

### **Summary (Team 71727)**

Traffic congestion isn't merely a nuisance, it is a significant cost to commuters in both money and time. In 2014, American commuters lost 42 hours, 19 gallons of fuel, and the equivalent of \$960 per commuter to excess travel time caused by traffic congestion. This totals to a yearly congestion cost of \$160 billion. Experts expect fully automated cars to arrive in the next 7 to 12+ years, however, it is unclear how the rise of automated vehicles will affect traffic congestion. Self driving cars may relieve congestion in a several ways: they may be less likely to become involved in accidents, and they may be able to coordinate with one another to plan more globally efficient routes or to improve road capacity by driving closer to one another than human drivers can. We focus on the latter effect, modeling a vehicle to vehicle (V2V) communication infrastructure which allows self driving cars to follow one another closely, even at high speeds, accelerating and decelerating in unison. Contrary to claims that coordination between self driving cars will not alleviate traffic until most or all cars are self driving, we show modest improvements in road capacity with modest adoption of self driving cars. Assuming that adoption of self driving cars does not increase the number of cars on the road at any time, we see a doubling of road capacity with a 75% share of self driving cars. However, some fear that self driving cars may worsen traffic congestion by making driving easier and more enjoyable, causing an increase in the average distance commuters drive. We explore scenarios where the number of car owners remains constant, but self driving car owners spend more time driving, and find that while low levels of self driving car adoption may make traffic worse due to increased driving, near complete adoption always improves traffic relative to wholly human traffic.

## **Letter to the Governor**

Dear Governor Insee,

Currently, traffic congestion causes commuters in Seattle to lose 63 hours per commuter each year. As self driving cars enter the market within the next two decades, they can be expected to have some effect on traffic congestion. Will self driving cars improve traffic congestion, by increasing efficiency and effective road capacity, or will they worsen traffic congestion by making it easier and more pleasant to drive, causing owners to drive more? We modeled the effect of self driving cars on traffic congestion, and found that, in general, self driving cars do have the potential to reduce traffic congestion by increasing effective road capacity. However, when including increased driving resulting from self driving car owners driving more, we found that introducing self driving cars initially worsens congestion. Even when taking this effect into account, and even assuming self driving car owners drive twice as much, we find that when a majority of drivers have self driving cars, congestion is improved.

Therefore, we urge the governor's office to implement policy which encourages the use of self driving cars. While it does not make sense to designate formal self-driving car lanes initially, we recommend allowing self driving cars to use HOV lanes. Since self driving cars in our model perform better when surround by other self driving cars, this will allow drivers in the HOV lane to benefit more from a modest proportion of self driving cars. As usage expands, more lanes should be dedicated to self driving cars. Eventually, HOV lanes should be again restricted to cars which actually contain multiple passengers, to promote carpooling in order to offset the increased driving caused by self driving cars.

Sincerely,

MCM Team #71727

# Highway Efficiency Gains From Self Driving Cars

## MCM Contest Question C

Team # 71727

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## 1 Introduction

According to the 2015 Texas Transportation Institute Urban Mobility Scorecard, traffic congestion causes Seattlites to lose \$1,491 and 63 hours per peak hour commuter per year [4]. As self driving cars are introduced, likely

within the next two decades [5], we seek to understand the effect of an increased share of self driving cars on congestion. As the percentage of self driving cars on the roads increases, does traffic congestion increase, decrease, or stay the same? Does an increase in the share of self driving cars benefit the public, the self driving car owners, or everyone?

## 2 Analytic Model

Self driving cars can be modeled as acting completely autonomously or as acting in a strongly coordinated manner. Completely autonomous self driving cars can be modeled as behaving similarly to human driven cars, but perhaps with faster reaction times and a lower probability of error. Alternately, self driving cars may be strongly connected to each other through vehicle to vehicle (V2V) or vehicle to infrastructure (V2I) communication systems. Some models which incorporate V2V or V2I infrastructure model cars as using information about local and global traffic patterns to choose routes which optimize total network efficiency, or maneuvering to form large platoons.

In some versions of our model, we assume a modest V2V coordination network: while self driving cars do not choose their routes to optimize global efficiency or maneuver to form platoons, we assume self-driving cars recognize when they are following one another. All members of a chain of self driving cars gain access to a secure communication channel. Therefore, when the first car in a chain perceives an obstacle, it can send information to each car in the chain. Therefore, the chain of cars can decelerate in unison. While human drivers and other non-coordinating cars need to maintain a buffer zone in order to allow following cars to avoid rear ending leading cars, with the length of the required buffer zone depending on speed, coordinating self driving cars need not face this constraint, and can instead, in effect, behave as coupled cars in a train. Thus, self driving cars in a chain can maintain a short, constant distance between one another.

