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Juan M. Morales · Antonio J. Conejo
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Integrating Renewables in Electricity Markets

Operational Problems

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To our families.

Preface

Electric energy systems in the past included almost solely *deterministic* production facilities, such us coal- or gas-fired units, or nuclear power plants. If a unit of this type is not out-of-order, its functioning depends only on the will of its owner.

However, electric energy systems throughout the world are currently incorporating an increasing number of *stochastic* production facilities, such as solar- or wind-based units. The functioning of a stochastic unit that is not out-of-order depends not only on the will of its owner, but also on the availability of the primary energy source, i.e., solar intensity or wind level.

The control, operations, planning, economics, and regulation of an electric energy system that includes an important number of stochastic renewable production facilities are considerably different than those of a system without such stochastic facilities. This is the direct result of the variability and the limited predictability of the production levels of the stochastic units, which make it necessary to count on flexible backup energy resources to compensate for the variable and uncertain nature of the power output of these units.

This book focuses on operational issues in electric energy systems that comprise a significant number of stochastic renewable producers, and provides models and algorithms for the efficient and secure operation of such systems. These models and algorithms pertain to the market operator, the stochastic producers, and the demand.

To efficiently cope with the inherent uncertainty and variability in the production of stochastic renewable units, the algorithms provided mostly rely on techniques of optimization under uncertainty, in particular, stochastic programming and robust optimization.

This book consists of nine chapters and five appendices.

Chapter 1 motivates the subject matter of this book by introducing the organization of the pool-based electric energy market that is considered and providing an overview of the main problems addressed in the remaining chapters.

Chapter 2 introduces different types of models and forecasts to characterize the behavior of stochastic renewable electricity production facilities, such as solar- and wind-based units.

Chapter 3 provides tools to clear the day-ahead auction, the most important component of a pool-based electricity market. Both stochastic programming and robust optimization algorithms are proposed and illustrated.

Chapter 4 provides clearing tools for the real-time or balancing auction, another important component of every electricity market that includes a significant level of stochastic production capacity.

Chapter 5 describes a number of flexibility measures to facilitate the integration of stochastic renewable production facilities. Such measures involve both the supply and the demand sides.

Chapter 6 provides a detailed characterization of the impact of stochastic renewable production units on market outcomes, including production and prices.

Chapter 7 adopts the view of a stochastic renewable producer and considers the problem of how to sell effectively its production in a pool-based market. Both analytic and computation procedures are described in detail.

Chapter 8 considers different associations of stochastic and non-stochastic production units, forming a so-called virtual power plant, to increase their competitive edge in the market.

Chapter 9 takes the perspective of the demand and analyzes a number of price-response actions to enable a higher integration of renewable energy and an overall economic improvement in the operation of the system as a whole.

Appendix A provides the fundamentals of random variables and stochastic processes, Appendix B describes some basics of optimization theory, Appendices C and D provide introductions to stochastic programming and robust optimization, respectively, and Appendix E compiles GAMS codes for a number of illustrative examples considered throughout the book.

The material in this book can be arranged in different manners to address the needs of graduate teaching in electricity markets and in the integration of stochastic renewable production in electric energy systems.

Chapters 3–5 and Appendices C and D constitute the core of a course on market-clearing procedures from the perspective of a market operator.

Chapters 7 and 8 and Appendices B–D provide the basic material for a course on trading strategies for producers.

Chapters 2 and 6, and Appendix A include important material for a course on modeling and forecasting stochastic renewable production and their impact on electricity markets.

Chapter 9 and Appendices B–D can form part of a course on demand-side management.

GAMS codes in Appendix E help students to develop the appropriate skills to code and use algorithms of the type discussed throughout this book.

Stochastic renewable production facilities are here to stay for a number of important reasons, including global warming, the depletion of fossil fuels and, generally, the achievement of a sustainable planet. The penetration of stochastic production facilities in electric energy systems throughout the world will progressively increase, eventually making such systems fully renewable.

This book opens the door to develop operational tools for electric energy systems dominated by stochastic renewable production facilities. Such tools will evolve in a non-trivial manner as electric energy systems approach the fully renewable status. This constitutes a fascinating route ahead involving important intellectual and practical challenges, and, definitively, a way to contribute to the sustainability of planet Earth.

Lastly, we would be remiss if we were to conclude this preface without expressing our gratitude to a number of people and institutions. We are thankful to the Technical University of Denmark for providing us with an invaluable research environment and to the Danish Council for Strategic Research and DONG Energy for the financial support. More particularly, during the preparation of this book, the authors have been partly funded through projects ENSYMORA (no. 10-093904/DSF), iPower (DSR-SPIR program 10-095378), and 5s (no. 12-132636/DSF). We would also like to thank Fred Hillier and Camille Price for enthusiastically supporting the writing of this book, and to Neil Levine and Matt Amboy, from Springer US, for all their help throughout the writing process. Our special thanks go to our families, to whom we dedicate this book. We could have never made it through the arduous yet fulfilling adventure of writing a book without their constant support and encouragement.

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August 2013

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Chapter 1
Introduction

1.1 Why Integrate Renewables?

Power systems researchers are very fortunate to live in probably the most exciting and challenging times since the late nineties, when the liberalization of the electricity sector took place.

The issue of climate change has sparked a flurry of discussion on the pressing need for limiting industrial emissions of greenhouse gases. Despite the fact that climate change can only be strictly appraised in terms of the projected impacts for the coming decades, centuries, or even millennia of increasing temperatures, rising sea levels, heat waves, droughts, reduced crop yields, and species extinction, there seems to be consensus on one thing: If the worst and irreversible damages of a plausible climate change are to be avoided, then global greenhouse gas emissions must begin to decline in the near future.

In addition to the worries caused by the envisioned tragic consequences of climate change, many state that the world is on the verge of an energy crisis due to the depletion of fossil fuels.

Against this background, renewable energy has come to occupy a prominent place on the agenda of governments in most industrialized countries during the recent years. This shift has been encouraged by international agreements aiming at reducing CO_2, spurred by the necessity for meeting the rising demand for energy and safeguarding the security of energy supply, and backed by larger and larger shares of the society. More particularly, renewable energy is presently paving the way for the decarbonization of the electricity sector, with a booming wind power industry at the forefront of the change [9]. This impressive development can partly be explained by the favorable incentives renewables were granted at the early stages of their deployment. Under these support policies, renewable power producers are allowed to contribute to power generation and at the same time sidestep most of the drawbacks and the risks implied by the participation in the market.

In parallel to their massive deployment, the per-unit cost of renewable energy has constantly decreased, and is approaching grid parity for some technologies such as wind and solar. As a result, renewables are able to, and asked to, compete in the marketplace with conventional energy production means, despite being fundamentally

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different from those. Indeed, generation technologies based on renewable sources, with the notable exception of hydro and biomass, are *nondispatchable*, i.e., their output cannot or can only partly be controlled at the will of the producer, and their production is *stochastic* and therefore, hard to predict in advance. Chapter 2 of this book elaborates on methods and models to describe the variable and uncertain power output of these renewable generation technologies.

1.2 Electricity Markets

Electricity is a fundamental resource in modern societies. We use it continuously to satisfy our basic household needs. Besides, electrical power is the backbone of our economy, as it is key for the activity in both manufacturing and service industries.

Because of the importance of electricity in our society, it is essential that the whole chain of processes from the generation to the delivery of power to the end consumers is managed in a reliable and cost-efficient manner. In a large number of industrialized countries, this is currently performed in the framework of electricity markets.

In the following, we first review briefly the history of liberalization of the electricity sector and describe the main characteristics of electricity markets in Sect. 1.2.1. Section 1.2.2 then provides a short overview of the impact of renewable energy on electricity markets, which is the subject of subsequent discussion in Chap. 6 of this book.

1.2.1 A Few Notes on the Historical Evolution and Main Features of Electricity Markets

Until the last two decades of the twentieth century, power systems worldwide were organized in a centralized fashion. State-owned, vertically-integrated utilities were in charge of the whole chain of activity related to electrical power: generation, transmission, distribution, and retail. Furthermore, such utilities acted as monopolies in all these fields.

The organization of power systems as state monopolies remained practically unchallenged until the end of the last century. The first steps towards the creation of modern electricity markets were taken by the Chicago Boys in Chile in 1982, during the Pinochet dictatorship, with the separation of generation and distribution activities (unbundling), the introduction of competition between producers, as well as of trading and pricing of electricity according to the production cost (marginal pricing).

As far as Europe is concerned, the first countries to liberalize the electricity sector were the UK, with the creation of an electricity market in England and Wales in 1990, and Norway in 1991. Australia and New Zealand (1996) were also among the first movers. The USA followed with the liberalization of markets in California (CalPX), New York (NYISO), and Pennsylvania, New Jersey, and Maryland (PJM) by the end of the century [25]. Electricity markets worldwide have been implemented in a variety of ways, which would be impossible to review here. However, these implementations share a number of common features.

The first common feature is the separation of generation, transmission, distribution, and retail activities. Markets promote competition in generation and retail, while transmission remains a monopoly managed by noncommercial organizations (system operators, in short SOs).

Trading of electricity is organized in pools or exchanges. Commonly, the preferred marketplace for short-term transactions is a *day-ahead market*, often referred to as *forward market* in the USA and as *spot market* in Europe. This market is organized as a two-sided auction where producers, retailers, and large consumers submit offers and bids for electricity delivery to and withdrawal from the grid, respectively, throughout the following day. Market participants must usually submit 24 selling offers/purchasing bids in total, i.e., one for each hour of the following day. In other markets, such as the New Zealand electricity market, offers are submitted, though, on a 30-minute basis. Each offer/bid is specified as a set of price–quantity pairs, indicating the amount of energy the participant is willing to sell/purchase at a given price. In order to clear the market, the market operator determines aggregate sale and purchase curves by sorting the sale offers according to increasing prices, and the purchase bids in the inverse order. If the transmission grid is not considered in the market-clearing procedure, the intersection between the two curves sets the system price and this price applies for all market participants. More specifically, the sale (purchase) offers whose price is not greater (lower) than this price are accepted, and this determines the day-ahead schedule. On the other hand, if the transmission network is considered in the market-clearing procedure, instead of a single system price, a *locational marginal price* (LMP) is associated with each node of the power system. LMPs may differ due to line congestion and line losses. Further information on the concept of LMPs can be found, for example, in [21].

Later adjustments of day-ahead contracts are possible in *intra-day markets*, also known under the more generic name of *adjustment markets*. These trading arenas are indicated by many as fundamental to allow the large-scale integration of renewables [24].

Finally, the *balancing market*, which is also called *real-time* or *regulation market*, is a last-resort market that ensures that production equals consumption at any point in time. This implies that all unwanted deviations from the production and consumption plans resulting from the day-ahead and intra-day markets are balanced by the activation of regulating power from other market participants. These unwanted energy deviations, or *imbalances*, are typically settled ex post according to the metered production and consumption of the market participant that deviates from its forward schedule.

Day-ahead, intra-day, and balancing markets are energy markets, in that the payment to or from the market operator is proportional to the amount of energy actually delivered to or withdrawn from the grid. In addition to energy markets, reserve capacity markets are in place in some countries to guarantee the availability of sufficient balancing power during the real-time operation of the power system. Producers participating in reserve capacity markets are paid proportional to the available capacity

(MW). The capacity price is often determined using rules similar to those employed to compute the energy price.

Day-ahead and reserve capacity markets are treated in Chap. 3 of this book, while Chap. 4 is exclusively dedicated to balancing markets.

It is worth mentioning that most electricity markets also provide clearing services for long-term financial contracts (forward, options, and derivatives), but these are outside the scope of this book.

In general, liberalization is considered to have improved the efficiency of power systems' management, leading to lower prices for electricity, and to have solved the overinvestment problem typical of centralized power systems [25]. However, the implementation of electricity markets has not been free of failures, see for example the crisis in California at the beginning of the century [2]. Currently, the traditional market design is challenged by the growth in installed renewable capacity, as we discuss next.

We refer the reader to [11], [22], [25] for a more detailed history and description of electricity markets.

1.2.2 Impact of Renewables on Electricity Markets

Since the marginal cost of renewable energy sources like wind and solar is very low, zero, or even negative if incentive schemes award price premia to renewables on top of the clearing price, the offer from renewable producers enters the aggregate supply curve from the left-hand side. In other words, renewables are scheduled before conventional power producers and their output directly influences the market price as a result. This phenomenon is known as *merit-order effect* and is comprehensively analyzed in Chap. 6.

Given that renewable power is *variable*, the aggregate supply curve for the system is shifted to the left in case of low production from renewables, and to the right in case of high renewable outturn. As illustrated in Fig. 1.1, this has an effect on the intersection between the supply and demand curves. In periods with high renewable power production, the amount of scheduled production and consumption increases, and the market price is low. On the contrary, periods with low renewable power production are characterized by higher prices and lower production and consumption schedules.

An implication of the effect of renewable power production on prices is that regions where a high output from renewables is forecast tend to have lower and more volatile prices than regions with lower renewable power production or penetration. As an example, Fig. 1.2 illustrates the 24-hourly day-ahead prices recorded in the Electricity Market of the Iberian Peninsula [12] on March 12, 2013. On that day, zero prices occurred for hours in which the renewable and hydro production exceeded energy demand.

Furthermore, the *uncertain* nature of renewable sources, such as wind and solar power, increases the need for backup power to cope with the unpredicted fluctuations of power production. This results in an increasing need for liquidity in markets whose

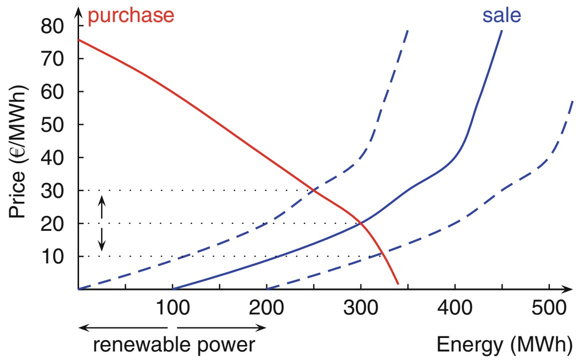

Fig. 1.1 The merit-order effect and the impact of renewables on the clearing price in an electricity exchange

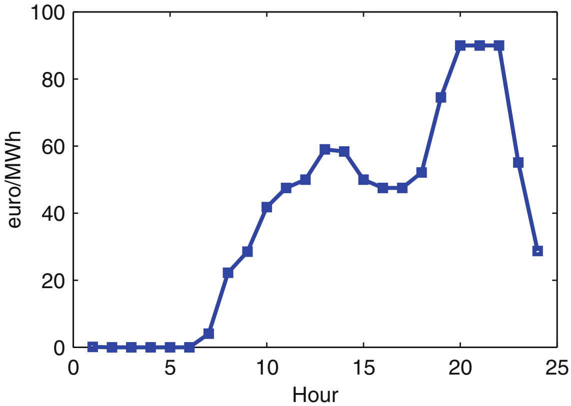

Fig. 1.2 Day-ahead electricity prices registered in the Spanish area of the Iberian Electricity Market on March 12, 2013

gate closure approaches real-time, as well as for an efficient use of the resources participating in these markets. In particular, the balancing market is of special importance to renewable power producers, as it allows them to adjust their contracts so that they match their actual output. Consequently, as the penetration of stochastic production in the electricity market grows, the share of balancing costs in the total system operation costs increases. This increase may become critical if the balancing market is not provided with enough flexible and competitive generation capacity able to cope with uncertainties during the real-time operation of the power system in an economical manner. To clarify this effect, consider the series of illustrations in Fig. 1.3, starting from the top-left drawing and finishing with the bottom-left one. In this series, the black plot represents the aggregated power supply curve of the system. This curve is built by collecting, one day in advance of the physical energy delivery, selling offers from producers, and sorting them out based on a merit-order criterion. Similarly, the grey plot is the aggregated demand curve, which is here represented by a vertical line on the assumption that such a demand is inelastic. Following a traditional market approach, the intercept between the aggregated power supply and demand curves leads to the day-ahead market price λ^{D} (Illustration 1). In principle, if all the market participants were completely flexible, these two curves

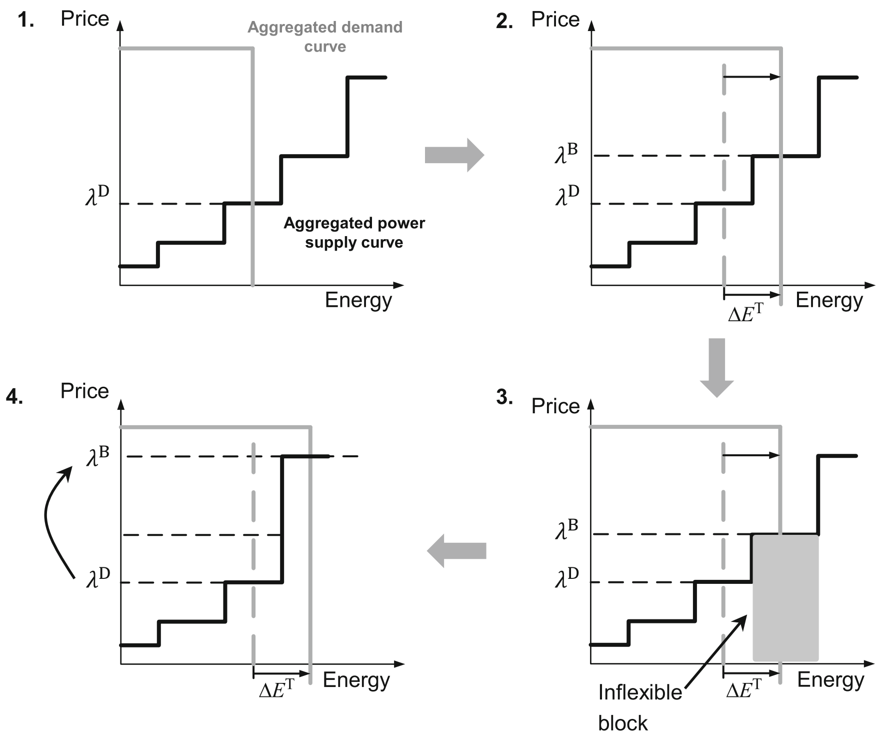

Fig. 1.3 Illustration of the effect of system inflexibility on the cost of balancing the power system and on the associated balancing market price

should also determine the balancing market price. Let us suppose that when it comes to balancing the power system, extra generation is required, because the amount of energy that is being produced is smaller than the amount of energy that is being consumed. Graphically this is represented by shifting the aggregated demand curve to the right by an amount equal to the system energy imbalance ΔE^{T} (Illustration 2). The new intercept is the balancing market price λ^{B}, which embodies the cost of the energy required to restore the system balance. Now let us put ourselves in the situation in which the block of energy that should, in principle, settle the balancing price belongs, in fact, to an inflexible generating unit (Illustration 3), i.e., this unit is technically unable to increase its power output within the required activation time. Therefore, such a block of energy cannot be used for balancing. It must be thus removed from the aggregated power supply curve of the system, and balancing costs and the corresponding balancing market price increase accordingly (Illustration 4).

Situations like the one illustrated in Fig. 1.3 can be prevented by coordinating the operation of the day-ahead and balancing markets [16], [19], introducing sufficiently liquid intra-day trading floors [24] and/or designing well-functioning reserve capacity markets [13]. In this vein, the market-clearing procedures introduced in Chap. 3 of this book aim at improving the overall market efficiency by dispatching the power system in a way that costly balancing actions are avoided.

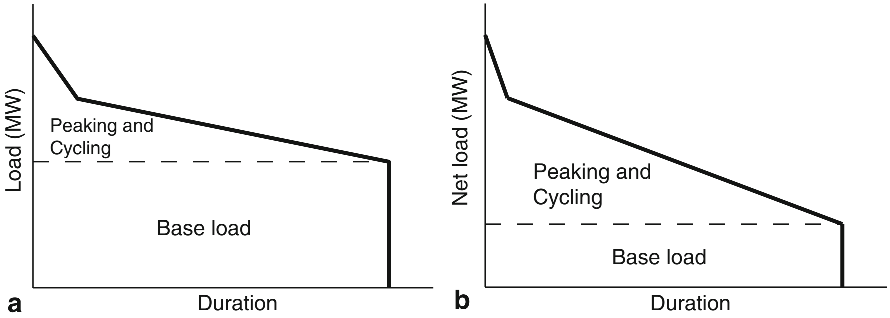

Fig. 1.4 Impact of renewable power integration on the load duration curve. **a** No variable production. **b** With variable production

1.3 Flexibility is a Must

The variability of renewables requires operating the power system with a higher degree of *flexibility* in order to follow the fluctuating *net load*, defined at each instant as the difference between the total energy consumption and the total variable renewable production. To accomplish this, a variety of solutions involving both producers and consumers are being assessed at present. We briefly summarize below some of these solutions. For further details, the reader is referred to Chap. 5 of this book, which elaborates on the components of a power system that are key to provide enhanced flexibility.

1.3.1 In the Pursuit of the Perfect Generation Mix

Undoubtedly, the ability of a power system to integrate renewable generation is strongly dependent on its generation mix. Production technologies able to efficiently alter their power output as required by the net load variations are of necessity to sustain an electricity supply paradigm primarily based on renewable, but variable energy sources. In fact, given the usually negative correlation between electricity consumption and some of the most attractive renewable energy sources, e.g., wind power, the need for conventional units operating in peaking and cycling regimes may potentially increase, as graphically illustrated in Fig. 1.4. Indeed, flexibility on the generation-side of a power system usually leads to operate conventional units at production levels higher or lower than their optimum in an attempt to accommodate the inherent variability of renewable generation by ramping up or down. Further, these ramping excursions may often end up with the start-up or shut-down of conventional units.

Consequently, if large variations of renewable generation are to be accommodated by conventional generation, this may result in conventional power plants operating in a less efficient way, thus reducing the emissions savings brought about by the contribution of renewables to the electricity supply [7].

Furthermore, since the increasing penetration of renewable energy sources will result in lower electricity prices, a question that remains open is whether electricity markets will provide by themselves the adequate economic incentives for electricity producers to invest in conventional generation, in particular, in the flexible production technologies required to continue integrating larger amounts of renewables without jeopardizing the security of supply.

1.3.2 The Role of Demand Response

If the seasonal patterns of renewable production were similar to those of demand, the variability of renewables would be *passively* absorbed by consumers, with the consequent reduction in net load fluctuations. Unfortunately, in many cases, demand and renewables availability are not favorably correlated, i.e., periods of high (low) demand do not often coincide with periods of high (low) renewable energy production. For instance, in Northern Europe, even if the historical records reveal that wind power production is greater in winter than in summer, there is not a significant coincidence of winter periods with high load and high wind generation [10]. Similarly, in the state of Texas, wind power production is negatively correlated with energy consumption. In this region, wind usually blows more in winter, spring, and autumn than in summer, when the energy demand for air-conditioning reaches its maximum; and more during off-peak hours than on-peak [1].

Therefore, if renewables and demand are not *naturally* correlated, how can we encourage loads to move the bulk of their electricity consumption to periods of high renewable power production?

A number of different initiatives have been proposed in this regard. Basically, these initiatives can be grouped into either *direct control* or *indirect control*. The former group comprises initiatives aimed at granting Transmission System Operators (TSOs), or other market entities with similar objectives, the right to directly modulate the demand by means of rationing or disconnecting individual consumers, or groups of. Typically consumers involved in these programmes are protected by a contract fixing how often they can be disconnected or rationed.

Indirect control implies the use of economic incentives so that demand adapts to the stochastic and variable production. In practice, this would be done by broadcasting time-varying prices to the consumers (dynamic real-time pricing). This topic is considered in Chap. 9 of this book.

The implementation of demand response requires the installment of infrastructure allowing communication to the consumers (one way or bidirectional), and of consumer appliances able to adapt their consumption to the broadcast signals. This upgrade of the grid and of the consumer appliances is often associated with the notion of *smart grid*.

Besides the technical challenges involved with the structural developments just mentioned, demand response poses difficult challenges also in terms of market design. First of all, the introduction of price-incentives will confer dynamic properties to the demand, by increasing its cross-elasticity across different time periods, which need to be modeled and accounted for by the policy-makers. Furthermore, demand response will require the coordination of a large number of consumers and distributed generators spread around the system, with further complications imposed by the tight capacity constraints that characterize the power grid at the distribution level. For these and other reasons, demand response is considered one of the most challenging topics of power systems research in the years to come.

We refer the interested reader to [23] for a detailed description of the demand response initiatives currently implemented and planned in Europe.

1.3.3 Large-Scale Electricity Storage: The Cure for All Ills?

The scientific community agrees that the massive integration of variable energy sources could be greatly facilitated by developing large-scale storage capabilities. Even though, currently, there exist a number of energy storage technologies that can contribute to this end (e.g., batteries, flywheels, electric vehicles, superconducting magnetic energy storage systems, compressed air, and molten-salt or pumped hydro storage systems) [6], it seems that new advances and more incentives in this field are still needed for the massive electricity storage to become a reality in the required magnitude [18].

1.3.4 Towards a Power System Based on Distributed Energy Resources

Since August 24, 1891, when three-phase alternating current (AC) was first transmitted from the hydroelectric power station at Lauffen to the International Exposition at Frankfurt (Germany), 175 km away, electrical power systems have been designed to take energy from high-voltage levels and distribute it to low-voltage networks. The emergence of AC grids allowed electricity to be efficiently transported over long distances, and economies of scale in electricity generation led to large power plants with lower per unit production costs. As a result, today's electric energy systems worldwide are colossal supply chains in which electricity is consumed far away from the big production centers and the energy essentially flows in one direction, i.e., from the transmission to the distribution systems.

However, in the last decade, technological innovations, particularly in the fields of renewables and smart grids—just think of photovoltaic (PV) cells, onshore wind turbines, internet technologies, and storage equipment, among others—have spurred increasing interest in taking full advantage of distributed energy sources. Five major factors are considered to be the key drivers of the change [17], [20], namely a dynamic and growing industry in distributed generation technology, constraints on the construction of new transmission lines, increased customer demand for highly reliable electricity, the deregulation of the electricity sector with the establishment of electricity markets, and as already mentioned, concerns about climate change. Indeed, distributed generation may help electricity suppliers to provide a more differentiated service to end-consumers and improve their overall quality and reliability of supply by way of voltage support, power factor corrections and, more generally, the provision of ancillary services. Distributed generation may also serve as a substitute for investments in transmission and distribution grid capacity and facilitate the utilization of cheap and locally available energy resources—for example, photovoltaic cells have the potential to enable consumers of all sizes to generate power by themselves. Furthermore, government policies aimed at promoting the use of renewables will also support the development of distributed generation technologies, since renewables, except for large hydro, have a decentralized and dispersed nature. The combined impact of all these factors is, in short, accelerating a global transition to distributed power generation based on renewable energy.

Consistent with these developments, many envision a future electric energy system studded with small-scale generators of different nature, ranging from distributed wind and PV generation systems to small biofuel-based production units; an electric energy system where the distribution grid will supply electricity to small businesses and residential consumers that are flexible enough to respond to the electricity cost and where the grid will also count on a certain amount of storage capability (e.g., batteries or mini-hydro power units). In this scenario, matching power consumption and weather-dependent power production efficiently will become an intricate task. Chapter 8 of this book provides insight into fundamental modeling aspects and techniques to develop an energy management tool for the optimal operation of an aggregation of distributed energy resources, i.e., for the efficient exploitation of a so-called *virtual power plant*.

1.3.5 Optimal Use and Expansion of the Transmission Network

Network bottlenecks may become a major obstacle in the large-scale integration of renewable energy for two main reasons. On the one hand, renewable energy sources are often plentiful in areas that are far away from the large demand centers, e.g., big cities. Consequently, a transmission infrastructure that enables renewable generating units to send their power production towards the consumption nodes is a determining factor in the successful development of renewables. On the other hand, increasing transmission capability improves the access to flexible energy sources. For example, renewable power could be much more easily integrated in a hypothetically well-interconnected Europe, where large production from renewables in one area could be backed by flexible generation available in others.

Unfortunately the transmission system expansion required to allow for the connection of renewable energy sources at sites far from the traditional load and generation centers usually involves significant lead-time and investment, aside from being exposed to the public reluctance to extensive new-built infrastructure. Consequently, procedures for the optimal allocation of renewable capacity are needed to make the most of existing transmission resources, especially, in the short run [3, 15]. Notwithstanding this, transmission expansion projects aimed to reinforce system interconnections can considerably increase the value of renewables [8].

1.3.6 Enhanced Tools for Power System Operations

There is a broad consensus in the research community that the stochastic and variable nature of some renewable energy sources like wind and solar will induce significant changes in the paradigms of power systems management. Indeed, today's electricity market design is the result of integrating traditional practices, such as unit commitment or economic dispatch within a competitive framework. These practices were conceived in view of a generation mix mostly formed by dispatchable plants, and are now to be reexamined so that stochastic producers can compete on equal terms.

Consider, for example, the case of wind generation, which is highly variable. Part of this variability can be predicted some hours or days ahead and as such, can be absorbed by an optimal forward scheduling of generating units and loads in accordance with their flexibility. The unpredictable part of this variability must be managed, however, with reserves. Basically, a generating unit or load that provides reserve capacity is willing to alter its production or consumption as required in a short period of time. The reserves needed to accommodate unforseen wind power fluctuations are fundamentally slow, i.e., with activation times around or greater than 15 minutes [5]. Even so, the deployment of reserves translates into relatively fast energy redispatches, which significantly increases the need for a more flexible power system operation. As a result, wind power uncertainty entails additional operating costs [13]. These costs, nevertheless, can be minimized by allocating and deploying reserves in an efficient manner. To this end, tools for the optimal energy and reserve scheduling are crucial to make the most of wind, avoid unnecessary wind energy curtailments, and reduce the costs derived from its variability, either unpredictable or not. In this vein, Chap. 3 introduces advanced tools for the optimal dispatch of energy and reserve capacity in presence of renewables that are based on techniques of optimization under uncertainty.

1.4 Selling Energy from Renewable Sources

When it comes to the energy-selling business, three basic aspects differentiate the renewable producer from the thermal producer, namely:

1. Renewables constitute nonpolluting energy sources and as such, their development is spurred on by governments willing to face the issue of climate change. Economically speaking, this usually translates into support schemes that increase the value of the MWh produced by renewable generating units above the market price.
2. Renewable generation technologies, except for those based on biomass, do not consume fuel and as a result, their associated fuel cost is zero.
3. Renewable power units are, in many cases, nondispatchable and characterized by the uncertainty inherent to the availability of the underlying renewable energy source. Decisions on the amount of renewable energy to be traded in the electricity market are therefore risky.

While the features indicated in points 1 and 2 above favor the integration of renewable production in power systems, the characteristic stated in point 3, i.e., the uncertainty associated with the availability of the corresponding renewable energy source, constitutes a major obstacle to the natural incorporation of renewable producers into a market framework. Indeed, when stochastic power producers offer in short-term electricity markets, they very rarely trade on a single market floor. Their production being uncertain, they most likely need to take corrective measures in the balancing markets after having traded in the day-ahead market, so as to align their actual output with their contracts for energy delivery. Since balancing energy is mostly provided by flexible generating units, often with high marginal costs, the competitiveness of renewable producers that highly depend on the balancing market to cope with their uncertain production may be considerably impaired.

Consequently, the offering strategy of a renewable producer should aim at maximizing the expected revenue from its participation in the different trading floors of an electricity market, while minimizing the cost of covering its eventual energy deviations in the balancing market.

Basically, the key factors conditioning the short-term offering strategy of a renewable power producer are:

1. *Uncertain production*, which increases the risk of selling renewable energy in markets that are cleared in advance of the physical energy delivery.
2. *Mechanism for the formation of the balancing market prices*, i.e., the mechanism through which the energy imbalances incurred by market participants are priced in the balancing market. There exist in this regard two main different designs for the balancing market, namely, the one-price and the two-price schemes. These are introduced in Chaps. 4 and 7 of this book.
3. *Certainty gain effect*, defined as the economic advantage for renewable producers that trade in a series of markets that are arranged sequentially closer to real-time operation [14]. This effect is related to the value of updating the forecasts of renewable energy generation [4].

Chapter 7 analyzes the short-term trading problem of a renewable power producer in detail.

1.5 Summary

The progress of renewable energy around the world in recent years has been impressive, and the policies intended to curb emissions of greenhouse gases causing global warming are expected to guarantee the steady growth of renewable power generation in the electricity sector, or even speed it up.

This introductory chapter serves to motivate the subject matter of this book by providing a brief overview of the main operational challenges arising from the massive penetration of variable and stochastic renewable energy in electricity markets.

References

1. Baldick, R.: Wind and energy markets: a case study of Texas. IEEE Syst. J. **6**(1), 27–34 (2012)
2. Borenstein, S.: The trouble with electricity markets: Understanding California's restructuring disaster. J. Econ. Perspect. **16**(1), 191–211 (2002)
3. Burke, D.J., O'Malley, M.J.: Maximizing firm wind connection to security constrained trasnmission networks. IEEE Trans. Power Syst. **25**(2), 749–759(2010)
4. Conejo, A.J., Morales, J.M., Martínez, J.A.: Tools for the analysis and design of distributed resources—Part III: Market studies. IEEE Trans. Power Deliv. **26**(3), 1663–1670 (2011)
5. DeMeo, E.A., Jordan, G.A., Kalich, C., King, J., Milligan, M.R., Murley, C., Oakleak, B., Schuerger, M.J.: Accommodating wind's natural behavior. IEEE Power & Energy Mag. **5**(6), 68–77 (2007)
6. Denholm, P., Ela, E., Kirby, B., Milligan, M.: The role of energy storage with renewable electricity generation. Tech. Rep. NREL/TP-6A2-47187, National Renewable Energy Laboratory (NREL) (2010)
7. Denny, E., O'Malley, M.: Wind generation, power system operation, and emissions reduction. IEEE Trans. Power Syst. **21**(1), 341–347 (2006)
8. Denny, E., O'Malley, M.: Quantifying the total net benefits of grid integrated wind. IEEE Trans. Power Syst. **20**(2), 605–615 (2007)
9. Global Wind Energy Council: Global wind report. Annual market update 2012. http://www.gwec.net/publications/global-wind-report-2/ (2013). Accessed 10 May 2013
10. Holttinen, H.: Impact of hourly wind power variations on the system operation in the Nordic countries. Wind Energy **8**(2), 197–218 (2005)
11. Kirschen, D.S., Strbac, G.: Fundamentals of Power System Economics. Wiley, Chichester (2004)
12. MIBEL: Iberian Electricity Market. http://www.mibel.com (2013). Accessed 13 May 2013
13. Morales, J.M., Conejo, A.J., Pérez-Ruiz, J.: Economic valuation of reserves in power systems with high penetration of wind power. IEEE Trans. Power Syst. **24**(2), 900–910 (2009)
14. Morales, J.M., Conejo, A.J., Pérez-Ruiz, J.: Short-term trading for a wind power producer. IEEE Trans. Power Syst. **25**(1), 554–564 (2010)
15. Morales, J.M., Pinson, P., Madsen, H.: A transmission-cost-based model to estimate the amount of market-integrable wind resources. IEEE Trans. Power Syst. **27**(2), 1060–1069 (2012)
16. Morales, J.M., Conejo, A.J., Liu, K., Zhong, J.: Pricing electricity in pools with wind producers. IEEE Trans. Power Syst. **27**(3), 1366–1376 (2012)
17. Pepermans, G., Driesen, J., Haeseldonckx, D., Belmans, R., D'haeseleer, W.: Distributed generation: Definition, benefits and issues. Energy Policy **33**(6), 787–798 (2005)
18. Pickard, W.F., Shen, A.Q., Hansing, N.J.: Parking the power: Strategies and physical limitations for bulk energy storage in supply–demand matching on a grid whose input power is provided by intermittent sources. Renew. Sust. Energ. Rev. **13**(8), 1934–1945 (2009)
19. Pritchard, G., Zakeri, G., Philpott, A.: A single-settlement, energy-only electric power market for unpredictable and intermittent participants. Oper. Res. **58**(4), 1210–1219 (2010)
20. Schleicher-Tappeser, R.: How renewables will change electricity markets in the next five years. Energy Policy **48**, 64–75 (2012)
21. Schweppe, F.C., Tabors, R.D., Caraminis, M., Bohn, R.E.: Spot Pricing of Electricity. Kluwer, Norwell (1988)
22. Stoft, S.: Power System Economics: Designing Markets for Electricity. Wiley–IEEE Press, New York (2002)
23. Torriti, J., Hassan, M.G., Leach, M.: Demand response experience in Europe: Policies, programmes and implementation. Energy **35**(4), 1575–1583 (2010)
24. Weber, C.: Adequate intraday market design to enable the integration of wind energy into the European power systems. Energy Policy **38**(7), 3155–3163 (2010)
25. Weron, R.: Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach. Wiley, Chichester (2006)

Chapter 2
Renewable Energy Sources—Modeling and Forecasting

2.1 Introduction

Forecasts are essential to the integration of renewable power generation in electricity markets operations, since markets ought to be cleared in advance, while market participants shall then make decisions even before that. This is true for all types of electricity markets, that is, from real-time to futures markets, via the more classical day-ahead (forward) ones. For the reference case of conventional generators, power production forecasts are straightforward since, except for unit failures, one actually controls future electricity generation. In such a case, forecasts directly consist of potential schedules, which then translate to supply offers in the market. When it comes to renewable power generation, one is mostly left with Nature deciding on the future schedule of the power plants: wind power is only there when the wind blows and solar energy when the sun shines. Only hydro power is more dispatchable as the water originating from rainfall and snow melt can be stored in gigantic reservoirs. The nonstorability of other types of renewable energy sources, at least in a technologically and economically efficient manner today, magnifies this need for appropriate forecasts of renewable power generation. Here emphasis will be mainly placed on wind energy, which has so far been the leading form of renewable energy. The ideas and concepts presented could be extended to the case of, e.g., solar and wave energy since, from a mathematical point of view, the modeling and forecasting problems share a high level of similarity. Solar energy is becoming increasingly popular and present in a number of countries like Spain and Germany, among others. Wave energy is finally envisaged to become a natural complement to wind energy in the offshore energy mix, based on a number of demonstration projects today in the UK and Portugal, for instance.

It is sometimes argued that forecasts are there mainly to comfort decision-makers—here, the market and network operators, as well as power producers, and potentially end-consumers—while they are not really used or at least not used in an optimal manner in daily operations. However, employing the appropriate forecasts in a well-defined decision-making problem can tremendously improve the decisions to be made, while allowing controlling the risk brought in by unforeseen events. Indeed, a crucial starting point of this chapter is that forecasts are always wrong to

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a certain extent. This should be accounted for in the various operational problems considered.

All aspects of renewable power forecasting cannot be covered within a single chapter of this book, nor can the necessary theoretical background on, e.g., stochastic processes, modeling, and estimation. Forecasting of renewable power generation relies on cross-disciplinary approaches taking roots in mathematics, statistics, meteorology, and power systems engineering. Most importantly, we aim at discussing here the various types of forecasts that exist for renewable power generation, being wind, solar or wave energy, and that are to be used as input to operational problems for electricity markets.

In Sect. 2.2, we introduce some of the necessary notation and definitions while placing ourselves in a stochastic process modeling framework. Necessary concepts related to stochastic processes are further developed in Appendix A. Subsequently, the various types of renewable energy forecasts that may be issued as input to decision-making problems are introduced in Sect. 2.3 based on examples, giving a pragmatic view of their characteristics. Emphasis is then placed in Sect. 2.4 on the quality of these forecasts, by covering their required and necessary properties, as well as some key scores and diagnostic tools for their evaluation. It is of utmost importance to fully appraise the quality of forecasts before to use them as input to decision-making and general operational problems. The way these forecasts may be generated from various sets of input data is then discussed in Sect. 2.5. Further readings are suggested at the end of this chapter.

2.2 Renewable Power Generation as a Stochastic Process

Even though referring to either renewable energy or power modeling and forecasting, focus is always placed on the power variable. This is because it is actually power which is measured at renewable energy generation plants. It is then straightforward to obtain energy values for given periods of time if necessary, by integrating power observations over these time periods.

Owing to the combination of a large number of complex physical processes, also mixed with additional uncertainties in our understanding of these processes, there may always be a part of randomness in our knowledge of energy generation from renewable energy sources. For instance, for a wind farm, even if having a perfect picture of the theoretical power curve of each and every turbine (as provided by the turbine manufacturer), it is close to impossible to know for sure what the power curve of the wind farm composed by all these turbines may be. This uncertainty originates from shadowing effects among the set of turbines, turbulence effects, dust and insects on the blades, etc.

Accepting the fact that there are uncertainties in the process of renewable energy generation, it is hence considered as a stochastic process. Necessary basics related to the definition of stochastic processes are introduced in Appendix A. Consequently, power generation from renewable energy sources, such as wind and solar, will be referred to as *stochastic power generation* in the subsequent chapters.

Definition 2.1 (The Renewable Energy Generation Stochastic Process). In the most general case,

$$\{Y_{r,s,t},\ r = r_1, \ldots, r_m,\ s = s_1, \ldots, s_n,\ t = 1, \ldots, T\}, \tag{2.1}$$

is a multivariate stochastic process in space and in time, observed at a set of n locations, $s = s_1, s_2, \ldots, s_n$, and for successive time points $t = 1, \ldots, T$, describing power generation from a number m of different renewable energy sources, $r = r_1, r_2, \ldots, r_m$. The corresponding realizations of that stochastic process are denoted by

$$\{y_{r,s,t},\ r = r_1, \ldots, r_m,\ s = s_1, \ldots, s_n,\ t = 1, \ldots, T\}. \tag{2.2}$$

This stochastic process may be univariate ($m = 1$, in the above definition) if considering one type of renewable energy only, or multivariate ($m > 1$) if jointly considering several forms of renewable energy generation, as for the example of wind and wave energy generation offshore. In the former case, the notation for the stochastic process may be simplified to $\{Y_{(s,t)}\}$. Similarly, while both the time and space dimensions may be jointly considered, it is often the case that *(i)* focus is on the spatial dimension only, e.g., as input to a power flow calculation, or *(ii)* focus is on the time dimension only, e.g., if dealing with renewable energy generation for a given location in an optimal storage operation problem. Notation would then simplify even more, by using the relevant subscript only, that is, $\{Y_s\}$ and $\{Y_t\}$ for the space and time cases, respectively.

This stochastic process can be normalized for simplification, hence taking values between 0 and 1 at any time, any location and for all types of renewable energy,

$$Y_{r,s,t} \in [0, 1], \quad \forall i, s, t. \tag{2.3}$$

The above then also necessarily applies to all realizations $y_{r,s,t}$. The normalization is done individually by the nominal capacity of that type of renewable energy at this location and at this point in time. While it is fairly obvious that nominal capacity depends upon the renewable energy plant and therefore its location, one should not forget that nominal capacity can vary in time, e.g., due to maintenance planning and decommissioning/recommissioning of renewable energy assets.

The concepts introduced in the above are illustrated in the following example describing a univariate case, with wind power generation only, though observed at a number of locations, and for a long period of time.

Example 2.1 (Wind Power Generation for 15 Control Zones in Western Denmark) A dataset with wind power generation over the control area (split into 15 control zones) of Western Denmark, operated by Energinet.dk for a total nominal capacity of 2.515 GW, will be used as a basis for illustration in this chapter. This control area is commonly referred to as DK-1. Wind power generation over these 15 control zones can be considered as a univariate stochastic process $\{Y_{s,t}\}$ in space and in time, in practice observed at 15 locations $s = s_1, s_2, \ldots, s_{15}$ only. Figure 2.1 depicts an episode with two days in mid-February 2006 of wind power observations at these 15 control zones, with a hourly temporal resolution. These wind power observations for every zone are normalized by the relevant nominal capacity values. The generation

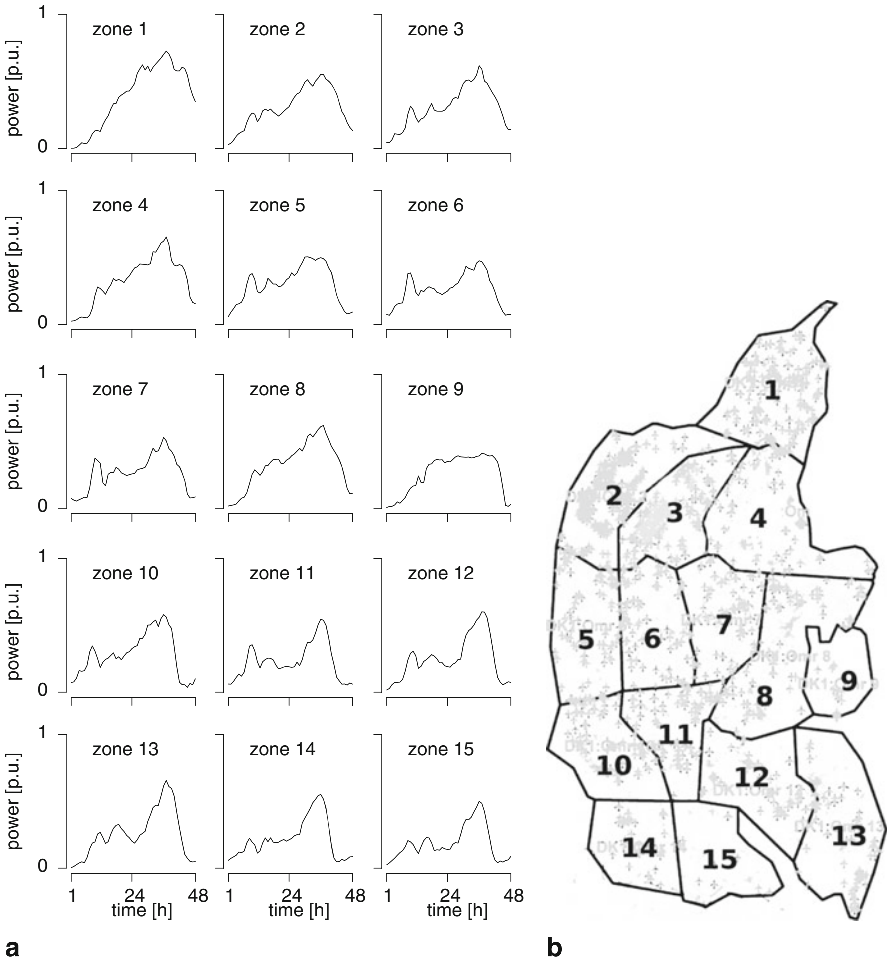

Fig. 2.1 Episode from mid-February 2006 with two days of wind power observations at the 15 control zones forming the control area DK-1 of Energinet.dk. These measurements have an hourly temporal resolution and are normalized by the respective nominal capacities at every control zone. **a** Normalized power observations. **b** 15 control zones

patterns for neighboring zones have similar characteristics, while there is also a clear temporal dependence. These are important aspects when it comes to the modeling and forecasting of such a stochastic process.

2.3 The Various Types of Renewable Power Forecasts

Predictions of renewable energy generation can be obtained and presented in a number of different manners. The choice for the type of forecasts and their presentation somewhat depends upon the process characteristics of interest to the decision-maker, and also upon the type of operational problem. For instance, a wind farm operator aiming to plan maintenance over the coming week may only be interested in simple deterministic-type of forecasts for wind and power generation at the level of this wind farm, and not in detailed space–time scenarios over the whole country.

The various types of renewable energy forecasts and their presentation are introduced below, starting from the most common point forecasts and building up towards the more advanced products that are probabilistic forecasts and scenarios. We finally mention some of the more exotic forecasts that are currently being issued with focus on predefined events.

2.3.1 Common Features of Renewable Power Forecasts

Forecasting is about foreseeing the future state of the process of interest, in this case, renewable energy generation, at a given location s or for a set of n locations $s = s_1, s_2, \ldots, s_n$, potentially with different forms of renewable energy sources at every location. Even though several locations and renewable energy forms may be considered, it is the temporal dimension that is of importance here. In contrast to spatial forecasts, we do not aim in this chapter at predicting the dynamics of the stochastic process at new locations. We do not attempt at issuing forecasts for new types of renewable energy sources either. The set of locations s and the energy mix are both fixed. Let us then place ourselves at time t and look at a future point in time $t + k$. For ease of notation, we only use time indices in the following when referring to values for the stochastic process. One should not forget that these may also be for several locations and types of renewable energy sources.

Emphasis is placed in the following on model-based approaches to forecasting. There exists a number of other approaches, e.g., based on expert judgments. For the example case of forecasting the electric demand (commonly referred to as load), it is often said that such expert judgments are very difficult to outperform by any model-based approach. For renewable energy forecasting, however, model-based approaches are to be preferred, since it would be much more difficult for experts to sharply foresee weather developments and their impact on corresponding renewable energy generation. Note that a difference should be made between a model, which comprises a mathematical representation of the processes considered, and a forecasting method, which is, instead, the process of issuing a prediction, based or not on a model.

Definition 2.2. A (model-based) forecast $\hat{\cdot}_{t+k|t}$ of renewable power generation is an estimate of some of the characteristics of the stochastic process Y_{t+k} (where Y is for all locations and types of renewable energy sources) given a chosen model g, an estimated set of parameters $\hat{\Theta}_t$ and the information set Ω_t gathering all data and knowledge about the processes of interest up to time t. That information set is commonly employed to identify a model g and the set of parameters Θ_t.

In the above definition, k is the *lead time*, though sometimes also referred to as *forecast horizon*. The 'hat' symbol expresses that $\hat{\cdot}_{t+k|t}$ is an estimate only: it reflects the presence of uncertainty both in our knowledge of the process and inherent to the process itself. The notation '$t + k|t$' is based on the conditional symbol '|' in probability theory. The forecast for time $t + k$ is conditional on our knowledge of the stochastic process up to time t, including the data used as input to the forecasting process, as well as the models identified and parameters estimated.

> Whatever the type of forecast, forecasting is to be seen as a form of extrapolation. A model is built and fitted to a set of data, then applied for prediction purposes on totally new data. This conditionality of forecasts makes that they should implicitly be formulated as: *"given the information set and assuming that the identified dynamics continue in the future, we can predict that ... "*. A forecaster somewhat makes the crucial assumption that the future will be like the past.

Forecasts are issued as series of consecutive values $\hat{\cdot}_{t+k|t}$, $k = 1, 2, \ldots, K$, that is, for regularly spaced lead times up to the *forecast length* K. That regular spacing is called the *temporal resolution* of the forecasts. This will be illustrated when introducing the various types of renewable energy forecasts below. For instance, when one talks of 48-hour ahead forecasts with hourly resolution, this means that forecasts actually consist in forecast series gathering predicted power values for each of the following 48 h. Similarly, if predictions were to be issued on a regular spatial grid, one would talk of their spatial resolution. Here forecasts are for specific locations, not uniformly distributed on a grid, and therefore, the concept of spatial resolution does not make much sense.

2.3.2 Point Forecasts

When the renewable energy forecast issued at time t for $t + k$ is single-valued, it is referred to as a point prediction and denoted by $\hat{y}_{t+k|t}$. The fact this forecast is single-valued makes that point forecasts issued in a deterministic or stochastic process framework look similar. However, they are not in essence. In a deterministic framework, the forecaster is somewhat sure that the prediction ought to realize—there is no uncertainty involved. In a stochastic process framework, instead, $\hat{y}_{t+k|t}$ is an estimate only, hence acknowledging the presence of uncertainty.

Definition 2.3. A point forecast $\hat{y}_{t+k|t}$ corresponds to the conditional expectation of Y_{t+k} given g, $\hat{\Theta}$, and the information set Ω_t,

$$\hat{y}_{t+k|t} = \mathbb{E}[Y_{t+k}|g, \Omega_t, \hat{\Theta}]. \tag{2.4}$$

In everyday words, the conditional expectation is the mean of all that may happen given our state of knowledge up to time t. Providing decision-makers with a forecast in the form of a conditional expectation translates to acknowledging the presence of uncertainty, even though it is not quantified and communicated.

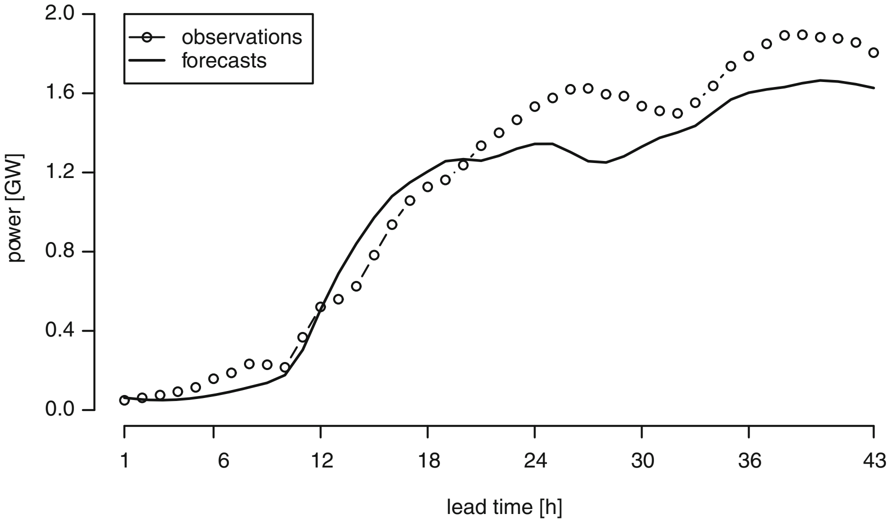

Fig. 2.2 Point forecasts of wind power generation issued on the 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day)

Example 2.2 (Point Forecasts of Wind Power Generation) Let us consider the example of point forecasts issued on 4th April 2007 at 00:00 UTC[1] for the whole onshore capacity of Western Denmark (2.515 GW at the time, see Example 2.1, depicted in Fig. 2.2, along with the corresponding observations obtained a posteriori. This forecast series has a hourly temporal resolution up to 43 h ahead.

It informs that the expected power generation on 5th April 2007 at 00:00 UTC should be 1.32 GW. There what the forecaster really says is that the predicted mean of all potential power production values is 1.32 GW. He or she is not telling about what could really happen, however. The actual power generated 24 h after the forecast is issued could range anywhere between 0 and 2.5 GW, and that would make a big difference! This will all depend upon the forecaster's skill and the inherent forecast uncertainty. In this case, the forecast error made a posteriori appears fairly small, since the observed power generation at that time was of 1.466 GW (still a 146 MW difference).

2.3.3 Probabilistic Forecasts

This shortcoming of point predictions not giving the full picture about what *could* happen is of crucial importance when it comes to operational problems, where the costs potentially induced by the whole potential range of realizations that are likely

[1] UTC actually stands for *Coordinated Universal Time*, which is a time standard by which we regulate time and clocks.

to occur is to be accounted for. This has therefore motivated the substantial research effort invested in the development of probabilistic forecasting methodologies for energy applications, with a strong emphasis on their optimal integration in operations research problems.

In contrast to point predictions, probabilistic forecasts aim at providing decision-makers with the full information about potential future outcomes. Let us use the same notation as before while dropping out the subscripts for location and type of renewable energy source. Recall that y_t is the power production measured at time t and corresponds to a realization of the random variable Y_t. Then let f_t and F_t be the *probability density function* (abbreviated pdf) and related *cumulative distribution function* (abbreviated cdf) of Y_t, respectively.

Definition 2.4. A probabilistic forecast issued at time t for time $t+k$ consists in a prediction of the pdf (or equivalently, the cdf) of Y_{t+k}, or of some summary features.

Deterministic forecasts may be reinterpreted in a probabilistic framework as probability masses of 1 placed on these values predicted for the future state of the process—there is no uncertainty. The various types of probabilistic forecasts are detailed below, from quantile to density forecasts, and through prediction intervals.

2.3.3.1 Quantile Forecasts

Let us now introduce the concept of quantile forecast based on the definition of the quantile of a cumulative distribution function as given in Def. A.6 of Appendix A.

Definition 2.5. A quantile forecast $\hat{q}^{(\alpha)}_{t+k|t}$ with *nominal level* α is an estimate, issued at time t for lead time $t+k$, of the quantile $q^{(\alpha)}_{t+k}$ for the random variable Y_{t+k}, given a model g, its estimated parameters $\hat{\Theta}_t$ and the information set Ω_t, i.e.,

$$\mathrm{P}[Y_{t+k} \leq \hat{q}^{(\alpha)}_{t+k|t} \mid g, \Omega_t, \hat{\Theta}] = \alpha. \tag{2.5}$$

By issuing a quantile forecast $\hat{q}^{(\alpha)}_{t+k|t}$, the forecaster tells at time t that there is a probability α that renewable energy generation will be less than $\hat{q}^{(\alpha)}_{t+k|t}$ at time $t+k$.

Quantile forecasts are of interest for a number of operational problems, since for a variety of loss functions (quantifying the cost of making a suboptimal decision, to be further introduced and discussed in Sect. 2.5.3), optimal decisions always relate to quantile forecasts with given nominal levels [2]. This is, for instance, the case for the design of optimal offering strategies by wind power producers, where optimal bids are quantile forecasts whose nominal level is a simple function of day-ahead and balancing market prices (see Chap. 7). Furthermore, quantile forecasts also define prediction intervals and, more generally, nonparametric probabilistic forecasts, as will be explained more extensively in the following. The concept of quantile forecasts is further illustrated below by building on the previous examples with wind power generation in Western Denmark.

Example 2.3 (Quantile Forecasts of Wind Power Generation) Fig. 2.3 depicts an example episode with quantile forecasts with a nominal level $\alpha = 0.5$ (i.e., the

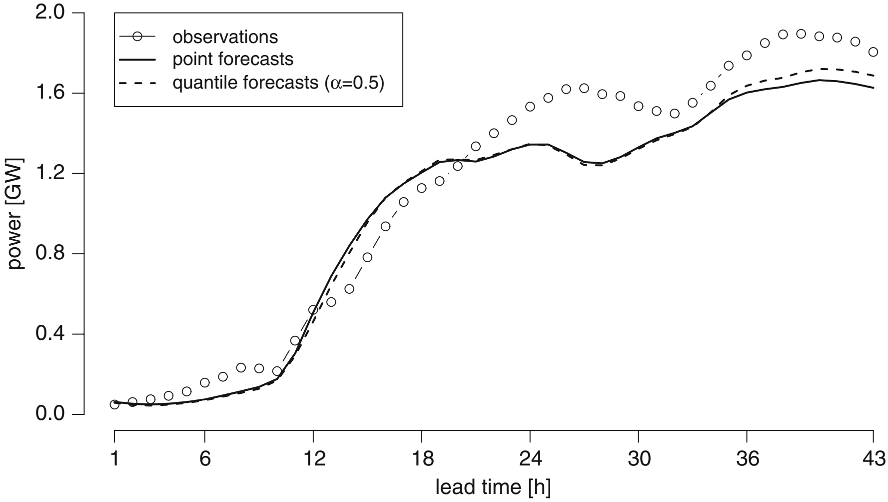

Fig. 2.3 Quantile forecasts of wind power generation with a nominal level of 0.5 (i.e., the median) issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown

median), for the same period than in Fig. 2.2 and the same set-up as introduced in Example 2.1. For each lead time, these forecasts tell that wind power generation has a 50 % probability of being below (and, therefore, also above) the value they indicate. Their interpretation is hence quite different from that of the point forecasts considered before, since point forecasts, as conditional expectations, are not associated to any form of probability level. Note that, if forecast uncertainty is perfectly symmetric around point predictions, then $\hat{q}^{(0.5)}_{t+k|t} = \hat{y}_{t+k|t}$.

In the present case, if looking more closely at the 42-hour ahead lead time, while the previously discussed point forecasts tell that the expected power generation is 1.646 GW, the quantile forecast informs there is a 50 % probability that power generation will be below (or above) 1.706 GW.

2.3.3.2 Prediction Intervals

Quantile forecasts give a probabilistic information about future renewable power generation, in the form of a threshold level associated with a probability. Even though they may be of direct use for a number of operational problems, they cannot provide forecast users with a feeling about the level of forecast uncertainty for the coming period. For that purpose, *prediction intervals* certainly are the most relevant type of forecasts. Furthermore, prediction intervals are frequently used to make decisions under uncertainty using robust optimization (see, e.g., Chaps. 8 and 9).

Definition 2.6. A *prediction interval* $\hat{I}^{(\beta)}_{t+k|t}$, issued at time t for time $t+k$, defines a range of potential values for Y_{t+k}, for a certain level of probability $(1-\beta)$, $\beta \in [0, 1]$, its *nominal coverage rate*,

$$\mathrm{P}[Y_{t+k} \in \hat{I}^{(\beta)}_{t+k|t} \mid g, \Omega_t, \hat{\Theta}] = 1 - \beta. \tag{2.6}$$

It is equivalently referred to as an *interval forecast*.

Such an interval $\hat{I}^{(\beta)}_{t+k|t}$ must be defined by its lower and upper bounds,

$$\hat{I}^{(\beta)}_{t+k|t} = [\hat{q}^{(\underline{\alpha})}_{t+k|t},\ \hat{q}^{(\overline{\alpha})}_{t+k|t}], \tag{2.7}$$

where these bounds are quantile forecasts whose nominal levels $\underline{\alpha}$ and $\overline{\alpha}$ verify that

$$\overline{\alpha} - \underline{\alpha} = 1 - \beta. \tag{2.8}$$

This general definition makes that a prediction interval is not uniquely defined by its nominal coverage rate. It is thus also necessary to decide on the way it should be centered on the probability density function. Commonly, it is chosen to center it (in probability) on the median, so that there is the same probability that an uncovered realization y_{t+k} lies below or above that interval. This translates to

$$\underline{\alpha} = 1 - \overline{\alpha} = \beta/2. \tag{2.9}$$

With this type of centering, the resulting intervals are called *central prediction intervals*. For example, central prediction intervals with a nominal coverage rate of 90 % (i.e., $(1-\beta) = 0.9$) are defined by quantile forecasts with nominal levels of 5 and 95 %. Other types of intervals exist, e.g., shortest-length intervals and highest-density regions among others [5], depending upon the way they are chosen to summarize information from the full probabilistic distribution. An illustration is given below, for the case of wind power generation in Western Denmark.

Example 2.4 (Central Prediction Intervals of Wind Power Generation) Central prediction intervals of wind power generation with a nominal coverage rate of 90 % (i.e., $(1-\beta) = 0.9$), issued for the whole onshore capacity of Western Denmark and for the same period than in Figs. 2.2 and 2.3, are depicted in Fig. 2.4. They give a range of possibilities of power generation for every lead time, for a certain probability level, and therefore tell about how confident one may be about the point forecasts originally provided—the tighter they are, the higher the confidence is. The advantage is that they give a very visual information on the expected range of future events.

In the present case, these intervals, for instance, inform that there is a 90 % probability that, 24 h in the future, wind power generation will be between 0.897 GW and 1.65 GW. There is only a 5 % probability that wind power generation will actually be less than 0.897 GW, and similarly, only a 5 % probability of being more than 1.65 GW.

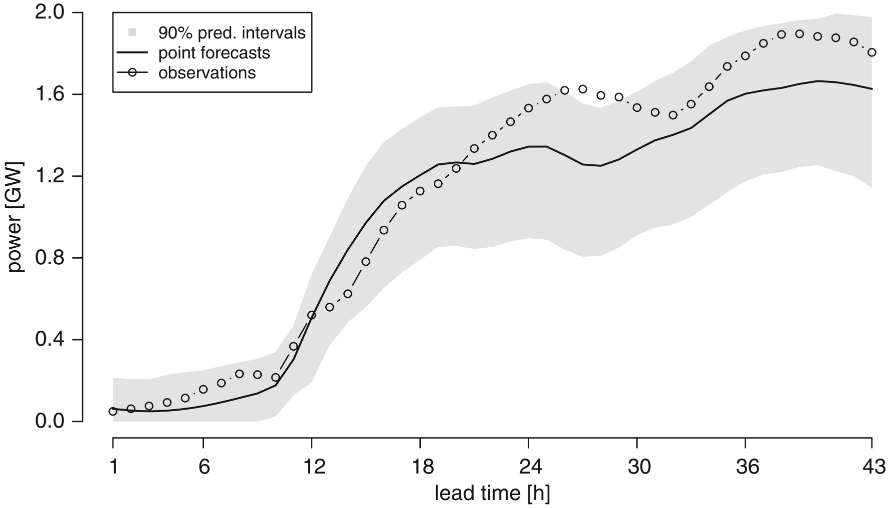

Fig. 2.4 Central prediction intervals of wind power generation with a nominal coverage rate of 90 % issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown

2.3.3.3 Density Forecasts

All the various types of predictions presented in the above, i.e., point, quantile, and interval forecasts, are in fact only partly describing the whole information about future renewable energy generation at every lead time. This whole information would be given by *density forecasts* for each point of time in the future.

Definition 2.7. (A *density forecast* $\hat{f}_{t+k|t}$ (or $\hat{F}_{t+k|t}$ if focusing on cdfs) issued at time t for time $t+k$, is a complete description of the pdf (or cdf) of Y_{t+k} conditional on a given model g, estimated parameters $\hat{\Theta}_t$ and the information set Ω_t.

For a wide range of decision-making problems related to renewable energy management and to its integration into electricity markets, density forecasts in the form of predictive densities $\hat{f}_{t+k|t}$ or predictive cdfs $\hat{F}_{t+k|t}$ are a necessary input to related operational problems. Representative examples include the design of optimal offering strategies, to be dealt with in Chap. 7, or the optimal quantification of reserve requirements, accounting for all uncertainties involved [11].

Example 2.5 (Density Forecasts of Wind Power Generation) Figure 2.5 displays density forecasts of wind power generation issued for the whole onshore capacity of Western Denmark and for the same period than in Figs. 2.2, 2.3, and 2.4. Density forecasts are visualized as a river-of-blood fan chart based on central prediction intervals with nominal coverage rates $(1-\beta) \in \{0.1, 0.2, \dots, 0.9\}$. Even though they may not be easy to interpret visually, they permit to fully characterize wind power generation for every individual lead time: for instance, Fig. 2.5 shows, for the case

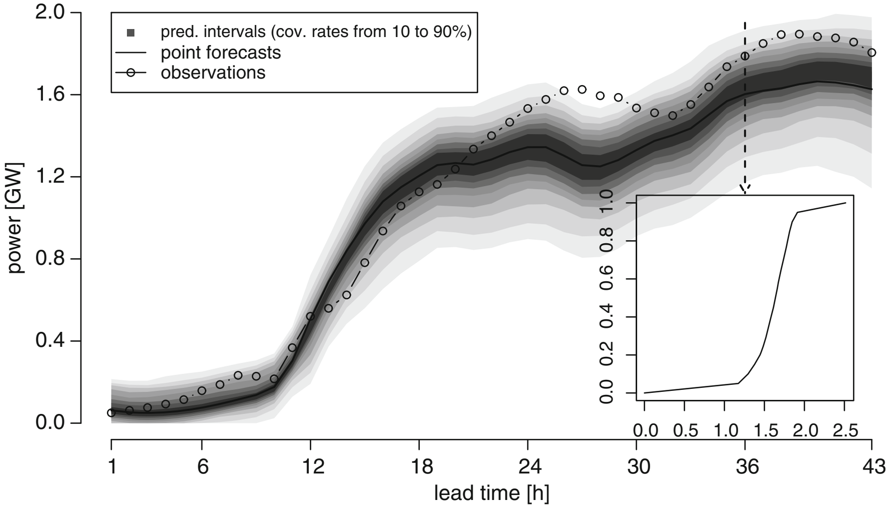

Fig. 2.5 Predictive densities of wind power generation issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown. The predicted cumulative distribution function for the 36-hour lead time is depicted as an example piece of information that can be extracted from such forecasts

of the 36-hour ahead lead time, the type of cumulative distribution function that can be extracted from the set of density forecasts, hence describing all possibilities for wind power generation at that lead time. Point forecasts and the corresponding measurements are also shown for comparison.

Remember that, for this example, the point forecasts told that the expected power generation for 5th April 2007 at 00:00 UTC was of 1.32 GW. Here the probabilistic forecasts inform that there is a 5 % probability that power production will be less than 896 MW, a 25 % probability of power being less than 1.181 GW, and a 25 % probability of being more than 1.477 GW.

Whatever the type of probabilistic forecasts considered, they may be obtained based on parametric or nonparametric approaches, which are discussed in a further section below.

2.3.4 Scenarios

Probabilistic forecasts give substantial information about the characteristics of the stochastic process of interest, i.e., renewable power generation, for the coming future. However, they only concentrate on detailing marginal densities for each lead time, location and renewable energy type, independently. For instance, if considering several locations and types of renewable energy, say, wind and wave energy, the interdependence between locations and renewable energy sources would not be described. Maybe, more importantly, the temporal dependence structure of potential forecast errors is disregarded. For instance, if forecasts errors are strongly correlated in time, it means that a large forecast error at time $t+k$ is most likely followed at time $t+k+1$ by another large error. In contrast, if that dependence is weak, forecast errors for future lead times may be seen as completely random.

Such information about dependence in time, space, and among types of renewable energy sources, may be crucial as input to a number of operational problems arising from an integrated management of their energy output. One may think of

(i) the optimal operation of a virtual power plant composed by a wind farm and storage,
(ii) optimal offshore maintenance planning that necessitates the joint forecasting of the power output of wind and wave energy devices at some offshore location,
(iii) stochastic unit commitment accounting for wind power capacities spread over a control zone,
(iv) market-clearing methods aiming to optimally accommodate the output of renewable energy sources.

Scenarios are first introduced here in their most simple form as time trajectories. They will be presented in a more generic manner in the modeling section below, by also looking at other dimensions as well. Emphasis is therefore placed on a single type of renewable energy source for a single location (or for a capacity aggregation). As preliminary, it is required to introduce the multivariate random variable

$$Z_t = \{Y_{t+k},\ k = 1, \dots, K\}. \tag{2.10}$$

which gathers the random variables characterizing the stochastic power generation process for the K following lead times, hence covering their marginal densities as well as their interdependence structure. Denote by F_{Z_t} its multivariate cdf.

Definition 2.8. Scenarios issued at time t and for a set of K successive lead times, i.e., with $k \in \{1, 2, \dots, K\}$, are samples of $\hat{F}_{Z_t}$, namely the predicted cdf of Z_t. They consist in a set of J time trajectories

$$\hat{z}_t^{(j)} = [\hat{y}_{t+1|t}^{(j)}, \hat{y}_{t+2|t}^{(j)}, \dots, \hat{y}_{t+K|t}^{(j)}]^\top \quad j = 1, \dots, J. \tag{2.11}$$

The corresponding observation is $z_t = [y_{t+1}, \dots, y_{t+K}]^\top$.

Indeed, even in the most simple case, where Z_t would be multivariate Gaussian, communicating a forecast distribution for Z_t is complex since it would consist in a set of conditional expectations for the successive lead times, associated with a conditional covariance matrix summarizing the second-order characteristics of Z_t. Overall, it may not even be possible to fully characterize the K-dimensional distribution of Z_t. This is why one, instead, issues time trajectories such as that in Eq. (2.11). If used as input to operational problems, they should be seen as equally likely samples of the predictive distribution of Z_t. The resulting time trajectories comprise scenarios like those commonly used as input to operational problems in

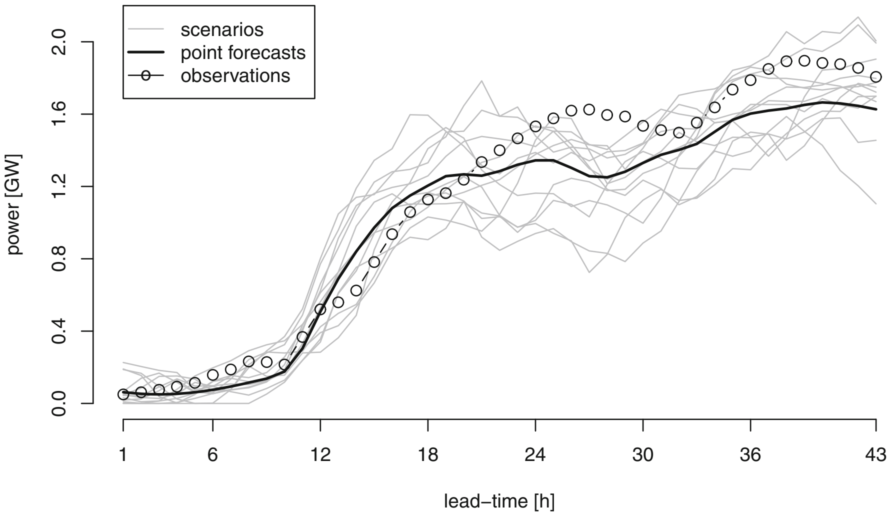

Fig. 2.6 Scenarios (12 time trajectories) of wind power generation issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown

a stochastic programming framework (see, e.g., Chap. 5). An illustration is given below based on the real-world example of Western Denmark.

Example 2.6 (Scenarios of Wind Power Generation) For the same period and conditions as in Fig. 2.5, scenarios of short-term wind power generation in the form of time trajectories are depicted in Fig. 2.6. These 12 scenarios jointly inform on the marginal densities for each lead time (even though having more scenarios would clearly be beneficial), while they also tell about the temporal correlation of the generation process, since they represent plausible alternative paths into the future. The original point forecasts, as well as the corresponding observations obtained a posteriori, are also shown for comparison.

The scenarios reflect some of the properties of predictive densities shown in Fig. 2.5, for instance, their uncertainty increasing when getting further in the future. For every lead time, however, they only give a discrete representation of these densities, here with 12 equally likely values of power generation.

The optimal set of scenarios, i.e., time, space–time, and/or multivariate trajectories, readily depends upon the type of operational problem to be solved, as well as the dependencies that are known to be of primary importance. Raising the complexity and dimension of these scenarios clearly has a cost—the simpler they are, the better.

2.3.5 Event-Based Predictions

Since the mid-2000s, a number of renewable power forecast users have expressed the need for more targeted types of predictions, based on specific events that have the most impact on electricity markets and power systems operations. This has therefore led to coining the term of *event-based* predictions, where the events may be the so-called ramps (large gradients of power production over a short time period), intense power variability (periods with power fluctuations of high magnitude), etc.

Event-based forecasting has its root in meteorology and climate sciences, where events are defined by setting a threshold on the value of a continuous variable, e.g., in our case, "renewable power generation being greater than 50 % of the portfolio's nominal capacity". The particularity of an event is that observations take values in $\{0, 1\}$ only, depending upon the event realizing or not. Related probability forecasts take values in $[0, 1]$, hence informing about the probability of that particular event realizing. These forecasts are evaluated for each observation time, potentially as a function of the lead time.

In a generic manner, write

$$g : z \rightarrow g(z; \theta), \tag{2.12}$$

a functional allowing to define an event based on a time trajectory z and a parameter set θ. A number of functionals could easily be defined for events relevant to renewable power forecasts users. The functional g may be generalized to be applied to the multivariate random variable Z_t of Eq. (2.10), with the probability of observing the event based on the random variable Z_t as output.

Example 2.7 (Ramp Events over 6-hour Time Windows) Let us place ourselves at time t. Given a window of size h centered on lead time k, one may introduce the functional g as

$$g(z_t; k) = 1\left\{\left(\max_{i\in\mathcal{S}} y_{t+i} \; - \; \min_{i\in\mathcal{S}} y_{t+i}\right) \geq 0.5\right\},$$
$$\mathcal{S} = \{k-3, k-2, \ldots, k+2, k+3\}, \tag{2.13}$$

where y_{t+i} is the i^{th} component of z_t. In the above, $1\{.\}$ is an indicator operator, being equal to 1 if the condition expressed within brackets realizes, and to 0 otherwise. This functional defines a ramp event over 6-hour time windows centered on the lead time k, with the maximum absolute variation in power generation being (or not) greater than 50 % of nominal capacity.

As an extension, it is now possible to introduce the concept of event-based prediction.

Definition 2.9. An event-based forecast issued at time t for time $t+k$ consists in a forecast $\hat{g}(Z_t; \theta)$ of the probability $g(Z_t; \theta)$ of observing the event defined by g.

Event-based forecasts can be directly issued based on statistical models such as generalized linear models (GLMs) [10]. Alternatively, they may be a filtered version of the complex scenarios presented before. In that case, the event-based forecasts $\hat{g}(Z_t; \theta)$ is obtained by applying the functional g to the predicted set of time trajectories,

$$\hat{g}(Z_t; \theta) = \frac{1}{J} \sum_{j=1}^{J} g(\hat{z}_t^{(j)}; \theta). \tag{2.14}$$

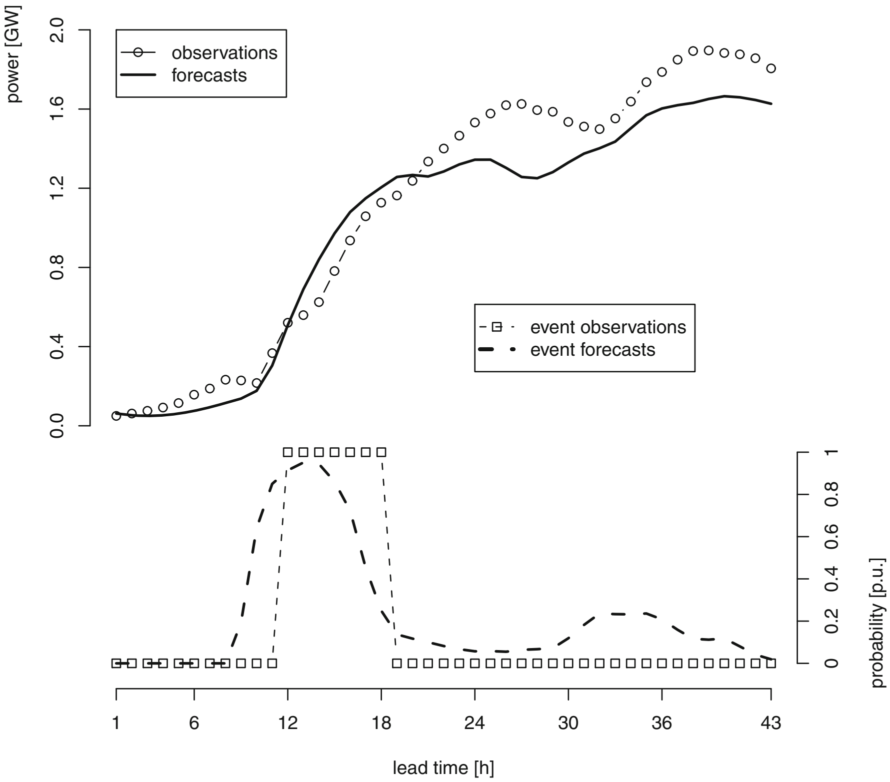

Fig. 2.7 Probability forecasts of ramp events issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day), along with the original point forecasts of wind power generation. Ramps are defined as a change of more than 500 MW in power generation within a 6-hour time window. Ramp forecasts are filtered from 1000 scenarios of short-term wind generation. Related observations, for both power generation and ramp events, are also shown

i.e., as the share of time trajectories predicting this event.

Event-based forecasts obtained from trajectories may be appealing in practice owing to the difficulty to process complex temporal, spatio-temporal, and multivariate dependencies with naked eyes. Working based on specific events of interest readily simplifies the analysis.

Example 2.8 (Ramp Probability Forecasts of Wind Power Generation) To finalize the series of examples based on wind power forecasts issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day), Fig. 2.7 shows ramp probability forecasts along with the original point forecasts for that period. The ramp forecasts are obtained from 1000 scenarios such as those shown in Fig. 2.6, where a ramp event is defined as a change of 500 MW (or more) in power generation within a time window of 6 h.

The forecasts tell that the probability of observing a ramp event increases steadily for lead times between 8 and 13 h ahead, while it decreases for further lead times. The predicted probability is then small for lead times greater than 28 h ahead, though it is more than 0, indicating that such an event could occur.

Even though these event-based forecasts may be appealing from a visual point of view, and also owing to their simplicity in comparison with scenarios, it is clear that their value as input to operational problems may be more limited. Scenarios of renewable energy generation, informing on the full characteristics of the stochastic processes involved, are a must-have when it comes to operational problems related to electricity markets and power systems operations. For that reason, event-based forecasts will not be extensively dealt with in the remainder of this chapter.

2.4 Aspects of Forecast Verification

Predictions ought to be evaluated prior to be used as input to operational problems. It would be particularly difficult to analyze the results obtained for the operational problem at hand if not knowing whether the input forecasts were good or not. After briefly covering general aspects of forecast verification below, focus is given to some of the key concepts, criteria, and diagnostic tools to be used for evaluation of point and probabilistic forecasts, as well as scenarios. An extensive overview of forecast verification concepts, although applied to meteorological forecasting, is available in [6].

2.4.1 General Framework

For predictions in any form, one must differentiate between their quality and their value. Forecast *quality* corresponds to the ability of the forecasts to genuinely inform of future events by mimicking the characteristics of the processes involved. Forecast *value* relates, instead, to the benefits from using forecasts in a decision-making process.

Consequently, forecast quality is independent of the operational problem at hand, while this is not the case of forecast value. In the remainder of the book, we will mainly seek to exploit the value of forecasts, insofar as these forecasts will be considered as input to operational problems. In contrast, here, our aim is to provide the reader with the basics necessary to appraise the quality of predictions.

Visually inspecting the forecasts so as to get a feel for their quality is most certainly advisable. The most relevant manner to assess forecast quality, nonetheless, is to carry out quantitative assessments based on a number of criteria, potentially complemented with a qualitative appraisal using related diagnostic tools. Such quantitative and qualitative assessments must be performed on an independent evaluation period, i.e., a period with data that have not been used in any way for model identification and estimation purposes. Over this period, the forecasts should also be issued as if in real operational conditions. In the following, it will be considered that predictions are verified on an evaluation period of length T with T sufficiently large. Typically, one wants to have several months of forecasts and corresponding measurements available for verification, so as to draw meaningful conclusions. Employing evaluation sets of limited lengths would bring significant uncertainty in the quality assessment.

Importantly for an objective and critical appraisal of evaluation results, benchmark approaches should be considered. A benchmark method is characterized by its apparent simplicity while already allowing to provide competitive forecasts. The most relevant ones are persistence and climatology. The former is based on the idea that the most recent observation is the best forecast for the coming future, while the latter considers that the best prediction is to be derived from knowledge of the long-term statistics.

Definition 2.10. Placing ourselves at time t and looking in the future at time $t + k$, the *persistence* forecast is given by the last available measurement,

$$\hat{y}_{t+k|t} = y_t, \tag{2.15}$$

while the *climatology* one is the mean of all available past measurements. If having m past observations, this yields

$$\hat{y}_{t+k|t} = \frac{1}{m}\sum_{i=0}^{m-1} y_{t-i}. \tag{2.16}$$

Illustrative examples of persistence and climatology-based forecasts are given below.

Example 2.9 (Persistence and Climatology Forecasts) The observed wind power generation at time t is equal to 1.25 GW, while the average of all past observations is 892 MW. A persistence forecast for any time $t + k$ in the future is 1.25 GW, while the climatology one is 892 MW.

At time $t+1$, when having a new observed power value of 1.08 GW, the persistence forecast for any time $t + 1 + k$ becomes 1.08 GW, while the climatology forecast is still 892 MW.

2.4.2 Point Forecasts

Point forecasts of renewable power generation are for a continuous variable. Consequently, the base quantity for evaluating point forecasts is the *forecast error*.

Definition 2.11. The *forecast error* $\varepsilon_{t+k|t}$ is the difference between observed and predicted values,

$$\varepsilon_{t+k|t} = y_{t+k} - \hat{y}_{t+k|t}. \tag{2.17}$$

The subscripts employed are the same as for the forecast it is linked to.

Note that the forecast error may be normalized so that verification results may be comparable for different renewable energy generation sites. If so, normalization is most commonly performed by the nominal capacity of the site of interest. Alternatively, some use the average power generation over the evaluation period. The former convention will be used here, while the terms "forecast error" and "normalized forecast error" will be used interchangeably.

Example 2.10 (Forecast Error and its Normalized Version) A forecast issued at time t indicates that the expected wind power generation at time $t+1$ is 1.25 GW. The observation, obtained a posteriori, is 1.08 GW. The corresponding forecast error is hence -170 MW. Given a nominal capacity of 2.515 GW, the normalized forecast error is $-6.76\,\%$ of that nominal capacity.

The first error criterion that may be computed is the *bias* of the forecasts, which corresponds to the systematic part of the forecast error. It may be corrected in a straightforward manner using simple statistical models.

Definition 2.12. The *bias* is the mean of all errors over the evaluation period of length T, considered indifferently. For lead time k, this writes

$$\text{bias}(k) = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_{t+k|t}. \tag{2.18}$$

This summary measure does not tell much about the quality of point forecasts, only about a systematic error that should be corrected. For a better appraisal of the forecasts, it is advised to use scores defined based on particular *loss functions* (to be further introduced and discussed in Sect. 2.5.3). Loss functions assign a penalty to forecast errors, as a proxy of the cost of these errors for those making decisions based on such forecasts. Considering a quadratic loss function yields the definition of the *root mean square error* (*RMSE*).

Definition 2.13. The *RMSE* is defined as the square root of the sum of squared errors over an evaluation set of length T,

$$\text{RMSE}(k) = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\varepsilon_{t+k|t})^2}. \tag{2.19}$$

Alternatively, the *mean absolute error* (*MAE*) criterion is based on a linear loss function.

Definition 2.14. The *MAE* is defined as the average of the absolute forecast errors over an evaluation set of length T,

$$\text{MAE}(k) = \frac{1}{T} \sum_{t=1}^{T} |\varepsilon_{t+k|t}|. \tag{2.20}$$

All the above error criteria are independent of the length of the evaluation set. They are normalized by its length. They may also be normalized by the nominal capacity of

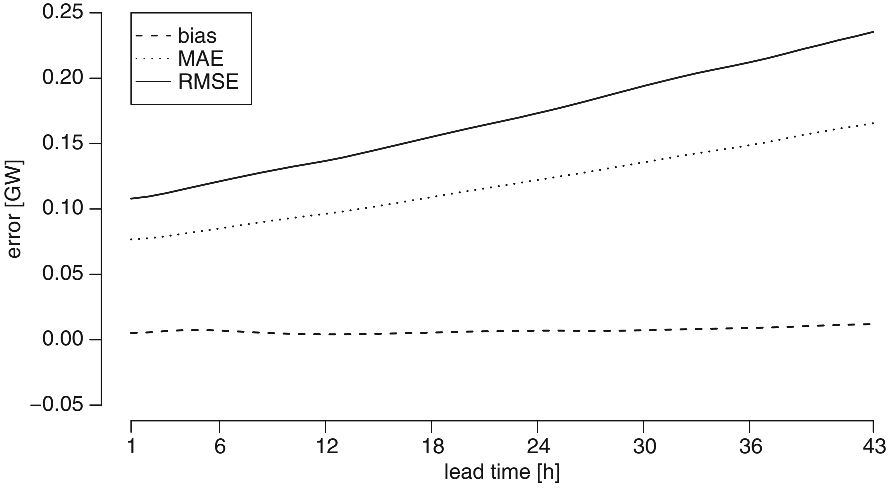

Fig. 2.8 Evaluation of point forecasts of wind power generation for Western Denmark with bias, MAE and RMSE criteria, over a period of almost two years

the renewable energy site (or aggregation) of interest, and then referred to as Nbias, NRMSE, and NMAE.

Example 2.11 (Evaluation of Point Forecasts of Wind Power Generation) Point forecasts of wind power generation issued for the whole Western Denmark area, with lead times between 1 and 43 h ahead, are evaluated over a period of almost two years. Figure 2.8 shows the result of such an evaluation, as the evolution of bias, MAE, and RMSE as a function of the lead time. Recall that the nominal capacity of that area is 2.515 GW.

The forecasts are almost unbiased, while the forecast accuracy degrades almost linearly as the lead time increases. RMSE values are greater than MAE values owing to the nature of their underlying loss functions (squared errors for the RMSE, and errors in absolute value for the MAE).

For the example of the 24-hour ahead lead time, the MAE and RMSE of the forecasts are 122 MW and 173 MW. If considering their normalized versions instead, NMAE and NMRSE values are 4.86 % and 6.89 % of the nominal capacity, respectively.

2.4.3 Probabilistic Forecasts

The quality of probabilistic forecasts of renewable energy is dominated by their probabilistic calibration and their overall skill. These aspects are treated one after the other in the following. They may be relevant for all types of probabilistic forecasts, i.e., quantile, interval and density forecasts. The quality of probabilistic forecasts is here generally evaluated in a nonparametric framework: no assumption is made about the shape of predictive densities. Indeed, a particular predictive density $\hat{f}_{t+k|t}$ issued at time t for time $t+k$ is simply characterized by sets of quantile forecasts $\hat{q}^{(\alpha_i)}_{t+k|t}$ with nominal levels α_i, $i = 1, \ldots, m$.

2.4.3.1 Probabilistic Calibration

The first requirement for probabilistic forecasts is for them to consistently inform about the probability of events. This directly leads to the concept of *probabilistic calibration*, also referred to as *reliability*. The general definition of probabilistic calibration is introduced for the most general density forecasts, while it may then be derived for quantile and interval forecasts in a straightforward manner.

Definition 2.15. Probabilistic forecasts for a given lead time k, defined by their predictive cdfs $\hat{F}_{t+k|t}$, are said to be probabilistically calibrated if

$$\hat{F}_{t+k|t}(Y_{t+k}) \sim U[0,1], \tag{2.21}$$

that is, if the forecast probabilities are consistent with the observed ones.

That definition is difficult to use in practice, since only one realization of Y_{t+k} is observed. One, therefore, employs a frequentist approach for the assessment of probabilistic calibration, based on an evaluation set of sufficient length. There it is verified that the *probability integral transform* (PIT) $\hat{F}_{t+k|t}(y_{t+k})$ of the measurements follows a uniform distribution over the unit interval. This may be performed in a nonparametric set-up by assessing the reliability of each of the defining quantile forecasts. For that purpose, the *indicator variable* $\xi^{(\alpha)}_{t,k}$ is introduced as follows:

Definition 2.16. The indicator variable $\xi^{(\alpha)}_{t,k}$, for a given quantile forecast $\hat{q}^{(\alpha)}_{t+k|t}$ and corresponding realization y_{t+k} is defined as

$$\xi^{(\alpha)}_{t,k} = 1\{y_{t+k} < \hat{q}^{(\alpha)}_{t+k|t}\} = \begin{cases} 1, & \text{if} \quad y_{t+k} < \hat{q}^{(\alpha)}_{t+k|t} \\ 0, & \text{otherwise} \end{cases}, \tag{2.22}$$

i.e., as a binary variable indicating if the quantile forecasts actually cover, or not, the renewable power measurements.

Example 2.12 (Indicator Variable and Quantile Forecasts) Quantile forecasts of wind power generation are issued at time t and time $t+1$, with lead time k and for a nominal level of 0.8. For instance, the forecasts for time $t+k$ and $t+k+1$ are 1.86 GW and 1.93 GW, respectively. The power measurements, obtained a posteriori, are 1.43 GW and 1.98 GW. As a consequence the indicator variable takes the value 1 for time $t+k$ and 0 for time $t+k+1$.

These '1's and '0's are nicknamed *hits* and *misses*. It is by studying the binary time-series $\{\xi^{(\alpha)}_{t,k}\}$ of indicator variable values that one can assess the reliability of quantile forecasts, for given nominal level α and lead time k. Adapting the definition of probabilistic calibration, quantile forecasts $\hat{q}^{(\alpha)}_{t+k|t}$ with nominal level α are probabilistically calibrated if

$$\xi_{t,k}^{(\alpha)} \sim B(\alpha), \tag{2.23}$$

that is, if the corresponding indicator variable $\xi_{t,k}^{(\alpha)}$ is distributed Bernoulli with chance of success α.

Using this indicator variable, the *empirical level* of quantile forecasts can be defined, estimated, and eventually compared with their nominal one.

Definition 2.17. The *empirical level* $\hat{a}_k^{(\alpha)}$, for a nominal level α and lead time k, is obtained by calculating the mean of the $\{\xi_{t,k}^{(\alpha)}\}$ time-series over an evaluation set of length T,

$$\hat{a}_k^{(\alpha)} = \frac{1}{T} \sum_{t=1}^{T} \xi_{t,k}^{(\alpha)} = \frac{n_{k,1}^{(\alpha)}}{n_{k,0}^{(\alpha)} + n_{k,1}^{(\alpha)}}, \tag{2.24}$$

where $n_{k,1}^{(\alpha)}$ and $n_{k,0}^{(\alpha)}$ correspond to the sum of hits and misses, respectively.

