 16: Log employment of entrants, by sector

(a) Goods



(b) Services



Notes: the figure shows average log employment of entrants in U.S. Census data by sector. Sector classifications correspond to two-digit NAICS codes.

C Model

C.1 Solving the dynamic firm problem

The HJB for a high productivity-type firm h reads[30]

$$
\begin{aligned}
&r_t V_t^h(n,[\mu_i],S_t) - \dot{V}_t^h(n,[\mu_i],S_t) = \\
&\quad \sum_{k=1}^{n} \pi(\mu_k) + \sum_{k=1}^{n} \tau_t \Big[V_t^h\left(n-1,[\mu_i]_{i\neq k},S_t\right) - V_t^h(n,[\mu_i],S_t) \Big] \\
&\quad + \max_{[x_k,I_k]} \Bigg\{ \sum_{k=1}^{n} I_k \Big[V_t^h\left(n,\left[[\mu_i]_{i\neq k},\mu_k\times\lambda\right],S_t\right) - V_t^h(n,[\mu_i],S_t) \Big] \\
&\quad + \sum_{k=1}^{n} x_k \Big[S_t V_t^h\left(n+1,\left[[\mu_i],\lambda\right],S_t\right) + (1-S_t) V_t^h\left(n+1,\left[[\mu_i],\lambda\times\varphi^h/\varphi^l\right],S_t\right) - V_t^h(n,[\mu_i],S_t) \Big] \\
&\quad - w_t \left[\mu_k^{-1} \frac{1}{\psi_I}(I_k)^\zeta + \frac{1}{\psi_x}(x_k)^\zeta \right] \Bigg\}
\end{aligned}
$$

The HBJ for a low productivity-type firm l reads

$$
\begin{aligned}
&r_t V_t^l(n,[\mu_i],S_t) - \dot{V}_t^l(n,[\mu_i],S_t) = \\
&\quad \sum_{k=1}^{n} \pi(\mu_k) + \sum_{k=1}^{n} \tau_t \Big[V_t^l\left(n-1,[\mu_i]_{i\neq k},S_t\right) - V_t^l(n,[\mu_i],S_t) \Big] \\
&\quad + \max_{[x_k,I_k]} \Bigg\{ \sum_{k=1}^{n} I_k \Big[V_t^l\left(n,\left[[\mu_i]_{i\neq k},\mu_k\times\lambda\right],S_t\right) - V_t^l(n,[\mu_i],S_t) \Big] \\
&\quad + \sum_{k=1}^{n} x_k \Big[S_t V_t^l\left(n+1,\left[[\mu_i],\lambda\times\varphi^l/\varphi^h\right],S_t\right) + (1-S_t) V_t^l\left(n+1,\left[[\mu_i],\lambda\right],S_t\right) - V_t^l(n,[\mu_i],S_t) \Big] \\
&\quad - w_t \left[\mu_k^{-1} \frac{1}{\psi_I}(I_k)^\zeta + \frac{1}{\psi_x}(x_k)^\zeta \right] \Bigg\}.
\end{aligned}
$$

I solve for the value function of a high-type firm, however the steps for the low-type firm are equivalent. For clarity, I suppress the dependence of the value function on S_t in the following. Guess that the value function of the firm consists of a component that is common to all lines and a line-specific component

$$V_t^h(n,[\mu_i]) = V_{t,P}^h(n) + \sum_{k=1}^{n} V_{t,M}^h(\mu_k).$$

[30] The notation follows Peters (2020).

Substituting the guess into the HJB, $V^h_{t,P}(n)$ and $V^h_{t,M}(\mu_k)$ solve the following differential equations

$$\begin{aligned} r_t V^h_{t,M}(\mu_i) - \dot{V}^h_{t,M}(\mu_i) =& \pi(\mu_i) - \tau_t V^h_{t,M}(\mu_i) \\ &+ \max_{I_i} \left\{ I_i \left[V^h_{t,M}(\mu_i \times \lambda) - V^h_{t,M}(\mu_i) \right] - w_t \mu_i^{-1} \frac{1}{\psi_I} (I_i)^\zeta \right\} \end{aligned} \tag{31}$$

and

$$\begin{aligned} & r_t V^h_{t,P}(n) - \dot{V}^h_{t,P}(n) = \sum_{k=1}^{n} \tau_t \left[V^h_{t,P}(n-1) - V^h_{t,P}(n) \right] \\ & + \max_{[x_k]} \left\{ \sum_{k=1}^{n} x_k \left[V^h_{t,P}(n+1) - V^h_{t,P}(n) + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) \right] - w_t \frac{1}{\psi_x} (x_k)^\zeta \right\}. \end{aligned} \tag{32}$$

Assume that in steady-state $V^h_{t,P}$ and $V^h_{t,M}$ grow at the constant rate g. Using this guess in eq. (31) and following Peters (2020), we obtain for $V^h_{t,M}(\mu_i)$

$$V^h_{t,M}(\mu_i) = \frac{\pi(\mu_i) + \frac{\zeta-1}{\psi_I}(I_i)^\zeta w_t \mu_i^{-1}}{\rho + \tau},$$

where I_i solves

$$I_i = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta - 1}{\psi_I} (I_i)^\zeta \right) \left(1 - \frac{1}{\lambda} \right) \frac{\psi_I}{\zeta(\rho + \tau)} \right)^{\frac{1}{\zeta - 1}}. \tag{33}$$

Eq. (33) shows that internal innovation rates I_i are time invariant, and independent of the product line and the productivity type of the firm, $I \equiv I^h = I^l$.