Because self driving cars can safely drive closer together, traffic containing a large fraction of self driving cars will be able to accommodate denser traffic at any given speed. We initially consider models where self driving cars maintain a human sized stopping distance behind human driven cars, but also consider models where self driving cars have a negligible reaction time, and thus tailgate human cars. Finally, we consider self driving cars without a V2V network which have a negligible reaction time and thus can safely tailgate all cars. How strong are these effects, and how do they de-

pend on the fraction of self driving cars?

## 2.1 Design

To answer this, we first determine how traffic density varies with speed in humans-only traffic. The safe stopping distance required by human and other non-cooperating cars is the sum of the distance traveled before the driver reacts to the stimulus (the perception reaction distance) and the distance traveled while braking (the braking deceleration distance). With speed  $s$ , reaction time  $r$ , and braking acceleration constant  $a$ , the stopping distance is

$$d_0(s) = rs + \frac{1}{2}as^2.$$

A reaction time of 1.5 seconds, or  $2.2 \frac{\text{ft}}{\text{mph}}$  and a braking acceleration constant of  $a = 0.096 \frac{\text{ft}}{\text{mph}^2}$  thus gives a total stopping distance of

$$d_0(s) = 2.2 \frac{\text{ft}}{\text{mph}}s + 0.048 \frac{\text{ft}}{\text{mph}^2}s^2.$$

For instance, this gives a stopping distance of 27 ft at 10 mph, 109 ft at 30 mph, and 304 ft. at 60 mph, in accordance with the safe driving guidelines presented in [3] In practice, at very low speeds, the distance between cars cannot go to zero, since some space must be occupied by the car itself, and some extra distance for added safety. Thus, we let (human) stopping distance be

$$d(s) = \max(d_0(s), 30\text{ft}).$$

For our models of platooning self driving cars, we set the distance between successive self driving cars at a constant  $c = 30\text{ft}$  regardless of speed. In all models, we assume self driving cars behind human drivers maintain a safe stopping distance, as we have assumed human drivers do. When we assume self driving cars have improved reaction times, we use the stopping distance

$$d'_0(s) = \frac{1}{2}as^2 + 0.048 \frac{\text{ft}}{\text{mph}^2}s^2,$$

taking computerized reaction time to be negligible. Otherwise, we use  $d(s)$  as with human drivers.

In a population composed entirely of human driven cars, the average distance between cars will be  $d(s)$ , while in a population composed entirely of self driving cars, the average distance between cars will be shorter.

How does the average distance between cars depend on the fraction of self driving cars? To get a sense of how this might play out in Seattle, we work with a simplified problem: what is the expected average distance between cars for a string of  $n$  cars, each of which has a probability  $p$  of being self-driving? To calculate this, we sum the average distance of each string of  $n$  cars, weighted by the probability of that string of cars. For instance, if we consider strings of two cars and platooning self driving cars with slow reaction times, the possibilities are:

Sequence	Probability	Distance
AA	$p^2$	$c$
AS	$p(1 - p)$	$d(s)$
SA	$(1 - p)p$	$d(s)$
SS	$(1 - p)^2$	$d(s)$

Thus the expected distance is

$$\hat{d}_p(s, p) = p^2c + (1 - p^2)d(s).$$

By induction, we can prove that this holds for strings of any length. Let  $\Lambda_n$  be the set of all strings of  $n$  cars,  $p(\lambda)$  be the probability of a string  $\lambda$ , and  $l(\lambda)$  be the length of a string  $\lambda$ . Then the expected length of a string of  $n$  cars,  $l(\Lambda_n)$  is

$$\mathbb{E}[l(\Lambda_n)] = \sum_{\lambda \in \Lambda} p(\lambda)l(\lambda)$$

By the linearity of expectation, the expected value of the length of a string of  $n + 1$  cars is the expected value of a string of  $n$  cars plus the expected value of the distance between the  $n^{th}$  car and the  $(n + 1)^{st}$  car. As an inductive hypothesis, assume  $l(\Lambda_n) = (n - 1)(p^2c + (1 - p^2)d(s))$ . Then

$$\begin{aligned} \mathbb{E}[l(\Lambda_{n+1})] &= \mathbb{E}(l(\Lambda_n)) + p^2c + (1 - p^2)d(s) \\ \mathbb{E}[l(\Lambda_{n+1})] &= (n - 1)(p^2c + (1 - p^2)d(s)) + p^2c + (1 - p^2)d(s) \\ \mathbb{E}[l(\Lambda_{n+1})] &= (n)(p^2c + (1 - p^2)d(s)) \end{aligned}$$

