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Homework 1

Question 1:

A:

1. $10011011_2 = 2^7 + 2^4 + 2^3 + 2^1 + 2^0 = 128 + 16 + 8 + 2 + 1 = 155_{10}$

2. $456_7 = 4 * 7^2 + 5 * 7^1 + 6 * 7^0 = 4*49 + 5*7 + 6 = 237_{10}$

3. $38A_{16} = 3*16^2 + 8*16 + 10 = 768 + 128 + 10 = 906_{10}$

4. $2214_5 = 2*5^3 + 2*5^2 + 5 + 4 = 309_{10}$

B:

1. $69_{10}$ = 69 / 2 = 34 R 1
34 / 2 = 17 R 0
17 / 2 = 8 R 1
8 / 2 = 4 R 0
4 / 2 = 2 R 0
2 / 2 = 1 R 0
1 / 2 = 0 R 1 = $1000101_2$

2. $485_{10}$ = 485 / 2 = 242 R 1
242 / 2 = 121 R 0
121 / 2 = 60 R 1
60 / 2 = 30 R 0
30 / 2 = 15 R 0
15 / 2 = 7 R 1
7 / 2 = 3 R 1
3 / 2 = 1 R 1
1 / 2 = 0 R 1 = $111100101_2$

3. $6D1A_{16}$ = Using 4 bit binary word to hexidecimal
$6_{16} = 0110_2$
$D_{16} = 1101_2$
$1_{16} = 0001_2$
$A_{16} = 1010_2$
= 110 1101 0001 1010

C:

1. $1101011_2 =$ Using 4 bit binary word to hexidecimal

   $(0110)_2 = 6_{16}$

   $(1011)_2 = B_{16}$

   $= 6B_{16}$

2. $895_{10} = 895/16 = 55$ R $15$

   $55/16 = 3$ R $7$

   $3 / 16 = 0$ R $3$

   $= 37F_{16}$

Question 2

1. $7566_8 + 4515_8 =$

$$
\begin{array}{rr}
 & 1111\phantom{_8} \\
 & 7566_8 \\
+ & 4515_8 \\
= & 14303_8
\end{array}
$$

2. $10110011_2 + 1101_2 =$

$$
\begin{array}{rr}
 & 11111\phantom{_2} \\
 & 10110011_2 \\
+ & 1101_2 \\
= & 11000000_2
\end{array}
$$

3. $7A66_{16} + 45C5_{16} =$

$$
\begin{array}{rr}
 & 11\phantom{66_{16}} \\
 & 7A66_{16} \\
+ & 45C5_{16} \\
= & C02B_{16}
\end{array}
$$

4. $3022_5 - 2433_5 =$

$$
\begin{array}{rr}
 & 291\phantom{_5} \\
 & 3022_5 \\
- & 2433_5 \\
= & 34_5
\end{array}
$$

Question 3:

A.

1. $124_{10}$ = 124 / 2 = 62 R 0
   62 / 2 = 31 R 0
   31 / 2 = 15 R 1
   15 / 2 = 7 R 1
   7 / 2 = 3 R 1
   3 / 2 = 1 R 1
   1 / 2 = 0 R 1 = $01111100_{\text{8 bit 2's comp}}$

2. $-124_{10}$ = Using additive inverse: = $10000100_{\text{8 bit 2's comp}}$

$$
\begin{array}{rrl}
 & 11111 & \\
 & 01111100 & (124) \\
+ & 10000100 & (-124) \\
= & 100000000 &
\end{array}
$$

3. $109_{10}$ = 109 / 2 = 54 R 1
   54 / 2 = 27 R 0
   27 / 2 = 13 R 1
   13 / 2 = 6 R 1
   6 / 2 = 3 R 0
   3 / 2 = 1 R 1
   1 / 2 = 0 R 1
   = $01101101_{\text{8bit 2's comp}}$

4. $-79_{10}$ = First converting $79_{10}$ to binary and then using additive inverse property:
   $79_{10}$ = 79 / 2 = 39 R 1
   39 / 2 = 19 R 1
   19 / 2 = 9 R 1
   9 / 2 = 4 R 1
   4 / 2 = 2 R 0
   2 / 2 = 1 R 0
   1 / 2 = 0 R 1
   = $01001111_{\text{8bit 2's comp}}$

$$
\begin{array}{rrl}
 & 1111111 & \\
 & 01001111 & (79_{10}) \\
+ & 10110001 & (-79_{10}) \\
= & 100000000 &
\end{array}
$$

$-79_{10}$ = $10110001_{\text{8bit 2's comp}}$

B.