With this at hand, we can turn back to the differential equation for $V^h_{t,P}(n)$ in eq. (32). In addition to the guess that $V^h_{t,P}(n)$ grows at rate g, conjecture that $V^h_{t,P}(n) = n \times v^h_t$. Combined with the Euler we get

$$(\rho + \tau) n v^h_t = \max_{[x_k]} \left\{ \sum_{k=1}^{n} x_k \left[v^h_t + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) \right] - w_t \frac{1}{\psi_x} (x_k)^\zeta \right\}. \tag{34}$$

The optimality condition for x_k is given by

$$v^h_t + S_t V^h_{t,M}(\lambda) + (1-S_t) V^h_{t,M}(\lambda \times \varphi^h/\varphi^l) = w_t \frac{\zeta}{\psi_x} (x_k)^{\zeta - 1}. \tag{35}$$

Several observations are noteworthy. First, eq. (35) shows that optimal expansion rates are independent of quality and productivity gaps in line k. We can hence drop the item indexation: $x_k = x^d$, where $d \in \{h, \ell\}$. Second, $v_t, V^h_{t,M}, w_t$ all grow at the same rate g, which implies that expansion rates are constant over time. We can hence write eq. (34) as

$$v^h_t = \frac{1}{(\rho+\tau)}\frac{\zeta-1}{\psi_x}(x^h)^\zeta w_t.$$

Gathering all terms, the value function is given by

$$\begin{aligned} V^h_t(n,[\mu_i]) =& V^h_{t,P}(n) + \sum_{k=1}^{n} V_{t,M}(\mu_k) \\ =& n v^h_t + \sum_{k=1}^{n} V_{t,M}(\mu_k) \\ =& n\frac{1}{(\rho+\tau)}\frac{\zeta-1}{\psi_x}(x^h)^\zeta w_t + \sum_{k=1}^{n}\frac{\pi(\mu_k)+\frac{\zeta-1}{\psi_I}I^\zeta w_t \mu_k^{-1}}{\rho+\tau}, \end{aligned} \tag{36}$$

which is the expression for the value function stated in the main text, Proposition 1. To see that high-type firms expand at different rates than low-type firms, assume that $x^h = x^\ell$. In this case, $v^h_t = v^\ell_t$, however $E[V^h_t(1,\mu_i)] > E[V^\ell_t(1,\mu_i)]$, because the value function is increasing in the markup. This is true because $Y - \frac{\zeta-1}{\psi_I}I^\zeta w > 0$, otherwise the optimal internal R&D rate defined in eq. (33) would be negative (or zero). The optimality condition for expansion R&D in eq. (35) relates the expected value of expanding into a new product market to the marginal cost of expanding. Given $E[V^h_t(1,\mu_i)] > E[V^\ell_t(1,\mu_i)]$, the marginal cost of expansion R&D (the right hand side of eq. (35)) must be larger for high-type than for low-type firms, which implies $x^h > x^\ell$. As in Lentz and Mortensen (2008), the fact that the marginal value of a product line increases in profits per line implies that firms' expansion rates increase with profitability (productivity).

Using the expression for v^h_t, write the optimality condition in eq. (35) as

$$\frac{\zeta-1}{\psi_x}(x^h)^\zeta + S_t\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I}I^\zeta\lambda^{-1}\right)+(1-S_t)\left(\frac{Y_t}{w_t}\left(1-\frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I}I^\zeta\lambda^{-1}\frac{\varphi^l}{\varphi^h}\right)$$
$$= (\rho+\tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta-1}.$$

Following the same steps for low-productivity firms, we obtain the optimality condition

$$\frac{\zeta-1}{\psi_x}(x^l)^\zeta + S_t\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right)+\frac{\zeta-1}{\psi_I}I^\zeta\lambda^{-1}\frac{\varphi^h}{\varphi^l}\right)+(1-S_t)\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right)+\frac{\zeta-1}{\psi_I}I^\zeta\lambda^{-1}\right)$$
$$=(\rho+\tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta-1}.$$

C.2 Joint distribution of quality and productivity gaps

I characterize the two-dimensional distribution of quality and productivity gaps along the BGP as a function of firm policies. This allows for optimal policies and the distribution to be solved jointly. I solve for the steady state distribution over quality and productivity gaps by setting the differential equations characterizing the law-of-motion in eq. (11) and (12) equal to zero. From this, one obtains the stationary mass of product lines with quality gap λ^Δ and productivity gap φ^i/φ^j

$$\nu\left(\Delta,\frac{\varphi^l}{\varphi^h}\right)=\left(\frac{I}{I+\tau}\right)^\Delta\frac{(1-S)x^lS+z(1-p^h)S}{I}$$
$$\nu\left(\Delta,\frac{\varphi^l}{\varphi^l}\right)=\left(\frac{I}{I+\tau}\right)^\Delta\frac{(1-S)x^l(1-S)+z(1-p^h)(1-S)}{I}$$
$$\nu\left(\Delta,\frac{\varphi^h}{\varphi^h}\right)=\left(\frac{I}{I+\tau}\right)^\Delta\frac{Sx^hS+zp^hS}{I}$$
$$\nu\left(\Delta,\frac{\varphi^h}{\varphi^l}\right)=\left(\frac{I}{I+\tau}\right)^\Delta\frac{Sx^h(1-S)+zp^h(1-S)}{I}.$$