Thus, the expected average distance between cars for a string of cars of any length with platooning and slow reaction times is

$$\hat{d}_p(s, p) = p^2c + (1 - p^2)d(s).$$

Under the assumption that self driving cars have negligible reaction time, and therefore can safely tailgate human driven cars, we obtain the following table of following distances:

Sequence	Probability	Distance
AA	$p^2$	$c$
AS	$p(1-p)$	$\frac{1}{2}as^2$
SA	$(1-p)p$	$d(s)$
SS	$(1-p)^2$	$d(s)$

This gives a distance function of

$$\hat{d}_{pt}(s, p) = p^2c + (1-p)rs + (1-p^2)\frac{1}{2}as^2.$$

Finally, assuming no V2V communication network, but that self driving cars have negligible reaction time, we obtain the following table of following distances:

Sequence	Probability	Distance
AA	$p^2$	$\frac{1}{2}as^2$
AS	$p(1-p)$	$\frac{1}{2}as^2$
SA	$(1-p)p$	$d(s)$
SS	$(1-p)^2$	$d(s)$

giving the distance function

$$\hat{d}_t(s, p) = \frac{1}{2}as^2 + (1-p)rs.$$

We are ultimately interested in the relationship between speed and the flux of cars, that is, the number of cars which pass any given point over a period of time. The flux along a highway gives a good proxy for the number of people commuting at a given time: if 100 drivers have to pass from point  $A$  to point  $C$  between 5:00 pm and 6:00 pm, and point  $B$  is between points  $A$  and  $C$ , then the flux at point  $B$  must be at least 100 cars/hour. This means that it is not surprising that flux is maximized at slow speeds: high traffic flux causes congestion, which leads to low speed. The flux is given by the product of the density of cars and the speed of the cars. Since the density of cars, or number of cars per unit length, is the inverse of the distance between cars, we have a form for the flux  $f(s, p)$  dependent on car speed

$$f(s, p) = \frac{s}{\hat{d}(s, p)}$$

We derive an number of other relationships from this equation, which we present plots of in the next section.

## 2.2 Results

For all of the plots which follow, solid black lines represent the fully human drivers reference model, dashed blue lines the platooning slow reaction model, dotted orange lines the non-platooning quick reaction model, and solid green lines the platooning quick reaction model.

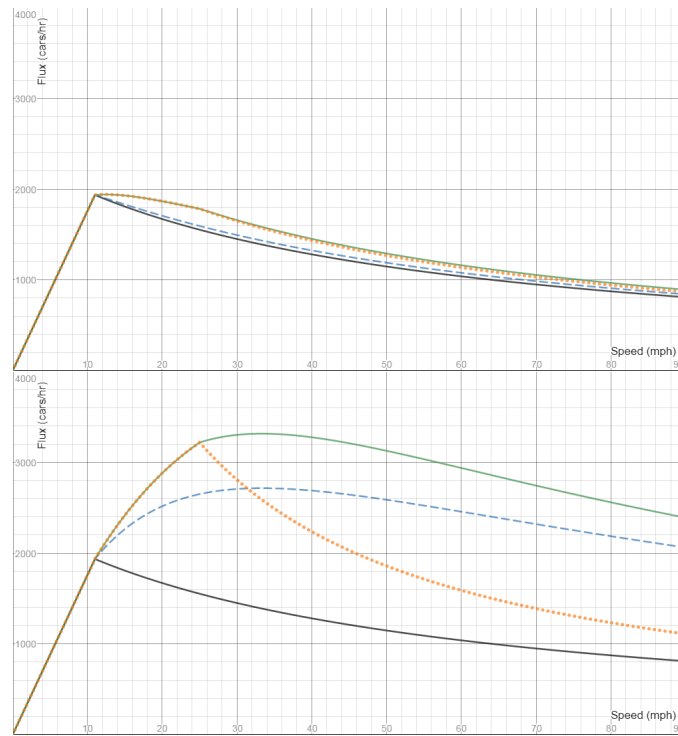


Figure 1: Plots of flux vs. speed for a 20% and 80% share of self driving cars.

A first analysis comes from examining the relationship between flux and speed directly. We can see that for low proportions of self driving cars, traffic flux rates are fairly comparable in all models, with the quick reaction times allowing for more flux at moderate speeds than the other models. Flux is maximized at the critical point of about 11 mph, which



is where the safe stopping distance exceeds our minimum distance of 30 feet—at the threshold of gridlock. For higher proportions of self driving cars, the gains are much more substantial, and maximum flux occurs at higher speeds than the critical stopping distance speed. The models with quick reaction times have a higher critical speed based on their modified stopping distance; at sufficiently high proportions of self driving cars the platooning models exceed even this.