More specifically, they are calculated as

$$n_{k,1}^{(\alpha)} = \#\{\xi_{t,k}^{(\alpha)} = 1\} = \sum_{t=1}^{T} \xi_{t,k}^{(\alpha)}, \tag{2.25}$$

$$n_{k,0}^{(\alpha)} = \#\{\xi_{t,k}^{(\alpha)} = 0\} = T - n_{k,1}^{(\alpha)}. \tag{2.26}$$

The difference between nominal and empirical levels of quantile forecasts is to be seen as a probabilistic bias.

Example 2.13 (Nominal and Empirical Levels of Quantile Forecasts) Let us build on Example 2.12, by considering quantile forecasts issued for a lead time k and for a nominal level of 0.8, over an evaluation set with 200 forecasts and corresponding observations. After counting the number of times the observations were lower or greater than the quantile forecasts, the sum of hits and misses are $n_{k,1}^{(0.8)} = 153$ and $n_{k,0}^{(0.8)} = 47$, yielding an empirical level of 0.765. It is slightly less than the nominal one.

Prediction intervals and nonparametric predictive densities are characterized by a set of quantile forecasts. Consequently, their reliability may be evaluated similarly by verifying the empirical level of their defining quantile forecasts. Such an approach can also be used for parametric probabilistic forecasts by extracting quantiles for given nominal levels. The overall deviation from perfect probabilistic calibration is quantified by the so-called *reliability* or *discrepancy* index.

Definition 2.18. For a given lead time k, the *reliability* (also called *discrepancy*) index $\Delta(k)$ corresponds to the average absolute difference between nominal and empirical levels,

$$\Delta(k) = \frac{1}{m} \sum_{i=1}^{m} |\alpha_i - \hat{a}_k^{(\alpha_i)}|, \tag{2.27}$$

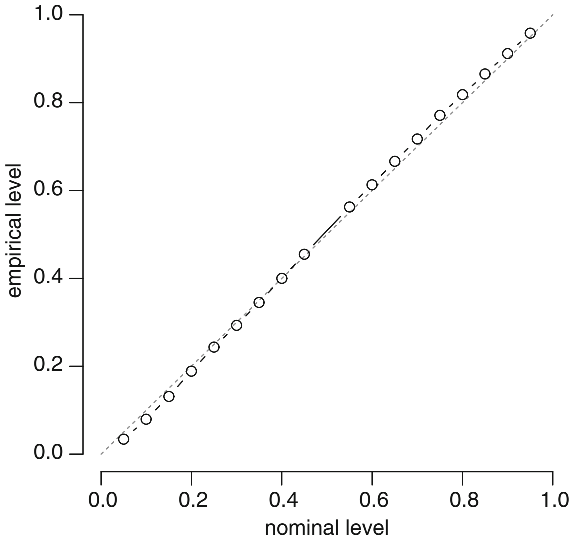

Fig. 2.9 Reliability diagram for the evaluation of the reliability of probabilistic forecasts for the whole onshore capacity of Western Denmark over a period of almost 2 years. The lead time considered here is 12-hour ahead. The probabilistic forecasts are defined by a number of quantile predictions, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median

for interval or density forecasts defined by a set of quantile forecasts with nominal levels α_i, $i = 1, \dots, m$.

Instead of calculating discrepancy indices, probabilistic calibration may be appraised visually based on reliability diagrams plotting empirical vs. nominal levels of the quantiles defining density forecasts.

Example 2.14 (Reliability Diagrams and Discrepancy Index) Probabilistic forecasts of wind power generation issued for the whole Western Denmark area, with lead times between 1 and 43 h ahead, are evaluated over a period of almost two years. These probabilistic forecasts take the form of predictive densities defined by a number of quantile predictions, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median.

The calibration of these probabilistic forecasts is performed by evaluating that of quantile forecasts for all nominal levels. The resulting comparison of empirical and nominal levels is summarized in the reliability diagram such as that in Fig. 2.9, for the example of 12-hour ahead forecasts. Their correspondence appears to be very good, with a low discrepancy index value of 0.012. This discrepancy index value corresponds to the average difference between what was observed (line with circle markers) and the ideal case of perfect calibration (dotted line). Results for other lead times may be looked at similarly.

2.4.3.2 Skill of Probabilistic Forecasts

As for the case of the bias for point forecasts, the assessment of probabilistic calibration only informs about a form of bias of probabilistic forecasts. The fact that they may be perfectly calibrated does not guarantee that the forecasts are really good, for instance, they may not be able to discriminate among situations with various uncertainty levels, while these aspects are of crucial importance in decision-making.

The overall quality of probabilistic forecasts may be assessed based on skill scores for quantile, interval and density forecasts. The first skill score that may be used is the *negative quantile-based score* (*NQS*).

Definition 2.19. For predictive densities $\hat{f}_{t+k|t}$ defined by sets of quantile forecasts $\hat{q}^{(\alpha_i)}_{t+k|t}$ with nominal levels α_i, $i = 1, \ldots, m$, and corresponding measurements y_{t+k}, the *NQS* is given by

$$\mathrm{NQS}(k) = \frac{1}{T}\frac{1}{m}\sum_{t=1}^{T}\sum_{i=1}^{m}(\alpha_i - \xi^{(\alpha_i)}_{t,k})(y_{t+k} - \hat{q}^{(\alpha_i)}_{t+k|t}), \tag{2.28}$$

over an evaluation set of length T.

This score is negatively oriented and admits a minimum value of 0 for perfect probabilistic predictions. It may be readily used for quantile forecasts (then the sum over i in Eq. (2.28) disappears) and for prediction intervals (with $m = 2$ in Eq. (2.28)).

When focus is on predictive densities only, the most widely used score is the *continuous ranked probability score* (*CRPS*).

Definition 2.20. The *CRPS* for predictive densities $\hat{F}_{t+k|t}$ and corresponding measurement y_{t+k}, is calculated as

$$\mathrm{CRPS}(k) = \frac{1}{T}\sum_{t=1}^{T}\int_x \left(\hat{F}_{t+k|t}(x) - H(x - y_{t+k})\right)^2 dx, \tag{2.29}$$

over an evaluation set of length T. $H(x)$ is the Heaviside step function, taking the value 1 for $x \geq y$ and 0 otherwise.

The CRPS evaluates the area between the predictive cdf and that of the observation (which would have been the perfect forecast). It is a proper skill score: it is minimal when the true distribution of events is used as predictive density. It is a negatively-oriented score (the lower the better) and has the same unit than the variable of interest, while taking a minimum of 0. Note that it can be directly compared to the MAE criterion (defined by Eq. (2.20)) used for point forecasts, since the CRPS is its generalization in a probabilistic forecasting framework.

The computation of the CRPS can directly rely on analytical formulas in the case of some parametric predictive densities, e.g., Gaussian (also for truncated and censored). In the nonparametric case, it is obtained using numerical integration through predictive cdfs as defined by the set of quantile forecasts. Asymptotically, as the number m of quantiles considered tends towards infinity, one exactly has CRPS = 2NQS.

Example 2.15 (Skill of Probabilistic Forecasts) For the same dataset with probabilistic forecasts of wind power generation as in Example 2.14, and for the same period as in Example 2.11, the skill of these forecasts is evaluated as a function of the lead time, with the CRPS criterion. The NQS criterion is not shown, though it

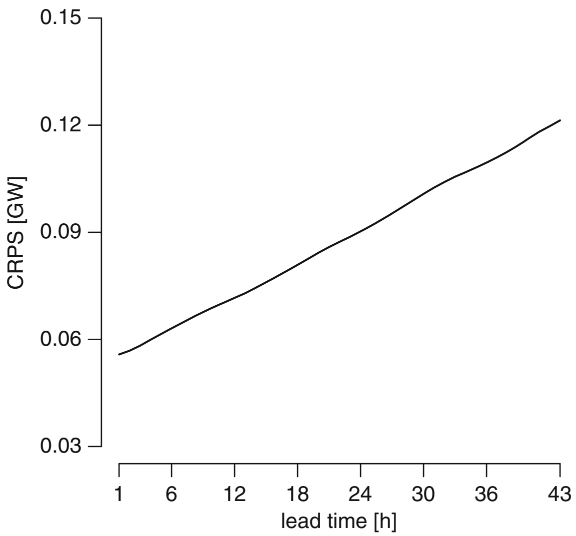

Fig. 2.10 Skill evaluation of probabilistic forecasts for the whole onshore capacity of Western Denmark over a period of almost 2 years, as given by the evolution of the CRPS score as a function of lead time. The probabilistic forecasts are defined by a number of quantile predictions, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median

could also be employed by summing quantile score values for all defining nominal levels. Remember that these probabilistic forecasts are given by a number of quantile forecasts, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median. Linear interpolation through these quantiles yields complete predictive densities and related cumulative distribution functions. The results of the forecast skill evaluation are displayed in Fig. 2.10.

The decrease in forecast skill is fairly linear, similar to the point forecast verification results shown and discussed in Example 2.11, from a CRPS value of 56 MW for the first lead time to a value of 121 MW for the 43-hour ahead lead time. These values are slightly less than the MAE values in Fig. 2.8, hence telling that these probabilistic forecasts have higher skill (and could be seen as more informative) than the previously studied point predictions.

2.4.4 Scenarios

Scenarios ought to be seen as an extension of probabilistic forecasts in a multivariate framework, in the form of sample trajectories from multivariate probabilistic predictions. They should therefore be evaluated in a manner consistent with that described for probabilistic forecasts in the above, based on probabilistic calibration and skill.

One the one hand, probabilistic calibration of scenarios may be assessed based on the multivariate generalization of calibration for probabilistic forecasts (Def. 2.15). Since it may be too difficult to directly use that definition for evaluating the probabilistic calibration of scenarios, alternative proposals were made in the literature, relying on the concept of ranks. Owing to the complexity of these approaches to the reliability assessment of scenarios, these aspects are overlooked here. The interested reader is referred to [14] and references therein for a complete treatment.

On the other hand, the skill of scenarios is more straightforward to appraise. Recall that the skill scores for probabilistic forecasts presented in Sect. 2.4.3.2 are to be computed for each lead time, location and type of renewable energy sources. They therefore overlook the interdependence structure that is modeled by scenarios. In order to assess skill, a multivariate generalization is necessary for the case of scenarios. Today, the lead skill score for that purpose is the *energy score*, which is a direct generalization of the CRPS defined by Eq. (2.29).

Definition 2.21. For a given set of time trajectories $\hat{z}_t^{(j)}\, j = 1, \ldots, J$, issued at time t, the *energy score* is given by

$$\mathrm{Es}_t = \frac{1}{J} \sum_{j=1}^{J} \|z_t - \hat{z}_t^{(j)}\|_2 - \frac{1}{2\,J^2} \sum_{i=1}^{J} \sum_{j=1}^{J} \|\hat{z}_t^{(i)} - \hat{z}_t^{(j)}\|_2, \tag{2.30}$$

where $\|.\|_2$ is the K-dimensional Euclidean norm (also called l^2 norm). It is then calculated and averaged over an evaluation set of length T,

$$\mathrm{Es} = \frac{1}{T} \sum_{t=1}^{T} \mathrm{Es}_t. \tag{2.31}$$

The energy score is a strictly proper skill score, in the sense that it is minimal only when the true distribution is used for generating trajectories. It is a negatively-oriented score (the lower the better), while having the same unit than the variable of interest.

Example 2.16 (Skill of Scenarios of Wind Power Generation) A set of 12 scenarios of wind power generation was shown in Fig. 2.6, as issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark, and for a nominal capacity of 2.515 GW. Applying the formula of Eq. (2.30) for assessing their quality based on the energy score yields a score value of 0.765 GW.

2.5 Model-Based Approaches to Generating Renewable Power Forecasts

Renewable power forecasting has its wealth of models and methods that have appeared in the scientific literature and in commercial software over the last couple of decades. The development of forecasting approaches intensified since the beginning of the new millennium with the boost in the deployment of renewable energy capacities. Most of these approaches were first introduced for wind energy, reflecting its leading role in this deployment. Since then, similar methodologies have appeared for, e.g., solar and wave energy.

As it is impossible (neither is it our aim) to describe all approaches to renewable energy forecasting, our objective is instead to cover some of the basic aspects of interest to readers aiming to use them as input to operational problems related to electricity markets. For that, we place ourselves in a probabilistic framework, where probabilistic forecasts comprise a basis product, while point forecasts and scenarios are to be derived from these probabilistic forecasts.

2.5.1 Overview of Forecasting Methodologies and Required Inputs

Power generation from all forms of renewable energy sources is directly influenced by some of the weather variables, e.g., wind (and air density to a lesser extent) for wind energy, solar irradiance, and temperature for solar energy, and finally, significant wave height and period for wave energy. In parallel, the power output of renewable energy devices is a function of the technology embedded, which impact the way the energy originally provided by the weather variables is transformed into electric energy. Consequently, for all forms of renewable energy sources, the core aspects of the prediction exercise include *(i)* the appraisal of external conditions that drive the energy conversion process, and *(ii)* the energy conversion process itself, that is, how the potential energy in external conditions is eventually converted to electrical power.

The mathematical modeling involved in these two core aspects may rely on physical and statistical concepts, optimally on a combination of both. The share of physical and statistical expertise to be blend in to get the best forecasts will depend on the lead times of interest, world location (linking to local climatology), and maybe, the level of expertise of those laying the mathematical models and forecasting method.

For the latter aspect, the effect of technology can be directly modeled based on physical and statistical approaches. For the former aspect however, one is bound to rely on weather forecast providers, since all the modeling and computing involved in predicting relevant weather variables may be too cumbersome for the intended application. Note, nevertheless, that some consider predicting these relevant weather variables themselves, most often based on statistical time-series models such as those encountered in the finance literature.

Two types of data are employed as input to the various existing forecasting methodologies. On the one hand, relevant observations at the site of interest (and potentially offsite) of power generation and of relevant meteorological variables permit to better appraise the local process dynamics. This set of observations is generically denoted ψ_t. On the other hand, forecasts of the relevant weather variables, as provided by any public or private weather forecasting office, permit to give an overall picture of how input weather variables may evolve in the coming hours to days. This set of input meteorological forecasts is denoted by Γ_t.

Example 2.17 In the case of wind power forecasting, the set ψ_t of observations available at time t may be such that

$$\psi_t = \{y_t, y_{t-1}, \dots, u_t, u_{t-1}, \dots, \omega_t, \omega_{t-1}, \dots\}, \tag{2.32}$$

where y is for power measurements, while u and ω are for observed wind speed and direction respectively. These variables will be different when looking at solar and wave energy prediction, e.g., solar irradiance and cloud cover for the former, and wave height and period for the latter. In parallel, the set Γ_t of meteorological forecasts available at a given time t may take the form of

$$\Gamma_t = \{\hat{u}_{t+1|t}, \hat{u}_{t+2|t}, \ldots, \hat{u}_{t+K|t}, \hat{\omega}_{t+1|t}, \hat{\omega}_{t+2|t}, \ldots, \hat{\omega}_{t+K|t}\}, \tag{2.33}$$

i.e., with forecasts of wind speed and direction for the site or portfolio of interest, for all lead times up to the forecast length K. Again, these variables will be different for the solar and wave energy cases.

Finally, in a generic manner, any prediction of renewable energy generation issued at time t, being point, probabilistic, scenario or event-based forecast is a linear or nonlinear function of these sets ψ_t and Γ_t (or of some of their subsets).

2.5.2 Issuing Probabilistic Forecasts

Probabilistic forecasts of renewable power generation are the most general form of prediction, informing on the whole range of potential power outcomes for every lead time. They were illustrated in the form of predictive densities in Fig. 2.5, as well as in the simpler version of prediction intervals in Fig. 2.4. The way they may be generated based on parametric and nonparametric approaches is presented below.

2.5.2.1 Parametric Approaches

The interest of parametric approaches is to rely on parametric assumptions, which can be seen as predefined shapes for predictive densities. These shapes are fully characterized by a few parameters only, hence easing subsequent estimation.

Definition 2.22. Following a parametric assumption, the distribution of the stochastic process at time $t + k$ is such that

$$Y_{t+k} \sim F(y; \theta_{t+k}), \tag{2.34}$$

where F is the parametric distribution of choice, and where θ_{t+k} is a set of parameters fully determining the distribution F. A probabilistic forecast issued at time t for Y_{t+k} is then obtained after predicting the value of the set of parameters θ_{t+k}, i.e.,

$$Y_{t+k} \sim \hat{F}_{t+k|t}, \tag{2.35}$$

where

$$\hat{F}_{t+k|t} = F(y; \hat{\theta}_{t+k|t}). \tag{2.36}$$

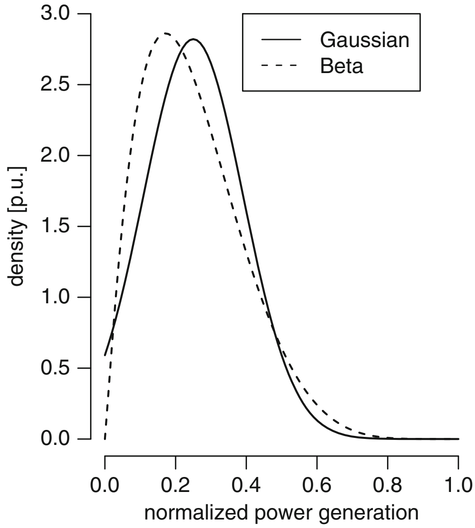

Fig. 2.11 Probabilistic forecasts of normalized wind power generation based on Gaussian and Beta distributions, with the same mean and variance of 25 % and 2 % of the nominal capacity

The choice for F comes from an expert guess of the forecaster or after a thorough empirical analysis of the stochastic process based on available data. Furthermore, it is the set of parameters θ_{t+k} that is to be modeled and then predicted based on the information available at a given time t.

Example 2.18 (Gaussian and Beta Predictive Densities with Same Mean and Variance) Consider forecasting wind power generation (normalized by nominal capacity) for a specific portfolio and for lead times of several hours ahead, based on a Gaussian assumption for predictive densities,

$$Y_{t+k} \sim \mathcal{N}(\hat{\mu}_{t+k}, \hat{\sigma}^2_{t+k}), \tag{2.37}$$

for instance, with a mean $\hat{\mu}_{t+k} = 25\,\%$ of nominal capacity and a variance $\hat{\sigma}^2_{t+k} = 2\,\%$. Alternatively, the forecast may be based on a Beta assumption,

$$Y_{t+k} \sim \text{Beta}(\hat{\alpha}_{t+k}, \hat{\beta}_{t+k}). \tag{2.38}$$

As an example, for the same mean and variance, the parameters of the Beta distribution would be $\hat{\alpha}_{t+k} = 2.09$ and $\hat{\beta}_{t+k} = 6.28$. Even if the mean and variance are the same, the resulting distributions look different, as illustrated in Fig. 2.11. This is because the skewness and kurtosis of Beta distributions are also a function of their mean and variance, while this is not the case for Gaussian distributions.

The choice for one assumption or the other will provide different types of information on the range of potential outcomes and their probability. In both cases, the set of parameters θ_{t+k} to be modeled and predicted is of dimension two only. Formulating a Gaussian assumption may not be the best choice, since it would not reflect the double-bounded nature of wind power generation (between zero and the nominal capacity of the portfolio).

Despite the fact that a wide range of candidate distributions F exist for the probabilistic forecasting of renewable energy generation, the Gaussian one (and its truncated and censored versions) certainly is the most commonly employed in practice. Other possibilities which are not necessarily more complex should be envisaged depending upon the type of renewable energy, the forecast range, or upon the size and geographical spreading of the portfolio more generally. As an example, Beta distributions are a good alternative to Gaussian ones when it comes to wind power. If more specifically focusing on very-short term fluctuations of wind power generation, Generalized logit-Normal (GL-Normal) should be preferred. Likewise, when forecasting wave energy flux and wave power generation, log-Normal and GL-Normal distributions, respectively, are relevant candidate distributions. Note that no particular results were reported so far for the case of solar energy.

Any chosen distribution F can be fully characterized by the set of parameters θ_{t+k}. Mathematical models are then used to describe the future evolution of this set of parameters, as a function of the information set discussed before. For the i^{th} element (out of l) of θ_{t+k}, this writes

$$\theta_{i,t+k} = g_i(\psi_t, \Gamma_t) + \varepsilon_{i,t+k}, \quad i = 1, \ldots, l, \tag{2.39}$$

with $\varepsilon_{i,t+k}$ a centered noise term, and with ψ_t and Γ_t the set of observations and forecasts available at time t. The g_i models may be linear or not depending upon the forecaster's expertise and the result of an empirical analysis of the data available.

Example 2.19 The previous example is continued here. Note that if choosing a Beta assumption, given that α_{t+k} and β_{t+k} may be directly linked to a mean μ_{t+k} and a variance σ^2_{t+k} parameter, it may still be more natural to work with these latter two quantities. The mean μ_{t+k} of predictive densities is commonly modeled as a nonlinear function of wind forecasts for that lead time,

$$\mu_{t+k} = g_\mu(\hat{u}_{t+k|t}, \hat{\omega}_{t+k|t}) + \varepsilon_{\mu,t+k}, \tag{2.40}$$

with $\varepsilon_{\mu,t+k}$ a centered noise term. This model reflects the power curve converting the available energy in the wind to electric power, for the wind portfolio of interest. In parallel, the variance σ^2_{t+k} could be modeled as a constant, in the most simple case,

$$\sigma^2_{t+k} = c_k + \varepsilon_{\sigma,t+k}, \tag{2.41}$$

with $\varepsilon_{\sigma,t+k}$ also a centered noise term. The constant c_k is most certainly different for every lead time k. For both models, the parameters of g_μ and the constant c_k would have to be estimated from data.

2.5.2.2 Nonparametric Approaches

In contrast with the parametric approaches presented in the above, the nonparametric ones do not rely on any specific assumption regarding the shape of predictive densities. As a consequence, it is necessary to fully characterize the distribution F

instead of having to model and predict a limited number of parameters only. Since it is not possible to do so, F is commonly summarized by a set of quantiles with appropriately chosen nominal levels.

Definition 2.23. In a nonparametric setup, the distribution of the stochastic process at time $t + k$ is such that

$$Y_{t+k} \sim F_{t+k}, \tag{2.42}$$

where F_{t+k} is summarized by a set of Q quantiles

$$F_{t+k} = \{q_{t+k}^{(\alpha_i)},\ 0 \leq \alpha_1 < \ldots < \alpha_i < \ldots < \alpha_Q \leq 1\}, \tag{2.43}$$

that is, with chosen nominal levels spread over the unit interval. A probabilistic forecast issued at time t for Y_{t+k} is then obtained based on a set of quantile forecasts for these Q nominal levels,

$$Y_{t+k} \sim \hat{F}_{t+k|t}, \tag{2.44}$$

where

$$\hat{F}_{t+k|t} = \{\hat{q}_{t+k|t}^{(\alpha_i)},\ 0 \leq \alpha_1 < \ldots < \alpha_i < \ldots < \alpha_Q \leq 1\}. \tag{2.45}$$

A model is to be proposed for each of the Q quantiles permitting to define nonparametric distributions,

$$q_{t+k}^{(\alpha_i)} = g_i(\psi_t, \Gamma_t) + \varepsilon_{i,t+k}, \quad i = 1, \ldots, Q, \tag{2.46}$$

with $\varepsilon_{i,t+k}$ a centered noise term, and with ψ_t and Γ_t the set of observations and forecasts available at time t. Here again, these g_i models may be linear or not depending upon the forecaster's expertise and the result of an empirical analysis of the data available. The number of models and the corresponding potential computational burden may rapidly increase in comparison with the parametric approaches presented before as the number Q of defining quantiles gets large. Indeed, while most parametric distributions may be fully characterized with two or three parameters only, nonparametric distributions will require a minimum of $Q = 18$–20 quantiles to provide a satisfactory description. This was, for instance, the case of the representation of probabilistic forecasts in Fig. 2.5, where 18 quantiles were used for describing distributions of wind power generation for every lead time, i.e., with $\alpha_i \in \{0.05, 0.1, \ldots, 0.45, 0.55, \ldots, 0.9, 0.95\}$.

Example 2.20 Similar to the parametric approach in the above, consider the issue of probabilistic forecasting of wind power generation for a given portfolio and lead time k. Models for the quantiles $q_{t+k}^{(\alpha_i)}$ may actually be formulated in a manner similar to that for μ_{t+k} in the parametric case, that is,

$$q_{t+k}^{(\alpha_i)} = g_{\alpha_i}(\hat{u}_{t+k|t}, \hat{\omega}_{t+k|t}) + \varepsilon_{i,t+k}, \quad i = 1, \ldots, Q, \tag{2.47}$$

with $\varepsilon_{i,t+k}$ a centered noise term. Such models will also have some shape similar to that of the power curve for the wind portfolio, except that it will reflect given thresholds in terms of potential outcomes of the stochastic process.

2.5.3 Extracting Single-Valued Forecasts

Because probabilistic forecasts may be too difficult to handle and interpret by a number of forecast users, it is often the case that single-valued forecasts are to be extracted from probabilistic density forecasts. Note, however, that they could also be directly generated if aiming to skip the step of producing full density forecasts of renewable power generation. This is the case if directly predicting the conditional expectation of renewable power generation (as in Eq. (2.40)), or if modeling and predicting quantiles for a specific nominal level (as in Eq. (2.47)).

Indeed, two types of single-valued forecasts were introduced in Sect. 2.3: those corresponding to the conditional expectation of the stochastic process for every lead time, and quantile forecasts that inform on a probabilistic threshold (for a given nominal level) for the range of outcomes to be expected. These various definitions of single-valued forecasts are to be linked to the so-called *loss function* of forecast users, already mentioned a few times in previous sections, which will then permit to extract single-valued predictions from density forecasts. Formally, a loss function $L(\hat{y}, y)$ gives the perceived loss of the forecast user if provided with the forecast $\hat{y}$ while the value y then materializes.

A special relevant case of a loss function is the quadratic one,

$$L_2(\hat{y}, y) = (y - \hat{y})^2. \tag{2.48}$$

At time t a point forecast $\hat{y}_{t+k|t}$ for time $t + k$ is the value of the process such that it minimizes the expected loss for the forecast user for all potential realizations of the process, given our state of knowledge at that time. This translates to

$$\hat{y}_{t+k|t} = \arg\min_{\hat{y}} \mathbb{E}[L_2(\hat{y}, y) \mid y \sim F_{t+k}, g, \hat{\Theta}_t]. \tag{2.49}$$

The value that is optimal in terms of this problem is that given in Eq. (2.4), that is, the conditional expectation of Y_{t+k}.

Quantile forecasts are related to another type of loss function $L(\hat{y}, y)$ to be assumed for the forecast user. This loss function is the piecewise linear one, sometimes nicknamed as the “pinball” loss,

$$L_\alpha(\hat{y}, y) = \begin{cases} \alpha|\hat{y} - y|, & \text{if } y \le \hat{y}, \\ (1-\alpha)|\hat{y} - y|, & \text{otherwise.} \end{cases} \tag{2.50}$$

Similar to the case of Eq. (2.49), quantile forecast can be defined as the solution of an optimization problem in a decision-theoretic framework,

$$\hat{q}^{(\alpha)}_{t+k|t} = \arg\min_{\hat{y}} \mathbb{E}[L_\alpha(\hat{y}, y) \mid y \sim F_{t+k}, g, \hat{\Theta}_t], \tag{2.51}$$

i.e., as the quantity minimizing the expected pinball loss for any potential realization of Y_{t+k} with cumulative distribution function F_{t+k}.

Example 2.21 (Extracting Single-Valued Forecasts from Predictive Densities) Let us build on Example 2.18, where predictive densities were issued based on Gaussian and

Beta parametric assumptions with the same mean and variance parameters ($\mu_{t+k} = 0.25$ and $\sigma^2_{t+k} = 0.02$).

Consider here two types of loss functions, based on which single-valued forecasts should be extracted from these predictive densities. These loss functions are the quadratic one introduced in Eq. (2.48), and the piecewise linear one of Eq. (2.50), with $\alpha = 0.5$.

In the case of the quadratic loss function, the optimal single-valued prediction to be extracted is given by optimization problem (2.49). It yields $\hat{y}_{t+k|t} = 0.25$ for both distributions (i.e., their expected value). Looking at the piecewise linear case instead, solving optimization problem (2.51) gives different single-valued forecasts for the two predictive densities: $\hat{y}_{t+k|t} = 0.25$ for the Gaussian distribution and $\hat{y}_{t+k|t} = 0.23$ for the Beta distribution. These single-valued forecasts correspond to their median. They are not the same owing to the different shape of these predictive densities, as can be seen from Fig. 2.11.

2.5.4 Issuing Scenarios

Scenarios of renewable power generation comprise the most important type of forecast input to operational problems when it comes to renewable energy in electricity markets. They ought to represent both

(i) the *marginal predictive densities* of power generation for each lead time, location and renewable energy source individually, and
(ii) the *interdependence structure* in power generation through time, space and type of renewable energy sources.

The central ideas about the generation of scenarios of renewable power generation are detailed below, with considerations related to building joint predictive densities from marginal ones, to the modeling of interdependence structures, and finally to the generation of scenarios themselves.

2.5.4.1 From Marginal to Joint Predictive Densities

In the most general set-up, let us assume that at any given time t, one aims at predicting the full set of characteristics of the multivariate random variable

$$Z_t = \{Y_{r,s,t+k},\ r = r_1, \ldots, r_m,\ s = s_1, \ldots, s_n,\ k = 1, \ldots, K\}. \tag{2.52}$$

This multivariate random variable jointly considers m types of renewable energy sources, $r = r_1, \ldots, r_m$, a set of locations $s = s_1, \ldots, s_n$, as well as the set of lead times $k = 1, \ldots, K$.

Based on the concepts presented before, predictive densities of renewable power generation are available for the random variables $Y_{r,s,t+k}$. That for the i^{th} type of renewable energy source r_i, the j^{th} location s_j and the k^{th} lead time is denoted $\hat{F}_{i,j,t+k|t}$.

The overall set of predictive densities is consequently written $\{\hat{F}_{i,j,t+k|t}\}$. These densities are referred to as *marginal* since being issued for each lead time, location, and energy source, individually, hence not informing about the interdependence structure of Z_t.

In order to overcome this lack of information about such interdependence, the set of marginal predictive densities may be augmented so as to become a joint predictive density.

Definition 2.24 (Joint Predictive Density). A joint predictive density $\hat{F}_{Z_t}$ issued at time t for m types of renewable energy sources $r = r_1, \dots, r_m$, a set of locations $s = s_1, \dots, s_n$ and a set of look-ahead times $k = 1, \dots, K$, can be obtained by complementing the set of marginal predictive densities $\{\hat{F}_{i,j,t+k|t}\}$ with a *copula* model,

$$\hat{F}_{Z_t} = \left\{\{\hat{F}_{i,j,t+k|t}\}, C(\delta r, \delta s, \delta k)\right\}, \tag{2.53}$$

where the copula model $C(\delta r, \delta s, \delta k)$ gives the whole information about the interdependence among sources, locations and lead times. δr, δs and δk are here to denote variations in these variables.

An exhaustive introduction to copulas and copula-based modeling is available in [13]. The above definition follows from the probabilistic calibration property of probabilistic forecasts introduced in Sect. 2.4.3. Indeed, in the case where every marginal predictive density $\hat{F}_{i,j,t+k|t}$ is calibrated,

$$U_{i,j,t+k|t} = \hat{F}_{i,j,t+k|t}{}^{-1}(Y_{i,j,t+k}), \quad U_{i,j,t+k|t} \sim U[0,1]. \tag{2.54}$$

One can then introduce at time t the multivariate random variable U_t with marginal distributions $U_{i,j,t+k|t}$, $\forall i, j, k$. The so-called copula function permits to define the joint cumulative distribution of U_t. Example contributions of joint predictive densities based on a Gaussian copula and various different forms of covariance models are given in the following.

2.5.4.2 Gaussian Copula and Alternative Covariance Models

Applying a Gaussian Copula

The simplest and most convenient copula to use is the Gaussian one. Its basis consists in transforming each uniform random variable $U_{i,j,t+k|t}$ to standard Gaussian with

$$\Phi^{-1}\left(U_{i,j,t+k|t}\right) \sim \mathcal{N}(0,1), \tag{2.55}$$

where Φ^{-1} is the inverse of the cumulative distribution function for a standard Gaussian distribution.

Consequently, the copula model $C(\delta l, \delta s, \delta k)$ permitting to summarize the whole interdependence structure reduces to a covariance structure. It can be similarly referred to as correlation structure, since the Gaussian variables involved have unit variance. In addition, considering that this covariance structure is separable, the multidimensional (and potentially complex) covariance $C(\delta r, \delta s, \delta k)$ reduces to

$$C(\delta r, \delta s, \delta k) = C_r(\delta r) C_s(\delta s) C_k(\delta k), \tag{2.56}$$

that is, as the direct product of three one-dimensional covariance functions.

Example 2.22 (The Wind-Wave Offshore Installation) One may look at a wind-wave offshore installation, for which multivariate scenarios are to be issued. For simplicity, only one lead time (say, 24 h ahead) is considered. Predictive densities of wind and wave power generation are produced for that lead time. Employing a Gaussian copula, the interdependence structure in Eq. (2.56), for wind and wave power, reduces to $C_r(\delta_r)$, with

$$C_r(\delta_r) = \rho(r_1, r_2), \tag{2.57}$$

which is a single correlation coefficient describing the interdependence between wind (r_1) and wave (r_2) power generation, at that location and for that lead time.

Example 2.23 (Spatio-Temporal Dependencies in Wind Power Generation) A more advanced case is that of spatio-temporal dependencies of wind power generation (following the setup of Sect. 2.2 and the test case of Western Denmark) for which multivariate scenarios may be required as input to problems for optimal power system operations. Predictive densities of wind power generation are available for all 15 control zones and for lead times between 1 and 43 h ahead. Based on Eq. (2.56), the interdependence structure is summarized as

$$C(\delta s, \delta k) = C_s(\delta s) C_k(\delta k), \tag{2.58}$$

for which the spatial and temporal dependence structures have to be described individually. This is dealt with in the following.

Covariance Modeling

Different strategies exist for the modeling and estimation of the covariance functions involved. In the case where the dimension is low, as for the offshore wind-wave energy setup of Example 2.22, it may be easier to summarize all information with a single correlation matrix $\Sigma = \{\rho_{ij}\}$, where each and every element ρ_{ij} may indifferently inform of dependence among locations, lead times and types of renewable energy sources.

Example 2.24 (Interdependence Among n-Locations) Let us focus on one single form of renewable energy generation, say, wind energy, for a given lead time (24 h ahead), at n locations $s = s_1, s_2, \dots, s_n$. Predictive densities of wind power generation are produced for that lead time and these n locations. The whole spatial covariance structure $C_s(\delta_s)$ can be summarized in the form of a correlation matrix $\Sigma = \{\rho_{ij}\}$ of dimension $n \times n$, where

$$\rho_{ij} = \rho(s_i, s_j), \quad i, j \in \{1, 2, \dots, n\}, \tag{2.59}$$

that is, by the value of the correlation coefficient for each pairwise association of sites.

When the dimension of the problem gets high, it may not be possible to handle the full description of the interdependence structure by evaluating all correlation values in Σ. It may then be beneficial to parametrize such a dependence with covariance functions.

Example 2.25 (Spatio-Temporal Dependencies in Wind Power Generation (Continued)) For the case of spatio-temporal dependencies of wind power generation (as a continuation of Example 2.23 with 15 control zones and 43 lead times, a correlation matrix allowing to summarize all interdependencies would have a dimension of 645×645 (i.e., 15 times 43). It would translate to 416,025 correlation values to be calculated in order to fully characterize the interdependence structure. One may propose covariance functions instead, for both spatial and temporal dimensions. For instance, assuming an exponential decay in time and in space, thus considering a rapidly decreasing interdependence with increasing δs and δk, this yields

$$C_s(\delta s) = \exp\left(\delta s / \nu_s\right), \tag{2.60}$$

and

$$C_k(\delta k) = \exp\left(\delta k / \nu_k\right), \tag{2.61}$$

where ν_s and ν_k are range parameters in space and in time, controlling the speed of the exponential decay. The model hence rely on two parameters only, which is considerably less than the original 416,025 correlations that should have been calculated.

Following this parametric covariance modeling, a covariance matrix may then be derived, if necessary, for the subsequent generation of scenarios. This is performed by evaluating the correlation values in Σ for all associations of locations and lead times.

The parameters of the covariance models can be estimated in varied statistical frameworks, for instance, with exponential smoothing for the tracking of the correlation matrix Σ, or with weighted least squares or maximum likelihood techniques for the parameters of covariance functions (for more details, see e.g., [14]).

2.5.4.3 Generation of Scenarios

Based on the above, one has at a given time t joint probabilistic forecasts $\hat{F}_{Z_t}$ covering all potential locations, lead times and types of renewable energy sources. Following Eq. (2.53), these are formed by marginal predictive densities $\{\hat{F}_{i,j,t+k|t}\}$ and by a copula model simplifying the covariance structure $C(\delta r, \delta s, \delta k)$.

For issuing a set of J scenarios at time t, one first needs to sample a number J of realizations $z^{(j)}$ from a multivariate Gaussian random variable Z, with mean 0 and covariance matrix Σ as estimated directly or derived from parametric modeling based on covariance functions,

$$Z \sim \mathcal{N}(0,\ \Sigma). \tag{2.62}$$

Using the cumulative distribution function Φ for a standard Gaussian random variable, as well as the predictive cumulative distribution functions $\{\hat{F}_{i,j,t+k|t}\}$ for every lead time, location and renewable energy source, these multivariate Gaussian realizations are transformed into trajectories of wind power generation having the appropriate marginal distributions,

$$\hat{z}_t^{(j)} = \hat{F}_{i,j,t+k|t}{}^{-1}\left(\Phi(z^{(j)})\right), \quad j = 1, \ldots, J. \tag{2.63}$$

This is illustrated below for the example of the Western Denmark dataset, already used in some of the previous examples.

Example 2.26 (Space–Time Trajectories for Western Denmark) We consider again the dataset with wind power generation at the 15 control zones of the Energinet.dk control area in Western Denmark, for which an episode with two days of wind power generation was depicted in Fig. 2.1.

Based on input meteorological forecasts and on statistical methods, nonparametric probabilistic forecasts of wind power production are generated every 6 h, with a forecast length of 43 h. In parallel, a parametric space–time covariance structure is chosen, being separable and with exponential decay in time and in space. The range parameters for these exponential covariance models in space and in time are of 125 km and 7 h, respectively. For comparison, the maximum distance between the centroid of the control zones is of 250 km, between zones 1 and 15. By combining the nonparametric probabilistic forecasts and the covariance structure, scenarios of wind power generation can be issued.

As an illustration, Fig. 2.12 gathers a set of 12 spatio-temporal scenarios for this dataset, issued on 15th January 2007 at 00:00 UTC and normalized by the nominal capacity of every control zone.

2.6 Summary and Conclusions

This chapter presents the most common inputs to operational problems related to renewable energy sources in electricity markets. It therefore places particular emphasis on various forms of forecasts, since markets are cleared a fair amount of time before actual operation.

These forecasts are introduced in their various forms, while insisting on the necessity to appraise them in a probabilistic framework. Indeed, renewable energy forecasts will always have a non-negligible share of uncertainty. Such uncertainty is, at the very least, contingent on external conditions (most generally, the weather conditions) and to the time-varying state of the energy conversion systems. The type

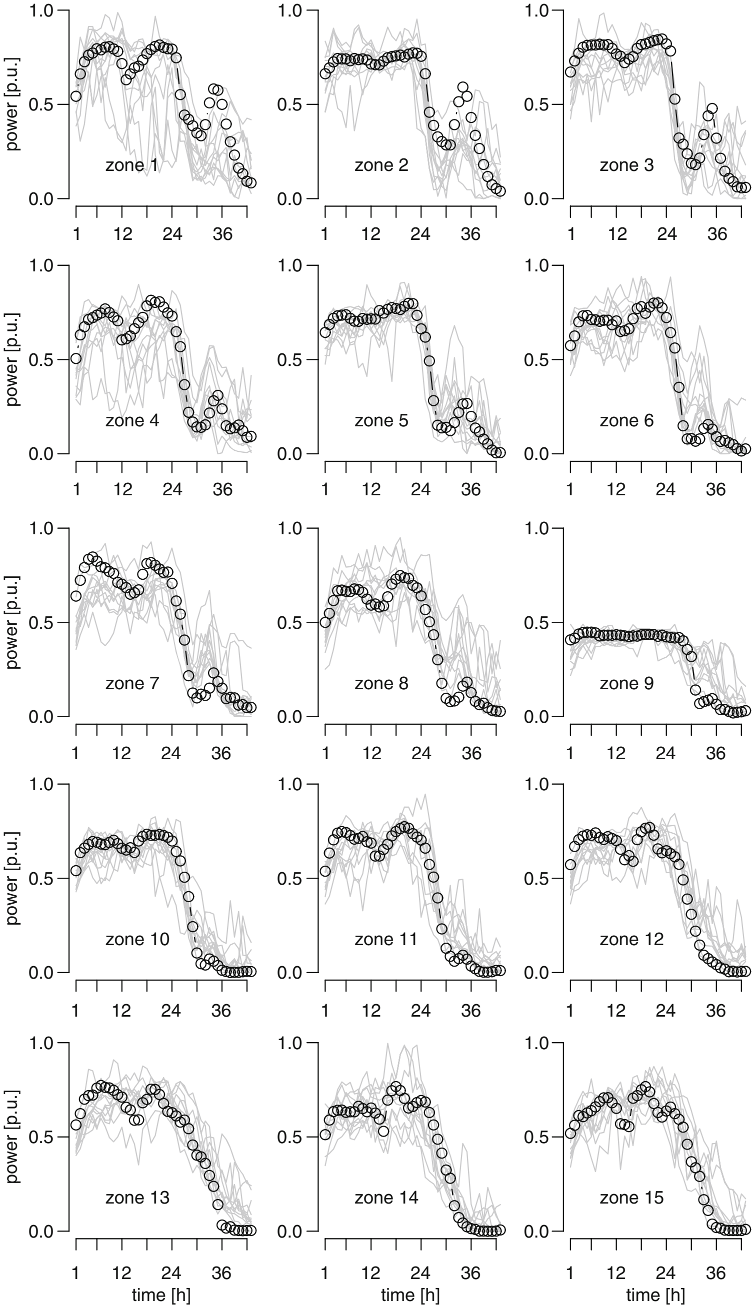

Fig. 2.12 Scenarios with 43-hour ahead wind power generation at the 15 control zones of Western Denmark issued on the 15th January 2007 at 00:00 UTC, normalized by nominal capacity. These are based on input nonparametric probabilistic forecasts and on a parametric separable space–time covariance structure with exponential decay. The decay parameters in space and in time are of 125 km and 7 h, respectively. Observations, obtained a posteriori, are also shown

of forecasts to be used as input to operational problems will depend upon the nature of the problem itself, and on the way the corresponding optimization problem is formulated. Not using the appropriate type of forecasts in operational problems may certainly lead to suboptimal decisions and policies, even though these may be more transparent and easier to understand.

Basics of forecast verification were covered, in order for the reader to acquire background knowledge on forecast quality. It is intuitively expected that higher-quality forecasts will yield better policies and decisions. Evaluating the quality of probabilistic forecasts is, however, a difficult task. A complete coverage would require some further reading, suggested below. Similarly, when aiming at building appropriate models for generating these forecasts, extensive literature exists. Only main principles were discussed in this chapter.

Nowadays, the renewable energy forecasting field is extremely active and dynamic. It is hence expected that new forecasting methods and products will be proposed and used in operational problems related to electricity markets in the coming years.

2.7 Further Reading

This chapter aimed at giving a compact overview of renewable power forecasting, laying down some of the main concepts and key characteristics of the various types of forecasts to be used as input to operational problems. For those interested in a more extensive coverage of renewable power forecasting, we refer to [7] for the case of wind energy, mainly focusing on a physical approach to the problem, hence overlooking some of the statistical aspects of relevance. In parallel, for the case of solar energy, a good overview can be obtained from [1] and [8]. Finally, some relevant approaches and discussion on forecasting for wave energy can be found in [17] and [18]. Advanced works on the generation of scenarios of renewable power production include those in [12] and [15].

Readers who are interested in learning more about the question of forecast quality, and about how to thoroughly evaluate prediction before to use them in operational problems, may find extensive information in [9] for point predictions, in [3] and [16] for probabilistic forecasts, and lastly, in [14] for scenarios of renewable power generation. More generally, [4] and [19] give a good overview on aspects of multivariate probabilistic forecast evaluation. Finally, a discussion on point forecasts and how to optimally extract them from probabilistic predictions is given in [2].

Exercises

2.1. Quantile forecasts with nominal level $\alpha = 0.3$ are issued at time t for the following 6 h, for power generation at a solar power plant with a nominal capacity of 12 MW:

$$\hat{q}^{(0.3)}_{t+1|t} = 2\text{MW}, \;\; \hat{q}^{(0.3)}_{t+2|t} = 9\text{MW}, \;\; \hat{q}^{(0.3)}_{t+3|t} = 11\text{MW}, \;\; \hat{q}^{(0.3)}_{t+4|t} = 10\text{MW}.$$

What is the predicted probability that solar power generation will be above 9 MW at time $t+2$?

2.2. A number of 1-hour ahead point forecasts are issued for power generation at this same solar power plant (in MW):

$$\{2,\ 3.5,\ 4.2,\ 5.6,\ 7.4,\ 5.6,\ 6.4,\ 5.3,\ 6.7,\ 8.6,\ 9.3,\ 4.7\}. \tag{2.64}$$

The corresponding observations are obtained a posteriori:

$$\{1.8,\ 3.9,\ 4,\ 5.1,\ 7.2,\ 6.1,\ 6.7,\ 5.9,\ 6.6,\ 8.3,\ 10.5,\ 6.2\}. \tag{2.65}$$

Evaluate these point forecasts with the common scores introduced in this chapter, i.e., bias, MAE, and RMSE, calculated both in MW and in their normalized version, i.e., as percentage of installed capacity.

2.3. A nonparametric predictive density for power generation at a wind farm of 35 MW is issued, at time t for time $t+k$, based on a set of quantile forecasts with increasing nominal levels, i.e.,

$$\begin{aligned}\hat{q}^{(0.05)}_{t+k|t} &= 6\text{MW},\ \hat{q}^{(0.25)}_{t+k|t} = 9\text{MW},\ \hat{q}^{(0.5)}_{t+k|t} = 11\text{MW},\ \hat{q}^{(0.75)}_{t+k|t} \\ &= 14\text{MW},\ \hat{q}^{(0.95)}_{t+k|t} = 17\text{MW}.\end{aligned}$$

Based on these forecasts, define central prediction intervals with nominal coverage rates of 50 % and 90 % for that lead time.

2.4. A parametric predictive density for the normalized power generation at a wind farm is issued at time t for time $t+k$ in the form of a Beta distribution such that

$$Y_{t+k} \sim \text{Beta}(6.28, 2.09).$$

Deduce from that predictive density a number of point and interval forecasts:

1. the point forecast that would be extracted under a quadratic loss function,
2. the point forecast that would be extracted under a pinball loss function, with $\alpha = 0.4$,
3. the central prediction interval with nominal coverage rate $(1-\beta) = 0.8$.

Finally, what is the (predicted) probability that the normalized power generation will be greater than 25 % of nominal capacity?

2.5. Two forecasters generate at time t different parametric predictive densities of normalized wind power generation for time $t+k$. The first one is

$$Y_{t+k} \sim \text{Beta}(2, 5),$$

while the second one is

$$Y_{t+k} \sim \mathcal{N}(0.4, 0.01).$$

Which of the two forecasters predict the highest probability that power generation will be greater than 50 % of nominal capacity at time $t + k$?

2.6. Parametric predictive densities for the normalized power generation at a wind farm are issued at time t for time $t + 1$ and $t + 2$, based on a Beta distribution assumption, with

$$Y_{t+1} \sim \text{Beta}(2, 5), \quad Y_{t+2} \sim \text{Beta}(5, 4).$$

In addition, the interdependence structure between the two lead times is modeled with a Gaussian copula with a covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}.$$

Based on these predictive densities and this interdependence structure, issue 100 scenarios of wind power generation and evaluate them with the energy score, given that the normalized power observations for times $t + 1$ and $t + 2$ are $y_{t+1} = 0.3$ and $y_{t+2} = 0.38$.

References

1. Bacher, P., Madsen, H., Nielsen, H.A.: Online short-term solar power forecasting. Sol. Energy **83**(10), 1772–1783 (2009)
2. Gneiting, T.: Quantiles as optimal point forecasts. Int. J. Forecasting **27**(2), 197–207 (2011)
3. Gneiting, T., Balabdaoui, F., Raftery, A.E.: Probabilistic forecasts, calibration and sharpness. J. Royal Stat. Soc. B **69**(2), 243–268 (2007)
4. Gneiting, T., Stanberry, L.I., Grimit, E.P., Held, L., Johnson, N.A.: Assessing probabilistic forecasts of multivariate quantities, with an application to ensemble predictions of surface winds. Test **17**(2), 211–235 (2008)
5. Hyndman, R.J.: Highest-density forecast regions for nonlinear and non-normal time series models. J. Forecasting **14**(5), 431–441 (1995)
6. Jolliffe, I.T., Stephenson, D.B.: Forecast Verification - A Practitioner's Guide in Atmospheric Science, 2nd edn. Wiley, New York (2012)
7. Lange, M., Focken, U.: Physical Approach to Short-term Wind Power Prediction. Springer, Berlin (2005)
8. Lorenz, E., Hurka, J., Heinemann, D., Beyer, H.G.: Irradiance forecasting for the power prediction of grid-connected photovoltaic systems. IEEE J. Sel. Top. Appl. Earth Observ. Remote Sens. **2**(1), 2–10 (2009)
9. Madsen, H., Pinson, P., Kariniotakis, G., Nielsen, H.A., Nielsen, T.S.: Standardizing the performance evaluation of short-term wind power prediction models. Wind Eng. **29**(6), 475–489 (2005)
10. Madsen, H., Thyregod, P.: Introduction to General and Generalized Linear Models. Chapman & Hall/CRC, Boca Raton (2011)
11. Matos, M.A., Bessa, R.J.: Setting the operating reserve using probabilistic wind power forecasts. IEEE T. Power Syst. **26**(2), 594–603 (2011)
12. Morales, J.M., Minguez, R., Conejo, A.J.: A methodology to generate statistically dependent wind speed scenarios. Appl. Energy **87**(3), 843–855 (2010)
13. Nelsen, R.B.: An Introduction to Copulas. Springer, New York (1999)

14. Pinson, P., Girard, R.: Evaluating the quality of scenarios of short-term wind power generation. Appl. Energy **96**, 12–20 (2012)
15. Pinson, P., Madsen, H., Nielsen, H.A., Papaefthymiou, G., Klöckl, B.: From probabilistic forecasts to statistical scenarios of short-term wind power production. Wind Energy **12**(1), 51–62 (2009)
16. Pinson, P., Nielsen, H.A., Møller, J.K., Madsen, H., Kariniotakis, G.N.: Non-parametric probabilistic forecasts of wind power: Required properties and evaluation. Wind Energy **10**(6), 497–516 (2007)
17. Pinson, P., Reikard, G., Bidlot, J.R.: Probabilistic forecasting of the wave energy flux. Appl. Energy **93**, 364–370 (2012)
18. Reikard, G., Pinson, P., Bidlot, J.R.: Forecasting ocean wave energy: The ECMWF wave model and time series methods. Ocean Eng. **38**(10), 1089–1099 (2011)
19. Wilks, D.S.: The minimum spanning tree histogram as a verification tool for multidimensional ensemble forecasts. Mon. Weather Rev. **132**(6), 1329–1340 (2004)

Chapter 3
Clearing the Day-Ahead Market with a High Penetration of Stochastic Production

3.1 Electricity Markets: Day-Ahead Market

Electricity markets are trading floors that allow electricity producers, on the one hand, and electricity consumers and retailers, on the other hand, to trade electricity.

Two trading floor categories are available depending on the immediacy of the trading: long-term trading (from 1 week to 1 year ahead of energy delivery), which takes place through *futures markets* and via private *bilateral contracts*; and short-term trading (from several minutes to 1 day ahead of energy delivery), which takes place through the *electricity pool*.

Futures markets allow the arrangement of long-term electricity trading through forward contracts and options.

A forward contract involves the trading of a prespecified amount of power during a future time period, e.g., 10 MW during the next week. A forward contract involves a seller that produces the energy sold and a buyer that consumes such energy.

An option associated with a forward contract provides the buyer of the option with the possibility of deciding at some future time whether to implement the forward contract. Buying an option involves a payment from the buyer of the option.

For instance, a producer may buy an option to sell 10 MW during 3 days in 2-week time to be decided in 1-week time, and pays for such an option a fee to the consumer willing to provide the buying flexibility required by the option.

Conversely, a consumer may acquire an option to buy 10 MW during 3 days in 2-week time to be decided in 1-week time, and pays for such an option a fee to the producer willing to provide the selling flexibility required by the option.

The electricity pool allows short-term trading and generally involves two trading arrangements, the day-ahead market and the balancing market.

The day-ahead market takes place the day prior to energy delivery, typically around noon. Producers submit to this market production offers (consisting of production quantities and minimum selling prices), while consumers and retailers submit consumption bids (consisting of consumption quantities and maximum buying prices). In turn, the market operator clears the market using a market-clearing tool that is generally an auction. This auction results in scheduled production and consumption levels and day-ahead market-clearing prices.

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International Series in Operations Research & Management Science 205,
DOI 10.1007/978-1-4614-9411-9_3,

The balancing market takes place several minutes before energy delivery and constitutes the last market mechanism to balance production and consumption. This market is particularly relevant for stochastic producers (e.g., wind and solar power producers) that cannot accurately predict their production levels prior to the closing of the day-ahead market. The balancing market is cleared by the market operator in a similar fashion as the day-ahead market through an auction. Its outcome involves production and consumption adjustments and balancing clearing prices.

Some electricity pools include intermediate market arrangements between the day-ahead and the balancing markets, intended to further hedge against uncertainty and to allow corrective actions in response to unexpected events and errors by market agents. These trading arrangements are generally called adjustment or intra-day markets.

This chapter focuses on the day-ahead market and provides clearing models particularly suited for markets including a significant number of stochastic producers. More specifically, the remainder of this chapter is structured as follows. Section 3.2 first introduces the concept of reserve capacity as a market commodity to cope with uncertainty in power systems, then briefly describes basic models for the dispatch of energy and reserve in electricity markets, and finally presents the market-clearing mechanism based on two-stage stochastic programming with recourse. Subsequently, Sect. 3.3 uses this two-stage stochastic programming model to derive consistent clearing prices for an energy-only market settlement. Alternatively to two-stage stochastic programming, Sect. 3.4 introduces a dispatch method for energy and reserve capacity that is built on adaptive robust optimization. Section 3.5 summarizes the chapter, and Sect. 3.6 provides a collection of selected readings on the topic. Finally, proofs related to some properties of the resulting stochastic and robust market-clearing procedures are provided in appendices at the end of the chapter.

It is worth mentioning that this chapter places emphasis on advanced methods for clearing electricity markets with a high penetration of stochastic generation. It is therefore highly recommended for the reader unfamiliar with the functioning of these markets to first learn the basics from more introductory manuals such as [6] and [16].

3.2 Clearing the Day-Ahead Market Under Uncertainty

Electricity markets for short-term energy transactions usually comprise, at least, two different trading stages in the form of a day-ahead energy exchange and a balancing market. The day-ahead energy exchange takes place 1 day in advance and settles contracts for the delivery of energy on an hourly basis. The balancing market serves to competitively settle the energy adjustments required to ensure the constant balance between electricity supply and demand.

The coexistence of both markets is well-justified. On the one hand, the day-ahead market is useful for those power plants that need advance planning in order to efficiently and reliably adjust their production levels. If major changes in the overall supply were left to be driven by the balancing market, some generating units would be limited or just unable to respond to market signals. On the other hand, if market participants were able to perfectly predict with enough lead time the amount of energy that they will produce or consume, there would be no need for taking balancing actions. However, there are always imbalances in practice, especially in power systems with a high penetration of stochastic production. The balancing market constitutes thus a competitive mechanism to efficiently cope with these energy imbalances by allowing flexible firms to adjust their day-ahead positions. No doubt that the balancing market is, therefore, of primary importance for stochastic producers given the limited predictability of their power production.

This chapter focuses on the day-ahead market, while Chap. 4 focuses on the balancing market.

3.2.1 Cooptimizing Energy and Reserve Capacity

In order to ensure that enough balancing resources are available during the real-time operation of the power system, the system operator allocates *reserve capacity* in advance. In practice, the procurement and *scheduling* of reserve capacity implies operating the system at less than its full capacity, while its use or *deployment* usually translates into the redispatch of units previously committed in the day-ahead market, the voluntary curtailment of loads, and/or the quick start-up of extra power plants to cover unexpected shortages of energy supply in real time.

There exist two schools of thought on how reserve should be traded in electricity markets. On the one hand, reserve capacity may be *sequentially* procured in a series of auctions run once the day-ahead energy dispatch has been determined. These auctions are organized to procure reserves with different activation times. The rationale behind this approach is that the free capacity that has not been successfully placed in one market can then be offered in the following auctions where the required activation time for the traded reserve is not as demanding. Consequently, reserve capacity offers that are successful in one market are not considered in the subsequent ones.

On the other hand, energy and reserve may be *simultaneously* procured in the same auction using a co-optimization algorithm that captures the strong coupling between the supply of energy and the provision of reserve capacity. The following illustrative example serves to get a more intuitive understanding of this coupling.

Example 3.1 (Cooptimization of Energy and Reserve) Consider an electricity market that solely includes two power producers, A and B. Each of these producers runs a power plant with a capacity of 100 MW. Producer A offers to sell energy at \$10/MWh, while producer B does it at \$30/MWh. A demand of 130 MWh is to be supplied.

Additionally, with the aim of dealing with unforseen events, the system operator estimates that 20 MW of reserve capacity are required. Producer A is willing to provide reserve at no cost, whereas producer B offers reserve capacity at \$25/MW.

To start with, let us suppose that energy and reserve capacity are *sequentially* settled in this order. Thus, the energy-only dispatch is first determined as follows:

$$\text{Min. } 10P_{\text{A}} + 30P_{\text{B}} \tag{3.1a}$$

$$\text{s.t. } P_{\text{A}} + P_{\text{B}} = 130, \tag{3.1b}$$

$$0 \leq P_{\text{A}} \leq 100, \tag{3.1c}$$

$$0 \leq P_{\text{B}} \leq 100, \tag{3.1d}$$

where P_{A} and P_{B} are the amounts of energy sold by producers A and B, respectively. Optimization problem (3.1) is trivial, and its solution is given by $P_{\text{A}}^* = 100$ MWh and $P_{\text{B}}^* = 30$ MWh. The clearing (marginal) price for energy, which is defined as the dual variable of constraint (3.1b), results in \$30/MWh.

Once the energy dispatch is determined, the reserve capacity market is cleared as follows:

$$\text{Min. } 0R_{\text{A}} + 25R_{\text{B}} \tag{3.2a}$$

$$\text{s.t. } R_{\text{A}} + R_{\text{B}} = 20, \tag{3.2b}$$

$$0 \leq R_{\text{A}} \leq 100 - P_{\text{A}}^*, \tag{3.2c}$$

$$0 \leq R_{\text{B}} \leq 100 - P_{\text{B}}^*, \tag{3.2d}$$

where R_{A} and R_{B} are the amounts of reserve capacity sold by producers A and B, respectively. Note that the reserve scheduling takes the energy dispatch $\left\{P_{\text{A}}^*, P_{\text{B}}^*\right\}$ as input. The solution to problem (3.2) is also trivial and is given by $R_{\text{A}}^* = 0$ and $R_{\text{B}}^* = 20$MW. That is, since producer A has been dispatched at full capacity in the energy market, reserve needs are entirely covered by producer B. Thus, the total system operation costs TC^{seq}, including both the procurement costs of energy and reserve capacity, are computed as

$$\begin{aligned} \text{TC}^{\text{seq}} &= 10P_{\text{A}}^* + 30P_{\text{B}}^* + 0R_{\text{A}}^* + 25R_{\text{B}}^* \\ &= 10 \times 100 + 30 \times 30 + 0 + 25 \times 20 = \$2400. \end{aligned} \tag{3.3}$$

The clearing (marginal) price for reserve capacity is \$25/MW, which is the value taken by the dual variable associated with the reserve requirement constraint (3.2b). Therefore, the profits made by producers A and B, respectively, under the sequential market organization are calculated as follows:

$$\text{Profit}_{\text{A}}^{\text{seq}} = (30 - 10)P_{\text{A}}^* + (25 - 0)R_{\text{A}}^* = 20 \times 100 + 25 \times 0 = \$2000, \tag{3.4a}$$

$$\text{Profit}_{\text{B}}^{\text{seq}} = (30 - 30)P_{\text{B}}^* + (25 - 25)R_{\text{B}}^* = 0. \tag{3.4b}$$

Let us now consider that energy and reserve capacity are *simultaneously* traded in the same auction. To this end, both commodities are jointly dispatched using optimization problem (3.5) below, which minimizes the total system operation costs.

$$\text{Min.}\ 10P_{\text{A}} + 30P_{\text{B}} + 0R_{\text{A}} + 25R_{\text{B}} \tag{3.5a}$$

$$\text{s.t.}\ P_{\text{A}} + P_{\text{B}} = 130, \tag{3.5b}$$

$$R_{\text{A}} + R_{\text{B}} = 20, \tag{3.5c}$$

$$P_{\text{A}} + R_{\text{A}} \leq 100, \quad P_{\text{A}} \geq 0, \quad R_{\text{A}} \geq 0, \tag{3.5d}$$

$$P_{\text{B}} + R_{\text{B}} \leq 100, \quad P_{\text{B}} \geq 0, \quad R_{\text{B}} \geq 0. \tag{3.5e}$$

The solution to this problem is $P_{\text{A}}^* = 80$ MWh, $R_{\text{A}}^* = 20$ MW, $P_{\text{B}}^* = 50$ MWh, and $R_{\text{B}}^* = 0$ MW. The total system operation costs in this case (TC^{sim}) are calculated as

$$\begin{aligned}\text{TC}^{\text{sim}} &= 10P_{\text{A}}^* + 30P_{\text{B}}^* + 0R_{\text{A}}^* + 25R_{\text{B}}^* \\ &= 10 \times 80 + 30 \times 50 + 0 \times 20 + 25 \times 0 = \$2300.\end{aligned} \tag{3.6}$$

Prices for energy and reserve capacity, defined as the dual variables of constraints (3.5b) and (3.5c), respectively, are \$30/MWh and \$20/MW in that order. Therefore, the profits made by producers A and B under the simultaneous market clearing of energy and reserve are given by

$$\text{profit}_{\text{A}}^{\text{sim}} = (30 - 10)P_{\text{A}}^* + (20 - 0)R_{\text{A}}^* = 20 \times 80 + 20 \times 20 = \$2000, \tag{3.7a}$$

$$\text{profit}_{\text{B}}^{\text{sim}} = (30 - 30)P_{\text{B}}^* + (20 - 25)R_{\text{B}}^* = 0 \times 50 - 5 \times 0 = 0, \tag{3.7b}$$

which turn out to be the same as the profits made by both producers in the sequential setup. However, the simultaneous dispatch of energy and reserve captures the coupling existing between these two commodities, thus reducing the total costs by \$100. Actually, in this illustrative example, the interaction between energy and reserve is inferred from the following results:

1. Producer A cannot sell as much energy as it might do otherwise. Indeed, this producer is committed to producing 80 MWh of energy, so that it can provide its spare capacity (20 MW) as reserve. Reserve requirements are thus satisfied.
2. On the contrary, producer B, which runs a more expensive power plant, has to produce more energy in order to meet the electricity demand.
3. The price for reserve capacity in the simultaneous arrangement (\$20/MW) does not correspond to any of the reserve offer costs submitted by the producers. It is, in fact, given by the difference between the marginal energy costs of producer B (\$30/MWh) and A (\$10/MWh). This is so because a 1-MW increase of the reserve needs in constraint (3.5c) is covered by producer A. To this end, this producer must decrease its energy production by 1 MWh, while producer B must increase it by the same amount. This action does not involve any additional reserve cost, but increases the cost of the energy dispatch by \$20.

3.2.2 Reserve Requirements

In developed countries, electricity has become a necessity of everyday life, an asset essential for the functioning of society and economy. Being deprived of electricity may be thus extremely costly and troublesome for many consumers. Therefore, determining the amount of reserve capacity necessary to ensure a secure and efficient balancing operation in real time is paramount. Furthermore, the reserve determination must comply with market principles, i.e., the procurement cost of reserve should match the value it provides to power system users.

When estimating reserve capacity needs, two different approaches, namely *deterministic* or *probabilistic*, can be adopted. The deterministic approach often relies on rule-of-thumb criteria such as procuring enough reserve capacity to cover the loss of the largest generating unit (the so-called $\mathrm{N}-1$ security criterion), or to supply a percentage of the hourly demand, or even a combination of these two. These standards, nevertheless, ignore the stochastic nature of the factors that call for balancing energy, and consequently, reserve requirements are estimated independent of the magnitude of the uncertainties affecting the power system and their impact on system operation costs. On the other hand, in the probabilistic approach, reserve needs are determined based on a probabilistic description of these uncertainties. This approach, therefore, exploits concepts and methods from stochastic process theory, such as those presented in Appendix A of this book, to quantify the optimal amount of reserve capacity to be procured from a market perspective.

A natural way to compute reserve needs using a probabilistic approach is by means of the *expected load not served* (ELNS). The ELNS is a stochastic security metric that represents the average amount of energy not supplied as a result of load-shedding actions. It is cast as a weighted average energy value that accounts for the probability of uncertain factors and the damage that these factors cause to the system in the form of involuntarily curtailed load. Moreover, the ELNS can be expressed linearly, and hence, easily included within a market-clearing problem. Indeed, as illustrated in the example below, the ELNS allows determining reserve requirements *endogenously*, i.e., as a byproduct of the dispatch problem itself.

Example 3.2 (Estimating Reserve Requirements) Consider again the electricity market described in Example 3.1. Recall that this market is a duopoly made up of producers A and B, in which reserve requirements are estimated by the system operator at 20 MW. The reason for this estimate is that the electricity demand may increase from 130 MWh to 150 MWh without prior notice, and the system operator decides to protect the electrical infrastructure against this unexpected growth of consumption by scheduling 20 MW of reserve capacity in advance. The probability of this happening is, though, relatively small, specifically 0.05.

Let us now rethink this problem using a probabilistic approach. For this purpose, note that, in response to a sudden increase of load, three different balancing actions may be taken, namely:

1. Producer A may increase its production from P_{A} to $P_{\text{A}}+r_{\text{A}}$. The energy increase r_{A} is obtained from the reserve capacity R_{A} scheduled beforehand for this producer.
2. Similarly, producer B may increase its production from P_{B} to $P_{\text{B}}+r_{\text{B}}$. The energy increase r_{B} results from deploying the reserve capacity R_{B} dispatched beforehand for this producer.
3. A part of the load increase, L^{shed}, may be simply not supplied. This action, however, entails huge economic losses, which are estimated at \$1000/MWh.

Based on these three possible balancing measures, the energy-reserve dispatch problem can be reformulated as follows:

$$\text{Min. } 10P_{\text{A}} + 30P_{\text{B}} + 0R_{\text{A}} + 25R_{\text{B}} + 0.05\left(10r_{\text{A}} + 30r_{\text{B}} + 1000\,L^{\text{shed}}\right) \tag{3.8a}$$

$$\text{s.t. } r_{\text{A}} + r_{\text{B}} + L^{\text{shed}} = 20, \tag{3.8b}$$

$$r_{\text{A}} \le R_{\text{A}}, \tag{3.8c}$$

$$r_{\text{B}} \le R_{\text{B}}, \tag{3.8d}$$

$$P_{\text{A}} + R_{\text{A}} \le 100, \tag{3.8e}$$

$$P_{\text{B}} + R_{\text{B}} \le 100, \tag{3.8f}$$

$$L^{\text{shed}} \le 20, \tag{3.8g}$$

$$P_{\text{A}},\ P_{\text{B}},\ R_{\text{A}},\ R_{\text{B}},\ r_{\text{A}},\ r_{\text{B}},\ L^{\text{shed}} \ge 0. \tag{3.8h}$$

The solution to this problem is $P_{\text{A}}^{*} = 80$ MWh, $R_{\text{A}}^{*} = 20$ MW, $P_{\text{B}}^{*} = 50$ MWh, $R_{\text{B}}^{*} = 0$ MW, $r_{\text{A}}^{*} = 20$ MWh, $r_{\text{B}}^{*} = 0$, and $L^{\text{shed}*} = 0$. Therefore, the energy and reserve capacity dispatches, i.e., $\left\{P_{\text{A}}^{*}, P_{\text{B}}^{*}\right\}$ and $\left\{R_{\text{A}}^{*}, R_{\text{B}}^{*}\right\}$, respectively, obtained from problem (3.8) are the same as those resulting from problem (3.5) in Example 3.1. This is just pure coincidence. Actually, dispatch models (3.5) and (3.8) are essentially different inasmuch as the following:

1. Market-clearing problem (3.8) takes into account explicitly both the probability of occurrence of the 20-MWh demand increase and its potential impact on system operation costs through the utilization of balancing resources. Indeed, the expression $0.05\left(10r_{\text{A}} + 30r_{\text{B}} + 1000\,L^{\text{shed}}\right)$ in (3.8a) represents the expected cost incurred at the balancing stage. This cost component is, in contrast, ignored in dispatch model (3.5).
2. The reserve dispatch yielded by market-clearing model (3.8) is directly determined based on how valuable this reserve is to consumers by including the cost of the expected load not served in objective function (3.8a), where this cost appears as $0.05 \times 1000 \times L^{\text{shed}}$. For the particular instance solved above, this cost is equal to zero, meaning that consumers are willing to pay for 20 MW of reserve capacity that can be deployed to satisfy a potential consumption increase, if needed. In contrast, if the probability of occurrence of the 20-MWh demand growth is small enough, say 0.005, or the value of lost load is sufficiently low,

e.g., \$100/MWh, no reserve capacity is dispatched, i.e., $\left\{R_{\mathrm{A}}^{*}, R_{\mathrm{B}}^{*}\right\} = \{0, 0\}$, and the whole demand increase is shed instead ($L^{\text{shed}*} = 20$ MWh), if it comes to it.

3. While the 20-MW reserve requirement enters dispatch model (3.5) as an *input* in constraint (3.5c), reserve needs are an *outcome* of market-clearing model (3.8). In fact, there is no reserve requirement constraint in this problem. Instead, we enforce constraint (3.8b), in which all the variables involved, namely r_{A}, r_{B}, and L^{shed}, represent balancing *energy* quantities. But if there is no such reserve requirement constraint, how do we determine the reserve capacity price? We will get to the answer of this question in due time.

3.2.3 A Two-Stage Stochastic Programming Approach

One of the main functions of the system operator is to ensure that enough reserve capacity is scheduled in advance so that a sufficient level of balancing resources are available in real time to cope with system uncertainties. In Example 3.1, we came to the conclusion that system operation costs are minimized if energy and reserve capacity are simultaneously dispatched in the day-ahead market, as the supply of energy and the provision of reserve capacity are complementary services. Subsequently, in Example 3.2, we showed that, by including the expectation of the balancing costs in the objective function of the dispatch problem, reserve needs are determined as a byproduct of the clearing process itself. This way, the amount of reserve capacity that is scheduled matches the value it provides to system users. Besides, the direct connection between reserve and system uncertainties bestows a *probabilistic* sense on this value.

The market-clearing model (3.8) in Example 3.2 is, in fact, a two-stage stochastic programming model, in which the *here-and-now* decisions make up the energy-reserve dispatch and the *wait-and-see* decisions correspond to the real-time operation. The reader is referred to Appendix C for a brief introduction to stochastic programming.

The objective function in (3.8) aims at minimizing the so-called *expected system operation costs*, which include both the cost related to the day-ahead energy-reserve dispatch and the expected cost of the *anticipated* balancing actions to be taken during the real-time operation of the power system. These costs are computed based on the energy and reserve capacity offers submitted by market participants to the day-ahead market.

This objective function is subject to three different sets of constraints, namely, the constraints involving energy and reserve capacity variables in the day-ahead dispatch; the equations constraining the utilization of balancing resources, some of which may involve day-ahead decision variables; and the constraints declaring the non-negative nature of energy- and reserve-related variables.

A generalization of such a market-clearing model is outlined below.

Minimize Day-ahead dispatch cost + Expected balancing cost

subject to
- Day-ahead market constraints:
  - Power balance equations at the day-ahead stage
  - Reserve capacity determination constraints
  - Bounds of reserve and energy offers
- Operation constraints:
  - Power balance equations at the balancing stage
  - Network constraints
  - Deployed reserve determination constraints
- Declarations of non-negative variables

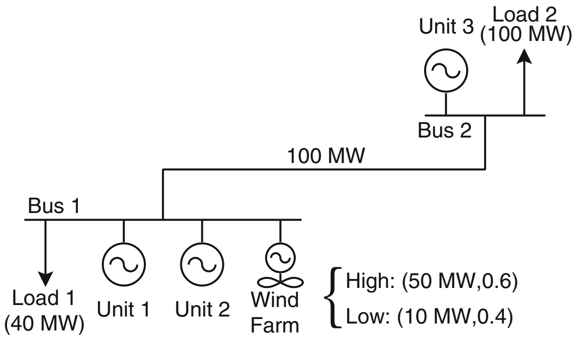
This approach, based on a two-stage stochastic programming model, naturally describes the interaction between the day-ahead and the real-time operation. In particular, the economic performance of the day-ahead energy-reserve dispatch is improved by implicitly accounting for its projected impact on the subsequent balancing costs. This way, enough flexible capacity is made available for balancing to efficiently cope with uncertain factors. The following illustrative example highlights the main features of this approach.

Example 3.3 (A Two-Stage Stochastic Programming Approach) Consider the two-node system in Fig. 3.1.

This small system includes three thermal units, two loads, and a 50-MW wind farm placed at bus 1. The single transmission line in the system has a per-unit reactance of 0.13. For a given time period in the future, the system operator must determine *here-and-now* both the energy dispatch and the reserve capacity needs. Naturally, reserve capacity is required to cope with the uncertain wind power production, which is represented via two scenarios, namely *high* (50 MW) and *low* (10 MW), with probability 0.6 and 0.4, respectively. Specifically, the sequence of decisions that the system operator has to face is as follows:

1. Determine the production levels of thermal units and the quantity and allocation of reserves to deal with the uncertain wind power production.
2. Deploy reserves in the form of balancing energy during the real-time operation of the power system to accommodate the actual realization of wind power production. Four different types of balancing actions can be undertaken for this purpose, namely:
   a) The power output of thermal unit i can be *increased* from P_i to $P_i + r_i^{\mathrm{U}}$, where r_i^{U} is the balancing energy obtained from the *upward* reserve capacity of unit i, denoted as R_i^{U}. This action entails a cost given by $C_i r_i^{\mathrm{U}}$, where C_i is the marginal production cost declared by unit i.
   b) Conversely, the power output of unit i can be *decreased* from P_i to $P_i - r_i^{\mathrm{D}}$, where r_i^{D} is the balancing energy resulting from the deployment of the

Fig. 3.1 Two-bus system



downward reserve capacity of unit i, represented by R_i^{D}. This action implies cost savings of $C_i r_i^{\mathrm{D}}$.

c) A part of the wind power production, W^{spill}, can be curtailed (spilled). This action is cost free, as long as the marginal cost of wind energy production is considered to be zero.

d) A part of the load j, L_j^{shed}, can be also curtailed. This action involves, though, the so-called value of lost load, V_j^{LOL}, which is estimated for this small example at \$200/MWh.

These balancing actions may be taken in either of the two wind power scenarios considered. Subscripts h and l are used to indicate to which scenario, *high* or *low*, respectively, each balancing action refers to. For example, W_h^{spill} is the amount of wind power production that is curtailed in scenario *high*.

The market-clearing process is driven by the minimization of the expected system operation cost, which is made up of the energy-reserve dispatch costs plus the expected cost involved in the balancing actions. These costs are computed from the energy and reserve offers submitted by market agents to the electricity market. In this illustrative example, we assume that each thermal unit offers a single block of energy and up- and down-reserve capacity at prices C, C^{RU}, and C^{RD}, respectively. The value of these offer prices are shown in Table 3.1 together with the maximum power output, $P^{\max}$, of every unit i. These units are assumed to be fully dispatchable between 0 and $P^{\max}$.

Objective Function: The expected system operation cost (EC) is calculated as

$$
\begin{aligned}
\mathrm{EC} = {} & \underbrace{\underbrace{10P_1 + 30P_2 + 35P_3}_{\text{Day-ahead energy costs}} + \underbrace{16R_1^{\mathrm{U}} + 15R_1^{\mathrm{D}} + 13R_2^{\mathrm{U}} + 12R_2^{\mathrm{D}} + 10R^{\mathrm{U}}{}_3 + 9R^{\mathrm{D}}{}_3}_{\text{Reserve capacity costs}}}_{\text{Day-ahead dispatch costs}} \\
& + 0.6\underbrace{\left[10\left(r_{1h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}}\right) + 30\left(r_{2h}^{\mathrm{U}} - r_{2h}^{\mathrm{D}}\right) + 35\left(r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}}\right) + 200\left(L_{1h}^{\mathrm{shed}} + L_{2h}^{\mathrm{shed}}\right)\right]}_{\text{Balancing costs in scenario } \textit{high}} \\
& + 0.4\underbrace{\left[10\left(r_{1l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}}\right) + 30\left(r_{2l}^{\mathrm{U}} - r_{2l}^{\mathrm{D}}\right) + 35\left(r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}}\right) + 200\left(L_{1l}^{\mathrm{shed}} + L_{2l}^{\mathrm{shed}}\right)\right]}_{\text{Balancing costs in scenario } \textit{low}}.
\end{aligned}
$$

Table 3.1 Unit data

Unit i	1	2	3
$P^{\max}$ (MW)	50	110	100
C (\$/MWh)	10	30	35
C^{RU} (\$/MW)	16	13	10
C^{RD} (\$/MW)	15	12	9

Day-Ahead Market Constraints: The day-ahead energy dispatch must satisfy the power balance equations, i.e.,

$$P_1 + P_2 + W^{\mathrm{S}} - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13} \qquad \text{(bus 1)},$$

$$P_3 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13} \qquad \text{(bus 2)},$$

where W^{S} is the amount of wind power production scheduled in the day-ahead market. We define bus 1 as the reference node by setting δ_1^0 to 0. The power flow resulting from the day-ahead energy dispatch must satisfy the transmission capacity limits, i.e.,

$$\frac{(\delta_1^0 - \delta_2^0)}{0.13} \leq 100, \tag{3.9a}$$

$$\frac{(\delta_2^0 - \delta_1^0)}{0.13} \leq 100. \tag{3.9b}$$

Furthermore, energy and reserve capacity are mutually exclusive. Therefore, it holds

$$P_1 + R_1^{\mathrm{U}} \leq 50, \tag{3.10a}$$

$$P_1 - R_1^{\mathrm{D}} \geq 0, \tag{3.10b}$$

$$P_2 + R_2^{\mathrm{U}} \leq 110, \tag{3.10c}$$

$$P_2 - R_2^{\mathrm{D}} \geq 0, \tag{3.10d}$$

$$P_3 + R_3^{\mathrm{U}} \leq 100, \tag{3.10e}$$

$$P_3 - R_3^{\mathrm{D}} \geq 0. \tag{3.10f}$$

Operation Constraints: Now we focus on the balancing market stage. Needless to say, balancing actions must ensure the real-time balance between supply and demand under each possible scenario, i.e.,

$$r_{1h}^{\mathrm{U}} + r_{2h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}} - r_{2h}^{\mathrm{D}} + L_{1h}^{\mathrm{shed}} + 50 - W^{\mathrm{S}} - W_h^{\mathrm{spill}} = \frac{(\delta_{1h} - \delta_1^0 + \delta_2^0 - \delta_{2h})}{0.13} \quad \text{(bus 1)},$$

$$r_{1l}^{\mathrm{U}} + r_{2l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}} - r_{2l}^{\mathrm{D}} + L_{1l}^{\mathrm{shed}} + 10 - W^{\mathrm{S}} - W_{l}^{\mathrm{spill}} = \frac{(\delta_{1l} - \delta_1^0 + \delta_2^0 - \delta_{2l})}{0.13} \quad \text{(bus 1)},$$

$$r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}} + L_{2h}^{\mathrm{shed}} = \frac{(\delta_{2h} - \delta_2^0 + \delta_1^0 - \delta_{1h})}{0.13} \quad \text{(bus 2)},$$

$$r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}} + L_{2l}^{\mathrm{shed}} = \frac{(\delta_{2l} - \delta_2^0 + \delta_1^0 - \delta_{1l})}{0.13} \quad \text{(bus 2)}.$$

We also consider bus 1 as the reference node in the balancing stage by setting $\delta_{1h} = \delta_{1l} = 0$. Due to the implementation of balancing actions, the power flowing between buses 1 and 2 is altered. The new power flow must also satisfy the transmission capacity limits. This is stated as follows:

$$\frac{(\delta_{1h} - \delta_{2h})}{0.13} \leq 100, \qquad \frac{(\delta_{2h} - \delta_{1h})}{0.13} \leq 100 \qquad \text{(scenario } \textit{high}\text{)},$$

$$\frac{(\delta_{1l} - \delta_{2l})}{0.13} \leq 100, \qquad \frac{(\delta_{2l} - \delta_{1l})}{0.13} \leq 100 \qquad \text{(scenario } \textit{low}\text{)}.$$

Clearly, the amount of wind power production that is curtailed under each scenario must be lower than or equal to the actual wind power output, i.e.,

$$W_h^{\mathrm{spill}} \leq 50 \quad \text{(scenario } \textit{high}\text{)},$$

$$W_l^{\mathrm{spill}} \leq 10 \quad \text{(scenario } \textit{low}\text{)}.$$

Similarly, the amount of load that is shed in each scenario has to be lower than or equal to the actual consumption value,

$$L_{1h}^{\mathrm{shed}} \leq 40, \qquad L_{1l}^{\mathrm{shed}} \leq 40 \qquad \text{(load 1)},$$

$$L_{2h}^{\mathrm{shed}} \leq 100, \qquad L_{2l}^{\mathrm{shed}} \leq 100 \qquad \text{(load 2)}.$$

The balancing energy comes from the reserve capacity that has been previously scheduled in the day-ahead market. Consequently, we have

$$r_{1h}^{\mathrm{U}} \leq R_1^{\mathrm{U}}, \quad r_{1h}^{\mathrm{D}} \leq R_1^{\mathrm{D}}, \quad r_{2h}^{\mathrm{U}} \leq R_2^{\mathrm{U}}, \quad r_{2h}^{\mathrm{D}} \leq R_2^{\mathrm{D}}, \quad r_{3h}^{\mathrm{U}} \leq R_3^{\mathrm{U}}, \quad r_{3h}^{\mathrm{D}} \leq R_3^{\mathrm{D}},$$

$$r_{1l}^{\mathrm{U}} \leq R_1^{\mathrm{U}}, \quad r_{1l}^{\mathrm{D}} \leq R_1^{\mathrm{D}}, \quad r_{2l}^{\mathrm{U}} \leq R_2^{\mathrm{U}}, \quad r_{2l}^{\mathrm{D}} \leq R_2^{\mathrm{D}}, \quad r_{3l}^{\mathrm{U}} \leq R_3^{\mathrm{U}}, \quad r_{3l}^{\mathrm{D}} \leq R_3^{\mathrm{D}}.$$

Declarations of Non-Negative Variables: Lastly, reserve, production, and consumption quantities in both the day-ahead and balancing stages must be non-negative,

$$R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}, R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}, P_1, P_2, P_3, W^{\mathrm{S}} \geq 0 \qquad \text{(day-ahead stage)},$$

$$r_{1h}^{\mathrm{U}}, r_{2h}^{\mathrm{U}}, r_{3h}^{\mathrm{U}}, r_{1h}^{\mathrm{D}}, r_{2h}^{\mathrm{D}}, r_{3h}^{\mathrm{D}}, L_{1h}^{\mathrm{shed}}, L_{2h}^{\mathrm{shed}}, W_h^{\mathrm{spill}} \geq 0 \quad \text{(balancing stage, scenario } \textit{high}\text{)},$$

$$r_{1l}^{\mathrm{U}}, r_{2l}^{\mathrm{U}}, r_{3l}^{\mathrm{U}}, r_{1l}^{\mathrm{D}}, r_{2l}^{\mathrm{D}}, r_{3l}^{\mathrm{D}}, L_{1l}^{\mathrm{shed}}, L_{2l}^{\mathrm{shed}}, W_l^{\mathrm{spill}} \geq 0 \qquad \text{(balancing stage, scenario } \textit{low}\text{)}.$$

Complete Model Formulation: We compile below all these constraints to formulate the two-stage stochastic programming problem to be solved.

$$\text{Min.}\ 10\mathrm{P}_1 + 30\mathrm{P}_2 + 35\mathrm{P}_3 + 16\mathrm{R}_1^{\mathrm{U}} + 15\mathrm{R}_1^{\mathrm{D}} + 13\mathrm{R}_2^{\mathrm{U}} + 12\mathrm{R}_2^{\mathrm{D}} + 10\mathrm{R}_3^{\mathrm{U}} + 9\mathrm{R}_3^{\mathrm{D}}$$
$$+ 0.6\big[10\left(r_{1h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}}\right) + 30\left(r_{2h}^{\mathrm{U}} - r_{2h}^{\mathrm{D}}\right) + 35\left(r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}}\right) + 200\left(L_{1h}^{\mathrm{shed}} + L_{2h}^{\mathrm{shed}}\right)\big]$$
$$+ 0.4\big[10\left(r_{1l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}}\right) + 30\left(r_{2l}^{\mathrm{U}} - r_{2l}^{\mathrm{D}}\right) + 35\left(r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}}\right) + 200\left(L_{1l}^{\mathrm{shed}} + L_{2l}^{\mathrm{shed}}\right)\big] \tag{3.11a}$$

$$\text{s.t.}\ \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{W}^{\mathrm{S}} - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13}, \tag{3.11b}$$

$$P_3 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13}, \tag{3.11c}$$

$$\frac{(\delta_1^0 - \delta_2^0)}{0.13} \le 100, \tag{3.11d}$$

$$\frac{(\delta_2^0 - \delta_1^0)}{0.13} \le 100, \tag{3.11e}$$

$$P_1 + R_1^{\mathrm{U}} \le 50, \tag{3.11f}$$

$$P_1 - R_1^{\mathrm{D}} \ge 0, \tag{3.11g}$$

$$P_2 + R_2^{\mathrm{U}} \le 110, \tag{3.11h}$$

$$P_2 - R_2^{\mathrm{D}} \ge 0, \tag{3.11i}$$

$$P_3 + R_3^{\mathrm{U}} \le 100, \tag{3.11j}$$

$$P_3 - R_3^{\mathrm{D}} \ge 0, \tag{3.11k}$$

$$r_{1h}^{\mathrm{U}} + r_{2h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}} - r_{2h}^{\mathrm{D}} + L_{1h}^{\mathrm{shed}} + 50 - W^{\mathrm{S}} - W_h^{\mathrm{spill}} = \frac{(\delta_{1h} - \delta_1^0 + \delta_2^0 - \delta_{2h})}{0.13}, \tag{3.11l}$$

$$r_{1l}^{\mathrm{U}} + r_{2l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}} - r_{2l}^{\mathrm{D}} + L_{1l}^{\mathrm{shed}} + 10 - W^{\mathrm{S}} - W_l^{\mathrm{spill}} = \frac{(\delta_{1l} - \delta_1^0 + \delta_2^0 - \delta_{2l})}{0.13}, \tag{3.11m}$$

$$r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}} + L_{2h}^{\mathrm{shed}} = \frac{(\delta_{2h} - \delta_2^0 + \delta_1^0 - \delta_{1h})}{0.13}, \tag{3.11n}$$

$$r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}} + L_{2l}^{\mathrm{shed}} = \frac{(\delta_{2l} - \delta_2^0 + \delta_1^0 - \delta_{1l})}{0.13}, \tag{3.11o}$$

$$\frac{(\delta_{1h} - \delta_{2h})}{0.13} \le 100, \quad \frac{(\delta_{2h} - \delta_{1h})}{0.13} \le 100, \tag{3.11p}$$

$$\frac{(\delta_{1l} - \delta_{2l})}{0.13} \le 100, \quad \frac{(\delta_{2l} - \delta_{1l})}{0.13} \le 100, \tag{3.11q}$$

$$\delta_1^0 = 0, \quad \delta_{1l} = 0, \quad \delta_{1h} = 0, \tag{3.11r}$$

$$r_{1h}^{\mathrm{U}} \le R_1^{\mathrm{U}}, \; r_{1h}^{\mathrm{D}} \le R_1^{\mathrm{D}}, \; r_{2h}^{\mathrm{U}} \le R_2^{\mathrm{U}}, \; r_{2h}^{\mathrm{D}} \le R_2^{\mathrm{D}}, \; r_{3h}^{\mathrm{U}} \le R_3^{\mathrm{U}}, \; r_{3h}^{\mathrm{D}} \le R_3^{\mathrm{D}}, \tag{3.11s}$$

$$r_{1l}^{\mathrm{U}} \le R_1^{\mathrm{U}}, \; r_{1l}^{\mathrm{D}} \le R_1^{\mathrm{D}}, \; r_{2l}^{\mathrm{U}} \le R_2^{\mathrm{U}}, \; r_{2l}^{\mathrm{D}} \le R_2^{\mathrm{D}}, \; r_{3l}^{\mathrm{U}} \le R_3^{\mathrm{U}}, \; r_{3l}^{\mathrm{D}} \le R_3^{\mathrm{D}}, \tag{3.11t}$$

$$W_h^{\mathrm{spill}} \le 50, \quad W_l^{\mathrm{spill}} \le 10, \tag{3.11u}$$

$$L_{1h}^{\mathrm{shed}} \le 40, \quad L_{1l}^{\mathrm{shed}} \le 40, \tag{3.11v}$$

$$L_{2h}^{\mathrm{shed}} \le 100, \quad L_{2l}^{\mathrm{shed}} \le 100, \tag{3.11w}$$

$$R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}, R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}, P_1, \; P_2, \; P_3, \; W^{\mathrm{S}} \ge 0, \tag{3.11x}$$

$$r_{1h}^{\mathrm{U}}, \; r_{2h}^{\mathrm{U}}, \; r_{3h}^{\mathrm{U}}, \; r_{1h}^{\mathrm{D}}, \; r_{2h}^{\mathrm{D}}, \; r_{3h}^{\mathrm{D}}, \; L_{1h}^{\mathrm{shed}}, \; L_{2h}^{\mathrm{shed}}, \; W_h^{\mathrm{spill}} \ge 0, \tag{3.11y}$$

$$r_{1l}^{\mathrm{U}}, \; r_{2l}^{\mathrm{U}}, \; r_{3l}^{\mathrm{U}}, \; r_{1l}^{\mathrm{D}}, \; r_{2l}^{\mathrm{D}}, \; r_{3l}^{\mathrm{D}}, \; L_{1l}^{\mathrm{shed}}, \; L_{2l}^{\mathrm{shed}}, \; W_l^{\mathrm{spill}} \ge 0. \tag{3.11z}$$

We can use stochastic programming model (3.11) to assess the impact of the uncertain wind power production on the expected value of the total system operation cost. Table 3.2 includes a breakdown of this cost into energy production and reserve capacity costs. For ease of comparison, Fig. 3.2 provides a graphical illustration of this cost breakdown, which is calculated for four different cases, namely:

Case a) The future wind power production is perfectly known, and it coincides with its expected value, given by $0.6 \times 50\text{MW} + 0.4 \times 10\text{MW} = 34\text{MW}$.