1. $00011110_{8\ \text{bit2's comp}} = 2 + 2^2 + 2^3 + 2^4 = 30_{10}$

2. $11100110_{8\ \text{bit 2's comp}}$ = Using additive inverse, find positive number and convert it to negative

$$\begin{array}{lll} & 111111 & \\ & 00011010_{8\ \text{bit 2's comp}} & (26_{10}) \\ + & 11100110_{8\ \text{bit 2's comp}} & (-26_{10}) \\ = & 100000000 & \end{array}$$

$11100110_{8\ \text{bit 2's comp}} = -26_{10}$

3. $00101101_{8\ \text{bit 2's comp}} = 1 + 2^2 + 2^3 + 2^5 = 32 + 8 + 4 + 1 = 45_{10}$

4. $10011110_{8\ \text{bit 2's comp}}$ = Using additive inverse, find positive number and convert it to negative

$$\begin{array}{lll} & 111111 & \\ & 01100010_{8\ \text{bit 2's comp}} & (98_{10}) \\ + & 10011110_{8\ \text{bit 2's comp}} & (-98_{10}) \\ = & 100000000 & \end{array}$$

$10011110_{8\ \text{bit 2's comp}} = -98_{10}$

Question 4:

1. Exercise 1.2.4, sections b, c

b. $\neg(p \lor q)$

| p | q | p v q | ¬(p ∨ q) |
|---|---|---|---|
| T | T | T | F |
| T | F | T | F |
| F | T | T | F |
| F | F | F | T |

c. $r \lor (p \land \neg q)$

| p | q | r | p ∧ ¬q | r ∨ (p ∧ ¬q) |
|---|---|---|---|---|
| T | T | T | F | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

2. Exercise 1.3.4, sections b, d

b. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

| p | q | (p → q) | (q → p) | (p → q) → (q → p) |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

d. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

| p | q | ¬q | (p ↔ q) | (p ↔ ¬q) | (p ↔ q) ⊕ (p ↔ ¬q) |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| F | T | F | F | T | T |
| F | F | T | T | F | T |

Question 5:

1. Exercise 1.2.7, sections b, c
    b. $(B \wedge D)$ v $(B \wedge M)$ v $(D \wedge M)$

    c. $B$ v $(D \wedge M)$

2. Exercise 1.3.7, sections b–e
    b. $(s \vee y) \rightarrow p$
    c. $p \rightarrow y$
    d. $p \leftrightarrow (s \wedge y)$
    e. $p \rightarrow (s \vee y)$

3. Exercise 1.3.9, sections c, d
    c. $c \rightarrow p$
    d. $c \rightarrow p$

Question 6:

1. Exercise 1.3.6, sections b-d
    - b. If Joe is eligible for the honors program then Joe maintains a B average.
    - c. If Rajiv can go on the roller coaster then he is at least four feet tall.
    - d. If he is at least four feet tall then Rajiv can go on the roller coaster.

2. Exercise 1.3.10, sections c–f

    p = true, q = false, r = unknown

c. $(p \lor r) \leftrightarrow (q \land r)$ = False, $(p \lor r)$ is true, since p is true. $(q \land r)$ is false since q is false. $(p \lor r) \leftrightarrow (q \land r)$ is false since both sides do not have the same truth values.

d. $(p \land r) \leftrightarrow (q \land r)$ = Unknown, $(q \land r)$ is false since q is false, but $(p \land r)$ is unknown since p is true and r is unknown. The entire expression is unknown since $(p \land r)$ is unknown.

e. $p \rightarrow (r \lor q)$ = Unknown, since p is true $(r \lor q)$ must be true for the expression to be true and false for the expression to be false. But $(r \lor q)$ is unknown since q is false and r is unknown.

f. $(p \land q) \rightarrow r$ = True, $(p \land q)$ is false since p is true and q is false. Since $(p \land q)$, the hypothesis is false, the conditional statement is true regardless of the truth value of the conclusion.

