Ming Kong
Extended CS Bridge-Winter 2021
Homework 8

Question 7:

a) Exercise 6.1.5, sections b-d

Section b:

(13 * (4 choose 3) * (12 choose 2) * (4 choose 1) * (4 choose 1)) / (52 choose 5)
= 0.0211284514

Section c:

(4 choose 1) (13 choose 5) / (52 choose 5) = 0.001981

Section d:

((13 choose 1) (4 choose 2) * 12 * 11 * 10 * 4^3) / (52 choose 5) = 0.4225690276

b) Exercise 6.2.4, sections a-d

Section a:

1 – ((39 choose 5) / (52 choose 5)) = 0.77846

Section b:

1 – ((13 choose 5) * 4^5) / (52 choose 5)) = 0.49292

Section c:

2 * ((13 * (39 choose 4)) / (52 choose 5)) – (13^2 * (26 choose 3) / (52 choose 5)) = 0.6537

Section d:

1 – ((26 choose 5) / (52 choose 5)) = 0.97468

Question 8:

a) Exercise 6.3.2, sections a-e

Section a:

p(A) = 6!/7! = 1/7

p(B) = $\left(5!\sum_{i=1}^{6} 7-i\right)/7!$ = (21*5!) / 7! = 1/2

p(C) = 5!/7! = 1/42

Section b:

p(A|C) = $p(A\cap C)/p(C)$

$p(A\cap C)$ = (2*3!)/7!

p(A|C) = (2(3!)/7!) / (1/42) = 1/10

Section c:

p(B|C) = $p(B\cap C)/p(C)$

$p(B\cap C)$ = $3!\sum_{i=1}^{4}(5-i)$ / 7!= (3! * 10) / 7! = 1/84

p(B|C) = (1/84) / (1/42) = 1/2

Section d:

p(A|B) = $p(A\cap B)/p(B)$

$p(A\cap B)$ = (3*5!) / 7! = 1/14

p(A|B) = (1/14) / (½) = 1/2

Section e:

No pair of events are independent because:
p(A|C) ≠ p(A)
p(B|C) ≠ p(B)
p(A|B) ≠ p(A)

b) Exercise 6.3.6, sections b, c

$p(H) = 1/3$
$p(T) = 2/3$

Section b:

$(1/3)^5(2/3)^5 = 0.00054$

Section c:

$(1/3)(2/3)^9 = 0.00867$

c) Exercise 6.4.2, section a

$p(6) = 0.25$
$p(1) = p(2) = p(3) = p(4) = p(5) = 0.15$

$p(F) = 1/2$, fair die is chosen

$p(X|\overline{F}) = (0.15)^4(0.25)^2$
$p(X|F) = (1/6)^6$

$p(F|X) = (p(X|F)p(F)) / (p(X|F)p(F) + p(X|\overline{F})p(\overline{F}))$
$= ((1/6)^6(1/2)) / ((1/6)^6(1/2) + (0.15)^4(0.25)^2(1/2)) = 0.404$

Question 9:

a) Exercise 6.5.2, sections a, b

Section a:

range of A = {0, 1, 2, 3, 4}

Section b:

distribution =

(0, (48 choose 5) / (52 choose 5) = (0, ~0.659)

(1, (4 * (48 choose 4) / (52 choose 5)) = (1, ~0.299)

(2, ((4 choose 2) * (48 choose 3))/ (52 choose 5)) = (2, ~0.0399)

(3, (4 * (48 choose 2) / (52 choose 5)) = (3, ~0.0017)

(4, 48 / (52 choose 5)) = (4, ~0.000018)

b) Exercise 6.6.1, section a

range = {0, 1, 2}

p(0) = (3 choose 2) / (10 choose 2) = 1/15

p(1) = ((7 choose 1) * (3 choose 1)) / (10 choose 2) = 7/15

p(2) = (7 choose 2) / (10 choose 2) = 7/15

E[G] = 0 p(0) + 1p(1) + 2 p(2)

E[G] = 1.4

c) Exercise 6.6.4, sections a, b

Section a:

p(1) = p(2) = p(3) = p(4) = p(5) = p(6) = 1/6

E[X] = p(1) + 2^2 p(2) + 3^2 p(3) + 4^2 p(4) + 5^2 p(5) + 6^2 p(6)

= 91/6 = 15.16666

Section b:

p(H) = p(T) = 1/2

range = {0, 1, 4, 9}

p(0) = $(1/2)^3$ = 1/8

p(1) = (3 choose 1) / 2^3 = 3/8

p(2) = (3 choose 2) / 8 = 3/8

p(3) = $(1/2)^3$ = 1/8

E[Y] = 0p(0) + 1p(1) + 4p(2) + 9p(3) = 3

d) Exercise 6.7.4, section a

Section a:

p(gets own coat) = 1/10

E[gets own coat] = 10 (1/10) = 1

Question 10:

a) Exercise 6.8.1, sections a-d

p(defect) = 0.01

Section a:

p(defect = 2) = (100 choose 2) $(0.01)^2(0.99)^{98}$ = 0.1849

Section b:

p(defect >= 2) = 1 – p(defect < 1) = 1 – p(defect=0) + p(defect=1)

= 1 – $(0.99^{100} + (100(0.01)^1(0.99)^{99})$ = 0.2642

Section c:

n = 100

p = 0.01

E[defects] = np = 1

Section d:

p = 0.01

n = 50

p(defect=0) = 0.99^{50}

p(defect >= 1) = 1 – p(defect < 1) = 1 – p(defect=0) = 1- 0.99^{50} = 0.395

E[defects] = np = 50 * 0.01 = 0.5, since each defect is 2 circuit boards, its 0.5 * 2

E[defects] = 1

Even though the expected number of circuit boards are the same between batches and singles, the probability of a defective board is greater.

b) Exercise 6.8.3, section b

fair coin: p(H) = p(T) = ½

biased coin: p(H) = 0.3

p(T) = 1 – p(H) = 0.7

given a biased coin getting at least 4 heads = incorrect conclusion

p(X >= 4) = 1 – p(X < 4)

= 1 – (p(X=0) + p(X=1) + p(X=2) + p(X=3))

= 1 – [(10 choose 0)$0.3^0 0.7^{10}$ + (10 choose 1)$0.3^1 0.7^9$ + (10 choose 2)$0.3^2 0.7^8$ +

(10 choose 3)$0.3^3 0.7^7$

=0.3504