Case b) The wind farm is removed from the system in Fig. 3.1. Therefore, loads are exclusively supplied by the thermal units.

Case c) Wind power production is uncertain in keeping with the two-scenario representation indicated in the beginning of this illustrative example. Results in this case are directly obtained by solving optimization problem (3.11).

Case d) Wind power production is uncertain as in case (c), and the capacity of the single transmission line in the system is reduced from 100 to 40 MW.

By comparing case (a) or (c) with case (b), it becomes clear that wind generation leads to a significant reduction in the costs of energy production, since a substantial portion of the electricity demand, which otherwise would be covered by thermal generation, is satisfied by renewable and free energy instead. However, the expected cost in case (a) is significantly smaller than in case (c). This difference is largely driven by the cost of the reserve capacity required to cope with wind production uncertainty. Case (d) highlights the key role played by the network in the integration of stochastic production into power systems. If the capacity of the single line in the system is not high enough to make the most of the wind energy produced at bus 1 in all scenarios, a part of the wind power production is likely to be wasted and the two-bus power system will not fully benefit from the cost-free generation of the wind farm.

We conclude this section by generalizing the two-stage stochastic programming model introduced in the previous illustrative example. For this purpose, we define the following sets and indices:

Table 3.2 Breakdown of the expected cost in dollars for four different cases. **a** No wind generation uncertainty. **b** No wind generation. **c** Uncertain wind generation. **d** Network congestion

Case	a	b	c	d
Energy	2180	3200	2260	2880
Reserve	0	0	360	180
Total	2180	3200	2620	3060

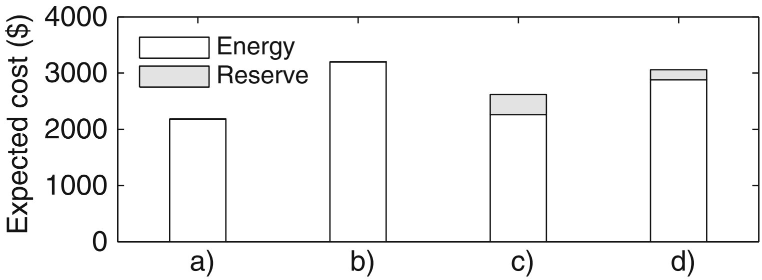


Fig. 3.2 Illustration of the expected cost breakdown. **a** No wind generation uncertainty. **b** No wind generation. **c** Uncertain wind generation. **d** Network congestion

I Set of conventional production units.
J Set of loads.
Q Set of stochastic production units.
N Set of buses.
Λ Set of transmission lines.
Ω Set of scenarios.
Φ_n^I Set of conventional units located at bus n.
Φ_n^J Set of loads located at bus n.
Φ_n^Q Set of stochastic production units located at bus n.
$e(\ell)$ Receiving-end bus of line ℓ.
$o(\ell)$ Sending-end bus of line ℓ.

In addition, we build the general formulation on the following assumptions:

A1 The day-ahead market is cleared using a single-period network-constrained auction. Therefore, inter-temporal constraints, such as ramping limits, are not included in the problem formulation. Hourly periods are considered, and thus, power and energy magnitudes, i.e., MW and MWh, are treated equivalently.
A2 A DC model is used to account for the transmission network; see [10].
A3 Electricity consumption is inelastic, with a large value of lost load. Thus, the maximization of the social welfare boils down to the minimization of the operating costs.
A4 Supply cost functions are linear.
A5 The uncertainty affecting the market-clearing process is assumed to be solely induced by stochastic producers.

A6 The uncertainty associated with the stochastic producers can be efficiently modeled through a finite set of outcomes or scenarios $\{(W_{q\omega}, \pi_\omega), \omega = 1, \ldots,$ and $card(\Omega)\}$, where $\{\pi_\omega, \forall\omega \in \Omega\}$ are their associated probabilities of occurrence and $card(\cdot)$ is a function that gives the cardinality of a set.

A7 Conventional units are considered to be fully dispatchable from zero to their maximum capacities.

Assumptions A1–A5 are mere simplifications for the purpose of rendering the subsequent model formulation easier to follow. Indeed, the day-ahead market model used in the following two-stage stochastic programming formulation can be extended to a multi-period setup that includes ramping constraints, a piecewise linear approximation of the supply cost functions, elastic demand, and other sources of uncertainty such as equipment failures and/or demand uncertainty. We refer the interested reader to Chap. 5 for further details on these extensions. Assumption A6 is typical in stochastic programming and is needed to cast the stochastic market-clearing formulation in a form manageable by optimization solvers, while exploiting the scenario generation tools presented in Chap. 2. Finally, assumption A7 will be actually needed in the following section, where we will use the two-stage stochastic programming model here introduced to price electricity in spot markets under uncertainty. This assumption allows us to sidestep the problem of pricing in markets with non-convexities, which is out of the scope of this book. For further information on this specific problem, the interested reader is referred to [13], [15] and references therein.

The two-stage stochastic programming model that results from the assumptions above is formulated as follows:

$$\begin{aligned}
\underset{\Xi}{\text{Min.}} \quad & \sum_{i \in I} \left(C_i P_i + C_i^{\text{RU}} R_i^{\text{U}} + C_i^{\text{RD}} R_i^{\text{D}} \right) \\
& + \sum_{q \in Q} C_q W_q^{\text{S}} + \sum_{\omega \in \Omega} \pi_\omega \left[\sum_{i \in I} \left(C_i^{\text{U}} r_{i\omega}^{\text{U}} - C_i^{\text{D}} r_{i\omega}^{\text{D}} \right) \right. \\
& \left. + \sum_{q \in Q} C_q \left(W_{q\omega} - W_q^{\text{S}} - W_{q\omega}^{\text{spill}} \right) + \sum_{j \in J} V_j^{\text{LOL}} L_{j\omega}^{\text{shed}} \right] \qquad (3.12\text{a})
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{i \in \Phi_n^I} P_i + \sum_{q \in \Phi_n^Q} W_q^{\text{S}} - \sum_{j \in \Phi_n^J} L_j - \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0 \right) \\
& + \sum_{\ell \in \Lambda | e(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0 \right) = 0 : \lambda_n^{\text{D}}, \quad \forall n \in N, \qquad (3.12\text{b})
\end{aligned}$$

$$\begin{aligned}
& \sum_{i \in \Phi_n^I} \left(r_{i\omega}^{\text{U}} - r_{i\omega}^{\text{D}} \right) + \sum_{j \in \Phi_n^J} L_{j\omega}^{\text{shed}} + \sum_{q \in \Phi_n^Q} \left(W_{q\omega} - W_q^{\text{S}} - W_{q\omega}^{\text{spill}} \right) \\
& + \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega} \right)
\end{aligned}$$

$$-\sum_{\ell \in \Lambda | e(\ell)=n} b_\ell \left(\delta^0_{o(\ell)} - \delta_{o(\ell)\omega} - \delta^0_{e(\ell)} + \delta_{e(\ell)\omega}\right) = 0 \colon \gamma_{n\omega}, \quad \forall n \in N, \ \forall \omega \in \Omega, \tag{3.12c}$$

$$b_\ell \left(\delta^0_{o(\ell)} - \delta^0_{e(\ell)}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \tag{3.12d}$$

$$b_\ell \left(\delta^0_{e(\ell)} - \delta^0_{o(\ell)}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \tag{3.12e}$$

$$b_\ell \left(\delta_{o(\ell)\omega} - \delta_{e(\ell)\omega}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \ \forall \omega \in \Omega, \tag{3.12f}$$

$$b_\ell \left(\delta_{e(\ell)\omega} - \delta_{o(\ell)\omega}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \ \forall \omega \in \Omega, \tag{3.12g}$$

$$\delta^0_1 = 0, \tag{3.12h}$$

$$\delta_{1\omega} = 0, \quad \forall \omega \in \Omega, \tag{3.12i}$$

$$W^{\mathrm{S}}_q \le W^{\max}_q, \quad \forall q \in Q, \tag{3.12j}$$

$$P_i + R^{\mathrm{U}}_i \le P^{\max}_i, \quad \forall i \in I, \tag{3.12k}$$

$$P_i - R^{\mathrm{D}}_i \ge 0, \quad \forall i \in I, \tag{3.12l}$$

$$R^{\mathrm{U}}_i \le R^{\mathrm{U,max}}_i, \quad \forall i \in I, \tag{3.12m}$$

$$R^{\mathrm{D}}_i \le R^{\mathrm{D,max}}_i, \quad \forall i \in I, \tag{3.12n}$$

$$r^{\mathrm{U}}_{i\omega} \le R^{\mathrm{U}}_i, \quad \forall i \in I, \ \forall \omega \in \Omega, \tag{3.12o}$$

$$r^{\mathrm{D}}_{i\omega} \le R^{\mathrm{D}}_i, \quad \forall i \in I, \ \forall \omega \in \Omega, \tag{3.12p}$$

$$L^{\mathrm{shed}}_{j\omega} \le L_j, \quad \forall j \in J, \ \forall \omega \in \Omega, \tag{3.12q}$$

$$W^{\mathrm{spill}}_{q\omega} \le W_{q\omega}, \quad \forall q \in Q, \ \forall \omega \in \Omega, \tag{3.12r}$$

$$P_i, R^{\mathrm{U}}_i, R^{\mathrm{D}}_i \ge 0, \ \forall i \in I; \quad r^{\mathrm{U}}_{i\omega}, r^{\mathrm{D}}_{i\omega} \ge 0, \ \forall i \in I, \ \forall \omega \in \Omega; \ W^{\mathrm{S}}_q \ge 0, \ \forall q \in Q,$$
$$W^{\mathrm{spill}}_{q\omega} \ge 0, \ \forall q \in Q, \ \forall \omega \in \Omega; \ L^{\mathrm{shed}}_{j\omega} \ge 0, \ \forall j \in J, \ \forall \omega \in \Omega, \tag{3.12s}$$

where $\Xi = \{P_i, R^{\mathrm{U}}_i, R^{\mathrm{D}}_i, r^{\mathrm{U}}_{i\omega}, r^{\mathrm{D}}_{i\omega}, W^{\mathrm{S}}_q, \ W^{\mathrm{spill}}_{q\omega}, \ L^{\mathrm{shed}}_{j\omega}, \delta^0_n, \delta_{n\omega}, \forall i \in I, \forall q \in Q, \forall j \in J, \forall n \in N, \forall \omega \in \Omega\}$ is the set of decision variables.

The objective function (3.12a) to be minimized is the sum of the day-ahead energy-reserve dispatch cost and the expected balancing costs. We distinguish here between the *upward* and *downward* reserve capacity costs, represented by C^{RU} and C^{RD}, respectively. We also make a distinction between the cost of the energy sold by conventional producers in the day-ahead market, denoted as C, and the costs of increasing/reducing generation by these producers at balancing time, represented by C^{U} and C^{D} in that order. Note that we have included the energy costs incurred by stochastic producers, calculated as

$$\sum_{q \in Q} C_q W^{\mathrm{S}}_q + \sum_{\omega \in \Omega} \pi_\omega \sum_{q \in Q} C_q \left(W_{q\omega} - W^{\mathrm{S}}_q - W^{\mathrm{spill}}_{q\omega}\right)$$

$$= \sum_{\omega \in \Omega} \pi_\omega \sum_{q \in Q} C_q \left(W_{q\omega} - W_{q\omega}^{\text{spill}} \right).$$

Furthermore, since the term $\sum_{\omega \in \Omega} \pi_\omega \sum_{q \in Q} C_q W_{q\omega}$ is constant, it can be removed from the objective function. Constraints (3.12b) and (3.12c) are power balance equations, with b_ℓ being the susceptance of line ℓ. In particular, constraints (3.12b) enforce the power balance on the day-ahead energy dispatch, while constraints (3.12c) do so on the energy redispatch resulting from real-time balancing. The group of equations (3.12d)–(3.12g) enforces the transmission capacity limits. Equations (3.12h) and (3.12i) set, without loss of generality, bus 1 as the reference node. Constraints (3.12j) limit the power dispatched for each stochastic producer q to its capacity, $W_q^{\max}$. Equations (3.12k) and (3.12l) model the physical coupling between energy and reserve capacity. Constraints (3.12m) and (3.12n) restrict the amount of upward and downward reserve capacity sold by each conventional producer i to its reserve offer limits, $R^{\text{U,max}}$ and $R^{\text{D,max}}$, respectively. The amount of additional energy that each conventional producer i produces for balancing in scenario ω, i.e., $r_{i\omega}^{\text{U}}$, is obtained from its upward reserve capacity R_i^{U}. This is stated by Eq. (3.12o). Analogously, the amount of energy reduction that each conventional producer i implements for balancing in each scenario ω, i.e., $r_{i\omega}^{\text{D}}$, is obtained from its downward reserve capacity R_i^{D}. This is enforced through Eq. (3.12p). As already mentioned in Example 3.3, constraints (3.12q) and (3.12r) are commonsense bounds according to which the amount of load that is involuntarily shed and the amount of stochastic production that is curtailed are smaller than or equal to the actual demand value and the actual stochastic production, respectively. The set of constraints (3.12s) constitutes non-negative variable declarations.

Lastly, we point out that the family of dual variables $\{\lambda_n^{\text{D}}, \forall n \in N\}$ and $\{\gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$ associated with the power balance equations (3.12b) and (3.12c), respectively, are explicitly indicated in optimization problem (3.12) after these equations, separated by a colon, because these dual variables will play a fundamental role in pricing electricity in spot markets that are cleared using a two-stage stochastic programming approach. This is indeed the subject matter of the following section.

3.3 Pricing Energy in the Day-Ahead Market Under Uncertainty

In the presence of a high penetration of stochastic production in electricity markets, balancing costs may become strongly dependent on the day-ahead dispatch. If this is such that insufficient flexible and competitive generation capacity is left to the balancing market, managing uncertainties during the real-time operation of the power system may become problematic and costly. The rationale behind the use of the two-stage stochastic programming approach (3.12) is to explicitly account for the potential impact of the day-ahead dispatch on the balancing operation with the aim of improving the overall system performance. The strong coupling between the day-ahead and balancing market stages does not only manifest itself through the system operation cost, but also through its dual counterpart: the *electricity price*.

In this section, we present a settlement scheme that supports the day-ahead dispatch given by the stochastic programming model (3.12) in an economic sense. In other words, we define a set of prices that make market participants satisfied with the day-ahead dispatch outcomes resulting from such a model.

3.3.1 Towards an Energy-Only Electricity Pricing

Reserve capacity is purchased by the system operator prior to balancing time to guarantee that enough flexible generation will be available to deal with system uncertainties in real time. In essence, the need for reserve capacity is actually a need for balancing energy.

In those markets where energy and reserve capacity are traded as different commodities, the day-ahead energy dispatch is altered by the provision of reserve capacity. This alteration is financially supported by reserve capacity payments (see, for instance, [16; 17]). The dual variable associated with the constraint enforcing the procurement of demand for reserve may serve as the reserve capacity price, as we saw in Example 3.1. The demand for reserve is estimated by the system operator based on the need for energy at the balancing stage.

The stochastic programming model (3.12) does not include, however, a reserve requirement constraint. In this formulation, the day-ahead energy dispatch is determined by explicitly modeling the balancing operation as the second-stage or recourse problem. For this purpose, optimization problem (3.12) exploits the information submitted by market participants about their flexibility and willingness to supply balancing energy. Reserve requirements can be computed *ex post* as a byproduct of model (3.12), inasmuch as the provision of reserve boils down to pre-positioning the system in a way that balancing energy can be traded as anticipated. Therefore, in an electricity market cleared using the two-stage stochastic programming approach (3.12), reserve capacity does not need to be a commodity anymore.

Optimization problem (3.12) does include, on the other hand, two different sets of power balance equations, namely (3.12b) and (3.12c). The former are enforced in the day-ahead market, while the latter are imposed on the energy deployed at the balancing stage. The dual variables associated with these two group of constraints, $\{\lambda_n^{\mathrm{D}}, \forall n \in N\}$ and $\{\gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$, respectively, are particularly meaningful from an economic point of view. Specifically,

λ_n^{D} accounts for the impact on the expected system operation costs of a marginal increase in the *forecast* load at bus n. Therefore, to supply this foreseen marginal increase in load, inflexible units can be used with advance planning;

$\gamma_{n\omega}$ accounts for the impact on the expected system operation costs of a marginal *uncertain* increase in the load at bus n under scenario ω. This marginal increase in load, for being uncertain, cannot be supplied by inflexible units, as they cannot provide balancing energy. Besides, this impact is weighted by the probability of occurrence π_ω of scenario ω.

Based on the economic interpretation of these dual variables, we build the following settlement scheme:

1. Each conventional producer i located at bus n is paid for its day-ahead energy dispatch P_i at a price λ_n^{D}.
2. Each consumer j located at bus n is charged for its scheduled energy consumption L_j at a price λ_n^{D}.
3. Each stochastic producer q located at bus n is paid for its day-ahead dispatch W_q^{S} at a price λ_n^{D}.
4. Each conventional producer i located at bus n is paid for the additional energy $r_{i\omega}^{\mathrm{U}}$ required for balancing in scenario ω at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
5. Each conventional producer i located at bus n is charged for the energy reduction $r_{i\omega}^{\mathrm{D}}$ required for balancing in scenario ω at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
6. Each stochastic producer q located at bus n with production surplus in scenario ω is paid for its excess of generation $W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}}$ at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
7. Each stochastic producer q located at bus n with generation shortage in scenario ω is charged for its production deficit $W_q^{\mathrm{S}} + W_{q\omega}^{\mathrm{spill}} - W_{q\omega}$ at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
8. Each consumer j located at node n suffering from a load curtailment $L_{j\omega}^{\mathrm{shed}}$ in scenario ω is compensated for this curtailment at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.

If we now define

$s(k)$ Index of the bus where market participant k is located;
E_k Energy sold, if positive, or energy purchased, if negative, by market participant k in the day-ahead market;
$\Delta E_{k\omega}$ Additional energy sold, if positive, or repurchased, if negative, by market participant k for balancing in scenario ω,

the previous settlement scheme can be concisely cast as

$$\lambda_{s(k)}^{\mathrm{D}} E_k + \lambda_{s(k)\omega}^{\mathrm{B}} \Delta E_{k\omega}, \tag{3.13}$$

where $\lambda_{s(k)\omega}^{\mathrm{B}} = \frac{\gamma_{s(k)\omega}}{\pi_\omega}$. In fact, $\lambda_{s(k)\omega}^{\mathrm{B}}$ is a prediction of the balancing market price in scenario ω and can be alternatively computed by solving the recourse stage of the stochastic programming model (3.12) with the day-ahead dispatch variables fixed to their optimal values, for the specific realization ω of the uncertain parameters.

Observe that the settlement scheme (3.13) is solely based on energy payments, and consequently, it leads to what we call an *energy-only electricity market*. Under such a market settlement, one may question the actual meaning of the reserve capacity costs $\sum_{i \in I} C_i^{\mathrm{RU}} R_i^{\mathrm{U}} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}}$ included in the objective function (3.12a). In principle, the provision of reserve capacity implies no extra cost to the electricity generation process other than the cost of its actual deployment in the form of balancing energy, which is already counted in (3.12a). Reserve capacity costs may be, nevertheless, justified

Table 3.3 Two possible sets of market outcomes for the two-bus system in Fig. 3.1. Powers in MW

	Unit	P_i	R_i^{U}	R_i^{D}	$r_{i\omega}^{\mathrm{U}}$		$r_{i\omega}^{\mathrm{D}}$	
					High	Low	High	Low
(a) Solution A ($W^{\mathrm{S}} = 10$)	1	50	0	0	0	0	0	0
	2	80	0	40	0	0	40	0
	3	0	0	0	0	0	0	0
(b) Solution B ($W^{\mathrm{S}} = 50$)	1	50	0	0	0	0	0	0
	2	40	40	0	0	40	0	0
	3	0	0	0	0	0	0	0

for the following reason. The clearing process (3.12) allows for the possibility of withdrawing some flexible capacity from the day-ahead market to have it available at the balancing stage. This action may potentially increase the risk exposure of flexible agents inasmuch as the capacity placed in the day-ahead market brings them *certain* profits, while the capacity committed to beforehand in the balancing market yields *uncertain* returns, the actual value of which depends on the eventual outcome of uncertainties. In this sense, reserve capacity costs provide an extra value to the flexible capacity that is allocated, *in advance*, to the balancing market.

A different way to increase the value of the energy for balancing, more aligned with an energy-only electricity market, is to impose a *price premium* on the electricity traded in the balancing market. In practice, this means that $C_i^{\mathrm{U}} > C_i$ and $C_i^{\mathrm{D}} < C_i$ in the objective function (3.12a) of the two-stage stochastic programming model (3.12). From a mathematical point of view, this price premium, or the aforementioned reserve capacity costs, is required for the market-clearing procedure (3.12) not to have multiple solutions. This is illustrated in the example below.

Example 3.4 (Multiplicity of Solutions) Let us consider again the two-bus system described in Example 3.3. The two-stage stochastic programming model (3.12) is now solved by setting the reserve capacity costs to zero, i.e., $C_i^{\mathrm{RU}} = C_i^{\mathrm{RD}} = 0$, for all the three conventional units in the system. That being so, optimization problem (3.12) has infinite solutions. Table 3.3 provides two possible sets of market outcomes leading to the same expected system operation cost, which results in \$2180. There is no need for load curtailment.

Note that, in terms of the expected system operation cost, the following two results are equivalent:

1. To dispatch unit 2 to 80 MW and the wind farm to 10 MW in the day-ahead market, and then redispatch unit 2 to either 40 or 80 MW at the balancing stage, depending on whether the eventual wind power production is 50 (scenario *high*) or 10 MW (scenario *low*), respectively. This is the solution given in Table 3.3(a) (Solution A).
2. To dispatch unit 2 to 40 MW and the wind farm to 50 MW at the scheduling stage, and then redispatch unit 2 to 80 MW in the balancing market if scenario *low* realizes. This is the solution shown in Table 3.3(b) (Solution B).

However, if we allow for reserve capacity costs in the market-clearing problem (3.12), these solutions are not equivalent anymore. In particular, suppose that $C_i^{\mathrm{RU}} = \$2/\mathrm{MW}$ and $C_i^{\mathrm{RD}} = \$1/\mathrm{MW}$ for all the three conventional units in the system. In this case, Solution B is not optimal, as it requires "contracting" more expensive reserve capacity. If we consider a price premium on the balancing energy, say $C_i^{\mathrm{U}} = C_i + 1$ and $C_i^{\mathrm{D}} = C_i - 1$ for all i, instead of reserve capacity costs, Solution B becomes optimal. Indeed, this solution requires the same amount of balancing energy as Solution A, but with a lower probability (i.e., the probability of scenario *low*, 0.4).

3.3.2 Features of the Settlement Scheme

The settlement scheme (3.13) exhibits two important features, namely as follows:

1. It is revenue adequate in expectation, i.e., the payments that the system operator must make to and receive from the participants do not cause it to incur a financial deficit. The term *expectation* comes into play here due to the stochastic approach on which the market-clearing tool (3.12) is built. Intuitively speaking, a market settlement is said to be revenue adequate in expectation provided that it does not cause the system operator to run a financial deficit over time if used repeatedly over many trading periods.
2. It guarantees that the expected profit of each producer, either conventional or stochastic, is greater than or equal to its operating costs.

These two properties are proved in an appendix to this chapter on page 92 and illustrated through the example below.

Example 3.5 (Features of the Settlement Scheme) Let us turn back to the two-bus system introduced in Example 3.3. This time, though, we assume that, comparatively speaking, unit 1 is cheap, but completely inflexible; unit 2 is relatively expensive, but moderately flexible; and unit 3 is expensive, but very flexible. The new data for the three conventional units are collated in Table 3.4. Reserve capacity costs are not considered, i.e., C_i^{RU} and C_i^{RD} are set to zero in market-clearing problem (3.12) for all i. Instead, we assume that the market settlement allows for a price premium on the balancing energy. This way, for example, unit 3 is willing to sell balancing energy at a price \$5/MWh higher than in the day-ahead market, i.e., $C_3^{\mathrm{U}} = \$40/\mathrm{MWh}$. Similarly, this unit is willing to purchase balancing energy at a cost \$1/MWh lower than its marginal cost of production, i.e., $C_3^{\mathrm{D}} = \$34/\mathrm{MWh}$.

The electricity market is cleared using the two-stage stochastic programming model (3.12). The results of the clearing process are provided in Table 3.5. The wind farm is dispatched in the day-ahead market to 10 MW, $W^{\mathrm{S}} = 10$ MW. No load-shedding events occur. Electricity prices are the same at the two buses of the system, as the transmission line connecting them does not become congested in any of the two scenarios, *high* and *low*, considered. Given the dispatched quantities in Table 3.5(a)

Table 3.4 Cost and technical data of conventional units in Example 3.5. Powers in MW and marginal costs in \$/MWh

	$P_i^{\max}$	$R_i^{\mathrm{U,max}}$	$R_i^{\mathrm{D,max}}$	C_i	C_i^{U}	C_i^{D}
Unit 1	50	0	0	10	—	—
Unit 2	110	20	30	30	50	20
Unit 3	100	100	100	35	40	34

and the day-ahead and balancing prices in Table 3.5(b), the profit made by each market participant can be computed. For instance, the payment to unit 3 in scenario *low* is given by $40 \times 30 = \$1200$. Considering that the energy production cost of this unit is equal to \$35/MWh (recall that we assume that the price premium in the balancing market does not reflect a cost intrinsic to the electricity generation process), the profit it makes in this scenario is $1200 - 40 \times 35 = -200\$/\mathrm{MWh}$. Table 3.6 provides the benefit obtained by each market participant both per scenario and in expectation. Note that, at the day-ahead market stage, the profit made by unit 3 can be seen as a random variable the expected value of which $(173.3 \times 0.6 - 200 \times 0.4)$ is greater than zero. The randomness of this profit stems from the uncertain character of the power produced by the wind farm. The settlement scheme (3.13) guarantees cost recovery for all producers in expectation, but this does not prevent unit 3 from incurring economic losses in scenario *low*. Indeed, unit 3 enters the day-ahead dispatch in a loss-making position (!), as its marginal production cost is equal to \$35/MWh, while the day-ahead market price is just \$30/MWh. This unit is dispatched to 40 MW in the day-ahead market with the aim that the system can benefit from its ability and willingness to decrease its production in the case that scenario *high* eventually realizes. It is actually in this scenario where unit 3 makes enough profit to guarantee the recovery of its production cost in expectation.

Table 3.5 Market outcomes for Example 3.5. Powers in MW and prices in \$/MWh

(a) Dispatch ($W^{\mathrm{S}} = 10$)

Unit	P_i	$r_{i\omega}^{\mathrm{U}}$		$r_{i\omega}^{\mathrm{D}}$	
		High	Low	High	Low
1	50	0	0	0	0
2	40	0	0	0	0
3	40	0	0	40	0

(b) Prices

$\lambda_n^{\mathrm{D}}, \forall n$	$\lambda_{n\omega}^{\mathrm{B}}, \forall n$	
	High	Low
30	25.67	35.75

Table 3.6 Profit of market participants in Example 3.5. Profit in \$

	Expected	Per scenario	
		High	Low
Unit 1	1000	1000	1000
Unit 2	0	0	0
Unit 3	24	173.3	−200
Load 1	−1200	−1200	−1200
Load 2	−3000	−3000	−3000
Wind farm	916	1326.7	300

Table 3.7 Market outcomes for Example 3.5 when line capacity is reduced to 50 MW. Powers in MW and prices in \$/MWh

(a) Dispatch ($W^{S} = 10$)

Unit	P_i	$r^{U}_{i\omega}$		$r^{D}_{i\omega}$	
		High	Low	High	Low
1	50	0	0	0	0
2	0	0	0	0	0
3	80	0	0	30	0

(b) Prices

	$\lambda^{D}_{n}, \forall n$	$\lambda^{B}_{n\omega}, \forall n$	
		High	Low
Bus1	14.6	0	36.5
Bus2	35	34	36.5

Revenue adequacy in expectation is also ensured for the system as a whole. To illustrate this, we calculate next the expected payments to conventional producers ($\widehat{\rho}_I$), the expected payment to the wind power producer ($\widehat{\rho}_Q$), and the expected payments from consumers ($\widehat{\rho}_J$), i.e.,

$$\widehat{\rho}_I = \underbrace{50 \times 30}_{Unit\ 1} + \underbrace{40 \times 30}_{Unit\ 2} + \underbrace{40 \times 30 - 0.6 \times 40 \times 25.67}_{Unit\ 3} = \$3284,$$

$$\widehat{\rho}_Q = 10 \times 30 + 0.6 \times (50 - 10) \times 25.67 + 0.4 \times (10 - 10) \times 35.75 = \$916,$$

$$\widehat{\rho}_J = \underbrace{40 \times 30}_{Load\ 1} + \underbrace{100 \times 30}_{Load\ 2} = \$4200.$$

Thus, the system is expected not to incur deficit since \$3284 + \$916 − \$4200 = 0.

To conclude this example, we reduce the capacity of the single line in the system to 50 MW. Dispatched quantities and electricity prices in this new variant are shown in Table 3.7. If the eventual wind power production is *high* (50 MW), the line becomes congested and the balancing price differs between buses, that is, it becomes a *locational marginal price* (LMP). In contrast, no network bottleneck occurs in scenario *low*, and the resulting balancing price is unique accordingly. However, it is worth noting that the day-ahead price is also a locational marginal price even though the optimal day-ahead dispatch $\{P_1^* = 50, P_2^* = 0, P_3^* = 80, W^{S*} = 10\}$ does not cause itself network congestion. This highlights the strong coupling between the day-ahead and the balancing prices, which is captured by the two-stage stochastic programming approach. Intuitively speaking, the day-ahead price *anticipates* probable line bottlenecks during the real-time operation of the power system.

The market-clearing procedure (3.12) is designed to produce a day-ahead dispatch $\{P_i^*, \forall i \in I;\ W_q^{S*}, \forall q \in Q\}$ by accounting for its potential impact on the system balancing costs using stochastic programming. The settlement scheme (3.13) underpins this dispatch in a financial sense. In particular, the day-ahead price λ_n^{D*} makes all market participants satisfied with their respective day-ahead positions, as long as they seek to maximize their expected profit and are willing to take the risk of incurring losses under certain scenarios.

3.4 Clearing the Day-Ahead Market Using Robust Optimization

Robust optimization is an alternative framework to stochastic programming for dealing with optimization problems under uncertainty. This approach aims at determining a solution that is feasible under any realization of the uncertain parameters involved in an optimization problem, and optimal in their worst-case realization. The reader is referred to Appendix D for a brief introduction to the topic. The framework of robust optimization is relevant for the problems considered in this chapter. Indeed, the determination of the optimal dispatch in electricity markets is, as we have seen so far in this chapter, a problem of optimization under uncertainty, where the uncertain parameters include production from renewable sources. Furthermore, *robustness* is a quality that is particularly sought after in these models, as an underestimation of the reserve needs may result in costly load-shedding events. In this section, we shall learn how dispatch problems in electricity markets can be tackled using robust optimization.

Let us recall the problem of determining the day-ahead energy and reserve dispatch considered in Sect. 3.2. As seen in that section, such a problem is a two-stage optimization problem, where the *here-and-now* decisions comprise day-ahead energy and reserve dispatch, while the redispatch at the balancing stage is a *wait-and-see* decision, which adapts to the realization of the uncertainty. In Sect. 3.2.3, such a problem is cast as a stochastic programming problem that aims at minimizing the total costs of energy dispatch, reserve capacity, and redispatch in expectation over a discrete set of scenarios. An alternative approach to this problem based on robust optimization can be sketched as follows:

Minimize Day-ahead dispatch cost + Worst-case balancing cost
subject to

- Day-ahead market constraints:
  - Power balance equations at the day-ahead stage.
  - Reserve capacity determination constraints.
  - Bounds of reserve and energy offers.
- Balancing market constraints:
  - Power balance equations at the balancing stage in the worst-case realization of the uncertain parameters.
  - Network constraints in the worst-case realization of the uncertain parameters.
  - Deployed reserve determination constraints in the worst-case realization of the uncertain parameters.
- Declarations of non-negative variables.

In practice, the mathematical formulation of an adaptive robust optimization problem, i.e., including recourse (*wait-and-see*) decisions, is more complex than the corresponding stochastic programming one. Indeed, the robust formulation

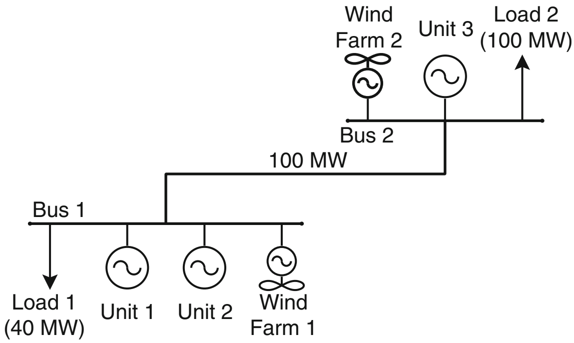


Fig. 3.3 Modified two-bus system

summarized in the box above involves the determination of the worst-case balancing cost, which confers the problem a *min-max-min* structure. This is illustrated in the following illustrative example.

Example 3.6 (An Adaptive Robust Optimization Approach to Energy and Reserve Dispatch). Let us consider a modified version of the two-node system considered in Example 3.3, which is illustrated in Fig. 3.3. Note that the modified two-node system includes two wind farms, one per each node of the network. All the parameters of the system, including cost, production, and transmission limits as well as demand are unchanged with respect to the ones used in Example 3.3.

Objective Function: The worst-case system operation cost (WCC) can be expressed as follows:

$$\mathrm{WCC} = \underbrace{\underbrace{10P_1 + 30P_2 + 35P_3}_{\text{Day-ahead energy costs}} + \underbrace{16R_1^{\mathrm{U}} + 15R_1^{\mathrm{D}} + 13R_2^{\mathrm{U}} + 12R_2^{\mathrm{D}} + 10R_3^{\mathrm{U}} + 9R_3^{\mathrm{D}}}_{\text{Reserve capacity costs}}}_{\text{Day-ahead dispatch costs}}$$
$$+ \underbrace{\mathcal{Q}(P_1, P_2, P_3, R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}, R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}, \delta_1^0, \delta_2^0)}_{\text{Worst-case energy redispatch costs}}. \tag{3.14}$$

We shall now explicitly write the constraints and the worst-case energy redispatch costs.

Day-Ahead Market Constraints: Similarly to Example 3.3, power balance is enforced at both nodes. However, here we assume that the day-ahead dispatch for the wind power producers is equal to their conditional mean forecast, which, for this particular example, is considered to be $\widehat{W}_1 = 15\,\mathrm{MWh}$ and $\widehat{W}_2 = 30\,\mathrm{MWh}$. This writes as follows:

$$P_1 + P_2 + 15 - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13} \qquad \text{(bus 1)},$$
$$P_3 + 30 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13} \qquad \text{(bus 2)},$$

We define bus 1 as the reference node by setting δ_1^0 to 0. Transmission and production capacity are enforced by (3.9) and (3.10), precisely as in the stochastic programming formulation in Example 3.3.

Before enforcing the constraints at the balancing stage, we formulate the worst-case cost of the balancing operation, which was implicitly defined in (3.14) as a function $\mathcal{Q}(\,\cdot\,)$ of the *here-and-now* decisions.

Worst-Case Balancing Cost:
For a given set of *here-and-now* decisions, Ξ^{D}, the following *max-min* formulation defines the worst-case balancing cost for the problem at hand:

$$\begin{aligned}\mathcal{Q}(\,\cdot\,) = \max_{\Delta W_1, \Delta W_2 \in \mathcal{W}} \; \min_{\Xi^{\mathrm{B}} \in \mathcal{B}(\Xi^{\mathrm{D}},\, \Delta W)} \; & \Big[10\left(r_1^{\mathrm{U}} - r_1^{\mathrm{D}}\right) + 30\left(r_2^{\mathrm{U}} - r_2^{\mathrm{D}}\right) \\ & + 35\left(r_3^{\mathrm{U}} - r_3^{\mathrm{D}}\right) + 200\left(L_1^{\mathrm{shed}} + L_2^{\mathrm{shed}}\right)\Big]. \end{aligned} \tag{3.15}$$

The outer maximization problem picks the worst-case realization of the deviations ΔW_1 and ΔW_2 of stochastic production from wind farms 1 and 2, respectively, from their conditional mean forecast. These deviations are to be chosen from within an uncertainty set $\mathcal{W}$, which we shall define later.

Once the worst-case realization of the uncertainty is fixed, the inner minimization problem determines the optimal recourse decision. Notice that the set Ξ^{B} of recourse decisions includes upward and downward redispatch, r^{U} and r^{D}, respectively, load-shedding, L^{shed}, wind power spillage, W^{spill}, as well as voltage angles, δ. These decision variables must be optimized within the feasibility set $\mathcal{B}$, which depends on the *here-and-now* decision set Ξ^{D} and the worst-case realization ΔW of the uncertainty. In turn, the feasibility set $\mathcal{B}$ is defined by the constraints modeling the balancing operation of the power system (i.e., the recourse problem). These constraints will be introduced later on.

Definition of the Uncertainty Set ($\mathcal{W}$): Typically, polyhedral uncertainty sets are chosen in problems of adaptive robust optimization. In this example, we consider symmetrical intervals for the deviation of wind power production from the conditional mean forecast:

$$|\Delta W_1| \leq 10, \tag{3.16a}$$

$$|\Delta W_2| \leq 20. \tag{3.16b}$$

Furthermore, we set a *budget of uncertainty* to limit the overall output deviation for the wind power producers in the network.

$$\frac{|\Delta W_1|}{10} + \frac{|\Delta W_2|}{20} \leq 1.4. \tag{3.17}$$

Basically, such a constraint guarantees that the output for the two wind farms cannot simultaneously be at the lower or upper bound of their respective feasible production intervals resulting from (3.16). Indeed, if the production from one wind farm is at

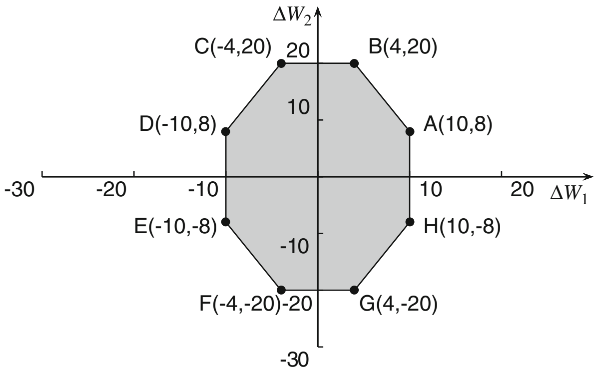


Fig. 3.4 Uncertainty set for the deviation of wind power production from the conditional mean forecast

the lower bound, then the deviation for the other must be at most equal to 40 % of its maximum value. This reflects nature's behavior.

The constraints (3.16) and (3.17) result in the polyhedral uncertainty set illustrated in Fig. 3.4.

Balancing Market Constraints: These constraints define the feasibility set $\mathcal{B}$ in (3.15), which determines the operating region of the power system in real time. Notice that in a robust optimization framework, it is sufficient to enforce one instance of the operation constraints, valid for the worst-case realization of the uncertainty; in this case, the deviation of stochastic power production. In contrast, we had to enforce one set of balancing constraints per scenario in the stochastic programming approach in Example 3.3.

The power balance at each node of the network is enforced by the following constraints:

$$r_1^{\mathrm{U}} + r_2^{\mathrm{U}} - r_1^{\mathrm{D}} - r_2^{\mathrm{D}} + L_1^{\mathrm{shed}} + \Delta W_1 - W_1^{\mathrm{spill}} = \frac{(\delta_1 - \delta_1^0 + \delta_2^0 - \delta_2)}{0.13} \quad \text{(bus 1)},$$

$$r_3^{\mathrm{U}} - r_3^{\mathrm{D}} + L_2^{\mathrm{shed}} + \Delta W_2 - W_2^{\mathrm{spill}} = \frac{(\delta_2 - \delta_2^0 + \delta_1^0 - \delta_1)}{0.13} \quad \text{(bus 2)}.$$

We consider again bus 1 as the reference node in the balancing stage by setting $\delta_1 = 0$.

The following constraints enforce the power transmission capacity between buses 1 and 2 at the balancing stage:

$$\frac{(\delta_1 - \delta_2)}{0.13} \leq 100, \qquad \frac{(\delta_2 - \delta_1)}{0.13} \leq 100.$$

The amount of spilled wind power production must be lower than, or equal to, the actual wind power production value. This is enforced by the following inequalities:

$$W_1^{\mathrm{spill}} \leq 15 + \Delta W_1 \qquad \text{(wind farm 1)},$$

$$W_2^{\mathrm{spill}} \leq 30 + \Delta W_2 \qquad \text{(wind farm 2)}.$$

In a similar fashion, load shedding must be lower than or equal to the actual consumption value:

$$L_1^{\text{shed}} \leq 40 \qquad \text{(load 1)},$$
$$L_2^{\text{shed}} \leq 100 \qquad \text{(load 2)}.$$

The additional energy redispatch is limited by the reserve capacity scheduled in the day-ahead market. This is ensured by the following constraints:

$$r_1^{\text{U}} \leq R_1^{\text{U}}, \quad r_1^{\text{D}} \leq R_1^{\text{D}}, \quad r_2^{\text{U}} \leq R_2^{\text{U}}, \quad r_2^{\text{D}} \leq R_2^{\text{D}}, \quad r_3^{\text{U}} \leq R_3^{\text{U}}, \quad r_3^{\text{D}} \leq R_3^{\text{D}}.$$

Declarations of Non-Negative Variables: Finally, we enforce the non-negativity of reserve, production, and consumption quantities in both the day-ahead and balancing stages:

$$R_1^{\text{U}},\ R_2^{\text{U}},\ R_3^{\text{U}},\ R_1^{\text{D}},\ R_2^{\text{D}},\ R_3^{\text{D}},\ P_1,\ P_2,\ P_3 \geq 0 \qquad \text{(day-ahead stage)},$$
$$r_1^{\text{U}},\ r_2^{\text{U}},\ r_3^{\text{U}},\ r_1^{\text{D}},\ r_2^{\text{D}},\ r_3^{\text{D}},\ L_1^{\text{shed}},\ L_2^{\text{shed}},\ W_1^{\text{spill}},\ W_2^{\text{spill}} \geq 0 \qquad \text{(balancing stage)}.$$

Complete Model Formulation: By joining the objective function with the constraints defined above, we get the following *min-max-min* problem formulation:

$$\begin{aligned}
\underset{\Xi^{\text{D}}}{\text{Min.}}\ & 10P_1 + 30P_2 + 35P_3 + 16R_1^{\text{U}} + 15R_1^{\text{D}} + 13R_2^{\text{U}} + 12R_2^{\text{D}} + 10R_3^{\text{U}} + 9R_3^{\text{D}} \\
& + \max_{\Delta W_1, \Delta W_2} \min_{\Xi^{\text{B}}} \Big[10\left(r_1^{\text{U}} - r_1^{\text{D}}\right) + 30\left(r_2^{\text{U}} - r_2^{\text{D}}\right) + 35\left(r_3^{\text{U}} - r_3^{\text{D}}\right) \\
& \qquad + 200\left(L_1^{\text{shed}} + L_2^{\text{shed}}\right) \Big] \qquad (3.18\text{a})
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } & r_1^{\text{U}} + r_2^{\text{U}} - r_1^{\text{D}} - r_2^{\text{D}} + L_1^{\text{shed}} \\
& \quad + \Delta W_1 - W_1^{\text{spill}} = \frac{(\delta_1 - \delta_1^0 + \delta_2^0 - \delta_2)}{0.13}, \qquad (3.18\text{b}) \\
& r_3^{\text{U}} - r_3^{\text{D}} + L_2^{\text{shed}} + \Delta W_2 - W_2^{\text{spill}} = \frac{(\delta_2 - \delta_2^0 + \delta_1^0 - \delta_1)}{0.13}, \qquad (3.18\text{c}) \\
& \quad -100 \leq \frac{(\delta_1 - \delta_2)}{0.13} \leq 100, \qquad (3.18\text{d}) \\
& \delta_1 = 0, \qquad (3.18\text{e}) \\
& W_1^{\text{spill}} \leq 15 + \Delta W_1, \qquad (3.18\text{f}) \\
& W_2^{\text{spill}} \leq 30 + \Delta W_2, \qquad (3.18\text{g}) \\
& L_1^{\text{shed}} \leq 40, \qquad (3.18\text{h}) \\
& L_2^{\text{shed}} \leq 100, \qquad (3.18\text{i}) \\
& r_1^{\text{U}} \leq R_1^{\text{U}},\ r_2^{\text{U}} \leq R_2^{\text{U}},\ r_3^{\text{U}} \leq R_3^{\text{U}}, \qquad (3.18\text{j})
\end{aligned}$$

$$r_1^{\mathrm{D}} \leq R_1^{\mathrm{D}},\; r_2^{\mathrm{D}} \leq R_2^{\mathrm{D}},\; r_3^{\mathrm{D}} \leq R_3^{\mathrm{D}}, \tag{3.18k}$$

$$r_1^{\mathrm{U}}, r_2^{\mathrm{U}}, r_3^{\mathrm{U}}, r_1^{\mathrm{D}}, r_2^{\mathrm{D}}, r_3^{\mathrm{D}}, L_1^{\mathrm{shed}}, L_2^{\mathrm{shed}}, W_1^{\mathrm{spill}}, W_2^{\mathrm{spill}} \geq 0, \tag{3.18l}$$

$$\text{s.t. } |\varDelta W_1| \leq 10, \tag{3.18m}$$

$$|\varDelta W_2| \leq 20, \tag{3.18n}$$

$$\frac{|\varDelta W_1|}{10} + \frac{|\varDelta W_2|}{20} \leq 1.4, \tag{3.18o}$$

$$\text{s.t. } P_1 + P_2 + 15 - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13}, \tag{3.18p}$$

$$P_3 + 30 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13}, \tag{3.18q}$$

$$-100 \leq \frac{(\delta_1^0 - \delta_2^0)}{0.13} \leq 100, \tag{3.18r}$$

$$\delta_1^0 = 0, \tag{3.18s}$$

$$P_1 + R_1^{\mathrm{U}} \leq 50, \tag{3.18t}$$

$$P_1 - R_1^{\mathrm{D}} \geq 0, \tag{3.18u}$$

$$P_2 + R_2^{\mathrm{U}} \leq 110, \tag{3.18v}$$

$$P_2 - R_2^{\mathrm{D}} \geq 0, \tag{3.18w}$$

$$P_3 + R_3^{\mathrm{U}} \leq 100, \tag{3.18x}$$

$$P_3 - R_3^{\mathrm{D}} \geq 0, \tag{3.18y}$$

$$R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}, R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}, P_1, P_2, P_3 \geq 0. \tag{3.18z}$$

In the formulation above, $\varXi^{\mathrm{D}}$ indicates the set of day-ahead decision variables, i.e., the energy dispatch, P_1, P_2, P_3, the dispatch of upward reserve $R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}$, and of downward reserve, $R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}$, as well as the voltage angles, δ_1^0 and δ_2^0, at this stage.

Notice that in the model above, we can introduce an auxiliary variable β representing the worst-case recourse cost $\mathcal{Q}(\,\cdot\,)$, which is the optimal objective function value of the inner *max-min* problem in (3.18a). We could then solve problem (3.18) as a single minimization problem after enforcing the following constraints:

$$\begin{aligned}
\beta \geq & 10\left[r_1^{\mathrm{U}}(\varDelta W_1, \varDelta W_2) - r_1^{\mathrm{D}}(\varDelta W_1, \varDelta W_2)\right] \\
& + 30\left[r_2^{\mathrm{U}}(\varDelta W_1, \varDelta W_2) - r_2^{\mathrm{D}}(\varDelta W_1, \varDelta W_2)\right] \\
& + 35\left[r_3^{\mathrm{U}}(\varDelta W_1, \varDelta W_2) - r_3^{\mathrm{D}}(\varDelta W_1, \varDelta W_2)\right] \\
& + 200\left[L_1^{\mathrm{shed}}(\varDelta W_1, \varDelta W_2) + L_2^{\mathrm{shed}}(\varDelta W_1, \varDelta W_2)\right], \qquad \forall \varDelta W_1, \varDelta W_2 \in \mathcal{W},
\end{aligned} \tag{3.19}$$

where we write the optimal recourse decision, i.e., the redispatch at the balancing stage, as a function of the deviation of wind power production. However, let us recall that the uncertainty set $\mathcal{W}$, defined by (3.16)–(3.17) and illustrated in Fig. 3.4, includes an infinite number of points. Then, there is an instance of each balancing market variable and constraint for each pair $(\Delta W_1, \Delta W_2) \in \mathcal{W}$.

Since the set $\mathcal{W}$ is uncountable, the reformulation described above would result in a problem with an infinite number of constraints and of variables for the balancing stage. In practice, however, we can get around this issue. Indeed, it can be shown that only the vertices $v = A, B, \ldots, H$ of the polyhedral set $\mathcal{W}$ illustrated in Fig. 3.4 can be part of a solution to the inner *max-min* problem in (3.18). The interested reader is referred to the appendix on page 95 at the end of this chapter for a proof of this property. As a result, we can consider only this finite number of vertices $v = A, B, \ldots, H$ of the uncertainty set, and cast problem (3.18) as follows:

$$\text{Min. } 10P_1 + 30P_2 + 35P_3 + 16R_1^{\mathrm{U}} + 15R_1^{\mathrm{D}} + 13R_2^{\mathrm{U}} + 12R_2^{\mathrm{D}} + 10R_3^{\mathrm{U}} + 9R_3^{\mathrm{D}} + \beta \tag{3.20a}$$

s.t. (3.18p)–(3.18z),

$$\beta \geq \Big[10\left(r_{1v}^{\mathrm{U}} - r_{1v}^{\mathrm{D}}\right) + 30\left(r_{2v}^{\mathrm{U}} - r_{2v}^{\mathrm{D}}\right) + 35\left(r_{3v}^{\mathrm{U}} - r_{3v}^{\mathrm{D}}\right) + 200\left(L_{1v}^{\mathrm{shed}} + L_{2v}^{\mathrm{shed}}\right)\Big], \quad v = A, \ldots, H, \tag{3.20b}$$

$$r_{1v}^{\mathrm{U}} + r_{2v}^{\mathrm{U}} - r_{1v}^{\mathrm{D}} - r_{2v}^{\mathrm{D}} + L_{1v}^{\mathrm{shed}} + \Delta W_{1v} - W_{1v}^{\mathrm{spill}} = \frac{(\delta_{1v} - \delta_1^0 + \delta_2^0 - \delta_{2v})}{0.13}, \quad v = A, \ldots, H, \tag{3.20c}$$

$$r_{3v}^{\mathrm{U}} - r_{3v}^{\mathrm{D}} + L_{2v}^{\mathrm{shed}} + \Delta W_{2v} - W_{2v}^{\mathrm{spill}} = \frac{(\delta_{2v} - \delta_2^0 + \delta_1^0 - \delta_{1v})}{0.13}, \quad v = A, \ldots, H, \tag{3.20d}$$

$$\frac{(\delta_{1v} - \delta_{2v})}{0.13} \leq 100, \quad v = A, \ldots, H, \tag{3.20e}$$

$$\frac{(\delta_{2v} - \delta_{1v})}{0.13} \leq 100, \quad v = A, \ldots, H, \tag{3.20f}$$

$$\delta_{1v} = 0, \quad v = A, \ldots, H, \tag{3.20g}$$

$$W_{1v}^{\mathrm{spill}} \leq 15 + \Delta W_{1v}, \quad v = A, \ldots, H, \tag{3.20h}$$

$$W_{2v}^{\mathrm{spill}} \leq 30 + \Delta W_{2v}, \quad v = A, \ldots, H, \tag{3.20i}$$

$$L_{1v}^{\mathrm{shed}} \leq 40, \quad v = A, \ldots, H, \tag{3.20j}$$

$$L_{2v}^{\mathrm{shed}} \leq 100, \quad v = A, \ldots, H, \tag{3.20k}$$

$$r_{1v}^{\mathrm{U}} \leq R_1^{\mathrm{U}},\ r_{2v}^{\mathrm{U}} \leq R_2^{\mathrm{U}},\ r_{3v}^{\mathrm{U}} \leq R_3^{\mathrm{U}}, \quad v = A, \ldots, H, \tag{3.20l}$$

$$r_{1v}^{\mathrm{D}} \leq R_1^{\mathrm{D}},\ r_{2v}^{\mathrm{D}} \leq R_2^{\mathrm{D}},\ r_{3v}^{\mathrm{D}} \leq R_3^{\mathrm{D}}, \quad v = A, \ldots, H, \tag{3.20m}$$

$$r_{1v}^{\mathrm{U}}, r_{2v}^{\mathrm{U}}, r_{3v}^{\mathrm{U}}, r_{1v}^{\mathrm{D}}, r_{2v}^{\mathrm{D}}, r_{3v}^{\mathrm{D}}, L_{1v}^{\mathrm{shed}}, L_{2v}^{\mathrm{shed}}, W_{1v}^{\mathrm{spill}}, W_{2v}^{\mathrm{spill}} \geq 0, \quad v = A, \ldots, H. \tag{3.20n}$$

Table 3.8 Energy (in MWh) and reserve schedule (in MW) obtained from the robust optimization model. The dispatched wind energy production is 45 MWh

	P	R^{U}	R^{D}
Unit 1	50	0	0
Unit 2	45	24	0
Unit 3	0	0	0

Notice that the reformulation above resembles the stochastic programming problem (3.11), although with two fundamental differences. Firstly, instead of employing scenarios ω, we consider the vertices v of the polyhedral feasible set for the deviation of wind power production defined by (3.16)–(3.17). Secondly, as a result of inequalities (3.20b), the objective function (3.20a) aims at minimizing the total worst-case cost of energy dispatch, reserve, and energy redispatch rather than its value in expectation.

Table 3.8 illustrates the day-ahead schedule determined using model (3.20). The following observations should be pointed out.

1. Robust optimization yields a rather conservative schedule in terms of upward reserve. Indeed, the scheduled value for this quantity is sufficient to cover any negative deviation of wind power production in the uncertainty set $\mathcal{W}$. Notice that the largest production deficit, equal to 24 MWh for the aggregation of the two nodes, is attained at vertex F in Fig. 3.4.
2. No downward reserve is scheduled. This stems from the combination of two facts. Firstly, robust optimization focuses on the worst-case realization of the uncertainty. Secondly, there is no cost associated with wind power spillage, while the penalty for load shedding is relatively large. This implies that the worst-case realization of the uncertainty is a negative deviation of wind power production from its mean forecast. Notice that for cases of production deficit, only upward reserve is needed. Therefore, scheduling downward reserve would unnecessarily increase the worst-case system cost.
3. Because of the focus on the worst-case realization of the uncertainty, robust optimization prioritizes the scheduling of reserves with the lowest sum of production and upward reserve cost. In this case, unit 2 is preferred to unit 3, since $C_2 + C_2^{\mathrm{U}} < C_3 + C_3^{\mathrm{U}}$. This may not be the case for the stochastic programming approach. Indeed, in the latter framework, the higher production cost C_3 is discounted by the probability of actually deploying reserve. On the contrary, the cost of buying reserve, which is lower for unit 3 than for unit 2, is fixed. As a result, scheduling reserve from unit 3 may be more beneficial if the probability of deploying that reserve is sufficiently low.

Example 3.7 (Comparing Robust Optimization and Stochastic Programming). Let us now compare the results obtained from the robust optimization model presented in the previous example with those of the stochastic programming model (3.11). In the implementation of the latter model, we employ a set of 100 scenarios sampled randomly from a uniform distribution the support of which is the uncertainty set illustrated in Fig. 3.4.

Table 3.9 Energy (in MWh) and reserve schedule (in MW) obtained from the stochastic programming model. The dispatched wind energy production is 27.72 MWh

	P	R^{U}	R^{D}
Unit 1	50	0	0
Unit 2	40.28	0	0
Unit 3	22	0	22

Table 3.10 Breakdown of system cost with the stochastic programming and the robust optimization approaches. Values in $

	Stochastic Programming		Robust Optimization	
Energy dispatch	2478.29		1850	
Reserve dispatch	197.95		312	
Total day-ahead	2676.24		2162	
	Expected	*Worst-case*	*Expected*	*Worst-case*
Energy redispatch	−521.75	0	144.92	720
Load shedding	26.03	1344.53	0	0
Total balancing	−495.72	1344.53	144.92	720
Total aggregate	2180.53	4020.78	2306.92	2882

Table 3.9 illustrates the day-ahead energy and reserve dispatch obtained for the conventional producers in the stochastic programming approach. Remarkably, reserve in this solution is assigned to unit 3 rather than to unit 2, as occurs in Table 3.8 for the robust optimization approach. This fact should be considered in view of observation 3 in the previous example. Furthermore, it should be noticed that the total day-ahead dispatch for the conventional units is larger in the stochastic programming approach. In turn, this results in a lower dispatch for the wind power producers, which totals 27.72 MWh, i.e., the amount needed to meet the total load (140 MWh). Notice that the dispatch for wind power producers is not constrained to be equal to the conditional mean forecast of production in model (3.11).

A breakdown of the system cost for the two approaches is provided in Table 3.10. The day-ahead cost is higher for the stochastic programming solution than for the robust optimization one, which dispatches more (zero-cost) wind. However, the former solution benefits from the possibility of redispatching unit 3 downward in the balancing stage, resulting in gains in expectation in this stage. The net effect of the combination of these two facts on the total aggregate expected cost is trivial: the total expected cost is lower in the stochastic programming approach than in the robust optimization one.

The situation is quite the opposite when looking at the worst-case realization of the stochastic production within the uncertainty set illustrated in Fig. 3.4. At the vertex F of this polyhedron, the two wind farms combined produce 24 MWh less than their expected output (45 MWh in total). In the stochastic programming solution, no upward reserve is available to cope with a day-ahead dispatch that is short by 6.72 MWh. Therefore, this amount becomes load shedding, which makes the worst-case balancing cost skyrocket to over \$1300. It should also be noticed that smaller load-shedding events take place in some of the scenarios used as input to the stochastic programming problem, which result in a load-shedding cost equal to roughly \$26 in expectation. In comparison, load shedding never takes place with the robust optimization solution.

We now give the general formulation for the market-clearing model for energy and reserve dispatch based on robust optimization. We employ the same notation as in Sect. 3.2.3. Furthermore, the assumptions made at the end of that section still hold with the exception of A6. Contrarily to this assumption, we now model the uncertainty through a polyhedral uncertainty set $\mathcal{W}$, where the stochastic parameters can take values in.

$$\begin{aligned}
\underset{\Xi^{\mathrm{D}}}{\text{Min.}} & \sum_{i \in I} \left(C_i P_i + C_i^{\mathrm{RU}} R_i^{\mathrm{U}} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}} \right) + \sum_{q \in Q} C_q \widehat{W}_q \\
& + \max_{\Delta W} \min_{\Xi^{\mathrm{B}}} \left[\sum_{i \in I} \left(C_i^{\mathrm{U}} r_i^{\mathrm{U}} - C_i^{\mathrm{D}} r_i^{\mathrm{D}} \right) + \sum_{q \in Q} C_q \left(\Delta W_q - W_q^{\mathrm{spill}} \right) + \sum_{j \in J} V_j^{\mathrm{LOL}} L_j^{\mathrm{shed}} \right]
\end{aligned} \tag{3.21a}$$

$$\begin{aligned}
\text{s.t.} & \sum_{i \in \Phi_n^I} \left(r_i^{\mathrm{U}} - r_i^{\mathrm{D}} \right) + \sum_{j \in \Phi_n^J} L_j^{\mathrm{shed}} + \sum_{q \in \Phi_n^Q} \left(\Delta W_q - W_q^{\mathrm{spill}} \right) \\
& + \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta^0{}_{e(\ell)} + \delta_{e(\ell)} \right) \\
& - \sum_{\ell \in \Lambda | e(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta_{e(\ell)}^0 + \delta_{e(\ell)} \right) = 0, \quad \forall n \in N,
\end{aligned} \tag{3.21b}$$

$$b_\ell \left(\delta_{o(\ell)} - \delta_{e(\ell)} \right) \leq C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21c}$$

$$- b_\ell \left(\delta_{o(\ell)} - \delta_{e(\ell)} \right) \leq C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21d}$$

$$\delta_1 = 0, \tag{3.21e}$$

$$r_i^{\mathrm{U}} \leq R_i^{\mathrm{U}}, \quad \forall i \in I, \tag{3.21f}$$

$$r_i^{\mathrm{D}} \leq R_i^{\mathrm{D}}, \quad \forall i \in I, \tag{3.21g}$$

$$L_j^{\mathrm{shed}} \leq L_j, \quad \forall j \in J, \tag{3.21h}$$

$$W_q^{\mathrm{spill}} \leq \widehat{W}_q + \Delta W_q, \quad \forall q \in Q, \tag{3.21i}$$

$$r_i^{\mathrm{U}}, r_i^{\mathrm{D}} \geq 0, \ \forall i \in I; \ W_q^{\mathrm{spill}} \geq 0, \ \forall q \in Q; \ L_j^{\mathrm{shed}} \geq 0, \ \forall j \in J, \tag{3.21j}$$

$$\text{s.t.} \ |\Delta W_q| \leq \Delta W_q^{\max}, \quad \forall q \in Q, \tag{3.21k}$$

$$\sum_{q \in Q} \frac{|\Delta W_q|}{\Delta W_q^{\max}} \leq \Gamma, \tag{3.21l}$$

$$\text{s.t.} \sum_{i \in \Phi_n^I} P_i + \sum_{q \in \Phi_n^Q} \widehat{W}_q - \sum_{j \in \Phi_n^J} L_j - \sum_{\ell \in \Lambda | o(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right)$$

$$+ \sum_{\ell \in \Lambda | e(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) = 0, \quad \forall n \in N, \tag{3.21m}$$

$$b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) \le C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21n}$$

$$-b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) \le C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21o}$$

$$\delta_1^0 = 0, \tag{3.21p}$$

$$P_i + R_i^{\mathrm{U}} \le P_i^{\max}, \quad \forall i \in I, \tag{3.21q}$$

$$P_i - R_i^{\mathrm{D}} \ge 0, \quad \forall i \in I, \tag{3.21r}$$

$$R_i^{\mathrm{U}} \le R_i^{\mathrm{U,max}}, \quad \forall i \in I, \tag{3.21s}$$

$$R_i^{\mathrm{D}} \le R_i^{\mathrm{D,max}}, \quad \forall i \in I, \tag{3.21t}$$

$$P_i, R_i^{\mathrm{U}}, R_i^{\mathrm{D}} \ge 0, \ \forall i \in I, \tag{3.21u}$$

where $\widehat{W}_q$ is the conditional mean forecast for the power generated by stochastic producer q and Γ is the budget of uncertainty, which limits the overall output deviation for stochastic producers, as in (3.17).

We conclude the section by mentioning that model (3.21) need not be solved by enumeration of all the vertices of the uncertainty set $\mathcal{W}$. Indeed, there exist iterative methods based on Benders decomposition, see [8], where vertices of the feasible polyhedron are generated on demand and a corresponding Benders cut is added at each iteration. In this way, the objective value as a function of the first-stage variables is constructed by sequential approximations, and consequently, more and more accurate estimates of the solution are obtained at each iteration. We refer the interested reader to [2] and [11] for further details on this solution technique.

Finally, note that it is not trivial to derive a pricing scheme for the robust optimization approach. Research is currently underway to develop pricing schemes with desirable short- and long-term properties, such as to convey proper marginal signals and to guarantee investment recovery, respectively.

3.5 Summary and Conclusions

This chapter describes market-clearing procedures for the day-ahead market under a large-scale penetration of stochastic renewable production sources.

Firstly, this procedure is formulated as a two-stage stochastic programming problem, which provides production and consumption levels, allocation of reserve capacity, and clearing prices. The clearing algorithm is of particular interest for markets with a significant number of stochastic producers.

To mimic the actual operation of electric energy systems, a two-stage decision framework, as the one proposed in this chapter, is required. However, such two-step decision framework is not that common in the technical literature.

The proposed stochastic programming framework allows anticipating the impact of the realization of uncertain events and, as a result, achieving the best possible pre-positioning of the market against such uncertain events, with the ultimate purpose of minimizing the expected system cost.

Following widely accepted marginal pricing theory, the algorithm proposed results in energy-only prices, which ensure cost recovery, on average, for all operating producers, and revenue adequacy for the system, also on average.

The algorithm proposed is computationally tractable provided that the number of scenarios required to describe the future realization of the uncertainty is small enough.

Finally, an alternative approach based on adaptive robust optimization is introduced. The market-clearing procedure developed in this framework aims at minimizing the total cost of system operation in the worst-case realization of the uncertain parameters, taking into account the operation at the balancing market.

3.6 Further Reading

Relevant manuals on electricity markets include [16] and [6]. A standard reference on power system reliability is [3], which elaborates on methods to estimate the amount of reserve required to attain a certain level of reliability. The modeling of reliability metrics within a mixed-integer linear programming formulation can be found in [5; 9]. The concepts of stochastic programming are provided in [4], and its applications to decision making under uncertainty in electricity markets, including the market-clearing problem, are presented in [7]. Settlement schemes based on the simultaneous dispatch of energy and reserve using stochastic optimization are discussed, for example, in [17]. Further details on pricing electricity in energy-only markets cleared using stochastic programming can be found in [12] and [14]. The reader is referred to [1] for a tutorial on robust optimization. Applications of adaptive robust optimization focusing on electricity markets, and in particular on unit commitment, comprise [2] and [11].

Appendix 1: Settlement Scheme Properties

The properties of the settlement scheme in Sect. 3.3.1 are formally stated below in the form of theorems. For this purpose, we define first the following indices:

$s(i)$ Index of the bus where conventional unit i is located.
$s(j)$ Index of the bus where load j is located.
$s(q)$ Index of the bus where stochastic production unit q is located.

Theorem 3.1 (Revenue adequacy in expectation). Consider the market-clearing procedure (3.12), built on a stochastic programming framework, and the resulting sets of dual variables $\left\{\lambda_n^{\mathrm{D}}, \forall n \in N\right\}$ and $\left\{\left(\lambda_{n\omega}^{\mathrm{B}}, \pi_\omega\right), \forall n \in N, \forall \omega \in \Omega\right\}$. The settlement scheme (3.13) is revenue adequate in expectation.

Proof Mathematically, the settlement scheme (3.13) is revenue adequate in expectation if, at the optimum, it holds

$$\sum_{n\in N}\lambda_n^{\mathrm{D}*}\left(\sum_{i\in\Phi_n^I}P_i^* + \sum_{q\in\Phi_n^Q}W_q^{\mathrm{S}*} - \sum_{j\in\Phi_n^J}L_j\right) + \sum_{\omega\in\Omega}\sum_{n\in N}\pi_\omega\lambda_{n\omega}^{\mathrm{B}*}\Big[\sum_{i\in\Phi_n^I}\left(r_{i\omega}^{\mathrm{U}*} - r_{i\omega}^{\mathrm{D}*}\right)$$
$$- \sum_{q\in\Phi_n^Q}\left(W_q^{\mathrm{S}*} + W_q^{\mathrm{spill}*} - W_{q\omega}\right) + \sum_{j\in\Phi_n^J}L_{j\omega}^{\mathrm{shed}*}\Big] \le 0, \tag{3.22}$$

where $\left\{\lambda_{n\omega}^{\mathrm{B}*} = \gamma_{n\omega}^*/\pi_\omega, \forall n \in N, \forall \omega \in \Omega\right\}$ are the probability-removed balancing prices and superscript "$*$" denotes optimal values.

Using the power balance equations (3.12b) and (3.12c), expression (3.22) can be equivalently rewritten as follows:

$$\sum_{n\in N}\lambda_n^{\mathrm{D}*}\left[\sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{e(\ell)}^{0*}\right) - \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{e(\ell)}^{0*}\right)\right]$$
$$- \sum_{\omega\in\Omega}\sum_{n\in N}\pi_\omega\lambda_{n\omega}^{\mathrm{B}*}\Big[\sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{o(\ell)\omega}^{*} - \delta_{e(\ell)}^{0*} + \delta_{e(\ell)\omega}^{*}\right)$$
$$- \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{o(\ell)\omega}^{*} - \delta_{e(\ell)}^{0*} + \delta_{e(\ell)\omega}^{*}\right)\Big] \le 0. \tag{3.23}$$

Let us consider the following partial Lagrangian function of problem (3.12):

$$\mathcal{L} = \sum_{i\in I}\left(C_iP_i + C_i^{\mathrm{RU}}R_i^{\mathrm{U}} + C_i^{\mathrm{RD}}R_i^{\mathrm{D}}\right) + \sum_{q\in Q}C_qW_q^{\mathrm{S}} + \sum_{\omega\in\Omega}\pi_\omega\Big[\sum_{i\in I}\left(C_i^{\mathrm{U}}r_{i\omega}^{\mathrm{U}} - C_i^{\mathrm{D}}r_{i\omega}^{\mathrm{D}}\right)$$
$$+ \sum_{q\in Q}C_q\left(W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}}\right) + \sum_{j\in J}V_j^{\mathrm{LOL}}L_{j\omega}^{\mathrm{shed}}\Big] - \sum_{n\in N}\lambda_n^{\mathrm{D}}\Big[\sum_{i\in\Phi_n^I}P_i + \sum_{q\in\Phi_n^Q}W_q^{\mathrm{S}}$$
$$- \sum_{j\in\Phi_n^J}L_j - \sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) + \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right)\Big]$$
$$- \sum_{\omega\in\Omega}\sum_{n\in N}\gamma_{n\omega}\Big[\sum_{i\in\Phi_n^I}\left(r_{i\omega}^{\mathrm{U}} - r_{i\omega}^{\mathrm{D}}\right) + \sum_{j\in\Phi_n^J}L_{j\omega}^{\mathrm{shed}} + \sum_{q\in\Phi_n^Q}\left(W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}}\right)$$
$$+ \sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right)$$
$$- \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right)\Big]. \tag{3.24}$$

Since problem (3.12) is linear and thus convex, $\mathcal{L}$ is minimized subject to the rest of constraints, i.e., constraints (3.12d)–(3.12s), at the optimum. Note that by moving the power balance equations (3.12b) and (3.12c) to the objective function (3.12a) to form the partial Lagrangian function $\mathcal{L}$, the resulting optimization problem {minimize (3.24), subject to (3.12d)–(3.12s)} can be decomposed into appropriate minimization subproblems for any given set of Lagrange multipliers $\{\lambda_n^{\mathrm{D}}, \forall n \in N; \gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$. In particular, the summation of the following terms, extracted from (3.24),

$$\sum_{n \in N} \lambda_n^{\mathrm{D}} \left[\sum_{\ell \in \Lambda | o(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) - \sum_{\ell \in \Lambda | e(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) \right]$$
$$- \sum_{\omega \in \Omega} \sum_{n \in N} \gamma_{n\omega} \left[\sum_{\ell \in \Lambda | o(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right) \right.$$
$$\left. - \sum_{\ell \in \Lambda | e(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right) \right] \tag{3.25}$$

is minimized subject to constraints (3.12d)–(3.12i) at the optimum.

A solution such that $\delta_n^0 = 0, \forall n \in \Omega$; $\delta_{n\omega} = 0, \forall n \in N, \forall \omega \in \Omega$, is feasible for the minimization subproblem {minimize (3.25), subject to (3.12d)–(3.12i)}, as long as the capacity of transmission lines is non-negative, i.e., $C_\ell^{\max} \geq 0$, $\forall \ell \in \Lambda$. This solution allows us to set the upper bound of expression (3.25) to zero. Therefore, inequality (3.23) holds, and this concludes the proof.

Theorem 3.2 (Cost recovery in expectation). Consider the market-clearing procedure (3.12), built on a stochastic programming framework, and the resulting sets of dual variables $\left\{\lambda_n^{\mathrm{D}}, \forall n \in N\right\}$ and $\left\{\left(\lambda_{n\omega}^{\mathrm{B}}, \pi_\omega\right), \forall n \in N, \forall \omega \in \Omega\right\}$. The settlement scheme (3.13) guarantees cost recovery for all market participants in expectation.

Proof The settlement scheme (3.13) ensures that both conventional and stochastic producers recover their energy production costs in expectation. Mathematically, this is expressed as follows:

$$C_i P_i^* + C_i^{\mathrm{RU}} R_i^{\mathrm{U}*} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}*} + \sum_{\omega \in \Omega} \pi_\omega \left(C_i^{\mathrm{U}} r_{i\omega}^{\mathrm{U}*} - C_i^{\mathrm{D}} r_{i\omega}^{\mathrm{D}*}\right) - \lambda_{s(i)}^{\mathrm{D}*} P_i^*$$
$$- \sum_{\omega \in \Omega} \pi_\omega \lambda_{s(i)\omega}^{\mathrm{B}*} \left(r_{i\omega}^{\mathrm{U}*} - r_{i\omega}^{\mathrm{D}*}\right) \leq 0, \quad \forall i \in I; \tag{3.26}$$

and

$$\sum_{\omega \in \Omega} \pi_\omega C_q \left(W_{q\omega} - W_{q\omega}^{\mathrm{spill}*}\right) - \lambda_{s(q)}^{\mathrm{D}*} W_q^{\mathrm{S}*}$$
$$- \sum_{\omega \in \Omega} \pi_\omega \lambda_{s(q)\omega}^{\mathrm{B}*} \left(W_{q\omega} - W_q^{\mathrm{S}*} - W_{q\omega}^{\mathrm{spill}*}\right) \leq 0, \forall q \in Q, \tag{3.27}$$

where superscript "$*$" denotes optimal values and $\lambda_{n\omega}^{\mathrm{B}*} = \frac{\gamma_{n\omega}^{*}}{\pi_{\omega}}, \forall n \in N, \forall \omega \in \Omega$.

Let us consider again the partial Lagrangian function (3.24). At the optimum, this function is minimized subject to constraints (3.12d)–(3.12s). As stated in the proof for revenue adequacy in expectation, the optimization problem {minimize (3.24), subject to (3.12d)–(3.12s) can be decomposed into appropriate minimization subproblems for any given set of shadow prices $\{\lambda_n^{\mathrm{D}}, \forall n \in N; \gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$. Specifically, the series of terms extracted from (3.24)

$$C_i P_i + C_i^{\mathrm{RU}} R_i^{\mathrm{U}} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}} + \sum_{\omega \in \Omega} \pi_\omega \left(C_i^{\mathrm{U}} r_{i\omega}^{\mathrm{U}} - C_i^{\mathrm{D}} r_{i\omega}^{\mathrm{D}} \right) - \lambda_{s(i)}^{\mathrm{D}} P_i - \sum_{\omega \in \Omega} \gamma_{s(i)\omega} \left(r_{i\omega}^{\mathrm{U}} - r_{i\omega}^{\mathrm{D}} \right), \tag{3.28}$$

and

$$\sum_{\omega \in \Omega} \pi_\omega C_q \left(W_{q\omega} - W_{q\omega}^{\mathrm{spill}} \right) - \lambda_{s(q)}^{\mathrm{D}} W_q^{\mathrm{S}} - \sum_{\omega \in \Omega} \gamma_{s(q)\omega} \left(W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}} \right), \tag{3.29}$$

are minimized, for all $i \in I$ and for all $q \in Q$, subject to the set of constraints {(3.12k)–(3.12p),(3.12s)} and {(3.12j),(3.12r),(3.12s)}, respectively.

The collection of decision variables such that $P_i = R_i^{\mathrm{U}} = R_i^{\mathrm{D}} = 0, \forall i \in I$ (here, we appeal to assumption A7, according to which conventional producers are fully dispatchable) and $r_{i\omega}^{\mathrm{U}} = r_{i\omega}^{\mathrm{D}} = 0, \forall i \in I, \forall \omega \in \Omega$, constitutes a feasible solution to the minimization subproblem made up of the objective function (3.28) and the group of constraints (3.12k)–(3.12p), and (3.12s). Likewise, the set of decision variables such that $W_q^{\mathrm{S}} = 0$, $\forall q \in Q$, and $W_{q\omega}^{\mathrm{spill}} = W_{q\omega}$, $\forall q \in Q$, $\forall \omega \in \Omega$, is a feasible solution to the minimization subproblem composed of the objective function (3.29) and constraints (3.12j),(3.12r), and(3.12s). This pair of solutions sets the upper bound of expressions (3.28) and (3.29) to zero. Consequently, inequalities (3.26) and (3.27) hold, which concludes the proof.

It is important to underline that the decomposition-based reasoning employed to prove Theorem 3.2 cannot be used, however, to prove cost recovery per scenario due to the day-ahead dispatch variables P_i and W_q^{S}, which link all the scenarios together. This is so because the settlement scheme (3.13) allows power producers to *incur economic losses in some scenarios* as long as they recover their production costs in expectation, i.e., in the long run and under similar conditions.

Appendix 2: Worst-Case Realization of Uncertain Production in Robust Optimization

In this appendix, we prove that the worst-case uncertainty realization for the adaptive robust optimization problem (3.21) occurs at an extreme point of the polyhedral uncertainty set $\mathcal{W}$ for the deviation of wind power production from its conditional mean forecast.

Let us consider the inner *max-min* problem in (3.21):

$$\underset{\Delta W}{\text{Max}} \ \underset{\Xi^{\text{B}}}{\min} \left[\sum_{i \in I} \left(C_i^{\text{U}} r_i^{\text{U}} - C_i^{\text{D}} r_i^{\text{D}} \right) + \sum_{q \in Q} C_q \left(\Delta W_q - W_q^{\text{spill}} \right) + \sum_{j \in J} V_j^{\text{LOL}} L_j^{\text{shed}} \right] \tag{3.30a}$$

$$\begin{aligned} \text{s.t.} \ & \sum_{i \in \Phi_n^I} \left(r_i^{\text{U}} - r_i^{\text{D}} \right) + \sum_{j \in \Phi_n^J} L_j^{\text{shed}} + \sum_{q \in \Phi_n^Q} \left(\Delta W_q - W_q^{\text{spill}} \right) \\ & + \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta_{e(\ell)}^0 + \delta_{e(\ell)} \right) \\ & - \sum_{\ell \in \Lambda | e(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta_{e(\ell)}^0 + \delta_{e(\ell)} \right) = 0 \quad : \lambda_n \,, \quad \forall n \in N, \end{aligned} \tag{3.30b}$$

$$b_\ell \left(\delta_{o(\ell)} - \delta_{e(\ell)} \right) \leq C_\ell^{\max} \quad : \sigma_\ell^{\text{U}}, \quad \forall \ell \in \Lambda, \tag{3.30c}$$

$$- b_\ell \left(\delta_{o(\ell)} - \delta_{e(\ell)} \right) \leq C_\ell^{\max} \quad : \sigma_\ell^{\text{D}}, \quad \forall \ell \in \Lambda, \tag{3.30d}$$

$$\delta_1 = 0 \quad : \nu, \tag{3.30e}$$

$$r_i^{\text{U}} \leq R_i^{\text{U}} \quad : \mu_i^{\text{U}}, \quad \forall i \in I, \tag{3.30f}$$

$$r_i^{\text{D}} \leq R_i^{\text{D}} \quad : \mu_i^{\text{D}}, \quad \forall i \in I, \tag{3.30g}$$

$$L_j^{\text{shed}} \leq L_j \quad : \epsilon_j^{\text{shed}}, \quad \forall j \in J, \tag{3.30h}$$

$$W_q^{\text{spill}} \leq \widehat{W}_q + \Delta W_q \quad : \epsilon_q^{\text{spill}} \quad \forall q \in Q, \tag{3.30i}$$

$$r_i^{\text{U}}, r_i^{\text{D}} \geq 0, \ \forall i \in I; \ W_q^{\text{spill}} \geq 0, \ \forall q \in Q; \ L_j^{\text{shed}} \geq 0, \ \forall j \in J, \tag{3.30j}$$

$$\text{s.t.} \ |\Delta W_q| \leq \Delta W_q^{\max}, \quad \forall q \in Q, \tag{3.30k}$$

$$\sum_{q \in Q} \frac{|\Delta W_q|}{\Delta W_q^{\max}} \leq \Gamma. \tag{3.30l}$$

Notice that we indicated the dual variables for the inner minimization problem on the right-hand side of the corresponding constraints, preceded by a colon.

First of all, we can reformulate inequalities (3.30k) and (3.30l) as follows:

$$- \Delta W_q^{\max} \leq \Delta W_q \leq \Delta W_q^{\max}, \qquad \forall q \in Q, \tag{3.31a}$$

$$\Delta W_q = \Delta W_q^{+} - \Delta W_q^{-}, \qquad \forall q \in Q, \tag{3.31b}$$

$$\sum_{q \in Q} \frac{\Delta W_q^{+} + \Delta W_q^{-}}{\Delta W_q^{\max}} \leq \Gamma, \tag{3.31c}$$

$$\Delta W_q^{+}, \Delta W_q^{-} \geq 0, \qquad \forall q \in Q. \tag{3.31d}$$

It is worth to point out that reformulation (3.31) is linear, on the contrary of (3.30k)–(3.30l).

Denoting the set of dual variables of the inner problem (3.30a)–(3.30j) with Ξ', we can replace the inner minimization problem with its dual (see Appendix B of the book). This renders the following problem:

$$\begin{aligned}
\underset{\Delta W}{\text{Max}}\ \underset{\Xi'}{\max} \sum_{n\in N}\Bigg[&\sum_{\ell\in\Lambda|o(\ell)=n} b_\ell\left(-\delta^0_{o(\ell)}+\delta^0_{e(\ell)}\right) - \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell\left(-\delta^0_{o(\ell)}+\delta^0_{e(\ell)}\right)\\
&- \sum_{q\in\Phi^Q_n}\Delta W_q\Bigg]\lambda_n + \sum_{\ell\in\Lambda} C^{\max}_\ell\left(\sigma^{\mathrm{U}}_\ell+\sigma^{\mathrm{D}}_\ell\right) + \sum_{i\in I}\left(R^{\mathrm{U}}_i\mu^{\mathrm{U}}_i + R^{\mathrm{D}}_i\mu^{\mathrm{D}}_i\right)\\
&+ \sum_{j\in J} L_j\epsilon^{\mathrm{shed}}_j + \sum_{q\in Q}\left[\left(\widehat{W}_q+\Delta W_q\right)\epsilon^{\mathrm{spill}}_q + C_q\Delta W_q\right] \qquad (3.32a)
\end{aligned}$$

$$\text{s.t. } \lambda_{s(i)} + \mu^{\mathrm{U}}_i \le C^{\mathrm{U}}_i, \quad \forall i\in I, \qquad (3.32b)$$

$$-\lambda_{s(i)} + \mu^{\mathrm{D}}_i \le -C^{\mathrm{D}}_i, \quad \forall i\in I, \qquad (3.32c)$$

$$\lambda_{s(j)} + \epsilon^{\mathrm{shed}}_j \le V^{\mathrm{LOL}}_j, \quad \forall j\in J, \qquad (3.32d)$$

$$-\lambda_{s(q)} + \epsilon^{\mathrm{spill}}_q \le -C_q, \quad \forall q\in Q, \qquad (3.32e)$$

$$\begin{aligned}
\nu - &\left(\sum_{\ell\in\Lambda|o(\ell)=1} b_\ell + \sum_{\ell\in\Lambda|e(\ell)=1} b_\ell\right)\lambda_1 + \sum_{\ell\in\Lambda|e(\ell)=1} b_\ell\lambda_{o(\ell)}\\
&+ \sum_{\ell\in\Lambda|o(\ell)=1} b_\ell\lambda_{e(\ell)} + \sum_{\ell\in\Lambda|o(\ell)=1} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right)\\
&- \sum_{\ell\in\Lambda|e(\ell)=1} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right) = 0, \qquad (3.32f)
\end{aligned}$$

$$\begin{aligned}
- &\left(\sum_{\ell\in\Lambda|o(\ell)=n} b_\ell + \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell\right)\lambda_n + \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell\lambda_{o(\ell)}\\
&+ \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell\lambda_{e(\ell)} + \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right)\\
&- \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right) = 0, \quad \forall n\in N\setminus\{1\}, \qquad (3.32g)
\end{aligned}$$

$$\sigma^{\mathrm{U}}_\ell, \sigma^{\mathrm{D}}_\ell \le 0,\ \forall \ell\in\Lambda;\ \mu^{\mathrm{U}}_i, \mu^{\mathrm{D}}_i \le 0,\ \forall i\in I; \qquad (3.32h)$$

$$\epsilon^{\mathrm{shed}}_j \le 0,\ \forall j\in J;\ \epsilon^{\mathrm{spill}}_q \le 0,\ \forall q\in Q, \qquad (3.32i)$$

$$\text{s.t. } -\Delta W^{\max}_q \le \Delta W_q \le \Delta W^{\max}_q, \quad \forall q\in Q, \qquad (3.32j)$$

$$\Delta W_q = \Delta W^+_q - \Delta W^-_q, \quad \forall q\in Q, \qquad (3.32k)$$

$$\sum_{q\in Q}\frac{\Delta W^+_q + \Delta W^-_q}{\Delta W^{\max}_q} \le \Gamma, \qquad (3.32l)$$

$$\Delta W^+_q, \Delta W^-_q \ge 0, \quad \forall q\in Q. \qquad (3.32m)$$

The following observations on model (3.32) are in order.

1. The two *max* operators can be merged. Therefore, (3.32) is a single maximization problem.
2. Constraints (3.32b)–(3.32m) are linear.
3. Objective function (3.32a) is bilinear, owing to the cross products between variables ΔW_q and λ_n as well as $\epsilon_q^{\text{spill}}$.

In view of the observations above, if we fix the variables in Ξ' at their optimal value, model (3.32) boils down to a linear programming problem in the decision variables ΔW_q, constrained by (3.32j)–(3.32m). For any linear program, at least one of the solutions (if it exists) is a vertex of the feasible set. Since the feasible set $\mathcal{W}$ is compact, at least an optimal solution of the bilinear program (3.32) is a vertex of $\mathcal{W}$.

Exercises

3.1 Reformulate the auction in Example 3.3 to include two time periods. Enforce ramping limits on the thermal generation units and analyze numerically the impact of such limits on market outcomes. Hint: the reader is advised to consult Sect. 5.3.3.

3.2 Consider multiple Gaussian distributed wind power production scenarios in the problem of Example 3.3. Analyze numerically the impact of increasing the number of scenarios on market outcomes. Compare these outcomes with those obtained considering solely the average value scenario.

3.3 Consider just two extreme scenarios (very-high wind production and no wind production), and analyze the outcomes of the auction in Example 3.3. Compare these outcomes with the outcomes obtained considering solely the average value scenario. What happens as scenarios become increasingly extreme?

3.4 Analyze the market-clearing algorithm in Example 3.3 in a case in which only wind producers are available. Study the behavior of prices, both day-ahead prices and balancing prices.

3.5 Consider the market-clearing algorithm in Example 3.3, but involving thermal plants with significantly high start-up costs. What happens with the clearing prices (both day-ahead and balancing) in such situation? Hint: you can get inspiration on how to model the start-up cost of a thermal power plant from Sect. 8.2.1 in the book.

3.6 Consider the market-clearing algorithm in Example 3.3, but involving thermal plants with minimum power outputs. What happens with the clearing prices (both day-ahead and balancing) in such situation? Hint: you can get inspiration from Sect. 5.3.2 for the modeling of capacity limits.

3.7 Consider the auction in Example 3.3, and solve it for a wide range of values of lost load. Study how market outcomes change as a result of an increasingly high unserved-energy value.

3.8 Consider wind production offers at non-zero price in the auction of Example 3.3. Analyze numerically how market outcomes change as wind offering prices increase.

3.9 Reformulate the auction in Example 3.3 to include two time periods involving highly different load levels, and a pumped storage plant. Is the availability of such pumped storage plant beneficial? Analyze how the impact of the pumped storage plant on market outcomes changes as the efficiency of the pumping-turbine cycle increases. Hint: Sect. 5.5 provides insight into how to model a pumped-storage power plant.

3.10 Reformulate the auction in Example 3.3 to include two time periods involving highly different load levels, and a pumped storage plant. Consider that the transmission line has such a low capacity that often leads to transmission bottleneck. Analyze the ability of the pumped storage plant to alleviate the detrimental effect of transmission bottlenecks.

3.11 Solve the robust optimization problem (3.20) considered in Example 3.6 for different values of the *budget of uncertainty* in (3.17). Start by enumerating the vertices of the polyhedral uncertainty set. What is the effect of increasing the uncertainty budget on the amount of dispatched reserve?

3.12 Include constraints of the following type

$$|\Delta W_1 - \Delta W_2| \leq \rho$$

in the definition of the uncertainty set for the dispatch model based on robust optimization presented in Example 3.6. Determine the uncertainty set and enumerate its vertices, then solve the dispatch problem.

3.13 Reformulate the robust optimization model (3.20) to include two time periods and a pumped storage plant. Consider the two-node system of Example 3.3, which includes only one wind power plant. Introduce intervals for the deviation of wind power production during each time period, and a *budget of uncertainty* to limit the total deviation of energy production over the two periods, similarly to (3.16) and (3.17). Analyze the effect on the robust dispatch of a storage facility with limited capacity.

References

1. Bertsimas, D., Brown, D.B., Caramanis, C.: Theory and applications of robust optimization. SIAM Rev. **53**(3), 464–501 (2011)
2. Bertsimas, D., Litvinov, E., Sun, X.A., Zhao, J., Zheng, T.: Adaptive robust optimization for the security constrainted unit commitment problem. IEEE Trans. Power Syst. **28**(1), 52–63 (2012)
3. Billinton, R., Allan, R.N.: Reliability Evaluation of Power Systems. Plenum Press, New York (1984)
4. Birge, J.R., Louveaux, F.: Introduction to Stochastic Programming, 2nd edn. Series in Operations Research and Financial Engineering. Springer, New York (2011)

5. Bouffard, F., Galiana, F.D., Conejo, A.J.: Market-clearing with stochastic security–Part I: Formulation. IEEE Trans. Power Syst. **20**(4), 1818–1826 (2005)
6. Chao, H., Huntington, H.: Designing Competitive Electricity Markets. International Series in Operations Research & Management Science. Kluwer, Boston (1998)
7. Conejo, A.J., Carrión, M., Morales, J.: Decision Making Under Uncertainty in Electricity Markets. International Series in Operations Research & Management Science. Springer, New York (2010)
8. Conejo, A.J., Castillo, E., Mínguez, R., García-Bertrand, R.: Decomposition Techniques in Mathematical Programming: Engineering and Science Applications, chap. 5, pp. 197–199. Springer, Berlin (2006)
9. Galiana, F.D., Bouffard, F.: An electricity market with a probabilistic spinning reserve criterion. IEEE Trans. Power Syst. **19**(1), 300–307 (2004)
10. Gómez-Expósito, A., Conejo, A.J., Cañizares, C.: Electric Energy Systems: Analysis and Operation. CRC Press, Boca Raton (2009)
11. Jiang, R., Wang, J., Guan, Y.: Robust unit commitment with wind power and pumped storage hydro. IEEE Trans. Power Syst. **27**(2), 800–810 (2012)
12. Morales, J.M., Conejo, A.J., Liu, K., Zhong, J.: Pricing electricity in pools with wind producers. IEEE Trans. Power Syst. **27**(3), 1366–1376 (2012)
13. O'Neill, R.P., Sotkiewicz, P.M., Hobbs, B.F., Rothkopf, M.H., Stewart, W.R.: Efficient market-clearing prices in markets with non-convexities. Eur. J. Oper. Res. **164**(1), 269–285 (2005)
14. Pritchard, G., Zakeri, G., Philpott, A.: A single-settlement, energy-only electric power market for unpredictable and intermittent participants. Oper. Res. **58**(4), 1210–1219 (2010)
15. Ruiz, C., Conejo, A.J., Gabriel, S.A.: Pricing non-convexities in an electricity pool. IEEE Trans. Power Syst. **27**(3), 1334–1342 (2012)
16. Stoft, S.: Power System Economics: Designing Markets for Electricity. Wiley–IEEE Press, New York (2002)
17. Wong, S., Fuller, J.D.: Pricing energy and reserves using stochastic optimization in an alternative electricity market. IEEE Trans. Power Syst. **22**(2), 631–638 (2007)

Chapter 4
Balancing Markets

4.1 Why Are Balancing Markets Needed?

Balancing markets are generally used to balance as closely as possible production and consumption prior (e.g., half an hour in advance) to energy delivery. This is important as there is no economical way to store large quantities of electric energy.

