UVAMT 2025 - Individual Round

I1. A two-digit positive integer x has the property that both $\sqrt{x}$ and $\sqrt[3]{x}$ are integers. Find x.

I2. A rectangle has perimeter 70 and area 300. How long is its diagonal?

I3. Utkarsh comes across some snowballs in a field. Every minute, Utkarsh choose three snowballs and combines them into one. Suppose that after 20 minutes, there are 25 separate snowballs left in the field. How many snowballs must there have been at the start?

I4. Utkarsh and Vincent are running laps around a long circular track with circumference 21 miles. Utkarsh runs at a constant pace of 6 minutes per mile, while Vincent runs at a constant pace of 8 minutes per mile in the opposite direction. If they started at the same point, how many miles did Vincent run before passing Utkarsh for the first time?

I5. Suppose there are 31 people signed up for UVAMT. Teams have a maximum size of 6, but any set of teams containing a total of at most 6 people can be combined together. However, teams cannot be broken up. Find the maximum possible number of teams (totaling 31 people) such that it is not possible to combine any of them.

I6. Vincent has 6 buckets, with capacities 2, 3, 4, 5, 6, and 7 pounds. He also has 6 bricks, weighing 1, 2, 3, 4, 5, and 6 pounds. He wants to put exactly one brick in each bucket such that no bucket is above capacity. How many ways are there for him to accomplish this?

I7. In Chickenville, each household has either 1, 2, or 3 children. 20% of the children are only children, but 35% of households have only one child. Determine the average number of children per household.

I8. A cylindrical container with radius 1 and height 4 is partially filled with water. If tilted 45 degrees from the upright position, no water will spill, but if tilted any more than that, water will start to spill. Determine the volume of water in the cup.

I9. A random number x is selected uniformly in $[0, 1]$. Mikhail can then apply as many moves to x as he wants, where a move is defined as rounding x to the decimal position of his choice. Mikhail wins if he can set x to 1 in finitely many moves. Find Mikhail's probability of winning.

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I10. A rectangular prism with positive integer side lengths has volume V and surface area $2V$. What is the sum of all possible values of V?

I11. Suppose that sets $S_1, S_2, \cdots, S_{12}$ satisfy the following properties:
  - Any three of these sets have exactly one element in common.
  - Any four of these sets have no elements in common.

Find the minimum possible value of $|S_1|$.

I12. Let S be a set of integers. Suppose that for any two distinct integers $x, y \in S$ and any nonnegative integer k, we have $2^k x \not\equiv y \pmod{2^{12} - 1}$. Find the maximum possible size of S.