MATH111-01: REVIEW SHEET 1 [Answers]

Summer Session II, 2017. Raritan Valley Community College

Open-Ended Questions

1. The **alternative hypothesis** is a statement we are trying to prove with data. The **null hypothesis** is a statement of a more skeptical position in opposition to the alternative hypothesis. Typically, null hypotheses are typically given by equations ($=$). And alternative hypotheses are given by inequalities ($<$, $>$, $\neq$)

2. A confidence interval is a range of numbers such that a givem population parameter (such as the population mean) is supposed to lie in with a specified **confidence level**. Common confidence levels include 90%, 95% and 99%. The **significance level** is 100% minus the confidence level. A (two-sided) confidence interval is typically written in the form

$$\text{point estimator} \pm \textbf{margin of error}.$$

   For example, a confidence interval for the population mean (confidence level $= 1 - \alpha$) is given by $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$. Here, $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is the margin of error.

3. The p-value of a hypothesis test is defined as the probability under the null hypothesis of obtaining a sample that is at least as favorable to the alternative hypothesis should a new, independent sample were to be collected. In symbols:

$$p - \text{value} = P(\text{obtaining more favorable result for } H_A | H_0).$$

   P-values are calculated from test statistics. This test-statistic takes a known distribution under the null hypothesis. The area under the tails of this distribution gives the p-value.

   P-values lower than the specified significance level (5% for many fields) indicate that the null hypothesis should be rejected in favor for the alternative hypothesis. Otherwise, the one can only fail to reject the null hypothesis.

4. The statistical power of a test is the probability of rejecting the null hypothesis given that the alternative hypothesis is true:

$$\text{power} = P(\text{reject } H_0 | H_A).$$

   Alternatively, the statistical power may be defined as 1 minus the Type II error rate. Statistical power is determined by (1) the true values of the unknown parameter(s), (2) the sample size, and (3) the specified significance level.

   Statistical power can be increased by increasing the significance level. But this has the negative effect of reducing the statistical significance of the result. Power can also be increased by increasing the sample size. But this has the negative effect of making the experiment more costly.

5. In the hypothesis tests thus discussed, we have made two assumptions: (1) observations must be independent and (2) the sampled populations must be normal (this can be relaxed with large samples). Furthermore:

   - In the pooled and nonpooled mean comparison tests as well as ANOVA, it is assumed that the samples are independent. This is different than the previously stated assumptions that the observations *within* each sample must be independent.
   - In the paired difference test, it is assumed that observations from each sample are paired. That is, they have a one-to-one correspondence.
   - In ANOVA and the pooled mean comparison tests, it is assumed that the variances of the sampled populations are equal. This is called **homoscedasticity**.
   - z-tests assume that σ is known. This condition may be relaxed with large sample sizes.

Calculating *p*-values

1. 0.1112
2. 0.0594
3. 0.2758
4. 0.9812
5. Between 0.025 and 0.05
6. Between 0.90 and 0.95
7. Between 0.10 and 0.20
8. Less than 0.005
9. Between 0.025 and 0.05
10. Greater than 0.10

Word Problems

1.

   a) $\bar{x} = 407.17$, $s_d = 113.68$.

   b) Margin of error = 58.94. Confidence interval = $(348.2, 466.1)$.

   c) Since the null value 300 lies outside the confidence interval, thus we reject the null hypothesis. We conclude that $\mu \neq 300$.

2.

   a) $t = -0.5973$, $df = 19$

   b) $p - \text{value} > 0.10$. Fail to reject the null.

   c) Lack of a control group. No evidence provided that the two tests are actually of similar difficulty.

3. (**NOTE:** These answers are for when $\bar{x}_{\text{NJ}} = 1356$)

   a) $t = 1.703$, $df = 48$. The $p - \text{value}$ is between 0.05 and 0.10.

   b) $t = 1.713$, $df = 58$. The p-value is between 0.05 and 0.10.

4.

   a) MSTR = 12, df = 2

   b) MSE = 3.111, df = 9

   c) $F = 3.857$, df = (2, 9), the p-value is between 0.05 and 0.10. We reject the null at 10% significance, but fail to do so at 5%.

   d) Jalapeño (try it!)