Summing over all Δ for a given productivity gap gives $S_{\varphi^l,\varphi^h}, S_{\varphi^l,\varphi^l}, S_{\varphi^h,\varphi^h}, S_{\varphi^h,\varphi^l}$ as stated in Proposition 1 in main text. It follows that

$$\Pr\left(\Delta\le d,\frac{\varphi^l}{\varphi^h}\right)=\sum_{i=1}^{d}\nu\left(i,\frac{\varphi^l}{\varphi^h}\right)=S_{\varphi^l,\varphi^h}\left(1-\left(\frac{I}{I+\tau}\right)^d\right)$$
$$\Pr\left(\Delta\le d,\frac{\varphi^l}{\varphi^l}\right)=\sum_{i=1}^{d}\nu\left(i,\frac{\varphi^l}{\varphi^l}\right)=S_{\varphi^l,\varphi^l}\left(1-\left(\frac{I}{I+\tau}\right)^d\right)$$
$$\Pr\left(\Delta\le d,\frac{\varphi^h}{\varphi^h}\right)=\sum_{i=1}^{d}\nu\left(i,\frac{\varphi^h}{\varphi^h}\right)=S_{\varphi^h,\varphi^h}\left(1-\left(\frac{I}{I+\tau}\right)^d\right)$$
$$\Pr\left(\Delta\le d,\frac{\varphi^h}{\varphi^l}\right)=\sum_{i=1}^{d}\nu\left(i,\frac{\varphi^h}{\varphi^l}\right)=S_{\varphi^h,\varphi^l}\left(1-\left(\frac{I}{I+\tau}\right)^d\right).$$

Focusing on product lines where a low-productivity incumbent faces a high-productivity second-best firm:

$$P\left(\Delta \leq d, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - e^{-d[\ln(I+\tau)-\ln I]}\right)$$

or

$$P\left(\ln\left(\lambda^\Delta\right) \leq d, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - e^{-\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}d}\right).$$

Conditional on the productivity gap, $\ln\left(\lambda^\Delta\right)$ is exponentially distributed with parameter $\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}$. Further

$$P\left(\lambda^\Delta \leq d, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - d^{-\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}}\right).$$

Conditional on the productivity gap, quality gaps follow a Pareto distribution with parameter $\frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}$. Denote $\theta = \frac{\ln(I+\tau)-\ln I}{\ln(\lambda)}$. We then have

$$P\left(\lambda^\Delta \leq m, \frac{\varphi^l}{\varphi^h}\right) = S_{\varphi^l,\varphi^h}\left(1 - m^{-\theta}\right).$$

Conditional on the productivity gap, quality gaps follow a Pareto distribution with parameter θ. As in Peters (2020), θ is affected by the rate of internal R&D I relative to creative destruction τ. The higher the rate of internal R&D, the more mass is in the tail of the quality gap distribution. The difference to Peters (2020) is that, in this model, quality gaps *conditional* on the productivity gap are Pareto distributed.

After repeating the same steps for lines with different productivity gaps, we obtain the aggregate labor income share as follows[31]

$$\begin{aligned}\Lambda = &\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int_1^\infty \frac{1}{\varphi_k/\varphi_n}\frac{1}{m}S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}dm \\ =&\frac{\theta}{\theta+1}\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\frac{1}{\varphi_k/\varphi_n}S_{\varphi_k,\varphi_n}.\end{aligned}$$

[31] For the derivation, I assume a continuous distribution of quality gaps.

The TFP misallocation statistic $\mathcal{M}$ is then given by

$$\mathcal{M} = \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int\left[\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\frac{1}{m}\right)S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}\right]dm}}{\Lambda}$$
$$= \frac{e^{\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\left[S_{\varphi_k,\varphi_n}\left(\ln\left(\frac{1}{\frac{\varphi_k}{\varphi_n}}\right)-\frac{1}{\theta}\right)\right]}}{\Lambda},$$

where I have made use of

$$\int_1^\infty \ln\left(\frac{1}{m}\right) S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}dm = \left[\frac{\theta\ln(m)+1}{\theta m^\theta}+C\right]_1^\infty = -\frac{1}{\theta}.$$

Alternatively note that this expression is equal to $-S_{\varphi_k,\varphi_n}E[\ln(\lambda^\Delta)|\varphi_k,\varphi_n]$. I have shown above that $\ln(\lambda^\Delta)$ conditional on the productivity gap is exponentially distributed with parameter θ. From the characteristics of an exponential distribution, its expected value is $1/\theta$.

The aggregate markup is then given by

$$E[\mu] = \sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\int_1^\infty \frac{\varphi_k}{\varphi_n} m S_{\varphi_k,\varphi_n}\theta m^{-(\theta+1)}dm$$
$$= \frac{\theta}{\theta-1}\sum_{k\in\{h,l\}}\sum_{n\in\{h,l\}}\frac{\varphi_k}{\varphi_n}S_{\varphi_k,\varphi_n}.$$

This concludes the derivations of the aggregate labor income share, TFP misallocation statistic and aggregate markup. In Peters (2020), the step size of innovation and the rate of creative destruction relative to internal R&D, captured by θ, fully characterize the aggregate labor income share, the misallocation measure, and the aggregate markup. In this model, these statistics further depend on the size and distribution of the productivity gaps. For example, a rise in the productivity gap or a reallocation of sales shares towards high-productivity firms lowers the aggregate labor income share and raises the markup, ceteris paribus.

