Ming Kong
Extended CS Bridge-Winter 2021
Homework 5

Question 3:

a) Exercise 4.1.3, sections b, c

Section b:

$f(x) = 1 / (x^2 – 4)$

Not a well defined function, when x = 2, the result is undefined.

Section c:

$f(x) = \sqrt{x^2}$

Not a well defined function, there is a positive and negative value f(x) for every x

Example, if x = 2 then $f(x) = \sqrt{2^2} = \pm 2$

b) Exercise 4.1.5, sections b, d, h, i, l

Section b:

Let A = {2, 3, 4, 5}.

f: A $\rightarrow$ **Z** such that $f(x) = x^2$.

Range: {4, 9, 16, 25}

Section d:

f: $\{0,1\}^5 \rightarrow$ **Z**. For $x \in \{0,1\}^5$, f(x) is the number of 1's that occur in x.

For example f(01101) = 3, because there are three 1's in the string "01101".

Range: {0,1,2,3,4,5}

Section h:

Let A = {1, 2, 3}.

f: A × A $\rightarrow$ **Z**×**Z**, where f(x,y) = (y, x)

Range: {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }

Section i:

Let A = {1, 2, 3}.

f: A × A $\rightarrow$ **Z**×**Z**, where f(x,y) = (x,y+1).

Range: {(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)}

Section l:

Let $A = \{1, 2, 3\}$.

$f: P(A) \rightarrow P(A)$. For $X \subseteq A$, $f(X) = X - \{1\}$

$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Range: $\{ \{\}, \{2\}, \{3\}, \{2,3\} \}$

Question 4:

I. Discrete Math zyBook:

a) Exercise 4.2.2, sections c, g, k

Section c:

h: $\mathbf{Z} \to \mathbf{Z}$. $h(x) = x^3$

Not onto, let y be an integer y = -2. There is no such x such that $x^3 = y$ and $x \in \mathbf{Z}$.
One to one.

Section g:

f: $\mathbf{Z} \times \mathbf{Z} \to \mathbf{Z} \times \mathbf{Z}$, $f(x, y) = (x+1, 2y)$

Not onto, there will be no odd integer values for y in $f(x,y) = (x+1, 2y)$
One to one

Section k:

f: $\mathbf{Z}^+ \times \mathbf{Z}^+ \to \mathbf{Z}^+$, $f(x, y) = 2^x + y$.

Not onto, there are no domain values that correspond to the target value of 1 or 2
Not, one to one. f(2, 1) = f(1, 4) = 6

b) Exercise 4.2.4, sections b, c, d, g

Section b:

f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.

Not onto, there are no domain values that gives us target values of {011, 000, 001, 010}
Not one to one, f(001) = f(101) = 101

Section c:

f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example f(011) = 110

Onto and one to one.

Section d:

f: $\{0, 1\}^3 \rightarrow \{0, 1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, f(100) = 1001.

Not onto, there are target values of y (ex. 1000, 0101), not in the range of f.
One to one.

Section g:

Let A be defined to be the set {1, 2, 3, 4, 5, 6, 7, 8} and let B = {1}. f: P(A) → P(A). For X ⊆ A, f(X) = X - B. Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.

Not onto, there are no domain values that gives us target values with a 1 in the set of f(x).

Not one to one. f({2,3}) = f({1,2,3}) = {2, 3}

II. Give an example of a function from the set of integers to the set of positive integers that is:

a. one-to-one, but not onto.

$f: Z \rightarrow Z^+$, $f(x) = \begin{cases} 2x + 3, & x >= 0 \\ -2x, & x < 0 \end{cases}$

b. onto, but not one-to-one.

$f: Z \rightarrow Z^+$, $f(x) = |x| + 1$

c. one-to-one and onto.

$f: Z \rightarrow Z^+$, $f(x) = \begin{cases} 2x + 1, & x >= 0 \\ -2x, & x < 0 \end{cases}$

d. neither one-to-one nor onto

$f: Z \rightarrow Z^+$, $f(x) = 5$

Question 5:

a) Exercise 4.3.2, sections c, d, g, i

Section c:

f: **R** → **R**. f(x) = 2x + 3

$f^{-1}(x) = (y-3) / 2$

Section d:

Let A be defined to be the set {1, 2, 3, 4, 5, 6, 7, 8}.
f: P(A) → {0, 1, 2, 3, 4, 5, 6, 7, 8}. For X ⊆ A, f(X) = |X|.
Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.

The function f is not one to one, because if X = {2,3} or X={4,5}, f({2,3}) = f({4,5}) = 2. Therefore f^{-1} is not well defined.

Section g:

f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example, f(011) = 110.

The function f is onto and one to one.
f^{-1}: $\{0, 1\}^3 \to \{0, 1\}^3$ The output of f^{-1} is obtained by taking the input string and reversing the bits

Section i:

f: **Z** × **Z** → **Z** × **Z**, f(x, y) = (x+5, y-2)

f^{-1}: **Z** × **Z** → **Z** × **Z**, $f^{-1}(x, y) = (x-5, y+2)$

b) Exercise 4.4.8, sections c, d

Section c:

$f \text{ o } h = f(h(x)) = 2(x^2+1) + 3$
$= 2x^2+2+3$
$= 2x^2 + 5$

Section d:

$h \text{ o } f = h(f(x)) = (2x + 3)^2 + 1$
$= 4x^2 + 12x + 9 + 1$

$= 4x^2 + 12x + 10$

c) Exercise 4.4.2, sections b-d

Section b:

f o h (52)= f(h(52))
h(52) = 11
f(11) = 121

Section c:

g o h o f (4) = g(h(f(4)))
f(4) = 16
h(16) = 4
g(4) = 16

Section d:

h o f = h(f(x)) = $\lceil x^2/5 \rceil$

d) Exercise 4.4.6, sections c-e

Section c:

What is h o f(010)?

f(010) = 110
h(110) = 111

h o f(010) = 111

Section d:

What is the range of h o f?

Range: {111, 101}

Section e:

What is the range of g o f?

Range: {001, 011, 101, 111}

e) Extra Credit Exercise 4.4.4 sections c, d

Let f: X → Y and g: Y → Z be two functions.

Section c:

Is it possible that f is not one-to-one and g o f is one-to-one?
Justify your answer. If the answer is "yes", give a specific example for f and g.

No. If f is not one to one that means there exists different x ∈ X that maps to same elements in Y.

Example functions:

f: Z →Z, f(x) = |x|
g: Z →Z, g(x) = x + 1
f o g (1) = f o g (-1) = 2

Section d:

Is it possible that g is not one-to-one and g o f is one-to-one?
Justify your answer. If the answer is "yes", give a specific example for f and g.

No if, g is not one to one that means there are different domains for g that gives the same values in the target of g.

Example functions:

g: Z →Z, g(x) = |x|
f: Z →Z, f(x) = x + 1

g o f (-2) = 1
g o f (0) = 1