We also note that for each flux below the maximum, there are two speeds which achieve it—a gridlocked slow speed, and a freer-flowing fast speed. It's possible that even if self driving cars are in small proportions, they could coordinate actions to achieve a transition to the higher-speed state. Unfortunately, the analytic model is not equipped to suggest how this may be accomplished.

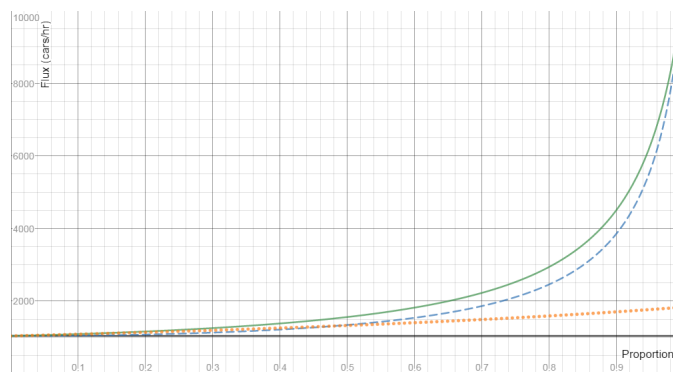


Figure 2: A plot of flux vs. the proportion of self driving cars at the speed limit, 60mph.

We can also examine the flux at a particular speed to compare the different scenarios. By examining the speed limit of 60 mph we can get an idea of the ideal capacity of the road when operating at speeds beneficial to individual drivers. It's clear that at large proportions of self driving cars, platooning allows for significantly more flux. It is likely that effects not accounted for by the simplicity of the analytic model like increased lane changes and merging near exits would prevent the flux from rising quite so high as suggested here, but the gains should still be substantial. The non-platooning model is much less dramatic, but still nearly doubles flux at high proportions.

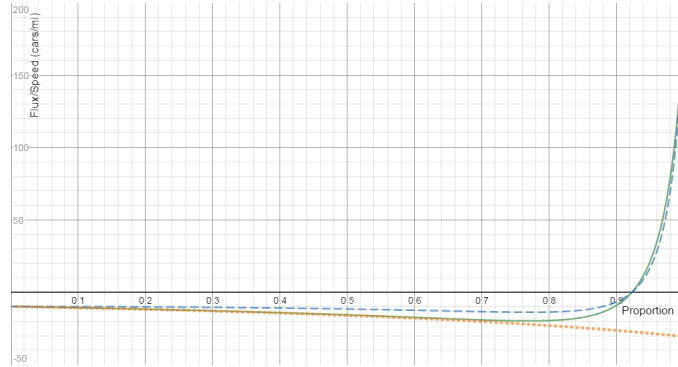


Figure 3: Derivative of flux with respect to speed  $\frac{df}{ds}$  plotted against proportion of self driving cars at the speed limit, 60 mph. Zeros correspond to proportions where maximum flux occurs at this speed.

We can also examine the derivatives of flux, extracting from their zeros the critical proportions beyond which maximum flux occurs under desirable conditions. With this kind of analysis, we can see that while the non-platooning self driving cars do not achieve high flux at high speeds, sufficiently many platooning cars do. In particular, at about 92% platooning cars (with or without quick reaction times) the flux maximum occurs at the speed limit. This means that if nearly all cars are self driving, optimum road use as measured by flux (quantity of people transported along the stretch of road) and speed (efficiency of each particular journey) coincide.

For another perspective, we invert the quadratic relationships between distance and speed in order to measure flux as a function of car density:

$$s(\hat{d}_t, p) = \frac{(1-p)r}{a} \left[ \sqrt{1 + \frac{2\hat{d}_t a}{(1-p)^2 r^2}} - 1 \right]$$

$$s(\hat{d}_p, p) = \frac{r}{a} \left[ \sqrt{1 + \frac{2(\hat{d}_p - c)a}{r^2}} - 1 \right]$$

$$s(\hat{d}_{pt}, p) = \frac{(1-p)r}{(1-p^2)a} \left[ \sqrt{1 + \frac{2(\hat{d}_{pt} - p^2 c)(1-p^2)a}{(1-p)^2 r^2}} - 1 \right]$$

From here, we use the relationship

$$f(\hat{d}, p) = \frac{s(\hat{d}, p)}{\hat{d}}$$

to examine the efficiency of the road system at prescribed densities.

