Ming Kong
Extended CS Bridge-Winter 2021
Homework 3

Question 7:

a) Exercise 3.1.1, sections a-g

A = { x ∈ **Z**: x is an integer multiple of 3 }
B = { x ∈ **Z**: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that $x = y^2$

Section a:

$27 \in A$, True

Section b:

$27 \in B$, False

Section c:

$100 \in B$, True

Section d:

$E \subseteq C$ or $C \subseteq E$, False

Section e:

$E \subseteq A$, True

Section f:

$A \subset E$, False

Section g:

$E \in A$, False

b) Exercise 3.1.2, sections a-e

A = { x ∈ **Z**: x is an integer multiple of 3 }

B = { x ∈ **Z**: x is a perfect square }

C = { 4, 5, 9, 10 }

D = { 2, 4, 11, 14 }

E = { 3, 6, 9 }

F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that $x = y^2$.

Section a:

15 ⊂ A, True

Section b:

{15} ⊂ A, False

Section c:

∅ ⊂ A, False

Section d:

A ⊆ A, True

Section e:

∅ ∈ B, False

c) Exercise 3.1.5, sections b, d

Section b:

{ 3, 6, 9, 12, ... } = {x ∈ **N**: x is an integer multiple of 3 and x > 0}, the set is infinite

Section d:

{ 0, 10, 20, 30, ..., 1000 } = {x ∈ **N**: x is an integer multiple of 10 and x includes 0: |x| = 101}

d) Exercise 3.2.1, sections a-k

Let X = {1, {1}, {1, 2}, 2, {3}, 4 }. Which statements are true?

Section a:

$2 \in X$, True

Section b:

$\{2\} \subseteq X$, True

Section c:

$\{2\} \in X$, False

Section d:

$3 \in X$, False

Section e:

$\{1, 2\} \in X$, True

Section f:

$\{1, 2\} \subseteq X$, True

Section g:

$\{2, 4\} \subseteq X$, True

Section h:

$\{2, 4\} \in X$, False

Section i:

$\{2, 3\} \subseteq X$, False

Section j:

$\{2, 3\} \in X$, False

Section k:

$|X| = 7$, False

Question 8:
Exercise 3.2.4, section b

Let A = {1, 2, 3}. What is $\{X \in P(A): 2 \in X\}$?

$P(A) = \{\varnothing, \{1\}, \{2\}, \{3], \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

$\{X \in P(A): 2 \in X\} = \{ \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\} \}$

Question 9:

a) Exercise 3.3.1, sections c-e

A = {-3, 0, 1, 4, 17}
B = {-12, -5, 1, 4, 6}
C = {x ∈ **Z**: x is odd}
D = {x ∈ **Z**: x is positive}

Section c:

$A \cap C = \{-3, 1, 17\}$

Section d:

$A \cup (B \cap C)$,

$B \cap C = \{-5, 1\}$

$A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

Section e:

$A \cap B \cap C$,

$A \cap C = \{-3, 1, 17\}$

$A \cap B \cap C = \{1\}$

b) Exercise 3.3.3,sections a, b, e, f

Section a:

$\{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\}$

$\{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\}$

$= \{1\}$

Section b:

$\{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\}$

$= \{1, 2, 3, 4, 5, 9, 16, 25\}$

Section e:

$\{-1/1 <= x <= 1/1\} \cap \{-1/2 <= x <= 1/2\} \cap \{-1/3 <= x <= 1/3\} \cap \ldots \cap \{-1/100 <= x <= 1/100\}$

$= \{x \in \mathbf{R}: -1/100 <= x <= 1/100\}$

c) Exercise 3.3.4, sections b, d

Use the set definitions A = {a, b} and B = {b, c} to express each set below.

Section b:

$P(A \cup B)$,

$A \cup B = \{a, b, c\}$

$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Section d:

$P(A) \cup P(B)$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Question 10:

a) Exercise 3.5.1, sections b, c

The sets A, B, and C are defined as follows:

- A = {tall, grande, venti}
- B = {foam, no-foam}
- C = {non-fat, whole}

Section b:

Write an element from the set $B \times A \times C$.

