Ming Kong
Extended CS Bridge-Winter 2021
Homework 7

Question 3:

a) Exercise 8.2.2 section b:
$f(n) = n^3 + 3n^2 + 4$. Prove that $f = \Theta(n^3)$.

$f = O(n^3)$,
Proof:
We can say that $f(n) = \Theta(g(n))$ if there exist positive real constants c_1, c_2 and a positive integer constant n_0 such that $c_2g(n) <= f(n) <= c_1g(n)$ for all $n >= n_0$.

We know that $n^3 <= n^3 + 3n^2 + 4$ for $n >= 0$
We also know that $5n^3 >= n^3 + 3n^2 + 4$ for $n >= 2$

If we pick $c_1 = 5$, $c_2= 1$, and $n_0=2$
It shows that:
$(1)n^3 <= n^3 + 3n^2 + 4 <= (5)n^3$ is true.

Therefore $f = \Theta(n^3)$

b) Exercise 8.3.5, sections a-e

Section a:
The algorithm will cause the sequence of numbers to have all numbers less than ‘p’ to the left of ‘p’ in the sequence and all numbers greater than or equal to ‘p’ on the right side of the sequence.

Section b:
The number of times the lines are executed depends on the length of the sequence. ‘i’ and ‘j’ will eventually meet and the sum of the count of both executions will be n - 1.

Section c:
The number of times the swap operation is executed depends on the actual numbers in the sequence. The minimum number of times a swap operation is executed is zero; if all number less than ‘p’ is to the left of p, and all numbers greater than or equal to ‘p’ on the right side of the sequence. The maximum number of swaps (n/2) will occur if the numbers in the sequence greater than p on the left side of the sequence and the numbers less than p on the right side of the sequence.

Section d:
$\Omega(n)$, since it will still need to iterate at least n-1 through the sequence.

Section e:
$O(n)$, the entire algorithm will iterate through the sequence n-1 times.

Question 4:

a) Exercise 5.1.1, sections b, c

Section b: Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters.

Number of choices per character = |Digits| + |Letters| + |Special characters|
= 10 + 26 + 4
= 40

Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters
= $7^{40}+8^{40}+9^{40}$

Section c:
Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters. The first character cannot be a letter.

If the first character cannot be a letter, then the cardinality is |Digits| + |Special Characters| = 14

Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters. The first character cannot be a letter.
= $14*(6^{40}+7^{40}+8^{40})$

b) Exercise 5.3.2 section a:
How many strings are there over the set {a, b, c} that have length 10 in which no two consecutive characters are the same? For example, the string "abcbcbabcb" would count and the strings "abbbcbabcb" and "aacbcbabcb" would not count.

The first character will have 3 possible choices, and every subsequent character will only have 2 choices since there can be no 2 consecutive characters.

= $3*(2^{9})$ = 1536

c) Exercise 5.3.3 sections b, c

Section b:
How many license plate numbers are possible if no digit appears more than once?

= $10 * 9 * 8 * 26^{4}$ = 329,022,720

Section c:
How many license plate numbers are possible if no digit or letter appears more than once?
= 10 * 9 * 8 * 26 * 25 * 24 * 23 = 258336000

d) Exercise 5.2.3, sections a, b

Section a:

$f: B^9 \rightarrow E_{10}$,

$f(x) = x0$ if $x \in E_9$

$= x1$, otherwise

$|B^9| = 512$, since $|B| = 2$, and $2^9 = 512$.

There are 1024 possible sequences for a 10 bit string of 0s and 1s. Since they are all either even 1s or odd 1s, there are 512 possible 'E_{10}' values. The function is a bijection because is has a well defined inverse. It is one to one as well as onto. All B^9 values will map to a E_{10} value, and different elements in B^9 will map to a different element in E_{10}

Ex:

Index	B^9	E_{10}
0	000000000	0000000000
1	000000001	0000000011
2	000000010	0000000101
3	000000011	0000000110
4	000000100	0000001001
...		
512	111111111	1111111111

Section b:

What is $|E_{10}|$?

$|E_{10}| = 512$

Question 5:

a) Exercise 5.4.2, sections a, b

Section a:

How many different phone numbers are possible?
$= 2 * (10^4) = 20{,}000$

Section b:

How many different phone numbers are there in which the last four digits are all different?
$= 2 * 10 * 9 * 8 * 7 = 10{,}080$

b) Exercise 5.5.3, sections a-g

How many 10-bit strings are there subject to each of the following restrictions?

Section a:

No restrictions $= 2^{10} = 1024$

Section b:

The string starts with 001 $= 2^7 = 128$

Section c:

The string starts with 001 or 10 $= 2^7 + 2^8 = 384$

Section d:

The first two bits are the same as the last two bits $= 2^2 * 2^6 = 2^8 = 256$

Section e:

The string has exactly six 0's. = C(10, 6) or C(10,4) = 210

Section f:

The string has exactly six 0's and the first bit is 1. = C(9,6)= 84

Section g:

There is exactly one 1 in the first half and exactly three 1's in the second half.
= C(5,1) * C(5,3) = 50

c) Exercise 5.5.5 section a:

There are 30 boys and 35 girls that try out for a chorus. The choir director will select 10 girls and 10 boys from the children trying out. How many ways are there for the choir director to make his selection?

= C(30,10) * C(35,10)

d) Exercise 5.5.8, sections c-f

Section c:

How many five-card hands are made entirely of hearts and diamonds?

= C(26, 5) = 65,780

Section d:

How many five-card hands have four cards of the same rank?

= C(13,1) * C(12,1) * C(4,1) = 624

Section e:

How many five-card hands contain a full house?

= C(13,1) * C(12,1) * C(4,3) * C(4,2) = 3744

Section f:

How many five-card hands do not have any two cards of the same rank?

= C(13,5) * 4^5 = 1,317,888

e) Exercise 5.6.6, sections a, b

|Demonstrators| = 44

|Repudiators| = 56

Section a:

How many ways are there to select a committee of 10 senate members with the same number of Demonstrators and Repudiators?

= C(44,5) * C(56,5) = 4,148,350,734,528

Section b:

Suppose that each party must select a speaker and a vice speaker. How many ways are there for the two speakers and two vice speakers to be selected?

= P(44,2) * P(56,2) = 5,827,360

Question 6:

a) Exercise 5.7.2, sections a, b

Section a:

How many 5-card hands have at least one club?

= C(52, 5) – C(39, 5) = 2,023,203

Section b:

How many 5-card hands have at least two cards with the same rank?

= C(52, 5) - (C(13,5) * 4^5) = 1,281,072

b) Exercise 5.8.4, sections a, b

20 different comic books will be distributed to five kids.

Section a:

How many ways are there to distribute the comic books if there are no restrictions on how many go to each kid (other than the fact that all 20 will be given out)?

= 20!

Section b:

How many ways are there to distribute the comic books if they are divided evenly so that 4 go to each kid?

= 20!/ (4!4!4!4!4!) = 305,540,235,000

Question 7:
How many one-to-one functions are there from a set with five elements to sets with the following number of elements?

a) 4

= 0

By definition of a one-to-one function, each element in the domain maps to a different element in the range. Assuming each set in the range is mapped to a domain, that means there can be no one to one function from a domain set of 5 elements to a range set of 4 elements.

b) 5

= 5! = 120

c) 6

= 6! = 620

d) 7

= 7! = 5040