Markets other than the balancing markets are cleared well in advance of energy delivery and thus the production and consumption levels scheduled in these markets can significantly differ from the actual production and consumption at balancing time. This is particularly so in markets with a significant amount of stochastic production units. Balancing markets bridge, or narrow, the balance gap between other forward markets and real-time energy delivery.

In the following we consider that the balancing market is an energy-only market, not including reserve or other ancillary services required for the proper functioning of the electric energy system. The analysis of reserves and ancillary services is outside the scope of this chapter.

4.1.1 Day-Ahead, Adjustment, and Balancing Markets

The day-ahead market allows electric energy trading one day ahead of energy delivery. It is typically cleared around noon the previous day to the day in which energy is to be delivered. Clearing at noon one day in advance implies 12-hour anticipation with respect to the first delivery hour and 35-hour anticipation with respect to the last delivery hour. Such anticipation is required by some production units (e.g., nuclear or coal plants) due to technical limitations on operation flexibility.

Adjustment or intraday markets allow electric energy trading after the day-ahead market clearing and typically up to one or few hours before energy delivery. For example in the Iberian Peninsula [3], four adjustment markets are arranged following the clearing of the day-ahead market for day d: the first adjustment market clears at 6 p.m. of day $d - 1$ and expands the 24 h of day d, the second one clears at midnight of day $d - 1$ and covers the last 23 h of day d, the third one clears at 6 a.m. of day

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d and spans the last 17 h of day d and finally, the fourth one clears at noon of day d and spans the last 11 h of day d.

Adjustment markets are particularly appropriate for nondispatchable, stochastic producers such as wind- or solar-based producers. Since such markets clear closer to actual energy delivery than the day-ahead market does, they allow stochastic producers to offer with increasing certainty on their production levels, which generally results in comparatively higher profits and reduced profit volatility. Intraday markets are implemented, although with different rules, in the majority of European markets. However, their liquidity is very limited [16], with the notable exception of the Iberian market.

The balancing market, also called real-time market, is the last market opportunity to balance production and consumption. The gate closure of this market is typically in the range between 30 min and 1 h before actual energy delivery.

4.1.2 Market Organization

Balancing markets are single-period markets as they take place just minutes before actual energy delivery. Since a multiperiod approach is not needed to clear the balancing market, time indexes are omitted in the formulations throughout this chapter.

Balancing markets should take into account network limitations as the network is a fundamental physical component of the electric energy system. However, in this chapter, for the sake of clarity and simplicity the network is not initially represented, i.e., we consider an unlimited transmission capacity. We revisit this assumption in Sect. 4.6.

In fact, balancing markets are partly needed due to transmission bottlenecks and transmission technical issues, which may straddle cheap production in some areas and force the use of expensive units elsewhere. These production allocation adjustments may actually take place in balancing markets.

In the following we consider that the market organization includes just two market floors:

1. The day-ahead market, which clears 12 h prior to the first hour of the delivery period.
2. A series of 24 balancing markets, each one cleared half an hour prior to energy delivery.

This market organization is sketched in Fig. 4.1. Notice that no adjustment market is considered for simplicity. If such a market exists, the day-ahead outcomes should be adjusted considering the adjustment market outcomes before analyzing the balancing markets.

Finally, it should be observed that hourly trading periods are considered in this chapter. For this reason, we will equivalently refer to 1 MWh of energy and 1 MW

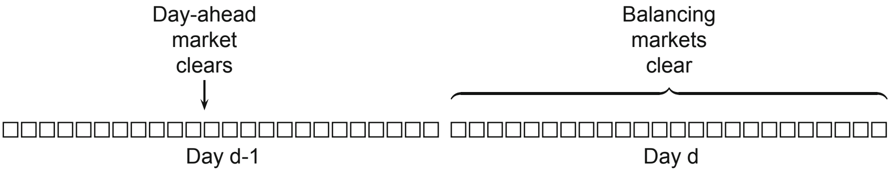


Fig. 4.1 Market decision timeline: one day-ahead market and 24 balancing markets

of power, since a constant power output of that magnitude generates 1 MWh in a 1-hour period.

4.2 Balancing Market Auction

In this section, the basic formulation of a balancing market auction is introduced. Such formulation refers to the so-called *one-price* or *single-price* imbalance settlement, which is adopted in many markets, especially in the USA.

4.2.1 Introduction

Once the day-ahead market is cleared, hourly production levels are assigned to generators and hourly consumption levels to demands. In addition, hourly clearing prices are derived. The reader is referred to Chap. 3 for a detailed description of the day-ahead market.

For the purpose of balancing markets, generators are divided into two sets:

1. Dispatchable generators that are denominated below *balancing generators*.
2. Stochastic generators, with no control over their production levels, e.g., wind- or solar-based generators.

The symbols used in the derivations below are defined in the following for clarity.

$P^{\mathrm{S}}_{\mathrm{B}i}$ is the scheduled power generation level for balancing generator i in the day-ahead market. Superscript S stands for *scheduled* and subscript B for *balancing*.

$P^{\mathrm{S}}_{\mathrm{B}}$ is the total scheduled power generation for all balancing generators in the day-ahead market.

$P^{\mathrm{S}}_{\mathrm{U}j}$ is the scheduled power generation level for stochastic generator j in the day-ahead market. Subscript U stands for *undispatchable*.

$P^{\mathrm{S}}_{\mathrm{U}}$ is the total scheduled power generation for all stochastic generators in the day-ahead market.

$P^{\mathrm{S}}_{\mathrm{D}k}$ is the scheduled power consumption level for demand k in the day-ahead market. Subscript D stands for *demand*.

$P^{\mathrm{S}}_{\mathrm{D}}$ is the total scheduled power consumption in the day-ahead market.

$P_{\mathrm{N}}^{\mathrm{S}}$ is the total scheduled net power consumption (demand minus stochastic production) in the day-ahead market. Subscript N stands for *net*.

λ^{S} is the clearing price in the day-ahead market.

Clearly,

$$\sum_{i \in \Xi_{\mathrm{B}}} P_{\mathrm{B}i}^{\mathrm{S}} = P_{\mathrm{B}}^{\mathrm{S}}, \tag{4.1a}$$

$$\sum_{j \in \Xi_{\mathrm{U}}} P_{\mathrm{U}j}^{\mathrm{S}} = P_{\mathrm{U}}^{\mathrm{S}}, \tag{4.1b}$$

$$\sum_{k \in \Xi_{\mathrm{D}}} P_{\mathrm{D}k}^{\mathrm{S}} = P_{\mathrm{D}}^{\mathrm{S}}, \tag{4.1c}$$

where Ξ_{B}, Ξ_{U}, and Ξ_{D} are the sets of indices of balancing generators, stochastic generators, and demands, respectively.

Since the day-ahead market is balanced, it holds that

$$P_{\mathrm{B}}^{\mathrm{S}} = P_{\mathrm{N}}^{\mathrm{S}} = P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}}. \tag{4.2}$$

The balancing generators producing electric energy have control over their production levels and can either sell additional energy in the balancing market or buy back the energy already sold in the day-ahead market. Needless to say, they do so for a profit. Market rules generally bind balancing generators to provide both up-regulation (selling additional energy) and down-regulation (buying back energy already sold).

We assume that the bid of balancing generator i in the balancing market is characterized by the following parameters:

$\lambda_{\mathrm{B}i}^{\mathrm{U}}$ is the cost offer of balancing generator i for additional production at the balancing stage. The superscript U stands for *up-regulation*.

$P_{\mathrm{B}i}^{\mathrm{U,max}}$ is the capacity of production of balancing generator i at the balancing stage.

$\lambda_{\mathrm{B}i}^{\mathrm{D}}$ is the price offer of balancing generator i for repurchase at the balancing stage of own production scheduled at the day-ahead market. The superscript D stands for *down-regulation*.

$P_{\mathrm{B}i}^{\mathrm{D,max}}$ is the maximum quantity of scheduled own production that balancing generator i offers to repurchase at the balancing stage.

The activation of an up-regulation bid from a balancing generator implies that production, originally left out of the day-ahead market dispatch, be increased with a relatively short notice. Since such production is costlier than the generation dispatched day-ahead, it must hold that

$$\lambda_{\mathrm{B}i}^{\mathrm{U}} \geq \lambda^{\mathrm{S}}. \tag{4.3}$$

On the other hand, the submission of a down-regulation bid for repurchase of energy implies that the producer is willing to decrease (repurchase) part of its production dispatched at the day-ahead stage. Clearly, such a repurchase pays off only if it is

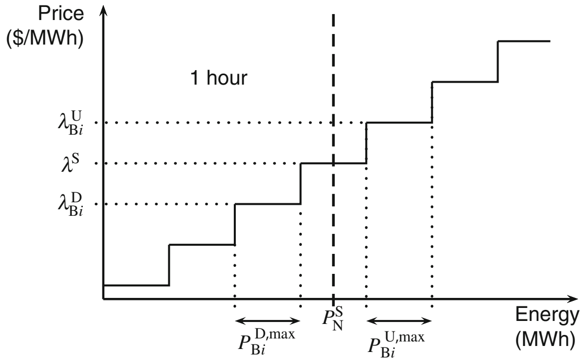


Fig. 4.2 Balancing supply function provided by all balancing generators

at a price not greater than the generation cost, which in turn must be lower than the day-ahead price λ^{S}. Therefore it holds that

$$\lambda_{Bi}^{D} \leq \lambda^{S}. \tag{4.4}$$

Considering jointly all balancing generators, the stepwise balancing supply function depicted in Fig. 4.2 is obtained. Observe that if the net demand level is P_{N}^{S}, the clearing price is λ^{S}.

Stochastic generators, e.g., wind- or solar-based generators, have no control over their production levels and thus they generally deviate from their scheduled productions in the day-ahead market. The production level at balancing time of stochastic generator j is denoted by P_{Uj}^{B}, and generally,

$$P_{Uj}^{B} \neq P_{Uj}^{S}. \tag{4.5}$$

Superscript B in the earlier expression stands for *balancing market*.

Demands are considered to have no control over their consumption levels. However, we revisit this assumption in Sect. 4.4. Demand k at balancing time has a demand level denoted by P_{Dk}^{B}, and generally,

$$P_{Dk}^{B} \neq P_{Dk}^{S}. \tag{4.6}$$

The total up- and down-regulation provided by the balancing generators, P_{B}^{U} and P_{B}^{D}, respectively, are

$$P_{B}^{U} = \sum_{i \in \Xi_{B}} P_{Bi}^{U}, \tag{4.7a}$$

$$P_{B}^{D} = \sum_{i \in \Xi_{B}} P_{Bi}^{D}. \tag{4.7b}$$

The total generation from balancing generators, P_{B}^{B}, from stochastic generators, P_{U}^{B}, the total demand, P_{D}^{B}, and the net demand, P_{N}^{B}, at balancing time are, respectively,

$$P_{\mathrm{B}}^{\mathrm{B}} = \sum_{i \in \Xi_{\mathrm{B}}} \left(P_{\mathrm{B}i}^{\mathrm{S}} + P_{\mathrm{B}i}^{\mathrm{U}} - P_{\mathrm{B}i}^{\mathrm{D}} \right) = P_{\mathrm{B}}^{\mathrm{S}} + P_{\mathrm{B}}^{\mathrm{U}} - P_{\mathrm{B}}^{\mathrm{D}}, \tag{4.8a}$$

$$P_{\mathrm{U}}^{\mathrm{B}} = \sum_{j \in \Xi_{\mathrm{U}}} P_{\mathrm{U}j}^{\mathrm{B}}, \tag{4.8b}$$

$$P_{\mathrm{D}}^{\mathrm{B}} = \sum_{k \in \Xi_{\mathrm{D}}} P_{\mathrm{D}k}^{\mathrm{B}}, \tag{4.8c}$$

$$P_{\mathrm{N}}^{\mathrm{B}} = P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{B}}. \tag{4.8d}$$

The different form of (4.8a) with respect to (4.8b) and (4.8c) depends on the fact that $P_{\mathrm{B}i}^{\mathrm{U}}$ and $P_{\mathrm{B}i}^{\mathrm{D}}$ represent *adjustments* at the balancing stage with respect to the day-ahead schedule. On the contrary, $P_{\mathrm{U}j}^{\mathrm{B}}$ and $P_{\mathrm{D}k}^{\mathrm{B}}$ denote the actual total production and consumption in the balancing market, respectively.

4.2.2 Auction Formulation

The simplest form of a balancing auction to match production and consumption and to minimize the balancing costs for the system is

$$\underset{P_{\mathrm{B}i}^{\mathrm{U}}, P_{\mathrm{B}i}^{\mathrm{D}}}{\text{Min.}} \quad \sum_{i \in \Xi_{\mathrm{B}}} \lambda_{\mathrm{B}i}^{\mathrm{U}} P_{\mathrm{B}i}^{\mathrm{U}} - \lambda_{\mathrm{B}i}^{\mathrm{D}} P_{\mathrm{B}i}^{\mathrm{D}} \tag{4.9a}$$

$$\text{s.t.} \quad P_{\mathrm{B}}^{\mathrm{S}} + \sum_{i \in \Xi_{\mathrm{B}}} P_{\mathrm{B}i}^{\mathrm{U}} - P_{\mathrm{B}i}^{\mathrm{D}} = P_{\mathrm{N}}^{\mathrm{B}} : \quad \lambda^{\mathrm{B}}, \tag{4.9b}$$

$$0 \leq P_{\mathrm{B}i}^{\mathrm{U}} \leq P_{\mathrm{B}i}^{\mathrm{U,max}}, \quad \forall i, \tag{4.9c}$$

$$0 \leq P_{\mathrm{B}i}^{\mathrm{D}} \leq P_{\mathrm{B}i}^{\mathrm{D,max}}, \quad \forall i, \tag{4.9d}$$

where λ^{B} is the dual variable associated with the balance constraint (4.9b), i.e., the marginal price, see Sect. B.2.

It is important to note that auction (4.9) clears the market for the total net demand, i.e., demand minus stochastic production, at the balancing stage. In a market without stochastic production, balancing markets would only serve to balance the deviations stemming from the demand and equipment (generating unit or transmission line) failures.

The solution of auction (4.9) provides each balancing generator i with the cleared production increase, $P_{\mathrm{B}i}^{\mathrm{U}}$, or decrease, $P_{\mathrm{B}i}^{\mathrm{D}}$, at the balancing stage, as well as the price λ^{B}. In a balancing market employing the one-price system as in this case, all market agents sell or buy energy at this marginal price.

Furthermore, from the balancing auction formulation (4.9), and because of the relationships (4.3) and (4.4), note that if $P_{\mathrm{N}}^{\mathrm{B}} > P_{\mathrm{N}}^{\mathrm{S}}$ (up-regulation), then $\lambda^{\mathrm{B}} \geq \lambda^{\mathrm{S}}$, else if $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$ (down-regulation), then $\lambda^{\mathrm{B}} \leq \lambda^{\mathrm{S}}$, and if $P_{\mathrm{N}}^{\mathrm{B}} = P_{\mathrm{N}}^{\mathrm{S}}$ (no imbalance), then $\lambda^{\mathrm{B}} = \lambda^{\mathrm{S}}$. Finally, note that auction (4.9) ensures power balance at balancing

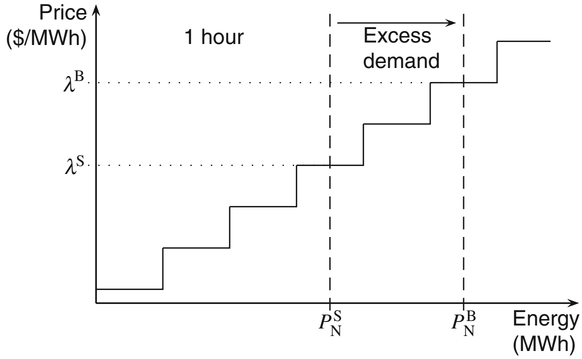


Fig. 4.3 Excess demand at balancing time: the net demand at balancing time (P_{N}^{B}) exceeds the scheduled net demand at the day-ahead market (P_{N}^{S})

time, i.e.,

$$P_{B}^{B} = P_{N}^{B} = P_{D}^{B} - P_{U}^{B}. \tag{4.10}$$

The different imbalance alternatives that can occur are analyzed in the two following subsections.

4.2.3 *Excess Consumption*

If at balancing time total net demand exceeds total scheduled generation, we have an *excess demand* situation. This situation is depicted in Fig. 4.3 and analyzed later.

In an excess demand situation, $P_{N}^{B} > P_{N}^{S}$ and $\lambda^{B} \geq \lambda^{S}$. The extra production needed, $\left(P_{N}^{B} - P_{N}^{S}\right)$, is produced by the balancing generators, whose total balancing revenue is

$$\left(P_{N}^{B} - P_{N}^{S}\right) \lambda^{B} > 0. \tag{4.11}$$

Regarding demands and stochastic generators, the four alternatives discussed later are possible.

1. If stochastic generator j deviates producing below its scheduled production in the day-ahead market, it has to buy energy. The amount

$$\left(P_{Uj}^{B} - P_{Uj}^{S}\right) \lambda^{B} < 0 \tag{4.12}$$

   is the negative revenue (payment) incurred by stochastic generator j for buying energy in the balancing market. Note that the additional energy required is bought at a higher price than the price in the day-ahead market, which entails an extra loss of revenue for the generator.
2. On the other hand, if stochastic generator j deviates producing above its scheduled production in the day-ahead market, it has to sell this excess energy. The amount

$$\left(P_{Uj}^{B} - P_{Uj}^{S}\right) \lambda^{B} > 0 \tag{4.13}$$

is the revenue achieved by stochastic generator j for selling its excess energy in the balancing market. Note that this additional energy is sold at a higher price than the price in the day-ahead market, which entails an additional revenue for the generator. This is so because this stochastic generator (although involuntarily) helps restoring the system balance.

3. If demand k deviates increasing its consumption, the amount

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} > 0 \tag{4.14}$$

represents a payment by this demand to buy the additional energy required. Note that this additional energy is bought at a higher price than the price in the day-ahead market, which entails an extra cost for the deviating demand.

4. On the other hand, if demand k deviates decreasing its consumption, the amount

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} < 0 \tag{4.15}$$

represents a negative payment (revenue) to demand k for selling back the excess energy. Note that this excess energy is sold back at a higher price than the price paid to buy it in the day-ahead market, which constitutes an extra revenue. This is so because this demand (although involuntarily) helps restoring the system balance.

To sum up, the required increase in energy production is

$$\begin{aligned}
&\sum_{k \in \Xi_{\mathrm{D}}} \left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) - \sum_{j \in \Xi_{\mathrm{U}}} \left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \\
&\quad = \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{D}}^{\mathrm{S}}\right) - \left(P_{\mathrm{U}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{S}}\right) \\
&\quad = \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{B}}\right) - \left(P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}}\right) \\
&\quad = P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}.
\end{aligned} \tag{4.16}$$

Therefore, the total payment due to deviations by demands and stochastic generators is

$$\left(P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}\right) \lambda^{\mathrm{B}}, \tag{4.17}$$

which coincides with the total payment for deviations to balancing generators given by (4.11).

4.2.4 Excess Production

If at balancing time total production exceeds total net demand, we have an *excess production* situation. This situation is depicted in Fig. 4.4. In an excess production situation, $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$ and $\lambda^{\mathrm{B}} \leq \lambda^{\mathrm{S}}$.

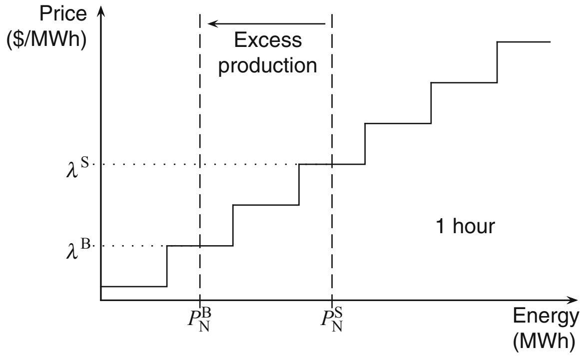


Fig. 4.4 Excess production at balancing time: the scheduled net demand (P_N^S) in the day-ahead market exceeds the net demand at balancing time (P_N^B)

As generally required by market rules, balancing generators need to buy back the excess production $P_N^S - P_N^B$ and pay for it the amount

$$\left(P_N^S - P_N^B\right)\lambda^B > 0. \tag{4.18}$$

Regarding demands and stochastic generators, the four alternatives discussed below are possible.

1. If stochastic generator j deviates producing below its scheduled production in the day-ahead market, it has to buy energy. The quantity

$$\left(P_{Uj}^B - P_{Uj}^S\right)\lambda^B < 0 \tag{4.19}$$

   is the negative revenue (payment) incurred by the stochastic generator for buying energy. Note that the additional energy required is bought at a lower price than the price in the day-ahead market, which constitutes an extra revenue. This is so because this stochastic generator (although involuntarily) helps restoring the system balance.
2. On the other hand, if stochastic generator j deviates producing above its scheduled production in the day-ahead market, it has to sell this excess energy. The amount

$$\left(P_{Uj}^{B} - P_{Uj}^S\right)\lambda^B > 0 \tag{4.20}$$

   is the revenue achieved by the stochastic producer for selling its excess energy. Note that the additional energy is sold at a lower price than the price in the day-ahead market, which constitutes an opportunity loss.
3. If demand k deviates increasing its consumption, the value

$$\left(P_{Dk}^B - P_{Dk}^S\right)\lambda^B > 0 \tag{4.21}$$

   represents a payment by the demand to buy the additional energy required. Note that this additional energy is bought at a lower price than the price in the day-ahead market, which ensures lower costs for the demand than if all the consumption was bought at the day-ahead market. This is so because this demand (although involuntarily) helps restoring the system balance.
4. On the other hand, if demand k deviates decreasing its consumption, the quantity

$$\left(P_{Dk}^B - P_{Dk}^S\right)\lambda^B < 0 \tag{4.22}$$

represents a negative payment (revenue) to the demand for selling back the excess energy. Note that this excess demand is sold back at a lower price than the price paid to buy it in the day-ahead market, which entails an opportunity loss.

To sum up, the required decrease in energy production is

$$\begin{aligned}
&\sum_{j \in \Xi_{\mathrm{U}}} \left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) - \sum_{k \in \Xi_{\mathrm{D}}} \left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \\
&\quad = \left(P_{\mathrm{U}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{S}}\right) - \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{D}}^{\mathrm{S}}\right) \\
&\quad = \left(P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}}\right) - \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{B}}\right) \\
&\quad = P_{\mathrm{N}}^{\mathrm{S}} - P_{\mathrm{N}}^{\mathrm{B}} > 0.
\end{aligned} \tag{4.23}$$

Thus, the total revenue by demands and stochastic generators for selling back energy due to deviations is

$$\left(P_{\mathrm{N}}^{\mathrm{S}} - P_{\mathrm{N}}^{\mathrm{B}}\right) \lambda^{\mathrm{B}}, \tag{4.24}$$

which coincides with the total payment due to deviations by balancing generators given in (4.18).

4.2.5 Payments and Revenues

Payment and revenues are calculated as follows.

1. The revenue for balancing generator i is given by

$$P_{\mathrm{B}i}^{\mathrm{S}} \lambda^{\mathrm{S}} + \left(P_{\mathrm{B}i}^{U} - P_{\mathrm{B}i}^{\mathrm{D}}\right) \lambda^{\mathrm{B}}. \tag{4.25}$$

2. The revenue for stochastic generator j is given by

$$P_{\mathrm{U}j}^{\mathrm{S}} \lambda^{\mathrm{S}} + \left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \lambda^{\mathrm{B}}. \tag{4.26}$$

3. The payment by demand k is given by

$$P_{\mathrm{D}k}^{\mathrm{S}} \lambda^{\mathrm{S}} + \left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{B}}. \tag{4.27}$$

The total payment by demands is

$$\begin{aligned}
&\lambda^{\mathrm{S}} \sum_{k \in \Xi_{\mathrm{D}}} P_{\mathrm{D}k}^{\mathrm{S}} + \lambda^{\mathrm{B}} \sum_{k \in \Xi_{\mathrm{D}}} \left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \\
&\quad = \lambda^{\mathrm{S}} P_{\mathrm{D}}^{\mathrm{S}} + \lambda^{\mathrm{B}} \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{D}}^{\mathrm{S}}\right).
\end{aligned} \tag{4.28}$$

The total revenue for generators is

$$\begin{aligned}\lambda^{\mathrm{S}} \sum_{i \in \Xi_{\mathrm{B}}} P_{\mathrm{B}i}^{\mathrm{S}} + \lambda^{\mathrm{B}} \sum_{i \in \Xi_{\mathrm{B}}} \left(P_{\mathrm{B}i}^{\mathrm{U}} - P_{\mathrm{B}i}^{\mathrm{D}}\right) & \\ + \lambda^{\mathrm{S}} \sum_{j \in \Xi_{\mathrm{U}}} P_{\mathrm{U}j}^{\mathrm{S}} + \lambda^{\mathrm{B}} \sum_{j \in \Xi_{\mathrm{U}}} \left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) & \\ = \lambda^{\mathrm{S}} \left(P_{\mathrm{B}}^{\mathrm{S}} + P_{\mathrm{U}}^{\mathrm{S}}\right) + \lambda^{\mathrm{B}} \left[\left(P_{\mathrm{B}}^{\mathrm{U}} - P_{\mathrm{B}}^{\mathrm{D}}\right) + \left(P_{\mathrm{U}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{S}}\right)\right] & \\ = \lambda^{\mathrm{S}} P_{\mathrm{D}}^{\mathrm{S}} + \lambda^{\mathrm{B}} \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{D}}^{\mathrm{S}}\right), & \qquad (4.29)\end{aligned}$$

where the last equality is a result of the balance Eqs. (4.2) and (4.9b). Since the right hand sides of (4.29) and of (4.28) coincide, the balancing market is revenue adequate, i.e., it does not involve losses for the market operator.

In some American balancing markets the whole energy interchange takes place at balancing prices, i.e.,

1. The revenue for balancing generator i is given by

$$P_{\mathrm{B}i}^{\mathrm{B}} \lambda^{\mathrm{B}}. \qquad (4.30)$$

2. The revenue for stochastic generator j is given by

$$P_{\mathrm{U}j}^{\mathrm{B}} \lambda^{\mathrm{B}}. \qquad (4.31)$$

3. The payment by demand k is given by

$$P_{\mathrm{D}k}^{\mathrm{B}} \lambda^{\mathrm{B}}, \qquad (4.32)$$

where

$P_{\mathrm{B}i}^{\mathrm{B}}$ is the actual power generation level for balancing generator i.
$P_{\mathrm{U}j}^{\mathrm{B}}$ is the actual power generation level for stochastic generator j.
$P_{\mathrm{D}k}^{\mathrm{B}}$ is the actual power consumption level for demand k.

The earlier pricing scheme also results in revenue adequacy.

Two examples that illustrate the functioning of a balancing market auction are provided below. The examples are based on a simplified power system, and consider the cases of excess consumption and production, respectively.

Example 4.1 (Balancing auction with excess consumption) To illustrate the functioning of a balancing auction, we consider a simple system with three dispatchable generators (units B1, B2, and B3), two stochastic generators (U1 and U2) and two loads (D1 and D2). The outcome of the day-ahead market clearing is shown in Table 4.1.

Half of the total load, $P_{\mathrm{D}}^{\mathrm{S}} = 140$ MWh, is covered by dispatchable producers, which are scheduled $P_{\mathrm{B}}^{\mathrm{S}} = 70$ MWh in total. Likewise, stochastic producers are scheduled $P_{\mathrm{U}}^{\mathrm{S}} = 70$ MWh. The scheduled net demand is therefore $P_{\mathrm{N}}^{\mathrm{S}} = P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}} =$ 70 MWh. Besides, it is useful to remark that the dispatchable producers' installed capacity is 50, 50, and 70 MW for units B1, B2, and B3, respectively. Hence, unit B1 is fully scheduled at the day-ahead market.

Table 4.1 Results of the day-ahead market clearing

Variable		Value
P^S (MWh)	Unit B1	50
	Unit B2	20
	Unit B3	0
	Unit U1	40
	Unit U2	30
	Load D1	40
	Load D2	100
λ^S (\$/MWh)		20

Table 4.2 Realization of stochastic production and consumption at the balancing stage, and deviation from the day-ahead dispatch

Participant	P^B (MWh)	$P^B - P^S$ (MWh)
Unit U1	50	10
Unit U2	20	– 10
Load D1	35	– 5
Load D2	120	20

Table 4.3 Up-regulation offers of the dispatchable generators in the balancing market

Unit	λ^U (\$/MWh)	$P^{U,max}$ (MWh)
B1	–	–
B2	30	10
B3	50	50

At the balancing stage, stochastic plants and loads produce and consume as shown in Table 4.2.

As the reader can notice, unit U1 is producing 10 MWh in excess of its day-ahead schedule. On the contrary, generator U2 has an underproduction of the same amount. As far as the loads are concerned, D1 is consuming 5 MWh less than scheduled day-ahead, while D2 exceeds the day-ahead demand schedule by 20 MWh. The total net demand at the balancing market is

$$P_N^B = \left(P_{D1}^B + P_{D2}^B\right) - \left(P_{U1}^B + P_{U2}^B\right) = 85\,\text{MWh}\,. \tag{4.33}$$

Since $P_N^B > P_N^S$, the system is in a situation of excess consumption.

The offers made by dispatchable producers in the balancing market are included in Table 4.3.

Since the system needs to increase the total production to restore balance, it suffices to consider only the sale offers in this example. Notice that unit B1 is not offering to sell any additional production, since it was fully dispatched (i.e., dispatched at its production capacity) at the day-ahead market.

Under these assumptions, the auction in (4.9) translates to

$$\min_{P_{B2}^U, P_{B3}^U} \quad 30P_{B2}^U + 50P_{B3}^U \tag{4.34a}$$

$$\text{s.t.} \quad 70 + P_{B2}^U + P_{B3}^U = 85 \quad : \lambda^B, \tag{4.34b}$$

$$0 \le P_{B2}^U \le 10, \tag{4.34c}$$

$$0 \le P_{B3}^U \le 50. \tag{4.34d}$$

Table 4.4 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	500	900
Unit B3	0	250	250
Unit U1	800	500	1300
Unit U2	600	– 500	100
Load D1	– 800	250	– 550
Load D2	– 2000	– 1000	– 3000

Clearly, the optimal solution consists in fully dispatching generator B2, which has the cheaper offer; the remainder of the consumption is to be covered by the costlier unit B3:

$$P^{\mathrm{U}}_{\mathrm{B2}} = 10\,\mathrm{MWh}, \tag{4.35}$$

$$P^{\mathrm{U}}_{\mathrm{B3}} = 5\,\mathrm{MWh}. \tag{4.36}$$

The dual price of (4.34b), i.e., the balancing market price, is equal to the per unit cost of the marginal generator, i.e.,

$$\lambda^{\mathrm{B}} = \lambda^{\mathrm{U}}_{\mathrm{B3}} = \$50\,/\mathrm{MWh}, \tag{4.37}$$

which is higher than the day-ahead market price λ^{S}.

Revenues and payments for all the producers and loads are summarized in Table 4.4.

The first column provides the financial results for the participants in the day-ahead market, which are obtained by multiplying the day-ahead schedule P^{S} with the market clearing price λ^{S} in that market. The results for the balancing stage, included in the second column, are calculated as the multiplication of the schedule deviation $P^{\mathrm{B}} - P^{\mathrm{S}}$ and the balancing market price λ^{B}. Since $\lambda^{\mathrm{B}} > \lambda^{\mathrm{S}}$, the balancing market rewards the producers that, voluntarily or not, are increasing their production at the balancing stage (B2, B3, and U1). Indeed, these producers are paid at the higher price $\lambda^{\mathrm{B}} = \$50\,/\mathrm{MWh}$ as compared to the day-ahead price $\lambda^{\mathrm{S}} = \$20\,/\mathrm{MWh}$. Similarly, load D1, which is decreasing its consumption, sells back 5 MWh of its scheduled consumption at the price λ^{B}, which is higher than the price of initial purchase λ^{S}. On the contrary, the unit U2 and the load D2, which are underproducing and overconsuming with respect to their day-ahead schedule, are penalized as they must purchase power at a higher cost than at the day-ahead market. In particular, unit U2 realizes a total revenue of only $100, despite producing 20 MWh. In comparison, the balancing unit B3, which produces only 5 MWh, receives a payment of $250 from the market operator. As a final remark, we underline that the total payments collected and made by the market operator are zero at both market stages. Indeed, the sums of the quantities in the first and the second columns of Table 4.4 are both zero.

Table 4.5 Realization of stochastic production and consumption at the balancing stage, and deviation from the day-ahead dispatch

Participant	P^{B} (MWh)	$P^{\mathrm{B}} - P^{\mathrm{S}}$ (MWh)
Unit U1	65	25
Unit U2	25	− 5
Load D1	22	− 18
Load D2	120	20

Table 4.6 Down-regulation bids of the dispatchable generators in the balancing market

Unit	λ^{D} (\$/MWh)	$P^{\mathrm{D,max}}$ (MWh)
B1	8	40
B2	15	10
B3	–	–

Example 4.2 (Balancing auction with excess production) Let us consider the simplified power system already introduced in Example 4.1, with the day-ahead market clearing summarized in Table 4.1.

In this case, at the balancing stage, the actual productions from stochastic generators and the demand realize as indicated in Table 4.5.

As in Example 4.1, there are an overproducing unit (U1) and an underproducing one (U2), a load that consumes below schedule (D1) and another load that consumes above schedule (D2). Differently from the previous example, though, the total net demand at the balancing market

$$P_{\mathrm{N}}^{\mathrm{B}} = \left(P_{\mathrm{D1}}^{\mathrm{B}} + P_{\mathrm{D2}}^{\mathrm{B}}\right) - \left(P_{\mathrm{U1}}^{\mathrm{B}} + P_{\mathrm{U2}}^{\mathrm{B}}\right) = 52\,\mathrm{MWh} \tag{4.38}$$

is lower than its corresponding quantity in the day-ahead market: $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$. Therefore, the system is in a situation of excess production.

Table 4.6 shows the bids submitted by balancing generators in the balancing market. In this case only the bids for down-regulation need to be shown.

Notice that unit B3 is not able to bid for repurchasing scheduled production, since it was not dispatched in the day-ahead market (Table 4.1).

In this case, the balancing market auction (4.9) writes as follows

$$\underset{P_{\mathrm{B1}}^{\mathrm{D}},P_{\mathrm{B2}}^{\mathrm{D}}}{\mathrm{Min.}} \quad -8P_{\mathrm{B1}}^{\mathrm{D}} - 15P_{\mathrm{B2}}^{\mathrm{D}} \tag{4.39a}$$

$$\text{s.t.} \quad 70 - P_{\mathrm{B1}}^{\mathrm{D}} - P_{\mathrm{B2}}^{\mathrm{D}} = 52 \quad : \lambda^{\mathrm{B}}, \tag{4.39b}$$

$$0 \le P_{\mathrm{B1}}^{\mathrm{D}} \le 40, \tag{4.39c}$$

$$0 \le P_{\mathrm{B2}}^{\mathrm{D}} \le 10. \tag{4.39d}$$

Since unit B2 has a higher benefit in repurchasing energy, the optimal clearing of the balancing market implies that this unit repurchases as much as possible (10 MWh), while the remainder of excess production is assigned to producer B1:

$$P_{\mathrm{B1}}^{\mathrm{D}} = 8\,\mathrm{MWh}, \tag{4.40}$$

$$P_{\mathrm{B2}}^{\mathrm{D}} = 10\,\mathrm{MWh}. \tag{4.41}$$

Table 4.7 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	– 64	936
Unit B2	400	– 80	320
Unit B3	0	0	0
Unit U1	800	200	1000
Unit U2	600	– 40	560
Load D1	– 800	144	– 656
Load D2	– 2000	– 160	– 2160

The balancing market price, obtained as the dual variable of (4.39b), is given by the benefit per unit of the marginal generator

$$\lambda^{\mathrm{B}} = \$8\,/\mathrm{MWh}, \tag{4.42}$$

which is lower than the day-ahead market price $\lambda^{\mathrm{S}} = \$20/\mathrm{MWh}$.

Table 4.7 summarizes revenues and payments for producers and loads, respectively.

The balancing units B1 and B2 have to pay the market operator for repurchasing energy at the balancing stage. Since this power was initially sold at the day-ahead market at the price $\lambda^{\mathrm{S}} = \$20\,/\mathrm{MWh}$, these producers realize a net profit on the energy they are asked not to produce, as the repurchasing price $\lambda^{\mathrm{B}} = \$8\,/\mathrm{MWh}$ is lower than the initial selling price. Regarding the stochastic generators, unit U1 is penalized in the balancing market, since it sells its production surplus at a lower price than it could have received at the day-ahead market. On the contrary, unit U2 is rewarded, since it repurchases energy at a lower price than it was sold. The situation for the loads is similar, with D2 being rewarded (its additional consumption is paid a lower rate) and D1 being penalized (it sells energy back at a lower price than the purchase price). As in the previous example, we point out that payments and revenues cancel out both at the day-ahead and at the balancing markets.

4.3 Two-Price Imbalance Settlement

The auction and pricing mechanism presented in the previous section, where all the power exchanged in the balancing market is priced at the marginal cost of the power balance constraint (4.9b), describe the functioning of markets based on the so-called one-price imbalance settlement.

The rationale behind one-price balancing markets implies that deviations from the day-ahead schedule are settled at a price that is more favorable than the day-ahead price if the sign of the participant's imbalance is opposite to the sign of the overall system deviation. Indeed, generators producing more power than contracted in the day-ahead market, and demands consuming less power, receive a higher price for their sale in the balancing market if the system is in excess of demand. On the contrary, generators that produce less power than contracted, and loads consuming more, when the system is in excess of production have to pay a price that is lower than the day-ahead market price, thus achieving a profit. The one-price imbalance settlement, therefore, rewards participants that help restore system balance with their deviations, regardless of whether such deviations are wanted or not.

Other balancing markets are designed according to the *two-price* (or *dual-price*) imbalance settlement principle. In this type of market design, only wanted deviations (i.e., from dispatchable producers) opposite in sign from the system imbalance are rewarded financially with a balancing market price that is more favorable than the day-ahead price. On the contrary, deviations from stochastic producers are either settled at the day-ahead price (if opposite to the system imbalance) or, just like in a one-price market, at the less favorable balancing market price (if in the same direction as the system imbalance). Such a design is common in European electricity markets.

The determination of the optimal dispatch is performed in the same fashion both for the one-price and the two-price balancing markets by solving an optimization problem, whose basic formulation is given in (4.9). While there is no difference in the determination of the clearing price either, which is the dual variable of the power balance constraint (4.9b), the two market designs differ as to how prices apply to conventional and stochastic producers.

In the remainder of this section, we analyze how producers are priced in a balancing market following the two-price design. Since the auction model for the balancing market is equal to that in the one-price case, the notation is the same as in the previous section.

4.3.1 Excess Consumption

As pointed out in Sect. 4.2.3, in the excess demand situation $P_{\mathrm{N}}^{\mathrm{B}} > P_{\mathrm{N}}^{\mathrm{S}}$ and $\lambda^{\mathrm{B}} \geq \lambda^{\mathrm{S}}$. The production needed to cover the difference $\left(P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}\right)$ is generated by the balancing generators. The pricing for these generators is exactly the same as in the one-price settlement, and their total balancing revenue is still given by

$$\left(P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} > 0. \tag{4.43}$$

Contrary to the case of balancing generators, the pricing of demands and stochastic generators in the two-price model differs from the one-price market. This is discussed in the remainder of this section.

1. Stochastic generator j that deviates producing less than its day-ahead schedule has to buy energy. Since such a generator is increasing the total system imbalance, this exchange is settled at the less favorable price λ^{B} for the producer, implying the following negative revenue (payment)

$$\left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} < 0 \tag{4.44}$$

   for repurchasing energy in the balancing market. As in the one-price model, energy is purchased at a higher price than the day-ahead market price, which entails a loss of revenue for the generator.

2. On the other hand, stochastic generator j producing above its scheduled production in the day-ahead market is involuntarily helping restoring the system balance. In the two-price model, the energy in excess is sold at the day-ahead market price, generating the following revenue

$$\left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right)\lambda^{\mathrm{S}} > 0. \tag{4.45}$$

   Differently from the one-price model, there is no price premium for selling this additional energy at the balancing market compared to the day-ahead market.
3. If demand k deviates increasing its consumption, thus increasing the system imbalance, it must pay the following amount

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right)\lambda^{\mathrm{B}} > 0 \tag{4.46}$$

   in order to buy the energy needed to balance its position. Once more, note that this additional energy is bought at a higher price, and thus less favorable, than the day-ahead market price.
4. If demand k decreases its consumption, thus reducing (although involuntarily) the overall system imbalance, its negative payment (revenue) is

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right)\lambda^{\mathrm{S}} < 0 \tag{4.47}$$

   for selling back the energy in excess. Contrary to the one-price system, the two-price settlement entails no extra revenue for the demand involuntarily reducing the system imbalance compared to the day-ahead market.

Differently from the one-price model, the total payment due to deviations by demands and stochastic generators is not a linear function of the required increase $P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}$ in energy production, but it depends on the sign and magnitude of the individual deviations. Denoting positive and negative part functions by, respectively,

$$[\,\cdot\,]^{+} = \max\left(\cdot, 0\right), \tag{4.48}$$

$$[\,\cdot\,]^{-} = -\min\left(\cdot, 0\right), \tag{4.49}$$

the net of the payments that the market operator receives from demands and stochastic generators is

$$\begin{aligned}
&\Bigg(\sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{+} + \sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{-}\Bigg)\lambda^{\mathrm{B}}\\
&\quad - \Bigg(\sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-} + \sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+}\Bigg)\lambda^{\mathrm{S}}\\
&\quad \geq \Bigg(\sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{+} + \sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{-}\Bigg)\lambda^{\mathrm{B}}\\
&\quad - \Bigg(\sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-} + \sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+}\Bigg)\lambda^{\mathrm{B}} = (P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}})\lambda^{\mathrm{B}},
\end{aligned} \tag{4.50}$$

where the inequality is due to the fact that $\lambda^{\mathrm{B}} \geq \lambda^{\mathrm{S}}$ in situations of excess of demand. Since the market operator may receive higher net payments than the total payment for deviations to balancing generators given by (4.43), the two-price model may entail a surplus at the balancing stage for the market operator in the case of excess consumption. More specifically, this surplus is generated whenever at least one stochastic generator produces more or one demand consumes less than scheduled at the day-ahead market.

4.3.1.1 Payments and Revenues

Payments and revenues in the case of excess demand in a two-price model are calculated as follows.

1. The revenue for balancing generator i is given by

$$P_{\mathrm{B}i}^{\mathrm{S}} \lambda^{\mathrm{S}} + \left(P_{\mathrm{B}i}^{\mathrm{B}} - P_{\mathrm{B}i}^{\mathrm{S}}\right) \lambda^{\mathrm{B}}. \tag{4.51}$$

2. The revenue for stochastic generator j is given by

$$P_{\mathrm{U}j}^{\mathrm{S}} \lambda^{\mathrm{S}} + \left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+} \lambda^{\mathrm{S}} - \left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{-} \lambda^{\mathrm{B}}. \tag{4.52}$$

3. The payment by demand k is given by

$$P_{\mathrm{D}k}^{\mathrm{S}} \lambda^{\mathrm{S}} + \left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{+} \lambda^{\mathrm{B}} - \left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-} \lambda^{\mathrm{S}}. \tag{4.53}$$

Because all the transactions at the day-ahead market are settled at price λ^{S}, and since the day-ahead market is balanced, see (4.2), the total day-ahead payment by demands always equals the total day-ahead revenue for generators

$$P_{\mathrm{D}}^{\mathrm{S}} \lambda^{\mathrm{S}} = (P_{\mathrm{B}}^{\mathrm{S}} + P_{\mathrm{U}}^{\mathrm{S}}) \lambda^{\mathrm{S}}. \tag{4.54}$$

Furthermore, the previous subsection showed that there is at least revenue adequacy (total revenue is higher than or equal to total cost) in the balancing market (see (4.50)), with the possibility for the market operator to realize a surplus. Therefore, we conclude that there is revenue adequacy for the aggregation of these two market floors in the case of excess demand.

4.3.2 Excess Production

As in Sect. 4.2.4, in an excess production situation $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$ and $\lambda^{\mathrm{B}} \leq \lambda^{\mathrm{S}}$. The price for balancing generators scheduled to buy the excess production is, just like in the single-price settlement, the marginal price λ^{B} of (4.9b). Therefore, these producers pay at the balancing stage the following amount

$$\left(P_{\mathrm{N}}^{\mathrm{S}} - P_{\mathrm{N}}^{\mathrm{B}}\right) \lambda^{\mathrm{B}} > 0. \tag{4.55}$$

The pricing of demands and stochastic producers follows the rules below.

1. If stochastic producer j produces below its day-ahead market dispatch, it has to repurchase energy at the balancing stage. Since such a deviation, opposite in sign to the overall system imbalance, is involuntary, the less favorable day-ahead market price λ^{S} is charged, resulting in the negative revenue (payment)
$$\left(P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right)\lambda^{\mathrm{S}} < 0 \tag{4.56}$$
for the repurchase. Compared to the case of the one-price settlement, there is no extra revenue involved in this transaction.
2. Else, if stochastic generator j overproduces with respect to the day-ahead market schedule, it has to sell this energy surplus. Since the deviation has the same sign as the overall system deviation, the price λ^{B} applies, resulting in the following revenue
$$\left(P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right)\lambda^{\mathrm{B}} > 0 \tag{4.57}$$
for selling the excess energy. Like in the one-price market settlement, the additional energy is sold at a lower price than the day-ahead market price, thus resulting in an opportunity loss.
3. If the consumption from demand k is higher than scheduled, the day-ahead price applies, resulting in the payment
$$\left(P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right)\lambda^{\mathrm{S}} > 0 \tag{4.58}$$
for purchasing the extra energy. Differently from the one-price settlement described in Sect. 4.2.4, there is no price premium for the demand. This is because the deviation, although contributing to restore the system balance, is involuntary.
4. Finally, if load k instead deviates decreasing its demand, it must sell back its unrealized consumption. Since the market is already in an excess of production, the price λ^{B}, less favorable than the day-ahead price λ^{S}, applies. The negative payment (revenue) for selling back the unrealized consumption is therefore
$$\left(P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right)\lambda^{\mathrm{B}} < 0. \tag{4.59}$$

Similar to the case of excess demand, the amount the market operator has to pay to demands and stochastic producers is not a linear function of the required decrease $P^{\mathrm{B}}_{\mathrm{N}} - P^{\mathrm{S}}_{\mathrm{N}}$ in energy production. This payment is given by

$$\begin{aligned}
&\Big(\sum_{j\in\Xi_{\mathrm{U}}}\left[P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right]^{+} + \sum_{k\in\Xi_{\mathrm{D}}}\left[P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right]^{-}\Big)\lambda^{\mathrm{B}} \\
&\qquad - \Big(\sum_{j\in\Xi_{\mathrm{U}}}\left[P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right]^{-} + \sum_{k\in\Xi_{\mathrm{D}}}\left[P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right]^{+}\Big)\lambda^{\mathrm{S}} \\
&\qquad \le \Big(\sum_{j\in\Xi_{\mathrm{U}}}\left[P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right]^{+} + \sum_{k\in\Xi_{\mathrm{D}}}\left[P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right]^{-}\Big)\lambda^{\mathrm{B}}
\end{aligned}$$

$$-\Big(\sum_{j\in\Xi_{\rm U}}\big[P^{\rm B}_{{\rm U}j}-P^{\rm S}_{{\rm U}j}\big]^{-}+\sum_{k\in\Xi_{\rm D}}\big[P^{\rm B}_{{\rm D}k}-P^{\rm S}_{{\rm D}k}\big]^{+}\Big)\lambda^{\rm B}=\big(P^{\rm S}_{\rm N}-P^{\rm B}_{\rm N}\big)\lambda^{\rm B}, \tag{4.60}$$

where the inequality is due to the fact that $\lambda^{\rm B} \leq \lambda^{\rm S}$ when the system is in surplus of production. Since the term on the right hand side of (4.60) is equal to the payment received from the balancing generators (4.55), it is clear that the market operator realizes a surplus whenever either one stochastic generator produces less than its day-ahead dispatch, or one load consumes more than planned.

4.3.2.1 Payments and Revenues

Payments and revenues in the case of excess production can be formulated as follows.

1. Balancing generator i receives

$$P^{\rm S}_{{\rm B}i}\lambda^{\rm S}+\big(P^{\rm B}_{{\rm B}i}-P^{\rm S}_{{\rm B}i}\big)\lambda^{\rm B}. \tag{4.61}$$

2. For stochastic generator j, the revenues is

$$P^{\rm S}_{{\rm U}j}\lambda^{\rm S}+\big[P^{\rm B}_{{\rm U}j}-P^{\rm S}_{{\rm U}j}\big]^{+}\lambda^{\rm B}-\big[P^{\rm B}_{{\rm U}j}-P^{\rm S}_{{\rm U}j}\big]^{-}\lambda^{\rm S}. \tag{4.62}$$

3. Demand k pays the following amount

$$P^{\rm S}_{{\rm D}k}\lambda^{\rm S}+\big[P^{\rm B}_{{\rm D}k}-P^{\rm S}_{{\rm D}k}\big]^{+}\lambda^{\rm S}-\big[P^{\rm B}_{{\rm D}k}-P^{\rm S}_{{\rm D}k}\big]^{-}\lambda^{\rm B}. \tag{4.63}$$

Following the same reasoning as in Sect. 4.3.1.1, it follows that there is always perfect balance between payments and revenues for the market operator at the day-ahead market stage. Furthermore, as shown in the previous section (see (4.60)), the balancing market, and therefore the aggregation of the two market floors, is always revenue adequate in the case of excess production, with the possibility that the market operator realizes a surplus.

In conclusion, a balancing market with two-price settlement is revenue adequate.

Example 4.1 is revisited below employing a two-price system for settling imbalances in the balancing market.

Example 4.3 (Balancing auction with excess consumption in a two-price market) Let us consider the day-ahead dispatch, the realizations of production and consumption and the regulation bids of Example 4.1, described in Tables 4.1, 4.2 and 4.3, respectively. Since the balancing market auction (4.9) is still valid in a two-price system, the clearing of the balancing market of Example 4.1, summarized in (4.35), (4.36) and (4.37), is still optimal.

Table 4.8 summarizes revenues and payments for all the market participants.

Clearly, the column regarding the day-ahead market is unchanged as compared to the one in Table 4.4. As far as payments and revenues in the balancing market are concerned, we notice that the situation changes with respect to Example 4.1

Table 4.8 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	500	900
Unit B3	0	250	250
Unit U1	800	200	1000
Unit U2	600	– 500	100
Load D1	– 800	100	– 700
Load D2	– 2000	– 1000	– 3000

Table 4.9 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	– 64	936
Unit B2	400	– 80	320
Unit B3	0	0	0
Unit U1	800	200	1000
Unit U2	600	– 100	500
Load D1	– 800	144	– 656
Load D2	– 2000	– 400	– 2400

only for the participants that are involuntarily restoring the system balance with their deviation. Indeed, producer U1, which is producing 10 MWh above its schedule, and load D1, which is consuming 5 MWh less than scheduled, are now only paid $\lambda^{S} = \$20$ /MWh for their deviation. This way, their revenues at the balancing market reduce to \$200 and \$100 from \$500 and \$250, respectively. Contrarily, revenues for the balancing generators (B1, B2, and B3) and payments for the participants that are increasing the total system deviations (U2 and D2) are the same as in Table 4.4 for the one-price market. Finally, we underline that the net balance of payments for the market operator is not 0 at the balancing stage. Indeed, while \$1500 are collected from U2 and D2, only \$1050 are paid to B2, B3, U1, and D1, resulting in a net profit of \$450.

Next, we revisit Example 4.2 employing the two-price system for settling imbalances in the balancing market.

Example 4.4 (Balancing auction with excess production in a two-price market) The day-ahead dispatch, the realization of production and consumption and the down-regulation bids of Example 4.2, included in Tables 4.1, 4.5, and 4.6, are considered again. Once more, we only need to consider revenues and payments, since the optimal market clearing is the same as in the one-price market case of Example 4.2, i.e., the quantities and prices in (4.40), (4.41), and (4.42).

Revenues and payments are summarized in Table 4.9.

Once again, the only difference with respect to Table 4.7 regards the balancing market results for the participants that are involuntarily decreasing the overall system imbalance. Indeed, generator U2, which is producing less than scheduled in the day-ahead market, and load D2, which is consuming more than planned, buy energy from the balancing market at the price $\lambda^{S} = \$20$ /MWh instead of at $\lambda^{B} = \$8$ /MWh. For this reason, their payment at the balancing market increase to \$100 and \$400 from \$40 and \$160, respectively. Again, the reader can notice that there is a net profit for the market operator at the balancing stage. Indeed, it receives in total \$644 from B1, B2, U2, and D2, but pays only \$344 to U1 and D1 and, therefore, realizes a net profit of \$300.

4.4 Balancing Auction with Proactive Demand

In this section, the balancing auction mechanism for a single-price market, introduced in Sect. 4.2, is extended to consider flexibility on the demand side.

Proactive demands counteract the net deviation and achieve a profit in doing so. In case of excess demand, a proactive demand sells back energy for a profit, and in doing so contributes to balancing the market and reduces the balancing price moving it toward the day-ahead clearing price, which benefits some balancing market participants.

On the contrary, in case of excess production a proactive demand buys energy at a lower price than the day-ahead price. This way, it contributes to balancing the market and as a result, to increasing the balancing price, which moves toward the day-ahead price. Again, this benefits some balancing market participants.

A balancing auction with proactive demand has the form

$$\underset{P^{\mathrm{U}}_{\mathrm{B}i},P^{\mathrm{D}}_{\mathrm{B}i},P^{\mathrm{U}}_{\mathrm{D}k},P^{\mathrm{D}}_{\mathrm{D}k}}{\text{Min.}} \quad \sum_{i\in\Xi_{\mathrm{B}}} \lambda^{\mathrm{U}}_{\mathrm{B}i} P^{\mathrm{U}}_{\mathrm{B}i} - \lambda^{\mathrm{D}}_{\mathrm{B}i} P^{\mathrm{D}}_{\mathrm{B}i} + \sum_{k\in\Xi_{\mathrm{C}}} \lambda^{\mathrm{U}}_{\mathrm{D}k} P^{\mathrm{U}}_{\mathrm{D}k} - \lambda^{\mathrm{D}}_{\mathrm{D}k} P^{\mathrm{D}}_{\mathrm{D}k} \tag{4.64a}$$

$$\text{s.t.} \quad P^{\mathrm{S}}_{\mathrm{B}} + \sum_{i\in\Xi_{\mathrm{B}}} P^{\mathrm{U}}_{\mathrm{B}i} - P^{\mathrm{D}}_{\mathrm{B}i} = P^{\mathrm{B}}_{\mathrm{N}} + \sum_{k\in\Xi_{\mathrm{C}}} \left(P^{\mathrm{D}}_{\mathrm{D}k} - P^{\mathrm{U}}_{\mathrm{D}k}\right) : \quad \lambda^{\mathrm{B}}, \tag{4.64b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}i} \le P^{\mathrm{U,max}}_{\mathrm{B}i}, \quad \forall i, \tag{4.64c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B}i} \le P^{\mathrm{D,max}}_{\mathrm{B}i}, \quad \forall i, \tag{4.64d}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{D}k} \le P^{\mathrm{U,max}}_{\mathrm{D}k}, \quad k \in \Xi_{\mathrm{C}}, \tag{4.64e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{D}k} \le P^{\mathrm{D,max}}_{\mathrm{D}k}, \quad k \in \Xi_{\mathrm{C}}, \tag{4.64f}$$

where

$P^{\mathrm{U}}_{\mathrm{D}k}$ represents how much demand k is asked to decrease its consumption at the balancing stage. The superscript U stands for *up-regulation*, since demand k is effectively selling power when accepting to reduce its consumption.

$P^{\mathrm{D}}_{\mathrm{D}k}$ represents how much demand k is asked to increase its consumption at the balancing stage. Because demand k is buying additional power, the superscript D stands for *down-regulation*.

$\lambda^{\mathrm{U}}_{\mathrm{D}k}$ is the per unit price offered by demand k to reduce its consumption.

$\lambda^{\mathrm{D}}_{\mathrm{D}k}$ is the per unit price offered by demand k to increase its consumption.

$P^{\mathrm{U,max}}_{\mathrm{D}k}$ is the maximum consumption decrease from demand k.

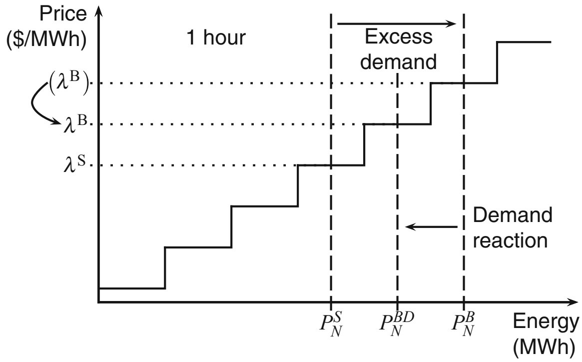


Fig. 4.5 Role of proactive demands in the case of excess consumption

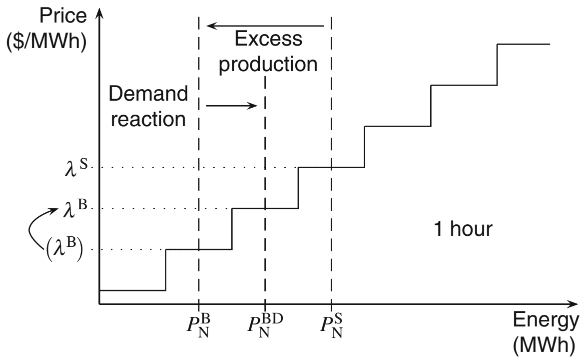


Fig. 4.6 Role of proactive demands in the case of excess production

$P_{Dk}^{D,max}$ is the maximum consumption increase from demand k.

Ξ_C is the set of indices of proactive demands.

The effect of proactive demands is illustrated in Figs. 4.5 and 4.6. Figure 4.5 depicts the action of proactive demands in a case of excess demand. Proactive demands reduce their respective consumptions. As a result, the net consumption including demand response (P_N^{BD}) and the balancing price drop.

Figure 4.6 refers to the excess production case. In that situation, proactive demands increase their respective consumptions. As a result, the net consumption including demand response (P_N^{BD}) and the balancing price rise.

The examples below revisit Examples 4.1 and 4.2, considering a simplified power system where all demands are proactive, and illustrate the functioning and characteristics of a balancing auction with proactive demands.

Example 4.5 (Balancing auction with excess consumption and proactive demands) We consider the same day-ahead dispatch as well as the realization of production and consumption of Example 4.1, included in Tables 4.1 and 4.2.

In addition to the bids submitted by the balancing generators shown in Table 4.3, bids are also submitted to the market by proactive demands. Table 4.10 shows the characteristics of these bids.

Table 4.10 Up-regulation bids of the proactive demands in the balancing market

Demand	λ^{U} (\$/MWh)	$P^{\text{U,max}}$ (MWh)
D1	22	5
D2	24	5

Both demands are willing to deviate marginally from their scheduled consumption for a small profit, given by the price difference between their offer and the day-ahead market price $\lambda^{\text{S}} = \$20\,/\text{MWh}$.

Under these assumptions, the balancing auction (4.64) is the following

$$\underset{P^{\text{U}}_{\text{B2}},P^{\text{U}}_{\text{B3}},P^{\text{U}}_{\text{D1}},P^{\text{U}}_{\text{D2}}}{\text{Min.}} \quad 30P^{\text{U}}_{\text{B2}} + 50P^{\text{U}}_{\text{B3}} + 22P^{\text{U}}_{\text{D1}} + 24P^{\text{U}}_{\text{D2}} \tag{4.65a}$$

$$\text{s.t.} \quad 70 + P^{\text{U}}_{\text{B2}} + P^{\text{U}}_{\text{B3}} = 85 - P^{\text{U}}_{\text{D1}} - P^{\text{U}}_{\text{D2}} \quad : \lambda^{\text{B}}, \tag{4.65b}$$

$$0 \le P^{\text{U}}_{\text{B2}} \le 10, \tag{4.65c}$$

$$0 \le P^{\text{U}}_{\text{B3}} \le 50, \tag{4.65d}$$

$$0 \le P^{\text{U}}_{\text{D1}} \le 5, \tag{4.65e}$$

$$0 \le P^{\text{U}}_{\text{D2}} \le 5. \tag{4.65f}$$

The optimal solution is again evident. Indeed, reducing the consumption from the proactive demands is significantly cheaper than activating up-regulation bids from balancing generators. Thus, the optimal dispatch at the balancing market is the following

$$P^{\text{U}}_{\text{B2}} = 5\,\text{MWh}, \tag{4.66}$$

$$P^{\text{U}}_{\text{D1}} = 5\,\text{MWh}, \tag{4.67}$$

$$P^{\text{U}}_{\text{D2}} = 5\,\text{MWh}. \tag{4.68}$$

The dual variable associated with (4.65b) is equal to the per unit cost of the marginal activated bid, i.e., the one from unit B2, and sets the balancing price

$$\lambda^{\text{B}} = \$30\,/\text{MWh}. \tag{4.69}$$

We underline that the price in (4.69) is significantly lower than the balancing price obtained in Example 4.1.

Clearly, since the balancing market clearing changes when demand response is introduced, revenues and payments are also different from the ones reported in Table 4.16. As Table 4.11 shows, the balancing generators obtain lower revenues in the balancing market than in Example 4.1. This is an effect both of the reduced balancing price and of the lower dispatch of the balancing units. Because the balancing price is lower, unit U1, overproducing with respect to its day-ahead schedule, receives a lower payment than in Example 4.1. On the contrary, underproducing unit U2 pays a lower amount to the market operator. Finally, as a result of their flexibility, loads D1 and D2 are rewarded with a higher revenue and a lower cost in the balancing market, respectively.

Table 4.11 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	150	550
Unit B3	0	0	0
Unit U1	800	300	1100
Unit U2	600	– 300	300
Load D1	– 800	300	– 500
Load D2	– 2000	– 450	– 2450

Table 4.12 Down-regulation bids of the proactive demands in the balancing market

Demand	λ^{D} ($/MWh)	$P^{\mathrm{D,max}}$ (MWh)
D1	16	5
D2	19	10

Example 4.6 (Balancing auction with excess production and proactive demands) Let us again consider the day-ahead dispatch, and the realization of production and consumption of Example 4.2, summarized in Tables 4.1 and 4.5.

The loads submit the bids for down-regulation illustrated in Table 4.12.

The offers submitted by the dispatchable generators in the balancing market are the ones shown in Table 4.6.

Auction (4.64) writes as follows:

$$\min_{P^{\mathrm{D}}_{\mathrm{B1}},P^{\mathrm{D}}_{\mathrm{B2}},P^{\mathrm{D}}_{\mathrm{D1}},P^{\mathrm{D}}_{\mathrm{D2}}} \quad -8P^{\mathrm{D}}_{\mathrm{B1}} - 15P^{\mathrm{D}}_{\mathrm{B2}} - 16P^{\mathrm{D}}_{\mathrm{D1}} - 19P^{\mathrm{D}}_{\mathrm{D2}} \tag{4.70a}$$

$$\text{s.t.} \quad 70 - P^{\mathrm{D}}_{\mathrm{B1}} - P^{\mathrm{D}}_{\mathrm{B2}} = 52 + P^{\mathrm{D}}_{\mathrm{D1}} + P^{\mathrm{D}}_{\mathrm{D2}} \quad : \lambda^{\mathrm{B}}, \tag{4.70b}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1}} \le 40, \tag{4.70c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2}} \le 10, \tag{4.70d}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{D1}} \le 5, \tag{4.70e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{D2}} \le 10. \tag{4.70f}$$

Since the loads have the highest benefit in purchasing the additional power available at the balancing stage, their bids are fully accepted. The extra supply left is bought back by the dispatchable generator B2. The resulting dispatch is

$$P^{\mathrm{D}}_{\mathrm{B2}} = 3\,\mathrm{MWh}, \tag{4.71}$$

$$P^{\mathrm{D}}_{\mathrm{D1}} = 5\,\mathrm{MWh}, \tag{4.72}$$

$$P^{\mathrm{D}}_{\mathrm{D2}} = 10\,\mathrm{MWh}. \tag{4.73}$$

The balancing market price, i.e., the dual variable of (4.70b), is determined on the basis of the benefit of the marginal generator, i.e., unit B2, and equal to

$$\lambda^{\mathrm{B}} = \$15\,/\mathrm{MWh}. \tag{4.74}$$

Table 4.13 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	– 45	355
Unit B3	0	0	0
Unit U1	800	375	1175
Unit U2	600	– 75	525
Load D1	– 800	195	– 605
Load D2	– 2000	– 450	– 2450

The reader should notice that this price is higher than the one in (4.42). This results from deploying demand response in a situation of excess production.

Revenues and payments are summarized in Table 4.13.

As a result of the increase in price for balancing power as compared to Example 4.2, sellers of energy, i.e., unit U1 and load D1, are less penalized when demand response is introduced.

4.5 Balancing Auction with Stepwise Offers

In the previous sections, for the sake of simplicity, producers were bound to submit a single price offer in the balancing market. This assumption is revisited in this section, where the auction model is extended so as to consider multiblock, stepwise offers from the producers.

Balancing generators offer generally a number of power blocks at increasing marginal offer costs, which represent the actual cost of energy production. This is recognized in the formulation of the balancing auction

$$\underset{P^{\mathrm{U}}_{\mathrm{B}i,s},P^{\mathrm{D}}_{\mathrm{B}i,s}}{\text{Min.}} \quad \sum_{i\in\Xi_{\mathrm{B}}}\sum_{s\in\Xi_{\mathrm{B}i}} \lambda^{\mathrm{U}}_{\mathrm{B}i,s}P^{\mathrm{U}}_{\mathrm{B}i,s} - \lambda^{\mathrm{D}}_{\mathrm{B}i,s}P^{\mathrm{D}}_{\mathrm{B}i,s} \tag{4.75a}$$

$$\text{s.t.} \quad P^{\mathrm{S}}_{\mathrm{B}} + \sum_{i\in\Xi_{\mathrm{B}}}\sum_{s\in\Xi_{\mathrm{B}i}} P^{\mathrm{U}}_{\mathrm{B}i,s} - P^{\mathrm{U}}_{\mathrm{B}i,s} = P^{\mathrm{B}}_{\mathrm{N}} : \quad \lambda^{\mathrm{B}}, \tag{4.75b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}i,s} \le P^{\mathrm{U,max}}_{\mathrm{B}i,s}, \quad \forall i,s, \tag{4.75c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B}i,s} \le P^{\mathrm{D,max}}_{\mathrm{B}i,s}, \quad \forall i,s, \tag{4.75d}$$

where

$P^{\mathrm{U}}_{\mathrm{B}i,s}$ is the power increase from the level scheduled within block s by balancing generator i.

$P^{\mathrm{D}}_{\mathrm{B}i,s}$ is the power decrease from the level scheduled within block s by balancing generator i.

$\lambda^{\mathrm{U}}_{\mathrm{B}i,s}$ is the production cost offer for block s of balancing generator i in the balancing market.

Table 4.14 Up-regulation bids of the dispatchable generators in the balancing market

Unit	Block	λ^{U} (\$/MWh)	$P^{\mathrm{U,max}}$ (MWh)
B1	–	–	–
B2	1	25	5
	2	45	5
B3	1	40	15
	2	60	35

$\lambda^{\mathrm{D}}_{\mathrm{B}i,s}$ is the price offer for block s of balancing generator i for repurchase of energy in the balancing market.

$\Xi_{\mathrm{B}i}$ is the set of indices of the power blocks of balancing generator i.

The objective function (4.75a) includes two summations, one pertaining to the balancing generators and the other one to the blocks of each of them. Similarly, a double summation appears in the balancing Eq. (4.75b). Constraints (4.75c) and (4.75d) impose bounds on the production blocks of all balancing generators.

Example 4.1 is modified next in order to allow the submission of stepwise offers in the balancing market.

Example 4.7 (Balancing Auction with Excess Consumption and Stepwise Offers) Once again, the day-ahead dispatch and the realization of production and consumption are the same as in Example 4.1, illustrated in Tables 4.1 and 4.2.

The dispatchable generators submit the stepwise up-regulation offers described in Table 4.14 in the balancing market.

The auction resulting from model (4.75) is

$$\min_{P^{\mathrm{U}}_{\mathrm{B}2,1},P^{\mathrm{U}}_{\mathrm{B}2,2},P^{\mathrm{U}}_{\mathrm{B}3,1},P^{\mathrm{U}}_{\mathrm{B}3,2}} \quad 25P^{\mathrm{U}}_{\mathrm{B}2,1} + 45P^{\mathrm{U}}_{\mathrm{B}2,2} + 40P^{\mathrm{U}}_{\mathrm{B}3,1} + 60P^{\mathrm{U}}_{\mathrm{B}3,2} \tag{4.76a}$$

$$\text{s.t.} \quad 70 + P^{\mathrm{U}}_{\mathrm{B}2,1} + P^{\mathrm{U}}_{\mathrm{B}2,2} + P^{\mathrm{U}}_{\mathrm{B}3,1} + P^{\mathrm{U}}_{\mathrm{B}3,2} = 85 : \quad \lambda^{\mathrm{B}}, \tag{4.76b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}2,1} \le 5, \tag{4.76c}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}2,2} \le 5, \tag{4.76d}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}3,1} \le 15, \tag{4.76e}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}3,2} \le 35. \tag{4.76f}$$

We notice that the balancing market clearing differs from the one in the basic auction of Example 4.1. This is because the first block of unit B3 is cheaper than the second block of unit B2, which results in the following optimal dispatch:

$$P^{\mathrm{U}}_{\mathrm{B}2} = P^{\mathrm{U}}_{\mathrm{B}2,1} = 5\,\mathrm{MWh}, \tag{4.77}$$

$$P^{\mathrm{U}}_{\mathrm{B}3} = P^{\mathrm{U}}_{\mathrm{B}3,1} = 10\,\mathrm{MWh}. \tag{4.78}$$

Furthermore, the balancing market price is now

$$\lambda^{\mathrm{B}} = \$40\,/\mathrm{MWh}. \tag{4.79}$$

Table 4.15 Down-regulation bids of the dispatchable generators in the balancing market

Unit	Block	λ^{D} (\$/MWh)	$P^{\mathrm{D,max}}$ (MWh)
B1	1	12	10
	2	5	30
B2	1	18	5
	2	10	5
B3	–	–	–

Example 4.2 is revisited next, with the inclusion of stepwise offers from the producers participating in the balancing market.

Example 4.8 (Balancing Auction with Excess Production and Stepwise Offers) In this example, we reconsider the day-ahead dispatch and the realization of production and consumption of Example 4.2, shown in Tables 4.1 and 4.5, respectively.

The stepwise down-regulation bids received from the balancing generators are illustrated in Table 4.15.

Under these assumptions, auction (4.75) translates to

$$\underset{P^{\mathrm{D}}_{\mathrm{B1,1}},P^{\mathrm{D}}_{\mathrm{B1,2}},P^{\mathrm{D}}_{\mathrm{B2,1}},P^{\mathrm{D}}_{\mathrm{B2,2}}}{\text{Min.}} \quad -12P^{\mathrm{D}}_{\mathrm{B1,1}} - 5P^{\mathrm{D}}_{\mathrm{B1,2}} - 18P^{\mathrm{D}}_{\mathrm{B2,1}} - 10P^{\mathrm{D}}_{\mathrm{B2,2}} \tag{4.80a}$$

$$\text{s.t.} \quad 70 - P^{\mathrm{D}}_{\mathrm{B1,1}} - P^{\mathrm{D}}_{\mathrm{B1,2}} - P^{\mathrm{D}}_{\mathrm{B2,1}} - P^{\mathrm{D}}_{\mathrm{B2,2}} = 52 : \quad \lambda^{\mathrm{B}}, \tag{4.80b}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1,1}} \le 10, \tag{4.80c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1,2}} \le 30, \tag{4.80d}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2,1}} \le 5, \tag{4.80e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2,2}} \le 5. \tag{4.80f}$$

Since the first block of unit B1 has a higher marginal benefit than the second block of unit B2, the former has preference over the latter in the optimal balancing market dispatch, which is therefore the following:

$$P^{\mathrm{D}}_{\mathrm{B1}} = P^{\mathrm{D}}_{\mathrm{B1,1}} = 10\,\mathrm{MWh}, \tag{4.81}$$

$$P^{\mathrm{D}}_{\mathrm{B2}} = P^{\mathrm{D}}_{\mathrm{B2,1}} + P^{\mathrm{D}}_{\mathrm{B2,2}} = (5+3)\,\mathrm{MWh} = 8\,\mathrm{MWh}. \tag{4.82}$$

The balancing market price is determined from the price offer of the marginal block called in, i.e., the second block of unit B2

$$\lambda^{\mathrm{B}} = \$10\,/\mathrm{MWh}. \tag{4.83}$$

4.6 Network-Constrained Balancing Auction

For the sake of simplicity, the auction models presented in the previous sections assume that there is infinite transmission capacity, so that the location of generators and demands in the power grid has no influence on the optimal balancing market dispatch and on the resulting prices.

In this section, network constraints are considered in the balancing auction through a DC load flow representation. The output of this optimization model is the optimal dispatch of balancing power, which results in flows that satisfy the network capacity limits. Besides, it yields *Locational Marginal Prices* (LMPs), i.e., the marginal cost of electricity at each node. These are used to price electricity in balancing markets designed according to the *nodal pricing* rationale, e.g., the majority of power exchanges in the USA.

This section describes a network-constrained balancing auction and removes the unlimited transmission capacity assumption previously used.

For the sake of simplicity, we assume that at most one stochastic generator, one balancing generator, and one demand is located at each node of the transmission network. These generators and demands are identified by the node at which they are located.

A network-constrained balancing auction can be formulated as

$$\underset{P^{\mathrm{U}}_{\mathrm{B}n}, P^{\mathrm{D}}_{\mathrm{B}n}}{\text{Min.}} \quad \sum_{n \in \Xi_{\mathrm{N}}} \lambda^{\mathrm{U}}_{\mathrm{B}n} P^{\mathrm{U}}_{\mathrm{B}n} - \lambda^{\mathrm{D}}_{\mathrm{B}n} P^{\mathrm{D}}_{\mathrm{B}n} \tag{4.84a}$$

$$\text{s.t.} \quad P^{\mathrm{S}}_{\mathrm{B}n} + P^{\mathrm{U}}_{\mathrm{B}n} - P^{\mathrm{D}}_{\mathrm{B}n} - P^{\mathrm{B}}_{\mathrm{N}n} = \sum_{m \in \Xi_{\mathrm{N}(n)}} B_{nm} \left(\delta_n - \delta_m\right) : \quad \lambda^{\mathrm{B}}_{n}, \quad \forall n, \tag{4.84b}$$

$$0 \leq P^{\mathrm{U}}_{\mathrm{B}n} \leq P^{\mathrm{U,max}}_{\mathrm{B}n}, \quad \forall n, \tag{4.84c}$$

$$0 \leq P^{\mathrm{D}}_{\mathrm{B}n} \leq P^{\mathrm{D,max}}_{\mathrm{B}n}, \quad \forall n, \tag{4.84d}$$

$$B_{nm} \left(\delta_n - \delta_m\right) \leq C^{\max}_{nm}, \quad \forall n, m \in \Xi_{\mathrm{N}(n)}, \tag{4.84e}$$

$$- B_{nm} \left(\delta_n - \delta_m\right) \leq C^{\max}_{nm}, \quad \forall n, m \in \Xi_{\mathrm{N}(n)}, \tag{4.84f}$$

$$\delta_1 = 0. \tag{4.84g}$$

where

- $P^{\mathrm{S}}_{\mathrm{B}n}$ is the production level of the balancing generator at node n as scheduled in the day-ahead market.
- $P^{\mathrm{U}}_{\mathrm{B}n}$ is the increase in production of the balancing generator at node n in the balancing market.
- $P^{\mathrm{D}}_{\mathrm{B}n}$ is the decrease in production of the balancing generator at node n in the balancing market.
- B_{nm} is the absolute value of susceptance (physical constant) of the interconnection between nodes n and m.
- δ_n is the voltage angle (state variable) at node n.
- $C^{\max}_{nm}$ is the capacity of the interconnection between nodes n and m.
- $P^{\mathrm{B}}_{\mathrm{N}n}$ is the net demand (demand minus stochastic production) at balancing time at bus n.
- Ξ_{N} is the set of nodes.
- $\Xi_{\mathrm{N}(n)}$ is the set of nodes adjacent to node n.

Objective function (4.84a) is the balancing cost. Equality constraints (4.84b) enforce energy balance per node of the transmission network and have associated dual variables $\lambda_n^{\mathrm{B}}, \forall n$, which are the LMPs. Constraints (4.84c) and (4.84d) enforce bounds on increments and decrements, respectively, of the production levels of the balancing generators. Inequalities (4.84e) and (4.84f) ensure that the power transfer between nodes n and m is below the capacity of the corresponding line. Finally, (4.84g) sets the first node as the reference bus with angle zero.

It is relevant to note that in case of network congestion, the payments made by demands exceed the revenues obtained by balancing generators, resulting in a merchandizing surplus to be administered by the market operator, e.g., for network improvement.

Example 4.1 is revised next considering a simple two-node network, where the power transfer capacity between the two buses is limited. The system is illustrated in Fig. 4.7. Balancing generators B1 and B2, stochastic unit U1 and demand D1 are located at node 1, while B3, U2, and D2 are located at node 2. The capacity of the interconnection between the two nodes is limited to 90 MW.

Example 4.9 (Balancing Auction with Excess Demand and Network Congestion) Let us consider again the day-ahead dispatch, the realization of production and consumption, and the up-regulation bids of Example 4.1, included in Tables 4.1, 4.2 and 4.3.

Denoting with $B = B_{12} = B_{21}$ the susceptance of the interconnection linking nodes 1 and 2, balancing auction (4.84) writes as

$$\underset{P_{\mathrm{B2}}^{\mathrm{U}}, P_{\mathrm{B3}}^{\mathrm{U}}}{\text{Min.}} \quad 30 P_{\mathrm{B2}}^{\mathrm{U}} + 50 P_{\mathrm{B3}}^{\mathrm{U}} \tag{4.85a}$$

$$\text{s.t.} \quad 70 + P_{\mathrm{B2}}^{\mathrm{U}} + 15 = -B\delta_2 : \quad \lambda_1^{\mathrm{B}}, \tag{4.85b}$$

$$0 + P_{\mathrm{B3}}^{\mathrm{U}} - 100 = B\delta_2 : \quad \lambda_2^{\mathrm{B}}, \tag{4.85c}$$

$$0 \leq P_{\mathrm{B2}}^{\mathrm{U}} \leq 10, \tag{4.85d}$$

$$0 \leq P_{\mathrm{B3}}^{\mathrm{U}} \leq 50, \tag{4.85e}$$

$$-B\delta_2 \leq 90, \tag{4.85f}$$

$$B\delta_2 \leq 90. \tag{4.85g}$$

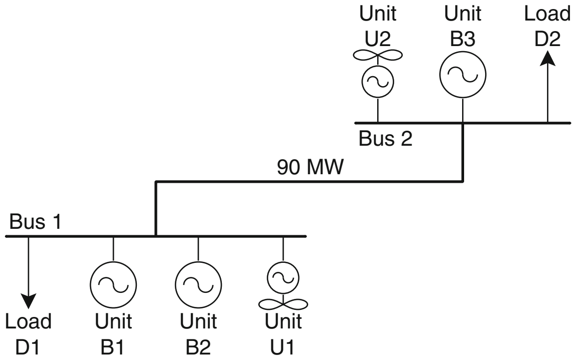
Since the capacity of the line is limited to 90 MW, generator B2 can only produce additionally 5 MWh at the balancing stage. The remaining excess of consumption, caused by the increase in load by D2 and the production decrease of U2, must be covered by the only dispatchable unit at node 2, i.e., B3. The optimal dispatch is then

$$P_{\mathrm{B2}}^{\mathrm{U}} = 5\,\text{MWh}, \tag{4.86}$$

$$P_{\mathrm{B3}}^{\mathrm{U}} = 10\,\text{MWh}. \tag{4.87}$$

Since the network is congested, the LMPs are different at the two nodes of the system. Indeed, the reader can notice that an increase in the right-hand side of (4.85b), i.e.,

Fig. 4.7 Two-bus system



an increase in the net demand at the balancing stage at node 1, could be balanced by an increase in production by dispatchable generator B2. The marginal increase of the system cost would then be the price bid by this generator, i.e., \$30 /MWh. Note that this is not possible when the net demand at node 2 is increased. Indeed, the upper bound to the power transfer from bus 1 to bus 2 ($-B\delta_2 \leq 90$) implies that any additional increase in demand at node 2 is to be balanced by the costlier generator B3, whose price bid is \$50 /MWh. The balancing prices at each node are therefore

$$\lambda_1^{\mathrm{B}} = \$30\,/\mathrm{MWh}, \tag{4.88}$$

$$\lambda_2^{\mathrm{B}} = \$50\,/\mathrm{MWh}. \tag{4.89}$$

Revenues and payments are summarized in Table 4.16.

Notice that λ_1^{B} applies to units B1, B2, U1, and load D1, while B3, U2, and D2 trade at price λ_2^{B}. The reader should also note that the market operator realizes a profit at the balancing stage. Indeed, it collects \$1500 from U2 and D2, but only pays \$1100 to B2, B3, U1, and D1.

The following example revisits Example 4.2 introducing network congestion.

Example 4.10 (Balancing Auction with Excess Production and Network Congestion) Let us consider one more time the day-ahead schedule and the realization of production and consumption of Example 4.2, summarized in Tables 4.1 and 4.5. As in the previous example, we consider a simplified power system with a two-bus network, depicted in Fig. 4.7.

In this example, we have to consider both the offers for up-regulation included in Table 4.3 and the ones for down-regulation summarized in Table 4.6.

In this case, auction (4.84) writes as

$$\underset{P_{\mathrm{B2}}^{\mathrm{U}}, P_{\mathrm{B3}}^{\mathrm{U}}, P_{\mathrm{B1}}^{\mathrm{D}}, P_{\mathrm{B2}}^{\mathrm{D}}}{\text{Min.}} \quad 30P_{\mathrm{B2}}^{\mathrm{U}} + 50P_{\mathrm{B3}}^{\mathrm{U}} - 8P_{\mathrm{B1}}^{\mathrm{D}} - 15P_{\mathrm{B2}}^{\mathrm{D}} \tag{4.90a}$$

$$\text{s.t.} \quad 70 + P_{\mathrm{B2}}^{\mathrm{U}} - P_{\mathrm{B1}}^{\mathrm{D}} - P_{\mathrm{B2}}^{\mathrm{D}} + 43 = -B\delta_2 : \quad \lambda_1^{\mathrm{B}}, \tag{4.90b}$$

$$0 + P_{\mathrm{B3}}^{\mathrm{U}} - 95 = B\delta_2 : \quad \lambda_2^{\mathrm{B}}, \tag{4.90c}$$

Table 4.16 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	150	550
Unit B3	0	500	500
Unit U1	800	300	1100
Unit U2	600	– 500	100
Load D1	– 800	150	– 650
Load D2	– 2000	– 1000	– 3000

$$0 \le P^{\mathrm{U}}_{\mathrm{B2}} \le 10, \tag{4.90d}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B3}} \le 50, \tag{4.90e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1}} \le 40, \tag{4.90f}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2}} \le 10, \tag{4.90g}$$

$$-B\delta_2 \le 90, \tag{4.90h}$$

$$B\delta_2 \le 90. \tag{4.90i}$$

Despite the system as a whole has a production surplus of 18 MWh, this does not hold true at node 2, where the net demand at the balancing stage (95 MWh) is greater than the one in the day-ahead market (70 MWh). Furthermore, the interconnection capacity is not sufficiently high to allow the large power production at node 1 to cover the net demand at node 2 completely. Therefore, the market operator has to activate down-regulation at node 1

$$P^{\mathrm{D}}_{\mathrm{B1}} = 13\,\mathrm{MWh}, \tag{4.91}$$

$$P^{\mathrm{D}}_{\mathrm{B2}} = 10\,\mathrm{MWh}, \tag{4.92}$$

and up-regulation at node 2

$$P^{\mathrm{U}}_{\mathrm{B3}} = 5\,\mathrm{MWh}. \tag{4.93}$$

The LMPs at the balancing market spread significantly in this case. Indeed, it is easy to verify that

$$\lambda^{\mathrm{B}}_1 = \$8\,/\mathrm{MWh}, \tag{4.94}$$

$$\lambda^{\mathrm{B}}_2 = \$50\,/\mathrm{MWh}. \tag{4.95}$$

Payments and revenues are summarized in Table 4.17.

We emphasize again that the market operator realizes a profit in the balancing market. Indeed, it collects $1434 from participants B1, B2, U2, and D2. On the other hand, the total payment to units B3, U1, and load D1 is only $594.

Table 4.17 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	– 104	896
Unit B2	400	– 80	320
Unit B3	0	250	250
Unit U1	800	200	1000
Unit U2	600	– 250	350
Load D1	– 800	144	– 656
Load D2	– 2000	– 1000	– 3000

4.7 Relevant Worldwide Experiences

Some North American and European balancing markets are briefly described in this section. The two-price settlement rule is most widespread in Europe while American systems generally use a single-price settlement scheme.

4.7.1 The Americas

Balancing markets in North America are generally denominated *real-time* markets and are organized as follows:

1. Some time before energy delivery, a security-constrained dispatch is carried out with the most updated information available. This dispatch is a sophisticated version of auction (4.84). This is an ex ante dispatch as it takes place prior to energy delivery.
2. A state estimation algorithm [6] is used after each dispatch period (e.g., every 5 min) to identify the actual production level of each unit and the actual consumption level of each demand. This is an ex post calculation.
3. A pricing algorithm is used in turn to determine the price to be paid by each demand and to be received by each generator for each dispatch period. This price is similar to the marginal price derived from auction (4.84) but ex post not ex ante.

Details of specific implementations of this procedure in the cases of ISO New England, PJM, and Midwest ISO are available in [9], [11], and [7], respectively.

4.7.2 Europe

Balancing markets in Europe are organized in a variety of ways. They are loosely characterized by the following features:

1. The *zonal pricing* criterion is generally preferred to the nodal one. As opposed to the latter one, which potentially can assign a different price, i.e., the locational marginal price, at each node in the network (see Sect. 4.6), markets with zonal pricing are divided in a number of areas, across which the price is uniform.
2. An ex-post calculation is carried out to determine payments and revenues for the market participants based on their metered power injection to or withdrawal from the grid.
3. The prevailing imbalance settlement scheme is the two-price system. However, the German EEX and the Dutch APX electricity markets are two remarkable exceptions where the one-price system is adopted, see [5] and [14], respectively.
4. An important issue in European markets is how imbalances are determined. Indeed, differently from the American electricity markets where day-ahead schedules are determined unit-wise, payments and revenues in the balancing markets are calculated on the basis of the total net imbalance of several units associated with a single market participant. For example, mixed load and generation portfolios are allowed in the Iberian MIBEL market [12]. Contrarily, the imbalances for load and generation are accounted for separately in the Danish balancing market [4].

4.8 Balancing Prices

The statistical characterization of clearing prices is important in all markets and thus in balancing markets too.

This statistical characterization and the actual forecasting of balancing prices have an important complication emanating from the fact that deviations are either excess demand, leading to an increase in price with respect to the day-ahead price, or excess production, leading to a decrease in price. This above/below phenomenon that has an impact on clearing prices is complex to model if using time series models [2]. A relevant reference describing a methodology for balancing price forecasting is [10]. Finally, note that further statistical characterization of balancing prices is beyond the scope of this chapter.

4.9 Summary and Conclusions

The remarks pertaining to balancing markets are worth mentioning:

1. Balancing markets are needed to balance as close as possible production and consumption just before energy delivery. This is so because electric energy cannot be stored in large quantities.
2. Balancing markets play an important role in case of equipment failures (transmission lines and/or production facilities) and in case of high penetration of stochastic production facilities.

3. Balancing auctions are conceptually and mathematically simple, but very important in practice.
4. Balancing auctions are easily formulated as simple optimization problems (e.g., linear programming problems) that can be solved in a robust and efficient manner.
5. Characterizing the behavior of balancing prices through time series models is a challenging research task.

4.10 Further Reading

Some relevant references on balancing markets are [17], [10], [1], [13], and [15]. Reference [17] describes the functioning of the balancing market of ISO New England. References [1] and [15] describe balancing market implementation and price derivation algorithms. References [13] and [8] consider the impact of balancing market on the strategies of wind producers. Balancing markets become more and more important as renewable stochastic producers increasingly penetrate the electric energy systems. Finally, reference [10] pertains to price forecasting in balancing markets.

Exercises

4.1. Extend the formulation of auction (4.9) to include a minimum power output level per balancing generator. Note that the minimum power output pertains to the total production (day-ahead and balancing) of any given generating unit.

4.2. Illustrate using an example the model in the previous exercise.

4.3. Extend the formulation of auction (4.75) to include proactive demands and a minimum power output levels for balancing generators.

4.4. Illustrate using an example the model in the previous exercise.

4.5. Within a four-bus network, use an instance of auction (4.84) to illustrate how clearing prices can vary throughout the network.

4.6. Extend the formulation of auction (4.84) to include multiblock production offers, a minimum power output level per balancing generator and proactive demands.

4.7. Illustrate using an example the model in the previous exercise.

4.8. Show revenue adequacy (the market administrator collects a higher amount of money that the amount it has to distribute) for auction (4.75).

