Ming Kong
Extended CS Bridge-Winter 2021
Homework 6

Question 5:

a) $5n^3 + 2n^2 + 3n = \Theta(n^3)$
Let $f(n) = 5n^3 + 2n^2 + 3n$, by definition, $f(n) = \Theta(g(n))$ if there exists positive real constants c_1, c_2 and a positive integer constant n_0 such that

$c_2 g(n) <= f(n) <= c_1 g(n)$ for all $n >= n_0$

We can pick $c_1 = 7$
Since:

$5n^3 + 2n^2 + 3n <= 5n^3 + 2n^3$
$5n^3 + 2n^2 + 3n <= 7n^3$

We can pick $c_2 = 5$
Since

$5n^3 + 2n^2 + 3n >= 5n^3$

To pick n_0 we set $2n^2 + 3n >= 0$

$n(2n + 3) >= 0$
$n >= 0$

We can pick $n_0 = 2$

For $n >= 2$ we can safely say:
$5n^3 <= 5n^3 + 2n^2 + 3n <= 7n^3$

Therefore $5n^3 + 2n^2 + 3n = \Theta(n^3)$

b) $\sqrt{(7n^2+2n-8)} = \Theta(n)$
Let $f(n) = \sqrt{(7n^2+2n-8)}$, by definition, $f(n) = \Theta(g(n))$ if there exists positive real constants c_1, c_2 and a positive integer constant n_0 such that

$c_2 g(n) <= g(n) <= c_1 g(n)$ for all $n >= n_0$

We can pick $c_1 = 9$
Since $\sqrt{(7n^2+2n-8)} <= 9n$

We can pick $c_2 = 2$
Since $\sqrt{(7n^2+2n-8)} >= 2n$

To pick n_0 we set $7n^2 + 2n - 8 >= 4n^2$

$3n^2 + 2n - 8 >= 0$
$n >= 1.7$

We pick $n_0 = 2$
For $n >= 2$, $2n <= \sqrt{(7n^2+2n-8)} <= 9n$
Therefore $\sqrt{(7n^2+2n-8)} = \Theta(n)$