HW Solution (Math111-01, July 13th, 2017)

In these problems, μ represents the population mean, $\bar{x}$ represents the sample mean, σ represents the population standard deviation (sometimes approximated by the sample standrd deviation), n is the sample size, μ_0 is the null value, and Z is a standard normal random variable.

Problem 4.24b

The hypothesis test is one-sided:

$$\begin{aligned} H_0&: \ \mu = 32 \\ H_A&: \ \mu < 32. \end{aligned}$$

The test statistic (z-score) is given by:

$$\begin{aligned} z-\text{score} &= \frac{\text{point estimate} - \text{null value}}{\text{standard error}} \\ &= \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \\ &\approx \frac{30.69-32}{4.31/\sqrt{36}} \\ &\approx -1.82 \end{aligned}$$

The p-value is given by

$$\begin{aligned} p-\text{value} &= P(Z<-1.82) \\ &\approx .0344. \end{aligned}$$

Since the p-value is so small, one rejects the null hypothesis with the significance level of 0.10.

Problem 4.25b

The hypothesis test is two-sided:

$$\begin{aligned} H_0&: \ \mu = 127 \\ H_A&: \ \mu \neq 127 \end{aligned}$$

The z-score is given as

$$\begin{aligned} z-\text{score} &= \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \\ &\approx \frac{137.5-127}{39/\sqrt{64}} \\ &\approx 2.15 \end{aligned}$$

The p-value is given by

$$\begin{aligned} p-\text{value} &\approx 2P(Z>2.15) \\ &\approx 2(.0158) \\ &= .0316. \end{aligned}$$

Since this is quite lower than the significance level of .05, we reject the null hypothesis that $\mu = 127$.

Problem 4.26a

The hypothesis test is two-sided:

$$\begin{aligned} H_0\!: &\quad \mu = 100 \\ H_0\!: &\quad \mu \neq 100. \end{aligned}$$

The z-score is

$$\begin{aligned} z - \text{score} &= \frac{118.2 - 100}{6.5/\sqrt{36}} \\ &\approx 16.8 \end{aligned}$$

The $p-$value is given by

$$\begin{aligned} p - \text{value} &\approx 2P(Z > 16.8) \\ &\approx 0. \end{aligned}$$

At any reasonable level of significance, the IQ of of mothers of gifted children is higher than the average IQ of the population at large.