4.9. Show revenue adequacy for auction (4.84).

4.10. Are balancing prices more difficult to characterize statistically than day-ahead prices? If so, why?

References

1. Choi, J.Y., Rim, S.H., Park, J.K.: Optimal real time pricing of real and reactive powers. IEEE Trans. Power Syst. **13**(4), 1226–1231 (1998)
2. Conejo, A.J., Contreras, J., Espinola, R., Plazas, M.A.: Forecasting electricity prices for a day-ahead pool-based electric energy market. Int. J. Forecast. **21**(3), 435–462 (2005)
3. Electricity market of the Iberian Peninsula, OMIE: http://www.omie.es/en (2013). Accessed 3 Nov 2013
4. Energinet.dk: Regulation C2: The balancing market and balance settlement. http://energinet.dk/EN/El/Forskrifter/Markedsforskrifter/ (2008). Accessed 3 Nov 2013
5. German transmission system operators: Internet platform for tendering control reserve. https://www.regelleistung.net/ip/action/static/chargesys. (2013). Accessed 3 Nov 2013
6. Gómez-Expósito, A., Conejo, A.J., Cañizares, C.: Electric Energy System. Analysis and Operation. CRC, Boca Raton (2009)
7. Ma, X., Chen, Y., Wan, J.: Midwest ISO co-optimization based real-time dispatch and pricing of energy and ancillary services. In Proc. 2009 IEEE Power Energy Soc. Gen. Meet. (2009)
8. Morales, J.M., Conejo, A.J., Perez-Ruiz, J.: Short-term trading for a wind power producer. IEEE Trans. Power Syst. **25**(1), 554–564 (2010)
9. New England Power Pool-Market Rule 1, Chapter 2: http://www.iso-ne.com (2013). Accessed 3 Nov 2013
10. Olsson, M., Söder, L.: Modeling real-time balancing power market prices using combined SARIMA and Markov processes. IEEE Trans. Power Syst. **23**(2), 443–450 (2008)
11. PJM Manual 11: Energy & Ancillary Services Market Operations: http://www.pjm.com/∼/media/documents/manuals/m11.ashx (2013). Accessed 3 Nov 2013
12. Project OPTIMATE Report D32: Current designs and expected evolutions of day-ahead, intra-day and balancing market/mechanisms in Europe: http://www.optimate-platform.eu/downloads/ (2011). Accessed 5 Nov 2013
13. Sioshansi, R.: Evaluating the impacts of real-time pricing on the cost and value of wind generation. IEEE Trans. Power Syst. **25**(2), 741–748 (2010)
14. TenneT: The imbalance pricing system. http://www.tennet.org. (2010). Accessed 3 Nov 2013
15. Wang, X., Song, Y.H., Lu, Q.: A coordinated real-time optimal dispatch method for unbundled electricity markets. IEEE Trans. Power Syst. **17**(2), 482–490 (2002)
16. Weber, C.: Adequate intraday market design to enable the integration of wind energy into the European power systems. Energy Policy **38**(7), 3155–3163 (2010)
17. Zheng, T., Litvinov, E.: On ex post pricing in the real-time electricity market. IEEE Trans. Power Syst. **26**(1), 153–164 (2011)

Chapter 5
Managing Uncertainty with Flexibility

5.1 Introduction

The incorporation of an increasing number or wind- and solar-based production facilities (as well as geothermal, wave-powered, and others) in electricity generation systems has resulted in a circumstance previously unknown: the presence of a significant amount of stochastic production capacity. That is, production capacity whose output depends on stochastic phenomena (e.g., wind level or solar radiation intensity) and not on the will of the owner of such capacity. The exploitation of this renewable capacity whose generation cost is generally low requires the availability of flexible production capacity as a backup, i.e., as reserve. Such flexible capacity ensures the production–consumption balance regardless of the output from the stochastic sources.

From the viewpoint of the Independent System Operator (ISO), this chapter visualizes the need for such flexible backup capacity and provides a detailed computational analysis of its effect on system operations.

Particularly, solar power plants do not produce during the night but do during the day if a sufficient level of solar intensity is available. The production of solar plants can be extended a few hours after the sunset if on-purpose storage facilities are available. On the other hand, wind power production is generally higher during the night than at midday. This is particularly so for wind farms located near the sea. Thus, solar- and wind-based facilities play a complementary role. However, wind power production generally decreases well in advance of the sunrise, that is, before significant level of solar power production is available, which poses a significant stress on regulating thermal units, e.g., combined cycle gas turbines (CCGTs).

Finally, it is important to note that flexibility is directly linked to the considered time span and it is naturally measured as a rate of change, given typically in megawatts per minute or megawatts per hour. For instance, a ramping-up rate of 120 MW/h means that the considered unit can increase its output power at the rate or 120 MW/h or 2 MW/min. Conversely, a demand drop rate of 138 MW/h means that the considered demand can drop at the rate of 138 MW/h or 2.3 MW/min.

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The remainder of this chapter is organized as follows. Section 5.2 provides a mathematical formulation for market clearing that allows studying flexibility. It is based on the formulation described in Chap. 3 just for one time period. Section 5.2 analyzes the flexibility provided by flexible (nonstochastic) production units. Section 5.4 considers flexibility measures from the demand. Section 5.5 describes the great impact on flexibility provided by storage facilities. Section 5.6 analyzes the impact on flexibility of the transmission network. Section 5.7 addresses the issue of how to measure flexibility in an accurate yet computationally efficient manner. Section 5.8 provides some remarks on the impact of flexibility on clearing prices. Finally, Sect. 5.9 summarizes the chapter and Sect. 5.10 provides a list of selected readings that further develop the topic of managing uncertainty in power systems with flexibility.

5.2 Flexibility: Mathematical Modeling

The extension of the market-clearing procedure explained in Chap. 3 to a multi-period case ($t = 1, \ldots, T$) is provided below:

$$\begin{aligned}\underset{\Xi}{\text{Min.}} \quad & \sum_{t=1}^{T} \sum_{i \in I} \left(C_i P_i(t) + C_i^{\text{RU}} R_i^{\text{U}}(t) + C_i^{\text{RD}} R_i^{\text{D}}(t) \right) + \sum_{t=1}^{T} \sum_{q \in Q} C_q W_q^{\text{S}}(t) \\ & + \sum_{t=1}^{T} \sum_{\omega \in \Omega} \pi_\omega \left[\sum_{i \in I} \left(C_i^{\text{U}} r_{i\omega}^{\text{U}}(t) - C_i^{\text{D}} r_{i\omega}^{\text{D}}(t) \right) \right. \\ & \left. + \sum_{q \in Q} C_q \left(W_{q\omega}(t) - W_q^{\text{S}}(t) - W_{q\omega}^{\text{spill}}(t) \right) + \sum_{j \in J} V_j^{\text{LOL}} L_{j\omega}^{\text{shed}}(t) \right] \end{aligned} \tag{5.1a}$$

s.t.

$$\begin{aligned} & \sum_{i \in \Phi_n^I} P_i(t) + \sum_{q \in \Phi_n^Q} W_q^{\text{S}}(t) - \sum_{j \in \Phi_n^J} L_j(t) - \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0(t) - \delta_{e(\ell)}^0(t) \right) \\ & \quad + \sum_{\ell \in \Lambda | e(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0(t) - \delta_{e(\ell)}^0(t) \right) = 0 \quad : \lambda_n^{\text{D}}(t), \ \forall n \in N, \ \forall t = 1, \ldots, T, \end{aligned} \tag{5.1b}$$

$$\begin{aligned} & \sum_{i \in \Phi_n^I} \left(r_{i\omega}^{\text{U}}(t) - r_{i\omega}^{\text{D}}(t) \right) + \sum_{j \in \Phi_n^J} L_{j\omega}^{\text{shed}}(t) + \sum_{q \in \Phi_n^Q} \left(W_{q\omega}(t) - W_q^{\text{S}}(t) - W_{q\omega}^{\text{spill}}(t) \right) \\ & \quad + \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0(t) - \delta_{o(\ell)\omega}(t) - \delta_{e(\ell)}^0(t) + \delta_{e(\ell)\omega}(t) \right) \\ & \quad - \sum_{\ell \in \Lambda | e(\ell) = n} b_\ell \left(\delta_{o(\ell)}^0(t) - \delta_{o(\ell)\omega}(t) - \delta_{e(\ell)}^0(t) + \delta_{e(\ell)\omega}(t) \right) = 0 \quad : \gamma_{n\omega}(t), \\ & \qquad \forall n \in N, \ \forall \omega \in \Omega, \ \forall t = 1, \ldots, T, \end{aligned} \tag{5.1c}$$

$$b_\ell \left(\delta^0_{o(\ell)}(t) - \delta^0_{e(\ell)}(t)\right) \leq C^{\max}_\ell, \quad \forall \ell \in \Lambda,\ \forall t = 1,\dots,T, \tag{5.1d}$$

$$b_\ell \left(\delta^0_{e(\ell)}(t) - \delta^0_{o(\ell)}(t)\right) \leq C^{\max}_\ell, \quad \forall \ell \in \Lambda,\ \forall t = 1,\dots,T, \tag{5.1e}$$

$$b_\ell \left(\delta_{o(\ell)\omega}(t) - \delta_{e(\ell)\omega}(t)\right) \leq C^{\max}_\ell, \quad \forall \ell \in \Lambda,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1f}$$

$$b_\ell \left(\delta_{e(\ell)\omega}(t) - \delta_{o(\ell)\omega}(t)\right) \leq C^{\max}_\ell, \quad \forall \ell \in \Lambda,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1g}$$

$$\delta^0_1(t) = 0, \quad \forall t = 1,\dots,T, \tag{5.1h}$$

$$\delta_{1\omega}(t) = 0, \quad \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1i}$$

$$W^{\mathrm{S}}_q(t) \leq W^{\max}_q, \quad \forall q \in Q\ \forall t = 1,\dots,T, \tag{5.1j}$$

$$P_i(t) + R^{\mathrm{U}}_i(t) \leq P^{\max}_i, \quad \forall i \in I,\ \forall t = 1,\dots,T, \tag{5.1k}$$

$$P_i(t) - R^{\mathrm{D}}_i(t) \geq 0, \quad \forall i \in I,\ \forall t = 1,\dots,T, \tag{5.1l}$$

$$R^{\mathrm{U}}_i(t) \leq R^{\mathrm{D,max}}_i, \quad \forall i \in I,\ \forall t = 1,\dots,T, \tag{5.1m}$$

$$R^{\mathrm{D}}_i(t) \leq R^{\mathrm{D,max}}_i, \quad \forall i \in I,\ \forall t = 1,\dots,T, \tag{5.1n}$$

$$r^{\mathrm{U}}_{i\omega}(t) \leq R^{\mathrm{U}}_i(t), \quad \forall i \in I,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1o}$$

$$r^{\mathrm{D}}_{i\omega}(t) \leq R^{\mathrm{D}}_i(t), \quad \forall i \in I,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1p}$$

$$L^{\mathrm{shed}}_{j\omega}(t) \leq L_j(t), \quad \forall j \in J,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1q}$$

$$W^{\mathrm{spill}}_{q\omega}(t) \leq W_{q\omega}(t), \quad \forall q \in Q,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1r}$$

$$P_i(t),\ R^{\mathrm{U}}_i(t),\ R^{\mathrm{D}}_i(t) \geq 0, \quad \forall i \in I,\ \forall t = 1,\dots,T, \tag{5.1s}$$

$$r^{\mathrm{U}}_{i\omega}(t),\ r^{\mathrm{D}}_{i\omega}(t) \geq 0, \quad \forall i \in I,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1t}$$

$$W^{\mathrm{S}}_q(t) \geq 0, \quad \forall q \in Q,\ \forall t = 1,\dots,T, \tag{5.1u}$$

$$W^{\mathrm{spill}}_{q\omega}(t) \geq 0, \quad \forall q \in Q,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1v}$$

$$L^{\mathrm{shed}}_{j\omega}(t) \geq 0, \quad \forall j \in J,\ \forall \omega \in \Omega,\ \forall t = 1,\dots,T, \tag{5.1w}$$

where $\Xi = \{P_i(t), R^{\mathrm{U}}_i(t), R^{\mathrm{D}}_i(t), r^{\mathrm{U}}_{i\omega}(t), r^{\mathrm{D}}_{i\omega}(t), W^{\mathrm{S}}_q(t), W^{\mathrm{spill}}_{q\omega}(t), L^{\mathrm{shed}}_{j\omega}(t), \delta^0_n(t), \delta_{n\omega}(t), \forall i \in I, \forall q \in Q, \forall j \in J, \forall n \in N, \forall \omega \in \Omega, \forall t = 1,\dots,T\}$ is the set of decision variables.

The notation used in the above formulation (5.1) is similar to that used in Chap. 3 but including the time-period index t that appears in parenthesis.

The explanations of the equations of problem 5.1 above are also similar to the explanations provided in Chap. 3 for the single-period formulation used in that chapter. We, therefore, refer the reader to Chap. 3.

A base case example is described below. Different flexibility/inflexibility measures are tested against this base case throughout the chapter.

Example 5.1 (Base case) The base case, which is presented in this example, is built on the electric energy system of Example 3.3 in Chap. 3. For clarity, this system is reproduced in Fig. 5.1. All the parameters in the system of Example 3.3 are

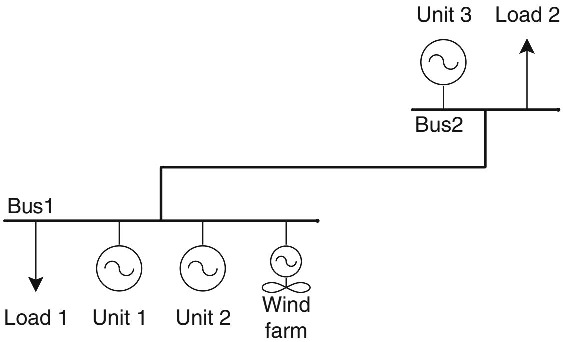

Fig. 5.1 Two-bus system for flexibility studies

Table 5.1 Unit data

	Unit 1	Unit 2	Unit 3
$P^{\max}$ (MW)	50	110	100
C (\$/MWh)	10	30	35
C^{RU} (\$/MW)	16	13	10
C^{RD} (\$/MW)	15	12	9

Table 5.2 Base case: data for consumption and wind power production in megawatts

(a) Consumption

	Demand 1	Demand 2
Period 1	40	100
Period 2	25	80
Period 3	45	95

(b) Wind power production

Scenario	High	Low
Period 1	50	10
Period 2	65	30
Period 3	35	15

unchanged. Table 5.1 reports capacity, generation cost, and reserve cost for each unit of the system.

The example is extended to a three-period study horizon, so that different intertemporal constraints can be implemented and analyzed. Note that period 1 coincides with the single-period case in Example 3.3.

The mathematical model used in this example consists of the minimization of objective function (5.1a), constrained by (5.1b)–(5.1l) and (5.1o)–(5.1w). Notice that Eqs. (5.1m) and (5.1n) are not enforced in this case, i.e., it is assumed that each unit is capable of offering as much upward and downward reserve as possible according to the production capacity limits (5.1k) and (5.1l).

Additional data required, which include demand values in megawatts per period and wind power production in megawatts per period and scenario, are provided in Table 5.2. As stated in Chap. 3, the probability of the high wind scenario is 0.6 while the probability of the low wind scenario is 0.4.

The results for this base case example are described below.

Table 5.3 provides the optimal power scheduled in this base case (results for period 1 are the same as those obtained in Example 3.3 of Chap. 3). Although the production cost of unit 3 is comparatively high, it is dispatched since it is needed to provide

Table 5.3 Base case: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	40	40	10
Period 2	50	0	25	30
Period 3	50	55	20	15

down-regulation. Note that unit 2 is shut down during period 2, which is possible because unit 2 is *highly flexible* and can be shut down in one hour and started up in the next one. It is also worth noting that the wind schedule coincides with the lowest wind power production scenario.

Table 5.4 provides the reserve scheduled to account for wind variability. Only down-reserve is scheduled to compensate high wind power production in all periods. Unit 3, which has the lowest reserve cost among the units in the system, is the sole provider of reserve. Notice that the downward reserve schedule for this unit equals the production schedule in Table 5.3.

Table 5.5 reports the actual deployment of downward reserve per scenario and time period. The downward reserve scheduled for unit 3 is fully deployed in all periods for wind scenario "high".

Table 5.6 provides the amount of wind spillage in megawatts per scenario and time period. Note that will spillage only occurs in the high wind scenario and period 2.

Table 5.7 provides the flows in megawatts through the line connecting the two buses of the system for each time period, both at the scheduling stage and at the balancing one for each scenario of wind power production.

Note that the power flow is comparatively higher in the high wind scenario during which a significant level of wind power is sent from bus 1 to bus 2.

Table 5.4 Base case: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	40
Period 2	0	0	0	0	0	25
Period 3	0	0	0	0	0	20

Table 5.5 Base case: down-reserve deployed in megawatts (up-reserve is always nil)

	Unit 1		Unit 2		Unit 3	
Scenario	High	Low	High	Low	High	Low
Period 1	0	0	0	0	40	0
Period 2	0	0	0	0	25	0
Period 3	0	0	0	0	20	0

Table 5.6 Base case: wind spillage in megawatts

Scenario	High	Low
Period 1	0	0
Period 2	10	0
Period 3	0	0

Table 5.7 Scheduled and scenario power flows through the line from bus 1 to 2 in megawatts

Stage	Scheduling	Balancing	
Scenario	–	High	Low
Period 1	60	100	60
Period 2	55	80	55
Period 3	75	95	75

Table 5.8 provides scheduling and scenario prices for all time periods. Note the 0 price at period 2, due to wind spillage in the high price scenario.

Each scenario price, which is the dual variable of the scenario energy balance equation, is implicitly multiplied by the probability of the corresponding scenario.

Also observe that each scheduling price is equal to the sum of the scenario prices, e.g., $30 = 7 + 23$. This is a direct consequence of the problem formulation and the proposed pricing scheme.

Finally, Table 5.9 provides the total expected cost, including the expected energy cost, which is broken down into its scheduling and reserve deployment components, and the cost for reserve capacity.

Example 5.1 above constitutes the base case against which the effect on market outcomes of different flexibility/inflexibility measures are tested throughout the chapter using a number of different but related examples.

5.3 Flexibility from the Supply Side

This section describes a number of flexibility actions that can be provided by dispatchable production units such as CCGTs.

Table 5.8 Base case: scheduling and balancing prices in dollars per megawatt hour. Balancing prices are implicitly multiplied by the corresponding scenario probabilities (0.6 for scenario "high" and 0.4 for scenario "low")

Stage	Scheduling		Balancing			
Scenario	–	–	High		Low	
Bus	1	2	1	2	1	2
Period 1	30	35	7	12	23	23
Period 2	23	23	0	0	23	23
Period 3	30	30	7	7	23	23

Table 5.9 Base case: expected cost in dollars

		Cost
	Schedule	7325
Energy	Deployed reserve (expected)	−1785
	Total (expected)	5540
Reserve capacity		765
Total (expected)		6305

5.3.1 Variables and Constants

One of the key variables to be considered to analyze system flexibility is the power output of unit i during period t and under scenario ω, defined as

$$P_{i\omega}(t) = P_i(t) + r^{\mathrm{U}}_{i\omega}(t) - r^{\mathrm{D}}_{i\omega}(t), \tag{5.2}$$

where

$P_{i\omega}(t) \geq 0$ is the actual power output of unit i during period t and scenario ω.
$P_i(t)$ is the scheduled power of unit i during period t.
$r^{\mathrm{U}}_{i\omega}(t)$ is the deployed up-reserve of unit i during period t and scenario ω.
$r^{\mathrm{D}}_{i\omega}(t)$ is the deployed down-reserve of unit i during period t and scenario ω.

Additionally, to analyze the flexibility of a power production unit we need to define the variables below:

$v_i(t) \in \{0, 1\}$ binary variable that is equal to 1 if unit i is on during period t and 0 otherwise.
$y_i(t) \in \{0, 1\}$ binary variable that is equal to 1 if unit i is started up at the beginning of period t and 0 otherwise.
$z_i(t) \in \{0, 1\}$ binary variable that is equal to 1 if unit i is shut down at the beginning of period t and 0 otherwise.

To accurately reproduce the up and down status transitions of production units, the logical constraints below need to be enforced:

$$v_i(t-1) - v_i(t) + y_i(t) - z_i(t) = 0, \quad \forall i,\ \forall t, \tag{5.3a}$$

$$y_i(t) + z_i(t) \leq 1, \quad \forall i,\ \forall t, \tag{5.3b}$$

$$v_i(t),\ y_i(t),\ z_i(t) \in \{0, 1\}, \quad \forall i,\ \forall t. \tag{5.3c}$$

5.3.2 Capacity Limits

The flexibility of a production unit is ultimately conditioned by its capacity (maximum power output) and minimum power output, as defined by the constants below:

$P^{\max}_i$ is the capacity (maximum power output) of unit i.
$P^{\min}_i$ is the minimum power output of unit i.

The actual limits are enforced by the constraints below:

$$P^{\min}_i\, v_i(t) \leq P_{i\omega}(t) \leq P^{\max}_i\, v_i(t), \quad \forall i,\ \forall t. \tag{5.4}$$

From the above expression, the following two observations are in order:

1. The higher the value of $P^{\max}_i$, higher is the production range and thus higher the flexibility of production unit i.

Table 5.10 Reduced flexible capacity: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	50	30	10
Period 2	50	0	25	30
Period 3	50	55	20	15

2. Conversely, the lower the value of $P_i^{\min}$, higher is the production range and thus higher the flexibility of production unit i.

The example below illustrates how the system becomes less flexible when the capacity of conventional units is reduced.

Example 5.2 (Reduced flexible capacity) The capacity of unit 3, which is the most flexible as it provides the cheapest reserve, is drastically reduced from 100 to 30 MW. The resulting optimization problem is more constrained than in the base case considered in Example 5.1, and therefore we can expect a higher objective function value.

The mathematical model used in this example consists of (5.1a)–(5.1j) and (5.1o)–(5.1w), including definition (5.2), logical constraints (5.3) and power limits at the balancing stage (5.4).

Table 5.10 provides the optimal power schedule in this case in which the capacity of the most flexible unit has been decreased. The dispatch for the first time period in this case differs from the one in the base case solution. Indeed, unit 3 cannot be dispatched at 40 MW any more, because of the reduction of its capacity to 30 MW. Thus, unit 2 is assigned 10 extra megawatts in the optimal dispatch for this case as compared to the base case solution.

Table 5.11 provides the scheduled reserve in MW. As in the base case, only down-reserve is scheduled. Unit 3 cannot provide more than 30 MW of down-reserve at period 1, so 10 MW of down-reserve are purchased from unit 2 at a higher cost.

The actual deployment of downward reserve is reported in Table 5.12. In period 1, both unit 2 and unit 3 are dispatched down by 10 and 30 MW, respectively, in

Table 5.11 Reduced flexible capacity: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	10	0	30
Period 2	0	0	0	0	0	25
Period 3	0	0	0	0	0	20

Table 5.12 Reduced flexible capacity: down-reserve deployed in megawatt hour (up-reserve is always nil)

	Unit 1		Unit 2		Unit 3	
Scenario	High	Low	High	Low	High	Low
Period 1	0	0	10	0	30	0
Period 2	0	0	0	0	25	0
Period 3	0	0	0	0	20	0

Table 5.13 Expected cost in dollars in the case with reduced flexible capacity and in the base case

(a) Reduced flexible capacity

		Cost
	Schedule	7275
Energy	Deployed reserve (exp.)	−1755
	Total (exp.)	5520
Reserve capacity		795
Total (exp.)		6315

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.14 Minimum power output: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	35	45	10
Period 2	50	0	25	30
Period 3	50	75	0	15

the high scenario for wind power production. Notice that the energy redispatch in periods 2 and 3 are equal to the one in the base case solution in Example 5.1.

Table 5.13 provides the expected operation cost in this case of reduced flexible capacity.

The lower capacity of unit 3, which is the most flexible, results in higher expected cost. The lower production cost of unit 2, which takes part of the energy and reserve dispatch that is assigned to unit 3 in the base case, reduces the costs for energy scheduling. However, the combined effect of rising reserve costs and decreasing benefits from the deployment of down-reserve result in an increase in the total expected cost by \$10 in comparison with the base case.

In Example 5.3 below conventional units are made less flexible by incorporating minimum power output values above zero.

Example 5.3 (Minimum power output) In this example the minimum power output for units 1, 2 and 3 is increased from 0 to 10, 10 and 5 MW, respectively, making the production system significantly more inflexible. The new constraints may not be binding in the day-ahead scheduling, but they are generally at balancing time. Unit 3 cannot provide all the down-regulation that the system needs, which results in comparatively higher wind spillage and expected cost.

The mathematical model used in this example consists of (5.1a)–(5.1j), (5.1o)–(5.1w), as well as (5.2)–(5.4). Note that the mathematical model is the one used in Example 5.2; however, the power limits appearing in (5.4) are different in this case.

Table 5.14 provides the optimal power scheduled for the three conventional units and the wind power plant. Note that 5 additional megawatts are needed from unit 3 in the schedule for period 1 to provide down-regulation. This increase is a consequence of the minimum power output increase (from 0 to 5 MW). Furthermore, as compared to the base case, it is cheaper to dispatch unit 2 instead of unit 3 in period 3. Notice that the case of this period is different from period 1, where unit 3 is still dispatched despite the increased cost involved with ensuring a minimum power output of 5 MW. In period 3, the variability of wind is lower, since the difference in production between the scenarios "low" and "high" is 20 MW, which in this case coincides with the

Table 5.15 Minimum power output: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	40
Period 2	0	0	0	0	0	20
Period 3	0	0	0	20	0	0

Table 5.16 Minimum power output: wind spillage in megawatts

Scenario	High	Low
Period 1	0	0
Period 2	15	0
Period 3	0	0

amount of reserve to be purchased by the market operator. As a consequence of the lower variability, it is not economically efficient to keep unit 3 on during period 3. Indeed, the beneficial effects of providing cheap down-reserve are outweighed by the higher dispatch cost, incurred to guarantee the minimum power output, when the amount of reserve to be purchased is limited as in this case. Put in mathematical terms, dispatching solely unit 2 yields a lower cost than dispatching both unit 2 and 3 whenever the following condition is satisfied:

$$\underbrace{(30+12)\Delta w}_{\text{energy + reserve cost}} \underbrace{-0.6\times 30\Delta w}_{\text{redispatch cost}} \leq \underbrace{(35+9)\Delta w}_{\text{energy + reserve cost}} \underbrace{-0.6\times 35\Delta w}_{\text{redispatch cost}} + \underbrace{(35-30)\times 5}_{\text{minimum output additional cost}}, \tag{5.5}$$

where Δw is the amount of extra wind power production in the high scenario, which is to be absorbed by deploying a correspondent amount of down-reserve, either from unit 2 (on the left side of the $\leq$ operator) or from unit 3 (on the right-hand side). The rightmost term in the above inequality is the additional cost incurred when reducing the schedule of unit 2 and increasing the one of unit 3 by 5 MW, so as to ensure the minimum power output for the latter unit. Solving (5.5) for Δw, one finds that it is cheaper to only dispatch unit 2 as long as the variation of wind power production between the two scenarios is lower than the following threshold:

$$\Delta w \leq 25. \tag{5.6}$$

Notice that the condition above is not fulfilled in period 1, where both units are scheduled, while it holds in period 3, where only unit 2 is on.

Table 5.15 provides the optimal schedule of reserve. Note that unit 2 provides expensive reserve at period 3. Indeed, unit 3 is scheduled to be off during this period as a result of the minimum power output, thus it cannot provide cheaper reserve.

Values of wind spillage for this example are included in Table 5.16. Wind spillage during the second period increases as compared to the base case.

Table 5.17 Expected cost in dollars in the case where a minimum power output is imposed and in the base case

(a) Minimum power output

		Cost
	Schedule	7250
Energy	Deployed reserve (exp.)	−1620
	Total (exp.)	5630
Reserve capacity		780
Total (exp.)		6410

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.17 provides the expected energy cost and the reserve capacity cost. As in this example we face a more constrained problem (resulting from a less flexible system) than the one corresponding to the base case, it exhibits higher expected costs.

5.3.3 Ramping Limits

The ramping-up and ramping-down limits of any production unit are formulated using the constants defined below. These constants pertain to the physical characteristics of the production unit, and relate to either thermal or hydro systems.

S_i^{U} is the maximum ramping-up rate.
S_i^{D} is the maximum ramping-down rate.
P_i^{SU} is the maximum power output during the period at the beginning of which unit i is started up.
P_i^{SD} is the maximum power output during the period prior to the one at the beginning of which unit i is shut down.

Note that $P_i^{\mathrm{SU}} \geq P_i^{\min}$ and $P_i^{\mathrm{SD}} \geq P_i^{\min}$ for being able to start up and shut down unit i.

The ramping-up limit of unit i is enforced through the constraints below:

$$P_{i\omega}(t) - P_{i\omega}(t-1) \leq S_i^{\mathrm{U}} v_i(t-1) + P_i^{\mathrm{SU}} y_i(t), \quad \forall i, \ \forall \omega, \ \forall t. \tag{5.7}$$

According to the above inequality, the upper bound S_i^{U} for the ramping-up is valid in general when unit i is on, while the more stringent upper bound P_i^{SU} becomes active only when it is started up.

Note that the higher the values of S_i^{U} and P_i^{SU}, higher is the flexibility of unit i, i.e., the higher its capability to adapt to changes as time evolves.

The following constraints enforce the ramping-down limit of unit i:

$$P_{i\omega}(t-1) - P_{i\omega}(t) \leq S_i^{\mathrm{D}} v_i(t) + P_i^{\mathrm{SD}} z_i(t), \quad \forall i, \ \forall \omega, \ \forall t. \tag{5.8}$$

Similarly to the case of ramping-up, the upper bound on the ramping-down is generally S_i^{D} when unit i is on. However, the more stringent upper bound P_i^{SD} is valid during shutdown.

Table 5.18 Ramping limits: limits for ramping-up and -down in megawatts per hour

	Unit 1	Unit 2	Unit 3
Ramping up/down limit	10	20	60

Table 5.19 Ramping limits: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	40	40	10
Period 2	50	20	5	30
Period 3	50	40	35	15

Table 5.20 Ramping limits: wind spillage in megawatts

Scenario	High	Low
Period 1	0	0
Period 2	30	0
Period 3	0	0

Like in the ramping-up case, the higher the values of S_i^{D} and P_i^{SD}, higher is the flexibility of unit i.

The example below illustrates the relationship between the system's flexibility and the ramping limits of conventional units. Specifically, it shows how tighter ramping limits result in a less flexible system, and thus in a higher expected cost.

Example 5.4 (Ramping limits) In this example, the rather tight ramping limits in Table 5.18 are considered. For the sake of simplicity, we do not consider different ramping limits during start up and shutdown of the units. Furthermore, we assume equal ramping-up and ramping-down limits, i.e., $S_i^{\mathrm{U}} = S_i^{\mathrm{D}}$. Notice that maximum and minimum power outputs are reset to the base case values.

The mathematical model used in this example consists of (5.1a)–(5.1l), (5.1o)–(5.1w), as well as the following simplified version of the ramping constraints (5.7) and (5.8):

$$P_{i\omega}(t) - P_{i\omega}(t-1) \leq S_i^{\mathrm{U}}, \quad \forall i,\ \forall \omega,\ \forall t, \tag{5.9}$$

$$P_{i\omega}(t-1) - P_{i\omega}(t) \leq S_i^{\mathrm{D}}, \quad \forall i,\ \forall \omega,\ \forall t. \tag{5.10}$$

Notice that binary variables are not needed in this case, since the minimum power output is zero for all units, and there are no special ramping-up and ramping-down limits at start up and shutdown. The initial production level is 50, 40, and 40 MW for units 1, 2, and 3, respectively.

Results are described below. Table 5.19 provides the optimal power scheduled for this case of tight ramping limits. Unit 2 cannot ramp down fast enough to be shut down in period 2, so it must be scheduled, thus reducing the dispatch level of unit 3. In period 3, unit 2 cannot ramp up as quickly as needed. Hence, unit 3 is dispatched at a higher level than in the base case in this period, so as to make up for the lower dispatch of unit 2, whose ramping limit is binding.

Table 5.20 provides the wind spillage resulting from the inflexible ramping limits of the production units. Note that the comparatively lower dispatch of unit 3 at time 2 implies smaller down-regulation capability: this results in higher wind spillage in the high wind scenario (spillage is 10 in the base case).

Table 5.21 Expected cost in $ in the case with ramping limits and in the base case

(a) Ramping limits

		Cost
	Schedule	7300
Energy	Deployed reserve (exp.)	−1365
	Total (exp.)	5935
Reserve capacity		585
Total (exp.)		6520

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.21 provides the expected cost for this case of tight ramping limits. The ramping constraints are not binding at the balancing stage, since all the reserve deployed is provided by flexible unit 3. Ramping constraints for unit 2 are, however, binding at the scheduling stage. The total expected costs increase as compared to the base case.

5.3.4 Minimum Up-Time and Down-Time

If a thermal production unit (e.g., coal-fired or gas-fired) is started up, for a number of technical reasons related to the functioning of its thermal system, it should be on for a minimum number of hours, known as *minimum up-time*. Similarly, if such unit is shutdown, it should remain down for a minimum number of hours, known as *minimum down-time*.

The constants needed to define the minimum up-time and down-time of production unit i are described in the following.

- T_i^{U} is the minimum number of hours that unit i needs to be up if started up.
- T_i^{D} is the minimum number of hours that unit i needs to be down if shut down.
- T_i^{U0} is the number of hours that unit i needs to be up from the beginning of the study horizon if it is up at its beginning (to comply with the minimum up-time condition).
- T_i^{D0} is the number of hours that unit i needs to be down from the beginning of the study horizon if it is down at its beginning (to comply with the minimum down-time condition).
- T is the number of hours of the study horizon.

Additionally, the auxiliary constants below are needed to define minimum up-time and down-time constraints:

$$T_i^{\mathrm{Ue}} = \min\{T, T_i^{\mathrm{U0}}\},$$
$$T_i^{\mathrm{De}} = \min\{T, T_i^{\mathrm{D0}}\}.$$

It is important to emphasize that the two relevant constants are T_i^{U} and T_i^{D}, while the remaining constants play an auxiliary role.

The lower the value of both T_i^{U} and T_i^{D}, the higher is the flexibility of unit i.

Enforcing the minimum up-time constraint of unit i requires the set of constraints below [14]:

$$\sum_{t=1}^{T_i^{\mathrm{Ue}}} v_i(t) = T_i^{\mathrm{Ue}}, \quad \forall i, \tag{5.11a}$$

$$\sum_{k=t}^{t+T_i^{\mathrm{U}}-1} v_i(k) \geq T_i^{\mathrm{U}} y_i(t), \quad \forall i,\ \forall t = T_i^{\mathrm{Ue}} + 1, \ldots, T - T_i^{\mathrm{U}} + 1, \tag{5.11b}$$

$$\sum_{k=t}^{T} [v_i(k) - y_i(t)] \geq 0, \quad \forall i,\ \forall t = T - T_i^{\mathrm{U}} + 2, \ldots, T. \tag{5.11c}$$

Constraints (5.11a) enforce the minimum up-time requirement if unit i is on-line at the beginning of the study horizon and the number of hours it should remain on-line is smaller than its minimum up-time. Complementarily, constraints (5.11b) enforce the minimum up-time requirement for all consecutive sets of hours of cardinality T_i^{U}. Finally, constraints (5.11c) enforce the minimum up-time requirement for the last T_i^{U} hours of the study horizon.

Analogously, enforcing the minimum down-time requirements of unit i calls for the set of constraints below [14]:

$$\sum_{t=1}^{T_i^{\mathrm{De}}} v_i(t) = 0, \quad \forall i, \tag{5.12a}$$

$$\sum_{k=t}^{t+T_i^{\mathrm{D}}-1} [1 - v_i(k)] \geq T_i^{\mathrm{D}} z_i(t), \quad \forall i,\ \forall t = T_i^{\mathrm{De}} + 1, \ldots, T - T_i^{\mathrm{D}} + 1, \tag{5.12b}$$

$$\sum_{k=t}^{T} [1 - v_i(k) - z_i(t)] \geq 0, \quad \forall i,\ \forall t = T - T_i^{\mathrm{D}} + 2, \ldots, T. \tag{5.12c}$$

Constraints (5.12a) enforce the minimum down-time requirement if unit i is off-line at the beginning of the study horizon and the number of hours it should remain off-line is smaller than its minimum down-time. Complementarily, constraints (5.12b) enforce the minimum down-time requirement for all consecutive sets of hours of cardinality T_i^{D}. Finally, constraints (5.12c) enforce the minimum down-time for the last T_i^{D} hours of the study horizon.

The example below illustrates the inflexibility brought to the production system by enforcing the minimum up-time and down-time limits of conventional units.

Example 5.5 (Minimum up-time and down-time) For the sake of a meaningful comparison, results from this example are not compared with those pertaining to the base case but with those pertaining to Example 5.3, in which minimum power outputs are different from zero.

Table 5.22 Minimum up-time/down-time: initial status, on or off, number of hours up from the beginning of the study horizon, and number of hours down from the beginning of the study horizon

	Unit 1	Unit 2	Unit 3
Initial status	on	on	on
Number of hours up	0	0	0
Number of hours down	0	0	0

Table 5.23 Minimum up-time/down-time: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	35	45	10
Period 2	50	25	0	30
Period 3	50	75	0	15

The setup of the system is identical to the one in Example 5.3, i.e., minimum power outputs of units 1, 2, and 3 are 10, 10, and 5 MW, respectively; and their capacities are 50, 110, and 100 MW, respectively. The three units have both minimum up-time and down-time equal to 2 h. The initial values for unit status and the number of hours the units must be up or down from the beginning of the study horizon are specified in Table 5.22.

The mathematical model used in this example consists of (5.1a)–(5.1j), (5.1o)–(5.1w), with the inclusion of (5.2)–(5.4), as well as of minimum up-time and down-time constraints (5.11) and (5.12).

Table 5.23 provides the optimal power scheduled in this case involving both minimum power output and minimum up-time/down-time. Contrarily to what happens in Example 5.3, unit 2 is not shut down during period 2. Shutting down unit 2 would be uneconomical because the unit would have to remain off-line during period 3 as well, as a result of its minimum down-time constraint.

Table 5.24 provides the optimal reserve scheduled. Notwithstanding that reserve cost for unit 2 is higher than for unit 3, reserve at periods 2 and 3 is solely provided by unit 2. The reason for this is that unit 3 is dispatched to zero in these two periods and consequently, cannot deploy down-regulation.

Table 5.25 contains the expected cost. Note that the expected cost increases in comparison to that in Example 5.3 (including minimum power output constraints but no minimum up-time/down-time constraints). Specifically, the energy dispatching cost is comparatively smaller but the (down) reserve deployment revenue is significantly smaller as well.

Table 5.24 Minimum up-time/down-time: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	40
Period 2	0	0	0	15	0	0
Period 3	0	0	0	20	0	0

Table 5.25 Expected cost in dollars in the case with minimum up-time and down-time requirements and the correspondent case of Example 5.3 without minimum up-time and down-time constraints

(a) Minimum up-/down-time

		Cost
	Schedule	7125
Energy	Deployed reserve (exp.)	−1470
	Total (exp.)	5655
Reserve capacity		780
Total (exp.)		6435

(b) No up-/down-constraints (Example 5.3)

		Cost
	Schedule	7250
Energy	Deployed reserve (exp.)	−1620
	Total (exp.)	5630
Reserve capacity		780
Total (exp.)		6410

Table 5.26 Full hydro energy availability: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Hydro	Wind
Period 1	50	0	30	50	10
Period 2	25	0	0	50	30
Period 3	50	5	20	50	15

5.3.5 Limited Hydro Energy Availability

Hydroelectric production units have generally high flexibility in terms of very high ramping rates and virtually no minimum power outputs. However, the amount of energy that can be produced within a given time period is limited, either for physical or strategic reasons.

Thus, the production of hydro unit h throughout the planning horizon for scenario ω is limited by the constraint below:

$$\sum_{t=1}^{T} P_{h\omega}(t) \leq E_h^{\max}, \quad \forall \omega, \tag{5.13}$$

where $E_h^{\max}$ is the energy available within the study horizon.

As a policy decision, condition (5.13) may also be imposed at scheduling time. However, we opt not to do so to achieve a more relaxed formulation that results in lower expected costs.

Note that the set of hydro units is a subset of the set I of production units and, as such, these units are subject to power and reserve limits (5.1k)–(5.1p).

The example below illustrates the flexibility brought to the system by relaxing the energy limits of hydroelectric units.

Example 5.6 (Limited hydro energy availability) In this example, we add a hydro power plant at bus 2. We consider that this plant offers at zero price both energy and reserve deployment, but it offers reserve at \$5 MW. The example includes: (1) a case with infinite energy availability and (2) a case with limited energy availability.

The mathematical model used in this example consists of (5.1a)–(5.1l), (5.1o)–(5.1w), as well as the energy limit (5.13) for the hydro plants in the system.

We consider first that the hydro plant has a capacity of 50 MW and infinite energy availability.

Table 5.26 gives the optimal power scheduled in megawatts in this case of infinite energy availability. Note that the hydro unit is fully dispatched and provides no

Table 5.27 Full hydro energy availability: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3		Hydro	
Reserve	Up	Down	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	30	0	0
Period 2	0	0	0	0	0	0	0	0
Period 3	0	0	0	0	0	20	0	0

Table 5.28 Expected cost in dollars in the case with full hydro energy availability and in the base case

(a) Full hydro energy availability

		Cost
	Schedule	3150
Energy	Deployed reserve (exp.)	−1050
	Total (exp.)	2100
Reserve capacity		450
Total (exp.)		2550

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.29 Limited hydro energy availability: optimal power schedules in megawatts

	Unit 1	Unit 2	Unit 3	Hydro	Wind
Period 1	50	0	40	40	10
Period 2	50	0	25	0	30
Period 3	50	35	20	20	15

reserve. This results from the following two assumptions: hydro power has zero production cost, while the reserve cost is positive. From an economical perspective, spilling wind is a more attractive option than dispatching down such a hydro unit, as the latter option involves purchasing reserve, while the first one implies no additional costs. In practice, a hydro power producer (whose energy availability is limited) would assign a certain value to the water stored in its dam, and therefore to its available energy. In that situation the provision of reserve would be economically meaningful for hydro power producers as well.

Table 5.27 provides the optimal reserve scheduled in megawatts for this case of full hydro energy availability. Down-reserve is provided in periods 1 and 3 by unit 3. No up-reserve is scheduled.

Table 5.28 provides the expected cost. Note that such cost drastically decreases as compared to the cost pertaining to the base case.

We consider next that the energy limit for the hydro plant is set to 60 MWh for the three periods of the study horizon.

Table 5.29 provides the optimal power scheduled for this case of limited hydro resources. Note that the hydro unit replaces unit 2 at periods 1 and 3. However, given its limited energy availability, the hydro unit is not dispatched at period 2, when demand is low and the wind power production peaks.

Table 5.30 provides the optimal reserve scheduled in megawatts for this case of limited hydro availability. Down-reserve is provided in all periods by unit 3. Observe

Table 5.30 Limited hydro energy availability: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3		Hydro	
Reserve	Up	Down	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	40	0	0
Period 2	0	0	0	0	0	25	0	0
Period 3	0	0	0	0	0	20	0	0

Table 5.31 Expected cost in dollars in the case with limited hydro energy availability and in the base case

(a) Limited hydro energy availability

		Cost
	Schedule	5525
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	3740
Reserve capacity		765
Total (exp.)		4505

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

that the reserve scheduled in this case of limited hydro availability is significantly higher than that scheduled in case of full hydro availability.

Table 5.31 provides the expected cost for this case with limited hydro energy availability. Note that the total expected cost is higher than that in the case with infinite energy availability, but lower than that in the base case.

The flexibility provided by the hydro unit consists in the possibility of shifting cheap power production in time. In this case, this unit is scheduled in periods 1 and 3, when wind power production is relatively low, to replace unit 2, which is less economically efficient than unit 3 when providing both power and reserve. From this point of view, hydro power complements the variability of stochastic renewable power production by supporting the system in periods with low production from stochastic units.

5.4 Flexibility from the Demand Side

Demands can contribute to system flexibility through a number of actions, namely:

1. By lowering the rate of increase at periods of high demand increase.
2. By lowering the rate of decrease at periods of high demand decrease.
3. By lowering the peak demand.
4. By increasing the valley demand.
5. By shifting energy from high demand to low demand periods.

For example, during the early morning (say around 7 a.m.), when wind power production is typically decreasing and solar power production has not picked up yet, a lower demand-increase rate helps the operation of the system, as the requirement to increase power imposed on regulating flexible units (e.g., CCGTs) decreases.

Conversely, during the late evening (say around 7 p.m.), when solar power production is typically decreasing and wind power production has not started to pick up yet, a slower drop-off of demand aids the operation of the system, as the requirement to decrease power imposed on regulating dispatchable units (e.g., CCGTs) decreases.

Reducing the demand at peak-demand hours contributes to lower the stress on peakers to supply the peak demand. Conversely, increasing the demand at valley-demand hours contributes to lower the stress on highly inflexible base-loaded units (e.g., nuclear power plants) to reduce their production levels.

Shiftable demands that can be transferred from high demand periods to low demand periods highly facilitate the operation of the system and reduce operation costs as base-loaded units produce comparatively higher quantities of energy and peaker intervention is reduced.

A simple mathematical model to illustrate how flexible demands contribute to system flexibility is discussed below.

5.4.1 Mathematical Model for Flexible Demands

This subsection provides a simple mathematical model to characterize flexible demands, which is similar to the one proposed in [5]. More detailed models considering demand scheduling (which involves on–off decisions) are provided in [9] and [8].

To formulate this mathematical model, we define first relevant constants that characterize the behavior of any flexible demand:

$D_k^{\min}(t)$ Minimum load required by flexible demand k during period t.
$D_k^{\max}(t)$ Maximum load that can be consumed by flexible demand k during period t.
D_k^{U} Maximum load pickup rate of flexible demand k.
D_k^{D} Maximum load drop rate of flexible demand k.
E_k^{day} Minimum daily energy consumption for flexible demand k.

On the other hand, relevant variables include:

$d_k(t)$ Scheduled load for flexible demand k during period t.
$d_{k\omega}(t)$ Actual load for flexible demand k during period t and scenario ω.
$c_{k\omega}^{\mathrm{D}}(t)$ Load increase of flexible demand k during period t and scenario ω, where the superscript D indicates that the demand is providing down-regulation.
$c_{k\omega}^{\mathrm{U}}(t)$ Load curtailment for flexible demand k during period t and scenario ω, where the superscript U stands for up-regulation.

Considering the definitions above, the actual load for flexible demand k in period t and scenario ω is expressed as

$$d_{k\omega}(t) = d_k(t) + c_{k\omega}^{\mathrm{D}}(t) - c_{k\omega}^{\mathrm{U}}(t), \tag{5.14}$$

where

$$0 \le c_{k\omega}^{\mathrm{D}}(t) \le D_k^{\max}(t) - d_k(t), \tag{5.15a}$$

$$0 \le c_{k\omega}^{\mathrm{U}}(t) \le d_k(t) - D_k^{\min}(t), \tag{5.15b}$$

which simply state that the change in the scheduled load of flexible demand k in period t is bounded above and below by its maximum and minimum load levels, respectively.

Additionally, the minimum daily energy consumption E_k^{day} of flexible demand k is enforced by

$$\sum_{t=1}^{T} d_{k\omega}(t) \geq E_k^{\text{day}}, \quad \forall \omega. \tag{5.16}$$

Also, the maximum load-pickup and load-drop rates for flexible demand k are stated as

$$d_{k\omega}(t) - d_{k\omega}(t-1) \leq D_k^{\text{U}}, \quad \forall t, \ \forall \omega, \tag{5.17a}$$

$$d_{k\omega}(t-1) - d_{k\omega}(t) \leq D_k^{\text{D}}, \quad \forall t, \ \forall \omega. \tag{5.17b}$$

The load shedding constraint pertaining to flexible demand k should be updated as well to:

$$L_{k\omega}^{\text{shed}}(t) \leq d_{k\omega}(t) + c_{k\omega}^{\text{D}}(t) - c_{k\omega}^{\text{U}}(t), \quad \forall t, \ \forall \omega. \tag{5.18}$$

Balancing conditions should also be reformulated, so they include the new variables for flexible demand.

Finally, note that modeling flexible demands requires expanding the objective function of the scheduling problem (5.1) to incorporate the utilities of the flexible demands, which is achieved by subtracting the term

$$\sum_{t=1}^{T} \sum_{k \in J} \left\{ U_k(t) d_k(t) + \sum_{\omega \in \Omega} \pi_\omega \left[U_k^{\text{D}}(t) c_{k\omega}^{\text{D}}(t) - U_k^{\text{U}}(t) c_{k\omega}^{\text{U}}(t) \right] \right\}, \tag{5.19}$$

where $U_k(t)$ is the utility of flexible demand k at time t at the dispatching stage, $U_k^{\text{D}}(t)$ and $U_k^{\text{U}}(t)$ the utilities of electricity purchase and sale, respectively, at the balancing stage, and J the set of all demands.

The example below illustrates the flexibility brought to the system by flexible demands.

Example 5.7 (Flexibility from the demand side) In this example, we upgrade the base case by including demand flexibility through demand bids that involve an underlying utility. We then increase demand flexibility by allowing it to provide regulation.

The optimization model for this example consists of the minimization of the subtraction of (5.19) from (5.1a), subject to constraints (5.1b)–(5.1l), (5.1o)–(5.1p), (5.1r)–(5.1w), as well as to constraints (5.14)–(5.18) for variables of flexible demand.

We first upgrade the base case by including utilities for flexible demands at the scheduling stage. However, the redispatch of demands at the balancing stage is not yet allowed, i.e., we enforce $c_{k\omega}^{\text{D}}(t) = c_{k\omega}^{\text{U}}(t) = 0, \forall k, \forall \omega, \forall t$.

Table 5.32 Demand flexibility: offer data including marginal utility in dollars per megawatt hour and minimum and maximum levels in megawatts

(a) Marginal utility

	Demand 1	Demand 2	Demand 3	Demand 4
Period 1	40	15	45	55
Period 2	25	15	30	40
Period 3	35	15	40	50

(b) Lower and upper limits

	Demand 1		Demand 2		Demand 3		Demand 4	
	Min	Max	Min	Max	Min	Max	Min	Max
Period 1	0	40	0	50	0	60	0	40
Period 2	0	25	0	45	0	35	0	45
Period 3	0	45	0	50	0	40	0	55

Table 5.33 Demand flexibility: offer data including minimum daily energy consumption in megawatt hour and pick-up/drop rate limits in megawatt per hour

	Minimum consumption	Pick-up/drop rate limit
Demand 1	100	25
Demand 2	0	15
Demand 3	120	30
Demand 4	130	20

Table 5.32 provides data pertaining to demand flexibility offers. Demands 1 and 2 are located at bus 1, while demands 3 and 4 are located at bus 2.

Table 5.33 provides the minimum daily energy consumption per demand and the demand pick-up and drop rates.

Table 5.34 provides the optimal power production/consumption scheduled in megawatts. Note that production and demand schedules are equal to the corresponding ones in the base case at both buses.

Table 5.34 Demand flexibility: optimal power production/consumption scheduled in megawatts
(a) Production schedule

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	40	40	10
Period 2	50	0	25	30
Period 3	50	55	20	15

(b) Consumption schedule

	Bus 1			Bus 2		
	Demand 1	Demand 2	Total	Demand 3	Demand 4	Total
Period 1	40	0	40	60	40	100
Period 2	25	0	25	35	45	80
Period 3	45	0	45	40	55	95

Table 5.35 Expected cost, utility and social welfare in dollars in the augmented base case with bids from the consumption side and no redispatch, and cost in the base case, where demand is inflexible

(a) Augmented base case (with consumption bids and no redispatch)

		Cost	Utility	Welfare
	Schedule	7325	15900	8575
Energy	Deployed reserve (exp.)	−1785	0	1785
	Total (exp.)	5540	15900	10360
Reserve capacity		765	–	−765
Total (exp.)		6305	15900	9595

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.35 provides expected costs and expected utilities. Notice that the production schedule in Table 5.34(a) coincides with the one in Table 5.3 for the base case, and so does the amount of purchased reserve. Therefore, the costs for scheduling production, purchasing and deploying reserve in the augmented base case with flexible demand are equal to the ones in the base case in Example 5.1. Additionally, the inclusion of consumption bids in the model permits us to calculate the total consumer utility associated with the consumption schedule in Table 5.34(b), as well as the social welfare, which is determined by subtracting the costs from the utility. The latter quantities will turn out to be useful as a comparison in the remainder of this example, where demands can be redispatched at the balancing stage.

Next, we allow demands to be redispatched within their physical limits. Purchasing/selling price is 5 % lower/higher than offering prices at the scheduling stage.

Table 5.36 provides the optimal power production/consumption scheduled in megawatts in this case in which loads can provide reserve.

Note that the optimal dispatch changes with respect to the case in which demand reserve is not allowed. Observe that demand 2 (whose utility is comparatively smaller) is scheduled and employed to provide up-reserve. As we shall see later in this example, this demand is ready to decrease its consumption, thus deploying up-reserve, in the scenario with low wind power production.

Table 5.37 provides the optimal reserve scheduled by thermal units in megawatts. The reserve level provided by thermal units decreases as a consequence of the flexibility on the demand side.

Table 5.38 provides the deployed up-reserve by demands. It is important to note that only up-reserve is deployed by flexible demands, which decrease their consumption in the scenario with low wind power production.

Table 5.36 Demand flexibility including redispatch: optimal power production/consumption scheduled in megawatts

(a) Production schedule

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	40	37.5	12.5
Period 2	50	0	0	65
Period 3	50	67.5	0	35

(b) Consumption schedule

	Bus 1			Bus 2		
	Demand 1	Demand 2	Total	Demand 3	Demand 4	Total
Period 1	40	0	40	60	40	100
Period 2	25	10	35	35	45	80
Period 3	45	12.5	57.5	40	55	95

Table 5.37 Demand flexibility including redispatch: optimal reserve schedule by thermal unit in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	37.5
Period 2	0	0	0	0	0	0
Period 3	0	0	0	0	0	0

Table 5.39 reports expected cost, expected utility and social welfare for this case including demand flexibility and reserve provided by demand, reported in Table 5.39(a), and for the case previously considered in this example, where demand is flexible but does not provide reserve, in Table 5.39(b). Note that the expected social welfare increases when demand is allowed to provide reserve. Indeed, this augments the flexibility of the system.

5.5 Flexibility from Storage Availability

Storage plants allow shifting in time the demand by consuming electricity at low-demand (low-price) periods and producing it during high-demand (high-price) periods. This demand shifting is not free, involving generally an overall efficiency ranging between 75 and 80 %.

Table 5.38 Demand flexibility including redispatch: up-reserve deployed by demands in megawatts

	Demand 1		Demand 2		Demand 3		Demand 4	
Scenario	High	Low	High	Low	High	Low	High	Low
Period 1	0	0	0	0	0	2.5	0	0
Period 2	0	7.5	0	10	0	7.5	0	10
Period 3	0	2.5	0	12.5	0	5	0	0

Table 5.39 Cost, utility and welfare in cases of flexible demand with and without the possibility of redispatching the consumption at the balancing stage

(a) Demand flexibility including redispatch

		Cost	Utility	Welfare
	Schedule	6037.5	16237.5	10200
Energy	Deployed reserve (exp.)	−787.5	−620	167.5
	Total (exp.)	5587.5	15617.5	10030
Reserve capacity		337.5	–	−337.5
Total (exp.)		5250	15617.5	10367.5

(b) Augmented base case (with consumption bids and no redispatch)

		Cost	Utility	Welfare
	Schedule	7325	15900	8575
Energy	Deployed reserve (exp.)	−1785	0	1785
	Total (exp.)	5540	15900	10360
Reserve capacity		765	–	−765
Total (exp.)		6305	15900	9595

Increasing consumption at low-demand periods allows inflexible production units with limited capability of reducing their power outputs to maintain their production levels throughout these low demand periods. Thus, pumped-storage plants add flexibility to the system at low demand periods. Conversely, producing from storage units at high demand periods reduce the need of reserve and the use of costly peakers, and thus pumped-storage plants add flexibility to the system at peak demand periods as well.

It is important to note that we consider that storage units do not actively participate in the market seeking maximum profit; instead, they are considered regulated facilities operated by the system operator for the benefit of the market as a whole. In this sense, storage units play a regulated role similar to that of the transmission system.

Pumped-storage is the storage technology most commonly found in practice. However, other storage technologies are also available, such as compressed air. In the following we consider a pumped-storage plant.

A simple mathematical model for the functioning of a pumped-storage unit is described below. It is important to note that time periods of 1 hour are considered for this analysis and thus no distinction is made between power and energy in each hour.

The constants characterizing the working of pumped-storage unit p are:

$Q_p^{\max}$ Maximum water flow in production/pumping mode.
σ_p^{T} Conversion factor water flow to produced power ($\sigma_p^{\mathrm{T}} < 1$).
σ_p^{P} Conversion factor water flow to consumed power ($\sigma_p^{\mathrm{P}} > 1$).
$V_p^{\mathrm{U,ini}}$ Initial water content of the upper reservoir.
$V_p^{\mathrm{U,min}}$ Minimum water content of the upper reservoir.
$V_p^{\mathrm{U,max}}$ Maximum water content of the upper reservoir.

$V_p^{\mathrm{L,ini}}$ Initial water content of the lower reservoir.
$V_p^{\mathrm{L,min}}$ Minimum water content of the lower reservoir.
$V_p^{\mathrm{L,max}}$ Maximum water content of the lower reservoir.

The variables needed to describe the functioning of pumped-storage unit p are:

$q_{p\omega}^{\mathrm{T}}(t)$ Water flow turbined during period t and scenario ω.
$q_{p\omega}^{\mathrm{P}}(t)$ Water flow pumped during period t and scenario ω.
$P_{p\omega}^{\mathrm{T}}(t)$ Power production during period t and scenario ω.
$P_{p\omega}^{\mathrm{P}}(t)$ Power consumption during period t and scenario ω.
$v_{p\omega}^{\mathrm{U}}(t)$ Water content of the upper reservoir at the beginning of period t and scenario ω.
$v_{p\omega}^{\mathrm{L}}(t)$ Water content of the lower reservoir at the beginning of period t and scenario ω.

The working of pumped-storage unit p is described by the constraints below:

$$P_{p\omega}^{\mathrm{T}}(t) = \sigma_p^{\mathrm{T}} \times q_{p\omega}^{\mathrm{T}}(t), \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20a}$$

$$P_{p\omega}^{\mathrm{P}}(t) = \sigma_p^{\mathrm{P}} \times q_{p\omega}^{\mathrm{P}}(t), \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20b}$$

$$v_{p\omega}^{\mathrm{U}}(t+1) = v_{p\omega}^{\mathrm{U}}(t) + q_{p\omega}^{\mathrm{P}}(t) - q_{p\omega}^{\mathrm{T}}(t), \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20c}$$

$$v_{p\omega}^{\mathrm{L}}(t+1) = v_{p\omega}^{\mathrm{L}}(t) + q_{p\omega}^{\mathrm{T}}(t) - q_{p\omega}^{\mathrm{P}}(t), \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20d}$$

$$V_p^{\mathrm{U,min}} \le v_{p\omega}^{\mathrm{U}}(t) \le V_p^{\mathrm{U,max}}, \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20e}$$

$$V_p^{\mathrm{L,min}} \le v_{p\omega}^{\mathrm{L}}(t) \le V_p^{\mathrm{L,max}}, \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20f}$$

$$v_{p\omega}^{\mathrm{U}}(t) \ge V_p^{\mathrm{U,ini}}, \quad t = T,\ \forall p,\ \forall \omega, \tag{5.20g}$$

$$v_{p\omega}^{\mathrm{L}}(t) \ge V_p^{\mathrm{L,ini}}, \quad t = T,\ \forall p,\ \forall \omega, \tag{5.20h}$$

$$0 \le q_{p\omega}^{\mathrm{T}}(t) \le Q_p^{\max}, \quad \forall p,\ \forall t,\ \forall \omega, \tag{5.20i}$$

$$0 \le q_{p\omega}^{\mathrm{P}}(t) \le Q_p^{\max}, \quad \forall p,\ \forall t,\ \forall \omega. \tag{5.20j}$$

Constraints (5.20a) define the power generated by pumped-storage unit p during period t and scenario ω as a function of the conversion factor (water flow to electric power) and the water extracted from the upper reservoir. Constraints (5.20b) define the power consumed by pumped-storage unit p to pump water from the lower to the upper reservoir during period t and scenario ω as a function of the conversion factor (electric power to water flow) and the amount of water pumped from the lower reservoir. Constraints (5.20c) and (5.20d) are water balance constraints involving the upper and lower reservoirs, respectively. Constraints (5.20e) and (5.20f) enforce the limits on water content in the lower and upper reservoirs, respectively. Constraints (5.20g) and (5.20h) fix minimum water levels in the upper and lower reservoirs, respectively, at the end of the study horizon. Note that levels other than the initial ones can be imposed. Constraints (5.20i) limit the extraction of water from the upper reservoir to generate electricity, while constraints (5.20j) limit the extraction of water from the lower reservoir for pumping.

Table 5.40 Storage availability: storage unit data

Parameter	Value
Maximum pumping/turbining flow in Hm^3/h	20
Factor water-flow to power in (MW)/(Hm^3/h)	0.8
Factor power to water-flow in (MW)/(Hm^3/h)	1.2
Upper reservoir initial volume in Hm^3	40
Upper reservoir minimum volume in Hm^3	0
Upper reservoir maximum volume in Hm^3	80
Lower reservoir initial volume in Hm^3	40
Lower reservoir minimum volume in Hm^3	0
Lower reservoir maximum volume in Hm^3	80

Note that the power delivered or consumed by a pumped-storage plant enters the power balance equations in the same fashion as conventional production and consumption. Notice also that power output limits are already enforced in (5.20).

The example below illustrates the flexibility brought to the system by pumped-storage units.

Example 5.8 (Storage availability) In this example, we consider a pumped-storage power plant located at bus 2 of the power system in the base-case described in Example 5.1. The parameter values for the pumped-storage power plant are provided in Table 5.40.

The mathematical model used in this example consists of (5.1a)–(5.1l) and (5.1o)–(5.1w), as well as of constraints (5.20) for the storage variables.

Results are given below. Table 5.41 provides the details of the operation of the pumped-storage plant at the balancing stage. At period 2, the pumped-storage unit consumes 10 MWh of wind power production that would otherwise be spilled in the high wind power scenario. At period 1, an equivalent amount of water is turbined. Considering the prices of the base case, see Table 5.8, we see that the storage performs arbitrage across time periods.

Table 5.42 provides the optimal schedule of thermal plants in megawatts. Note that the schedule for period 1 changes from that in the base case so as to give unit 3 the capability of further reducing its output if the pumped-storage unit turbines.

Table 5.43 provides the optimal reserve allocated to thermal units in megawatts. Reserve requirement at time 1 increases. This so because in the high wind scenario the pumped-storage unit feeds power into the grid at period 1, requiring unit 3 to decrease its production.

Table 5.44 reports the expected cost. As expected, this cost decreases in comparison with the base case due to the ability of the pumped-storage unit to transfer consumption from high-cost periods to low-cost ones.

5.6 Flexibility from Enhancing the Transmission Network

The power flow through transmission line ℓ, with origin and end buses $o(\ell)$ and $e(\ell)$, respectively, during time t is enforced at the scheduling stage by (5.1d)–(5.1e). Similarly, the transmission constraints during time period t and for scenario ω at

Table 5.41 Storage availability: pumped-storage plant operation outcome including production and consumption in megawatts, turbined and pumped flow in Hm^3/h, upper and lower reservoir content in Hm^3

(a) Power operation

	Turbined power		Pumped power	
Scenario	High	Low	High	Low
Period 1	6.67	0	0	0
Period 2	0	0	10	0
Period 3	0	0	0	0

(b) Flow operation

	Turbined flow		Pumped flow	
Scenario	High	Low	High	Low
Period 1	8.33	0	0	0
Period 2	0	0	8.33	0
Period 3	0	0	0	0

(c) Reservoir content

	Upper reservoir		Lower reservoir	
Scenario	High	Low	High	Low
Period 1	31.67	40	48.33	40
Period 2	40	40	40	40
Period 3	40	40	40	40

Table 5.42 Storage availability: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	33.33	46.67	10
Period 2	50	0	25	30
Period 3	50	55	20	15

Table 5.43 Storage availability: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	46.67
Period 2	0	0	0	0	0	25
Period 3	0	0	0	0	0	20

the balancing stage are modeled by (5.1f)–(5.1g). In constraints of this type, $C_\ell^{\max}$ represents the line capacity (i.e., the maximum power flow that can be transmitted), b_ℓ its susceptance, and $\delta^0_{o(\ell)}(t)$ ($\delta_{o(\ell)\omega}(t)$) and $\delta^0_{e(\ell)}(t)$ ($\delta_{e(\ell)\omega}(t)$) the voltage angles at the sending and receiving ends, respectively, in the scheduling (balancing) stage.

Note that the transmission constraints at the scheduling stage (5.1d)–(5.1e) may be discarded without hindering the actual operation of the power system. Indeed, the actual power flows at the balancing stage may still satisfy the transmission limits in

Table 5.44 Expected cost in dollars in the case of storage availability and in the base case

(a) Storage availability

		Cost
	Schedule	7358.33
Energy	Deployed reserve (exp.)	−1925
	Total (exp.)	5433.33
Reserve capacity		825
Total (exp.)		6258.33

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.45 Reduced transmission capacity: optimal production schedule in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	20	60	10
Period 2	50	0	25	30
Period 3	50	40	35	15

Table 5.46 Reduced transmission capacity: flows through the line connecting buses 1 and 2 in megawatts

Stage	Scheduling	Balancing	
Scenario	–	High	Low
Period 1	40	80	40
Period 2	55	80	55
Period 3	60	80	60

any realization of the stochastic power production, although the scheduled flows do not.

The system flexibility generally increases if the capacity limits of its transmission lines increase or if the transmission system is expanded, allowing eventually sending larger quantities of energy from areas with excess of generation to areas with excess of demand. For instance, this may facilitate the transmission of large quantities of wind power (at times of high wind power production) from its generation area to areas of expensive generation.

The example below illustrates the relationship between the system flexibility and its transmission capacity. As we shall see in the example, reducing the transmission capacity decreases flexibility, hence leading to higher expected cost for the operation of the system.

Example 5.9 (Reduced transmission capacity) The mathematical model used in this example is the one of Example 5.1, consisting of the minimization of objective function (5.1a), subject to constraints (5.1b)–(5.1l) and (5.1o)–(5.1w).

The transmission capacity between buses 1 and 2 is reduced from 100 MW to 80 MW so that the system incurs in more severe congestion. Results are explained below.

Table 5.45 provides the optimal power dispatch in megawatts. The power output of unit 3 needs to increase as compared to the base case since this unit is the only one located at bus 2, which also accommodates a large share of the total system demand.

Table 5.46 provides the flows through the line in megawatts, including scheduled and realized flows in both scenarios. Note that if wind power production is high, the transmission capacity is reached and the system becomes congested.

Table 5.47 Expected costs with reduced transmission capacity and in the base case

(a) Reduced transmission capacity

		Cost
	Schedule	7500
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5715
Reserve capacity		765
Total (exp.)		6480

(b) Base case

		Cost
	Schedule	7325
Energy	Deployed reserve (exp.)	−1785
	Total (exp.)	5540
Reserve capacity		765
Total (exp.)		6305

Table 5.47 provides expected costs. As foreseen, a more constrained problem results in increased expected costs as compared to the base case.

5.7 Measuring Flexibility via Sensitivity

A key issue to assess the flexibility of a given electric energy system is to to be able to provide an accurate measure of such flexibility. This section aims at providing such measure.

Any of the optimization problems considered throughout in this chapter can be formulated as a general *primal* problem of the form

$$\underset{x}{\text{Min.}}\ z_P = f(x,a) \tag{5.21a}$$

$$\text{s.t. } h(x,a) = b \quad : \lambda, \tag{5.21b}$$

$$g(x,a) \leq c \quad : \mu, \tag{5.21c}$$

where $x \in \mathbb{R}^n$, $a \in \mathbb{R}^p$, $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$, $h : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^\ell$, $g : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^m$, with $h(x,a) = (h_1(x,a), \ldots, h_\ell(x,a))^\top$, $g(x,a) = (g_1(x,a), \ldots, g_m(x,a))^\top$ and $f, h, g \in C^2$, i.e., their derivatives up to order 2 exist and are continuous.

The *dual* problem of problem (5.21) is

$$\underset{\lambda,\mu}{\text{Max.}}\ z_D = \underset{x}{\text{Infimum}}\{\mathcal{L}(x,\lambda,\mu,a,b,c)\} \tag{5.22a}$$

$$\text{s.t. } \mu \geq 0, \tag{5.22b}$$

where

$$\mathcal{L}(x,\lambda,\mu,a,b,c) = f(x,a) + \lambda^\top(h(x,a) - b) + \mu^\top(g(x,a) - c) \tag{5.23}$$

is the Lagrangian function associated with problem (5.21), and λ and μ are dual variable vectors of dimensions ℓ and m, respectively.

Under convexity and regularity assumptions [11], if primal problem (5.21) has an optimal solution x^*, then dual problem (5.22) also has an optimal solution (λ^*, μ^*), and the optimal values of the objective functions of the two problems coincide.

Considering problems (5.21) and (5.22), the following questions regarding sensitivity are in order:

1. What is the local sensitivity of $z_P^* = f(x^*, a)$ to changes in a, b and c? That is, the sensitivity of the objective function value at the optimum with respect to the input parameters.
2. What is the local sensitivity of x^* to changes in a, b and c? That is, the sensitivity of the primal variables at their optimal values with respect to the input parameters.
3. What are the local sensitivity of λ^* and μ^* to changes in a, b and c? That is, the local sensitivities of the dual variables at their optimal values with respect to the data.

Specifically, we are interested in:

1. The sensitivity (as a measure of flexibility) of the optimal social cost (objective function optimal value) with respect to both production parameters and transmission parameters. That is, to generation unit capacity limits, ramping limits, minimum up-time and down-time parameters, energy capacity limits; to pumped-storage unit reservoir capacities and other pumping/turbining limits; and to transmission line capacity limits, resistances, reactances and shunt susceptances.
2. The sensitivities (as measures of flexibility) of the scheduled production and consumption energy levels, reserve levels and the actual reserve deployment levels at each scenario with respect to the generation unit parameters and transmission line parameters.
3. The sensitivities (as measures of flexibility) of certain dual variables, particularly those representing locational marginal prices, with respect to both generation unit parameters and transmission line parameters.

It is important to note that a given sensitivity, e.g., the sensitivity of the objective function value at the optimum with respect to the ramping-up limit of a given CCGT, provides a local measure of the actual value of the flexibility. In other words, a high figure of such sensitivity indicates that rendering more flexible the ramping-up limit of the CCGT has a high (local) impact on system operation costs. In turn, this information can be used to implement practical flexibility actions, such as relaxing the ramping-up limit of the aforementioned CCGT within reasonable technical bounds.

The above sensitivity analysis to measure flexibility can be carried out as stated in [2]. As an example, the analysis of the sensitivity of locational marginal prices with respect to system parameters is analyzed in [3].

Notwithstanding the fact that any sensitivity analysis requires continuity and is essentially local, such analysis may provide initial insight into problems as the ones considered throughout this chapter that are not continuous. Therefore, the sensitivity results should be understood as approximate and used with care.

Table 5.48 Pricing under nonconvexity: initial status for the units

	Unit 1	Unit 2	Unit 3
Initial status	on	on	off
Number of hours up	0	0	0
Number of hours down	0	0	0

Table 5.49 Pricing under nonconvexity: optimal power scheduled in megawatts

	Unit 1	Unit 2	Unit 3	Wind
Period 1	50	35	45	10
Period 2	50	10	15	30
Period 3	50	50	25	15

Table 5.50 Pricing under nonconvexity: optimal reserve scheduled in megawatts

	Unit 1		Unit 2		Unit 3	
Reserve	Up	Down	Up	Down	Up	Down
Period 1	0	0	0	0	0	40
Period 2	0	0	0	0	0	10
Period 3	0	0	0	0	0	20

5.8 On Pricing

The problems considered in this chapter are mixed-integer and linear. It is important to note that once the optimal solution of one of these mixed-integer linear programming problems is derived (by branch-and-cut algorithms or the like), binary variables can be substituted in this problem by their optimal values, which results in a (continuous) linear programming problem. Day-ahead market clearing prices as well as real-time prices are then derived as stated in Chap. 3. This is common practice in a number of electricity markets, such as ISO New England [7], Midwest ISO [12] and PJM [16].

However, such prices do not generally "clear" the market, i.e., some production units may experience negative profits under such prices. Therefore, compensation mechanisms need to be devised and implemented to keep these units in the market. Such compensation mechanisms are based on uplifts [13] or others [17].

It is important to note that an increasing system flexibility generally results in comparatively low LMPs, as no active inequality constraint pushes prices up.

The example below illustrates the effect of flexibility on clearing prices, which may not guarantee cost recovery for the producers when the optimal dispatch problem includes binary variables.

Example 5.10 (Pricing under nonconvexity) In this example, we revisit the setup of Example 5.5 with minimum power outputs and minimum up-time and down-time requirements. Like in that example, the employed mathematical model consists of (5.1a)–(5.1j), (5.1o)–(5.1w), (5.2)–(5.4) and (5.11) and (5.12).

Table 5.48 provides the initial status of the units, on or off. Notice that unit 3 is initially off. Both the minimum up-time and down-time for unit 1, 2, and 3 are set to 2, 2, and 3, respectively.

Table 5.49 provides the optimal power scheduled in megawatts. All units need to be on for the entire planning horizon due to their limited flexibility.

Table 5.50 provides the optimal reserve schedule in megawatts. All reserve is provided by unit 3. Notice that at period 2 the contracted down-reserve is not sufficient

Table 5.51 Pricing under nonconvexity: prices in dollars per megawatt hour. Scenario prices are multiplied by the corresponding scenario probability

Stage	Scheduling		Balancing			
Scenario	–	–	High		Low	
Bus	1	2	1	2	1	2
Period 1	30	30	7	7	23	23
Period 2	23	23	0	0	23	23
Period 3	30	30	7	7	23	23

to absorb the extra production in the high wind power scenario (i.e., there is wind spillage). Unit 2 cannot provide down-reserve at this period because of its minimum power output (10 MW). Since unit 2 provides no reserve, it is not redispatched at any time period.

Table 5.51 provides scheduling and scenario prices (dual variables of scenario balance equations). Scenario prices are implicitly multiplied by the corresponding scenario probability. It is important to note that this table reports the dual variables of the balancing constraints for a problem in which the binary variables have been set to their respective optimal values.

Regarding periods 1 and 3 at scheduling stage, observe that a marginal increase in load would be covered by unit 2, whose marginal cost is \$30/MWh, without consequences on the operation at the balancing stage, i.e., reserve from unit 3 is already sufficient to cover the increase of wind power production in the high wind scenario. Hence the resulting price is \$30/MWh.

During periods 1 and 3 in the high wind scenario, observe that 1 extra megawatt of load would be covered by reducing the down regulation from unit 3, thus increasing operation costs by 0.6×35 (probability times cost); however, a saving of \$5 would be achieved by dispatching unit 2 instead of unit 3 at the scheduling stage and \$9 from buying less down-reserve. Therefore, the change in the objective function value is $0.6 \times 35 - 5 - 9 = 7$, and thus the balancing price is \$7/MWh.

As far as periods 1 and 3 are concerned in the low wind scenario, observe that an additional load would be covered by unit 3, implying an expected redispatch cost of 0.4×35 and an extra reserve cost of 9, totalling \$23/MWh. Hence the balancing price is \$23/MWh.

Regarding period 2 at scheduling stage, observe that an additional load would be covered by unit 3, which would increase its capability to provide down-regulation. The extra dispatch is 35 and for reserve 9; however, we save 0.6×35 in expectation at the balancing stage. The net extra cost is \$23/MWh. Hence the balancing price is \$23/MWh.

During period 2 in the high wind scenario, since there is wind spillage, a load increase can be served at 0 cost, and thus the balancing price is \$0/MWh.

In the low wind scenario during the same period, the same conclusions as for periods 1 and 3 apply.

Finally, it is important to note that unit 2 incurs a negative profit (it provides no reserve): at the scheduling stage it sells energy at its marginal cost at time 1 and 3. At time 2, it sells energy below its marginal cost, resulting in the loss $10 \times 23 - 10 \times 30 = -70$.

5.9 Summary and Conclusions

As stochastic production facilities, such as those based on wind, solar, geothermal, or wave resources, are increasingly integrated into an electric energy system, the following observations become more and more relevant:

1. Transmission *overcapacity* is increasingly important, as significant volumes of stochastic energy need to be transmitted throughout the network, particularly if high stochastic production occurs in some areas of the network but not throughout it.
2. Storage facilities play an increasingly important role as they allow shifting demand from periods with low stochastic production to periods with high stochastic production, easing the operation of the production system.
3. Other flexibility components related to thermal units, such as high ramping ratios, small minimum up-time and down-time and low minimum power output, play an important but secondary role.

5.10 Further Reading

A pioneering reference on electric energy system flexibility is [10]. MILP models representing the on-off status and operating regions of production units, which characterize their flexibility, are analyzed in detail in [14], [1] and [6]. Demand response models are described in [5], [9] and [8]. The impact of storage availability on system operation is studied, for instance, in [15], while the effect of transmission congestion is described, for instance, in [4]. Pricing and nonconvex pricing issues are considered in [17] and [13]. Sensitivity analysis is comprehensively analyzed in [2] and a case study is provided in [3]. Basic background on optimization models is provided in [11], while information on some electric energy markets is available at the web sites referenced in [7], [12] and [16].

Exercises

5.1 Consider a single-bus system including two thermal production units and one demand operating over a two-period time horizon. Visualize the effect on the operation outcomes of increasingly restrictive ramping limits (up and down) on one of the thermal units.

5.2 Substitute in Exercise 5.1 above the thermal unit with no ramping limits with a wind power plant characterized by three production scenarios. With such configuration, visualize the effect on the operation outcomes of increasingly restrictive ramping limits on the thermal unit.

5.3 Consider a single-bus system including a thermal unit, a pumped storage one and a demand operating over a horizon involving two time periods. Analyze the effect of increasing the upper and lower reservoir sizes on the operation outcomes of the pumped-storage unit.

5.4 Substitute the thermal plant in Exercise 5.3 above by a concentrating solar thermal plant characterized by three production scenarios. Analyze the effect on the operation outcomes of increasing the reservoir sizes (upper and lower) of the pumped-storage unit.

5.5 Consider a two-bus electric energy system operating over a two-period time horizon. A combined-cycle gas turbine is available at bus 1 while a wind power plant and a solar thermal power plant are located at bus 2. The demand is located at bus one. Each stochastic power plant is characterized by two scenarios. Analyze how locational marginal prices change with the capacity of the line.

5.6 Consider a single bus system including two thermal units and one demand operating over a horizon comprising two time periods. The capacity of the cheaper thermal unit allows supplying 95 % of the load at peak demand while the expensive thermal unit covers the remaining 5 %. Analyze the impact on market outcomes of a reduction in the peak load of the demand. What is the impact on clearing prices?

5.7 A single bus electric energy system includes a solar thermal plant and a combined cycle gas turbine to supply a constant demand. Both production units can operate from 0 to maximum power output. The operation of the solar thermal plant is characterized by two scenarios. Carry out a sensitivity analysis (as in [3]) to find out how clearing prices change as the production scenarios of the solar thermal plant change.

5.8 A single demand is supplied by a hydro plant and an expensive thermal unit over a horizon involving three time periods. The hydro plant has enough capacity to supply the whole demand but the energy content of its reservoir is limited. Analyze the impact on market outcomes (particularly on the total social cost) of an increasing availability of stored energy.

5.9 Consider a single bus system including a demand, a combined cycle gas turbine and a concentrating solar power plant with and without storage, operating over a three-period time horizon. The concentrating solar thermal plant is characterized by four production scenarios. Study how the operation flexibility of the system changes with the size of the storage facility and its operation policy.

5.10 Which flexibility elements should incorporate a fully renewable electric energy system, involving solar, wind and biomass power units?

References

1. Arroyo, J., Conejo, A.: Optimal response of a thermal unit to an electricity spot market. IEEE Trans. Power Syst. **15**(3), 1098 –1104 (2000)
2. Conejo, A., Castillo, E., Minguez, R., Garcia-Bertrand, R.: Decomposition Techniques in Mathematical Programming: Engineering and Science Applications. Springer, Berlin (2006)

3. Conejo, A., Castillo, E., Minguez, R., Milano, F.: Locational marginal price sensitivities. IEEE Trans. Power Syst. **20**(4), 2026 – 2033 (2005)
4. Conejo, A., Milano, F., Garcia-Bertrand, R.: Congestion management ensuring voltage stability. IEEE Trans. Power Syst. **21**(1), 357 – 364 (2006)
5. Conejo, A., Morales, J., Baringo, L.: Real-time demand response model. IEEE Trans. Smart Grid **1**(3), 236 –242 (2010)
6. Gomez-Exposito, A., Conejo, A., Canizares, C.: Electric Energy Systems: Analysis and Operation. Electric Power Engineering Series. Taylor & Francis, Boca Raton, FL (2008)
7. ISO New England: http://www.iso-ne.com (2012)
8. Karangelos, E., Bouffard, F.: Towards full integration of demand-side resources in joint forward energy/reserve electricity markets. IEEE Trans. Power Syst. **27**(1), 280 –289 (2012)
9. Khodaei, A., Shahidehpour, M., Bahramirad, S.: SCUC with hourly demand response considering intertemporal load characteristics. IEEE Trans. Smart Grid **2**(3), 564 –571 (2011)
10. Lannoye, E., Flynn, D., O'Malley, M.: Evaluation of power system flexibility. IEEE Trans. Power Syst. **27**(2), 922 –931 (2012)
11. Luenberger, D., Ye, Y.: Linear and Nonlinear Programming, 3rd edn. International Series in Operations Research & Management Science. Springer, New York (2008)
12. Midwest ISO: https://www.midwestiso.org (2012)
13. O'Neill, R., Sotkiewicz, P., Hobbs, B., Rothkopf, M., Stewart, W.: Efficient market-clearing prices in markets with nonconvexities. Eur. J. Oper. Res. **164**(1), 269–285 (2005)
14. Ostrowski, J., Anjos, M., Vannelli, A.: Tight mixed integer linear programming formulations for the unit commitment problem. IEEE Trans. Power Syst. **27**(1), 39 –46 (2012)
15. Pandzic, H., Morales, J.M., Conejo, A.J., Kuzle, I.: Offering model for a virtual power plant based on stochastic programming. Appl. Energy **105**, 282–292 (2013)
16. PJM: http://www.pjm.com/ (2012)
17. Ruiz, C., Conejo, A., Gabriel, S.: Pricing nonconvexities in an electricity pool. IEEE Trans. Power Syst. **27**(3), 1334 –1342 (2012)

Chapter 6
Impact of Stochastic Renewable Energy Generation on Market Quantities

6.1 Introduction

One of the most traditional systems for the commercial exchange of electricity production from stochastic renewable energy sources is that of feed-in tariffs, for which a price per generated energy unit is set by regulatory authorities. These energy producers are then given a payment that is directly proportional to the generation from their portfolio, regardless of variability and predictability characteristics. Stochastic renewable energy generation as a whole is consequently seen as a negative load that only magnifies the variability of the actual electric load, while decreasing its level of predictability. With negligible penetration of stochastic renewable energy in a power system, such an approach to their management makes sense. In contrast, as this penetration increases, it may be more appropriate to incentivize power producers whose portfolio includes a share of stochastic power generation to participate in the electricity market, in order to facilitate power systems operations.

In an electricity market environment, however, the impact of stochastic power generation on market characteristics is only negligible as long as its share in the electricity mix stays limited. If and when that penetration reaches significant levels as in, e.g., Denmark, Spain, and certain parts of the US, this impact becomes clear, though complex, with consequences not only on the overall costs of power system operations and revenues of the various market participants (and hence, on their capacity for further investment), but also on the approaches adopted by market agents to decision-making.

In this chapter, we first discuss the reasons for the impact of stochastic renewable energy generation on electricity markets in Sect. 6.2, based on considerations related to meteorological, economic, and power systems aspects. The various types of effects on the different markets, i.e., day-ahead and balancing ones, are then described and exemplified in Sect. 6.3. A methodology is then presented in Sect. 6.4 to perform statistical analyses of data originating from electricity markets highly penetrated by stochastic energy sources, such as wind and solar power, in a bottom-up manner. The market chosen for illustration is the Scandinavian Nord Pool. Particular focus is given, though, to the Western Denmark area, where today wind power generation provides 30 % of the yearly electric energy needs.

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6.2 Why Do Stochastic Renewable Energy Sources Impact Electricity Markets?

The major reasons behind the impact of stochastic renewable energy generation on electricity markets are sorted here in various categories. The first one concerns the economic aspects and, more particularly, the way stochastic renewable energy generation shapes the global supply curve in electricity markets. Then, the meteorological origins of variability in stochastic energy generation, e.g., from wind and solar power plants, is dealt with by drawing a parallel with seasonality and dynamics of electricity demand. More complex aspects linked to the power conversion process itself (nonlinear and bounded), and with the predictability of stochastic power generation are finally presented.

6.2.1 The Merit-Order Effect

In an auction-based market mechanism, participants on the supply side place offers in the electricity market that are defined in terms of energy quantity and price. The characteristics of such offers may vary depending upon the market rules and mechanisms, but overall, the price part directly relates to a short-run marginal cost, i.e., the cost of generating an extra unit of energy. All generation offers may be seen as forming a global *supply curve* which serves as a basis for clearing the market. Note that, for simplification, the presentation of the merit-order effect in the following disregards the additional effect of considering spatial and temporal constraints at the time of clearing the market. What we refer to as spatial constraints are those related to the network topology and transmission capacity, since these are accounted for in certain electricity markets, like those in the USA, for instance. Discussions on how network constraints enter the market-clearing problem, and their potential impact on market quantities, can be found in Chaps. 3 and 4. In parallel, the temporal constraints relate to multiple period dependencies, for instance, induced by ramping capabilities, or minimum up and down times.

This global supply curve is built following a *merit order* principle: supply offers are ranked in a straightforward manner based on their short-run marginal costs. For renewable energy producers, this cost of producing an extra unit of energy is generally zero. It can actually be negative if support mechanisms rewarding on the basis of every unit of generated energy are considered. Therefore, even when entering competitive market mechanisms, renewable energy generation is still somehow prioritized, since it is the first energy source to be used to meet the demand. This leads to the so-called merit-order effect of renewable energy generation.

Example 6.1 (Supply and demand curves from the day-ahead electricity market in the Scandinavian Nord Pool) An example supply curve is depicted in Fig. 6.1 for the day-ahead market of the Nord Pool in Scandinavia (called Elspot) on the 15 October 2012, for the market time unit between 7:00 and 8:00. All supply offers are

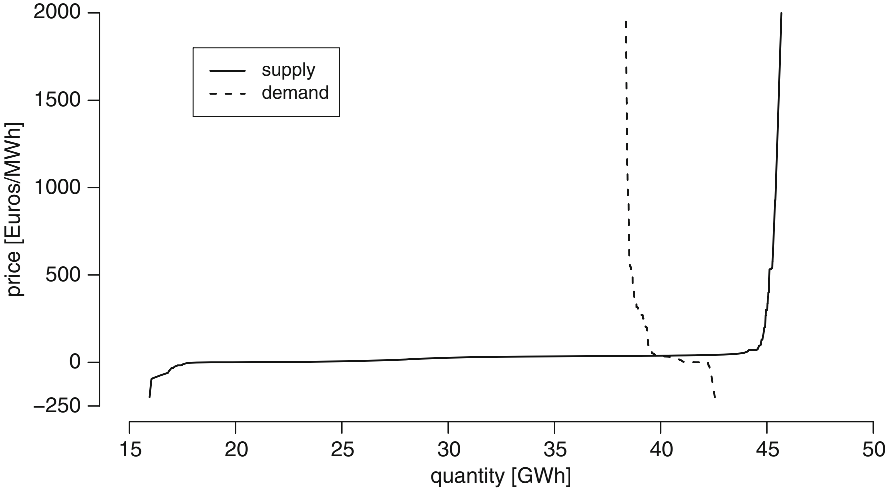

Fig. 6.1 Supply curve for the day-ahead market (Elspot) of the Nord Pool in Scandinavia on the 15 October 2012, for the market time unit between 7:00 and 8:00. The corresponding demand curve is also shown

ranked following a merit order, with a smooth increase in price for lower volumes followed by a much sharper one for high energy quantities. Renewable energy offers are located on the very left side of the supply curve, in the stack of negative and near-zero prices, along with the must-run conventional units.

6.2.2 Meteorological Origins of Variability in Energy Generation

All stochastic renewable energy sources are directly influenced by the weather, making their energy generation a nonlinear function of atmospheric variables. For instance, wind power production is linked to wind speed, but also to air density to a lesser extent. Similarly, solar power is mostly influenced by solar irradiance, though surface temperature may also affect the efficiency of the power conversion process. A consequence of that meteorological influence is that power production from stochastic renewable energy sources is highly variable at all time scales relevant for electricity markets. These energy sources are often referred to as *intermittent* for that very reason of sometime being available, sometime not.

Illustrating this variability, it is unlikely that solar energy production stays the same from one hour to the next, or during the day and at night. At these time scales, variability and periodic patterns in stochastic renewable energy generation tend to have an effect similar to that for the demand side. Electric demand indeed has a clear diurnal pattern, with a morning increase and a late-afternoon peak, then being of lower magnitude through the night. Wholesale electricity prices are known to follow

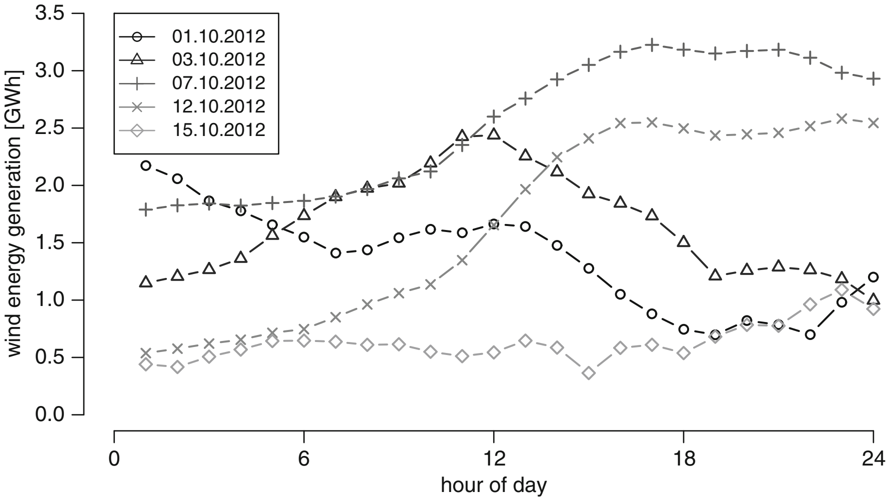

Fig. 6.2 Hourly energy generation from all wind power capacities in Denmark for five example days over the period from 1 to 15 October 2012, as reported by the Nord Pool in Scandinavia

a parallel trend, naturally induced by the quantity-price relationship and the merit order used to build the market supply curve.

Similarly, at the time scales of the balancing market (typically 5–15 min), it is also observed that the level of wind and solar energy can radically change from one time step to the next. Most importantly, in contrast to conventional generation capacities, renewable energy sources cannot be dispatched. They could be curtailed though, meaning that their maximum power generation would be kept below a maximum acceptable level. As a consequence, any unforeseen variation in renewable energy generation has to be corrected through the balancing market.

Globally, the combination of the merit-order effect and the natural variability of stochastic renewable energy generation at all time scales of relevance in power systems operations can be seen as the key driver of their influence on the various electricity markets and their characteristics.

Example 6.2 (Daily patterns of wind power generation for the whole Denmark) In order to illustrate the variability of stochastic renewable energy generation in electricity markets, Fig. 6.2 depicts the series of hourly wind energy generation for the whole Denmark as reported by the Nord Pool in Scandinavia for five example days over the period from 1 to 15 October 2012. The variability from one hour to the next, but also from one day to the other, is highly significant. The patterns also differ depending upon prevailing meteorological conditions. These variable patterns will contribute to shaping the evolution of the various variables in day-ahead and balancing markets.

6.2.3 Nonlinear and Bounded Generation Process

For all stochastic renewable energy sources, power production is a direct function of some atmospheric variables. The conversion process is generically characterized by the so-called *power curve*, representing the function relating these atmospheric variables and power output. Power curves differ if considering one single generation unit (a wind turbine, a solar panel, etc.), a group of them (as for the example of a wind farm), or a geographically distributed portfolio. It is necessarily nonlinear since the potential power that can be extracted from meteorological variables varies with their level in a complex manner. For instance, for the case of wind energy, while the potential energy in the wind is proportional to the cube of the wind speed, the relationship between wind speed and power is made even more complex in practice by the aerodynamic and technological limitations of wind turbines. In parallel, power curves are bounded since power generation is to be within zero and the nominal capacity of the generation unit or portfolio of interest.

While the nonlinear and bounded nature of stochastic renewable energy generation may not seem to be that different from the case of conventional units (their output is also bounded, and their efficiency is not linear with the level at which they operate), it magnifies the effects underlined in Sect. 6.2.1 and Sect. 6.2.2. When renewable energy generation is low, or expected to be low, it can only increase. Analogously, when close to nominal capacity, it can only decrease. This asymmetry also impacts electricity markets, and more specifically the need for balancing. It is enhanced by predictability issues, which are covered in the following. As an example, a compact discussion on the nonlinear and bounded nature of the wind-to-power conversion process, and on its impact on variability and predictability of wind power generation, is available in [8].

