

Problem: Implement a min heap with `extract_min` and `insert` methods.

Constraints

- Can we assume the inputs are ints?
 - Yes
- Can we assume this fits memory?
 - Yes

Test Cases

- Extract min of an empty tree
- Extract min general case
- Insert into an empty tree
- Insert general case (left and right insert)

- - ```
 5
 _
 / \
 20 15
 /\ /\
 22 40 25
```

- - `extract_min(): 5`

- - ```
      15
     _
    / \
   20 25
  /\  /\
 22 40
```

- - `insert(2):`

- - ```
 2
 _
 / \
 20 5
 /\ /\
 22 40 25 15
```

## Algorithm

A heap is a complete binary tree where each node is smaller than its children.

### extract\_min

- ```
      5
     / \
    20 15
   /\  /\
  22 40 25
```
-
- Save the root as the value to be returned: 5
- Move the right most element to the root: 25
-
- ```
 25
 / \
 20 15
 /\ /\
 22 40
```
- 
- Bubble down 25: Swap 25 and 15 (the smaller child)
- 
- ```
      15
     / \
    20 25
   /\  /\
  22 40
```
-
- Return 5

We'll use an array to represent the tree, here are the indices:

- ```
 0
 / \
 1 2
 /\ /\
 3 4
```

To get to a child, we take  $2 \text{ index} + 1$  (left child) or  $2 \text{ index} + 2$  (right child).

For example, the right child of index 1 is  $2 * 1 + 2 = 4$ .

Complexity:

- Time:  $O(\lg(n))$

- Space:  $O(\lg(n))$  for the recursion depth (tree height), or  $O(1)$  if using an iterative approach

## insert

- $\begin{array}{c} 5 \\ \_ \\ / \quad \backslash \\ 20 \quad 15 \\ / \backslash \quad / \backslash \\ 22 \quad 40 \quad 25 \end{array}$
- 
- insert(2):
- Insert at the right-most spot to maintain the heap property.
- 
- $\begin{array}{c} 5 \\ \_ \\ / \quad \backslash \\ 20 \quad 15 \\ / \backslash \quad / \backslash \\ 22 \quad 40 \quad 25 \quad 2 \end{array}$
- 
- Bubble up 2: Swap 2 and 15
- 
- $\begin{array}{c} 5 \\ \_ \\ / \quad \backslash \\ 20 \quad 2 \\ / \backslash \quad / \backslash \\ 22 \quad 40 \quad 25 \quad 15 \end{array}$
- 
- Bubble up 2: Swap 2 and 5
- 
- $\begin{array}{c} 2 \\ \_ \\ / \quad \backslash \\ 20 \quad 5 \\ / \backslash \quad / \backslash \\ 22 \quad 40 \quad 25 \quad 15 \end{array}$
- 

We'll use an array to represent the tree, here are the indices:

- $\begin{array}{c} 0 \\ \_ \\ / \quad \backslash \\ 1 \quad 2 \\ / \backslash \quad / \backslash \\ 3 \quad 4 \quad 5 \quad 6 \end{array}$

To get to a parent, we take  $(\text{index} - 1) // 2$ .

For example, the parent of index 6 is  $(6 - 1) // 2 = 2$ .

Complexity:

- Time:  $O(\lg(n))$
- Space:  $O(\lg(n))$  for the recursion depth (tree height), or  $O(1)$  if using an iterative approach
-