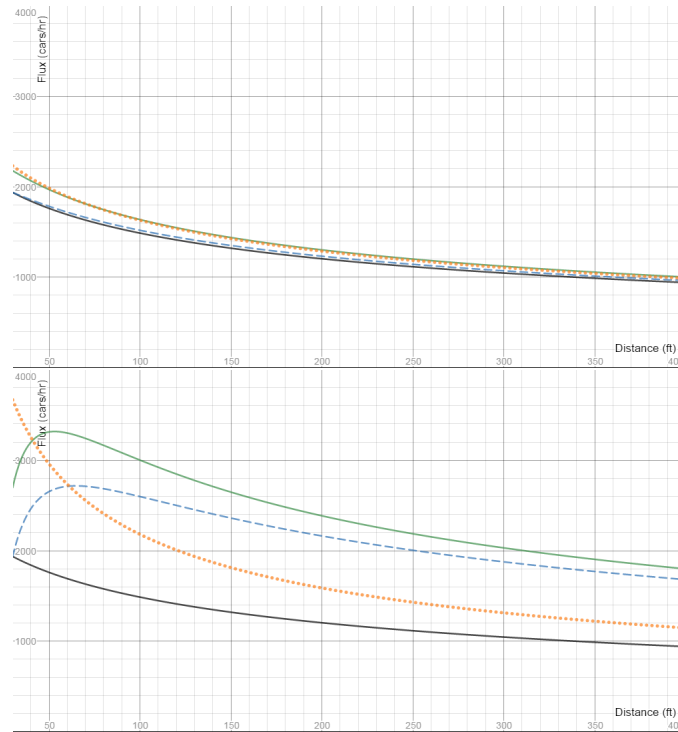


Figure 4: Plots of flux vs. distance between cars, with 20% self driving cars and 80% self driving cars, respectively. Note that for human driven cars, the maximum flux happens at the distance where cars are as close to each other as possible—in other words, in apparently heavy traffic. As more self driving cars are added into the mix, flux is optimized at higher speeds, so the density decreases.

We can again study the derivative of flux to obtain the critical proportions at which the optimum flux state becomes pleasant. Here, we are looking to see where it occurs at a density lighter than that of gridlock; again, non-platooning cars do not achieve this, but at 40% (for slow reactions)

to 49% (for quick reactions), maximum flux occurs with average distance greater than 30 feet.

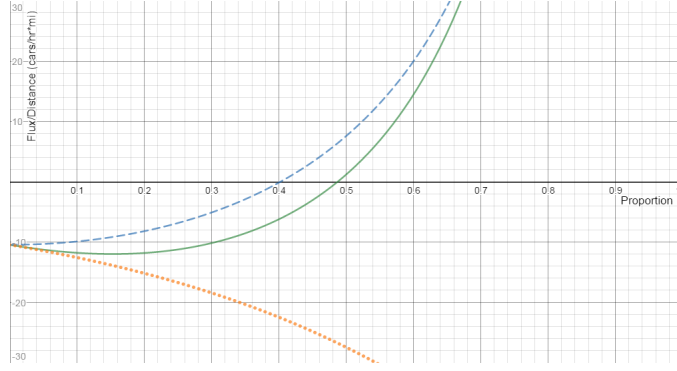


Figure 5: Derivative of flux with respect to average distance  $\frac{df}{dd}$  plotted against proportion of self driving cars at the minimum distance, 30 feet. Zeros correspond to proportions where this distance maximizes flux.

This last result requires a bit of interpretation, as the average distance tends to decrease as proportion increases. However, past these points the flux maximum state is such that gaps begin to form between human driven cars and the platoons. Far enough past this point, at optimum flux a given car occupant is either in a self driving car in a platoon, or has a open zone of road about them, hopefully providing an atmosphere of non-congestion.

Finally, we can compute the proportion of self-driving cars necessary to achieve a prescribed speed and average distance. This again comes from inverting a quadratic relationship, and results in

$$p(s, \hat{d}_t) = \frac{rs + \frac{1}{2}as^2 - \hat{d}_t}{rs}$$

$$p(s, \hat{d}_p) = \sqrt{\frac{rs + \frac{1}{2}as^2 - \hat{d}_p}{rs + \frac{1}{2}as^2 - c}}$$

$$p(s, \hat{d}_{pt}) = \frac{rs}{as^2 - 2c} \left[ \sqrt{1 + \frac{2(rs + \frac{1}{2}as^2 - \hat{d}_t)(as^2 - 2c)}{r^2s^2}} - 1 \right]$$

The regions where these functions take on certain values can be used to determine what traffic characteristics are possible given constraints on

the number of self driving cars, or to estimate the proportion of them given current traffic conditions.