6.2.4 Predictability of the Renewable Energy Generation Process

The question of the *predictability* of energy generation from stochastic renewable energy sources was extensively discussed in Chap. 2. The various types of forecasts and their characteristics were introduced, independently of the way these forecasts may be used eventually. One should bear in mind that all forecasts contain a part of error. Therefore, when using forecasts in order to design optimal offering strategies, or as input to market-clearing mechanisms, both the quality of the forecasts and the inherent predictability of renewable energy generation are to influence the market outcomes.

While the variability in energy generation directly impacts day-ahead electricity markets (see Sect. 6.2.2), it is not the actual energy generation that impacts the cleared energy volumes and system prices, as well as their dynamics. This is since, at the moment offers are placed and markets are cleared, these values are not known, they are only predicted. Therefore, it is not the observed energy generation which influences markets, but our knowledge about future power generation and related uncertainty [7]. Similarly, for the case of balancing markets, the potential need for balancing comes from the difference between day-ahead supply offers and actual generation. Consequently, there again, it is not the actual renewable energy generation that drives the balancing direction, prices and quantities, but the forecasting errors instead.

6.3 Expected Influence on Various Markets and Their Characteristics

The different markets are influenced by stochastic renewable energy generation in different ways, by combining some of the effects discussed previously. The underlying mechanisms are presented in the following. Emphasis is placed on an auction-based type of electricity market, where these effects can be simplified. They could be extended and generalized to markets with LMPs and additional constraints related to, e.g., grid and ramping limitations. Finally, for more advanced market mechanisms, such as those described in Chap. 3, which are based on stochastic programming or robust optimization, these effects would become more complex, even if some of the basics concepts would remain. Our presentation of these conceptual aspects begin with the case of day-ahead markets, then followed by balancing markets.

6.3.1 Day-Ahead Market

The specifics of day-ahead markets were introduced in Chap. 3, to which the reader is referred to for an overview of basic concepts. We place ourselves in the most simple, though realistic, set-up for a day-ahead electricity market. It consists in an auction-based mechanism with price-quantity offers for every time unit in that market, typically each hour of the day. This framework simplification will still allow us to introduce and illustrate the underlying mechanics of the impact of stochastic renewable energy generation on the various variables in that market. The outcome of the market-clearing mechanism consists of an overall energy volume and a system price. On the supply side, all the accepted offers are those for which the original price offer is less or equal to the system price, while the opposite is true on the demand side.

For a given time unit in the market, each of the N_{S} market participants with dispatchable units places a supply offer $(P^{\mathrm{S}}_{\mathrm{B}i}, \lambda^{\mathrm{S}}_{\mathrm{B}i})$ for a quantity $P^{\mathrm{S}}_{\mathrm{B}i}$ and a price $\lambda^{\mathrm{S}}_{\mathrm{B}i}$, corresponding to their short-run marginal costs. Following the notations in previous chapters, the superscript 'S' is for 'scheduled' while the subscript 'B' is to denote dispatchable generators that have balancing capabilities. The global supply curve in the market is determined by ranking the various price-quantity offers by increasing prices,

$$\{(P^{\mathrm{S}}_{\mathrm{B}i}, \lambda^{\mathrm{S}}_{\mathrm{B}i}) \mid \lambda^{\mathrm{S}}_{\mathrm{B}i} \leq \lambda^{\mathrm{S}}_{\mathrm{B}i+1},\ i \geq 1\}. \tag{6.1}$$

If disregarding any must-run capacities that would still be prioritized over stochastic renewable energy production, the above supply curve is extended to

$$\{(P^{\mathrm{S}}_{\mathrm{B}i}, \lambda^{\mathrm{S}}_{\mathrm{B}i}) \mid \lambda^{\mathrm{S}}_{\mathrm{B}i} \leq \lambda^{\mathrm{S}}_{\mathrm{B}i+1},\ i \geq 0\}, \tag{6.2}$$

where $\{(P^{\mathrm{S}}_{\mathrm{B0}}, \lambda^{\mathrm{S}}_{\mathrm{B0}})\}$ corresponds to the overall quantity of stochastic renewable energy generation at that time unit in the market, at a price which is 0 at most, $\lambda^{\mathrm{S}}_{\mathrm{B0}} \leq 0$. $P^{\mathrm{S}}_{\mathrm{B0}}$ is, therefore, the same as P^{S}_{U} in the notation used in Chap. 4, where the principles of balancing markets are introduced.

Following the argument in Sect. 6.2.4, the quantity $P^{\mathrm{S}}_{\mathrm{U}}$ cannot be given by observations of stochastic renewable energy generation (since these are not available at the time of clearing the day-ahead market), but instead by the latest forecasts issued when placing offers. These forecasts might have been modified by traders or operators depending upon their trading strategies and appraisal of forecast quality. The various offers $(P^{\mathrm{S}}_{\mathrm{B}i}, \lambda^{\mathrm{S}}_{\mathrm{B}i})$, $i \geq 1$, are to meet the net demand, i.e., demand minus the non-disptachable production from stochastic renewable energy sources.

The demand curve is determined in a symmetric manner, namely, by sorting a number N_d of quantity-price offers $\{(P^{\mathrm{S}}_{\mathrm{D}i}, \lambda^{\mathrm{S}}_{\mathrm{D}i})\}$ (the subscript 'D' being for 'demand'), though with decreasing prices, i.e.,

$$\{(P^{\mathrm{S}}_{\mathrm{D}i}, \lambda^{\mathrm{S}}_{\mathrm{D}i}) \mid \lambda^{\mathrm{S}}_{\mathrm{D}i} \geq \lambda^{\mathrm{S}}_{\mathrm{D}i+1},\ i \geq 1\}. \tag{6.3}$$

According to an auction-based market mechanism, the market-clearing process consists in finding the point where the supply and demand curves meet. The clearing price λ^{S} (also referred to as system price) is then given by the price of the supply offer corresponding to this crossing point, while the total scheduled power consumption (and production) $P^{\mathrm{S}}_{\mathrm{D}}$ is given by the overall energy volume reached at that point. They are denoted $\lambda^{\mathrm{S}}(P^{\mathrm{S}}_{\mathrm{U}})$ and $P^{\mathrm{S}}_{\mathrm{D}}(P^{\mathrm{S}}_{\mathrm{U}})$ so as to discuss their variation as a function of the level of stochastic renewable energy generation $P^{\mathrm{S}}_{\mathrm{U}}$. Given the formulation of the global supply curve in Eq. (6.2), we have that, for the same demand curve and the same supply curve for all generators placed on the right side of renewable energy generation (for which $\lambda^{\mathrm{S}}_{\mathrm{B}i} > 0$), different quantities of renewable energy readily yield different market outcomes, both in terms of energy quantities and system prices.

The increasing nature of the market supply curve, combined with the decreasing nature of the demand one, leads to the following properties for the outcome of the auction-based market-clearing mechanism on the day-ahead market with the presence of stochastic renewable energy generation. First of all, the cleared volume is at least as much as in the case of no renewable energy,

$$P^{\mathrm{S}}_{\mathrm{D}}(P^{\mathrm{S}}_{\mathrm{U}}) \geq P^{\mathrm{S}}_{\mathrm{D}}(0), \quad \forall P^{\mathrm{S}}_{\mathrm{U}} \geq 0. \tag{6.4}$$

Furthermore, the system price necessarily decreases with increased renewable energy penetration,

$$\lambda^{\mathrm{S}}(P^{\mathrm{S}}_{\mathrm{U}}) \leq \lambda^{\mathrm{S}}(0), \quad \forall P^{\mathrm{S}}_{\mathrm{U}} \geq 0. \tag{6.5}$$

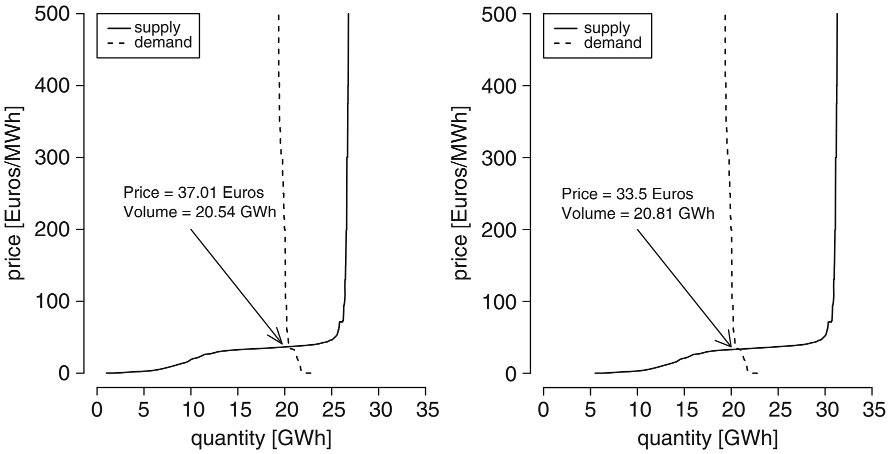

Fig. 6.3 Supply and demand curves for two different levels of predicted stochastic renewable energy generation: 1,000 MWh (*left*), and 5,500 MWh (*right*). These lead to different volumes and system prices as a result of the market-clearing procedure

This effect of stochastic renewable energy generation on the outcome of the day-ahead market through forecasts is illustrated in the following, based on the simulated day-ahead electricity market inspired by the Nord Pool in Example 6.1. It shows how the cleared energy volumes and system prices may vary depending upon the quantity of wind energy that enters the global supply curve.

Example 6.3 (Impact of wind power on the clearing of an auction-based day-ahead market) Consider a time unit in an auction-based day-ahead market mechanism, where the supply and demand curves are inspired by the example of Fig. 6.1. For simplicity, only supply and demand offers with prices between 0 and 500 €/MWh are considered. To illustrate the impact of renewable energy on day-ahead prices and volumes, two different quantities of renewable energy production are added to the supply curve for that time unit, namely, 1 GWh and 5.5 GWh, with a short-run marginal cost of 0 €/MWh. The resulting market supply curves are represented in Fig. 6.3. Other things being strictly equal, this difference in renewable generation induces an increase in the cleared energy volume of 270 MWh and a reduction of 3.51 €/MWh in the system price. As a reference in that example, if there was no renewable energy production at all for that time unit, the cleared market outcome would consist of a volume of 20.46 GWh and a system price of 37.82 €/MWh.

The above example considers the case of a single market time unit only. In order to further illustrate how the variability and predictability of renewable energy generation may influence price profiles on the day-ahead market, it is necessary to extend the previous example to multiple periods and various types of weather conditions.

Example 6.4 (Impact of wind power on daily price profiles) We extend here Example 6.3 with its specific supply and demand curves, by looking at how the wind profiles of Example 6.2 (in Fig. 6.2, with a hourly time resolution) would impact

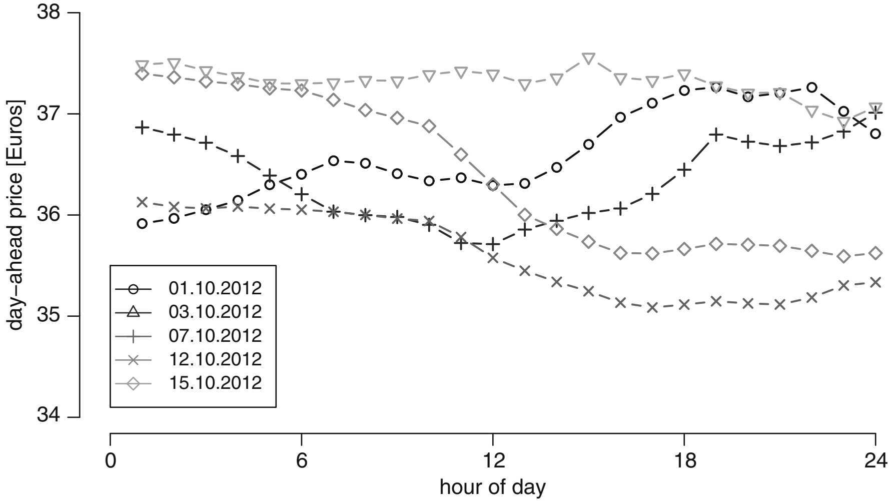

Fig. 6.4 Price profiles for 5 example days over the period from 1 to 15 October 2012, as induced by the wind profiles of Fig. 6.2

hourly prices in this day-ahead market, if they were the forecasts used as input to the market clearing. For simplicity, and for better highlighting the effect of wind power generation alone, the supply and demand curves are considered as constant for the 24 h of the day. Only the supply curve is shifted left and right depending on the wind power blocks.

Remember that if there was no wind generation, the system price would always be equal to 37.82 €/MWh. In fact, it is almost at this level for the whole day of 15 October 2012, where wind power generation was predicted to be very low. On the contrary, on 7 October 2012, wind power generation remained fairly high throughout the day, then exercising a downward pressure on prices, as shown in Fig. 6.4.

If we wanted to have a more integrated view of the impact of stochastic renewable energy generation on day-ahead electricity markets, we should account for the fact that the predicted weather jointly impacts electric load (e.g., temperature and wind may drive electric heating demand) and energy generation. Consequently, the interdependence between uncertainty in demand and renewable energy generation would need to be characterized, as well as its combined impact on electricity prices.

6.3.2 Balancing Market

The case of balancing markets is slightly more complicated than that of the day-ahead one, though relying on the same basic principles. There may be a number of reasons motivating the need for balancing, for instance, related to errors in load forecasts, or outages with transmission lines and conventional generators. Such reasons, which are not directly related to renewable energy generation are disregarded here in order to simplify things. A difference should be made between balancing direction on the one hand, that is, the need for upward or downward balancing, as well as the balancing volumes and prices on the other hand.

The impact of stochastic renewable energy generation on balancing markets is dealt with by first introducing imbalances as random variables related to errors in renewable energy forecasts. The way this can be translated to a probabilistic characterization of balancing direction, volumes and prices, is subsequently described.

6.3.2.1 Imbalances as Random Variables

Similarly to the case of the day-ahead market in Eqs. (6.1) and (6.2), if we place ourselves in an auction-based mechanism for the balancing market, its global supply curve is defined by a set of quantity-price offers for balancing. These can be sorted into N_{D} offers for downward balancing,

$$\{(P^{\mathrm{D}}_{\mathrm{B}i}, \lambda^{\mathrm{D}}_{\mathrm{B}i}) \mid \lambda^{\mathrm{D}}_{\mathrm{B}i} \geq \lambda^{\mathrm{D}}_{\mathrm{B}i+1},\ \lambda^{\mathrm{D}}_{\mathrm{B}i} \leq \lambda^{\mathrm{S}},\ i \geq 1\}, \tag{6.6}$$

where, following the notations in previous chapters, the superscript ‘D’ is for ‘downward’ balancing while the subscript ‘B’ is for dispatchable generators that have balancing capabilities. Similarly, one has N_{U} offers for upward balancing,

$$\{(P^{\mathrm{U}}_{\mathrm{B}i}, \lambda^{\mathrm{U}}_{\mathrm{B}i}) \mid \lambda^{\mathrm{U}}_{\mathrm{B}i} \leq \lambda^{\mathrm{U}}_{\mathrm{B}i+1},\ \lambda^{\mathrm{U}}_{\mathrm{B}i} \geq \lambda^{\mathrm{S}},\ i = 1, \ldots, N_{\mathrm{U}}\}, \tag{6.7}$$

where the superscript ‘U’ is for ‘upward’ balancing. These offers actually form offering curves similar to that for demand and supply, respectively, in the day-ahead electricity market. However, the actual demand is, in this case, directly defined by the need for balancing, not by a demand curve and its elasticity, since it is the balance between production and consumption that is sought after in the balancing market. Note that offers for downward and upward balancing may come from both the supply and demand side, in the former case by modifying the power level of generation units, and in the latter through load shedding and other forms of demand response mechanisms.

Using notations from and inspired by Chap. 4, the overall balancing volume is here given by $P^{\mathrm{S\text{-}B}}_{\mathrm{N}} = (P^{\mathrm{S}}_{\mathrm{N}} - P^{\mathrm{B}}_{\mathrm{N}})$, i.e., by the difference between net demand in the day-ahead market ($P^{\mathrm{S}}_{\mathrm{N}}$) and net demand in the balancing market ($P^{\mathrm{B}}_{\mathrm{N}}$). Remember that the term “net” means that non-dispatchable generation was subtracted from the actual demand. This balancing volume is a continuous random variable, which, if other potential origins of imbalances are disregarded, is directly related to the error in forecasts of stochastic renewable energy generation.

A forecasting error is a stochastic quantity which can be characterized by its distribution. This distribution is conditional on the power level of the forecast itself, on the forecasting approach originally employed, and on external conditions that inherently affect predictability. The imbalance distribution is equal to the distribution of errors of the forecasts used at the time of clearing the day-ahead market. For a market time unit t, for which the day-ahead market was cleared at time $t-k$, this writes

$$f_{P_{\mathrm{N}}^{\text{S-B}}}(p) = f_{\varepsilon_{t|t-k}}(\varepsilon), \tag{6.8}$$

where $\varepsilon_{t|t-k}$ is the error from the renewable energy forecasts, formally defined in Eq. (2.17). The variables ε and p are here used to generally denote forecast error and imbalance, respectively. The extension to the case of additional causes of imbalances, e.g., load forecasting errors and asset outages, can be obtained by convolution of this distribution of forecast errors with the distributions representing the uncertainty in load forecasts and potential outages for the various assets involved (lines, conventional generators, etc.).

Subsequently, in order to assess the influence of stochastic renewable energy on balancing markets, three aspects are of importance:

1. What is the impact on the balancing direction (upward or downward balancing), defined by the sign of $P_{\mathrm{N}}^{\text{S-B}}$?
2. What is the impact on the necessary balancing volumes for each direction?
3. What is the impact on the balancing price λ^{B} in single and two-price settlement mechanisms?

These various points are sequentially dealt with in the following.

6.3.2.2 Balancing Direction

For each time unit in the balancing market, the system may be subject to positive or negative imbalance. It may also happen that no balancing is needed. This does not mean that the system will be in exact balance at the moment of operation, but, instead, that this is to be handled outside of the balancing market owing to the very limited magnitude of the imbalance in question.

The imbalance sign, also defining the balancing direction, can be modeled with a discrete variable I^{B} such that

$$I^{\mathrm{B}} = \frac{P_{\mathrm{N}}^{\text{S-B}}}{|P_{\mathrm{N}}^{\text{S-B}}|}, \quad |P_{\mathrm{N}}^{\text{S-B}}| > \tau, \tag{6.9}$$

$$I^{\mathrm{B}} = 0, \quad \text{otherwise}, \tag{6.10}$$

making that I^{B} takes values in $\{-1, 0, 1\}$. In the above, τ is a small number representing a cap on absolute imbalance values, under which it is considered that the balancing is handled outside of the balancing market, e.g. with frequency-based reserve or other load-following capabilities. Remember also that the temporal resolution of the balancing market may be higher than that of the day-ahead market, for instance, hourly for the latter and with 15-min time steps for the former. The prediction error may not translate to uniformly distributed imbalances over the different time units of the balancing market corresponding to a single day-ahead market time unit. As an example, within a given hour, owing to the variability in the power output from a stochastic renewable energy source, the need for balancing may be positive at the beginning of this hour, and then negative for the remainder of that hour. Similarly, there may be successive periods within the hour with minimal and then substantial needs for balancing.

The case of $I^{\mathrm{B}} = 1$ corresponds to that of a positive imbalance with supply being greater than demand, hence requiring downward balancing, while, inversely, for $I^{\mathrm{B}} = -1$ one faces a negative imbalance requiring upward balancing. Knowing the balancing direction might be of particular importance since, e.g., in two-price settlement balancing markets, participants are "penalized" if their own imbalance is of the same sign as the system imbalance as a whole. Finally, for $I^{\mathrm{B}} = 0$ no imbalance is to be settled through the balancing market, leading to a no-balancing situation.

Following from the definition in Eq. (6.8) of the imbalance as a continuous random variable, the sign for the balancing direction is a categorical random variable such that

$$\mathrm{P}[I^{\mathrm{B}} = 1] = \int_{p>\tau} f_{P_{\mathrm{N}}^{\text{S-B}}}(p)dp, \tag{6.11}$$

$$\mathrm{P}[I^{\mathrm{B}} = 0] = \int_{p=-\tau}^{p=\tau} f_{P_{\mathrm{N}}^{\text{S-B}}}(p)dp, \tag{6.12}$$

$$\mathrm{P}[I^{\mathrm{B}} = -1] = \int_{p<\tau} f_{P_{\mathrm{N}}^{\text{S-B}}}(p)dp. \tag{6.13}$$

A necessary condition on these three probabilities is that

$$\mathrm{P}[I^{\mathrm{B}} = -1] + \mathrm{P}[I^{\mathrm{B}} = 0] + \mathrm{P}[I^{\mathrm{B}} = 1] = 1. \tag{6.14}$$

A consequence of the above relationship between balancing sign and distribution of forecast errors is that these three probabilities are directly related to the shape of this distribution and some of its key characteristics. The wider the distribution is, for a given τ value, the larger $\mathrm{P}[I^{\mathrm{B}} = 1]$ and $\mathrm{P}[I^{\mathrm{B}} = -1]$ will be, while reducing the probability of no-balancing $\mathrm{P}[I^{\mathrm{B}} = 0]$. In parallel, the more asymmetric (i.e., skewed) the error distribution, the more $\mathrm{P}[I^{\mathrm{B}} = 1]$ and $\mathrm{P}[I^{\mathrm{B}} = -1]$ will differ. This is further illustrated below through an example of the impact of wind power forecasting errors on probabilities for the balancing sign.

Example 6.5 (Balancing direction probabilities induced by wind power forecasting errors) Figure 6.5 shows an example of a distribution of forecast errors, say, for wind power generation, for a given time unit in the balancing market. This distribution is right-skewed, which, independently of the magnitude of the errors themselves, translates to a higher probability of positive imbalances (leading to a downward balancing situation) than of negative ones (resulting in a need for upward balancing). These probabilities are of 0.56 and 0.33, respectively. The no-balancing case corresponds

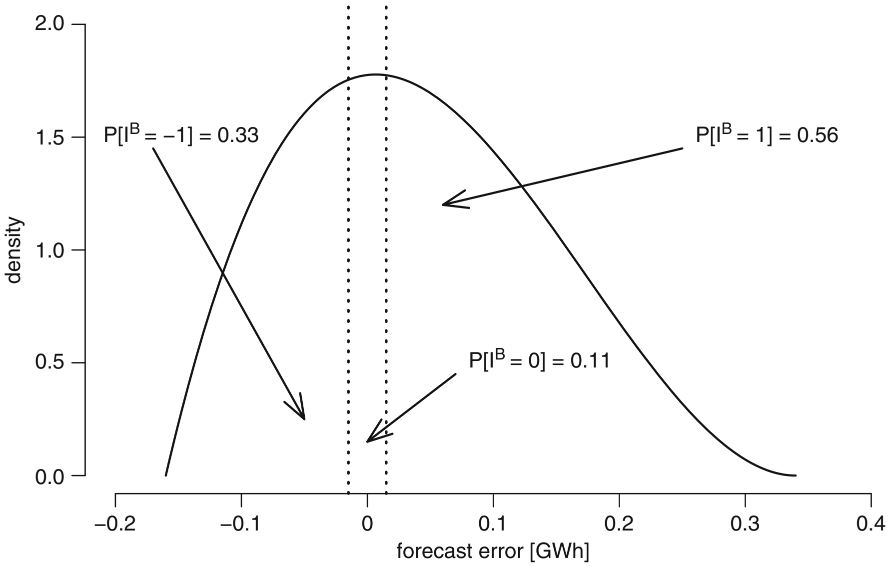

Fig. 6.5 Probabilities for the imbalance sign (and hence, for the balancing direction) in relation to the error distribution of renewable energy predictions. Here τ is arbitrarily set to 50 MWh

to the central band, with a probability (here, 0.11) that is not significantly affected by the skewness of the distribution, but by its width instead. If the distribution of forecast errors was sharper, this probability would increase, while the sum of probabilities for upward and downward balancing would be lower.

6.3.2.3 Balancing Volumes

Clearly, when looking at balancing volumes, only the case of downward and upward balancing are relevant, since the balancing volume in case of no-balancing is considered to be zero.

In view of their definition, balancing volumes are also to be seen as random variables, determined by the overall imbalance distribution, conditioned on the balancing direction. This yields

$$P_{\mathrm{B}}^{\mathrm{D}} = P_{\mathrm{N}}^{\text{S-B}} | (P_{\mathrm{N}}^{\text{S-B}} > \tau), \tag{6.15}$$

$$P_{\mathrm{B}}^{\mathrm{U}} = P_{\mathrm{N}}^{\text{S-B}} | (P_{\mathrm{N}}^{\text{S-B}} < -\tau), \tag{6.16}$$

where τ is a small value as in Sect. 6.3.2.2, corresponding to the threshold for which any imbalance (in absolute value) greater than τ leads to a balancing situation. The ‘|’ symbol in the Eqs. (6.15) and (6.16) should be read as “given that” or “conditional on”.

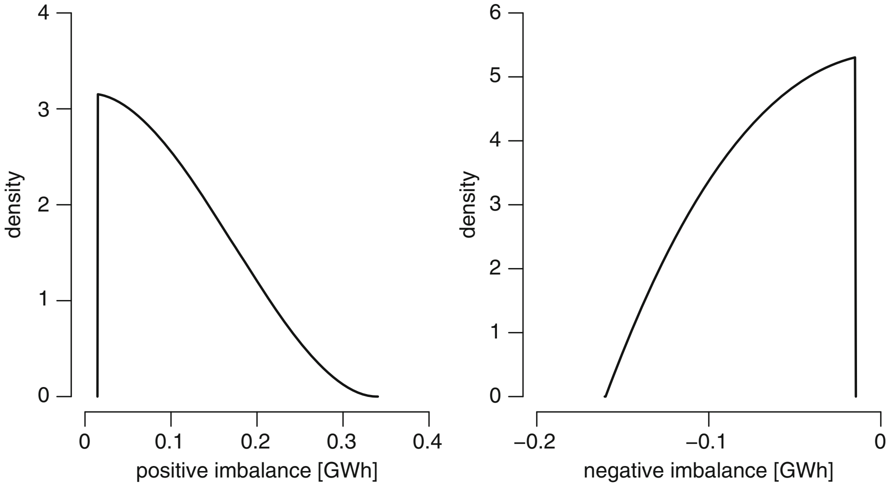

Fig. 6.6 Distributions of positive and negative imbalances corresponding to balancing volumes for downward and upward balancing, respectively. These are obtained from the example distribution of errors in renewable energy forecasts in Fig. 6.5

It follows that the *conditional* random variables for the balancing volumes can be described by two corresponding conditional densities

$$f_{P_{\mathrm{B}}^{\mathrm{D}}}(p) = \frac{f_{P_{\mathrm{N}}^{\text{S-B}}}(p)1\{p > \tau\}}{P[I^{\mathrm{B}} = 1]}, \quad I^{\mathrm{B}} = 1, \tag{6.17}$$

$$f_{P_{\mathrm{B}}^{\mathrm{U}}}(p) = \frac{f_{P_{\mathrm{N}}^{\text{S-B}}}(p)1\{p < -\tau\}}{P[I^{\mathrm{B}} = -1]}, \quad I^{\mathrm{B}} = -1, \tag{6.18}$$

which comprise the positive and negative parts of the whole distribution of imbalances $f_{P_{\mathrm{N}}^{\text{S-B}}}$, rescaled to a unit integral so as to make them proper density functions. In Eqs. (6.17) and (6.18), $1\{.\}$ is an indicator function, that is, equal to 1 if the condition within the brackets realizes, and to 0 otherwise. Positive and negative parts of imbalance distributions are separated, since downward and upward balancing situations are dealt with based on different offering curves, defined in Eqs. (6.6) and (6.7), respectively. The definitions of these conditional density functions for balancing volumes are illustrated below, again based on an example where balancing needs are induced by wind power forecasting errors.

Example 6.6 (Distributions of balancing volumes induced by wind power forecasting errors) Let us consider Example 6.5, where a given distribution of errors in wind power forecasts is used to show the impact on the balancing direction. This same distribution of Fig. 6.5 translates to two conditional distributions for positive and negative imbalances, represented in Fig. 6.6. These are readily given by the positive and negative parts of the distribution of forecast errors (with the interval $[-\tau, \tau]$ defining the no-balancing area).

6.3.2.4 Balancing Prices

If the offering curves for upward and downward balancing in the balancing market were continuous, instead of defined by quantity-price blocks, they could be described by two continuous functions, i.e.,

$$\lambda_{\mathrm{B}}^{\mathrm{D}} = g_{\mathrm{D}}(P_{\mathrm{B}}^{\mathrm{D}}), \tag{6.19}$$

for down-regulation, and

$$\lambda_{\mathrm{B}}^{\mathrm{U}} = g_{\mathrm{U}}(P_{\mathrm{B}}^{\mathrm{U}}), \tag{6.20}$$

for up-regulation. Such continuous curves can be seen as a generalization, or as a simplification, of the piecewise ones defined in Eqs. (6.6) and (6.7).

Given that g_{D} and g_{U} are strictly decreasing and increasing functions, respectively, the distributions of $\lambda_{\mathrm{B}}^{\mathrm{D}}$ and $\lambda_{\mathrm{B}}^{\mathrm{U}}$ are readily linked to those of $P_{\mathrm{B}}^{\mathrm{D}}$ and $P_{\mathrm{B}}^{\mathrm{U}}$ as

$$f_{\lambda_{\mathrm{B}}^{\mathrm{D}}}(\lambda) = f_{P_{\mathrm{B}}^{\mathrm{D}}}\left({g_{\mathrm{D}}}^{-1}(\lambda)\right), \tag{6.21}$$

and

$$f_{\lambda_{\mathrm{B}}^{\mathrm{U}}}(\lambda) = f_{P_{\mathrm{B}}^{\mathrm{U}}}\left({g_{\mathrm{U}}}^{-1}(\lambda)\right). \tag{6.22}$$

In the practical case of having quantity-price blocks in the balancing market, the distributions of $\lambda_{\mathrm{B}}^{\mathrm{D}}$ and $\lambda_{\mathrm{B}}^{\mathrm{U}}$ are stepwise functions where the probabilities of seeing prices $\lambda_{\mathrm{B}i}^{\mathrm{D}}$ and $\lambda_{\mathrm{B}i}^{\mathrm{U}}$ is given by the probability of the i-th generator being the marginal one setting the price, conditional on the balancing direction. This writes

$$\mathrm{P}\left[\lambda_{\mathrm{B}}^{\mathrm{D}} = \lambda_{\mathrm{B}i}^{\mathrm{D}}\right] = \int_{p_{i+1}^{\mathrm{D}}}^{p_i^{\mathrm{D}}} f_{P_{\mathrm{B}}^{\mathrm{D}}}(p)dp, \tag{6.23}$$

with

$$p_i^{\mathrm{D}} = \sum_{j=1}^{i-1} P_{\mathrm{B}j}^{\mathrm{D}}, \quad i = 2, \ldots, N_{\mathrm{D}} + 1, \tag{6.24}$$

$$p_1^{\mathrm{D}} = 0. \tag{6.25}$$

Similarly, one has

$$\mathrm{P}\left[\lambda_{\mathrm{B}}^{\mathrm{U}} = \lambda_{\mathrm{B}i}^{\mathrm{U}}\right] = \int_{p_i^{\mathrm{U}}}^{p_{i+1}^{\mathrm{U}}} f_{P_{\mathrm{B}}^{\mathrm{U}}}(p)dp, \tag{6.26}$$

with

$$p_i^{\mathrm{U}} = \sum_{j=1}^{i-1} P_{\mathrm{B}j}^{\mathrm{U}}, \quad i = 2, \ldots, N_{\mathrm{U}} + 1, \tag{6.27}$$

$$p_1^{\mathrm{U}} = 0. \tag{6.28}$$

Table 6.1 Upward and downward balancing offers of dispatchable generators in the balancing market. The last line is for the no-regulation situation for which $\lambda^B = \lambda^S$

Unit	λ^{U} (\$/MWh)	$P^{\mathrm{U,max}}$	λ^{D} (\$/MWh)	$P^{\mathrm{D,max}}$	Probability
B1	30	100	–	–	0.29
B2	40	60	–	–	0.04
B3	60	50	–	–	0
B4	–	–	15	150	0.44
B5	–	–	12	120	0.11
B6	–	–	10	80	0.01
–	–	–	–	–	0.11

The probability of the various balancing prices defined above are for the case when the balancing direction is known. They can be seen as conditional probabilities, for instance, telling about the probability of observing a particular balancing price, given that we are in a downward balancing situation. However, in a more general setup, the balancing direction is also a random variable, as discussed in Sect. 6.3.2.2. Consequently, the probabilities of different balancing directions are to be accounted for in order to obtain the probability for the overall balancing price value λ^{B}. This yields

$$\mathrm{P}\left[\lambda^{\mathrm{B}} = \lambda^{\mathrm{S}}\right] = \mathrm{P}[I^{\mathrm{B}} = 0], \tag{6.29}$$

$$\mathrm{P}\left[\lambda^{\mathrm{B}} = \lambda^{\mathrm{U}}_{\mathrm{B}i}\right] = \mathrm{P}\left[\lambda^{\mathrm{U}}_{\mathrm{B}} = \lambda^{\mathrm{U}}_{\mathrm{B}i}\right]\mathrm{P}[I^{\mathrm{B}} = -1], \tag{6.30}$$

$$\mathrm{P}\left[\lambda^{\mathrm{B}} = \lambda^{\mathrm{D}}_{\mathrm{B}i}\right] = \mathrm{P}\left[\lambda^{\mathrm{D}}_{\mathrm{B}} = \lambda^{\mathrm{D}}_{\mathrm{B}i}\right]\mathrm{P}[I^{\mathrm{B}} = 1]. \tag{6.31}$$

The determination of the probabilities of observing various balancing prices is exemplified below, based on a simple setup with a reduced number of generators, and where the need for balancing is induced by wind power forecasting errors.

Example 6.7 (Probabilities of balancing prices as induced by wind power forecasting errors) Examples 6.5 and 6.6 are continued here by considering their extension to the case of prices in the balancing market. The probabilities for the balancing direction are given in Fig 6.5, while the distributions of potential balancing needs are depicted in Fig. 6.6. Besides, the offers made by dispatchable producers for upward and downward balancing are gathered in Table 6.1. For information, the day-ahead price after market clearing is \$20/MWh.

By integrating the density function for negative imbalances (Eqs. (6.26)–(6.28)), the probabilities that B1, B2 and B3 will be the marginal generator, if upward balancing was to be necessary, can be computed. These probabilities are of 0.87, 0.13, and 0 for B1, B2, and B3, respectively. In parallel, the probabilities that B4, B5, and B6 will be the marginal generator, in the case where downward balancing would be necessary, are similarly obtained by integrating the density function of positive imbalances (Eqs. (6.23)–(6.25)). These are of 0.78, 0.2, and 0.02 for B4, B5, and B6, respectively.

Finally, these probability numbers have to be combined with the probabilities for the balancing direction, so as to obtain the final probabilities for the various balancing prices. These are collated in the most right column of Table 6.1. For instance, the overall probability that the balancing price is determined by the offer of the generator B1 is of $0.87 \times 0.33 = 0.29$, while the probability that it is the offer of B4 instead is of $0.78 \times 0.56 = 0.44$. Finally, as for the no-balancing situation (last line of Table 6.1), there is a probability of 0.11 that $\lambda^{B} = \lambda^{S}$.

6.4 Empirical Analysis of the Impact of Renewable Energy Sources

The conceptual analysis presented in the above could be extended to further look at the detailed influence of stochastic renewable energy generation, forecasts and their uncertainty, on various markets and their characteristics, for different market designs, rules, generation mix, etc. However, in practice, the most relevant manner to demonstrate and illustrate this impact is through an *ex-post* analysis. By that, it is meant that by gathering renewable energy generation and corresponding market data, it may be possible to better uncover the effect of renewables as it was actually experienced.

With that objective in mind, the methodological aspects of such type of ex-post analysis are first presented further. This is since most of the effects to be highlighted could be nonlinear, thus calling for more advanced approaches than the more traditional correlation analysis that could be readily performed. Moreover, the different market variables to be studied may be of different nature, i.e., continuous and bounded, or categorical, then requiring a different treatment. That ex-post analysis methodology is subsequently applied to the case of the Nord Pool in Scandinavia over a 2-year period, with results on day-ahead market prices, as well as balancing direction, volumes and prices.

6.4.1 Methodology for Empirical Data Analysis

Market data are to be analyzed in a regression framework, hence focusing on the static relationship between a set of *explanatory* variables and one or more *response variables*. Dynamical aspects, which can be the focus of more advanced studies, are not treated here.

Firstly, linear and nonlinear regression for continuous variables, such as market prices and volumes, are described. Subsequently, emphasis is placed on regression for categorical variables like the balancing direction.

6.4.1.1 General Regression Framework

Whatever the variable of interest, the set of observations consists of time series of measurements. These are observations pertaining to some electricity market variable for successive points in time. We denote by

$$\{y_t\} = \{y_1, y_2, \ldots, y_T\} \tag{6.32}$$

the observed time series for the response variable, and by

$$\{x_t\} = \{x_1, x_2, \ldots, x_T\} \tag{6.33}$$

that for the explanatory variable. In certain cases, multiple explanatory variables are to be considered simultaneously, and then each observation x_t is replaced by a vector of observations. In a practical setup, the response variable may be the day-ahead electricity price or the balancing direction, while the explanatory variable may be generated wind power or the hour of the day, for instance.

It is aimed at uncovering a relationship between explanatory and response variables that would be valid for any time t, without any consideration for what happened over the previous time steps. In a *general regression* framework, this translates to assuming a functional relationship between these variables of the form

$$y_t \;=\; f(x_t) + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{6.34}$$

where ε_t is a noise term representing a potential (random) deviation at every time t between the modeled relationship and what was actually observed. This noise term is seen as a set of realizations from a sequence of independent and identically distributed (i.i.d.) random variables, with a zero mean and finite variance. The functional relationship in Eq. (6.34) may be modeled in either a linear or a nonlinear framework. In addition, a regression model may allow to describe the mean trend or some of the quantiles in the data, depending upon the method and loss function chosen to fit the model of Eq. (6.34).

6.4.1.2 Linear Regression for Continuous Variables

In the most simple case of performing a linear correlation analysis, the relationship between explanatory and response variables is modeled with a *linear regression* model,

$$y_t = \theta_0 + \theta_1 x_t + \varepsilon_t = [1 \;\; x_t]\theta + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{6.35}$$

where $\theta = [\theta_0 \;\; \theta_1]^\top$ is the set of linear regression parameters. Additionally, ε_t is a noise term with zero mean and finite variance, here for potential deviations from the modeled linear relationship and actual observations of the response variable.

For a given dataset with T series of observations for explanatory and response variables, the values for θ allowing to optimally describe the data characteristics are obtained as the result of the following optimization problem:

$$\hat{\theta} = \arg\min_{\theta} \sum_{t=1}^{T} \rho\left(y_t - [1 \;\; x_t]\theta\right). \tag{6.36}$$

When aiming to identify the mean trend in the data, the loss function ρ is chosen as quadratic, i.e., $\rho(x) = x^2$. The optimization problem translates to minimizing a sum of squared deviations between model and observations. The resulting model estimate is referred to as the *Least Squares (LS)* estimate $\hat{\theta}$ of θ. A known result [9] is that it is readily given by

$$\hat{\theta} = (X^\top X)^{-1} X^\top Y, \tag{6.37}$$

where the matrix X and vector Y gather all observations for the explanatory and response variables, respectively,

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_T \end{bmatrix}, \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}. \tag{6.38}$$

By using a LS estimation approach for the parameters in a linear model, one obtains the conditional mean relationship between explanatory and response variables. In other words, it tells that if using the model of Eq. (6.35) with the parameters obtained from Eq. (6.37) for a given value of x, the obtained value for the response y would be the average value to be observed in practice.

The relationship between explanatory variables and the mean response can be highly informative when analyzing electricity market data. However, in view of the uncertainties induced by stochastic power generation, it is of the utmost interest to also be able to describe the distribution of the data around this mean response. Such a detailed description of the density of the response data, conditional on some explanatory variable, is obtained by modeling its *quantiles*. For a linear regression model such as that in Eq. (6.35), and a given nominal proportion $\alpha \in [0, 1]$, the corresponding *quantile regression* line tells that there is a probability α to observe values of y below that line, and inversely, a probability $(1-\alpha)$ of observing y values above that line. This allows for some form of probabilistic inference on the data available, for instance, by defining intervals within which the observations may lie a certain percentage of the times.

The parameters in a linear quantile regression model are also obtained by solving an optimization problem of the form of Eq. (6.35), except that the loss function ρ is now an asymmetric linear function instead of a quadratic one, i.e.,

$$\rho(x) = x(\alpha - 1\{x < 0\}) = \begin{cases} \alpha x, & \text{if } x \geq 0, \\ (\alpha - 1)x, & \text{if } x < 0. \end{cases} \tag{6.39}$$

There does not exist an analytical formula to this optimization problem similar to that in Eq. (6.37) for the LS regression case. Numerical solvers should be used instead.

Classical linear and linear quantile regression models and their fitting to data are illustrated below based on a simulated dataset.

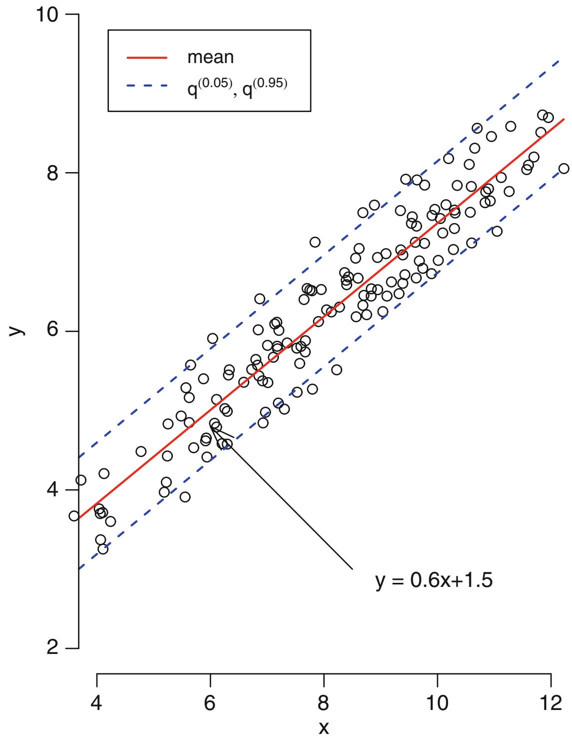

Fig. 6.7 Scatterplot with 150 observations for an explanatory and related response variable, along with fitted linear regression models (mean, as well as quantiles with nominal proportions $\alpha = 0.05$ and $\alpha = 0.95$)

Example 6.8 (Linear regression) Let us consider the case where one would have gathered a dataset with $T = 150$ observations for an explanatory variable x and for the corresponding response variable y. The scatterplot of x and y values is depicted in Fig. 6.7. Assuming that the relationship between these two variables is linear, the linear regression model is fitted to this cloud of data (with the LS estimation method). The regression line obtained has the equation $y = 0.6x + 1.5$. Linear quantile regression lines are also given for two nominal proportions ($\alpha = 0.05$ and $\alpha = 0.95$) in order to show the range of potential deviations around the mean trend. They then define intervals within which the observations lie 90 % of the times.

6.4.1.3 Extension to Nonlinear Regression

When empirically analyzing electricity market data, it is quite unlikely that simple linear regressions models are sufficient for describing the relationship between explanatory and response variables. As a motivating example, the impact of renewable energy generation on day-ahead prices will typically change depending on the production level, owing to the nonlinear shape of the overall supply curve.

The linear regression models presented before can be generalized by assuming that there exists a general functional relationship between explanatory and response variables as expressed in Eq. (6.34). This relationship is in most cases difficult to model directly. It is hence preferable to approximate it locally for a number of values of x. For that purpose, we describe here an easily applicable framework using local polynomial regression, which constitutes a straightforward generalization of the linear case. Indeed, with *local polynomial regression*, it is assumed that f may be locally approximated by k-order polynomials. Most common instances include kernel smoothing ($k = 0$) and local linear regression ($k = 1$). This approach is applicable as long as the relationship between explanatory and response variables is fairly smooth. The interested reader is referred to [1] for an overview of local polynomial regression and applications.

The function f is approximated at a number of so-called *fitting points*, chosen based on a rule of thumb or after consideration of the distribution of the values of the explanatory variable. Focusing on a single fitting point $\tilde{x}$, a k-order local polynomial approximation of the observation x_t, in the vicinity of the fitting point $\tilde{x}$, is given by

$$p_{\tilde{x}}^{k}(x_t) = [1 \;\; x_t - \tilde{x} \;\; \ldots \;\; (x_t - \tilde{x})^k]^\top. \tag{6.40}$$

For instance, if $k = 1$, $p_{\tilde{x}}^{k}(x_t) = [1 \;\; x_t - \tilde{x}]^\top$.

In parallel, write

$$\theta_{\tilde{x}} = [\theta_{\tilde{x},0} \;\; \theta_{\tilde{x},1} \;\; \ldots \;\; \theta_{\tilde{x},k}]^\top, \tag{6.41}$$

the vector of local coefficients at $\tilde{x}$, so that the following linear model

$$y_t = \theta_{\tilde{x}}^\top p_{\tilde{x}}^{k}(x_t), \quad t = 1, \ldots, T, \tag{6.42}$$

is a local approximation of the function f in the vicinity of $\tilde{x}$.

The linear model of Eq. (6.42) is fitted by minimizing a weighted loss function, i.e.,

$$\hat{\theta}_{\tilde{x}} = \arg\min_{\theta} \sum_{t=1}^{T} w_t \rho\left(y_t - \theta^\top p_{\tilde{x}}^{k}(x_t)\right), \tag{6.43}$$

with the weights w_t assigned by a Kernel function,

$$w_t = K(x_t, \tilde{x}) \; = \; \omega\left(\frac{|x_t - \tilde{x}|}{\hbar}\right), \tag{6.44}$$

and where ρ is a cost function similar to that in Eq. (6.36) for the linear regression case. It may be of quadratic form, if focus is given to the mean behavior in the data, or asymmetric linear in the case of quantile regression.

In the above, $\hbar$ is the so-called bandwidth, which controls the size of the neighborhood over which the model is fitted. As an example, ω can be defined as a tricube function,

$$\omega(v) = \begin{cases} (1 - v^3)^3, & v \in [0, 1], \\ 0 & , v > 1, \end{cases} \tag{6.45}$$

or alternatively with a standard Gaussian kernel function,

$$\omega(v) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right). \tag{6.46}$$

Note that the choice for a weight function or another can result from an expert choice or after a comparison of the quality of the fit of the corresponding models to the data being analyzed. The purpose of using such weight functions is to define the neighborhood around the fitting points at which models are to be fitted.

Focusing on the mean behavior in the data by defining ρ as a quadratic function in Eq. (6.43), the local LS estimate $\hat{\theta}_{\tilde{x}}$ is readily obtained as

$$\hat{\theta}_{\tilde{x}} = (X^\top W X)^{-1} X^\top W Y, \tag{6.47}$$

where the matrix X and vector Y gather all observations as in Eq. (6.38), while W is a diagonal matrix of weights,

$$W = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_T \end{bmatrix}. \tag{6.48}$$

The only difference with the LS-estimate of model parameters in Eq. (6.37) is the diagonal weight matrix W, which is the key to the local fitting of the polynomial models. As for linear regression, no analytical solution exists for the quantile regression case, and numerical solvers should be employed instead.

As an extension of the above regression framework, the estimation of the local coefficients $\theta_{\tilde{x}}$ can actually be made adaptive in order to account for smooth temporal changes in the functional relationship of Eq. (6.34). This is highly relevant if aiming, for instance, at accounting for seasonal variations in the relationship between renewable energy generation and market prices related to water levels in hydro reservoirs. The weights in Eq. (6.43) then include a time decay, e.g., in the form of exponential forgetting, in order to gradually discount older observations.

Finally, when the local coefficients are calculated at all fitting points (say, 100 for instance, uniformly distributed on the X-axis), the complete coefficient functions $\hat{\theta}(x)$ can be obtained by linear or spline interpolation of the local coefficients $\hat{\theta}_{\tilde{x}}$. This will be illustrated in the example applications below.

Example 6.9 (Local linear regression) Following on Example 6.8, another dataset with $T = 450$ observations for an explanatory variable x and for a response variable y is gathered. The scatterplot of x and y values in Fig. 6.8 does not show a linear relationship as for Fig. 6.7. Employing a local polynomial regression approach, with polynomials of order 1 (hence the name of local linear regression), linear models are fitted at a number of fitting points, 100 here. The example models obtained for the specific fitting points with $\tilde{x} = 7$, $\tilde{x} = 9$, $\tilde{x} = 11$ illustrate how such a local approximation may allow for a description of the nonlinear relationship between x and y. Standard Gaussian kernels and a bandwidth of $\hbar = 0.5$ were used. In addition

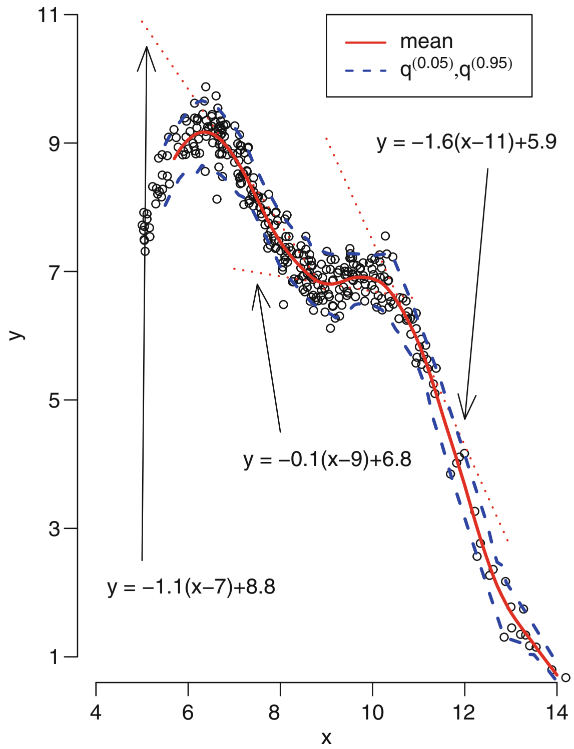

Fig. 6.8 Scatterplot with $T = 450$ observations for an explanatory and related response variable, along with the fitted local linear models and the final nonlinear regression curve (LS and quantile regression with nominal proportions $\alpha = 0.05$ and $\alpha = 0.95$)

to the LS regression curve giving the mean trend in the data, quantile regression with nominal proportions $\alpha = 0.05$ and $\alpha = 0.95$ was used to define intervals informing on the potential uncertainty around the LS regression curve.

6.4.1.4 Logistic Regression for Categorical Variables

The regression approaches described in the above, linear and nonlinear, are well suited for continuous variables such as market prices and volumes. They cannot be directly applied, however, to the case of categorical variables like balancing direction. In contrast, another form of regression should be used, the so-called *logistic regression*, which is a special instance of a generalized linear model. An extensive coverage of the various types of logistic regression models introduced in the following is available in [6].

A categorical variable π can only take values in $\{0, 1\}$. For a set of T successive observations $\{\pi_t\}$ for this response variable, as well as for a corresponding explanatory variable $\{x_t\}$, the logistic regression model writes

$$\log\left(\frac{\pi_t}{1-\pi_t}\right) = \theta_0 + \theta_1 x_t + \varepsilon_t, \qquad t = 1, \ldots, T. \tag{6.49}$$

where ε_t is a noise term in the form of a sequence of realizations from independent and identically distributed random variables with zero mean and finite variance. A logistic regression model for the categorical variable π models the probability of the event $\pi = 1$ as a function of the explanatory variable x. This explanatory variable can be the load (predicted or observed), stochastic renewable energy generation (also predicted or observed), or the hour of the day. Similarly to regression with continuous variables, the logistic regression model in Eq. (6.49) can be extended in a straightforward manner to considering more explanatory variables at once. For the estimation of parameters in logistic regression models, there does not exist an exact analytical solution. Consequently, numerical approaches should be employed, for instance, with iterative optimization in a maximum likelihood framework.

The model in Eq. (6.49) is linear with the explanatory variable. Its nonlinear generalization is obtained in a spirit similar to the local polynomial regression described in Sect. 6.4.1.3, by considering a set of fitting points where the logistic regression model of Eq. (6.49) is to be fitted. For a fitting point $\tilde{x}$, this model is reformulated as

$$\log\left(\frac{\pi_t}{1-\pi_t}\right) = \theta_{\tilde{x}}^\top p_{\tilde{x}}^k(x_t) + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{6.50}$$

where $p_{\tilde{x}}^k(x_t)$ was defined in Eq. (6.40) and with $\theta_{\tilde{x}}$ a vector of local coefficients at $\tilde{x}$. The estimation of the model parameters is to be carried out using similar numerical approaches, with an additional weighting of observations being a function of their distance to the fitting point considered, based on a Kernel function as in Eq. (6.44).

Logistic regression is introduced for the case of two categories only, that is, for the response variable following a Bernouilli process. This choice is suitable for a number of studies analyzing the impact of renewable energy generation on electricity markets. In certain cases, more than two categories need to be considered. For instance, if the response variable is I^B, it may take three different values. Such a situation calls for another generalization of logistic regression approaches, referred to as *multinomial logistic regression*, which is not presented and discussed here, for simplicity.

6.4.2 Example Application to the Scandinavian Nord Pool

To further illustrate the impact of renewable energy generation described in Sect. 6.3, an ex-post analysis is performed for the case of the *Nord Pool* market in Scandinavia. More precisely, we look at the Western Denmark area of this market, where the share of wind energy in the electricity mix is substantial. The setup and available data are firstly described, followed by two illustrative ex-post analyses on *(i)* the impact of the predicted wind power penetration on day-ahead prices, and *(ii)* its impact on balancing needs and direction. Overall, it should be noted that such empirical analyses of stochastic renewable energy generation, load, and market data should be performed extensively and with great care, so as to better understand their real-world interdependencies.

6.4.2.1 Setup and Available Data

The Western Denmark control area of the Nord Pool is that of the Scandinavian electricity market with the highest wind power penetration. The area covers the Jutland peninsula and the Island of Funen. Emphasis is placed on a 3-year period covering 2010–2012.

Long records of market quantities (day-ahead and balancing prices, balancing direction, etc.) are available for those markets. The day-ahead market is an auction-based mechanism with hourly resolution, cleared 12 h before the day of operation. The balancing market is a two-price imbalance settlement, where only market participants contributing to the overall system imbalance are penalized. In contrast, market participants who are in imbalance, though contributing to put the system back to balance, are not penalized. For instance, in a case of production surplus for the whole system (positive imbalance), market participants who are producing more than their day-ahead schedule are penalized, while those producing less than their day-ahead schedule are not.

In parallel, measurements and forecasts of the load and wind power generation for the area as a whole are available through the TSO in Denmark, Energinet.dk. They are both issued based on meteorological forecasts of some of the relevant variables: temperature for the load, as well as wind speed and direction for wind power generation. They have an hourly temporal resolution, consistent with that of the market variables under study. Forecasts are point forecasts such as those presented and discussed in Sect. 2.3.2 of Chap. 2.

6.4.2.2 Impact of Predicted Wind Power Penetration on Day-Ahead Prices

In line with the presentation in Sect. 6.3.1 of the expected impact of stochastic renewable energy generation on prices and volumes in the day-ahead market, a first ex-post analysis concentrates on assessing that impact based on an empirical study of the available data for the Western Denmark area of the Nord Pool, over the period 2010–2012. Following our previous discussion on the merit-order effect, one expects day-ahead electricity prices to decrease as wind power generation in this market area increases. Actually, rather than wind power generation, it is the predicted share of wind power production permitting to meet the electric demand that should be seen as the main driver of day-ahead electricity prices. This so-called *predicted wind power penetration* is defined for a given market time unit as the ratio between predicted wind power production and predicted load.

For the dataset available, local polynomial regression is applied for uncovering the empirical relationship between predicted wind power penetration and day-ahead prices. Market time units are considered individually, since the general load level is also to directly influence prices, hence as a function of the hour of the day. Figure 6.9 gathers example results for market time units from 7:00 to 8:00 and 18:00 to 19:00. Nonlinear regression models are fitted for the mean behavior of prices, as well as for quantiles with nominal proportions of 0.05 and 0.95.

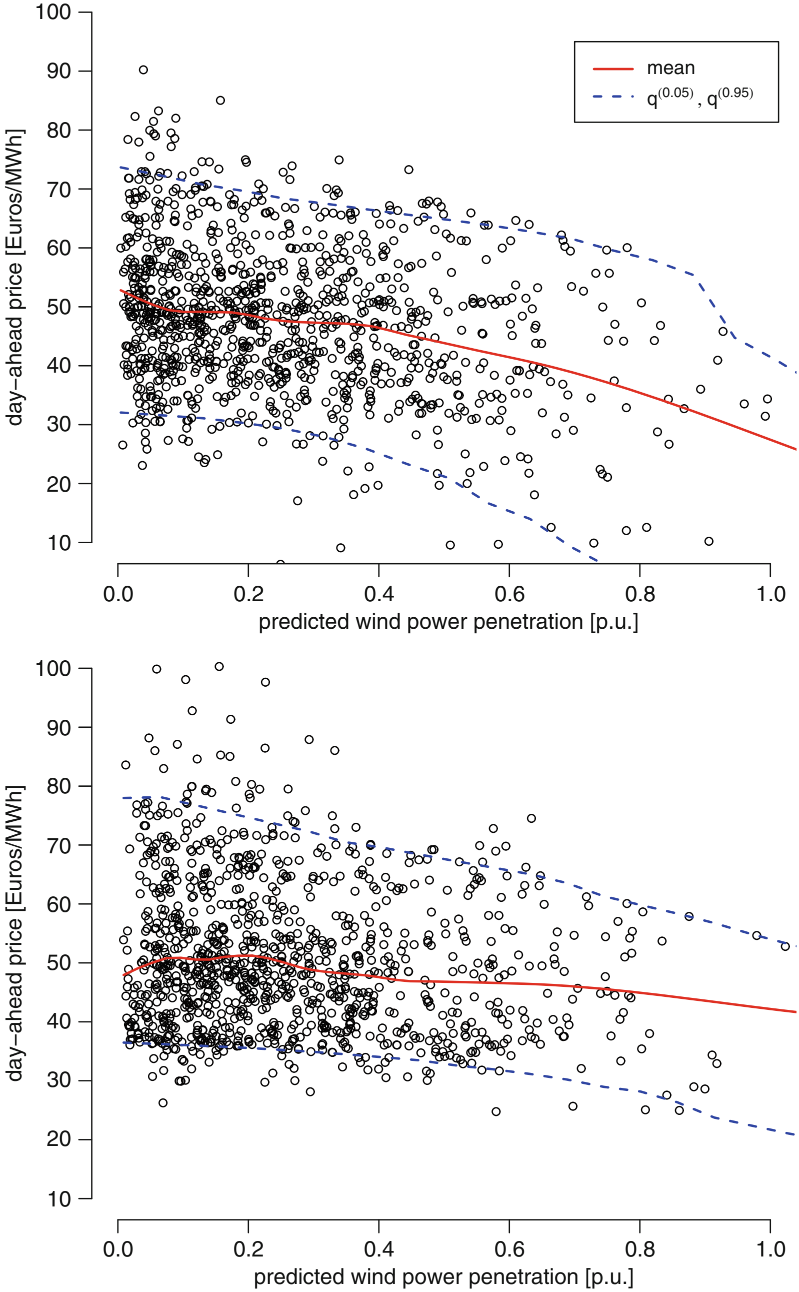

Fig. 6.9 The impact of predicted wind power penetration in Western Denmark on day-ahead prices in the Nord Pool. These results are for the period 2010–2012, and for two specific time units in the market: from 7:00 to 8:00 (*top*) and from 18:00 to 19:00. Raw observations as well as the empirical mean and quantile trends are given

There is a clear tendency for prices to decrease on average as the predicted wind power penetration gets larger. The magnitude of that reduction appears to vary during the day, however, here being more pronounced for the morning hour in comparison with the evening one. Probably, more importantly, the effect of predicted wind power penetration is more significant on high prices, as shown by the evaluation of the quantile with nominal proportion 0.95, since those seem to vanish with larger predicted wind power penetration. In parallel, a similar downward pressure can be noticed for the lower end of electricity prices.

6.4.2.3 Impact of Predicted Wind Power Penetration on the Need for Balancing and Its Direction

Next, emphasis is placed on the balancing market of the Nord Pool for this same Western Denmark area. It is aimed at uncovering the effect of predicted wind power penetration on balancing. The investigation is split in two parts, by first looking at how it influences the need for balancing (i.e., $I^B = 0$ for no-balancing against $I^B \neq 0$ for balancing), and then at how it impacts balancing direction. For the latter, it translates to assessing how it affects the conditional probabilities of $I^B = 1$ and $I^B = -1$, given that $I^B \neq 0$.

As an illustrative example, let us look at the market hour between 12:00 and 13:00, for the data collected over the period 2010–2012. Other market time units may be considered similarly. Analysis results for this dataset are collated in Fig. 6.10. Observations are given by circles taking integer values, while the regression lines are for the trends induced by the predicted wind power generation over the Western Denmark area. They are obtained by locally fitting logistic regression models where the predicted wind power generation is the only explanatory variable.

In the first case of analyzing the impact of predicted wind power penetration on the need for balancing, there is an observed frequency of 65 % of needing balancing when no wind is predicted to contribute to the generation mix. This observed frequency increases to 78 % as the predicted wind power penetration reaches 67 %, while it tends to decrease again for higher penetration values. This can be explained by the fact that the uncertainty in wind power forecasts is lowest for low power values (and also for power levels close to nominal capacity) and highest for medium power levels. It then directly impacts the need for balancing, in one direction or the other.

In addition, the predicted wind power penetration may also have an impact on the balancing direction, in relation to the skewness of the distribution of forecast errors. These distributions tend to be right-skewed for low predicted power values (i.e., asymmetric with a longer tail in the direction of higher power values), and inversely, left-skewed for higher predicted power values. This effect is confirmed by the empirical analysis of Fig. 6.10 (bottom), with a higher share of positive imbalances (need for downward balancing) for predicted penetration values of less than 20 %. Clearly, for higher penetration values, there is much greater share of negative imbalances, then confirming the overall influence of renewable energy generation on the balancing market.

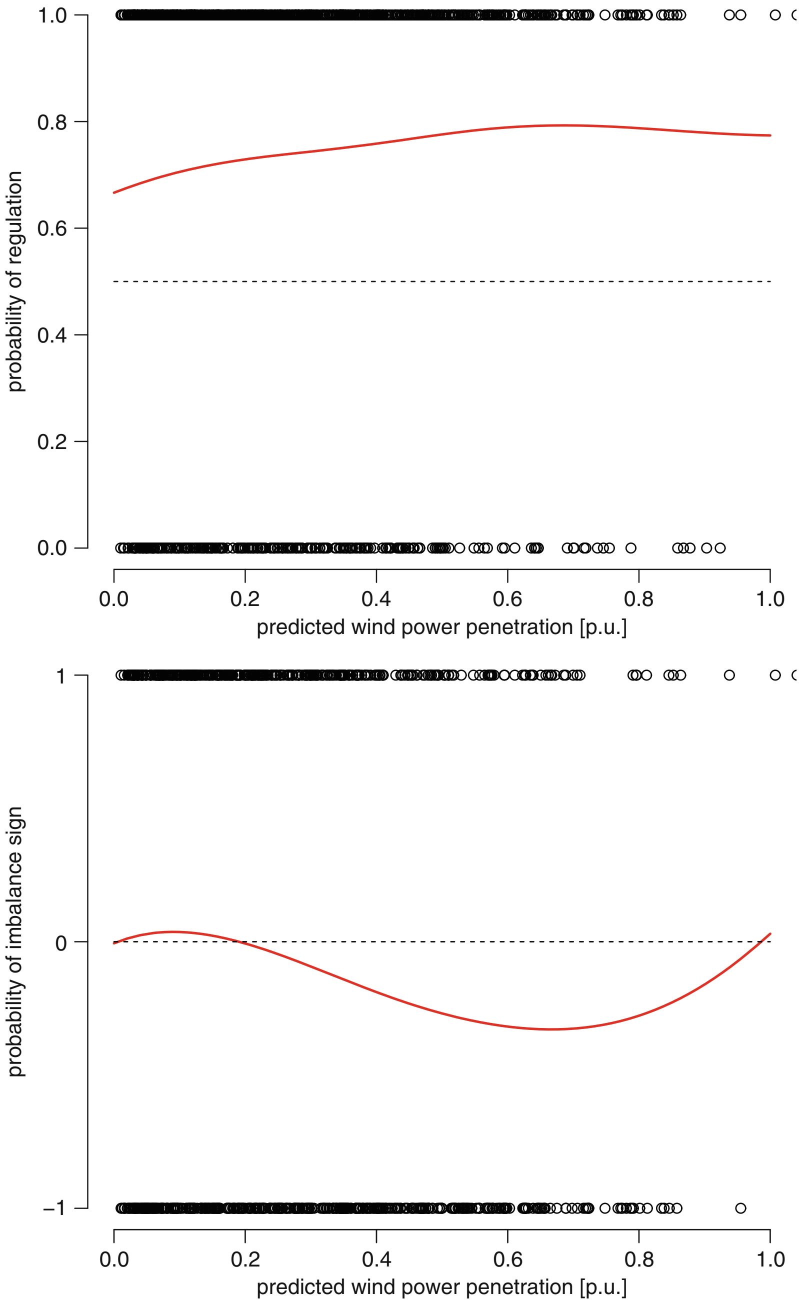

Fig. 6.10 The impact of predicted wind power penetration in Western Denmark on the need for balancing (*top*) and its direction (*bottom*). These results are for the period 2009–2011, and for the market time unit from 12:00 to 13:00. Raw observations are the −1, 0 and 1 dot values. The regression lines describe the evolution of the probability of needing balancing, and of its direction, as a function of predicted wind power penetration

6.5 Summary and Conclusions

As the penetration of stochastic renewable energy sources in electricity markets increases, it is expected that they are to impact the outcome of market clearing mechanisms, namely, price and quantities. This chapter covered both the origins of this impact and its mechanisms. The impact on day-ahead and balancing markets is different, the former being mainly affected by the predicted levels of renewable power generation, while the latter is affected by the uncertainty (and eventually, the errors) of these forecasts.

The impact of stochastic renewable energy sources on electricity markets can be illustrated based on simulations and toy models of increasing complexity. For simplicity, network effects were disregarded here. They may be of crucial importance, however, in a number of real-world cases, as for the example of the US. In addition, this impact on market may become more complex for more advanced market clearing mechanisms such as those presented in previous chapters, when coupling day-ahead and balancing markets, or when considering pro-active demand at the balancing stage. Designing markets that can dampen the impact of renewables is a current research problem that attracts a lot of attention and which should lead to potentially radical changes to electricity markets as we know them today.

While discussing how renewable energy generation affects electricity markets is more easily done based on simulations, its real influence ought to be evaluated empirically based on the increasing quantity of market data available. With that objective in mind, a number of simple linear and nonlinear models were introduced here, with focus on common regression for the mean response, but also on quantile regression to evaluate uncertainties in the impact of renewable energy generation on market variables. These statistical concepts and related models were applied to the test case of the western Denmark area of the Nord Pool for illustration purposes. Performing more of these empirical investigations would allow gaining substantial knowledge on the actual impact of renewable energy sources on market variables for different markets around the world.

6.6 Further Reading

The impact of wind power and of its predictability is now regularly accounted for in, e.g., network expansion and future offshore grid studies. It is therefore of utmost importance to properly characterize and model the effects of wind, and more generally stochastic renewable energy, on markets and power systems operations. Such analyses combining the meteorological, network and market aspects, can be found in the relevant literature. Recent examples of system studies going in that direction include [3] for the case of the UK system in 2020 and [5] concentrating on the Swiss power system at the horizon 2030. More specifically, for the case of the impact of stochastic renewable energy on market quantities, example ex-post studies concentrating on prices, balancing direction and volumes, were for instance performed by [12] and [7] for the Nord Pool in Scandinavia and by [2] for the Spanish electricity market. Similarly, a detailed analysis of the impact of German wind power generation on the German electricity market (EEX), as well as on all European cross-border power flows, can be found in [13].

As an extension of the static impact of stochastic renewable energy on electricity markets analyzed here, dynamical effects also ought to be uncovered and quantified. Toy model simulations already allow highlighting some of these effects, as for the example of [4], looking at the combination of feedback mechanisms, competition among market participants, and stochasticity from wind power generation. Another example is that of [14], where a simplified power system, in which wind dynamics and market mechanisms are modeled, is employed to describe the impact of wind power generation on market variables. A more advanced simulation study on the impact of wind energy on LMPs is described in [11]. Simplified systems and toy models may, however, mask some of the effects that are aimed at being uncovered. This is the reason why, maybe counter-intuitively, statistical ex-post analyses of some of the key variables can already give a fair picture, without looking at a complete modeling of all meteorological, market and network effects. A good description of the general regression framework, covering linear and generalized linear models, can be found in [10].

Exercises

6.1 Give a simple and intuitive proof of the properties of the impact of stochastic power generation on market-clearing outcomes (system price and volume) expressed by Eqs. (6.4) and (6.5).

6.2 Place yourself in a single-auction market-clearing procedure. Propose your own supply curve with 10 production offers (quantity and prices), one of them being for wind power generation at a short-run marginal price of zero. Similarly, define a demand curve with 12 consumption offers (quantity and prices). Obtain the market-clearing outcome, that is, the resulting system price and quantity.

6.3 As an extension of the above, replace the "fixed" wind power generation offer by the realization of a Gaussian random variable, with mean and standard deviation of your choice. Simulate the market-clearing outcome (resulting system price and quantity) for 100 different realizations from that random variable. Visualize the histograms of prices and quantities. Are they also Gaussian? Why wouldn't they be? Subsequently, repeat the same procedure with different mean and standard deviation values for the Gaussian random variable, and analyze the resulting distributions of system prices and volumes.

6.4 Consider the distribution of potential wind power generation by a Beta random variable Beta$(1, 4)$ (units being gigawatt hour), with a forecast of 0.2 GWh used at the time of market clearing. What is the distribution of forecast errors, following the discussion in Sect. 6.3.2? Given a threshold $\tau = 20$ MWh, find the probabilities for I^B of positive and negative imbalances, as well as no balancing needed. Then, visualize the distributions of potential positive and negative imbalances.

6.5 Consider the same conditions and distributions as in the Exercise 6.4. Follow the reasoning of Example 6.7, with the same upward and downward balancing offers in the balancing market as in Table 6.7 and recalculate the probabilities of each of the participants B1–B6 being the marginal generator. What is the probability that there will not be enough balancing power?

References

1. Fan, J., Gejbels, I.: Local Polynomial Modelling and its Applications. Chapman & Hall/CRC, London, UK (1996)
2. Gelabert, L., Labandeira, X., Linares, P.: An ex-post analysis of the effect of renewables and cogeneration on Spanish electricity prices. Energy Econ. **33**, 59–65 (2011)
3. Gerber, A., Qadrdan, M., Chaudry, M., Ekanayake, J., Jenkins, N.: A 2020 GB transmission network study using dispersed wind farm power output. Renew. Energy **37**(1), 124–132 (2012)
4. Giabardo, P., Zugno, M., Pinson, P., Madsen, H.: Feedback, competition and stochasticity in a day-ahead electricity market. Energy Econ. **32**(2), 292–301 (2010)
5. Hagspiel, S., Papaemannouil, A., Schmid, M., Andersson, G.: Copula-based modeling of stochastic wind power in Europe and implications for the Swiss power grid. Appl. Energy **96**, 33–44 (2012)
6. Hosmer, D., Lemeshow, S., Sturdivant, R.: Applied Logistic Regression, 3rd edn. John Wiley & Sons, New Jersey, USA (2013)
7. Jónsson, T., Pinson, P., Madsen, H.: On the market impact of wind energy forecasts. Energy Econ. **32**(2), 313–320 (2010)
8. Lange, M.: On the uncertainty of wind power predictions - Analysis of the forecast accuracy and statistical distribution of errors. J. Sol. Energy Eng. Trans.-ASME **127**(2), 177–184 (2005)
9. Lawson, C., Hanson, R.: Solving Least Squares Problems. SIAM, Philadelphia, USA (1995)
10. Madsen, H., Thyregod, P.: Introduction to General and Generalized Linear Models. Chapman & Hall/CRC, London, UK (2011)
11. Morales, J., Conejo, A., Pérez-Ruiz, J.: Simulating the impact of wind production on locational marginal prices. IEEE Trans. Power Syst. **26**(2), 820–828 (2011)
12. Morthorst, P.: Wind power and the conditions at a liberalized power market. Wind Energy **6**(3), 297–308 (2003)
13. Pinson, P., Jónsson, T., Zugno, M., Morales, J., Madsen, H.: Statistical analysis of the impact of wind power on market quantities and power flows. In: Proc. IEEE PES General Meeting 2012, San Diego, California, USA (2012)
14. Zeineldin, H., El-Fouly, T., El-Saadany, E., Salama, M.: Impact of wind farm integration on electricity market prices. IET Renew. Power Gener. **3**(1), 84–95 (2009)

Chapter 7
Trading Stochastic Production in Electricity Pools

7.1 Introduction and Decision Framework

Recent years have witnessed an exceptional technological development that, along with increasing political pressure to cut CO_2 emissions and to create local jobs, spurred an unprecedented growth in installed production capacity from renewable sources. To sustain such growth, national governments have put in place special support schemes (tax credits, feed-in tariffs, etc.) for easing the market participation of renewable producers.

As the energy cost of renewables constantly decreases approaching grid parity, green power needs smaller and smaller incentives for being competitive. For this reason, renewable power producers are increasingly required to participate in electricity markets under the same rules as conventional power producers. In particular, as opposed to feed-in tariff schemes, they are more and more frequently subject to market prices and assigned *balance responsibility*. The former implies that renewable electricity producers are subject, like any other power producer, to price risk. Besides, the latter implies that they are financially accountable for the additional balancing costs incurred by the system operators, which in practice means that they need to correct their energy imbalances by trading in the balancing market.

Although renewable producers are asked to participate in the market in the same way as conventional producers, trading green energy presents substantial differences when compared to the case of conventional power sources. Firstly, the actual production is variable and uncertain at the time of offering. This uncertainty, coupled with the stochastic nature of power prices, results in uncertain returns depending on the realization of both power production and prices. Secondly, renewable producers are *forced* to participate in multiple markets, because markets with early gate closures—day-ahead- markets—have more stable prices and, in parallel, deviations of the actual production from the contractual positions at the day-ahead and adjustment markets must be settled at the balancing market.

As the market design often—although not always—penalizes real-time deviations from (and in general later corrections of) the day-ahead schedule, renewable power producers are in a disadvantaged position compared to conventional producers. As a partial solution to these disadvantages, renewable power producers can trade

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strategically, in the attempt to get the most out of the information they possess on the uncertain variables in play. This chapter focuses on the determination of optimal trading strategies for renewable power producers participating in electricity pools. Owing to the stochastic features and the multiple market layers described above, this is a multistage problem of decision-making under uncertainty.

Two basic assumptions are made in this chapter. The first one entails that incentives such as price premia added on top of market prices, feed-in tariffs, etc., are discarded. As a result, renewable energy is traded under exactly the same rules as the conventional one. This simplification is introduced for the sake of clarity and generality as different markets have different incentive schemes. Nevertheless, the tools developed here could be extended so as to account for these incentives with little effort. The second assumption is that renewable power producers participate in the electricity pool on their own. This means that the optimal trading strategies developed in this chapter do not account for possible associations with other market entities, e.g., owners of storage devices or flexible demand, which are treated in Chaps. 8 and 9, respectively.

This chapter is structured as follows. Section 7.2 presents the basic problem formulation and introduces some common concepts in decision-making under uncertainty. Optimal strategies are determined analytically in Sect. 7.3 for offering in the day-ahead and balancing markets considering deterministic electricity prices. Section 7.4 develops different strategies in a series of cases with stochastic market prices. Section 7.5 considers the case of a risk-averse stochastic producer. Thereafter, Sect. 7.6 models the problem in the framework of stochastic programming, which is more versatile as it can accommodate any type of probabilistic multivariate distribution of the uncertainty as well as different risk metrics. Finally, Sect. 7.7 concludes the chapter.

7.2 Revenue and Imbalance Cost: Concept and Definition

In this section, we introduce the basic formulation of the problem of optimal trading when the production volume is uncertain, which is the case for renewable energy sources such as wind and solar. In parallel, some concepts of decision-making under uncertainty are presented.

In order to set up the problem in a simple framework, we introduce the following assumptions. Later on in the development of this chapter, assumptions A1–A3 will be gradually removed.

- A1 The stochastic producer trades only at the day-ahead and at the balancing market, while adjustment markets are discarded from the analysis.
- A2 The only uncertainty is related to the production volume, while market prices are deterministic and known in advance.
- A3 The producer is risk-neutral, meaning that it aims at the maximization of the expected profits with disregard of possible losses.
- A4 The stochastic producer is a price-taker, i.e., market prices do not depend on the employed offering strategy.

Assumptions A1 and A4 are critical for obtaining an analytical solution to the trading problem. On the other hand, assumption A2, which is exploited in the derivations in Sect. 7.3, is mainly for presentation convenience. Indeed, Sect. 7.4.1 shows that, under the assumption that production and market prices are uncorrelated, prices can be substituted by their expected values. Analytical solutions are still available under less restrictive assumptions on the correlation between day-ahead and balancing market prices; they are presented in Sect. 7.4.1. Further results obtained considering the correlation between prices and production are presented in [6]. The risk-neutrality assumption A3 is justified by the relatively high frequency with which the producers offer in electricity markets. This implies that the possible losses incurred in a single trading period are small if aggregated over a reasonable time-span. Section 7.5 presents some analytical results available when considering a risk-averse power producer.

As we show in Sect. 7.6, the stochastic programming framework provides the necessary flexibility to treat the trading problem for stochastic power producers disregarding the assumptions A1–A3 above.

To get rid of the price-taker assumption A4, one can employ mathematical programs with equilibrium constraints (MPEC), see Appendix B and [7]. Given its complexity, this topic is not covered here. However, we refer the interested reader to [2] and [18], where stochastic MPECs are applied to trading problems of renewable suppliers.

In the remainder of this section, we consider the cases of one-price and two-price markets separately.

Before getting started, two important clarifications should be made. Firstly, we assume that the renewable electricity producer is located at a certain bus of the transmission network. For the sake of simplicity, we always use the term "market price." However, if considering a market with nodal pricing, "market price" has the meaning of "locational marginal price (LMP) at the bus of interest." Similarly in a market with zonal pricing, such term would refer to the relevant zonal price. Notice that the generalization of the models presented in this chapter to the case of a producer whose plants are located in different nodes, or zones, of the power network is straightforward.

Secondly, there are no intertemporal constraints in the problems we analyze. This means that we focus on single-period models, and thus time indexes are dropped from the mathematical formulations.

7.2.1 One-Price Market

In a one-price balancing market, deviations from day-ahead contracts are traded at a unique balancing price regardless of the sign of producer and system imbalances. Generally, the balancing market price is higher than the day-ahead price if the system is in up-regulation, i.e., when the system is in deficit of power production as a result of all the deviations from producers and consumers with respect to their day-ahead

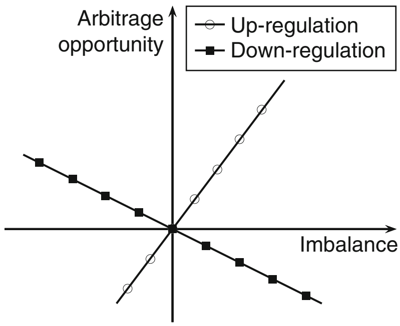

Fig. 7.1 Producer imbalance and arbitrage opportunity between balancing and day-ahead markets in the one-price system

positions. Conversely, in the down-regulation case (i.e., when the system has a surplus of generation) the balancing price is lower than the day-ahead price.

Due to the pricing rules described above, the participation at both the day-ahead and the balancing market opens arbitrage opportunities for power producers. When the producer's real-time imbalance with respect to the day-ahead contract is in the opposite direction compared to the overall system imbalance, power producers receive a more favorable price at the balancing market. Indeed, they can sell excess energy (positive imbalance) compared to their day-ahead position at a higher price than the day-ahead price when the system is in up-regulation, and repurchase their production deficit (negative imbalance) at a lower price in the down-regulation case. On the other hand, the balancing price is less favorable when the producer and the system deviations are in the same direction. Figure 7.1 summarizes these comments by showing the arbitrage opportunity as a function of the producer imbalance in the two regulation cases. Clearly, because the arbitrage opportunity is equal to the imbalance times the difference in price between the balancing and the day-ahead market, the relation is linear.

We begin our derivation by writing down the total profit during a single trading period for the stochastic power producer, which is equal to the product between the exchanged energy volume and the respective price, summed over all the market stages. Under assumption A1 in Sect. 7.1, only the day-ahead and the balancing markets are considered here.

Let us indicate prices with λ and traded production with E, with the superscripts D and B when these quantities refer to the day-ahead and to the balancing market, respectively. In a one-price system, a single balancing market price λ^{B} is applied for both sale and purchase of energy in real-time. Therefore, profits write down as

$$\widetilde{\rho} = \underbrace{\lambda^{\mathrm{D}} E^{\mathrm{D}}}_{\text{day-ahead market}} + \underbrace{\lambda^{\mathrm{B}} \widetilde{E}^{\mathrm{B}}}_{\text{balancing market}}. \tag{7.1}$$

The only decision variable in this formulation is E^{D}. This is because prices are exogenous variables under the price-taker assumption A4 made in the previous section. Furthermore, there are no degrees of freedom in the choice of the energy exchange $\widetilde{E}^{\mathrm{B}}$ at the balancing market. Indeed, this quantity is bound to match the difference between the day-ahead schedule and the actual production, i.e.,

$$\widetilde{E}^{\mathrm{B}} = \widetilde{E} - E^{\mathrm{D}}. \tag{7.2}$$

Being dependent on the uncertain production $\widetilde{E}$, the real-time exchange $\widetilde{E}^{\mathrm{B}}$ is stochastic, and so is the profit.

A relevant question when a decision-maker is exposed to stochastic profits is the definition of the objective of the optimal strategy. Owing to the risk-neutrality assumption A3, the producer is in this case interested in maximizing its profits *in expectation*, regardless of whether the shape of the profit distribution entails the possibility of incurring large losses. As we discuss in the following sections, decision-makers are not always risk-neutral, since in some circumstances possible losses are large enough to cause them financial problems.

In decision theory, the expected value of the profits for a certain decision goes under the name of *expected monetary value (EMV)*. Replacing (7.2) into (7.1), and taking the expectation yields the following expression for the EMV:

$$\mathbb{E}\{\widetilde{\rho}\} = \left(\lambda^{\mathrm{D}} - \lambda^{\mathrm{B}}\right) E^{\mathrm{D}} + \lambda^{\mathrm{B}} \widehat{E}, \tag{7.3}$$

where $\widehat{E}$ is the expected value of power production in the trading period considered.

7.2.2 *Two-Price Market*

In a two-price market, real-time deviations are priced differently depending on the imbalance sign. Deviations that are in the opposite direction to the overall system imbalance, which help the system restore the balance between production and consumption, are priced at the day-ahead market price. On the contrary, imbalances of the same sign as that of the system are settled at the clearing price of the balancing market. Let us denote the up-regulation and down-regulation prices with λ^{UP} and λ^{DW}, respectively, while the clearing price at the balancing market is still λ^{B}. The pricing rule in a two-price balancing market implies the following

$$\lambda^{\mathrm{UP}} = \begin{cases} \lambda^{\mathrm{B}} & \text{if} \quad \lambda^{\mathrm{B}} \geq \lambda^{\mathrm{D}}, \\ \lambda^{\mathrm{D}} & \text{if} \quad \lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}, \end{cases} \tag{7.4}$$

$$\lambda^{\mathrm{DW}} = \begin{cases} \lambda^{\mathrm{D}} & \text{if} \quad \lambda^{\mathrm{B}} \geq \lambda^{\mathrm{D}}, \\ \lambda^{\mathrm{B}} & \text{if} \quad \lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}. \end{cases} \tag{7.5}$$

As Fig. 7.2 shows, and contrarily to the one-price system, there is no arbitrage opportunity when the producer deviation is in the opposite direction to the overall system imbalance, since the price obtained at the balancing market is equal to the day-ahead market price. On the contrary, there is still an opportunity loss when producer and system deviations have the same sign.

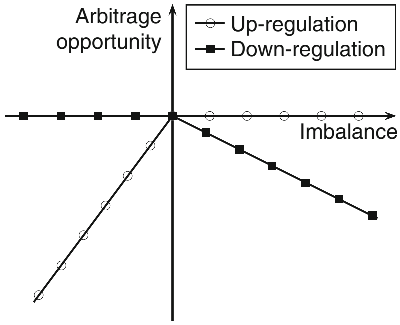

Fig. 7.2 Producer imbalance and arbitrage opportunity between balancing and day-ahead markets in the two-price system

As a result of this pricing rule, the term accounting for the balancing market profit in (7.1) splits in two when considering such a market settlement, resulting in the following formulation for the total profits

$$\widetilde{\rho} = \underbrace{\lambda^{\mathrm{D}} E^{\mathrm{D}}}_{\text{day-ahead market}} + \underbrace{\lambda^{\mathrm{UP}} \widetilde{E}^{\mathrm{UP}} + \lambda^{\mathrm{DW}} \widetilde{E}^{\mathrm{DW}}}_{\text{balancing market}}. \tag{7.6}$$

The symbols $\widetilde{E}^{\mathrm{UP}}$ and $\widetilde{E}^{\mathrm{DW}}$ refer to energy up-regulation and down-regulation for the producer at the balancing market, respectively. We recall that the power producer has to purchase upward regulation power at the balancing market when its actual production $\widetilde{E}$ is lower than the day-ahead position, E^{D}, while downward regulation is to be sold when $\widetilde{E}$ is larger than E^{D}. In mathematical terms, this writes

$$\widetilde{E}^{\mathrm{UP}} = \begin{cases} \widetilde{E} - E^{\mathrm{D}} & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} \leq 0, \\ 0 & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} > 0, \end{cases} \tag{7.7}$$

$$\widetilde{E}^{\mathrm{DW}} = \begin{cases} 0 & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} \leq 0, \\ \widetilde{E} - E^{\mathrm{D}} & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} > 0. \end{cases} \tag{7.8}$$

From the definition of up-regulation and down-regulation, it follows that

$$\widetilde{E} - E^{\mathrm{D}} = \widetilde{E}^{\mathrm{UP}} + \widetilde{E}^{\mathrm{DW}}. \tag{7.9}$$

Solving the previous equation for E^{D} and substituting the resulting expression in (7.6) yields

$$\widetilde{\rho} = \lambda^{\mathrm{D}} \widetilde{E} - \left[\left(\lambda^{\mathrm{D}} - \lambda^{\mathrm{UP}}\right) \widetilde{E}^{\mathrm{UP}} + \left(\lambda^{\mathrm{D}} - \lambda^{\mathrm{DW}}\right) \widetilde{E}^{\mathrm{DW}} \right]. \tag{7.10}$$

We notice that the first term of the sum in (7.10) is not under control of the power producer, in that neither λ^{D} nor $\widetilde{E}$ are dependent on its decisions. Furthermore, both terms inside brackets are nonnegative. This is because in a two-price system, pricing rules (7.4) and (7.5) entail that $\lambda^{\mathrm{UP}} \geq \lambda^{\mathrm{D}}$ and $\lambda^{\mathrm{DW}} \leq \lambda^{\mathrm{D}}$, and because (7.7) and (7.8) imply that $\widetilde{E}^{\mathrm{UP}} \leq 0$ and $\widetilde{E}^{\mathrm{DW}} \geq 0$.

The first term in (7.10) represents the profit that could be obtained by the producer, in a two-price system, if it had *perfect information* on the future realization of the stochastic production $\widetilde{E}$. Clearly in this case, the producer would sell the (certain) production entirely at the day-ahead market, since as Fig. 7.2 shows there is never a strictly positive arbitrage opportunity between the balancing market and the day-ahead one. The term within brackets in (7.10) is the *imbalance costs* that the producer faces when settling its regulation volume at the balancing market. This sum represents an *opportunity loss*, in that it quantifies the missing profits stemming from not being able to sell the uncertain production entirely, or for selling more than the actual production, at the day-ahead market.

Taking the expectation on both sides of (7.10), we get the following

$$\underbrace{\mathbb{E}\{\widetilde{\rho}\}}_{\text{EMV}} = \lambda^{\text{D}}\widehat{E} - \underbrace{\mathbb{E}\left\{\left(\lambda^{\text{D}} - \lambda^{\text{UP}}\right)\widetilde{E}^{\text{UP}} + \left(\lambda^{\text{D}} - \lambda^{\text{DW}}\right)\widetilde{E}^{\text{DW}}\right\}}_{\text{EOL}}, \qquad (7.11)$$

where $\widehat{E}$ is the expected power production and therefore, $\lambda^{\text{D}}\widehat{E}$ is the expected profit given perfect information. We will refer to the expectation term on the right-hand side of (7.11) equivalently as the expected imbalance costs or as the *expected opportunity loss (EOL)*.

Equation (7.11) states a general fact of decision-making under uncertainty: for any strategy, the sum of the expected monetary value (EMV) and the EOL is constant and equal to the expected profit obtained with perfect information. Therefore, we can alternatively look at the optimal strategy of a risk-neutral decision-maker either as the strategy maximizing the EMV, or as the decision minimizing the EOL. In the next section, we will determine the optimal offer for a stochastic power producer in the two-price system by formulating the problem as the minimization of the EOL.

As a final remark of this section, we point out that the minimum value of the EOL among all the feasible decisions is of particular importance. Such quantity is referred to as the *expected value of perfect information (EVPI)*. The EVPI represents the profit improvement that the decision-maker would experience if it held perfect information on the realization of the uncertainty, compared to the best performance achievable with the employed characterization of the stochastic parameters. Hence, this quantity also indicates how much the decision-maker would pay *at most* for obtaining perfect information on the contingent process. As such, the EVPI represents an upper bound to the value of an improved forecasting model for the uncertainty.

7.3 Trading with Deterministic Prices: Bidding Quantities

This section presents some analytical results for the determination of optimal trading strategies for stochastic producers when market prices are deterministic, i.e., known in advance with certainty. Similarly to the previous section, results for the one-price and the two-price markets are presented separately.

7.3.1 One-Price Market

The final result in Sect. 7.2.1 was the formulation (7.3) for the expected profits of a stochastic power producer in a one-price market. Equation (7.3) reveals the triviality of the determination of the optimal offer in a one-price system when prices are deterministic. Indeed, the second term on the right-hand side of (7.3) does not depend on the producer's decision. Therefore, this constant term can be discarded from the determination of the optimum. As far as the optimization of the first term is concerned, the following cases can happen, all with a trivial solution.

1. If $\lambda^{\mathrm{D}} < \lambda^{\mathrm{B}}$, the power producer sells nothing at the day-ahead market, and waits to place all its production at the balancing market, where the price λ^{B} is higher.
2. If $\lambda^{\mathrm{D}} > \lambda^{\mathrm{B}}$, the power producer sells as much energy as possible at the day-ahead market, eventually buying back at the balancing market the energy needed to cover the difference between day-ahead trade and actual production (7.2). As $\lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}$, the producer realizes a surplus on the energy that is sold at the day-ahead market but not delivered.
3. If $\lambda^{\mathrm{D}} = \lambda^{\mathrm{B}}$, the power producer is indifferent since any decision on E^{D} would yield the same profit.

Given that, in electricity markets, producers are usually imposed not to offer above their installed capacity $\overline{E}$, we have the following result.