C.3 Deriving the steady-state growth rate of aggregate variables

The growth rate of Q_t determines the growth rate of aggregate variables

$$g = \frac{\dot{Q}_t}{Q_t} = \frac{\partial\ln(Q_t)}{\partial t}.$$

Quality of a product in a given product line increases through internal R&D, expansion R&D or firm entry. For the growth rate of Q_t over a discrete time interval Δ, we have

$$\ln(Q_{t+\Delta}) = \int_0^1 \Big[(\Delta I + \Delta S x^h + \Delta(1-S)x^l + \Delta z)\ln(\lambda) + \ln(q_{t,i})\Big]\, di$$

so that

$$\frac{\ln(Q_{t+\Delta}) - \ln(Q_t)}{\Delta} = \Big(I + Sx^h + (1-S)x^l + z\Big)\ln(\lambda).$$

For $\Delta \to 0$, $g = \Big(I + Sx^h + (1-S)x^l + z\Big)\ln(\lambda)$ as stated in Proposition 1.

C.4 Solving for the steady state equilibrium

In the model, there are the seven unknown variables $x^h, x^l, I, z, \tau, \frac{Y_t}{w_t}, S$ and the markup distribution $\nu()$ in seven equations plus the system of differential equations characterizing $\nu()$.

Optimality condition for the internal innovation rate

$$I = \left(\left(\frac{Y_t}{w_t} - \frac{\zeta - 1}{\psi_I} I^\zeta\right)\left(1 - \frac{1}{\lambda}\right)\frac{\psi_I}{\zeta(\rho + \tau)}\right)^{\frac{1}{\zeta - 1}}$$

Optimality condition for high-productivity type expansion rate

$$\frac{\zeta - 1}{\psi_x}(x^h)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + \frac{\zeta - 1}{\psi_I} I^\zeta \lambda^{-1}\right) + (1-S)\left(\frac{Y_t}{w_t}\left(1 - \frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right) + \frac{\zeta - 1}{\psi_I} I^\zeta \lambda^{-1}\frac{\varphi^l}{\varphi^h}\right) = (\rho + \tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta - 1}$$

Optimality condition for low-productivity type expansion rate

$$\frac{\zeta - 1}{\psi_x}(x^l)^\zeta + S\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right) + \frac{\zeta - 1}{\psi_I l} I^\zeta \lambda^{-1}\frac{\varphi^h}{\varphi^l}\right) + (1-S)\left(\frac{Y_t}{w_t}\left(1 - \frac{1}{\lambda}\right) + \frac{\zeta - 1}{\psi_I l} I^\zeta \lambda^{-1}\right) = (\rho + \tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta - 1}$$

Free entry condition

$$p^h\Big(SV_t^h(1,\lambda) + (1-S)V_t^h(1, \lambda \times \varphi^h/\varphi^l)\Big) + (1-p^h)\Big(SV_t^l(1, \lambda \times \varphi^l/\varphi^h) + (1-S)V_t^l(1,\lambda)\Big) = \frac{1}{\psi_z} w_t,$$

where

$$V_t^d(1,\mu) = \frac{1}{(\rho+\tau)} \frac{\zeta-1}{\psi_x}(x^d)^\zeta w_t + \frac{Y_t\left(1-\frac{1}{\mu}\right) + \frac{\zeta-1}{\psi_I} I^\zeta w_t \mu^{-1}}{\rho+\tau}$$

Labor market clearing condition

$$1 = \frac{Y_t}{w_t} \sum_{\frac{\varphi_j}{\varphi_{j'}}} \sum_i \frac{1}{\lambda^i \frac{\varphi_j}{\varphi_{j'}}} \nu\left(i, \frac{\varphi_j}{\varphi_{j'}}\right) + \frac{1}{\psi_I} I^\zeta \sum_{\frac{\varphi_j}{\varphi_{j'}}} \sum_i \frac{1}{\lambda^i \frac{\varphi_j}{\varphi_{j'}}} \nu\left(i, \frac{\varphi_j}{\varphi_{j'}}\right) + S\frac{1}{\psi_x}(x^h)^\zeta + (1-S)\frac{1}{\psi_x}(x^l)^\zeta + \frac{z}{\psi_z}$$

Creative destruction

$$\tau = z + Sx^h + (1-S)x^l$$

Share of high productivity type

$$S = \sum_{i=1}^{\infty} \left[\nu\left(i, \frac{\varphi^h}{\varphi^h}\right) + \nu\left(i, \frac{\varphi^h}{\varphi^l}\right)\right],$$

where ν, the stationary distribution of quality and productivity gaps, is characterized by

$$0 = \dot{\nu}\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right) = I\nu\left(\Delta-1, \frac{\varphi_j}{\varphi_{j'}}\right) - \nu\left(\Delta, \frac{\varphi_j}{\varphi_{j'}}\right)(I+\tau) \quad \text{for} \quad \Delta \geq 2$$

and for the case of a unitary quality gap

$$\begin{aligned}
0 = \dot{\nu}\left(1, \frac{\varphi^l}{\varphi^h}\right) &= (1-S)x^l S + z_t(1-p^h)S - \nu\left(1, \frac{\varphi^l}{\varphi^h}\right)(I+\tau) \\
0 = \dot{\nu}\left(1, \frac{\varphi^l}{\varphi^l}\right) &= (1-S)x^l(1-S) + z_t(1-p^h)(1-S) - \nu\left(1, \frac{\varphi^l}{\varphi^l}\right)(I+\tau) \\
0 = \dot{\nu}\left(1, \frac{\varphi^h}{\varphi^h}\right) &= Sx^h S + z_t p^h S - \nu\left(1, \frac{\varphi^h}{\varphi^h}\right)(I+\tau) \\
0 = \dot{\nu}\left(1, \frac{\varphi^h}{\varphi^l}\right) &= Sx^h(1-S) + z_t p^h(1-S) - \nu\left(1, \frac{\varphi^h}{\varphi^l}\right)(I+\tau).
\end{aligned}$$