### 3 Caveats

#### 3.1 The Fundamental Law of Road Congestion

The problem statement assumes that traffic congestion is caused by limitations on the number of traffic lanes. In this model, delays happen when the volume of traffic exceeds the capacity of the road. Self driving cars alleviate traffic because they increase the capacity of all road segments, without increasing the number of lanes. However, substantial evidence indicates that increasing road capacity does not decrease traffic congestion, an effect termed the "fundamental law of road congestion"[2]. Instead, an increase in road capacity, and even an increase in public transit capacity, causes more people to travel by car at peak times, leading to a constant level of traffic congestion.

Thus, if an increase in the adoption of self driving cars acts like an increase in road capacity, we may not actually see a decrease in traffic congestion. There are some reasons to believe that increasing the proportion of self driving cars may be a more effective way of reducing congestion than increasing physical road capacity. Under our model, a 75% share of self driving cars would more than double the possible car-flux at 60 mph, a 90% share would more than quadruple it, and a 99% share would increase it by a factor of 20. Thus, the increase in road capacity promised by self driving cars under our model is much greater than any increase that could be achieved with increased infrastructure. Thus, the increase in capacity may be larger than any possible increase in demand, and the fundamental law of road congestion may not apply. However, our model is somewhat simplistic, and does not take into account behavior such as entering or exiting the freeway. While at low self-drivingness shares, it is likely that congestion caused by human reaction time and car spacing is the main effect, at higher self-drivingness, it is likely that other effects will come to dominate, therefore, reduction in congestion unlikely to be as large as our model predicts. Therefore, the fundamental law may still be able to absorb all any decrease in congestion caused by the adoption of self driving cars.

Furthermore, the fundamental law of road congestion may actually underpredict the increase in demand due to self driving cars. Since self driving cars are expected to make commuting by car more enjoyable and to allow

people who are currently unable drive to use cars, increased capacity due to increased self driving car adoption may increase demand for road space more than increased capacity due to expanded road infrastructure. Unfortunately, very little data on how self driving cars will affect demand for road space is available.

### 3.2 Modeling Road Demand

Here, we investigate a simple model of how self driving cars may affect road demand. We imagine a fixed number  $n$  of car owners,  $np$  of which own self driving cars. A human driven car has probability  $m_h$  of being on the road at any given time and a self-driving car has probability  $m_s$  of being on the road at any given time. Thus, the total number of cars on the road is

$$npm_s + n(1 - p)m_h = n(m_h + p(m_s - m_h)).$$

This ignores an additional increase in demand likely to result from an increased driving population, however, it also ignores a possible decrease in demand if communally owned self driving cars used for efficient carpooling become popular. It seems reasonable to assume that the density on a typical stretch of road is linearly related to the total number of cars on the road.

Using the coordinating, non-tailgating self driving car model, we have

$$\begin{aligned} \frac{1}{\hat{d}(s)} &\sim n(m_h + p(m_s - m_h)) \\ \frac{1}{\hat{d}(s)} &= k(m_h + p(m_s - m_h)) \end{aligned}$$

where  $k$  is linearly related to  $n$ , and represents the number of car owners for each foot of roadway. Since setting  $k$  greater than 1 results in gridlock for most reasonable values of  $m_h$  and  $m_s$ , we let  $k$  range from 0 to 1. A value  $k \cong 0$  can be understood as "very heavy traffic relative to capacity" and a value  $k \cong 1$  can be understood as "constant gridlock".

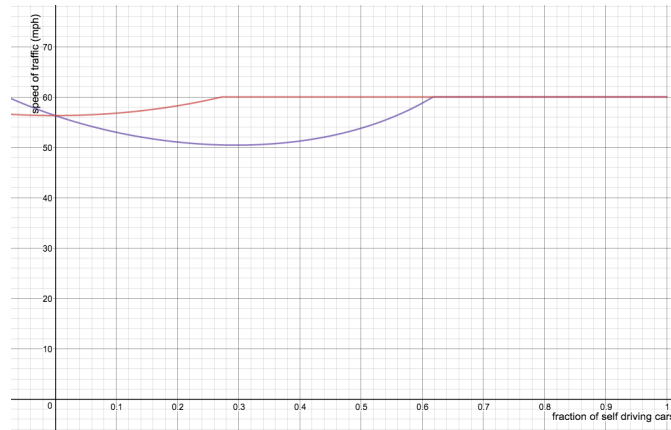


Figure 6: The upper red line represents speed of travel as a function of the fraction of self driving cars if we assume that self driving car owners do not drive more. The lower purple line represents speed of travel as a function of fraction of self driving cars if we assume that self driving car owners drive twice as much. This model represents light traffic relative to capacity.