$\{foam, tall, non\text{-}fat\} \in B \times A \times C$

Section c:

Write the set $B \times C$ using roster notation.

$B \times C = \{ \{foam, non\text{-}fat\}, \{foam, whole\}, \{no\text{-}foam, non\text{-}fat\}, \{no\text{-}foam, whole\} \}$

b) Exercise 3.5.3, sections b, c, e

Section b:

$\mathbf{Z}^2 \subseteq \mathbf{R}^{2,}$ True

Section c:

$\mathbf{Z}^2 \cap \mathbf{Z}^3 = \emptyset$, True

Section e:

For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$., True

c) Exercise 3.5.6, sections d, e

Section d:

$\{xy$: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2\}$

$\{0\}^2 = \{ (00) \}$

$\{1\}^2 = \{ (11) \}$

$\{0\} \cup \{0\}^2 = \{0, 00\}$

$\{1\} \cup \{1\}^2 = \{1, 11\}$

$xy = \{01, 011, 001, 0011\}$

Section e:

$\{xy$: $x \in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2\}$

$\{a\} \cup \{a\}^2 = \{a, aa\}$

$xy = \{ aaa, aaaa, aba, abaa\}$

d) Exercise 3.5.7, sections c, f, g

$A = \{a\}$
$B = \{b, c\}$
$C = \{a, b, d\}$

Section c:

$(A \times B) \cup (A \times C)$

$(A \times B) = \{ab, ac\}$

$(A \times C) = \{aa, ab, ad\}$

$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

Section f:

$P(A \times B)$

$A \times B = \{ab, ac\}$

$P(A \times B) = \{ \varnothing, \{ab\}, \{ac\}, \{ab, ac\} \}$

Section g:

P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

P(A) = { ∅, {a} }

P(B) = { ∅, {b}, {c}, {b, c} }

P(A) × P(B) = {(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c}),

Question 11:

a) Exercise 3.6.2, sections b, c

Section b:

$(B \cup A) \cap (B \cup A) = A$

$(B \cup A) \cap (\overline{B} \cup A)$	
$(A \cup B) \cap (A \cup \overline{B})$	Commutative law
$A \cup (B \cap \overline{B})$	Distributive Law
$A \cup \emptyset$	Complement Law
A	Identity Laws

Section c:

$\overline{(A \cap \overline{B})} = \overline{A} \cup B$

$\overline{(A \cap \overline{B})}$	
$\overline{A} \cup \overline{\overline{B}}$	DeMorgan's Law
$\overline{A} \cup B$	Double Complement Law

b) Exercise 3.6.3, sections b, d

Section b:

$A - (B \cap A) = A$

If $A = \{1, 2\}$ and $B = \{2, 3\}$, then $B \cap A = \{2\}$.

Then $A - (B \cap A) = \{1\}$, which is not equal A

Section d:

$(B - A) \cup A = A$

If $A = \{1, 2\}$ and $B = \{2, 3\}$, then $B-A = \{3\}$.

Then $(B - A) \cup A = \{1,2,3\}$ which is not equal to A.

c) Exercise 3.6.4, sections b, c

Section b:

$A \cap (B - A) = \emptyset$

$A \cap (B - A)$	
$A \cap (B \cap \overline{A})$	Set subtraction law
$(A \cap \overline{A}) \cap B$	Associative laws
$\emptyset \cap B$	Complement laws
$\emptyset$	Domination laws

Section c:

$A \cup (B - A) = A \cup B$

$A \cup (B - A)$	
$A \cup (B \cap \overline{A})$	Set subtraction law
$(A \cup B) \cap (A \cup \overline{A})$	Distributive law
$(A \cup B) \cap U$	Complement Law
$A \cup B$	Identity law