To simplify the system of equations, first rewrite the rate of creative destruction

$$z = (\tau - Sx^h - (1-S)x^l)$$

such that z can be substituted out from the remaining equations. Second, based on Proposition 1, we know

$$S = S_{\varphi^h,\varphi^h} + S_{\varphi^h,\varphi^l} = \frac{Sx^h + zp^h}{\tau}.$$

Third, the optimality conditions for expansion rates (multiplied by p^h and $(1-p^h)$) and the free entry condition together imply

$$\frac{1}{\psi_x}p^h(x^h)^{\zeta-1} + \frac{1}{\psi_x}(1-p^h)(x^l)^{\zeta-1} = \frac{1}{\psi_z\zeta}.$$

The system of equilibrium conditions can hence be reduced to:

Optimality condition for the internal innovation rate

$$I = \left(\left(\frac{Y_t}{w_t}\psi_I - (\zeta-1)I^\zeta\right)\frac{\left(1-\frac{1}{\lambda}\right)}{\zeta(\rho+\tau)}\right)^{\frac{1}{\zeta-1}}$$

Optimality condition for high-productivity type expansion rate

$$\begin{aligned}\frac{\zeta-1}{\psi_x}(x^h)^\zeta + S\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right) + (\zeta-1)I^\zeta\lambda^{-1}\frac{1}{\psi_I}\right) + (1-S)\left(\frac{Y_t}{w_t}\left(1-\frac{\varphi^l}{\varphi^h}\frac{1}{\lambda}\right) + (\zeta-1)I^\zeta\lambda^{-1}\frac{\varphi^l}{\psi_I\varphi^h}\right)\\ = (\rho+\tau)\frac{\zeta}{\psi_x}(x^h)^{\zeta-1}\end{aligned}$$

Optimality condition for low-productivity type expansion rate

$$\begin{aligned}\frac{\zeta-1}{\psi_x}(x^l)^\zeta + S\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\frac{\varphi^h}{\varphi^l}\right) + (\zeta-1)I^\zeta\lambda^{-1}\frac{\varphi^h}{\psi_I l\varphi^l}\right) + (1-S)\left(\frac{Y_t}{w_t}\left(1-\frac{1}{\lambda}\right) + (\zeta-1)I^\zeta\lambda^{-1}\frac{1}{\psi_I l}\right)\\ = (\rho+\tau)\frac{\zeta}{\psi_x}(x^l)^{\zeta-1}\end{aligned}$$

Free entry

$$p^h\frac{(x^h)^{\zeta-1}}{\psi_x} + (1-p^h)\frac{(x^l)^{\zeta-1}}{\psi_x} = \frac{1}{\psi_z\zeta}$$

Labor market clearing condition

$$1 = \frac{Y_t}{w_t}\Lambda + \Lambda_I + S\frac{1}{\psi_x}(x^h)^\zeta + (1-S)\frac{1}{\psi_x}(x^l)^\zeta + \frac{\tau - Sx^h - (1-S)x^l}{\psi_z},$$

where

$$\Lambda = \frac{\theta}{\theta+1} \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \frac{1}{\varphi_k/\varphi_n} S_{\varphi_k,\varphi_n}$$
$$\Lambda_I = \frac{1}{\psi_I} I^{\zeta} \frac{\theta}{\theta+1} \sum_{k\in\{h,l\}} \sum_{n\in\{h,l\}} \frac{1}{\varphi_k/\varphi_n} S_{\varphi_k,\varphi_n}$$
$$\theta = \frac{\ln(I+\tau)-\ln(I)}{\ln(\lambda)}$$

Share of high productivity type

$$S = \frac{Sx^h + (\tau - Sx^h - (1-S)x^l)p^h}{\tau}$$

The expressions related to the labor market clearing condition are derived in Section C.2. This constitutes a system of seven equations in seven unknowns $(x^h, x^l, I, \tau, \frac{Y_t}{w_t}, S)$, which I solve using a root finder.

C.5 Firm markups

Firm markups are defined by $\mu_f = \frac{py_f}{wl_f} = \left(\frac{1}{n}\sum_{k=1}^{n} \mu_{kf}^{-1}\right)^{-1}$. Therefore

$$\ln \mu_f = -\ln\left(\frac{1}{n}\sum_{k=1}^{n} \mu_k^{-1}\right).$$

Rewrite the term in brackets (for a high-productivity firm) as

$$\frac{1}{n}\sum_{k=1}^{n} \mu_k^{-1} = \frac{1}{n}\sum_{k=1}^{n} e^{-\ln\mu_k} = \frac{1}{n}\left(\sum_{i=1}^{n_i} e^{-\ln\frac{\varphi^h}{\varphi^l} - \Delta_i \ln\lambda} + \sum_{j=1}^{n_j} e^{-\Delta_j \ln\lambda}\right), \tag{37}$$

where i indexes the product lines where the high productivity firm faces a low productivity second best producer, j the lines where it faces a high productivity second best producer and $n_i + n_j = n$. A two-dimensional linear Taylor expansion around $\ln\lambda = 0$ and $\ln\frac{\varphi^h}{\varphi^l} = 0$ gives

$$\frac{1}{n}\left(\sum_{i=1}^{n_i} e^{-\ln\frac{\varphi^h}{\varphi^l} - \Delta_i \ln\lambda} + \sum_{j=1}^{n_j} e^{-\Delta_j \ln\lambda}\right) \approx 1 - \left(\frac{1}{n}\sum_{k=1}^{n} \Delta_k\right)\ln\lambda - \frac{n_i}{n}\ln\left(\frac{\varphi^h}{\varphi^l}\right)$$