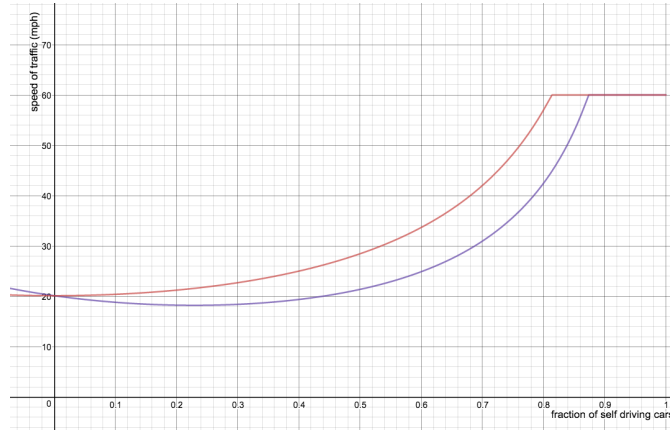


Figure 7: The upper red line represents speed of travel as a function of the fraction of self driving cars if we assume that self driving car owners do not drive more. The lower purple line represents speed of travel as a function of fraction of self driving cars if we assume that self driving car owners drive twice as much. This model represents heavy traffic relative to capacity.

Plugging in our value for  $\hat{d}(s)$  and solving for  $s$ , we have

$$\frac{1}{p^2c + (1 - p^2)(rs + \frac{1}{2}as^2)} = k(m_1 + p(m_2 - m_1))$$

$$p^2c + (1 - p^2)(rs + \frac{1}{2}as^2) = \frac{1}{k(m_1 + p(m_2 - m_1))}$$

$$s = \frac{-(1 - p^2)r + \sqrt{r^2(1 - p^2)^2 + 2a(1 - p^2)(p^2c + \frac{1}{k(m_1 + p(m_2 - m_1))})}}{(1 - p^2)a}$$

Overall, we see that for high-congestion scenarios, increasing the proportion of self-driving cars mostly relieves congestion as car ownership is held constant, though we still see a slight increase in congestion at low fractions of self driving car ownership. In low congestion scenarios, increasing the proportion of self driving cars causes a more noticeable decrease in traffic speed, and self driving cars only improve traffic flow when making up a majority of the cars on the road. Therefore, while self driving cars may relieve severe congestion, they are liable to make light or moderate congestion worse until they are widely adopted.



Possible ways to mitigate this effect include promotion of car sharing and smart carpooling. Since self driving cars would be able to move without a driver, they may be able to organize efficient carpools by picking up sets of people with similar commutes. This might mean that, while self driving cars are on the road for an even greater fraction of the time, the number of self driving cars needed to serve a population might be much less than the number of human driven cars currently needed.

### 3.3 Free Riders

In our model, we have assumed that human driven and self driving cars share the same lanes. Under these assumptions, any increase in road capacity caused by increased usage of self driving cars will benefit all commuters, not just those who use self driving cars. In this sense, self driving cars might be similar to fuel efficient or electric cars—though there is little personal benefit to owning one, society is better off if a larger proportion of drivers use them. Of course, some self interested reasons to own self driving cars exist; however, if these are not strong, the government might want to promote increased self driving car use to reduce traffic congestion.

One way to do this might be by designating self driving car lanes, or allowing self driving cars to use existing carpool lanes. Even assuming that self driving car lanes are assigned proportional to the fraction of self driving cars in use, autonomous vehicle users will experience reduced congestion because they will be able to travel the speed limit even at high densities. In this scenario, we do not expect non self driving car users to benefit at all from self driving car usage. However, given that there are a number of selfish reasons to own a self driving car and that our models show speedups for all drivers even considering modest fractions of self driving cars, it is likely that lane segregation is not necessary.

### 3.4 Simplicity of Our Model

Our analytical model assumes a long, one-lane stretch of road which drivers are not entering or exiting, and assumes that the only cause of traffic congestion is the relationship between speed and stopping distance. Therefore, while our models show huge improvements in road capacity with the introduction of self driving cars, in reality, other behaviors such as lane changes, turns, and avoidance of non-car obstacles may mean improvements will not be this large. However, we are able to explore a lot of interesting behavior in this model, and think it should be relatively accurate on roadways

which it approximates well, such as major highways between exits.