> In a one-price system, under the assumption of deterministic market prices, the optimal offer for a risk-neutral stochastic power producer is price-inelastic and equal to zero volume if the balancing price is higher than the day-ahead price, while it is equal to the nominal capacity $\overline{E}$ if the balancing price is lower than the day-ahead price; if such prices are equal, any offer is optimal.

The expression "price-inelastic" in the previous sentence means that the optimal offer is either zero or the nominal capacity, regardless of the value of the day-ahead price. As we shall see in Sect. 7.4.2, this is particularly meaningful as market rules allow producers to specify their offer as a price-quantity curve.

The solution obtained in this simplified case is trivial and, besides, completely decoupled from the forecast of the stochastic power production, as it is only dependent on the forecast of the arbitrage opportunity between the day-ahead and the balancing markets.

Furthermore, it should be pointed out that the statement that the rightmost term in (7.3) is not under the control of the power producer is not completely true. In principle, power producers could spill their excess production if economically attractive. This has little impact on the optimal day-ahead strategy described above, but it would imply that, should the price λ^{B} be negative, the producer would rather spill its power production and gain additional profits when repurchasing power.

7.3.2 Two-Price Market

It is relevant to recall that the optimal strategy for a risk-neutral stochastic producer minimizes the expected imbalance costs, i.e., the EOL in (7.11). Before carrying out the necessary algebra, we define the following penalties, both nonnegative

$$\psi^{\mathrm{UP}} = \lambda^{\mathrm{UP}} - \lambda^{\mathrm{D}}, \tag{7.12}$$

$$\psi^{\mathrm{DW}} = \lambda^{\mathrm{D}} - \lambda^{\mathrm{DW}}. \tag{7.13}$$

It is important to notice that ψ^{UP} and ψ^{DW} represent the opportunity loss per energy unit, i.e., the profits lost by exchanging up-regulation and down-regulation energy at the balancing market instead of at the day-ahead stage. Substituting the above quantities in the EOL term in (7.11), we get the following expression

$$\mathrm{EOL} = \mathbb{E}\left\{-\psi^{\mathrm{UP}}\widetilde{E}^{\mathrm{UP}} + \psi^{\mathrm{DW}}\widetilde{E}^{\mathrm{DW}}\right\}. \tag{7.14}$$

We determine the optimal day-ahead offer in the following way: first, we expand the expectation in (7.14) into an integral in the probability space of uncertain production; then, the first-order stationarity condition is enforced.

The terms inside the expectation operator in (7.14) can be expanded. Owing to the piecewise definitions of up-regulation and down-regulation in (7.7) and (7.8), respectively, each term in (7.14) expands into an integration in a half-space of the set of feasible day-ahead offers $\left[0, \overline{E}\right]$, split in two halves by the day-ahead offer E^{D}, i.e.,

$$\mathrm{EOL} = -\int_0^{E^{\mathrm{D}}} \psi^{\mathrm{UP}}\left(E - E^{\mathrm{D}}\right) p_{\widetilde{E}}(E)dE + \int_{E^{\mathrm{D}}}^{\overline{E}} \psi^{\mathrm{DW}}\left(E - E^{\mathrm{D}}\right) p_{\widetilde{E}}(E)dE, \tag{7.15}$$

where $p_{\widetilde{E}}(\cdot)$ is the probability density function (pdf) of the stochastic power production $\widetilde{E}$. The reader is referred to Appendix A for an introduction to random variables and to the concept of probability density function.

To determine the optimum, we enforce the first-order stationarity condition by taking the derivative of expression (7.15) with respect to the day-ahead offer E^{D} and setting it equal to 0. Carrying out the differentiation under the integral sign, see [9], yields the following

$$\begin{aligned}\frac{d\mathrm{EOL}}{dE^{\mathrm{D}}} =& -\psi^{\mathrm{UP}}\int_0^{E^{\mathrm{D}}} -p_{\widetilde{E}}(E)dE + \psi^{\mathrm{DW}}\int_{E^{\mathrm{D}}}^{\overline{E}} -p_{\widetilde{E}}(E)dE \\ =& \psi^{\mathrm{UP}}F_{\widetilde{E}}(E^{\mathrm{D}}) + \psi^{\mathrm{DW}}\left(F_{\widetilde{E}}(E^{\mathrm{D}}) - 1\right) = 0,\end{aligned} \tag{7.16}$$

where $F_{\widetilde{E}}(\cdot)$ indicates the cumulative distribution function (cdf) of power production, see Appendix A. Solving (7.16) for the day-ahead offer E^{D} readily yields the following expression

$$E^{\mathrm{D}*} = F_{\widetilde{E}}^{-1}\left(\frac{\psi^{\mathrm{DW}}}{\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}}}\right), \tag{7.17}$$

which involves the inverse $F_{\widetilde{E}}^{-1}(\,\cdot\,)$ of the production cdf, i.e., the quantile function, which is defined in Appendix A. The optimality of $E^{\mathrm{D}*}$ in (7.17) can be easily checked by noticing that the second order derivative of EOL with respect to E^{D} is nonnegative everywhere

$$\frac{d^2\mathrm{EOL}}{dE^{\mathrm{D}2}}(E^{\mathrm{D}}) = \psi^{\mathrm{UP}} p_{\widetilde{E}}(E^{\mathrm{D}}) + \psi^{\mathrm{DW}} p_{\widetilde{E}}(E^{\mathrm{D}}) \geq 0. \tag{7.18}$$

This is because probability density functions are nonnegative by definition, and so are the penalties according to (7.12) and (7.13). As a consequence of (7.18) and under mild continuity assumptions, the EOL is convex with respect to E^{D}, which implies that the first-order stationarity condition (7.16) is sufficient to ensure that $E^{\mathrm{D}*}$ is a minimum. The results obtained so far can be summarized in the following statement.

> In a two-price system, under the assumption of deterministic market prices, the optimal day-ahead offer for a risk-neutral stochastic power producer is price-inelastic and equal to the quantile of the power production distribution corresponding to a probability equal to the down-regulation penalty divided by the sum of the up-regulation and down-regulation penalties.

Example 7.1 (Optimal bid in a two-price settlement with deterministic prices) Let us consider the following deterministic penalties due to the less favorable price at the balancing market

$$\psi^{\mathrm{UP}} = \$9/\mathrm{MWh},$$
$$\psi^{\mathrm{DW}} = \$4/\mathrm{MWh}.$$

An analyst provides us with a probabilistic forecast of the stochastic power production at trading period t of the following day. We refer the reader to Chap. 2 for an introduction to forecasting the production from renewable sources. According to the analyst, power production follows a uniform distribution, see Appendix A, between 100 MWh and 150 MWh. The cumulative distribution function is therefore

$$F_{\widetilde{E}}(E) = \begin{cases} 0 & \text{if } E < 100, \\ \dfrac{E-100}{50} & \text{if } 100 \leq E < 150, \\ 1 & \text{if } E \geq 150. \end{cases} \tag{7.19}$$

According to (7.17), the optimal offer must satisfy

$$F_{\widetilde{E}}(E^{\mathrm{D}*}) = \frac{\psi^{\mathrm{DW}}}{\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}}} = \frac{4}{13}, \tag{7.20}$$

which yields, by inverting the function in the second case in (7.19)

$$E^{\mathrm{D}*} = 100 + 50 \times \frac{4}{13}\,\mathrm{MWh} = 115.38\,\mathrm{MWh}. \tag{7.21}$$

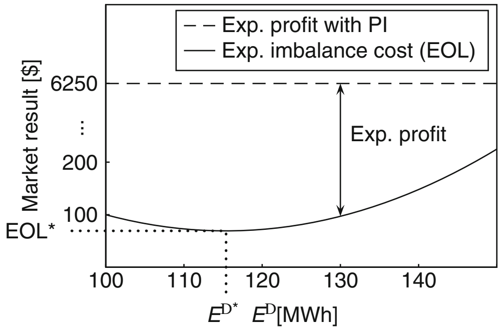

Fig. 7.3 Expected imbalance cost (EOL) as a function of the day-ahead offer. The expected profit is the difference between the expected profit with perfect information (PI) and the EOL

It is worth noticing that this result makes sense also from an intuitive point of view. Indeed, the optimal quantity offer is lower than 125 MWh, which is both the mean and the median of the power production distribution. This is in accordance with the fact that the penalty ψ^{UP} for underproducing with respect to the day-ahead market contract is higher than the penalty ψ^{DW} for overproducing, which makes a long position at the day-ahead market more attractive.

Figure 7.3 shows the expected imbalance cost (expected opportunity loss) as a function of the day-ahead offer. According to (7.11), the expected profit obtained with a given day-ahead offer E^{D} is the difference between the expected profit with perfect information (PI) and the EOL. The optimal quantity offer $E^{\mathrm{D}*}$ is a minimum point for the expected imbalance cost and, as a consequence, a maximum point for the expected profit. Observe that the expected profit with perfect information is calculated with a day-ahead price $\lambda^{\mathrm{D}} = \$50/\mathrm{MWh}$.

It is relevant to note that the closed formula (7.17) for the optimal day-ahead offer rests on the assumption that the cumulative distribution function $F_{\widetilde{E}}(\,\cdot\,)$ be invertible. In fact, we already know that $F_{\widetilde{E}}(\cdot)$ is, by definition of cdf, monotonically increasing, though the monotonicity may be nonstrict. In the latter case, the closed formula (7.17) is still valid using the generalized inverse function

$$F_{\widetilde{E}}^{-1}(\alpha) = \inf\left\{x \in \left[0,\ \overline{E}\right] : F_{\widetilde{E}}(x) \geq \alpha\right\}. \tag{7.22}$$

In fact, any amount E^{D} such that $F_{\widetilde{E}}(E^{\mathrm{D}}) = \psi^{\mathrm{DW}}/(\psi^{\mathrm{DW}} + \psi^{\mathrm{UP}})$ is optimal in this case.

Finally, we underline that expression (7.17) always makes sense, in that the ratio $\psi^{\mathrm{DW}}/(\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}})$ is always included in the interval $[0,\ 1]$. This follows trivially from the nonnegativity of ψ^{UP} and ψ^{DW}.

7.4 Trading with Stochastic Prices

In the previous section, we derived some results on the optimal trading of stochastic power producers under simplified assumptions, among which was the restrictive hypothesis of deterministic prices. In what follows, we relax this simplification by assuming that prices are also uncertain.

The trading problem presents an increased level of difficulty if prices are stochastic. This additional difficulty stems from the fact that taking expectations of the power producer's profit in (7.1) and (7.10) entails the integration in two variables, namely the power production and a market price (or penalty), which requires knowledge of their joint probability density function. Furthermore, because market rules allow power producers to differentiate the offered quantity depending on the day-ahead clearing price, these expectations are to be conditioned on the latter quantity.

As usual, we increase the difficulty of the problem gradually. Section 7.4.1 treats the special case of trading with stochastic prices, where the difference between the balancing price and the day-ahead price (i.e., the penalty) is uncorrelated both with the day-ahead price itself and with the stochastic power production. The results obtained in the previous section still hold, though with some formal modifications. In Sect. 7.4.2, the case where the penalty is uncorrelated with the power production, but not with the day-ahead price, is considered.

7.4.1 Stochastic Generalizations of Quantity Bidding

We now turn our focus to the analysis of a special case, where the results obtained in the previous section still hold under the assumption of uncertain prices—provided that deterministic prices are replaced by their expected values. The critical assumption here is that the difference between the balancing market price and the day-ahead price is uncorrelated both with the day-ahead price itself and with the stochastic power production. As customary, the cases of the one-price and the two-price markets are considered separately.

7.4.1.1 One-Price Market

Resuming the analysis in Sect.7.2.1, the equivalent of the expected profit in (7.3) if prices are stochastic is given by

$$\mathbb{E}\left\{\widetilde{\rho}\right\} = \mathbb{E}\left\{\left(\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right) E^{\mathrm{D}}\right\} + \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}}\widetilde{E}\right\}. \tag{7.23}$$

The last term in (7.23) is not under control of the stochastic power producer. Therefore, the optimal bid must maximize the first term on the right-hand side of (7.23). Assuming that the difference $\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}$ is uncorrelated with the day-ahead price, there is no additional benefit in differentiating the offered quantity with respect to the day-ahead price through a bidding curve. In other words, the optimal offer still consists of a single quantity. Taking the expectation of (7.23), we obtain

$$\mathbb{E}\left\{\widetilde{\rho}\right\} = \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} E^{\mathrm{D}} + \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}}\widetilde{E}\right\}. \tag{7.24}$$

The conclusion on the optimal day-ahead offer is similar to the one in Sect. 7.3.1. The following cases can happen.

1. If $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} < 0$, the optimal bid is 0.
2. If $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} > 0$, the optimal bid is the nominal capacity.
3. If $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} = 0$, the power producer is indifferent since any decision on E^{D} would yield the same profit in expectation.

> In a one-price system with stochastic prices, under the assumption that the difference between the balancing and the day-ahead prices is uncorrelated with both the day-ahead price and the power production, the optimal offer for a stochastic power producer is price-inelastic and equal to zero volume if the expectation of the balancing price is higher than the expected day-ahead price, while it is equal to the nominal capacity $\overline{E}$ in the opposite case; if the expected prices are equal, any offer is optimal.

We remark that the assumption of the difference between the day-ahead and the balancing market prices being independent from the day-ahead price itself is rather restrictive. If this assumption is relaxed, as we show in the following part of this section, the optimal offer is no longer a single quantity.

Example 7.2 (Optimal bid in a one-price settlement with uncorrelated price difference and day-ahead price) The price at the balancing market is equal to

$$\widetilde{\lambda}^{\mathrm{B}} = \widetilde{\lambda}^{\mathrm{D}} + \widetilde{u}, \tag{7.25}$$

where $\widetilde{u}$ is uniformly distributed in the interval between −\$20/MWh and \$30 /MWh. As a result, the difference between the day-ahead and the balancing market prices is independent of and therefore uncorrelated with the day-ahead price itself. The expected value of the price difference is

$$\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}} - \widetilde{\lambda}^{\mathrm{D}}\right\} = \mathbb{E}\left\{\widetilde{u}\right\} = \int_{-20}^{30} u\frac{1}{50}du = \$5/\mathrm{MWh}. \tag{7.26}$$

As the price difference is positive in expectation, the profit is maximized by bidding 0 at the day-ahead market and placing all the production at the balancing market.

7.4.1.2 Two-Price Market

In a two-price market, under the assumption that the market penalties (7.12) and (7.13) are stochastic and uncorrelated with the day-ahead price and the power production, the expectation of the imbalance costs in (7.15) writes as

$$\begin{aligned} \mathrm{EOL} = &-\int_0^{E^{\mathrm{D}}}\int_0^{\infty} \psi^{\mathrm{UP}}\left(E - E^{\mathrm{D}}\right) p_{\widetilde{\psi}^{\mathrm{UP}},\widetilde{E}}(\psi^{\mathrm{UP}}, E)d\psi^{\mathrm{UP}}dE \\ &+ \int_{E^{\mathrm{D}}}^{\overline{E}}\int_0^{\infty} \psi^{\mathrm{DW}}\left(E - E^{\mathrm{D}}\right) p_{\widetilde{\psi}^{\mathrm{DW}},\widetilde{E}}(\psi^{\mathrm{DW}}, E)d\psi^{\mathrm{DW}}dE. \end{aligned} \tag{7.27}$$

Exploiting the definition of conditional probability (see Appendix A), (7.27) can be rewritten as follows

$$\begin{aligned}\mathrm{EOL} = &-\int_0^{E^{\mathrm{D}}}\left(\int_0^{\infty}\psi^{\mathrm{UP}} p_{\widetilde{\psi}^{\mathrm{UP}}|\widetilde{E}}(\psi^{\mathrm{UP}}|E)d\psi^{\mathrm{UP}}\right)\left(E - E^{\mathrm{D}}\right) p_{\widetilde{E}}(E)dE \\ &+\int_{E^{\mathrm{D}}}^{\overline{E}}\left(\int_0^{\infty}\psi^{\mathrm{DW}} p_{\widetilde{\psi}^{\mathrm{DW}}|\widetilde{E}}(\psi^{\mathrm{DW}}|E)d\psi^{\mathrm{DW}}\right)\left(E - E^{\mathrm{D}}\right) p_{\widetilde{E}}(E)dE.\end{aligned} \tag{7.28}$$

We notice that the integrals inside the parentheses in the above equations are the expectation of the market penalties conditional on the realization of the stochastic production. Since market penalties and power production are uncorrelated, the terms inside the brackets are equal to their expected values $\widehat{\psi}^{\mathrm{UP}}$ and $\widehat{\psi}^{\mathrm{DW}}$. At this point, the derivation follows precisely the same steps as in Sect. 7.3.2, yielding the following optimal quantity offer

$$E^{\mathrm{D}*} = F_{\widetilde{E}}^{-1}\left(\frac{\widehat{\psi}^{\mathrm{DW}}}{\widehat{\psi}^{\mathrm{UP}} + \widehat{\psi}^{\mathrm{DW}}}\right). \tag{7.29}$$

This result is similar to the one obtained in Sect. 7.3.2 and can be summarized in the following statement.

In a two-price market with stochastic prices, the optimal offer for a stochastic power producer under the assumption that imbalance penalties are uncorrelated with its power production and the day-ahead price, is price-inelastic and equal to the quantile of the power distribution corresponding to a probability equal to the expected value of the down-regulation penalty divided by its sum with the expected value of the up-regulation penalty.

Example 7.3 (Optimal bid in a two-price settlement with uncorrelated price difference and day-ahead price) We assume that the clearing price at the balancing market is distributed as the balancing price $\widetilde{\lambda}^{\mathrm{B}}$ in Example 7.2, and that the stochastic power production is distributed as in Example 7.1.

Figure 7.4 shows the probability density function of the balancing market price, and the resulting distributions for the up-regulation and down-regulation prices $\widetilde{\lambda}^{\mathrm{UP}}$, $\widetilde{\lambda}^{\mathrm{DW}}$ in the two-price system. According to the pricing rules (7.4) and (7.5), the up-regulation price $\widetilde{\lambda}^{\mathrm{UP}}$ follows the same distribution as $\widetilde{\lambda}^{\mathrm{B}}$ for values greater than the day-ahead price λ^{D}, while the part of the density function on the left of the day-ahead price λ^{D} is compressed and placed at λ^{D}, which thus has a positive probability of occurrence $\mathrm{P}\{\widetilde{\lambda}^{\mathrm{UP}} = \lambda^{\mathrm{D}}\} = \mathrm{P}\{\widetilde{\lambda}^{\mathrm{B}} \leq \lambda^{\mathrm{D}}\} = 0.4$. In a similar fashion, $\mathrm{P}\{\widetilde{\lambda}^{\mathrm{DW}} = \lambda^{\mathrm{D}}\} = \mathrm{P}\{\widetilde{\lambda}^{\mathrm{B}} \geq \lambda^{\mathrm{D}}\} = 0.6$. The expected values of the up-regulation and down-regulation penalties are given by

$$\widehat{\psi}^{\mathrm{UP}} = 0.4 \times 0 + \int_0^{30} \psi^{\mathrm{UP}}\frac{1}{50}d\psi^{\mathrm{UP}} = \$9/\mathrm{MWh}, \tag{7.30}$$

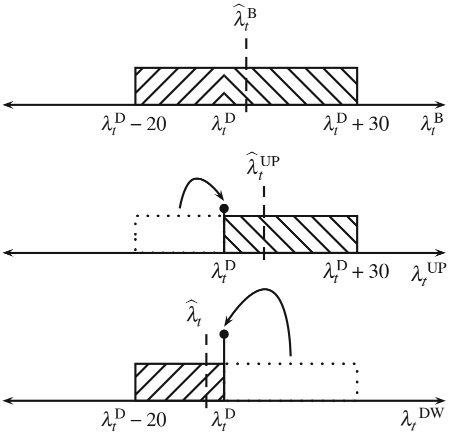

Fig. 7.4 Probability density function of a uniformly distributed clearing price $\widetilde{\lambda}^{\mathrm{B}}$ at the balancing market (*top axes*), and the resulting distributions of the up-regulation and down-regulation prices $\widetilde{\lambda}^{\mathrm{UP}}$, $\widetilde{\lambda}^{\mathrm{DW}}$ in a two-price settlement (*middle* and *bottom* axes, respectively)

$$\widehat{\psi}^{\mathrm{DW}} = \int_0^{20} \psi^{\mathrm{DW}} \frac{1}{50} d\psi^{\mathrm{DW}} + 0.6 \times 0 = \$4/\mathrm{MWh}. \tag{7.31}$$

Noticing that the expected values of the imbalance penalties are equal to their deterministic values in Example 7.1, and that the uncertain power production follows the same distribution, the optimal day-ahead offer is again $E^{\mathrm{D}*} = 115.38\,\mathrm{MWh}$.

7.4.2 Correlated Penalties and Day-Ahead Price: Bidding Curves

Electricity market rules allow power producers to submit supply curves rather than single quantities at day-ahead markets. Indeed, they can specify a certain number of production-price pairs, where they declare how much power they are willing to deliver at every price level indicated in the offer. By doing so, generators can offer power produced by units employing different technologies, while being confident that cost recovery is guaranteed for any realization of the stochastic price.

As such offer curves were conceived as an instrument for conventional power producers, and due to technical reasons related to market pricing, supply-curves must be nondecreasing, i.e., production-price pairs are to be ordered increasingly in price. Since the ordering of production-price pairs determines the scheduling preference, i.e., which supply blocks are chosen first when determining the power dispatch, a nondecreasing supply curve entails that blocks with a lower offered price are scheduled first. This is intuitively consistent with the preference of conventional power producers. Indeed, under the assumption that bids reflect the true marginal cost of generation, and disregarding non-convexities such as startup costs, conventional power producers would obviously prefer the scheduling of units with the lowest marginal cost to the more expensive ones, as this guarantees higher profits. Figure 7.5 shows an example of a supply curve for a producer employing different conventional generation technologies. Only the first two (cheapest) blocks on the left, indicated

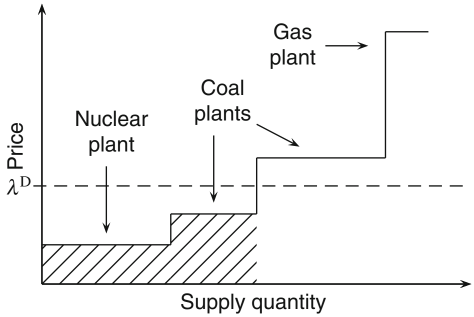

Fig. 7.5 Supply curve offered by a conventional power producer in the day-ahead market

with dashed fill and whose price offer is not greater than the cleared day-ahead price λ^{D}, are dispatched.

Owing to the fact that the marginal cost of power generation from stochastic sources such as wind and solar is null (or close to zero), it may appear that supply curves are not relevant for producers solely employing such technologies. Indeed, the optimal supply curve for producers of firm (deterministic) power is, under the price-taker assumption, the marginal cost of generation. From a simplistic analysis, one may expect that the same holds for stochastic power producers, who would thus be willing to sell the optimal quantity determined in the previous section at any (positive) price. This holds true in the special case presented in Sect. 7.4.1, but not in the general case.

The possibility of submitting a supply curve allows stochastic power producers to define in advance the quantity to be delivered to the market as a function $E^{\mathrm{D}}(\lambda^{\mathrm{D}})$ of the realization of the stochastic day-ahead price. Clearly, the determination of several quantity-price pairs is a far less constrained decision problem than the determination of a single quantity to be offered for any realization of the day-ahead price. This implies that the expected profit obtained with the optimal supply curve is at least not lower than the one resulting from bidding a fixed quantity. As we shall see in the following, the extent of this improvement depends on the level of correlation between imbalance penalties, power production, and the day-ahead price. In the case of uncorrelated variables, the optimal supply curve boils down to the fixed optimal quantity already determined in Sect. 7.4.1. In a more general case, the expected balancing prices, and therefore the optimal quantile of the power distribution, are correlated with the day-ahead price.

A last remark before going deeper into the problem regards the shape of the optimal curve. From the discussion above, it is clear that the requirement that the supply curve be nondecreasing is not restrictive for a conventional power producer. Indeed, it follows naturally from economic considerations that cheaper production blocks are offered first, as the dispatch preference of the producer is completely aligned with the increasing marginal costs of its production blocks. On the contrary, an optimal bidding curve for a stochastic power producer does not obviously follow this requirement in the general case. Determining the optimal nondecreasing supply curve analytically is not a trivial problem. As we shall see in Chap. 8, this problem can be solved by employing stochastic programming.

In the remainder of the section, we deal with the closed-form determination of the optimal bidding curve, individually for the one-price and the two-price case.

7.4.2.1 One-Price Market

In the general case where the quantities involved in the determination of the expected profit (7.23) are correlated with the day-ahead price $\widetilde{\lambda}^{\mathrm{D}}$, the stochastic power producer can benefit from specifying the quantity offered at the day-ahead market as a function of the cleared price, i.e., $E^{\mathrm{D}}(\lambda^{\mathrm{D}})$. The expected profit for the producer conditioned on the realization of the day-ahead price $\widetilde{\lambda}^{\mathrm{D}} = \lambda^{\mathrm{D}}$ writes as

$$\mathbb{E}\left\{\widetilde{\rho}|\lambda^{\mathrm{D}}\right\} = \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}|\lambda^{\mathrm{D}}\right\} E^{\mathrm{D}}(\lambda^{\mathrm{D}}) + \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}}\widetilde{E}|\lambda^{\mathrm{D}}\right\}. \tag{7.32}$$

As in the previous sections, the second term on the right-hand side of (7.32) is not dependent on the choice of the offered quantity $E^{\mathrm{D}}(\lambda^{\mathrm{D}})$. In the absence of constraints on the offer, the optimal quantity to be offered at the day-ahead price λ^{D} would be either 0 or nominal capacity, depending on whether the conditional expectation of $\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}$ is negative or positive, respectively. The determination of the optimal bidding curve can then be carried out in a pointwise fashion for any value λ^{D}.

Example 7.4 (Optimal bidding curve in a one-price settlement) The expectation of the price difference between the day-ahead and the balancing market, conditional on the day-ahead market price, is

$$\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}|\lambda^{\mathrm{D}}\right\} = -17.5 + \frac{1}{4}\lambda^{\mathrm{D}}. \tag{7.33}$$

We also expect the day-ahead price to be between \$20/MWh and \$80/MWh. Therefore, the day-ahead offer curve should specify a production value for each of these prices.

We notice that the value in (7.33) is zero for $\lambda^{\mathrm{D}} = \$70/\text{MWh}$, and strictly negative (positive) for day-ahead prices lower (greater) than this value. Therefore, the optimal offering curve prescribes to offer a zero quantity for $\$20\,/\text{MWh} \le \lambda^{\mathrm{D}} < \$70/\text{MWh}$ and the nominal capacity for $\$70\,/\text{MWh} < \lambda^{\mathrm{D}} \le \$80/\text{MWh}$. For $\lambda^{\mathrm{D}} = \$70/\text{MWh}$, any offer maximizes the expected revenues, as the expected value of the balancing market price is equal to the price at the day-ahead stage.

Figure 7.6 illustrates the conditional expectation of the price difference and the optimal offer curve for a unit with generation capacity equal to 200 MW.

Notice that if the expected day-ahead price is \$50 /MWh, the expected value of the balancing market price exceeds the former quantity by \$5 /MWh, just like in Example 7.2. However, bidding a zero quantity at any price in this case is suboptimal.

In the general case, the optimal bidding curve resulting from the pointwise calculation (7.32) may not fulfill the requirement that the supply curve be nondecreasing in its domain. Indeed, it is easy to realize that the optimal bidding curve determined in a pointwise fashion is not a supply curve whenever $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}|\lambda^{\mathrm{D}}\right\}$ switches in

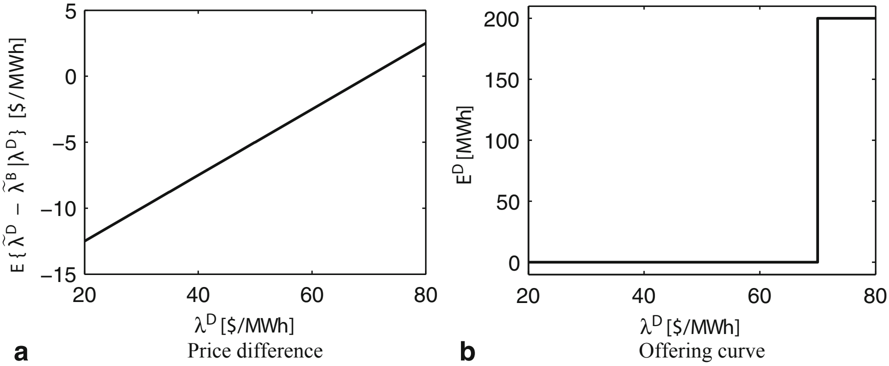

Fig. 7.6 Expected price difference between balancing and day-ahead markets, conditional on the day-ahead price (**a**), and resulting optimal offering curve (**b**)

sign from strictly positive to strictly negative for some values of λ^{D}. The stochastic programming framework can be used to determine the optimal offering curve in that case. We refer the reader to Chap. 8, where a similar offering problem including bidding curves is presented for a virtual power plant using stochastic programming.

7.4.2.2 Two-Price Market

The expected opportunity loss under the day-ahead price λ^{D} writes, by replacing the probability density functions of the penalties in (7.27) by probability distributions conditional on λ^{D}, as

$$\mathrm{EOL}(\lambda^{\mathrm{D}}) = -\int_0^{E^{\mathrm{D}}(\lambda^{\mathrm{D}})} \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|E,\lambda^{\mathrm{D}}\right\}\left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE$$
$$+\int_{E^{\mathrm{D}}(\lambda^{\mathrm{D}})}^{\overline{E}} \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|E,\lambda^{\mathrm{D}}\right\}\left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE. \quad (7.34)$$

By exploiting the fact that the imbalance penalties are uncorrelated with the stochastic power production, we can bring the conditional expectations of the penalties out of the integral operator

$$\mathrm{EOL}(\lambda^{\mathrm{D}}) = -\,\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\}\int_0^{E^{\mathrm{D}}(\lambda^{\mathrm{D}})} \left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE$$
$$+\,\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}\int_{E^{\mathrm{D}}(\lambda^{\mathrm{D}})}^{\overline{E}} \left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE. \quad (7.35)$$

Requiring that the first order derivative of the imbalance cost in (7.35) be equal to 0 yields the following expression for the optimal bidding curve

$$E^{\mathrm{D}*}(\lambda^{\mathrm{D}}) = F_{\widetilde{E}}^{-1}\left(\frac{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}}{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\}+\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}}\right). \quad (7.36)$$

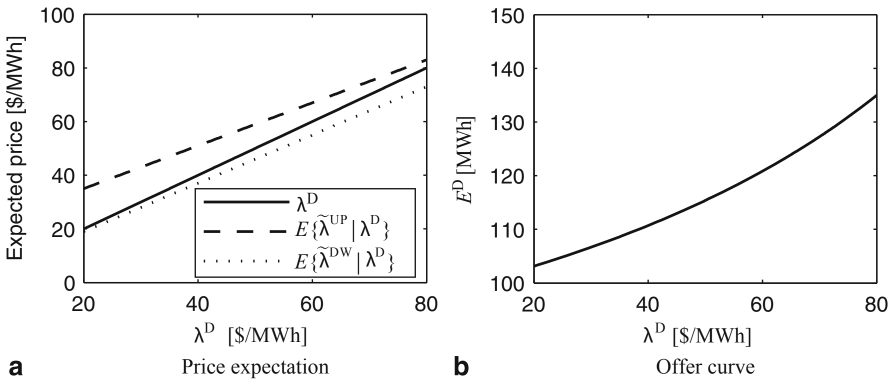

Fig. 7.7 Expected balancing prices, conditional on the day-ahead price (**a**), and resulting optimal offering curve (**b**)

As in the one-price market case, we remark that the optimal curve resulting from the pointwise calculation of the optimal quantities in (7.36) does not necessarily yield a valid nondecreasing supply curve in the general case.

Example 7.5 (Optimal bidding curve in a two-price settlement) Let us once again consider that the stochastic power production is uniformly distributed between 100 MWh and 150 MWh. The expectations of the up-regulation and down-regulation penalties, conditional on the realization of the day-ahead price, are affine functions of the latter quantity, defined as follows

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\} = 19 - \frac{1}{5}\lambda^{\mathrm{D}}, \tag{7.37}$$

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\} = -1 + \frac{1}{10}\lambda^{\mathrm{D}}. \tag{7.38}$$

The expected up-regulation and down-regulation prices at the balancing market are shown in Fig. 7.7(a) as functions of the day-ahead price.

It is worth to notice that $\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\}$ and $\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}$ are a decreasing and an increasing function of λ^{D}, respectively. According to (7.36), and due to the zero correlation between power production and day-ahead price, the optimal bidding curve for the producer is given by

$$E^{\mathrm{D}*}(\lambda^{\mathrm{D}}) = F_{\widetilde{E}}^{-1}\left(\frac{-1 + \frac{1}{10}\lambda^{\mathrm{D}}}{19 - \frac{1}{5}\lambda^{\mathrm{D}} - 1 + \frac{1}{10}\lambda^{\mathrm{D}}}\right) = F_{\widetilde{E}}^{-1}\left(\frac{-1 + \frac{1}{10}\lambda^{\mathrm{D}}}{18 - \frac{1}{10}\lambda^{\mathrm{D}}}\right). \tag{7.39}$$

The resulting optimal bidding curve is shown in Fig. 7.7(b). It is assumed that the support of the distribution of the day-ahead price $\widetilde{\lambda}^{\mathrm{D}}$ is included in the interval between \$20 /MWh and \$80/MWh.

We point out that, since the argument of the quantile function in (7.39) is an increasing function of the day-ahead price, it results that the optimal bidding curve is a valid offer (i.e., nondecreasing) at the day-ahead market.

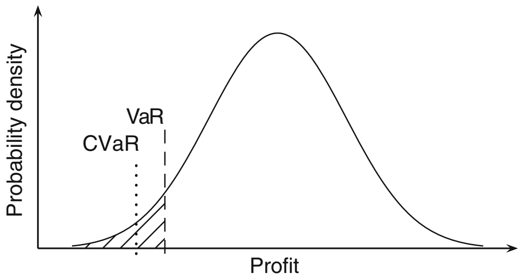

Fig. 7.8 Example of profit distribution and its relative VaR and CVaR

To conclude the example, we notice that, if the distribution of the day-ahead price $\widetilde{\lambda}^{\mathrm{D}}$ has mean \$50/MWh, the expected values $\widehat{\psi}^{\mathrm{UP}}$, $\widehat{\psi}^{\mathrm{DW}}$ of the up-regulation and down-regulation penalties are \$9/MWh and \$4/MWh, respectively, as in Example 7.3. The fixed quantity offer, though, is suboptimal in this case.

7.5 Modeling Risk-Aversion

When the power producer is not risk-neutral, a different objective than the maximization of the expected profit is sought. Generally speaking, a suitable objective function for a risk-averse power producer penalizes the lowest profit, i.e., the tail on the left-hand side of the profit distribution.

Two metrics widely used to quantify risk are the *Value at Risk (VaR)* and the *Conditional Value at Risk (CVaR)*. For a confidence level $0 \leq \alpha < 1$, $\mathrm{VaR}_{1-\alpha}$ is the $(1-\alpha)$-quantile of the profit. Denoting the (uncertain) profit with $\widetilde{\rho}$ and the support of its distribution with R, $\mathrm{VaR}_{1-\alpha}(\widetilde{\rho})$ is defined as

$$\mathrm{VaR}_{1-\alpha}(\widetilde{\rho}) = \max\left\{\rho \in \mathrm{R} : \mathrm{P}\left(\widetilde{\rho} < \rho\right) \leq 1-\alpha\right\}. \tag{7.40}$$

The definition of $\mathrm{CVaR}_{1-\alpha}$ is related to the previous definition [16]. For continuously distributed profits, it is the expected value of the profits that are lower than or equal to $\mathrm{VaR}_{1-\alpha}$:

$$\mathrm{CVaR}_{1-\alpha}(\widetilde{\rho}) = \mathbb{E}\left\{\widetilde{\rho} | \widetilde{\rho} \leq \mathrm{VaR}_{1-\alpha}(\widetilde{\rho})\right\} = \frac{1}{1-\alpha}\int_0^{\mathrm{VaR}_{1-\alpha}(\widetilde{\rho})} \rho p_{\widetilde{\rho}}(\rho) d\rho, \tag{7.41}$$

where $p_{\widetilde{\rho}}(\,\cdot\,)$ is the probability density function of the profit. According to this definition, CVaR often goes under the name of *expected shortfall*. Appendix C includes further details on these two risk measures and valid definitions in the case of discretely distributed profits.

In the recent years, CVaR has gained increasing attention, partly due to the fact that, differently from VaR, CVaR satisfies some properties that make it a *coherent* risk measure [1]. Figure 7.8 shows VaR and CVaR for an example of profit distribution. The dashed area in the illustration measures $1-\alpha$. VaR is the $(1-\alpha)$-quantile of the profit, while CVaR is the expected value of the profits falling below VaR.

An intuitive approach when defining a risk-averse strategy consists in seeking a compromise between the maximization of the expected profit and a term accounting for the chosen risk metric. Employing CVaR at the α confidence level, a suitable objective function z is

$$z = (1-k)\mathbb{E}\{\widetilde{\rho}\} + k \times \mathrm{CVaR}_{1-\alpha}(\widetilde{\rho}). \tag{7.42}$$

For $k = 0$, the objective function consists in the maximization of the expected profit, which corresponds to the risk-neutral case. For increasing values of k, the second term in the objective function weighs more and more, implying that the maximization of the worst outcomes has more and more importance. When $k = 1$, the decision is completely risk-averse. It is worth noticing that the objective defined in (7.42) depends on two parameters arbitrarily set by the decision-maker: α and k.

An alternative approach for a risk-averse decision-maker is the direct maximization of the CVaR, i.e.,

$$z = \mathrm{CVaR}_{1-\alpha}(\widetilde{\rho}). \tag{7.43}$$

Setting $\alpha = 0$ yields the risk-neutral case; increasing values of α represent situations with higher aversion to risk.

In the next section, we consider the short-term trading problem of a risk-averse stochastic power producer employing the objective function (7.43).

7.5.1 *Risk-Averse Strategy in a Two-Price Market*

A relevant application of the risk criteria described above in the trading problem for stochastic power producers is the case of a two-price settlement for the balancing market with deterministic penalties. In this case, the only risk is introduced by the uncertainty in power production.

We make the further assumption that market prices are always nonnegative. In this situation, it holds that the higher the realized power production, the higher the profit. If objective (7.43) is employed, the α fraction of highest returns corresponds to the α fraction of highest production, which is therefore discarded from the objective function. Taking into account the expression of the producer's returns (7.10), the expectation of the $1-\alpha$ lowest profits is given by

$$\begin{aligned} z = &\int_0^{E^{1-\alpha}} \lambda^{\mathrm{D}} E \frac{p_{\widetilde{E}}(E)}{1-\alpha} dE + \int_0^{E^{\mathrm{D}}} \psi^{\mathrm{UP}} \left(E - E^{\mathrm{D}}\right) \frac{p_{\widetilde{E}}(E)}{1-\alpha} dE \\ &- \int_{E^{\mathrm{D}}}^{E^{1-\alpha}} \psi^{\mathrm{DW}} \left(E - E^{\mathrm{D}}\right) \frac{p_{\widetilde{E}}(E)}{1-\alpha} dE, \end{aligned} \tag{7.44}$$

where $E^{1-\alpha}$ is the $(1-\alpha)$-quantile of the power production distribution, i.e., it satisfies $\mathrm{P}\{\widetilde{E} \leq E^{1-\alpha}\} = 1-\alpha$. Observe that in Eq. (7.44), it is assumed that the

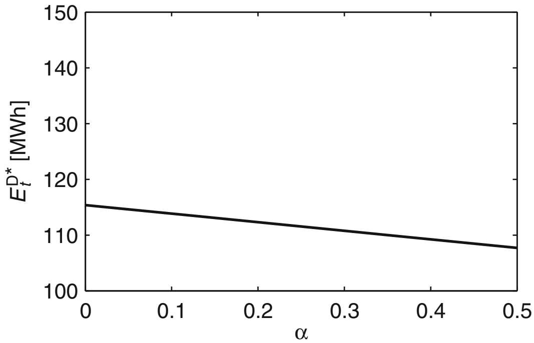

Fig. 7.9 Optimal day-ahead bid for a risk-averse power producer as a function of the risk-aversion parameter α

optimal value of the day-ahead offer E^{D} is not greater than the quantile $E^{1-\alpha}$, which makes sense for small values of α.

Proceeding in a similar way as in Sect. 7.3.2, the stationary point must satisfy

$$\frac{1}{1-\alpha}\psi^{\mathrm{UP}}F_{\widetilde{E}}(E^{\mathrm{D}}) + \frac{1}{1-\alpha}\psi^{\mathrm{DW}}\left(F_{\widetilde{E}}(E^{\mathrm{D}}) - (1-\alpha)\right) = 0, \tag{7.45}$$

which readily yields the optimal risk-averse bid

$$E^{\mathrm{D}*} = F_{\widetilde{E}}^{-1}\left\{(1-\alpha)\frac{\psi^{\mathrm{DW}}}{\psi^{\mathrm{UP}}+\psi^{\mathrm{DW}}}\right\}. \tag{7.46}$$

The result (7.46) is rather intuitive. Indeed, since the rightmost part of the power production distribution is discarded from the objective function (7.44), the risk-averse power producer is more concerned about negative (deficit) deviations from the day-ahead position. The coefficient $1-\alpha$ in the argument of the quantile function in (7.46) scales the optimal quantile, reducing the quantity placed at the day-ahead market and therefore decreasing the possibility and the size of power deficits at the balancing market.

Example 7.6 (Risk-averse offering strategy in a two-price market) We consider the same market penalties and the same distribution of power production as in Example 7.1. From (7.46) and the cdf (7.19) it follows that the optimal day-ahead bid for the risk-averse power producer is

$$E^{\mathrm{D}*} = 100 + 50 \times \frac{4}{13} \times (1-\alpha)\,\mathrm{MWh}. \tag{7.47}$$

Figure 7.9 shows the optimal quantity bid as a function of the risk-aversion parameter α. For $\alpha = 0$, the bid is the same as in the risk-neutral case, while it decreases for higher values of the parameter.

The expected value and the $\mathrm{CVaR}_{5\%}$ of the profit (i.e., the expectation of the 5% lowest profits), obtained with the day-ahead price $\lambda^{\mathrm{D}} = \$20$/MWh, are shown in Fig. 7.10. The expected profit in Fig. 7.10(a) decreases as α grows, signaling that risk-averse strategies achieve worse financial results in expectation than the

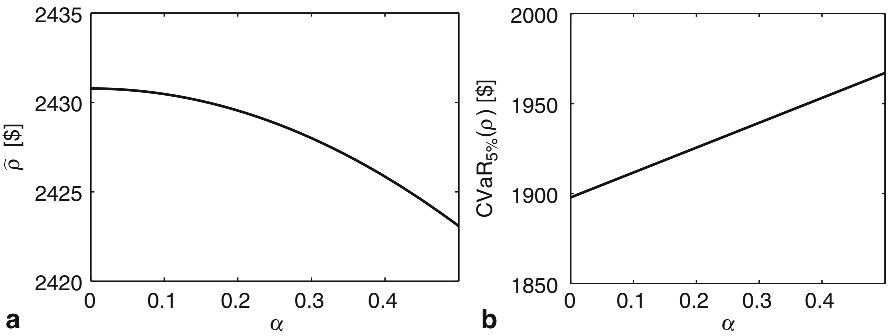

Fig. 7.10 Expected profit (**a**) and $\text{CVaR}_{5\%}$ of the profit (**b**) as functions of the risk-aversion parameter α

risk-neutral strategy. In turn, the $\text{CVaR}_{5\%}$ of the profit, shown in Fig. 7.10(b), increases, which implies that high values of α better hedge the power producer.

Notice that such a direct application of risk management to the case of a one-price market with deterministic prices is not particularly interesting. Indeed, a price-inelastic zero offer maximizes $\text{CVaR}_{1-\alpha}(\widetilde{\rho})$ for any α when $\lambda^{\text{B}} > \lambda^{\text{D}}$. Similarly, nominal capacity is still the optimal risk-averse offer if $\lambda^{\text{B}} < \lambda^{\text{D}}$ for any level of risk aversion.

The risk-averse strategy presented in this section, due to the use of deterministic prices, is only aimed at hedging the power generator from the uncertainty in power production. In practice, when imbalance penalties are stochastic, the risk stemming from both prices and production should be considered. The determination of the optimal risk-averse strategy would then involve double integrals with possibly joint probability distributions. Obviously, such calculations can be rather cumbersome. As we shall see in Sect. 7.6, stochastic programming can be used to account for risk in a simple and intuitive manner.

Before turning to the stochastic programming approach, we summarize the analytical results obtained so far in Table 7.1.

7.6 Bidding Strategies: Stochastic Programming Approach

The existence of an analytical solution to the short-term trading problem of a stochastic producer is limited to a number of simplified cases, all relying on at least one of the assumptions stated in Sect. 7.2 of this chapter. Such a solution is thus no longer available as soon as the dependence structure exhibited by market prices and production volume becomes more intricate and/or the trading problem is enriched with new elements and features.

In this section, we present an alternative approach to solving the trading problem of a stochastic producer. This approach is based on stochastic programming, which provides us with a powerful and flexible modeling framework to easily account for all the relevant factors in this problem. The stochastic programming approach starts from

Table 7.1 Summary of the analytical results

Case	Market	Optimal offer	Offering rule
Deterministic prices	1-price	$\begin{cases} 0 & \text{if} \quad \lambda^{\mathrm{B}} > \lambda^{\mathrm{D}} \\ \overline{E} & \text{if} \quad \lambda^{\mathrm{B}} < \lambda^{\mathrm{D}} \end{cases}$	fixed quantity to be offered at any day-ahead price λ^{D}
	2-price	$F_{\widetilde{E}}^{-1}\left(\frac{\psi^{\mathrm{DW}}}{\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}}}\right)$	
Stochastic prices, no penalties/day-ahead price correlation	1-price	$\begin{cases} 0 & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} < 0 \\ \overline{E} & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} > 0 \end{cases}$	fixed quantity to be offered at any day-ahead price λ^{D}
	2-price	$F_{\widetilde{E}}^{-1}\left(\frac{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}\right\}}{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}\right\} + \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}\right\}}\right)$	
Stochastic prices, nonzero penalties/day-ahead price correlation	1-price	$\begin{cases} 0 & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}} \mid \lambda^{\mathrm{D}}\right\} < 0 \\ \overline{E} & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}} \mid \lambda^{\mathrm{D}}\right\} > 0 \end{cases}$	price-quantity curve; valid only if nondecreasing
	2-price	$F_{\widetilde{E}}^{-1}\left(\frac{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}} \mid \lambda^{\mathrm{D}}\right\}}{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}} \mid \lambda^{\mathrm{D}}\right\} + \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}} \mid \lambda^{\mathrm{D}}\right\}}\right)$	
Deterministic prices, risk-averse	1-price	$\begin{cases} 0 & \text{if} \quad \lambda^{\mathrm{B}} > \lambda^{\mathrm{D}} \\ \overline{E} & \text{if} \quad \lambda^{\mathrm{B}} < \lambda^{\mathrm{D}} \end{cases}$	fixed quantity to be offered at any day-ahead price λ^{D}
	2-price	$F_{\widetilde{E}}^{-1}\left((1-\alpha)\frac{\psi^{\mathrm{DW}}}{\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}}}\right)$	

the premise that the uncertain parameters influencing the decision-making process faced by the stochastic producer can be efficiently approximated by a finite set Ω of plausible outcomes or *scenarios*.

For example, consider the random variable $\widetilde{E}$ describing the uncertain production in a future time period and let E_ω denote the realization of this random variable under scenario ω. The set $\{(E_\omega, \pi_\omega), \omega \in \Omega\}$, such that $\sum_{\omega \in \Omega} \pi_\omega = 1$ and $\pi_\omega \geq 0$ for all ω, is a discrete approximation of the probability distribution of $\widetilde{E}$, with π_ω being the probability of occurrence assigned to realization E_ω. Similarly, one might construct a scenario set $\{(E_\omega, \psi_\omega^{\mathrm{UP}}, \psi_\omega^{\mathrm{DW}}, \pi_\omega), \omega \in \Omega\}$ to model, if needed, the interdependence structure between the uncertain production volume $\widetilde{E}$ and the imbalance penalties $\widetilde{\psi}^{\mathrm{UP}}$ and $\widetilde{\psi}^{\mathrm{DW}}$. Be that as it may, the stochastic programming approach to the trading problem assumes that this scenario set is available. Chapter 2 provides the concepts and tools required to properly construct scenarios for the stochastic processes of interest within the scope of this book.

Next, we illustrate the stochastic programming approach to the offering problem of a stochastic producer using a small example. For a brief introduction to stochastic programming, we refer the interested reader to Appendix C.

Example 7.7 (Stochastic programming approach) Let us try to solve Example 7.1 using the concept of *scenario*. Recall that the imbalance penalties, ψ^{UP} and ψ^{DW}

are considered here deterministic and equal to \$9/MWh and \$4/MWh, respectively. Besides, the stochastic power production at trading period t, i.e., $\widetilde{E}$, is given by a uniform distribution between 100 MWh and 150 MWh.

The stochastic programming solution approach requires that this uniform distribution be approximated by N_Ω scenarios, i.e., $\widetilde{E} \approx \{(E_1, \pi_1), \ldots, (E_\omega, \pi_\omega), \ldots, (E_{N_\Omega}, \pi_{N_\Omega})\}$, with $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$ and $\pi_\omega \geq 0$. By means of this discretization, the offering problem of the stochastic producer can be cast as the linear programming problem (7.48), which can be readily processed by optimization solvers.

$$\text{Min.} \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(-\psi^{\text{UP}} E_\omega^{\text{UP}} + \psi^{\text{DW}} E_\omega^{\text{DW}}\right) \tag{7.48a}$$

$$\text{s.t. } 0 \leq E^{\text{D}} \leq \overline{E}, \tag{7.48b}$$

$$E_\omega - E^{\text{D}} = E_\omega^{\text{UP}} + E_\omega^{\text{DW}}, \forall \omega, \tag{7.48c}$$

$$E_\omega^{\text{UP}} \leq 0, \quad E_\omega^{\text{DW}} \geq 0, \quad \forall \omega. \tag{7.48d}$$

Variables E^{D}, E_ω^{UP}, and E_ω^{DW}, for all ω, are the decisions to be optimized. In stochastic programming terminology, the amount of energy sold by the stochastic producer in the day-ahead market, E^{D}, is referred to as a *here-and-now decision variable*, because it must be decided before knowing the eventual realization of the stochastic production $\widetilde{E}$. Consequently, E^{D} is independent of the scenario index ω. On the other hand, the amounts of up-regulation and down-regulation energy acquired by the stochastic producer in the balancing market, i.e., E_ω^{UP} and E_ω^{DW} are called *wait-and-see decision variables*, because they are decided after knowing the specific realization E_ω of the stochastic production $\widetilde{E}$. Therefore, E_ω^{UP} and E_ω^{DW} do depend on the scenario index ω.

It should be noticed that the objective function (7.48a) is the equivalent scenario-based formulation of the expected opportunity loss as expressed in (7.14). Analogously, Eq. (7.48c) is the scenario-based definition of the imbalance of the stochastic producer, also stated in (7.9) in a more general form.

Needless to say, the solution to the stochastic programming problem (7.48) is strongly dependent on the scenario set that we use to approximate the uniformly distributed power output $\widetilde{E}$. In a first attempt, we can just consider a set of one single scenario consisting in the expected power production, i.e., $\widehat{E} = \mathbb{E}\{\widetilde{E}\} = (100 + 150)/2 = 125$ MWh, with a probability of 1. In such a case, problem (7.48) becomes

$$\text{Min.} \; -9E_1^{\text{UP}} + 4E_1^{\text{DW}} \tag{7.49a}$$

$$\text{s.t. } 0 \leq E^{\text{D}} \leq 150, \tag{7.49b}$$

$$125 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.49c}$$

$$E_1^{\text{UP}} \leq 0, \quad E_1^{\text{DW}} \geq 0, \tag{7.49d}$$

whose solution is trivial. Indeed, the minimum of (7.49) is attained at $E^{\text{D}^*} = 125$ MWh, which zeroes the objective function (7.49a). However, we know from

Example 7.1 that the actual optimal strategy is to offer 115.38 MWh in the day-ahead market. Obviously, the difference is caused by the scenario-based representation of the stochastic power production.

In order to enhance the accuracy of the stochastic programming solution approach, we can clearly build a better scenario set. For instance, we can approximate the uncertain power output $\widetilde{E}$ this time using a two-scenario model that contains the two extremes of the associated uniform distribution with the same probability, i.e., $\widetilde{E} \approx \{(E_1, \pi_1), (E_2, \pi_2)\} = \{(100\text{ MWh}, 0.5), (150\text{ MWh}, 0.5)\}$. Thus the stochastic programming problem (7.48) becomes

$$\text{Min. } 0.5\left(-9E_1^{\text{UP}} + 4E_1^{\text{DW}}\right) + 0.5\left(-9E_2^{\text{UP}} + 4E_2^{\text{DW}}\right) \tag{7.50a}$$

$$\text{s.t. } 0 \le E^{\text{D}} \le 150, \tag{7.50b}$$

$$100 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.50c}$$

$$150 - E^{\text{D}} = E_2^{\text{UP}} + E_2^{\text{DW}}, \tag{7.50d}$$

$$E_1^{\text{UP}}, E_2^{\text{UP}} \le 0, \quad E_1^{\text{DW}}, E_2^{\text{DW}} \ge 0, \tag{7.50e}$$

which results in $E^{\text{D}*} = 100$ MWh, still far from the actual optimal bid of 115.38 MWh. We can further increase the size of the scenario set by adding the expected power production $\widehat{E}$ to the previous two-scenario model. That is, we approximate $\widetilde{E}$ by $\{(100\text{ MWh}, 1/3), (125\text{ MWh}, 1/3), (150\text{ MWh}, 1/3)\}$, which leads to the following stochastic programming problem:

$$\text{Min. } \frac{1}{3}\left(-9E_1^{\text{UP}} + 4E_1^{\text{DW}}\right) + \frac{1}{3}\left(-9E_2^{\text{UP}} + 4E_2^{\text{DW}}\right) + \frac{1}{3}\left(-9E_3^{\text{UP}} + 4E_3^{\text{DW}}\right) \tag{7.51a}$$

$$\text{s.t. } 0 \le E^{\text{D}} \le 150, \tag{7.51b}$$

$$100 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.51c}$$

$$125 - E^{\text{D}} = E_2^{\text{UP}} + E_2^{\text{DW}}, \tag{7.51d}$$

$$150 - E^{\text{D}} = E_3^{\text{UP}} + E_3^{\text{DW}}, \tag{7.51e}$$

$$E_1^{\text{UP}}, E_2^{\text{UP}}, E_3^{\text{UP}} \le 0, \quad E_1^{\text{DW}}, E_2^{\text{DW}}, E_3^{\text{DW}} \ge 0. \tag{7.51f}$$

However, problem (7.51) also yields $E^{\text{D}*} = 100$ MWh.

Finally, we construct a set of $N+1$ equiprobable scenarios uniformly spaced over the interval [100, 150] MWh, i.e., we model the stochastic production $\widetilde{E}$ by $\{(E_1, \pi_1), \ldots, (E_\omega, \pi_\omega), \ldots, (E_{N+1}, \pi_{N+1})\}$, where $E_\omega = 100 + (\omega - 1)(150 - 100)/N$ and $\pi_\omega = 1/(N+1)$, for all $\omega = 1, \ldots, N+1$. Figure 7.11 illustrates the optimal offer $E^{\text{D}*}$ given by the stochastic programming problem (7.48) as a function of N. Observe that as the size of the scenario set increases, the stochastic programming solution converges to the optimal bid that was obtained analytically in Example 7.1, namely 115.38 MWh.

In short, the reliance of the stochastic programming solution approach on the scenario set used to model the uncertain parameters is both its greatest virtue and its

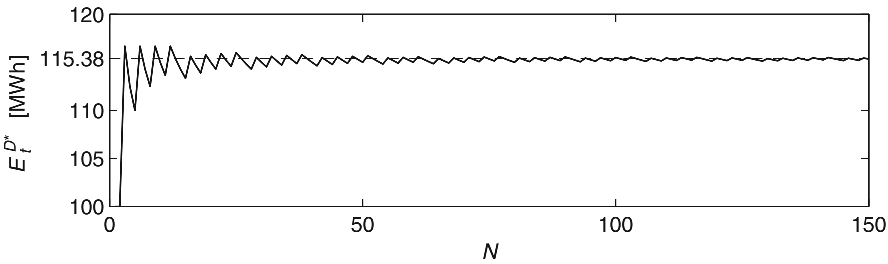

Fig. 7.11 Energy offer $E^{\mathrm{D}*}$ (in megawatt-hour) obtained from the stochastic linear programming problem (7.48) as a function of the size of the scenario set that approximates the uniformly distributed power production $\widetilde{E}$. Note that this offer approaches the analytical solution (115.38 MWh) as N increases

Achilles' heel. On the one hand, the scenario-based formulation of the trading problem allows us to determine the optimal offering strategy of the stochastic producer by means of an equivalent (deterministic) optimization problem that can be directly tackled by conventional optimization solvers. On the other, the actual value of the solution provided by this optimization problem becomes strongly contingent on the quality of the scenario set. This may render the stochastic solution approach computationally prohibitive if, for example, the quality of the scenario set is conditional on its size.

What is certain, though, is that the stochastic programming solution approach allows us to easily consider other facets of the trading problem. For instance, it is straightforward to determine risk-averse offering strategies using the conditional value at risk within a stochastic programming framework. In the following, we build on Example 7.7 to illustrate how to manage risk in the trading problem of a stochastic producer using stochastic programming.

Example 7.8 (Risk management via stochastic programming) Problem (7.48) in Example 7.7 determines the optimal offering strategy of a risk-neutral stochastic producer. We can now extend this problem to account for risk-averse behavior as follows:

$$\text{Max.}\ (1-k)\sum_{\omega\in\Omega}\pi_\omega\rho_\omega + k\left(\zeta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta_\omega\right) \tag{7.52a}$$

$$\text{s.t.}\ 0 \le E^{\mathrm{D}} \le \overline{E}, \tag{7.52b}$$

$$\rho_\omega = \lambda^{\mathrm{D}}E_\omega - \left(-\psi^{\mathrm{UP}}E_\omega^{\mathrm{UP}} + \psi^{\mathrm{DW}}E_\omega^{\mathrm{DW}}\right)\forall\omega, \tag{7.52c}$$

$$E_\omega - E^{\mathrm{D}} = E_\omega^{\mathrm{UP}} + E_\omega^{\mathrm{DW}}, \quad \forall\omega, \tag{7.52d}$$

$$\zeta - \rho_\omega \le \eta_\omega, \quad \forall\omega, \tag{7.52e}$$

$$E_\omega^{\mathrm{UP}} \le 0, \ \ E_\omega^{\mathrm{DW}} \ge 0, \ \ \eta_\omega \ge 0 \quad \forall\omega, \tag{7.52f}$$

where, for simplicity, we have omitted the time subscript t. Note that, as opposed to (7.48a), the new objective function (7.52a) is formulated in terms of profits, not opportunity costs. Accordingly, the newly defined variable ρ_ω represents the profit made by the stochastic producer in scenario ω, as stated by (7.52c), where variable ρ_ω is computed scenario-wise as the profit with perfect information minus the opportunity cost. At the optimum, the term $\left(\zeta^* - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta^*_\omega\right)$ in the objective function coincides with the conditional value at risk at confidence level α ($\mathrm{CVaR}_{1-\alpha}$), which here represents the average value of the $1-\alpha$ cases with lowest profits. Variables ζ and η_ω are auxiliary. For further details on how to model the conditional value at risk within a stochastic programming optimization problem, we refer the reader to Appendix C.

Note that the objective function (7.52a) establishes the tradeoff between the expected value and the conditional value at risk of the profit distribution. This tradeoff is resolved by means of the user-defined constant $k \in [0, 1]$, which is usually referred to as *risk-aversion parameter*. The higher the value of k, the more risk averse the stochastic producer is, as it becomes more concerned with the maximization of the lowest profits. As seen later, the multi-objective form of (7.52a) permits us to introduce the concept of *efficient frontier* in a very intuitive manner.

Now we assume the same imbalance penalties and the same distribution of power production as in Example 7.7. Likewise, we approximate such a distribution using a set of 1000 scenarios, built as explained in that example. We consider the conditional value at risk at a confidence level α of 95%, i.e., $\mathrm{CVaR}_{5\%}$, which accounts for the 5% of scenarios with lowest profits. The day-ahead market price λ^{D} is assumed to be equal to \$20/MWh.

Figure 7.12(a) shows the optimal energy offer $E^{\mathrm{D}*}$ in the day-ahead market as a function of the risk-aversion parameter k. Observe that higher values of k lead to lower values of E^{D}. Under risk aversion, poorer outcomes are weighted more than good outcomes. In the trading problem of a stochastic producer with positive prices, the cases with the lowest profits correspond to the scenarios in which the stochastic producer is short and therefore, must purchase its generation deficit in the balancing market. Figure 7.12(b) represents the so-called *efficient frontier*, which is made up of the optimal pairs ($\mathrm{CVaR}_{5\%}, \widehat{\rho}$) for different degrees of risk aversion. True to form, the stochastic producer can reduce its risk exposure—by increasing k in problem (7.52)—at the expense of decreasing the expected monetary value of its optimal sale offer in the day-ahead market.

The stochastic programming approach to the trading problem of a stochastic producer becomes particularly attractive in those instances for which an analytical solution is not available. This occurs, for example, in the case where the stochastic producer has the opportunity to participate in one or several adjustment markets. Generally speaking, these markets allow consumers and producers to adapt their forward consumption or production schedule to unplanned eventualities such as equipment failures, technical constraints, or sudden changes in load. For this reason, adjustment markets are placed in between the clearing of the day-ahead and balancing markets. Since the magnitude of the forecast error of stochastic production is usually strongly dependent on the lead time—the forecasts of wind/solar power issued

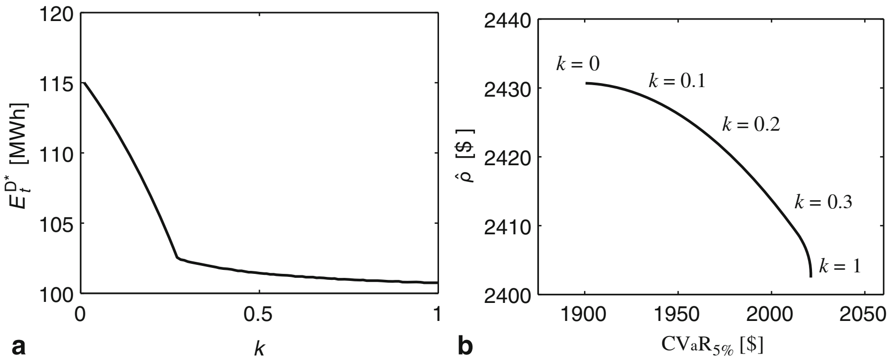

Fig. 7.12 The optimal amount of energy to be sold in the day-ahead market by the stochastic producer decreases as the degree of risk aversion increases (**a**). Offering strategies aimed at increasing the profit associated with the least favorable production outcomes are possible at the expense of decreasing the expected profit (**b**)

one hour ahead tend to be much more accurate than the forecast issued, e.g., 40 h ahead—stochastic producers may largely benefit from adjustment markets as they can trade in these markets with a lower degree of uncertainty on their eventual power production. In practice, this means that the future stochastic production is known better in the adjustments markets than in the day-ahead market. In the following example, we illustrate how to deal with an adjustment market in the trading problem of a stochastic producer by means of stochastic programming.

Example 7.9 (Adjustment market) Let us consider a stochastic producer that participates in an electricity market with the structure and time framework represented in Fig. 7.13. The day-ahead market is cleared at time t^{D}. After the closure of the day-ahead market, bidding in the adjustment market is allowed until Δt time units prior to the energy delivery period t. In the balancing market, the energy deviations incurred by the stochastic producer during the delivery period t are determined with respect to the dispatch program agreed in the day-ahead and adjustment markets, and priced accordingly.

Suppose that the day-ahead and adjustment market prices, i.e., λ^{D} and λ^{A}, are equal to \$20/MWh and \$19/MWh, respectively. Besides, the imbalance penalties, ψ^{UP} and ψ^{DW}, are \$9/MWh and \$4/MWh in that order, which means that the balancing market prices for upward and downward regulation, i.e., λ^{UP} and λ^{DW}, are given by $\lambda^{\mathrm{UP}} = \lambda^{\mathrm{D}} + \psi^{\mathrm{UP}} = \$29/\mathrm{MWh}$ and $\lambda^{\mathrm{DW}} = \lambda^{\mathrm{D}} - \psi^{\mathrm{DW}} = \$16/\mathrm{MWh}$.

The stochastic producer owns a 100-MW wind farm whose power output in time period t can be described as follows:

- The amount of energy $E_{t-\Delta t}$ produced by the wind farm in time period $t - \Delta t$ may be *high* (60 MWh) with a probability of 0.4 or low (30 MWh) with a probability of 0.6.

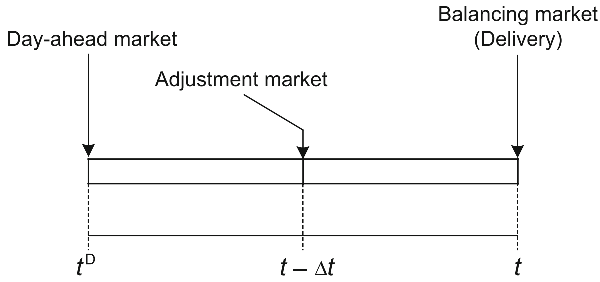

Fig. 7.13 Market organization and time framework including day-ahead, adjustment and balancing markets

- If the wind power production $E_{t-\Delta t}$ is *high* (60 MWh), the amount of energy E produced during the delivery period t may be *extremely high* (100 MWh) with a probability of 0.75 or *relatively high* (50 MWh) with a probability of 0.25.
- If the wind power production $E_{t-\Delta t}$ is *low* (30 MWh), the amount of energy E produced during the delivery period t may be *extremely low* (0 MWh) or *relatively low* (40 MWh), both with a probability of 0.5.

The sequence of stages and decisions that the stochastic producer has to face is described below.

1. Decide the amount of energy E^{D} to be sold in the day-ahead market with inaccurate information on the eventual power output of its wind farm in the delivery period t.
2. Decide the amount of energy E^{A} to be traded in the adjustment market. At this stage, the energy produced by its wind farm in time period $t-\Delta t$ is known and this information improves the stochastic producer's knowledge of its power production in the delivery period t.
3. Lastly, once the wind power production in the delivery period t becomes known, the stochastic producer must cover the amount of energy deviating from that scheduled in the day-ahead and adjustment markets by selling or purchasing its generation surplus or shortage, respectively, in the balancing market.

The decision-making process faced by the stochastic producer can be represented in the form of a tree, as depicted in Fig. 7.14. Each scenario ω in the tree is characterized by a certain wind power outcome in time periods $t-\Delta t$ and t, i.e., $E_{t-\Delta t\omega}$ and E_{ω}, with π_{ω} being its probability of occurrence. The scenario tree is made up of three sets of nodes corresponding to the three stages of the stochastic producer decision-making process. Each stage represents the trading in a different market.

For comparison purposes, let us first suppose that the stochastic producer ignores the adjustment market. In this case, its optimal energy offer in the day-ahead market is obtained as the solution to the opportunity loss minimization problem (7.48) stated in Example 7.7, that is

$$\text{Min. } 0.3\left(-9E_1^{\mathrm{UP}}+4E_1^{\mathrm{DW}}\right)+0.1\left(-9E_2^{\mathrm{UP}}+4E_2^{\mathrm{DW}}\right)+ \tag{7.53a}$$

$$0.3\left(-9E_3^{\mathrm{UP}}+4E_3^{\mathrm{DW}}\right)+0.3\left(-9E_4^{\mathrm{UP}}+4E_4^{\mathrm{DW}}\right) \tag{7.53b}$$

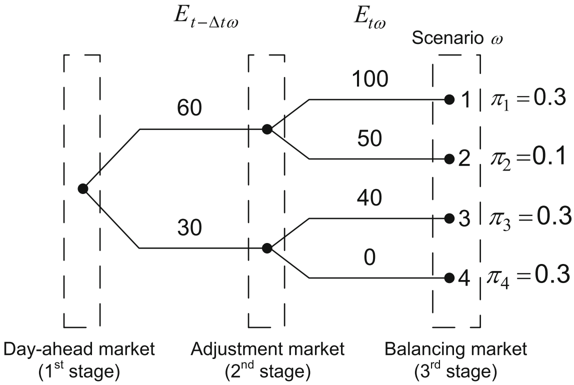

Fig. 7.14 Scenario tree describing the three-stage decision-making process faced by the stochastic producer. The nodes represent points in time where trading decisions are to be made. The branches represent the realization of wind power

$$\text{s.t. } 0 \leq E^{\mathrm{D}} \leq 150, \tag{7.53c}$$

$$100 - E^{\mathrm{D}} = E_1^{\mathrm{UP}} + E_1^{\mathrm{DW}}, \tag{7.53d}$$

$$50 - E^{\mathrm{D}} = E_2^{\mathrm{UP}} + E_2^{\mathrm{DW}}, \tag{7.53e}$$

$$40 - E^{\mathrm{D}} = E_3^{\mathrm{UP}} + E_3^{\mathrm{DW}}, \tag{7.53f}$$

$$0 - E^{\mathrm{D}} = E_4^{\mathrm{UP}} + E_4^{\mathrm{DW}}, \tag{7.53g}$$

$$E_1^{\mathrm{UP}}, E_2^{\mathrm{UP}}, E_3^{\mathrm{UP}}, E_4^{\mathrm{UP}} \leq 0, \quad E_1^{\mathrm{DW}}, E_2^{\mathrm{DW}}, E_3^{\mathrm{DW}}, E_4^{\mathrm{DW}} \geq 0, \tag{7.53h}$$

which yields $E^{\mathrm{D}*} = 40$ MWh. The expected opportunity loss (EOL) associated with this bid is \$184. Therefore, the expected profit $\widehat{\rho}$ made by the stochastic producer can be calculated as $\widehat{\rho} = \lambda^{\mathrm{D}} \sum_{\omega=1}^{4} \pi_\omega E_\omega - \mathrm{EOL} = \$20/\mathrm{MWh} \times 47\mathrm{MWh} - \$184 =$ \$756.

Let us now consider that the stochastic producer trades in the adjustment market as well. In this case, the best strategy the stochastic producer can adopt is given by the following expected profit maximization problem,

$$\text{Max. } \lambda^{\mathrm{D}} E^{\mathrm{D}} + \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(\lambda^{\mathrm{A}} E_\omega^{\mathrm{A}} + \lambda^{\mathrm{UP}} E_\omega^{\mathrm{UP}} + \lambda^{\mathrm{DW}} E_\omega^{\mathrm{DW}} \right) \tag{7.54a}$$

$$\text{s.t. } 0 \leq E^{\mathrm{D}} \leq \overline{E}, \tag{7.54b}$$

$$0 \leq E^{\mathrm{D}} + E_\omega^{\mathrm{A}} \leq \overline{E}, \quad \forall \omega, \tag{7.54c}$$

$$E_\omega - E^{\mathrm{D}} - E_\omega^{\mathrm{A}} = E_\omega^{\mathrm{UP}} + E_\omega^{\mathrm{DW}}, \forall \omega, \tag{7.54d}$$

$$E_1^{\mathrm{A}} = E_2^{\mathrm{A}}, \quad E_3^{\mathrm{A}} = E_4^{\mathrm{A}}, \tag{7.54e}$$

$$E_\omega^{\mathrm{UP}} \leq 0, \quad E_\omega^{\mathrm{DW}} \geq 0, \forall \omega, \tag{7.54f}$$

where E^{A}_{ω} is the amount of energy sold, if positive, or purchased, if negative, in the adjustment market by the stochastic producer. The objective function (7.54a) to be maximized is the expected profit, which includes three terms: (i) the profit made in the day-ahead market, $\lambda^{\mathrm{D}} E^{\mathrm{D}}$; (ii) the expected profit obtained in the adjustment market, $\lambda^{\mathrm{A}} \sum_{\omega=1}^{N\Omega} \pi_{\omega} E^{\mathrm{A}}{}_{\omega}$; and the expected profit made in the balancing market, $\sum_{\omega=1}^{N\Omega} \pi_{\omega} \left(\lambda^{\mathrm{UP}} E^{\mathrm{UP}}_{\omega} + \lambda^{\mathrm{DW}} E^{\mathrm{DW}}_{\omega}\right)$. Note that the last two terms referring to the adjustment and balancing markets may actually represent a cost in those scenarios where the stochastic producer purchases electricity from these markets. Logically, the energy deviations incurred by the stochastic producer are to be computed here with respect to the forward production program resulting from the day-ahead and adjustment markets, as stated by the set of constraints (7.54d). Equations (7.54e) enforce the nonanticipatory character of the information, which requires the amount of energy traded in the adjustment market be unique irrespective of the wind production outcome in the future delivery period t. However, the energy offer in the adjustment market may be indeed conditional on the wind power produced in time period $t - \Delta t$. In stochastic programming, constraints of the nature of (7.54e) are usually called *nonanticipativity constraints*.

By replacing parameters in optimization problem (7.54) with their actual values, we end up with the following optimization problem,

$$
\begin{aligned}
\text{Max.}\ & 20E^{\mathrm{D}} + 0.3\left(19E^{\mathrm{A}}_1 + 29E^{\mathrm{UP}}_1 + 16E^{\mathrm{DW}}_1\right) + \\
& 0.1\left(19E^{\mathrm{A}}_2 + 29E^{\mathrm{UP}}_2 + 16E^{\mathrm{DW}}_2\right) + 0.3\left(19E^{\mathrm{A}}_3 + 29E^{\mathrm{UP}}_3 + 16E^{\mathrm{DW}}_3\right) \qquad (7.55a)\\
& 0.3\left(19E^{\mathrm{A}}_4 + 29E^{\mathrm{UP}}_4 + 16E^{\mathrm{DW}}_4\right)\\
\text{s.t.}\ & 0 \le E^{\mathrm{D}} \le 100, \qquad (7.55b)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_1 \le 100, \qquad (7.55c)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_2 \le 100, \qquad (7.55d)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_3 \le 100, \qquad (7.55e)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_4 \le 100, \qquad (7.55f)\\
& 100 - E^{\mathrm{D}} - E^{\mathrm{A}}_1 = E^{\mathrm{UP}}_1 + E^{\mathrm{DW}}_1, \qquad (7.55g)\\
& 50 - E^{\mathrm{D}} - E^{\mathrm{A}}_2 = E^{\mathrm{UP}}_2 + E^{\mathrm{DW}}_2, \qquad (7.55h)\\
& 40 - E^{\mathrm{D}} - E^{\mathrm{A}}_3 = E^{\mathrm{UP}}_3 + E^{\mathrm{DW}}_3, \qquad (7.55i)\\
& 0 - E^{\mathrm{D}} - E^{\mathrm{A}}_4 = E^{\mathrm{UP}}_4 + E^{\mathrm{DW}}_4, \qquad (7.55j)\\
& E^{\mathrm{A}}_1 = E^{\mathrm{A}}_2,\ \ E^{\mathrm{A}}_3 = E^{\mathrm{A}}_4, \qquad (7.55k)\\
& E^{\mathrm{UP}}_1, E^{\mathrm{UP}}_2, E^{\mathrm{UP}}_3, E^{\mathrm{UP}}_4 \le 0,\ \ E^{\mathrm{DW}}_1, E^{\mathrm{DW}}_2, E^{\mathrm{DW}}_3, E^{\mathrm{DW}}_4 \ge 0, \qquad (7.55l)
\end{aligned}
$$

which results in $E^{\mathrm{D}^*} = 100$ MWh, $E^{\mathrm{A}^*}_1 = E^{\mathrm{A}^*}_2 = -50$ MWh, $E^{\mathrm{A}^*}_3 = E^{\mathrm{A}^*}_4 = -100$ MWh, and $\widehat{\rho} = \$912$. This trading strategy, therefore, entails an increase in the expected profit equal to \$912 − \$756 = \$156—a 20.64% increment in relative terms—with respect to the expected profit associated with the offering strategy derived from problem (7.53), which ignores the adjustment market. This increase is partly due to the fact that the future wind power production in the delivery period t is known with higher accuracy in the adjustment market. Indeed, from the day-ahead market, the stochastic producer foresees a future energy generation in period t ranging from 0 to 100 MWh. In contrast, from the adjustment market, the stochastic producer forecasts an energy production varying either from 50 to 100 MWh or from 0 to 40 MWh depending on whether the wind energy produced in time period $t - \Delta t$, i.e., $E_{t-\Delta t}$, is *high* (60 MWh) or *low* (30 MWh). This way, if $E_{t-\Delta t}$ is *high*, the stochastic producer purchases 50 MWh in the adjustment market, while if it is *low*, its energy purchase in this market is increased up to 100 MWh.

In short, the fact that the clearing of the adjustment market is closer in time to the energy delivery period enhances the profitability of the stochastic producer. Indeed, if the pair of constraints (7.54e) is replaced with the single equation $E_1^{\mathrm{A}} = E_2^{\mathrm{A}} = E_3^{\mathrm{A}} = E_4^{\mathrm{A}}$, in order to disregard this effect, the expected profit made by the stochastic producer drops to \$852, which represents a 6.6% decrease with regard to the expected profit resulting from problem (7.54).

In reality, the electricity prices in the day-ahead and adjustment markets are also uncertain. Furthermore, since the trading volume in adjustments markets is generally lower than in the day-ahead market, the electricity price in adjustment markets is often more volatile. As a result, the stochastic producer must face a tradeoff between selling in the day-ahead market at less volatile prices and trading in the adjustment market with a reduced level of uncertainty about its future energy production. This tradeoff can be resolved by means of the stochastic programming solution approach presented in this section.

7.7 Summary and Conclusions

Renewable energy producers are increasingly required to participate in electricity markets under the same rules as conventional power producers. However, the competitive sale of renewable energy by stochastic producers is cursed with the weather dependency of the underlying energy source, e.g., sunlight or wind. Stochastic producers are thus *forced* to channel part of their business into the balancing market, where they can take part with perfect knowledge of their production.

Compared to the electricity prices in markets with early gate closures, such as day-ahead and adjustment markets, balancing market prices are generally less predictable and/or less competitive. Consequently, as the stochastic producer becomes more and more dependent on the balancing market, its profitability may rapidly deteriorate. Therefore, the stochastic producer must smartly decide its involvement in day-ahead, adjustment, and balancing markets according to its best guess on market prices and future power production.

The trading problem of a stochastic producer is described in this chapter as a multi-stage decision-making problem under uncertainty. We identify the assumptions that render this problem analytically solvable and provide the corresponding exact solutions. For those cases in which these assumptions prove to be unrealistic, simplistic or too constraining, a more general modeling approach based on stochastic programming and the concept of scenarios is introduced to determine the best trading strategy for the stochastic producer.

7.8 Further Reading

For a basic introduction to statistical decision theory, the reader is referred to [14]. From the same author, [15] offers a more complete treatment of the subject, including the *newsvendor problem*, which is the general formulation of the power trading problem.

Further reading on analytical results on the trading problem for stochastic power producers consists mainly of research articles focusing on the case of wind power producers. The optimal strategy based on the offer of a quantile of the wind power distribution is proposed and tested in [4; 12; 17]. Further analytical results including and extending part of the results presented in this chapter can be found in [3; 6]. Finally, [8] provides a general discussion, including an application to electricity markets, on the relationship between loss functions and the optimal forecasts.

On the other hand, the short-term trading problem of a wind power producer is tackled in [10; 11; 13] using stochastic programming. Reference [5] also describes a number of trading models for retailers, consumers and producers (including nondispatchable agents) that are built upon stochastic programming, and provides insight into risk management and the scenario-based modeling of the uncertainties affecting trading decisions.

The trading problem for price-maker producers employing renewable sources with variable and stochastic nature is considered in [2; 18], making use of stochastic Mathematical Programs with Equilibrium Constraints.

Exercises

7.1 A forecaster predicts that the energy production $\widetilde{E}$ of a wind farm during a given trading period t is characterized by the following probability density function $p_{\widetilde{E}}(\cdot)$:

$$p_{\widetilde{E}}(E) = \begin{cases} \dfrac{E-100}{625}, & 100 \leq E < 125, \\ \\ \dfrac{150-E}{625}, & 125 \leq E < 150, \\ 0, & \text{elsewhere.} \end{cases} \tag{7.56}$$

1. Determine the cumulative distribution function $F_{\widetilde{E}}(\cdot)$ for wind power production.
2. Assume that the imbalance penalties in a two-price market are $\psi^{\mathrm{UP}} = \$9$ /MWh and $\psi^{\mathrm{DW}} = \$4$/MWh. Determine the optimal offer and the resulting expectation of the imbalance cost.
3. Now determine the optimal offer and the resulting expectation of the imbalance cost assuming that the imbalance penalties are $\psi^{\mathrm{UP}} = \$4$/MWh and $\psi^{\mathrm{DW}} = \$9$/MWh.

7.2 Consider the definition of $\widetilde{u}$ in Example 7.2. Let us assume that the probability density function for $\widetilde{u}$ is

$$p_{\widetilde{u}}(u) = \begin{cases} \dfrac{u+20}{625}, & -20 \leq u < 5, \\ \dfrac{30-u}{625}, & 5 \leq u < 30, \\ 0, & \text{elsewhere.} \end{cases} \tag{7.57}$$

1. Determine the expected value of the balancing price in a one-price market. If the forecast for wind power production is the same as in Exercise 7.1, what is the optimal offer?
2. Determine the expected values of the imbalance penalties $\widehat{\psi}^{\mathrm{UP}}$ and $\widehat{\psi}^{\mathrm{DW}}$ in a two-price market. If the forecast for wind power production is the same as in Exercise 7.1, what is the optimal offer?

7.3 In a two-price market, the expectation of the imbalance penalties, conditional on the day-ahead price, is given by the following.

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\} = 10 - \frac{1}{10}\lambda^{\mathrm{D}}, \tag{7.58}$$

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\} = \frac{1}{8}\lambda^{\mathrm{D}}. \tag{7.59}$$

Let us consider a uniform distribution for the stochastic production with lower and upper bounds equal to 100 MWh and 150 MWh, respectively. Determine the optimal offering curve, assuming that the day-ahead price is nonnegative and can take values up to \$100/MWh.

7.4 Consider the distribution of wind power production in Exercise 7.1, and the deterministic penalties $\psi^{\mathrm{UP}} = \$9$/MWh and $\psi^{\mathrm{DW}} = \$4$/MWh. Determine the risk-averse bid with parameter $\alpha = 0.2$. Assuming a day-ahead price $\lambda^{\mathrm{D}} = \$50$/MWh, determine the expected value of the profit, the *value at risk* ($\mathrm{VaR}_{5\%}$) and the *conditional value at risk* ($\mathrm{CVaR}_{5\%}$) of the profit.

7.5 Suppose that the energy production of a certain wind farm in a given trading period t can be modeled by a uniform distribution between 0 MWh and 50 MWh.

1. Construct a set of four equiprobable and uniformly spaced scenarios that approximates this uniform distribution.
2. Based on this scenario set and knowing that the imbalance penalties, ψ^{UP} and ψ^{DW}, are deterministic and equal to \$3/MWh and \$6/MWh, respectively, for-

mulate and solve a stochastic programming model to calculate the energy offer in the day-ahead market that minimizes the expected imbalance cost of the wind farm.

7.6 Reformulate the stochastic programming model of the previous exercise to account for the risk aversion of the wind power producer using the Conditional Value-at-Risk of its profit distribution at a confidence level of 99%. Then, obtain the optimal bid of the wind power producer in the day-ahead market as a function of a risk-aversion parameter and draw the resulting efficient frontier.

7.7 Consider a solar power producer that participates in an electricity market with the structure and time framework depicted in Fig. 7.13 of Example 7.9. The solar producer owns a 50-MW photovoltaic power plant whose energy output in time period t is stochastic and can be described as follows:

- The amount of energy $E_{t-\Delta t}$ produced by the solar power plant in time period $t - \Delta t$ may be *high* (45 MWh) with a probability of 0.7 or low (5 MWh) with a probability of 0.3.
- If the solar power production $E_{t-\Delta t}$ is *high* (45 MWh), the amount of energy E produced during the delivery period t may be *extremely high* (50 MWh) with a probability of 0.35 or *relatively high* (41 MWh) with a probability of 0.65.
- If the solar power production $E_{t-\Delta t}$ is *low* (5 MWh), the amount of energy E produced during the delivery period t may be *extremely low* (0 MWh) with a probability of 0.70 or *relatively low* (10 MWh) with a probability of 0.30.

Furthermore, the day-ahead and adjustment market prices, i.e., λ^{D} and λ^{A}, are known to be equal to \$45/MWh and \$46/MWh, respectively, and the imbalance penalties, ψ^{UP} and ψ^{DW}, equal to \$10/MWh and \$8/MWh, in that order.

1. Construct a scenario tree similar to that in Fig. 7.14 of Example 7.9 to describe the decision-making process faced by the solar power producer.
2. Compute the optimal bid of the solar power producer in the day-ahead market if the adjustment market is disregarded.
3. Compute the optimal bid of the solar power producer in the day-ahead market if the adjustment market is taken into account and calculate how much the expected profit of the solar power producer increases with respect to the previous case. Determine also how much of this increase is due to the fact that the future energy production of the photovoltaic power plant in period t is known with higher accuracy in the adjustment market than in the day-ahead market.

7.8 Consider a 60-MW solar power plant whose power output in a given trading period t can be either 35 and 60 MW, with probabilities 0.6 and 0.4, respectively. It is known that:

- The price in the day-ahead market for period t can be either \$25/MWh or \$50/MWh, with probabilities 0.3 and 0.7, in that order.
- If the day-ahead market price is equal to \$25/MWh, the imbalance penalties for upward and downward balancing energy are \$10/MWh and \$2/MWh, respectively.

- In contrast, if the day-ahead market price is equal to \$50/MWh, these imbalance penalties take on the values \$5/MWh and \$4/MWh instead.

Use the information about offering curves provided in Sect. 8.4 of Chap. 8 (see, in particular, Example 8.9 in this chapter) to formulate and solve a stochastic programming model that computes, as an increasing function of the day-ahead electricity price, the optimal energy offer in the day-ahead market that maximizes the expected profit of the solar power producer.

Based on the comments and explanations in Sect. 7.4.2, justify the obtained solution.

7.9 Repeat Exercise 7.8 for the case that:

- If the day-ahead market price is \$25/MWh, the imbalance penalties for upward and downward balancing energy are \$5 /MWh and \$4/MWh, respectively.
- If the day-ahead market price is equal to \$50/MWh, the imbalance penalties are \$10/MWh and \$2/MWh instead.

7.10 Try to solve Exercises 7.8 and 7.9 analytically. Can both problems be solved this way? If not, motivate why.

References

1. Artzner, P., Delbaen, F., Eber, J.M., Heath, D.: Coherent measures of risk. Math. Financ. **9**(3), 203–228 (1999)
2. Baringo, L., Conejo, A.J.: Strategic offering for a wind power producer. IEEE Trans. Power Syst. (2013). In press
3. Bitar, E., Rajagopal, R., Khargonekar, P., Poolla, K., Varaiya, P.: Bringing wind energy to market. IEEE Trans. Power Syst. **27**(3), 1225–1235 (2012)
4. Bremnes, J.B.: Probabilistic wind power forecasts using local quantile regression. Wind Energy **7**(1), 47–54 (2004)
5. Conejo, A.J., Carrión, M., Morales, J.: Decision Making Under Uncertainty in Electricity Markets. International Series in Operations Research & Management Science. Springer, New York (2010)
6. Dent, C., Bialek, J.W., Hobbs, B.F.: Opportunity cost bidding by wind generators in forward markets: Analytical results. IEEE Trans. Power Syst. **26**(3), 1600–1608 (2011)
7. Gabriel, S.A., Conejo, A.J., Fuller, J.D., Hobbs, B.F., Ruiz, C.: Complementarity Modeling in Energy Markets. International Series in Operations Research & Management Science. Springer, New York (2013)
8. Gneiting, T.: Quantiles as optimal point forecasts. Int. J. Forecast. **27**(2), 197–207 (2011)
9. Kaplan, W.: Advanced Calculus, 5th edn., chap. 4, pp. 253–257. Addison-Wesley Higher Mathematics. Addison-Wesley, Reading, MA (2002)
10. Matevosyan, J., Söder, L.: Minimization of imbalance cost trading wind power on the short-term power market. IEEE Trans. Power Syst. **21**(3), 1396–1404 (2006)
11. Morales, J.M., Conejo, A.J., Pérez-Ruiz, J.: Short-term trading for a wind power producer. IEEE Trans. Power Syst. **25**(1), 554–564 (2010)
12. Pinson, P., Chevalier, C., Kariniotakis, G.: Trading wind generation from short-term probabilistic forecasts of wind power. IEEE Trans. Power Syst. **22**(3), 1148–1156 (2007)
13. Rahimiyan, M., Morales, J.M., Conejo, A.J.: Evaluating alternative offering strategies for wind producers in a pool. Appl. Energy **88**(12), 4918–4926 (2011)

14. Raiffa, H.: Decision Analysis: Introductory Lectures on Choices Under Uncertainty. Addison-Wesley, Reading, MA (1968)
15. Raiffa, H., Schlaifer, R.: Applied Statistical Decision Theory. Division of Research – Harvard Business school, Boston, MA (1964)
16. Rockafellar, R.T., Uryasev, S.: Optimization of Conditional Value-at-Risk. J. Risk **2**, 21–41 (2000)
17. Zugno, M., Jónsson, T., Pinson, P.: Trading wind energy on the basis of probabilistic forecasts both of wind generation and of market quantities. Wind Energy **16**, 909–926 (2012).
18. Zugno, M., Morales, J.M., Pinson, P., Madsen, H.: Pool strategy of a price-maker wind power producer. IEEE Trans. Power Syst. **28**(3), 3440–3450 (2013)

Chapter 8
Virtual Power Plants

8.1 What is a Virtual Power Plant?

Under the current power system organization characterized by the centralized control of production units and the passive operation of the distribution grid, distributed generation may well displace a considerable amount of the energy produced by large conventional power plants. Yet, these plants will be still required to guarantee the secure functioning of the power system by providing support services such as load following, frequency and voltage regulation, and reserves. In parallel, the provision of these services by central power units will call for substantial investments at both the transmission and the distributions levels.

Therefore, in order to make the most of distributed energy resources (DERs), a new paradigm for power system operation in which the joint coordination of distributed generators and flexible loads can fully replace conventional plants should be put in place. This new operating paradigm should allow DERs to interact with the transmission system and participate in both energy markets and system management.

On the other hand, the future distribution grids are expected to accommodate thousands, if not millions, of distributed generators and flexible loads. Consequently, the communication of each distributed unit with the transmission system operator is, needless to say, out of reach, owing to the high costs that the required information and technology capability would entail. Instead, a decentralized management based on the aggregation of a number of DERs can potentially overcome the apparently inevitable increase in complexity of system operation, and here is where the concept of *Virtual Power Plant* (VPP) comes into the picture.

Formally, a VPP, also referred to as *Virtual Utility*, can be defined as a cluster of dispersed generating units, flexible loads, and storage systems that are grouped in order to operate as a single entity. The generating units in the VPP can employ both fossil and renewable energy sources. The primary purpose of a VPP is thus to coordinate the production and consumption of its constituent parts with a view to maximizing their performance.