such that

$$E\left[\ln\mu_f|\text{firm age}=a_f,\varphi^h\right]\approx E\left[\frac{1}{n}\sum_{k=1}^{n}\Delta_k|\text{firm age}=a_f,\varphi^h\right]\ln\lambda+(1-S)\ln\left(\frac{\varphi^h}{\varphi^l}\right),$$

where I have used the fact that (in expectation) the share of the firm's product lines with a low productivity second best producer is equal to the aggregate share of product lines where the incumbent is of the low productivity type. From Peters (2020), we know that

$$E\left[\frac{1}{n}\sum_{k=1}^{n}\Delta_k|\text{firm age}=a_f,\varphi^h\right]\ln\lambda=\left(1+I\times E[a_P^h|a_f]\right)\ln\lambda,$$

where $E[a_P^h|a_f]$ denotes the average product age of a high-productivity type firm conditional on firm age a_f and

$$\begin{aligned}
E[a_P^h|a_f]&=\frac{1}{x^h}\left(\frac{\frac{1}{\tau}\left(1-e^{-\tau a_f}\right)}{\frac{1}{x^h+\tau}\left(1-e^{-(x^h+\tau)a_f}\right)}-1\right)\left(1-\phi^h(a_f)\right)+a_f\phi^h(a_f)\\
\phi^h(a)&=e^{-x^h a}\frac{1}{\gamma^h(a)}\ln\left(\frac{1}{1-\gamma^h(a)}\right)\\
\gamma^h(a)&=\frac{x^h\left(1-e^{-(\tau-x^h)a}\right)}{\tau-x^h e^{-(\tau-x^h)a}},
\end{aligned}$$

which gives the expression in the main text.

For a firm of the low-productivity type, the last term in eq. (37) reads

$$\frac{1}{n}\left(\sum_{i=1}^{n_i}e^{-\ln\Delta_i\ln\lambda}+\sum_{j=1}^{n_j}e^{-\ln\frac{\varphi^l}{\varphi^h}-\ln\Delta_j\ln\lambda}\right),$$

where i indexes the product lines where the low-productivity producer faces a low-productivity second best producer, j the lines where it faces a high-productivity second best producer and $n_i+n_j=n$. Following the same steps as for a high-productivity firm, this time linearizing around $\ln\frac{\varphi^l}{\varphi^h}=0$ (and $\ln\lambda=0$) gives

$$E\left[\ln\mu_f|\text{firm age}=a_f,\varphi^l\right]\approx\left(1+I\times E[a_P^l|a_f]\right)\ln\lambda+S\ln\left(\frac{\varphi^l}{\varphi^h}\right),$$

where again I have made use of the fact that (in expectation) the share of the firm's product lines with a high-productivity second best producer is equal to the aggregate share of product lines where the incumbent is of the high-productivity type. $E[a_P^l|a_f]$ is exactly defined as

$E[a_P^h|a_f]$ with x^h replaced by x^l in the above expressions.

C.6 Firm size distribution

The mass of high- and low-productivity type firms with $n \geq 2$ products follows the differential equations

$$\begin{aligned}\dot{M}_t^h(n) &= (n-1)x_t^h M_t^h(n-1) + (n+1)\tau_t M_t^h(n+1) - n(x_t^h + \tau_t)M_t^h(n) \\ \dot{M}_t^l(n) &= (n-1)x_t^l M_t^l(n-1) + (n+1)\tau_t M_t^l(n+1) - n(x_t^l + \tau_t)M_t^l(n),\end{aligned} \tag{38}$$

whereas the mass of firms with one product evolves according to

$$\begin{aligned}\dot{M}_t^h(1) &= z_t p^h + 2\tau_t M_t^h(2) - (x_t^h + \tau_t)M_t^h(1) \\ \dot{M}_t^l(1) &= z_t(1-p^h) + 2\tau_t M_t^l(2) - (x_t^l + \tau_t)M_t^l(1).\end{aligned} \tag{39}$$

The mass of firms with n products increases through firms with $n-1$ products expanding to size n at rate x_t^h or x_t^l per product or through firms with $n+1$ products losing a product at the rate of aggregate creative destruction τ_t. The mass of firms with n products decreases through firms with n products either gaining or losing a product through expansion or creative destruction. The mass of firms with one product additionally increases through firm entry.

Proposition 2. *The stationary firm size distribution along the balanced growth path is characterized as follows.*

1. *The mass of high and low productivity firms with n products is*

$$\begin{aligned}M^h(n) &= \frac{(x^h)^{n-1} z p^h}{n\tau^n} = \frac{zp^h}{x^h}\frac{1}{n}\left(\frac{x^h}{\tau}\right)^n \\ M^l(n) &= \frac{(x^l)^{n-1} z(1-p^h)}{n\tau^n} = \frac{z(1-p^h)}{x^l}\frac{1}{n}\left(\frac{x^l}{\tau}\right)^n.\end{aligned}$$

2. *The total mass of firms with n products is*

$$M(n) = M^h(n) + M^l(n) = \frac{(x^h)^{n-1}zp^h + (x^l)^{n-1}z(1-p^h)}{n\tau^n}.$$

3. The mass of firms of each productivity type is

$$M^h = \sum_{n=1}^{\infty} M^h(n) = \frac{zp^h}{x^h} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x^h}{\tau}\right)^n = \frac{zp^h}{x^h} \ln\left(\frac{\tau}{\tau - x^h}\right)$$
$$M^l = \sum_{n=1}^{\infty} M^l(n) = \frac{z(1-p^h)}{x^l} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x^l}{\tau}\right)^n = \frac{z(1-p^h)}{x^l} \ln\left(\frac{\tau}{\tau - x^l}\right)$$

4. The total mass of firms is

$$M = M^h + M^l.$$

Proof. These results follow from setting the time derivatives in equations (38) and (39) equal to zero and solving the system of equations. □

For each firm type, the share of firms with n products, $M^h(n)/M^h$ and $M^l(n)/M^l$, follows the PDF of a logarithmic distribution with parameter x^h/τ and x^l/τ as in Lentz and Mortensen (2008). The firm size distribution is highly skewed to the right.