Question 7:
Exercise 1.4.5, sections b–d

j: Sally got the job.

l: Sally was late for her interview

r: Sally updated her resume.

b. If Sally did not get the job, then she was late for interview or did not update her resume.

$= \neg j \to (l \vee \neg r)$

If Sally updated her resume and was not late for her interview, then she got the job.

$= (r \wedge \neg l) \to j$

| **j** | **l** | **r** | **$(l \vee \neg r)$** | **$(r \wedge \neg l)$** | **$\neg j \to (l \vee \neg r)$** | **$(r \wedge \neg l) \to j$** |
|---|---|---|---|---|---|---|
| T | T | T | T | F | T | T |
| T | T | F | T | F | T | T |
| T | F | T | F | T | T | T |
| T | F | F | T | F | T | T |
| F | T | T | T | F | T | T |
| F | T | F | T | F | T | T |
| F | F | T | F | T | F | F |
| F | F | F | T | F | T | T |

Logically equivalent

c. If Sally got the job then she was not late for her interview.

$= j \to \neg l$

If Sally did not get the job, then she was late for her interview.

$= \neg j \to l$

| **j** | **l** | **$j \to \neg l$** | **$\neg j \to l$** |
|---|---|---|---|
| T | T | F | T |
| T | F | T | T |
| F | T | T | T |
| F | F | T | F |

Not logically equivalent

d. If Sally updated her resume or she was not late for her interview, then she got the job. $= (r \vee \neg l) \to j$

If Sally got the job, then she updated her resume and was not late for her interview.

$= j \to (r \wedge \neg l)$

| j | l | r | $(r \vee \neg l)$ | $(r \vee \neg l) \rightarrow j$ | $j \rightarrow (r \wedge \neg l)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | F | T | F |
| T | F | T | T | T | T |
| T | F | F | T | T | T |
| F | T | T | T | F | F |
| F | T | F | F | T | F |
| F | F | T | T | F | F |
| F | F | F | T | F | F |

Not logically equivalent

Question 8:

1. Exercise 1.5.2, sections c, f, I

c. $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(\neg p \lor q) \land (p \to r)$, Conditional Identity

$(\neg p \lor q) \land (\neg p \lor r)$, Conditional Identity

$\neg p \lor ( q \land r )$, Distributive Law

$p \to ( q \land r )$, Conditional Identity

f. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg p \land \neg(\neg p \land q)$, DeMorgan's Law

$\neg p \land(\neg\neg p \lor \neg q)$, DeMorgan's Law

$\neg p \land (p \lor \neg q)$, Double Negation Law

$(\neg p \land p) \lor (\neg p \land \neg q)$, Distributive Law

$F \lor (\neg p \land \neg q)$, Complement Law

$\neg p \land \neg q$, Domination Law

i. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$\neg(p \land q) \lor r$, Conditional Identity

$(\neg p \lor \neg q) \lor r$, DeMorgan's law

$\neg q \lor (\neg p \lor r)$, Associative Law

$(\neg p \lor r) \lor \neg q$, Commutative Law

$\neg(\neg p \lor r) \to \neg q$, Conditional Identity

$(\neg\neg p \lor \neg r) \to \neg q$, DeMorgan's Law

$(p \lor \neg r) \to \neg q$, Double Negation Law

2. Exercise 1.5.3, sections c, d

c. $\neg r \lor (\neg r \to p)$

$\neg r \lor (\neg\neg r \lor p)$, Conditional Identity

$\neg r \lor ( r \lor p)$, Double Negation Law

$(\neg r \lor r) \lor p$, Associative Law

$T \lor p$, Complement Law

$T$ , Domination Law

d. $\neg(p \to q) \to \neg q$

$\neg\neg(p \to q) \lor \neg q$ , Conditional Identity

$(p \to q) \lor \neg q$, Double Negation Law

$(\neg p \lor q) \lor \neg q$, Conditional Identity

$\neg p \lor (q \lor \neg q)$, Associative Law

$\neg p \lor T$, Complement Law

$T$ , Domination Law

Question 9:

1. Exercise 1.6.3, sections c, d
   - c. There is a number that is equal to its square.
     - $\exists x\ (x = x^2)$
   - d. Every number is less than or equal to its square.
     - $\forall x\ (x <= x^2)$

2. Exercise 1.7.4, sections b-d
   - b. Everyone was well and went to work yesterday
     - $\forall x\ (\neg S(x) \land W(x))$
   - c. Everyone who was sick yesterday did not go to work.
     - $\forall x\ (S(x) \rightarrow \neg W(x))$
   - d. Yesterday someone was sick and went to work.
     - $\exists x\ (S(x) \land W(x))$

Question 10:

1.Exercise 1.7.9, sections c-i

| | **P(x)** | **Q(x)** | **R(x)** |
|---|---|---|---|
| **a** | T | T | F |
| **b** | T | F | F |
| **c** | F | T | F |
| **d** | T | T | F |
| **e** | T | T | T |

c. $\exists x((x = c) \rightarrow P(x))$
  False, (x=c), True, but P(c) = False
d. $\exists x(Q(x) \wedge R(x))$
  True, see row 'e'
e. $Q(a) \wedge P(d)$
  True, Q(a) = True, P(d) = True
f. $\forall x ((x \neq b) \rightarrow Q(x))$
  True, All truth values for Q(x) are true except when x = b
g. $\forall x (P(x) \vee R(x))$
  False, Counterexample see row 'c', truth values for P(x) and R(x) are both False
h. $\forall x (R(x) \rightarrow P(x))$
  True, all values of R(x) are false except for row 'e', and P(x) has a True value for row e.
I. $\exists x(Q(x) \vee R(x))$
  True, see row 'a', 'c', 'd', 'e'

2.Exercise 1.9.2, sections b-i
b. $\exists x \forall y Q(x, y)$
  True, Q(2, y) all have True values
c. $\exists x \forall y P(y, x)$
  True, P(1, x) all have True values
d. $\exists x \exists y S(x, y)$
  False, all values in the S(x,y) predicate are false.
e. $\forall x \exists y Q(x, y)$
  False, Q(1, y) all have False values
f. $\forall x \exists y P(x, y)$
  True, for all x values there exists a y that is True, example: P(1,1), P(2,1), P(3,1)
g. $\forall x \forall y P(x, y)$
  False, counterexample P(1,2) = False
h. $\exists x \exists y Q(x, y)$
  True, example: Q(2,1)
i. $\forall x \forall y \neg S(x, y)$
  True, the entire S(x,y) predicate is false, $\neg S(x, y)$ is True

Question 11:

1.Exercise 1.10.4, sections c-g

c. There are two numbers whose sum is equal to their product.

$\exists x \exists y (x + y = xy)$

d. The ratio of every two positive numbers is also positive.

$\forall x \forall y ( (x > 0 \wedge y > 0) \rightarrow ( (x/y > 0) \wedge (y/x > 0) ) )$

e. The reciprocal of every positive number less than one is greater than one.

$\forall x ( (x < 1) \rightarrow ( 1/x > 1 ) )$

f. There is no smallest number.

$\neg\exists x\forall y(x<=y)$

g. Every number besides 0 has a multiplicative inverse.

$\forall x ( (x \neq 0) \rightarrow ( x * (1/x) = 1) )$

2.Exercise 1.10.7, sections c-f

c. There is at least one new employee who missed the deadline.

$\exists x (N(x) \wedge D(x))$

d. Sam knows the phone number of everyone who missed the deadline.

$\forall y (P(Sam, y) \wedge D(y))$

e. There is a new employee who knows everyone's phone number.

$\exists x\forall y (N(x) \wedge P(x, y))$

f. Exactly one new employee missed the deadline.

$\exists x [N(x) \wedge D(x)] \wedge \forall y [(x \neq y) \rightarrow (D(y) \wedge \neg N(y)) \vee (\neg D(y) \wedge N(x))]$

3.Exercise 1.10.10, sections c–f

c. Every student has taken at least one class besides Math 101.

$\forall x \exists y\ T(x, y \neq Math\ 101)$

d. There is a student who has taken every math class besides Math 101.

