

# **Study on modeling and monitoring the spread of COVID19 virus**

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# 1 Derivation of spreading coefficient limit of declining infection

In this document I try to develop a model that would help me to understand the essential figures and numbers that we should be monitored and measured in order to distinguish the effective and ineffective measures in limiting the spread of the COVID19 virus.

I start the derivation from the measured reproduction number  $R_0$ . We know that the nature of the growth of the infection is exponential which inherently means the disease is spread by infected persons infecting the uninfected ones over certain period of time.

With COVID19 there is lots of statistics about number of infected persons, daily new cases, deaths and recoveries. Based on these statistics we may derive the estimate of the contamination time during which the infected patient spreads the disease in such a manner that the daily proportional growth equals the monitored reproduction number.

Under assumption that infected persons spread the disease with constant fraction of cases per day, the newly infected persons tomorrow can be expressed as

$$y(n+1) = Ky(n) \quad (1.1)$$

$$K = \frac{y(n+1)}{y(n)} = \frac{y(n+1) - y(n)}{y(n)} + 1 \quad (1.2)$$

where  $y(n)$  is the number of infected persons on day  $n$ . Without limiting measures  $K$  is assumed here to be relatively constant.

We may express  $K$  as

$$K = 1 + S \quad (1.3)$$

$$S = \frac{y(n+1) - y(n)}{y(n)}, \quad (1.4)$$

where  $S$  is spreading coefficient, the *ratio of daily increase of cases to active cases*. It should be noted that any estimation error that is in fixed proportion to actually confirmed cases is canceled in calculation of  $S$ . Thus we can state that any measure that gives us proportionally relatively stable information will provide us accurate information about  $S$ .

As shown before,  $K_t$  and  $S$  can be measured. So can be the reproduction number  $R_0$ , which for the COVID19 have been estimated to be  $R_0 = 2.06 \dots 2.52$  [1]. These two expansion mechanisms are related, and they should match at least in early phases of epidemic, when deaths, recoveries, countermeasures, and developing immunity do not hinder the spreading. The relation can be expressed as

$$K^{T_c} = 1 + R_0 \quad (1.5)$$

$$T_c = \frac{\log(R_0 + 1)}{\log(K)}, \quad (1.6)$$

where  $T_c$  is the contamination time, i.e. the effective time during which the patients spread the disease. It can be thought as a time before patient get's so visibly sick that he will be isolated/quarantined from the community and thus does not spread the disease. This time is specific to disease and we may assume here it is relatively constant.

We may now denote the spreading coefficient, measured as the ratio of increase to the active cases, in the beginning of the epidemic as  $S_0$ . Thus

$$T_c = \frac{\log(R_0 + 1)}{\log(1 + S_0)} \quad (1.7)$$

For present COVID19,  $S_0$  seems to be, with reasonable number of cases, between 0.3 and 0.2, thus we may use  $S_0 = 0.25$  in our example calculations. We may now preform an example calculation for contamination period

$$S_0 = 0.25 \quad (1.8)$$

$$R_0 = 2.5 \quad (1.9)$$

$$T_c = \frac{\ln(1 + R_0)}{\ln(1 + S_0)} = 5,6. \quad (1.10)$$

Thus the estimate of the contamination time is 5,6 days, and we assume that to remain constant.

Under ongoing epidemic, public authorities and private persons perform measures to reduce  $S$  in order to hinder the spread of the disease. However, as  $S$  and  $R_0$  are connected, we may try to calculate *how low we should reduce  $S$  in order to stop the epidemic under assumption that  $T_T$  does not change*. Stopping the epidemic means  $R_0 < 1$ . We may now calculate the limit  $S_l$  that should result in declining numbers of active patients.

$$(1 + S_l)^{T_c} < 2 \quad (1.11)$$

$$S_l < 2^{\frac{\ln(1+S_0)}{\ln(1+R_0)}} - 1. \quad (1.12)$$

With the given values for current COVID19 outbreak  $S_l < 0.13$ .

In Table 1.1 the obtained values under various assumptions are presented. These values reveal that with given parameter spread there is no *practical* difference in contamination time, it lies between 3,7 to 6,2 days. This means practically two days difference, easily masked by individual variation and real life uncertainties. Furthermore we may quite certainly say there is no chance of epidemic to stop with spreading factors larger than 0,2. Assuming 0.15 might be wishful thinking but may work, and 0.1 should be quite safe. Now it is possible to evaluate the theory by comparing it to existing data.

Table 1.1: Spreading coefficient limit value under various assumptions on reproduction number  $R_0$  and initial spreading coefficient  $S_0$

$R_0$	$S_0$	$T_c$	$S_l$
2	0.25	4,9	0.15
2	0.3	4,2	0.18
2	0.4	3,7	0.21
2.5	0.25	5.6	0.13
2.5	0.3	4.8	0.16
2.5	0.4	4.2	0.18
3	0.25	6.2	0.11
3	0.3	5.3	0.14
3	0.4	4.6	0.18

## **Bibliography**

- [1] S. Zhang, M. Diao, W. Yu, L. Pei, Z. Lin, and D. Chen, "Estimation of the reproductive number of novel coronavirus (COVID-19) and the probable outbreak size on the Diamond Princess cruise ship: A data-driven analysis," *International Journal of Infectious Diseases*, vol. 93, pp. 201–204, 2020.