Since there is a continuum of mass one of products and each product is mapped to one firm $\sum_{i=1}^{\infty} M(n) \times n = 1$. Further, the mass of high-productivity type firms producing n products is related to the share of product lines operated by high-type firms, S, as follows

$$S = \sum_{n=1}^{\infty} M^h(n) \times n = \frac{zp^h}{\tau - x^h}.$$

D Computation of transition dynamics

In this section, I lay out the numerical procedure to solve for the transition path. Since time is continuous, I solve a discretized version of the model where the solution converges to the one in continuous time for small enough time intervals. As shown in Appendix C, value functions are additive across product lines. Therefore, I solve the problem of two representative one-product firms: one of the high productivity type and one of the low productivity type.

I normalize the value function by the wage w_t to obtain a stationary problem. The value function for the high-type firm (in discrete time) reads

$$\begin{aligned}
&\frac{V_t^h(1,\mu_i,S_t)}{w_t} = \frac{Y_t}{w_t}\left(1-\frac{1}{\mu_i}\right)dt \\
&-\tau_t \exp(-r_t dt)\frac{V_{t+dt}^h(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}dt \\
&+\max_{x_t^h}\left\{x_t^h \exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^h(1,\lambda,S_{t+dt})}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^h(1,\lambda\frac{\varphi^h}{\varphi^l},S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_x}(x_t^h)^\zeta dt\right\} \\
&+\max_{I_t^h}\left\{I_t^h \exp(-r_t dt)\left(\frac{V_{t+dt}^h(1,\mu_i\lambda,S_{t+dt})}{w_{t+dt}} - \frac{V_{t+dt}^h(1,\mu_i,S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_I}\mu_i^{-1}(I_t^h)^\zeta dt\right\} \\
&+\exp(-r_t dt)\frac{V_{t+dt}^h(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}. \qquad (40)
\end{aligned}$$

The value function for the low-type firm reads

$$\begin{aligned}
&\frac{V_t^l(1,\mu_i,S_t)}{w_t} = \frac{Y_t}{w_t}\left(1-\frac{1}{\mu_i}\right)dt \\
&-\tau_t \exp(-r_t dt)\frac{V_{t+dt}^l(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}dt \\
&+\max_{x_t^l}\left\{x_t^l \exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^l(1,\lambda\frac{\varphi^l}{\varphi^h},S_{t+dt})}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^l(1,\lambda,S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_x}(x_t^l)^\zeta dt\right\} \\
&+\max_{I_t^l}\left\{I_t^l \exp(-r_t dt)\left(\frac{V_{t+dt}^l(1,\mu_i\lambda,S_{t+dt})}{w_{t+dt}} - \frac{V_{t+dt}^l(1,\mu_i,S_{t+dt})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t}dt - \frac{1}{\psi_I}\mu_i^{-1}(I_t^h)^\zeta dt\right\} \\
&+\exp(-r_t dt)\frac{V_{t+dt}^l(1,\mu_i,S_{t+dt})}{w_{t+dt}}\frac{w_{t+dt}}{w_t}. \qquad (41)
\end{aligned}$$

From this, one obtains the first order conditions for the policy functions. For the optimal expansion R&D rate of the high type firm x_t^h (again suppressing the dependence of the value function on S_t):

$$\exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^h(1,\lambda)}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^h(1,\lambda\frac{\varphi^h}{\varphi^l})}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_x}(x_t^h)^{\zeta-1} \qquad (42)$$

and for the low type firm x_t^l:

$$\exp(-r_t dt)\left(S_{t+dt}\frac{V_{t+dt}^l(1,\lambda\frac{\varphi^l}{\varphi^h})}{w_{t+dt}} + (1-S_{t+dt})\frac{V_{t+dt}^l(1,\lambda)}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_x}(x_t^l)^{\zeta-1}. \qquad (43)$$

Both are independent of the markup μ_i. For the optimal internal R&D rates of the high type, I_t^h, one obtains

$$\exp(-r_t dt)\left(\frac{V_{t+dt}^h(1,\mu_i\lambda)}{w_{t+dt}} - \frac{V_{t+dt}^h(1,\mu_i)}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_I}\mu_i^{-1}(I_t^h)^{\zeta-1} \tag{44}$$

and similarly for I_t^l

$$\exp(-r_t dt)\left(\frac{V_{t+dt}^l(1,\mu_i\lambda)}{w_{t+dt}} - \frac{V_{t+dt}^l(1,\mu_i)}{w_{t+dt}}\right)\frac{w_{t+dt}}{w_t} = \frac{\zeta}{\psi_I}\mu_i^{-1}(I_t^l)^{\zeta-1}. \tag{45}$$