$\exists x \forall y\ T(x, y \neq Math\ 101)$

e. Everyone besides Sam has taken at least two different math classes.

$\forall x \exists y \exists z\ T(x \neq Sam, y \neq z) \wedge T(x \neq Sam, z \neq y)$

f. Sam has taken exactly two math classes.

$[\exists x \exists y\ T(Sam, x) \wedge T(Sam, y)] \wedge [\forall z\ (z \neq x) \wedge (z \neq y) \wedge \neg T(Sam, z)]$

Question 12:

1. Exercise 1.8.2, sections b-e

b. Every patient was given the medication or the placebo or both.

$\forall x\ (P(x) \vee D(x)\ )$

Negation: $\neg\forall x\ (P(x) \vee D(x)\ )$

DeMorgan's Law: $\exists x\ \neg(P(x) \vee D(x))$

DeMorgan's Law: $\exists x\ \ \neg P(x) \wedge\ \ \neg D(x)$

English: There exists a patient that did not get the placebo and did not get the medication

c. There is a patient who took the medication and had migraines.

$\exists x\ (D(x) \wedge M(x))$

Negation: $\neg\exists x\ (D(x) \wedge M(x))$

DeMorgan's Law: $\forall x\ \neg\ (D(x) \wedge M(x))$

DeMorgan's Law: $\forall x\ \neg D(x) \vee \neg M(x)$

English: Every patient was not given the medication or did not have migraines or both

d. Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \rightarrow q \equiv \neg p \vee q$.)

$\forall x\ (P(x) \rightarrow M(x))$

Negation: $\neg\forall x\ (P(x) \rightarrow M(x))$

DeMorgan's Law: $\exists x\ \neg(P(x) \rightarrow M(x))$

Conditional Identity: $\exists x\ \neg(\neg P(x) \vee M(x))$

DeMorgan's Law: $\exists x\ (\neg\neg P(x) \wedge \neg M(x))$

Double Negation Law: $\exists x\ (P(x) \wedge \neg M(x))$

English: There exists a patient that was given a placebo and did not have migraines.

e. There is a patient who had migraines and was given the placebo.

$\exists x\ (M(x) \wedge P(x))$

Negation: $\neg\exists x\ (M(x) \wedge P(x))$

DeMorgan's Law: $\forall x\ (\neg M(x) \vee \neg P(x))$

English: Every patient did not have a migraine and was not given a placebo.

2. Exercise 1.9.4, sections c-e

c. $\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$

$\neg\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$, Negation

$\forall x\ \exists y\ \neg(P(x, y) \rightarrow Q(x, y))$, DeMorgan's Law

$\forall x\ \exists y\ \neg(\neg P(x,y) \vee Q(x, y))$, Conditional Identity

$\forall x\ \exists y\ (\neg\neg P(x,y) \wedge\ \ \neg Q(x, y))$, DeMorgan's Law

$\forall x\ \exists y\ (P(x,y) \wedge\ \ \neg Q(x, y))$, Double Negation Law

d. $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$

$\neg\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$, Negation

$\forall x\ \exists y\ \neg(P(x, y) \leftrightarrow P(y, x))$, DeMorgan's Law

$\forall x\ \exists y\ \neg\ [\ (P(x, y) \rightarrow P(y, x)) \wedge (P(y, x) \rightarrow P(x, y))]$, Conditional Identity

$\forall x\ \exists y\ \neg\ [\ (\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y))\ ]$, Conditional Identity

$\forall x\ \exists y\ [\ \neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y))]$, DeMorgan's Law

$\forall x\ \exists y\ \ (\neg\neg P(x,y) \wedge \neg P(y,x)) \vee (\neg\neg P(y,x) \wedge \neg P(x,y))$, DeMorgan's Law

$\forall x \exists y (P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y))$, Double Negation Law

e. $\exists x \exists y P(x, y) \land \forall x \forall y Q(x, y)$

$\neg(\exists x \exists y P(x, y) \land \forall x \forall y Q(x, y))$ Negation

$(\neg\exists x \exists y P(x, y)) \lor (\neg\forall x \forall y Q(x, y))$, DeMorgan's Law

$\forall x \forall y \neg P(x, y) \lor \exists x \exists y Q(x, y)$, DeMorgan's Law