## 4 Computer Model

### 4.1 Design

In addition to the analytic model described above, we created a computer model of the effect of self-driving cars on highway traffic. Our computer model is of a stretch of a highway with multiple lanes and with no cars entering or exiting the highway and no cars merging from one lane to another. Our model divides the highway into a series of connected "cells"—segments of road that are one lane wide—and simulates highway traffic as an exchange of cars among these cells. We use a cellular automaton model similar to that used by Daganzo [1], in which the simulation is composed of a series of steps, each of which represents a discrete interval of time. In each step, the number of cars moving from their current cell to the next one ( $\Delta N$ ) in one time interval ( $\Delta t$ ) is determined by the equation

$$\Delta N = \min\{(N_{\max} - N), s\Delta t\},$$

where  $N_{\max}$  is the maximum number of cars that can fit the next cell,  $N$  is the number of cars currently in the next cell, and  $s$  is the speed limit (60 mph in this model). That is, cars move into the next cell at the ideal pace of traffic until there is no longer room to do so, at which point time is lost to the delay and traffic slows down.

The number of cars that can fit in a given cell is determined by the length of a cell ( $L$ ), the length of a car ( $l$ ), and the required stopping distance of an average car ( $d_{\text{avg}}$ ), as shown below

$$N_{\max} = \frac{L}{l + d_{\text{avg}}},$$

With the average stopping distance calculated in much the same way as in the analytical model.

### 4.2 Results

We ran this model both with and without platooning behavior between self-driving cars. As can be seen in the graphs below, our results with the computer model are generally consistent with the analytical model.

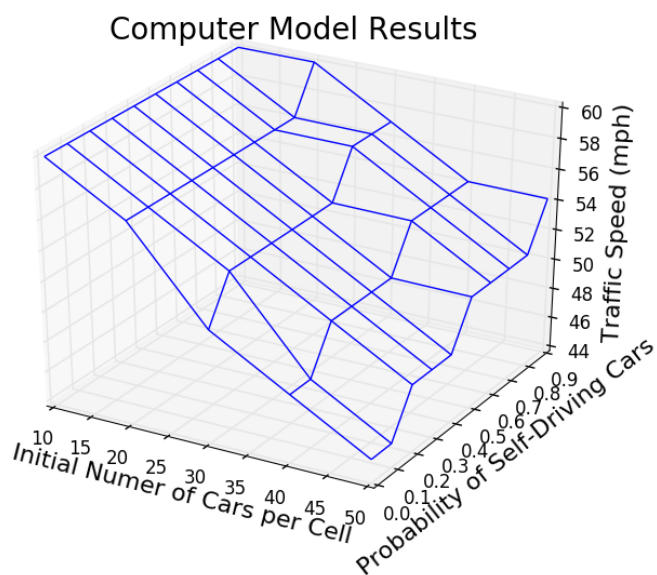


Figure 8: The results from the computer model without platooning. Note that the plot of speed against probability of a car being self-driving is roughly the same in low traffic density but is significantly steeper in higher traffic density.

Computer Model Results with Platooning

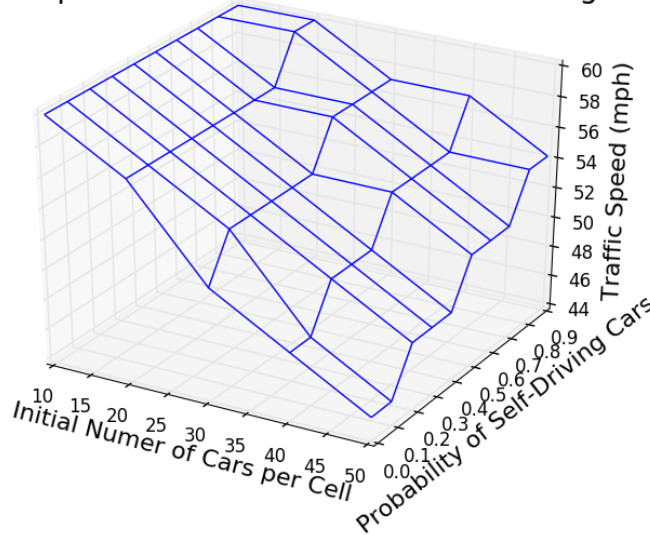


Figure 9: The results from the computer model with platooning. The model measured the effects of car density and probability of car being self-driving on the speed of traffic.

## 5 Conclusion

Our analytic and computational models show that as the proportion of self driving cars increases, traffic flow improves. Even accounting for self driving car owners driving more, we find that the large scale adoption of self driving car technology decreases traffic congestion, allowing all drivers to travel at or near the speed limit, even on heavily used roads. However, our model does not account for complex road patterns or increased car ownership, so it may overstate the case for self driving cars.

To increase self driving car usage, regulators may want to designate self driving car only lanes, or to allow self driving cars to use HOV lanes. Since communicating self driving cars work much more efficiently when following other self driving cars, and lane capacity increases nonlinearly with proportion of self driving cars, segregated self driving car lanes offer benefit to their users and improve average speed.

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