Equations (40) to (45) characterize the firm problem in discrete time. These equations are supplemented by the law of motion for the two dimensional distribution of quality and productivity gaps

$$\nu_{t+dt}\left(\Delta,\frac{\varphi_j}{\varphi_{j'}}\right) - \nu_t\left(\Delta,\frac{\varphi_j}{\varphi_{j'}}\right) = dt\left[I_{\mu_i,t}\nu_t\left(\Delta-1,\frac{\varphi_j}{\varphi_{j'}}\right) - \nu_t\left(\Delta,\frac{\varphi_j}{\varphi_{j'}}\right)(I_{\mu_i,t}+\tau_t)\right] \quad \text{for} \quad \Delta \geq 2 \tag{46}$$

and for product lines with a unitary quality gap, $\Delta = 1$,

$$\begin{aligned}
\nu_{t+dt}\left(1,\frac{\varphi^l}{\varphi^h}\right) - \nu_t\left(1,\frac{\varphi^l}{\varphi^h}\right) &= dt\left[(1-S_t)x_t^l S_t + z_t(1-p^h)S_t - \nu_t\left(1,\frac{\varphi^l}{\varphi^h}\right)(I_{\mu_i,t}+\tau_t)\right]\\
\nu_{t+dt}\left(1,\frac{\varphi^l}{\varphi^l}\right) - \nu_t\left(1,\frac{\varphi^l}{\varphi^l}\right) &= dt\left[(1-S_t)x_t^l(1-S_t) + z_t(1-p^h)(1-S_t) - \nu_t\left(1,\frac{\varphi^l}{\varphi^l}\right)(I_{\mu_i,t}+\tau_t)\right]\\
\nu_{t+dt}\left(1,\frac{\varphi^h}{\varphi^h}\right) - \nu_t\left(1,\frac{\varphi^h}{\varphi^h}\right) &= dt\left[S_t x_t^h S_t + z_t p^h S_t - \nu_t\left(1,\frac{\varphi^h}{\varphi^h}\right)(I_{\mu_i,t}+\tau_t)\right]\\
\nu_{t+dt}\left(1,\frac{\varphi^h}{\varphi^l}\right) - \nu_t\left(1,\frac{\varphi^h}{\varphi^l}\right) &= dt\left[S_t x_t^h(1-S_t) + z_t p^h(1-S_t) - \nu_t\left(1,\frac{\varphi^h}{\varphi^l}\right)(I_{\mu_i,t}+\tau_t)\right]
\end{aligned} \tag{47}$$

and a standard Euler equation

$$\frac{C_{t+dt}}{C_t} = \exp(-\rho dt)(1 + r_{t+dt}dt). \tag{48}$$

Further, the (static) free entry and labor market clearing conditions remain unchanged and are characterized in the main text by equations (9) and (10).

The algorithm to compute the transition path assumes that an initial and ending balanced growth path has been solved for including the (stationary) two-dimensional distribution of quality and productivity gaps. I choose $dt = 0.02$ and set the transition period to 100 years (T), after which I assume the economy has reached its new balanced growth path. I further truncate the two dimensional distribution of quality and productivity gaps along the quality dimension at $\Delta = 30$, implying a maximum quality gap of λ^{30}. No mass reaches this state during the transition such that this assumption is satisfied. I then compute the transition path as follows:

1. Guess a path of interest rates r_t and wage growth $\frac{w_{d+dt}}{w_t}$ over the transition (equal to their values in the final balanced growth path)
   (a) Guess a path for S_t over the transition (equal to its value in the final balanced growth path).
      i. Starting backwards in period T, solve for optimal policy functions in $T - dt$ using equations (42)-(45).[32]
      ii. Solve for τ_{T-dt} that ensures that the free entry condition (9) holds.
      iii. Compute the value function in $T - dt$ using equations (40) and (41).
      iv. Iterate backwards until the first time period.
      v. Starting from the initial balanced growth path, simulate S_t forward using[33]
      $$S_{t+dt} = S_t + dt\left[S_t x_t^h(1 - S_t) - (1 - S_t)x_t^l S_t + z_t(p^h(1 - S_t) - (1 - p^h)S_t)\right],$$
      where z_t can be substituted out by equation (18).

   (b) Update the guess for S_t from step v and go back to step i. Iterate until the guessed path for S_t converges to the implied one.
2. Starting from the initial balanced growth path, simulate the two dimensional distribution of quality and productivity gaps forward using equations 46 and 47.
3. Solve for the sequence of $\frac{Y_t}{w_t}$ from the labor market clearing condition.

[32] I solve for the optimal I_i (at each point in time) over a two-dimensional grid of quality and productivity gaps.

[33] One could already simulate the entire two-dimensional distribution of quality and productivity gaps forward here. However, for the inner loop, it is sufficient to iterate on S_t.

4. Compute the sequence of quality growth using

$$\frac{Q_{t+dt}}{Q_t} = \exp\left(\left[\int_0^1 I_{\mu_i,t}di + S_t x_t^h + (1-S_t)x_t^l + z_t\right] dt \ln(\lambda)\right).$$

5. Compute the sequence of aggregate productivity growth using

$$\frac{\Phi_{t+dt}}{\Phi_t} = \left(\frac{\varphi^h}{\varphi^l}\right)^{S_{t+dt}-S_t}.$$

6. Using the two dimensional distribution of quality and productivity gaps, compute the sequence of $\mathcal{M}_t$ defined in equation (8).
7. Compute the sequence of production labor L_{Pt} using equation (6).
8. Compute the sequence of aggregate output growth $\frac{Y_{t+dt}}{Y_t}$ using equation (8).
9. With the path of aggregate output growth, obtain the implied path of interest rates from the Euler equation (48).
10. With the paths of aggregate output growth and Y_t/w_t, obtain the implied path of wage growth $\frac{w_{d+dt}}{w_t}$.
11. Update the guesses for the interest rate and wage growth and go back to step (a). Iterate until the guessed and implied paths converge.