HW Solution (July 13)

Problem 4

The z-score is given by

$$\begin{aligned} z-\text{score} &= \frac{9.5-10}{4/\sqrt{225}} \\ &= -1.875. \end{aligned}$$

The corresponding p-values for alternative hypothesis is given as follows (Z being a standard normal random variable):

- $\mu > 10$: $p-\text{value} = P(Z > -1.875) \approx 0.9696$.
- $\mu < 10$: $p-\text{value} = P(Z < -1.875) \approx 0.0304$.
- $\mu \neq 10$: $p-\text{value} {\approx} 2 \times 0.0304 {=} 0.0608$.

(Note: to calculate $P(Z < -1.875)$ with the table provided, one could average $P(Z < -1.88)$ and $P(Z < -1.87)$).

Problem 5

The sample mean is given by

$$\begin{aligned} \bar{x} &= \frac{\sum x_i}{n} \\ &= \frac{58+65+\ldots+100}{16} \\ &= \frac{1361}{16} \\ &= 85\frac{1}{16}. \end{aligned}$$

To calculate the sample standard deviation, we first calculate the sum of squared errors $(x_i - \bar{x})^2$. This is easy to do with a spreadsheet of the form (this is especially easy using a spreadsheet application):

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
58	-27$\frac{1}{16}$	732.4
65	-20$\frac{1}{16}$	402.5
$\vdots$	$\vdots$	$\vdots$
100	14$\frac{15}{16}$	223.1

The sum of the third column is $\sum (x_i - \bar{x})^2 = 2326\frac{15}{16}$. Thus the sample standard deviation is

$$\begin{aligned} s &= \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})} \\ &= \sqrt{\frac{1}{16-1}\left(2326\frac{15}{16}\right)} \\ &\approx 12.46. \end{aligned}$$

The $t-$statistic is given by

$$\begin{aligned}
t-\text{statistic} &= \frac{\bar{x}-\mu_0}{s/\sqrt{16}} \\
&= \frac{85\frac{1}{16}-85}{12.46/\sqrt{16}} \\
&\approx 0.020.
\end{aligned}$$

The p-value is $2 \times P(T > 0.020)$, where T is a t-distributed random variable with $16-1=15$ degrees of freedom. Note that the t-statistic of 0.020 is very small, so the $p-$value $2 \times P(T > 0.020)$ is very large.

Using the table provided in the book (and distributed in class), the smallest t-statistic provided for 15 degres of freedom is $t_{0.10} = 1.341$. This corresponds to a p-value of 20% (since it is a two-tailed test). So the our p-value is *at least* 0.20. Using statistical software, one can calculate the precise $p-$value as 0.9843, which is very large. Thus, we must fail to reject the null hypothesis at the 10% significance. We cannot conclude that the test is not of the desired difficulty (the average score being 85).

The 90% confidence interval is given by

$$\begin{aligned}
\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n}} &\approx 85\frac{1}{16} \pm 1.753\frac{12.46}{\sqrt{16}} \\
&\approx 85\frac{1}{16} \pm 5.46 \\
&\approx (79.60, 90.42).
\end{aligned}$$

The null value $\mu_0 = 85$ is within the interval, which confirms the previous analysis that we reject the null at the 10% significance level.