This chapter is organized as follows. In Sect. 8.2, we introduce simplified mathematical models for the basic different components of a VPP. These models are subsequently used to build a tool for the operation of a VPP within the framework

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of an electricity market in Sect. 8.3. The resulting offering model is extended first to cope with uncertain information in Sect. 8.4 and then to account for a multi-market framework in Sect. 8.5. Last, Sect. 8.6 summarizes the chapter and Sect. 8.7 provides a list of selected readings for the interested reader to build on some of the most important concepts and models introduced herein.

8.2 Modeling the Components of a Virtual Power Plant

From a modeling point of view, the components of a VPP can be of four basic types: dispatchable power plants, flexible loads, storage equipment, and stochastic generating units (often referred to as intermittent energy sources).

The *dispatchable power plants* in a VPP are typically small power stations that burn fossil fuels, such as natural gas or oil, to produce electricity. Gas turbines and diesel generators, for instance, fall into this category. Furthermore, dispatchable power plants based on the combustion of biomass sources, e.g., biodiesel or biogas, are also becoming relevant in the generation mix. Besides, cogeneration—or more specifically, the use of a power station to produce both electricity and heat—is gaining momentum as an efficacious practice to maximize the performance of the thermal units in a VPP, thus making more attractive the exploitation of Combined Heat and Power (CHP) plants.

The *flexible loads* may fundamentally comprise both residential and industrial electricity consumers that are equipped with the communication and control infrastructure needed to adjust their consumption patterns at the command of the VPP. As we will see in Chap. 9, demand elasticity within the VPP can be induced by means of direct control or price signals (the so-called *dynamic pricing methods*).

The *storage equipment* is comprised of devices or physical media that store energy at a given point in time to make a "better use" of it at a later time. Therefore, storage units allow transferring electricity from one time period to another with the ultimate aim of getting a benefit out of this operation. The first utility-scale electricity storage systems were installed more than 30 years ago and consisted of pumped hydro power plants (all over the world), low-speed flywheels (in Germany), and lead-acid batteries (in the UK). Today, the spectrum of storage technologies is broader and also covers storage devices based on electrochemical capacitors, compressed air, superconducting magnetic fields, and fuel cells, among others [10].

Last, the *stochastic generating units* are facilities that produce electricity out of weather-driven energy sources. This is the case, for example, of wind turbines and photovoltaic cells, which obtain electricity from the energy contained in the wind and the sunlight, respectively. Therefore, the power output of these facilities is inherently uncertain, as are the weather phenomena that determine the availability of the exploited energy sources. Furthermore, these generating units are nondispatchable, which implies that any energy deviation from a preestablished production plan must be covered by back-up units.

In the following sections, we introduce basic mathematical descriptions of these four archetypes of VPP components.

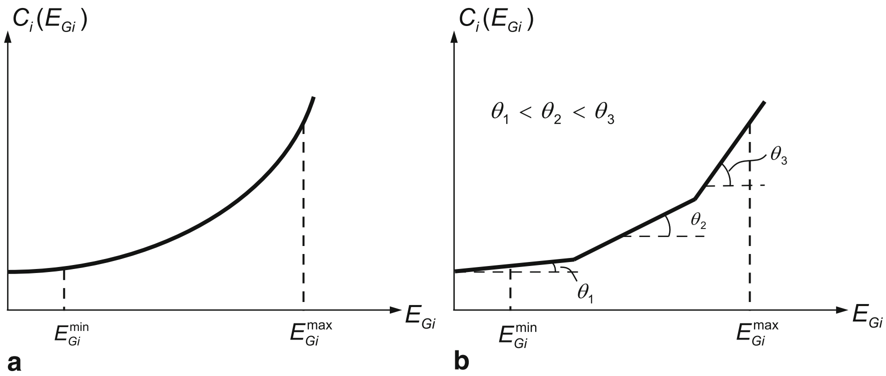

Fig. 8.1 Examples of functions describing the production cost of a dispatchable power plant. **a** Quadratic. **b** Piecewise linear

8.2.1 Dispatchable Power Plants

Each dispatchable unit i in the VPP is characterized by a cost function, $C_i(E_{Gi})$, which provides the cost C_i (e.g., in dollars) of generating a certain amount of electricity E_{Gi} (e.g., in megawatt hour). This function is obtained by multiplying the heat rate curve, which gives the amount of fuel needed to produce E_{Gi}, by the cost of the consumed fuel.

The cost $C_i(\,\cdot\,)$ is typically modeled as a convex quadratic or piecewise linear function of the energy output E_{Gi}, as illustrated in Fig. 8.1. Parameters $E_{Gi}^{\max}$ and $E_{Gi}^{\min}$ represent, respectively, the maximum and minimum amounts of electricity that can be generated by the dispatchable power plant, which are logically dependent on the length of the considered time period. Indeed, if we consider a certain time frame τ, it holds that $E_{Gi}^{\max} = P_{Gi}^{\max} \times \tau$ and $E_{Gi}^{\min} = P_{Gi}^{\min} \times \tau$, where $P_{Gi}^{\max}$ and $P_{Gi}^{\min}$ are the power capacity and the minimum power output of the plant.

Let $P_{Gi}(t)$ denote the power output of the dispatchable unit at a specific point in time t. $P_{Gi}(t)$ must be either zero—when the power unit is idle—or take on a value within the closed interval $\left[P_{Gi}^{\min}, P_{Gi}^{\max}\right]$—when the power unit is actually generating electricity. The range of power values in $\left(0, P_{Gi}^{\min}\right)$ is, therefore, technically infeasible. Mathematically, the functional state of a generating unit can be easily modeled using a binary variable $v_i(t)$ that is equal to 0 or 1 depending on whether the unit is *off* or *on*, respectively. This way the disjunctive operating region of the power plant can be formulated as follows:

$$v_i(t)P_{Gi}^{\min} \le P_{\mathrm{Gi}}(t) \le v_i(t)P_{Gi}^{\max}. \tag{8.1}$$

In practice, the change of $v_i(t)$ from 0 to 1, or from 1 to 0, involves the start-up, or the shutdown, of the unit. Both transitions entail additional costs to those already accounted for in the cost function $C_i(E_{Gi})$. If we now discretize the time horizon over which the operation of the VPP is to be optimized into a set of $T + 1$ samples, i.e., $\{0, 1, 2, \dots, t-1, t, t+1, \dots, T\}$, the start-up and shutdown costs incurred by the dispatchable plant i at time point t, denoted by $C_i^{\mathrm{SU}}(t)$ and $C_i^{\mathrm{SD}}(t)$, respectively, can be modeled as

$$C_i^{\mathrm{SU}}(t) \geq S_i^{\mathrm{U}}\,(v_i(t) - v_i(t-1)), \tag{8.2a}$$

$$C_i^{\mathrm{SU}}(t) \geq 0, \tag{8.2b}$$

and

$$C_i^{\mathrm{SD}}(t) \geq S_i^{\mathrm{D}}\,(v_i(t-1) - v_i(t)), \tag{8.3a}$$

$$C_i^{\mathrm{SD}}(t) \geq 0, \tag{8.3b}$$

where S_i^{U} and S_i^{D} are, in that order, the costs derived from the start-up and shutdown processes of the unit. On this point, it is worth clarifying that, in reality, variables $C_i^{\mathrm{SU}}(t)$ and $C_i^{\mathrm{SD}}(t)$ will only take the actual values of the start-up and shutdown costs incurred by the power plant provided that these costs are to be minimized. This is indeed the case when it comes to operating a VPP in a cost-efficient manner.

Furthermore, a unit may also be characterized by the so-called *no-load cost*, which is a fixed expense related to the fuel that needs to be consumed to sustain a 0-MW power output at synchronous generator speed, i.e., to keep the plant online. Such a cost is given by the y-intercept of the cost function, $C_i(0)$, and occurs only if the unit is on for the relevant time period, i.e.,

$$C_i^{\mathrm{NL}}(t) = C_i(0)v_i(t). \tag{8.4}$$

Note that keeping the unit online without producing electricity for a short period of time may be cheaper than shutting down and restarting the unit.

As it appears from the observations above, operating a VPP not only calls for decisions on how much power every dispatchable unit has to deploy, but also requires the calculation of an optimum time schedule for the start-up and shutdown of these units. In other words, since the power plants' start-up and shutdown costs may be significant, on/off decisions $v_i(t)$ must be optimally determined in coordination with the continuous generation outputs $P_{Gi}(t)$. We shall take up this issue in Sect. 8.3 below.

Besides the generation limits (8.1), a dispatchable power plant often has other technical constraints that shape its operating feasible region. Ramping constraints (8.5a) and (8.5b), for example, limit how much the power output of a generating unit may be increased or decreased between two consecutive time samples $t-1$ and t. In these constraints, constant Δ_{Gi} represents the maximum ramp-up/-down rate of the unit (e.g., in megawatts per hour) and τ the time elapsed between $t-1$ and t (e.g., in hours).

$$P_{Gi}(t) - P_{Gi}(t-1) \leq \Delta_{Gi} \times \tau, \tag{8.5a}$$

$$P_{Gi}(t-1) - P_{Gi}(t) \leq \Delta_{Gi} \times \tau. \tag{8.5b}$$

Other restrictions have to do with the time during which a unit must remain on (off) after being started up (shutdown), i.e., the so-called *minimum up-time and down-time*

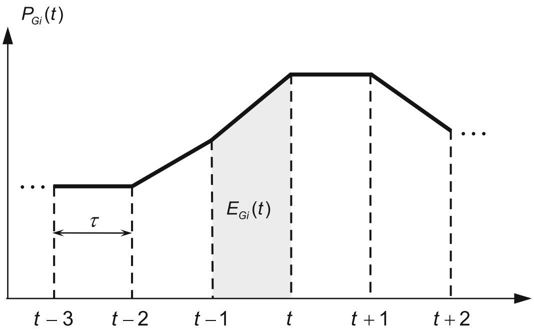

Fig. 8.2 Linear time trajectory followed by the power output $P_{Gi}(t)$ of the dispatchable plant. The shaded area represents the energy produced in the period in between time samples $t-1$ and t. The length of each period is τ

constraints. These constitute an important source of inflexibility when it comes to operating a power system with a high penetration of stochastic energy sources, as studied in Chap. 5 (p. 149) of this book, where the reader is referred to for a detailed formulation of these constraints.

Lastly, it should be noticed that Eqs. (8.1), (8.5a), and (8.5b) are imposed on the power output $P_{Gi}(t)$, while the production cost $C_i(\,\cdot\,)$ is a function of the amount of energy that is actually produced, E_{Gi}. We need, therefore, an additional equation that converts power into energy. This conversion depends on the time trajectory followed by the power output P_{Gi} in between time samples. If we assume that this trajectory is linear, as illustrated in Fig. 8.2, we can compute the energy generated by the dispatchable plant between $t-1$ and t as

$$E_{Gi}(t) = \frac{P_{Gi}(t-1) + P_{Gi}(t)}{2}\tau, \tag{8.6}$$

which is arbitrarily assigned to time sample t for convenience. Observe that Eq. (8.6) corresponds to the area of a trapezoid with parallel sides of length $P_{Gi}(t-1)$ and $P_{Gi}(t)$ and height given by τ.

The following example illustrates the modeling of a dispatchable power plant.

Example 8.1 (Modeling of a dispatchable power plant) Consider a dispatchable power plant consisting of a gas turbine (GT) with a rated capacity of 5 MW and a technical minimum of 1 MW, i.e., $P_G^{\max} = 5$ MW and $P_G^{\min} = 1$ MW, respectively. In addition, it is known that the maximum ramp rate of the GT unit is 2 MW/h, while its minimum up and down times are both zero.

The dispatchable power plant is to be operated over a time horizon comprising three hourly periods, that is, $T = 3$ and $\tau = 1$ h. Besides, the power output of the GT unit is initially 2 MW.

The operation of the plant is characterized by the convex quadratic cost function depicted in Fig. 8.3, which is given by $C(E_G) = 5E_G^2 + 10E_G + 50$, with cost expressed in dollars and energy in megawatt hours. The y-intercept of the cost function, $C(0) = \$50$, represents the no-load cost, which takes place only during time periods where the plant is online. Therefore, this term must be multiplied by the binary variable $v(t)$ representing the status of the unit in the cost function. For example, the

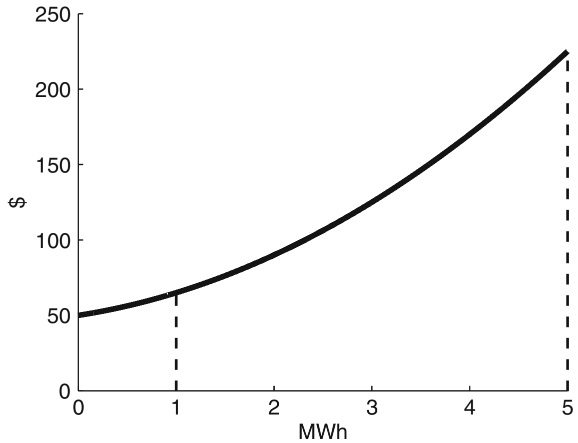

Fig. 8.3 Convex quadratic curve representing the cost function of the gas turbine. The analytical expression of the curve is $C(E_G) = 5E_G^2 + 10E_G + 50$

no-load cost for time period $t = 1$ is given by

$$C^{\mathrm{NL}}(1) = 50v(1). \tag{8.7}$$

Note also that the minimum and maximum amounts of energy that the GT unit can produce when it is on in a given time period are equal to $E_G^{\min} = P_G^{\min} \times \tau = 1\mathrm{MW} \times 1\mathrm{h} = 1$ MWh and $E_G^{\max} = P_G^{\max} \times \tau = 5\mathrm{MW} \times 1\mathrm{h} = 5$ MWh, respectively.

Aside from the expenses directly associated with fuel consumption during the electricity generation process and the no-load cost, the GT unit incurs an additional cost of \$10 every time it is started up. In contrast, its shutdown does not entail any extra cost.

We have now all the necessary information to build a mathematical model for the operation of the dispatchable power plant. To begin with, let us focus on the first period of the time horizon, i.e., $t = 1$. The constraints enforcing the power output limits of the GT unit in this period are stated as follows:

$$v(1) \leq P_G(1) \leq 5v(1), \tag{8.8a}$$

where $v(1)$ indicates the unit status (off/on) in period 1. This 0/1 variable is also useful to identify when the power plant has been started up and hence to model the associated start-up cost, which is indeed formulated as

$$C^{\mathrm{SU}}(1) \geq 10\,(v(1) - 1), \tag{8.8b}$$

$$C^{\mathrm{SU}}(1) \geq 0. \tag{8.8c}$$

The "1" in the right-hand side of inequality (8.8b) comes from the fact that the GT unit is producing 2 MW at the beginning of the considered time horizon, i.e., at $t = 0$, and therefore it must hold that $v(0) = 1$.

Since the shutdown cost of the GT unit is zero ($S^{\mathrm{D}} = 0$), constraints (8.3) are not needed.

The next set of technical constraints to be modeled are the ramping limits (8.5), which for $t = 1$ are written as follows:

$$P_G(1) - 2 \leq 2, \tag{8.8d}$$

$$2 - P_G(1) \leq 2, \tag{8.8e}$$

where we have taken into account that the power plant is initially generating 2 MW and that its maximum ramp rate is 2 MW/h, i.e., $P_G(0) = 2$ MW and $\Delta_G = 2$ MW/h.

Given that the minimum up and down times of the GT unit are both zero, the only equation needed to complete the power plant model for $t = 1$ is the one establishing the equivalence between energy and power, i.e.,

$$E_G(1) = 1 + \frac{P_G(1)}{2}. \tag{8.8f}$$

Now we only need to proceed analogously for the remaining two periods of the study horizon. We present below the corresponding two sets of constraints.

Period $t = 2$:

$$v(2) \leq P_G(2) \leq 5v(2), \tag{8.8g}$$

$$C^{\mathrm{SU}}(2) \geq 10\,(v(2) - v(1)), \tag{8.8h}$$

$$C^{\mathrm{SU}}(2) \geq 0, \tag{8.8i}$$

$$P_G(2) - P_G(1) \leq 2, \tag{8.8j}$$

$$P_G(1) - P_G(2) \leq 2, \tag{8.8k}$$

$$E_G(2) = \frac{P_G(1) + P_G(2)}{2}. \tag{8.8l}$$

Period $t = 3$:

$$v(3) \leq P_G(3) \leq 5v(3), \tag{8.8m}$$

$$C^{\mathrm{SU}}(3) \geq 10\,(v(3) - v(2)), \tag{8.8n}$$

$$C^{\mathrm{SU}}(3) \geq 0, \tag{8.8o}$$

$$P_G(3) - P_G(2) \leq 2, \tag{8.8p}$$

$$P_G(2) - P_G(3) \leq 2, \tag{8.8q}$$

$$E_G(3) = \frac{P_G(2) + P_G(3)}{2}. \tag{8.8r}$$

8.2.2 Flexible Loads

A flexible demand has the ability to reduce, increase, or defer its electricity consumption in response to high market prices or market incentives. In its simplest and

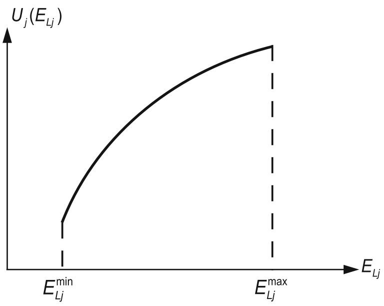

Fig. 8.4 Example of an utility function describing the benefit obtained by a flexible load from the electricity consumption, which is bounded from above and below by $E_{Lj}^{\max}$ and $E_{Lj}^{\min}$, respectively

most stylized form, the mathematical modeling of a flexible consumer resembles that of a dispatchable power unit. Each flexible load j in the VPP is characterized by an utility function, $U_j(E_{Lj})$, which provides the benefit (e.g., in dollars) that the consumer obtains out of the amount of electricity, E_{Lj}, it consumes. For example, the owner of a greenhouse that uses artificial lighting to control or increase the growth of its plants is making a profit out of it—its plants grow faster!—and thereby, confers some *utility* on the electricity consumed for lighting. Likewise, we all give some value to the electricity consumed by our refrigerators as these allow us to keep our food fresh longer. However, neither the owner of the greenhouse, nor ourselves, are willing to consume electricity at any cost. An utility function serves thus to express our willingness to consume electricity in economic terms.

Figure 8.4 illustrates a concave quadratic utility function. Parameters $E_{Lj}^{\max}$ and $E_{Lj}^{\min}$ represent, respectively, the maximum and minimum amounts of electricity that can be consumed by the flexible load. As in the case of a dispatchable power plant, these parameters depend on the length of the considered time period. That is, given a certain time interval τ, one can write $E_{Lj}^{\max} = P_{Lj}^{\max} \times \tau$ and $E_{Lj}^{\min} = P_{Lj}^{\min} \times \tau$, where $P_{Lj}^{\max}$ and $P_{Lj}^{\min}$ are constants characterizing the maximum and minimum power consumption of the load. More specifically, $P_{Lj}^{\max}$ represents the maximum power that can be demanded by the flexible load—individual households, for instance, usually have an upper bound to the power they can withdraw from the net—while $P_{Lj}^{\min}$ refers to the part of the load that is fixed, i.e., the consumption that cannot be increased, reduced or deferred at will. This demand category may include, for example, cooking appliances, critical lighting, and computers. Therefore, if we denote the power demanded by the flexible load j at a given time point t by $P_{Lj}(t)$, we have that

$$P_{Lj}^{\min} \leq P_{Lj}(t) \leq P_{Lj}^{\max}. \tag{8.9}$$

Similarly to the ramping constraints of a generating unit, the speed at which a flexible load can increase or decrease its consumption is limited by the so-called *pickup/drop rate*, Δ_{Lj} (e.g., in megawatt per hour), that is,

$$P_{Lj}(t) - P_{Lj}(t-1) \leq \Delta_{Lj} \times \tau, \tag{8.10a}$$

$$P_{Lj}(t-1) - P_{Lj}(t) \leq \Delta_{Lj} \times \tau. \tag{8.10b}$$

Likewise, if we now consider that the time trajectory of $P_{Lj}(t)$ is piecewise linear, we can compute the electricity consumed by the flexible load within the time samples $t-1$ and t as

$$E_{Lj}(t) = \frac{P_{Lj}(t-1) + P_{Lj}(t)}{2}\tau, \tag{8.11}$$

where τ represents the time elapsed between time points $t-1$ and t.

Lastly, constraint (8.12) below establishes a floor E^{f}_{Lj} for the total amount of energy that must be consumed during the T time periods of the scheduling horizon:

$$\sum_{t=1}^{T} E_{Lj}(t) \geq E^{\mathrm{f}}_{Lj}. \tag{8.12}$$

The following example serves to illustrate the modeling of a flexible load.

Example 8.2 (Modeling of a flexible load) Consider a factory that uses milling machines for gear manufacturing. As the dispatchable power plant in Example 8.1, the gear factory is to be operated over a time horizon spanning three hourly periods, that is, $T = 3$ and $\tau = 1$ h.

Due to ancillary services such as lighting, space cooling or heating, and monitoring and surveillance equipment, the gear factory requires a minimum power consumption of 0.5 MW to guarantee a 100 % safe and comfortable operation. Besides, the factory uses 2 MW of power when functioning at its full potential. Therefore, it follows that

$$0.5 \leq P_L(1) \leq 2, \tag{8.13a}$$

$$0.5 \leq P_L(2) \leq 2, \tag{8.13b}$$

$$0.5 \leq P_L(3) \leq 2. \tag{8.13c}$$

For both security reasons and a proper machinery maintenance, the factory cannot increase or decrease its power consumption at a rate higher than 1 MW/h. Knowing that it was consuming 1.5 MW at the beginning of the time horizon, we can write that

$$P_L(1) - 1.5 \leq 1, \tag{8.13d}$$

$$1.5 - P_L(1) \leq 1, \tag{8.13e}$$

$$P_L(2) - P_L(1) \leq 1, \tag{8.13f}$$

$$P_L(1) - P_L(2) \leq 1, \tag{8.13g}$$

$$P_L(3) - P_L(2) \leq 1, \tag{8.13h}$$

$$P_L(2) - P_L(3) \leq 1. \tag{8.13i}$$

If we now assume that the series of power values $\{P_L(0), P_L(1), P_L(2), P_L(3)\}$ describes a piecewise linear trajectory, the amount of energy consumed by the gear factory in each time period can be calculated as

$$E_L(1) = \frac{1.5 + P_L(1)}{2}, \tag{8.13j}$$

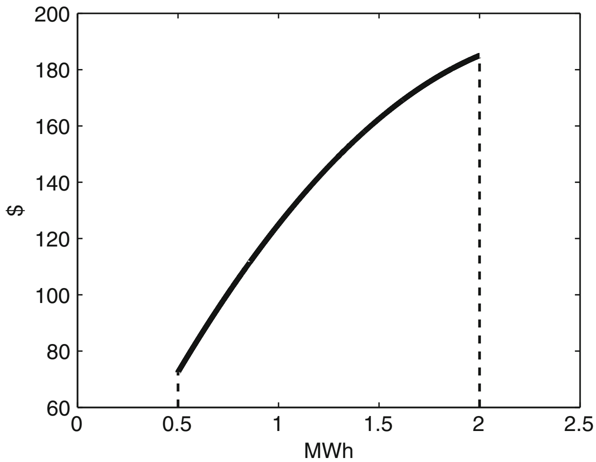

Fig. 8.5 Concave quadratic curve representing the utility function of the gear factory. The analytical expression of the curve is $U(E_L) = -30E_L^2 + 150E_L + 5$

$$E_L(2) = \frac{P_L(1) + P_L(2)}{2}, \tag{8.13k}$$

$$E_L(3) = \frac{P_L(2) + P_L(3)}{2}. \tag{8.13l}$$

It turns out that the demand for gears during the last couple of weeks has been remarkably buoyant, so the owner of the factory has decided to impose a minimum energy consumption during the 3 h horizon of 2.5 MWh in order to keep the gear stock above a sensible limit, that is,

$$E_L(1) + E_L(2) + E_L(3) \geq 2.5. \tag{8.13m}$$

Finally, the owner of the factory has estimated, based on the market price for gears, that the benefit the factory is obtaining out of the electricity it consumes is given by the concave quadratic utility function shown in Fig. 8.5, whose analytical expression is $U(E_L) = -30E_L^2 + 150E_L + 5$. The function is displayed for the interval $[E_L^{\min}, E_L^{\max}] = [P_L^{\min} \times \tau, P_L^{\max} \times \tau] = [0.5, 2]$ MWh.

In many real-life instances, the mathematical modeling of a flexible load exhibits a higher degree of complexity. This is especially the case when it comes to residential consumers, for whom an utility function is difficult to define and most likely, time-dependent. Think, for example, of a heat pump that keeps the indoor temperature of a household within certain limits. We can take advantage of the thermal inertia of the house to shift the electricity consumption of the heat pump in time. However, this should be done in such a way that the *comfort perception* of the inhabitants is not jeopardized. Chapter 9 provides further insight into this relevant issue.

8.2.3 Storage Units

The modeling of a storage unit k is essentially based on a state-transition equation that defines its energy content at every time sample t as a function of the amount of power charged into or drawn from the unit. Mathematically, the state-transition equation for the energy level in the storage device at time t, i.e., $E_{Sk}(t)$, can be stated as

$$E_{Sk}(t) = E_{Sk}(t-1) + \eta_k^c P_{Sk}^c(t)\tau - \frac{1}{\eta_k^d} P_{Sk}^d(t)\tau, \tag{8.14}$$

where $P_{Sk}^c(t)$ and $P_{Sk}^d(t)$ are, respectively, the charging and discharging powers, which are both positive and, in practice, limited by certain upper bounds, $P_{Sk}^{c,\max}$ and $P_{Sk}^{d,\max}$, indicating how fast the storage unit can be charged and discharged, that is,

$$0 \leq P_{Sk}^c(t) \leq P_{Sk}^{c,\max}, \tag{8.15}$$

$$0 \leq P_{Sk}^d(t) \leq P_{Sk}^{d,\max}. \tag{8.16}$$

Parameters η_k^c and η_k^d in the state-transition function (8.14) are efficiency factors that account for the energy losses associated with the charging and discharging processes. Thus, it holds that $\eta_k^c < 1$ and $\eta_k^d < 1$. Again, constant τ represents the time span between two time samples $t-1$ and t and is needed here to convert power magnitudes into energy values.

Finally, the storage unit is characterized by a finite capacity $E_{Sk}^{\max}$ and the energy level in the storage device is often kept above a prespecified threshold $E_{Sk}^{\min}$. Therefore, we can write

$$E_{\mathrm{Sk}}^{\min} \leq E_{\mathrm{Sk}}(t) \leq E_{\mathrm{Sk}}^{\max}. \tag{8.17}$$

The lower bound $E_{Sk}^{\min}$ is, for example, typical of electric batteries, where this limit is imposed to prevent the early wear of the battery and the consequent reduction of its lifetime.

In the following, we introduce a small example to illustrate the modeling of a storage unit.

Example 8.3 (Modeling of a storage unit) Let us assume that we own a small pumped-storage plant (PSP) that is to be run over a 3-h time horizon ($T = 3$ and $\tau = 1$ h). The equivalent energy capacity of the PSP upper basin is 1 MWh and, for both economic and technical reasons, it cannot be drained below 0.2 MWh. These initial data lead to the following set of inequalities:

$$0.2 \leq E_S(1) \leq 1, \tag{8.18a}$$

$$0.2 \leq E_S(2) \leq 1, \tag{8.18b}$$

$$0.2 \leq E_S(3) \leq 1. \tag{8.18c}$$

When the PSP is operated in turbine mode, i.e., when water is released from the upper basin, the maximum power output is 0.5 MW. Likewise, when the PSP is run in pumping regime, the maximum power consumption for refilling the upper basin is 0.3 MW. This information is expressed in the form of constraints as

$$0 \leq P_S^c(1) \leq 0.3, \tag{8.18d}$$

$$0 \le P_S^c(2) \le 0.3, \tag{8.18e}$$

$$0 \le P_S^c(3) \le 0.3, \tag{8.18f}$$

$$0 \le P_S^d(1) \le 0.5, \tag{8.18g}$$

$$0 \le P_S^d(2) \le 0.5, \tag{8.18h}$$

$$0 \le P_S^d(3) \le 0.5. \tag{8.18i}$$

Last, knowing that the efficiency of our PSP, both in turbine and pumping modes, is equal to 80 % and that the water contained in its upper reservoir at the beginning of the 3-h time horizon is equivalent to 0.4 MWh, the time evolution of the energy stored in the PSP can be modeled as

$$E_S(1) = 0.4 + 0.8P_S^c(1) - 1.25P_S^d(1), \tag{8.18j}$$

$$E_S(2) = E_S(1) + 0.8P_S^c(2) - 1.25P_S^d(2), \tag{8.18k}$$

$$E_S(3) = E_S(2) + 0.8P_S^c(3) - 1.25P_S^d(3). \tag{8.18l}$$

A more comprehensive model of a pumped-storage unit can be found in Sect. 5.5.

8.2.4 Stochastic Generating Units

The term "stochastic generating units" refers here to all those means to produce electricity from renewable energy sources that are dependent on short-term weather conditions, e.g., sunlight, wind, and waves, and as a consequence, are nondispatchable. Consequently, the *forward* estimate of the amount of energy that can be extracted from these sources is inherently uncertain in time and quantity, and any *forward* dispatch of stochastic generating units proves to be erroneous in the end. In view of this situation, to model the operation of a stochastic generating unit within a VPP, we need to resort to *forecasts* of its power output.

Chapter 2 of this book centers on the different types of forecasts most commonly used when dealing with renewable energy sources. Here we will just focus on three of them.

If we describe the uncertain power output of a stochastic generating unit q as the series of random variables $\{\widetilde{P}_{Wq}(1), \widetilde{P}_{Wq}(2), \ldots, \widetilde{P}_{Wq}(T)\}$, successive in time, the input information we need to operate a VPP that includes the stochastic generating unit q may fall into the following three classes:

1. The *point forecasts* $\{\widehat{P}_{Wq}(1), \ldots, \widehat{P}_{Wq}(T)\}$, where $\widehat{P}_{Wq}(t)$, for all $t = 1, 2, \ldots, T$, is the expected value of random variable $\widetilde{P}_{Wq}(t)$, i.e., $\widehat{P}_{Wq}(t) = \mathbb{E}\{\widetilde{P}_{Wq}(t)\}$.
2. The series of *intervals* $\{[P_{Wq}^{\mathrm{lo}}(1), P_{Wq}^{\mathrm{up}}(1)], \ldots, [P_{Wq}^{\mathrm{lo}}(T), P_{Wq}^{\mathrm{up}}(T)]\}$, where, in defining each interval $[P_{Wq}^{\mathrm{lo}}(t), P_{Wq}^{\mathrm{up}}(t)]$, we assume that the eventual outcome of random variable $\widetilde{P}_{Wq}(t)$ will be in between $P_{Wq}^{\mathrm{lo}}(t)$ and $P_{Wq}^{\mathrm{up}}(t)$.

Table 8.1 Modeling of the PV power output in the form of point forecasts, intervals, and scenarios. Powers are in megawatts

Period t	Point forecast	Interval	Scenario ω		
			1	2	3
1	3.30	[2.0, 6.0]	2.5	6.0	2.0
2	2.55	[1.1, 4.0]	4.0	4.0	1.1
3	3.00	[1.5, 6.0]	6.0	3.5	1.5
Probability π_ω	–	–	0.2	0.3	0.5

3. The set of *scenarios* $\{[P_{Wq\omega}(1), \ldots, P_{Wq\omega}(T)], \omega = 1, 2, \ldots, N_\Omega\}$, each with probability of occurrence $\pi_\omega \geq 0$ such that $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$.

It is important to mention that point forecasts, intervals, and scenarios are *conditional* on the past power outputs of stochastic generating unit q that are available at the beginning of the time horizon, i.e., at $t = 0$, and more generally, on all the information available at the time for any other explanatory variable used to issue such forecasts. As seen in Chap. 2, this should be indicated by adding the condition "$|t = 0$" in every single power parameter above. For example, the (conditional) expected value of random variable $\widetilde{P}_{Wq}(t)$ should be properly written as $\widehat{P}_{Wq}(t|t = 0)$. However, we drop here this notation for simplicity and conciseness.

Since the stochastic generating units we look at in this chapter produce electrical power from renewable energy sources that are abundant and free, their associated generating cost is usually very low. In particular, in this chapter, this cost is assumed to be zero.

We conclude this section with an example that illustrates the modeling of a stochastic generating unit.

Example 8.4 (Modeling of a stochastic generating unit) Consider a photovoltaic power plant (PV) with a rated capacity of 6 MW. Table 8.1 provides the modeling of the PV power output over a 3-h time horizon ($T = 3$, $\tau = 1$ h) in the form of point forecasts, intervals, and scenarios.

It should be noticed that, in this particular example, both the point forecasts and the intervals can be directly derived from the scenarios, which highlights that the latter carry more information on the uncertain PV power output than the former. Indeed, it holds that $\widehat{P}_{Wq}(t) = \sum_{\omega=1}^{3} \pi_\omega P_{Wq\omega}(t)$ and $[P^{\text{lo}}_{Wq}(t), P^{\text{up}}_{Wq}(t)] = [\min_\omega\{P_{Wq\omega}(t)\}, \max_\omega\{P_{Wq\omega}(t)\}]$, for all t. As we shall see later, this has important implications when managing a VPP that includes the photovoltaic unit of this example.

8.3 Energy Management in a Virtual Power Plant

Consider a VPP that consists of a set I of dispatchable power units, a set J of flexible loads, a set K of storage devices, and a set Q of stochastic generators. The VPP seeks to optimally operate these energy resources over a time horizon $\{1, \ldots, t, \ldots, T\}$ of T periods of length τ each.

Operating optimally the VPP translates into satisfying the flexible loads (maximizing their utilities) at the minimum cost. For this purpose, the VPP not only counts on its own energy sources, but also on the *electricity market*. Indeed, from the point of view of the VPP, the electricity market can be regarded as an infinite power supplier or consumer that is willing to sell or purchase any amount of energy in time period t at a cost $\lambda^{\mathrm{D}}(t)$, i.e., the electricity market price (we denote here the electricity market price by λ_t^{D}, because it will be subsequently associated with the day-ahead energy market). Therefore, the VPP can also earn profit from the market and as a result, its objective reduces to maximizing its profit, ρ, which is given by

$$\rho = \sum_{t=1}^{T} \left[\lambda^{\mathrm{D}}(t) P^{\mathrm{D}}(t)\tau - \sum_{i \in I} \left[C_i(E_{Gi}(t)) + C_i^{\mathrm{SU}}(t) \right] + \sum_{j \in J} U_j(E_{Lj}(t)) \right], \quad (8.19)$$

where

$P^{\mathrm{D}}(t)$	is the amount of power exchanged with the electricity market in time period t and $\lambda^{\mathrm{D}}(t) \times P^{\mathrm{D}}(t) \times \tau$ is the associated revenue. Variable $P^{\mathrm{D}}(t)$ is positive if the power is sold in the market or negative if bought from it.
$\sum_{i \in I} C_i(E_{Gi}(t))$	is the total production cost of the dispatchable power plants in the VPP in time period t and
$\sum_{i \in I} C_i^{\mathrm{SU}}(t)$	is the total cost due to startups in the same period.
$\sum_{j \in J} U_j(E_{Lj}(t))$	is the total utility derived from the energy consumed by the flexible loads in the VPP in time period t.

The tandem made up of the VPP and the electricity market constitutes a closed energy system that must be balanced at every time period t, which basically means that the amount of energy that is generated from both dispatchable and stochastic power plants and drawn from the storage units must be equal to the amount of energy that is consumed by the flexible loads, used to charge the storage units, and sold in the market for each time interval t. That is,

$$\sum_{i \in I} E_{Gi}(t) + \left(\sum_{q \in Q} \widetilde{P}_{Wq}(t) + \sum_{k \in K} P_{Sk}^{d}(t) \right) \tau = \sum_{j \in J} E_{Lj}(t) + \left(\sum_{k \in K} P_{Sk}^{c}(t) + P^{\mathrm{D}}(t) \right) \tau. \quad (8.20)$$

The energy balance equation, as it stands in (8.20), poses a challenging problem, as it mixes variables of different nature, namely:

1. The power production from each stochastic generating unit, $\widetilde{P}_{Wq}(t)$, is noncontrollable and only known in real time, as the underlying renewable energy source is weather-driven. Therefore, $\widetilde{P}_{Wq}(t)$ is an input random variable to the VPP operation problem, not a decision to be made.
2. The variables pertaining to the remaining VPP components, i.e., $E_{Gi}(t)$, $E_{Lj}(t)$, $P_{Sk}^{d}(t)$, and $P_{Sk}^{c}(t)$, are to be decided once the power output of each stochastic generating unit becomes known. This way, the dispatchable power plants, flexible

loads, and storage units in the VPP are operated to efficiently accommodate the uncertain power production from the stochastic generators. As a result, $E_{Gi}(t)$, $E_{Lj}(t)$, $P^d_{Sk}(t)$, and $P^c_{Sk}(t)$ are decisions to be made conditional on the eventual outcome of $\widetilde{P}_{Wq}(t)$.

3. The power exchanged with the electricity market, $P^{\mathrm{D}}(t)$, must be usually decided before knowing the eventual realization of $\widetilde{P}_{Wq}(t)$. For example, selling offers and purchasing bids in the day-ahead market, which is nowadays the most relevant trading floor in an electricity market, must be submitted around 14-38 h before the physical energy delivery and withdrawal. Consequently, $P^{\mathrm{D}}(t)$ is a decision variable, independent of the outcome of $\widetilde{P}_{Wq}(t)$, that requires *planning*.

In fact, since $E_{Gi}(t)$, $E_{Lj}(t)$, $P^d_{Sk}(t)$, and $P^c_{Sk}(t)$ can be adapted, and thus optimized, to fit any particular realization of $\widetilde{P}_{Wq}(t)$, it is the finding of the optimal value of $P^{\mathrm{D}}(t)$ which turns the VPP operational problem into a *decision-making problem under uncertainty*.

We can try different strategies to compute an "optimal" value for variable $P^{\mathrm{D}}(t)$ given that $\widetilde{P}_{Wq}(t)$ is uncertain. These strategies are linked to the information on $\widetilde{P}_{Wq}(t)$ to be taken into account and exploited when optimizing $P^{\mathrm{D}}(t)$, namely point forecasts, intervals, or scenarios (see Sect. 8.2.4). The easiest way to proceed is to replace $\widetilde{P}_{Wq}(t)$ in (8.20) with its conditional expectation $\widehat{P}_{Wq}(t)$. In doing so, the power traded by the VPP in the electricity market, $P^{\mathrm{D}}(t)$, can be obtained from the following profit-maximization problem:

$$\underset{\Xi}{\text{Max.}} \ \rho = \sum_{t=1}^{T}\Bigg[\lambda^{\mathrm{D}}(t)P^{\mathrm{D}}(t)\tau - \sum_{i\in I}[C_i(E_{Gi}(t)) + C_i^{\mathrm{SU}}(t) + C_i^{\mathrm{SD}}(t)] + \sum_{j\in J} U_j(E_{Lj}(t))\Bigg] \tag{8.21a}$$

s.t.

$$\sum_{i\in I} E_{Gi}(t) + \left(\sum_{q\in Q}\widehat{P}_{Wq}(t) + \sum_{k\in K} P^d_{Sk}(t)\right)\tau = \sum_{j\in J} E_{Lj}(t) + \left(\sum_{k\in K} P^c_{Sk}(t) + P^{\mathrm{D}}(t)\right)\tau, \ \ \forall t = 1,\dots,T, \tag{8.21b}$$

$$G_i\left(P_{Gi}, E_{Gi}, v_i\right) \le 0, \ \ \forall i \in I, \tag{8.21c}$$

$$v_i \in \{0,1\}^T, \ \ \forall i \in I, \tag{8.21d}$$

$$L_j\left(P_{Lj}, E_{Lj}\right) \le 0, \ \ \forall j \in J, \tag{8.21e}$$

$$S_k\left(P^c_{Sk}, P^d_{Sk}, E_{Sk}\right) \le 0, \ \ \forall k \in K, \tag{8.21f}$$

where the blocks of Eqs. (8.21c)–(8.21d), (8.21e) and (8.21f) define the feasible operational regions of dispatchable power plants, flexible loads, and storage units, respectively, as studied in Sects. 8.2.1, 8.2.2, and 8.2.3. Observe that in (8.21c)–(8.21f), the variables not depending on the time index t indicate vectors collecting the relevant variables at all time periods. For clarity and completeness, these blocks of equations are provided in extensive form below.

Feasible operational region of dispatchable power plant i, $G_i\left(P_{Gi}, E_{Gi}, v_i\right) \leq 0$, $v_i \in \{0,1\}^T$:

$$v_i(t) P_{Gi}^{\min} \leq P_{Gi}(t) \leq v_i(t) P_{Gi}^{\max}, \ \ t = 1, \ldots, T, \tag{8.22a}$$

$$C_i^{\mathrm{SU}}(t) \geq S_i^{\mathrm{U}}\left(v_i(t) - v_i(t-1)\right), \ \ t = 1, \ldots, T, \tag{8.22b}$$

$$C_i^{\mathrm{SU}}(t) \geq 0, \ \ t = 1, \ldots, T, \tag{8.22c}$$

$$C_i^{\mathrm{SD}}(t) \geq S_i^{\mathrm{D}}\left(v_i(t-1) - v_i(t)\right), \ \ t = 1, \ldots, T, \tag{8.22d}$$

$$C_i^{\mathrm{SD}}(t) \geq 0, \ \ t = 1, \ldots, T, \tag{8.22e}$$

$$P_{Gi}(t) - P_{Gi}(t-1) \leq \Delta_{Gi} \times \tau, \ \ t = 1, \ldots, T, \tag{8.22f}$$

$$P_{Gi}(t-1) - P_{Gi}(t) \leq \Delta_{Gi} \times \tau, \ \ t = 1, \ldots, T, \tag{8.22g}$$

$$E_{Gi}(t) = \frac{P_{Gi}(t-1) + P_{Gi}(t)}{2} \tau, \ \ t = 1, \ldots, T, \tag{8.22h}$$

$$v_i(t) \in \{0, 1\}, \ \ t = 1, \ldots, T. \tag{8.22i}$$

Feasible operational region of flexible load j, defined by $L_j\left(P_{Lj}, E_{Lj}\right) \leq 0$:

$$P_{Lj}^{\min} \leq P_{Lj}(t) \leq P_{Lj}^{\max}, \ \ t = 1, \ldots, T, \tag{8.23a}$$

$$P_{Lj}(t) - P_{Lj}(t-1) \leq \Delta_{Lj} \times \tau, \ \ t = 1, \ldots, T, \tag{8.23b}$$

$$P_{Lj}(t-1) - P_{Lj}(t) \leq \Delta_{Lj} \times \tau, \ \ t = 1, \ldots, T, \tag{8.23c}$$

$$E_{Lj}(t) = \frac{P_{Lj}(t-1) + P_{Lj}(t)}{2} \tau, \ \ t = 1, \ldots, T, \tag{8.23d}$$

$$\sum_{t=1}^{T} E_{Lj}(t) \geq E_{Lj}^{\mathrm{f}}. \tag{8.23e}$$

Feasible operational region of storage device k, defined by $S_k\left(P_{Sk}^c, P_{Sk}^d, E_{Sk}\right) \leq 0$:

$$E_{Sk}(t) = E_{Sk}(t-1) + \eta_k^c P_{Sk}^c(t) \tau - \frac{1}{\eta_k^d} P_{Sk}^d(t) \tau, \ \ t = 1, \ldots, T, \tag{8.24a}$$

$$0 \leq P_{Sk}^c(t) \leq P_{Sk}^{c,\max}, \ \ t = 1, \ldots, T, \tag{8.24b}$$

$$0 \leq P_{Sk}^d(t) \leq P_{Sk}^{d,\max}, \ \ t = 1, \ldots, T, \tag{8.24c}$$

$$E_{Sk}^{\min} \leq E_{Sk}(t) \leq E_{Sk}^{\max}, \ \ t = 1, \ldots, T. \tag{8.24d}$$

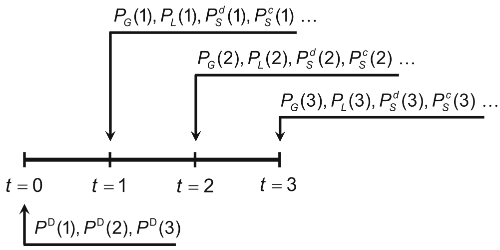

Fig. 8.6 Time framework and decision-making process faced by the manager of the VPP. The energy traded in the electricity market must be decided at t = 0, while decisions on the operation of the VPP components can be made as the power output of the PV unit becomes known

Therefore, the set Ξ of decision variables to be optimized is $\{E_{Gi}(t), E_{Lj}(t), P_{Sk}^d(t), P_{Sk}^c(t), P^{\mathrm{D}}(t), v_i(t), P_{Gi}(t), C_i^{\mathrm{SU}}(t), C_i^{\mathrm{SD}}(t), P_{Lj}(t), E_{Sk}(t)\}$.

Furthermore, the subset of market-related decisions $\{P^{\mathrm{D}*}(1), \ldots, P^{\mathrm{D}*}(T)\}$, solution to problem (8.21), is hereinafter referred to as *expected-value trading strategy*. This trading strategy, based on point forecasts, is illustrated next using a small example.

Example 8.5 (Expected-value trading strategy) Consider a VPP that includes the gas turbine, the gear factory, the PSP, and the photovoltaic unit of Examples 8.1, 8.2, 8.3, and 8.4, respectively. The VPP is to be operated over a 3 h time horizon ($T = 3$, $\tau = 1$ h).

As sketched in Fig. 8.6, the manager of the VPP has to decide *here-and-now* (at $t = 0$) how much energy to sell in or purchase from the electricity market for delivery in time periods $t = 1$, 2, and 3. The benefit or cost per megawatt-hour exchanged with the market is determined by the market price, $\lambda^{\mathrm{D}}(t)$, which is known and equal to $[\lambda^{\mathrm{D}}(1), \lambda^{\mathrm{D}}(2), \lambda^{\mathrm{D}}(3)] = \$[20, 80, 45]$/MWh. In contrast, the manager of the VPP can *wait-and-see* the amount of power that is finally available from the PV unit before making any decision on how to operate the remaining VPP components.

The operator of the VPP must thus decide on variables $P^{\mathrm{D}}(1)$, $P^{\mathrm{D}}(2)$, and $P^{\mathrm{D}}(3)$ under uncertainty. For this purpose, he is provided with Table 8.1 in Example 8.4, where information on the future PV power production is presented in the form of point forecasts, intervals, and scenarios. (We refer to the VPP manager as "he" for convenience and not to imply anything about the gender of VPP managers). After some thinking time, he finally opts to rely on point forecasts to compute $P^{\mathrm{D}}(1)$, $P^{\mathrm{D}}(2)$, and $P^{\mathrm{D}}(3)$ by solving the following optimization problem:

$$\begin{aligned}\text{Max.}\rho = {} & 20P^{\mathrm{D}}(1) + 80P^{\mathrm{D}}(2) + 45P^{\mathrm{D}}(3) - \left[5\left(E_G(1)^2 + E_G(2)^2 + E_G(3)^2\right)\right.\\ & \left.+10\left(E_G(1) + E_G(2) + E_G(3)\right) + 50\left(v(1) + v(2) + v(3)\right)\right]\\ & -\left(C^{\mathrm{SU}}(1) + C^{\mathrm{SU}}(2) + C^{\mathrm{SU}}(3)\right)\\ & -30\left(E_L^2(1) + E_L^2(2) + E_L^2(3)\right) + 150\left(E_L(1) + E_L(2) + E_L(3)\right) + 5\end{aligned} \tag{8.25a}$$

s.t.

$$E_G(1) + 3.30 + P_S^d(1) = E_L(1) + P_S^c(1) + P^{\mathrm{D}}(1),$$

$$E_G(2) + 2.55 + P_S^d(2) = E_L(2) + P_S^c(2) + P^{\mathrm{D}}(2),$$
$$E_G(3) + 3.00 + P_S^d(3) = E_L(3) + P_S^c(3) + P^{\mathrm{D}}(3), \tag{8.25b}$$

$$v(1) \leq P_G(1) \leq 5v(1),\;\; v(2) \leq P_G(2) \leq 5v(2),\;\; v(3) \leq P_G(3) \leq 5v(3), \tag{8.25c}$$
$$C^{\mathrm{SU}}(1) \geq 10\,(v(1) - 1),\;\; C^{\mathrm{SU}}(2) \geq 10\,(v(2) - v(1)),$$
$$C^{\mathrm{SU}}(3) \geq 10\,(v(3) - v(2)), \tag{8.25d}$$
$$C^{\mathrm{SU}}(1),\;\; C^{\mathrm{SU}}(2), C^{\mathrm{SU}}(3) \geq 0, \tag{8.25e}$$
$$P_G(1) - 2 \leq 2, P_G(2) - P_G(1) \leq 2,\;\; P_G(3) - P_G(2) \leq 2, \tag{8.25f}$$
$$2 - P_G(1) \leq 2\;\; P_G(1) - P_G(2) \leq 2,\;\; P_G(2) - P_G(3) \leq 2, \tag{8.25g}$$
$$E_G(1) = 1 + \frac{P_G(1)}{2},\;\; E_G(2) = \frac{P_G(1) + P_G(2)}{2},$$
$$E_G(3) = \frac{P_G(2) + P_G(3)}{2}, \tag{8.25h}$$
$$v(1),\;\; v(2),\;\; v(3 \in \{0, 1\}, \tag{8.25i}$$

$$0.5 \leq P_L(1) \leq 2,\;\; 0.5 \leq P_L(2) \leq 2,\;\; 0.5 \leq P_L(3) \leq 2, \tag{8.25j}$$
$$P_L(1) - 1.5 \leq 1,\;\; P_L(2) - P_L(1) \leq 1,\;\; P_L(3) - P_L(2) \leq 1, \tag{8.25k}$$
$$1.5 - P_L(1) \leq 1,\;\; P_L(1) - P_L(2) \leq 1,\;\; P_L(2) - P_L(3) \leq 1, \tag{8.25l}$$
$$E_L(1) = \frac{1.5 + P_L(1)}{2},\;\; E_L(2) = \frac{P_L(1) + P_L(2)}{2},$$
$$E_L(3) = \frac{P_L(2) + P_L(3)}{2}, \tag{8.25m}$$
$$E_L(1) + E_L(2) + E_L(3) \geq 2.5, \tag{8.25n}$$

$$E_S(1) = 0.4 + 0.8P_S^c(1) - 1.25P_S^d(1),$$
$$E_S(2) = E_S(1) + 0.8P_S^c(2) - 1.25P_S^d(2),$$
$$E_S(3) = E_S(2) + 0.8P_S^c(3) - 1.25P_S^d(3), \tag{8.25o}$$
$$0.2 \leq E_S(1) \leq 1,\;\; 0.2 \leq E_S(2) \leq 1,\;\; 0.2 \leq E_S(3) \leq 1, \tag{8.25p}$$
$$0 \leq P_S^c(1) \leq 0.3,\;\; 0 \leq P_S^c(2) \leq 0.3,\;\; 0 \leq P_S^c(3) \leq 0.3, \tag{8.25q}$$
$$0 \leq P_S^d(1) \leq 0.5,\;\; 0 \leq P_S^d(2) \leq 0.5,\;\; 0 \leq P_S^d(3) \leq 0.5. \tag{8.25r}$$

Note that the blocks of Eqs. (8.25c)–(8.25i), (8.25j)–(8.25n), and (8.25o)–(8.25r) pertain, respectively, to the modeling of the GT unit, the gear factory, and the PSP introduced in Examples 8.1, 8.2, and 8.3.

Table 8.2 VPP operation based on the expected-value solution strategy. Profit ρ = $814.04. Powers in megawatt and energy in megawatt-hour

Period t	$P^{\mathrm{D}}(t)$	$v(t)$	$P_G(t)$	$E_G(t)$	$P_L(t)$	$E_L(t)$	$P_S^c(t)$	$P_S^d(t)$	$E_S(t)$
1	4.25	1	4	3.0	2	1.75	0.3	0	0.64
2	5.90	1	5	4.5	1	1.50	0	0.35	0.20
3	5.50	1	3	4.0	2	1.50	0	0	0.20

The solution to the profit-maximization problem (8.25) is $P^{\mathrm{D}}(1) = 4.25$ MW, $P^{\mathrm{D}}(2) = 5.90$ MW, and $P^{\mathrm{D}}(3) = 5.50$ MW. In other words, the manager of the VPP should sell 4.25, 5.90, and 5.50 MW in the market in periods 1, 2, and 3, respectively. Furthermore, based on the point forecasts of the PV power output, he expects to operate the VPP components as described in Table 8.2 and makes a profit $\rho = \$814.04$. Observe that the PSP pumps water in period 1, when the market price is the lowest, and operates in turbine mode in period 2, when the market price is the highest. This way, the PSP brings economic benefits to the VPP. Furthermore, since the market price may signal the amount of renewable energy in the power system—as explained in Chap. 6, electricity production from renewable sources tends to drive the market price down—storage units may facilitate the large-scale integration of renewable energy within a market context by absorbing energy in those periods when renewable production is abundant and the electricity price low, and moving it to periods when renewable production is scarce and the electricity price high.

The behavior of both the GT unit and the gear factory is also largely driven by the market price. In particular, the amount of energy produced by the GT unit tends to increase in high-price periods and decrease in low-price ones, while the reverse is true for the amount of energy consumed by the gear factory. Logically, these general trends are partly distorted as a result of the technical constraints.

Point forecasts provide information on the expected power output of stochastic generating units, but they do not give any hint on how wrong this expectation can be. The mismatch between the actual production from these units and the point forecasts, i.e., the so-called *forecast error*, must be consequently accommodated by re-dispatching the VPP components. Since these components have limited flexibility, the practical implementation of the expected-value solution strategy to optimize the operation of a VPP may prove to be particularly problematic and inefficient, as illustrated in the following example.

Example 8.6 (Problems with the expected-value solution) The manager of the VPP in Example 8.5 is aware that the point forecasts he gets from forecasters are always wrong to some extent. So he wonders what may happen if he sells 4.25, 5.90, and 5.50 MW in the market in periods 1, 2, and 3, respectively, as suggested by the expected-value solution calculated in Example 8.5, but the PV power output that eventually realizes is different from the point forecasts. To shed some light on this issue, he decides to test the expected-value solution on the three scenarios of PV power production contained in Table 8.1. For this purpose, he solves the following optimization problem three times, each for a particular scenario ω:

$$\begin{aligned}
\text{Max. } \rho_\omega = {} & 20P^{\mathrm{D}*}(1) + 80P^{\mathrm{D}*}(2) + 45P^{\mathrm{D}*}(3) \\
& - \Big[5\left(E_{G\omega}(1)^2 + E_{G\omega}(2)^2 + E_{G\omega}(3)^2\right) + 10\big(E_{G\omega}(1) + E_{G\omega}(2) \\
& + E_{G\omega}(3)\big) + 50\big(v_\omega(1) + v_\omega(2) + v_\omega(3)\big)\Big] - \big(C_\omega^{\mathrm{SU}}(1) + C_\omega^{\mathrm{SU}}(2) \\
& + C_\omega^{\mathrm{SU}}(3)\big) - 30\left(E_{L\omega}(1)^2 + E_{L\omega}(2)^2 + E_{L\omega}(3)^2\right) \\
& + 150\left(E_{L\omega}(1) + E_{L\omega}(2) + E_{L\omega}(3)\right) + 5
\end{aligned} \tag{8.26a}$$

s.t.

$$\begin{aligned}
& E_{G\omega}(1) + P_{W\omega}(1) + P_{S\omega}^{d}(1) = E_{L\omega}(1) + P_{S\omega}^{c}(1) + P^{\mathrm{D}*}(1), \\
& E_{G\omega}(2) + P_{W\omega}(2) + P_{S\omega}^{d}(2) = E_{L\omega}(2) + P_{S\omega}^{c}(2) + P^{\mathrm{D}*}(2), \\
& E_{G\omega}(3) + P_{W\omega}(3) + P_{S\omega}^{d}(3) = E_{L\omega}(3) + P_{S\omega}^{c}(3) + P^{\mathrm{D}*}(3),
\end{aligned} \tag{8.26b}$$

$$\begin{aligned}
& v_\omega(1) \le P_{G\omega}(1) \le 5v_\omega(1), \;\; v_\omega(2) \le P_{G\omega}(2) \le 5v_\omega(2), \\
& v_\omega(3) \le P_{G\omega}(3) \le 5v_\omega(3),
\end{aligned} \tag{8.26c}$$

$$\begin{aligned}
& C_\omega^{\mathrm{SU}}(1) \ge 10\left(v_\omega(1) - 1\right), \;\; C_\omega^{\mathrm{SU}}(2) \ge 10\left(v_\omega(2) - v_\omega(1)\right), \\
& C_\omega^{\mathrm{SU}}(3) \ge 10\left(v_\omega(3) - v_\omega(2)\right),
\end{aligned} \tag{8.26d}$$

$$C_\omega^{\mathrm{SU}}(1), \;\; C_\omega^{\mathrm{SU}}(2), \;\; C_\omega^{\mathrm{SU}}(3) \ge 0, \tag{8.26e}$$

$$P_{G\omega}(1) - 2 \le 2, \;\; P_{G\omega}(2) - P_{G\omega}(1) \le 2, \;\; P_{G\omega}(3) - P_{G\omega}(2) \le 2, \tag{8.26f}$$

$$2 - P_{G\omega}(1) \le 2, \;\; P_{G\omega}(1) - P_{G\omega}(2) \le 2, \;\; P_{G\omega}(2) - P_{G\omega}(3) \le 2, \tag{8.26g}$$

$$\begin{aligned}
& E_{G\omega}(1) = 1 + \frac{P_{G\omega}(1)}{2}, \;\; E_{G\omega}(2) = \frac{P_{G\omega}(1) + P_{G\omega}(2)}{2}, \\
& E_{G\omega}(3) = \frac{P_{G\omega}(2) + P_{G\omega}(3)}{2},
\end{aligned} \tag{8.26h}$$

$$v_\omega(1), \;\; v_\omega(2), \;\; v_\omega(3) \in \{0, 1\}, \tag{8.26i}$$

$$0.5 \le P_{L\omega}(1) \le 2, \;\; 0.5 \le P_{L\omega}(2) \le 2, \;\; 0.5 \le P_{L\omega}(3) \le 2, \tag{8.26j}$$

$$P_{L\omega}(1) - 1.5 \le 1, \;\; P_{L\omega}(2) - P_{L\omega}(1) \le 1, \;\; P_{L\omega}(3) - P_{L\omega}(2) \le 1, \tag{8.26k}$$

$$1.5 - P_{L\omega}(1) \le 1, \;\; P_{L\omega}(1) - P_{L\omega}(2) \le 1, \;\; P_{L\omega}(2) - P_{L\omega}(3) \le 1, \tag{8.26l}$$

$$\begin{aligned}
& E_{L\omega}(1) = \frac{1.5 + P_{L\omega}(1)}{2}, \;\; E_{L\omega}(2) = \frac{P_{L\omega}(1) + P_{L\omega}(2)}{2}, \\
& E_{L\omega}(3) = \frac{P_{L\omega}(2) + P_{L\omega}(3)}{2},
\end{aligned} \tag{8.26m}$$

$$E_{L\omega}(1) + E_{L\omega}(2) + E_{L\omega}(3) \ge 2.5, \tag{8.26n}$$

Table 8.3 VPP operation based on the expected-value solution strategy under scenario 1. Profit ρ_1 = \$994.01. Powers in megawatt and energy in megawatt-hour

Period t	$P^{D*}(t)$	$v_1(t)$	$P_{G1}(t)$	$E_{G1}(t)$	$P_{L1}(t)$	$E_{L1}(t)$	$P^c_{S1}(t)$	$P^d_{S1}(t)$	$E_{S1}(t)$
1	4.25	1	4	3	1.00	1.25	0	0.00	0.40
2	5.90	1	2	3	1.50	1.25	0	0.16	0.20
3	5.50	0	0	1	1.50	1.50	0	0	0.20

$$E_{S\omega}(1) = 0.4 + 0.8P^c_{S\omega}(1) - 1.25P^d_{S\omega}(1),$$

$$E_{S\omega}(2) = E_{S\omega}(1) + 0.8P^c_{S\omega}(2) - 1.25P^d_{S\omega}(2),$$

$$E_{S\omega}(3) = E_{S\omega}(2) + 0.8P^c_{S\omega}(3) - 1.25P^d_{S\omega}(3), \tag{8.26o}$$

$$0.2 \le E_{S\omega}(1) \le 1, \;\; 0.2 \le E_{S\omega}(2) \le 1, \;\; 0.2 \le E_{S\omega}(3) \le 1, \tag{8.26p}$$

$$0 \le P^c_{S\omega}(1) \le 0.3, \;\; 0 \le P^c_{S\omega}(2) \le 0.3, \;\; 0 \le P^c_{S\omega}(3) \le 0.3, \tag{8.26q}$$

$$0 \le P^d_{S\omega}(1) \le 0.5, \;\; 0 \le P^d_{S\omega}(2) \le 0.5, \;\; 0 \le P^d_{S\omega}(3) \le 0.5, \tag{8.26r}$$

where $P^{D*}(1)$, $P^{D*}(2)$, and $P^{D*}(3)$ are here *fixed* at the expected-value solution, that is, $P^{D*}(1) = 4.25$ MW, $P^{D*}(2) = 5.90$ MW, and $P^{D*}(3) = 5.50$ MW.

Each scenario ω is characterized by a vector $[P_{W\omega}(1), P_{W\omega}(2), P_{W\omega}(3)]$ that describes a plausible realization of the PV power output over the 3-h time horizon. Scenario $\omega = 1$ is given by $[P_{W1}(1), P_{W1}(2), P_{W1}(3)] = [2.5, 4.0, 6.0]$ MW. If this scenario materializes in the end, the VPP should be operated as indicated in Table 8.3. The forecast error associated with this scenario is computed as $[2.50, 4.00, 6.00] - [3.30, 2.55, 3.00] = [-0.80, 1.45, 3.00]$ MW. The 0.80-MW deficit in time period 1 would then be compensated for by decreasing the energy consumption of the gear factory and canceling the pumping of water to the upper reservoir of the pumped-storage system, both with respect to the operational schedule shown in Table 8.2. The power surplus in the remaining two periods would be mostly absorbed by decreasing the power production of the GT unit. Since, overall, scenario 1 involves a PV power production higher than the point forecast, the occurrence of this scenario would lead to a profit increase of 994.01 − 814.04 = \$179.97, which is directly linked to the consequent reduction of the GT power output.

The forecast error associated with scenario $\omega = 2$ is [6.00, 4.00, 3.50] − [3.30, 2.55, 3.00] = [2.70, 1.45, 0.50] MW. This scenario is, therefore, characterized by a much higher power production from the PV unit than expected, particularly, in the first two time periods. Optimization problem (8.26) turns out to be infeasible under this scenario, for the reason that the GT unit, the PSP and the gear factory are not flexible enough to absorb the corresponding extra amount of energy from the PV unit. However, the manager of the VPP is not concerned about scenario 2, because, in case it finally realizes, there is one corrective measure, technically easy to implement, that he can take: to curtail PV production. Mathematically, one can enable generation curtailment in optimization problem (8.26) by rewriting the energy

Table 8.4 VPP operation based on the expected-value solution strategy under scenario 2. Profit $\rho_2 = \$942.59$. Powers in megawatts and energy in megawatt-hour

Period t	$P^{D*}(t)$	$v_2(t)$	$P_{G2}(t)$	$E_{G2}(t)$	$P_{L2}(t)$	$E_{L2}(t)$	$P^c_{S2}(t)$	$P^d_{S2}(t)$	$E_{S2}(t)$	$P^{\text{curt}}_{W2}(t)$
1	4.25	1	2.19	2.09	2.00	1.75	0.3	0	0.64	1.79
2	5.90	1	4.19	3.19	1.27	1.64	0	0.35	0.20	0
3	5.50	1	3.09	3.64	2.00	1.64	0	0	0.20	0

balance Eqs. (8.26b) as follows:

$$\begin{aligned} E_{G\omega}(1) + P_{W\omega}(1) - P^{\text{curt}}_{W\omega}(1) + P^d_{S\omega}(1) &= E_{L\omega}(1) + P^c_{S\omega}(1) + P^{D*}(1),\\ E_{G\omega}(2) + P_{W\omega}(2) - P^{\text{curt}}_{W\omega}(2) + P^d_{S\omega}(2) &= E_{L\omega}(2) + P^c_{S\omega}(2) + P^{D*}(2),\\ E_{G\omega}(3) + P_{W\omega}(3) - P^{\text{curt}}_{W\omega}(3) + P^d_{S\omega}(3) &= E_{L\omega}(3) + P^c_{S\omega}(3) + P^{D*}(3), \end{aligned} \tag{8.27}$$

where $P^{\text{curt}}_{W\omega}(t)$ represents the amount of PV production that is curtailed in time period t. Since the curtailment $P^{\text{curt}}_{W\omega}(t)$ can neither be negative, nor exceed the actual amount of PV production, the following constraints must be enforced as well:

$$0 \le P^{\text{curt}}_{W\omega}(t) \le P_{W\omega}(t), \quad \forall t. \tag{8.28}$$

Table 8.4 specifies how the VPP should be operated if scenario 2 comes true. Observe that the limited flexibility of the VPP components would make it impossible to fully exploit the PV power production under this scenario and consequently, 1.79 MWh of solar energy would have to be wasted in period 1. This operation plan would result in a profit $\rho_2 = \$942.59$ that is \$51.42 lower than that of scenario 1, even though the availability of energy from the PV unit is 1 MWh higher in scenario 2.

Last, the forecast error associated with scenario 3 is [2.00, 1.10, 1.50] − [3.30, 2.55, 3.00] = −[1.30, 1.45, 1.50] MW, which is significantly negative all over the time horizon. Optimization problem (8.26) is also infeasible under this scenario. In this case, the GT unit, the gear factory, and the PSP are not flexible enough to cover such a shortage of PV generation. The manager of the VPP does not like this scenario at all, because if it eventually happens, he only comes up with one possible way to remedy the situation for now: to resort to the old 0.5-MW diesel generating set he owns and uses in emergency situations. This diesel set is, unfortunately, remarkably inefficient and costly, and its production cost amounts to \$200 per each megawatt-hour of electricity it generates. There is, however, a good thing about the diesel generator, namely, it is so flexible that it has no other technical constraint than its 0.5-MW capacity limit.

Mathematically, this new generating unit can be easily modeled by introducing a new group of variables $E^{\text{diesel}}_{G\omega}(t)$, $t = 1, 2, \ldots, T$, into constraints (8.27) as follows:

$$\begin{aligned} E_{G\omega}(1) + E^{\text{diesel}}_{G\omega}(1) + P_{W\omega}(1) - P^{\text{curt}}_{W\omega}(1) + P^d_{S\omega}(1) &= E_{L\omega}(1) + P^c_{S\omega}(1) + P^{D*}(1),\\ E_{G\omega}(2) + E^{\text{diesel}}_{G\omega}(2) + P_{W\omega}(2) - P^{\text{curt}}_{W\omega}(2) + P^d_{S\omega}(2) &= E_{L\omega}(2) + P^c_{S\omega}(2) + P^{D*}(2),\\ E_{G\omega}(3) + E^{\text{diesel}}_{G\omega}(3) + P_{W\omega}(3) - P^{\text{curt}}_{W\omega}(3) + P^d_{S\omega}(3) &= E_{L\omega}(3) + P^c_{S\omega}(3) + P^{D*}(3). \end{aligned} \tag{8.29}$$

Table 8.5 VPP operation based on the expected-value solution strategy under scenario 3. Profit ρ_3 = \$386.54. Powers in megawatts and energy in megawatt-hour

Period t	$P^{D*}(t)$	$v_3(t)$	$P_{G3}(t)$	$E_{G3}(t)$	$P_{L3}(t)$	$E_{L3}(t)$	$P^c_{S3}(t)$	$P^d_{S3}(t)$	$E_{S3}(t)$	$E^{\text{diesel}}_{G3}(t)$
1	4.25	1	4	3.0	0.5	1.0	0.22	0	0.58	0.47
2	5.90	1	5	4.5	0.5	0.5	0	0.30	0.20	0.50
3	5.50	1	5	5.0	1.5	1.0	0	0	0.20	0

Logically, the amount of electricity that is produced by the diesel set, $E^{\text{diesel}}_{G\omega}(t)$, must be positive and bounded from above by 0.5MW $\times$ 1h, i.e.,

$$0 \leq E^{\text{diesel}}_{G\omega}(t) \leq 0.5. \tag{8.30}$$

Then, by (i) substituting (8.29) for (8.26b), (ii) including (8.28) and (8.30) into model (8.26), (iii) adding the generating cost term $-200E^{\text{diesel}}_{G\omega}(t)$ to the objective function (8.26a), and (iv) solving the resulting profit-maximization problem, the manager of the VPP concludes that, under scenario 3, he should operate the VPP as described in Table 8.5.

Note that the diesel generating set is utilized at almost full capacity in the first two periods. Besides, in order to minimize the use of this costly generator, the VPP would have to implement the following three actions:

1. To increase GT production as much and fast as possible.
2. To reduce the power consumption of the gear factory to its minimum in periods 1 and 2.
3. To pump water to the upper basin of the pumped-storage system in period 1 to release it back in period 2, thus contributing to fulfill the contract obligation of selling 5.90 MWh in the market in this period.

Despite these corrective measures, scenario 3 would lead to a profit ρ_3 equal to \$386.54, which is far lower than the profit the VPP would make in scenarios 1 and 2, where the diesel set is not needed.

Since each scenario ω has an associated probability of occurrence π_ω (see Table 8.1), the manager of the VPP can now estimate the expected profit $\mathbb{E}\{\rho\}$ derived from the implementation of the expected-value solution strategy, that is,

$$\begin{aligned}\mathbb{E}_\omega\{\rho\} = \sum_{\omega=1}^{N_\Omega} \pi_\omega \rho_\omega = 0.2 \times \$994.01 + 0.3 \times \$942.59 \\ + 0.5 \times \$386.54 = \$674.85.\end{aligned} \tag{8.31}$$

8.4 Managing Stochastic Energy Sources in a Virtual Power Plant

Before we go any further into this chapter, let us consider the decision-making process faced by the manager of a VPP. This process basically consists of two stages, namely:

1. First, the manager of the VPP must decide on the amount of energy to sell in or buy from the electricity market.
2. Once the energy exchange with the market has been settled, the manager has to operate the VPP on a real-time basis to accommodate the variability of its stochastic energy sources, while meeting contract obligations.

Therefore, the second stage of this decision-making process, which has to do with the actual operation of the VPP, is highly influenced by both the trading decisions made in the first stage and the uncertainties related to the stochastic energy sources that form part of it. Since point forecasts fail to provide any information on the magnitude of these uncertainties, the associated expected-value solution may underestimate the VPP operating costs or lead to the nonfulfilment of the market obligations.

In order to overcome these drawbacks, we describe in this section the fundamentals of how to manage a VPP using *robust optimization* and *stochastic programming*. Both mathematical frameworks provide support to make decisions under uncertainty. In particular, robust optimization uses information on how "big" uncertainties are to construct trading decisions (contract obligations) that are *immune* to them, i.e., that are feasible under any plausible realization of these uncertainties. The trading strategy based on robust optimization is illustrated by means of the following example.

Example 8.7 (Robust trading strategy) The manager of the VPP is very unsatisfied with the market decisions resulting from the expected-value solution. More specifically, he is aware that the diesel generating set is awfully inefficient. For this reason, he is determined to come up with a more *robust* trading strategy that avoids the utilization of the diesel set at all costs.

After some thought, the manager of the VPP comes to the conclusion that, since the PV generation can be curtailed with no trouble, the use of the diesel generator can be easily avoided *in any case* if the VPP operation is optimized by assuming a sufficiently low value for the PV power output. It turns out, besides, that he has a proper estimate of such a value: the minimum power production, $P_W^{\mathrm{lo}}(t)$, that, according to forecasters, one can expect from the PV unit in each time period t. This information is indeed available as part of the forecast intervals given in Table 8.1.

Therefore, to obtain a robust set of trading decisions $P^{\mathrm{D}}(1)$, $P^{\mathrm{D}}(2)$, and $P^{\mathrm{D}}(3)$, all he has to do is to solve an optimization problem similar to (8.25) where the point forecast $[\widehat{P}_W(1), \widehat{P}_W(2), \widehat{P}_W(3)] = [3.30, 2.55, 3.00]$ MW is instead replaced with the vector $[P_W^{\mathrm{lo}}(1), P_W^{\mathrm{lo}}(2), P_W^{\mathrm{lo}}(3)] = [2.0, 1.1, 1.5]$ MW, that is,

$$\begin{aligned}
\text{Max.}\ \ \rho = {} & 20P^{\mathrm{D}}(1) + 80P^{\mathrm{D}}(2) + 45P^{\mathrm{D}}(3) - \Big[5\left(E_G(1)^2 + E_G(2)^2 + E_G(3)^2\right) \\
& +10\left(E_G(1) + E_G(2) + E_G(3)\right) + 50\left(v(1) + v(2) + v(3)\right)\Big] \\
& - \left(C^{\mathrm{SU}}(1) + C^{\mathrm{SU}}(2) + C^{\mathrm{SU}}(3)\right) \\
& - 30\left(E_L^2(1) + E_L^2(2) + E_L^2(3)\right) + 150\left(E_L(1) + E_L(2) + E_L(3)\right) + 5
\end{aligned} \tag{8.32a}$$

s.t.

$$E_G(1) + 2.0 - P_W^{\text{curt}}(1) + P_S^d(1) = E_L(1) + P_S^c(1) + P^{\text{D}}(1),$$
$$E_G(2) + 1.1 - P_W^{\text{curt}}(2) + P_S^d(2) = E_L(2) + P_S^c(2) + P^{\text{D}}(2),$$
$$E_G(3) + 1.5 - P_W^{\text{curt}}(3) + P_S^d(3) = E_L(3) + P_S^c(3) + P^{\text{D}}(3), \tag{8.32b}$$

$$v(1) \le P_G(1) \le 5v(1), \;\; v(2) \le P_G(2) \le 5v(2), \;\; v(3) \le P_G(3) \le 5v(3), \tag{8.32c}$$
$$C^{\text{SU}}(1) \ge 10\,(v(1) - 1), \;\; C^{\text{SU}}(2) \ge 10\,(v(2) - v(1)),$$
$$C^{\text{SU}}(3) \ge 10\,(v(3) - v(2)), \tag{8.32d}$$
$$C^{\text{SU}}(1), \;\; C^{\text{SU}}(2), \;\; C^{\text{SU}}(3) \ge 0, \tag{8.32e}$$
$$P_G(1) - 2 \le 2, \;\; P_G(2) - P_G(1) \le 2, \;\; P_G(3) - P_G(2) \le 2, \tag{8.32f}$$
$$2 - P_G(1) \le 2, \;\; P_G(1) - P_G(2) \le 2, \;\; P_G(2) - P_G(3) \le 2, \tag{8.32g}$$
$$E_G(1) = 1 + \frac{P_G(1)}{2}, \;\; E_G(2) = \frac{P_G(1) + P_G(2)}{2},$$
$$E_G(3) = \frac{P_G(2) + P_G(3)}{2}, \tag{8.32h}$$
$$v(1), \;\; v(2), \;\; v(3) \in \{0, 1\}, \tag{8.32i}$$

$$0.5 \le P_L(1) \le 2, \;\; 0.5 \le P_L(2) \le 2, \;\; 0.5 \le P_L(3) \le 2, \tag{8.32j}$$
$$P_L(1) - 1.5 \le 1, \;\; P_L(2) - P_L(1) \le 1, \;\; P_L(3) - P_L(2) \le 1, \tag{8.32k}$$
$$1.5 - P_L(1) \le 1, \;\; P_L(1) - P_L(2) \le 1, \;\; P_L(2) - P_L(3) \le 1, \tag{8.32l}$$
$$E_L(1) = \frac{1.5 + P_L(1)}{2}, \;\; E_L(2) = \frac{P_L(1) + P_L(2)}{2},$$
$$E_L(3) = \frac{P_L(2) + P_L(3)}{2}, \tag{8.32m}$$
$$E_L(1) + E_L(2) + E_L(3) \ge 2.5, \tag{8.32n}$$

$$E_S(1) = 0.4 + 0.8 P_S^c(1) - 1.25 P_S^d(1),$$
$$E_S(2) = E_S(1) + 0.8 P_S^c(2) - 1.25 P_S^d(2),$$
$$E_S(3) = E_S(2) + 0.8 P_S^c(3) - 1.25 P_S^d(3), \tag{8.32o}$$
$$0.2 \le E_S(1) \le 1, \;\; 0.2 \le E_S(2) \le 1, \;\; 0.2 \le E_S(3) \le 1, \tag{8.32p}$$
$$0 \le P_S^c(1) \le 0.3, \;\; 0 \le P_S^c(2) \le 0.3, \;\; 0 \le P_S^c(3) \le 0.3, \tag{8.32q}$$
$$0 \le P_S^d(1) \le 0.5, \;\; 0 \le P_S^d(2) \le 0.5, \;\; 0 \le P_S^d(3) \le 0.5, \tag{8.32r}$$

$$0 \le P_W^{\text{curt}}(1) \le 2.0, \;\; 0 \le P_W^{\text{curt}}(2) \le 1.1, \;\; 0 \le P_W^{\text{curt}}(3) \le 1.5, \tag{8.32s}$$

Table 8.6 VPP operation based on the robust-solution strategy. Powers in megawatts and energy in megawatt-hour

(a) Scenario 1, profit $\rho_1 = \$933.69$

Period t	$P^{D*}(t)$	$v_1(t)$	$P_{G1}(t)$	$E_{G1}(t)$	$P_{L1}(t)$	$E_{L1}(t)$	$P_{S1}^c(t)$	$P_{S1}^d(t)$	$E_{S1}(t)$	$P_{W1}^{curt}(t)$
1	2.95	1	3.0	2.5	2	1.75	0.3	0	0.64	0
2	4.45	1	1.2	2.1	2	2.00	0	0.35	0.20	0
3	4.00	0	0	0.6	2	2.00	0	0	0.20	0.6

(b) Scenario 2, profit $\rho_2 = \$865.14$

Period t	$P^{D*}(t)$	$v_2(t)$	$P_{G2}(t)$	$E_{G2}(t)$	$P_{L2}(t)$	$E_{L2}(t)$	$P_{S2}^c(t)$	$P_{S2}^d(t)$	$E_{S2}(t)$	$P_{W2}^{curt}(t)$
1	2.95	1	1.0	1.5	2.0	1.75	0.3	0	0.64	2.5
2	4.45	1	3.0	2.0	1.8	1.90	0	0.35	0.20	0
3	4.00	1	1.8	2.4	2.0	1.90	0	0	0.20	0

(c) Scenario 3, profit $\rho_3 = \$604.54$

Period t	$P^{D*}(t)$	$v_3(t)$	$P_{G3}(t)$	$E_{G3}(t)$	$P_{L3}(t)$	$E_{L3}(t)$	$P_{S3}^c(t)$	$P_{S3}^d(t)$	$E_{S3}(t)$	$P_{W3}^{curt}(t)$
1	2.95	1	4	3.0	2	1.75	0.3	0	0.64	0
2	4.45	1	5	4.5	1	1.50	0	0.35	0.20	0
3	4.00	1	3	4.0	2	1.50	0	0	0.20	0

where we have introduced the family of decision variables $P_W^{curt}(1)$, $P_W^{curt}(2)$, and $P_W^{curt}(3)$ to allow for the curtailment of PV generation, if needed.

The solution to the profit-maximization problem (8.32) yields $P^D(1) = 2.95$ MW, $P^D(2) = 4.45$ MW, and $P^D(3) = 4.00$ MW.

In the same manner as he did before with the expected-value solution in Example 8.6, the operator of the VPP wants to test the performance of the new set of trading decisions on the three scenarios of PV power output listed in Table 8.1. To this end, he solves again the ω-parameterized family of optimization problems (8.26) in Example 8.6, but this time with the input vector of trading decisions $[P^{D*}(1), P^{D*}(2), P^{D*}(3)]$ fixed at $[2.95, 4.45, 4.00]$ MW, i.e., the robust solution. Also, remember that the equality constraints (8.26b) must be transformed so as to allow for curtailment of PV generation, $P_{W\omega}^{curt}(t)$, by subtracting the latter quantity from the left-hand side of the equations, as in (8.27).

Tables 8.6(a), 8.6(b) and 8.6(c), in that order, describe how the VPP should be operated under scenarios 1, 2, and 3, if the robust trading strategy is implemented. Compared to the expected-value solution, the robust approach results in less energy being sold in the market. This way the utilization of the expensive diesel set can be avoided even if scenario 3, where the PV power output turns out to be considerably lower than expected, realizes in the end. Consequently, the profit made by the VPP in this scenario proves to be significantly higher under the robust trading policy, in particular, $100 \times (604.54 - 386.54)/386.54 = 56.4$ % higher. On the other hand, the robust solution leads to a larger amount of PV power production being wasted in scenarios 1 and 2, which in turn reduces the profit achieved by the VPP in these two scenarios with respect to the expected-value solution.

To gain further insight into the overall performance of the robust approach, we can compute the resulting expected profit, which is given by

$$\begin{aligned}\mathbb{E}_{\omega}\{\rho\} = \sum_{\omega=1}^{N_{\Omega}} \pi_{\omega}\rho_{\omega} &= 0.2 \times \$933.69 + 0.3 \times \$865.14 \\ &+ 0.5 \times \$604.54 = \$748.55. \end{aligned} \tag{8.33}$$

This result makes the manager of the VPP very happy, because it means an improvement of $100(748.55 - 674.85)/674.85 = 10.9\,\%$ over the expected profit estimated for the expected-value solution in Example 8.6, while averting any potential use of the diesel generating set.

Next, we generalize the robust model introduced in the previous illustrative example for a VPP that includes a set I of dispatchable units, a set J of flexible loads, a set K of storage devices, and a set Q of stochastic generating units. If the uncertain power output of each stochastic generator q is described by the sequence of (conditional) forecast intervals $\{[P^{\mathrm{lo}}_{Wq}(1), P^{\mathrm{up}}_{Wq}(1)], [P^{\mathrm{lo}}_{Wq}(2), P^{\mathrm{up}}_{Wq}(2)], \ldots, [P^{\mathrm{lo}}_{Wq}(T), P^{\mathrm{up}}_{Wq}(T)]\}$, the robust trading strategy is the solution $[P^{\mathrm{D}*}(1), P^{\mathrm{D}*}(2), \ldots, P^{\mathrm{D}*}(T)]$ of the following profit-maximization problem:

$$\begin{aligned}\underset{\Xi}{\mathrm{Max.}}\;\; \rho = \sum_{t=1}^{T} \Bigg[\lambda^{\mathrm{D}}(t) P^{\mathrm{D}}(t)\tau - \sum_{i \in I} \big[C_i(E_{Gi}(t)) \\ + C_i^{\mathrm{SU}}(t) + C_i^{\mathrm{SD}}(t)\big] + \sum_{j \in J} U_j(E_{Lj}(t)) \Bigg] \end{aligned} \tag{8.34a}$$

s.t.

$$\begin{aligned}\sum_{i \in I} E_{Gi}(t) + \left(\sum_{q \in Q} P^{\mathrm{lo}}_{Wq}(t) - P^{\mathrm{curt}}_{Wq}(t) + \sum_{k \in K} P^{d}_{Sk}(t) \right) \tau = \sum_{j \in J} E_{Lj}(t) \\ + \left(\sum_{k \in K} P^{c}_{Sk}(t) + P^{\mathrm{D}}(t) \right) \tau, \;\; \forall t = 1, \ldots, T, \end{aligned} \tag{8.34b}$$

$$G_i\left(P_{Gi}, E_{Gi}, v_i\right) \leq 0, \;\; \forall i \in I, \tag{8.34c}$$

$$v_i \in \{0, 1\}^T, \;\; \forall i \in I, \tag{8.34d}$$

$$L_j\left(P_{Lj}, E_{Lj}\right) \leq 0, \;\; \forall j \in J, \tag{8.34e}$$

$$S_k\left(P^{c}_{Sk}, P^{d}_{Sk}, E_{Sk}\right) \leq 0, \;\; \forall k \in K, \tag{8.34f}$$

$$0 \leq P^{\mathrm{curt}}_{Wq} \leq P^{\mathrm{lo}}_{Wq}, \;\; \forall q \in Q, \tag{8.34g}$$

where, in the same way as in the expected-value model (8.21), the set $\varXi = \{E_{Gi}(t), E_{Lj}(t), P^{d}_{Sk}(t), P^{c}_{Sk}(t), P^{\mathrm{D}}(t), v_i(t), P_{Gi}(t), C_i^{\mathrm{SU}}(t), C_i^{\mathrm{SD}}(t), P_{Lj}(t), E_{Sk}(t), P^{\mathrm{curt}}_{Wq}(t)\}$ comprises the decision variables to be optimized and the blocks of Eqs. (8.34c)–(8.34d), (8.34e), and (8.34f) correspond to the modeling of dispatchable power plants, flexible loads, and storages units, respectively, as explained in Sects. 8.2.1, 8.2.2, and 8.2.3. Furthermore, constraints (8.34g) enforce the limits for the curtailment of stochastic generation. In fact, the key difference between the robust formulation (8.34) and the expected-value model (8.21) lies in the energy balance equations (8.34b), where the lower end $P^{\mathrm{lo}}_{Wq}(t)$ of the forecast interval replaces the conditional expectation $\widehat{P}_{Wq}(t)$ in (8.21b).

The robust approach to operating a VPP and, more particularly, determining its involvement in the electricity market, guarantees that the acquired contract obligations $[P^{\mathrm{D}*}(1), P^{\mathrm{D}*}(2), \ldots, P^{\mathrm{D}*}(T)]$ can be fulfilled under any outcome of random variables $\{\widetilde{P}_{Wq}(1), \ldots, \widetilde{P}_{Wq}(T)\}$ covered by the intervals $\{[P^{\mathrm{lo}}_{Wq}(1), P^{\mathrm{up}}_{Wq}(1)], \ldots, [P^{\mathrm{lo}}_{Wq}(T), P^{\mathrm{up}}_{Wq}(T)]\}$. *Stochastic programming*, in contrast, ensures feasibility by explicitly modeling, and thus *anticipating*, the operation of the VPP for every plausible realization $[P_{Wq\omega}(1), P_{Wq\omega}(2), \ldots, P_{Wq\omega}(T)]$ of uncertainties, or at least a representative group of them. Furthermore, since each of these realizations leads to a certain profit value ρ_ω, stochastic programming focuses on optimizing a certain characteristic of the resulting profit distribution, typically, its expected value, i.e., $\mathbb{E}_\omega[\rho]$.

Therefore, if we now consider a group of N_Ω outcomes—or scenarios—for the series of random variables $\{\widetilde{P}_{Wq}(1), \ldots, \widetilde{P}_{Wq}(T)\}$, indexed by $\omega = 1, \ldots, N_\Omega$ and each with a probability of occurrence π_ω such that $\pi_\omega \geq 0$ and $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$, the stochastic programming solution to the VPP management problem is obtained from the following expected-profit-maximization problem:

$$\begin{aligned}\underset{\varXi^{\mathrm{F}} \cup \varXi^{\mathrm{S}}}{\mathrm{Max.}} \ \mathbb{E}[\rho] = \sum_{t=1}^{T} \Big[\lambda^{\mathrm{D}}(t) P^{\mathrm{D}}(t)\tau + \sum_{\omega=1}^{N_\Omega} \pi_\omega \Big(\sum_{j \in J} U_j(E_{Lj\omega}(t)) \\ - \sum_{i \in I} \big[C_i(E_{Gi\omega}(t)) + C^{\mathrm{SU}}_{i\omega}(t) + C^{\mathrm{SD}}_{i\omega}(t) \big] \Big) \Big] \end{aligned} \tag{8.35a}$$

s.t.

$$\begin{aligned}\sum_{i \in I} E_{Gi\omega}(t) + \left(\sum_{q \in Q} P_{Wq\omega}(t) - P^{\mathrm{curt}}_{Wq\omega}(t) + \sum_{k \in K} P^{d}_{Sk\omega}(t) \right) \tau = \sum_{j \in J} E_{Lj\omega}(t) \\ + \left(\sum_{k \in K} P^{c}_{Sk\omega}(t) + P^{\mathrm{D}}(t) \right) \tau, \ \forall t = 1, \ldots, T, \ \forall \omega = 1, \ldots, N_\Omega, \end{aligned} \tag{8.35b}$$

$$G_i\left(P_{Gi\omega}, E_{Gi\omega}, v_{i\omega}\right) \leq 0, \ \forall i \in I, \ \forall \omega = 1, \ldots, N_\Omega, \tag{8.35c}$$

$$v_{i\omega} \in \{0,1\}^T, \ \forall i \in I, \ \forall \omega = 1, \dots, N_\Omega, \tag{8.35d}$$

$$L_j\left(P_{Lj\omega}, E_{Lj\omega}\right) \le 0, \ \forall j \in J, \ \forall \omega = 1, \dots, N_\Omega, \tag{8.35e}$$

$$S_k\left(P^c_{Sk\omega}, P^d_{Sk\omega}, E_{Sk\omega}\right) \le 0, \ \forall k \in K, \ \forall \omega = 1, \dots, N_\Omega, \tag{8.35f}$$

$$0 \le P^{\text{curt}}_{Wq\omega} \le P^{\text{curt}}_{Wq\omega}, \ \forall q \in Q, \ \forall \omega = 1, \dots, N_\Omega, \tag{8.35g}$$

where we distinguish between two different sets of optimization variables, namely:

1. The set $\Xi^{\text{F}} = \{P^{\text{D}}(t)\}$, which comprises all those decisions that need to be made *before* the power output of stochastic generating units becomes known. These decisions are, therefore, independent of random variables $\{\widetilde{P}_{Wq}(1), \dots, \widetilde{P}_{Wq}(T)\}$ and as such, are written without the subscript ω. In stochastic programming, optimization variables in the set Ξ^{F} are referred to as *first-stage* or *here-and-now* decisions (see Appendix C). These correspond to the energy exchanged by the VPP with the market, $P^{\text{D}}(t)$, in each period t, which is to be determined at the beginning of the time horizon.
2. The set $\Xi^{\text{S}} = \{E_{Gi\omega}(t), E_{Lj\omega}(t), P^d_{Sk\omega}(t), P^c_{Sk\omega}(t), v_{i\omega}(t), P_{Gi\omega}(t), C^{\text{SU}}_{i\omega}(t), C^{\text{SD}}_{i\omega}(t), P_{Lj\omega}(t), E_{Sk\omega}(t), P^{\text{curt}}_{Wq\omega}(t)\}$, which includes all those decisions that are made *as* the generation from stochastic generators becomes known. Consequently, these decisions are contingent on the eventual realization of random variables $\{\widetilde{P}_{Wq}(1), \dots, \widetilde{P}_{Wq}(T)\}$ and are augmented with the scenario index ω, accordingly. In stochastic programming, these type of variables are called *second-stage*, *wait-and-see*, or *recourse* decisions (see Appendix C). In our context, the variables in the set Ξ^{S} define the operation of the VPP, which is to be determined throughout the time horizon once a particular scenario realizes.

In the sequel, the set of optimal decisions $\{P^{\text{D}*}(1), P^{\text{D}*}(2), \dots, P^{\text{D}*}(T)\}$ resulting from optimization problem (8.35) is referred to as *stochastic trading strategy*. The following example illustrates how to compute the optimal involvement of a VPP in the electricity market using stochastic programming.

Example 8.8 (Stochastic trading strategy) It has just dawned on the VPP operator: so far, he has been using the three scenarios in Table 8.1 to test both the expected-value and robust solutions, and compute their associated expected profit. Then, why not to directly exploit these scenarios to design a *better* trading strategy? He still wonders why he had not thought of it sooner. Anyway, it is with this idea in mind that the manager of the VPP has built optimization problem (8.36)–(8.39r), where the blocks of constraints (8.37a)–(8.37r), (8.38a)–(8.38r), and (8.39a)–(8.39r), respectively, model and thus, anticipate the *future* operation of the VPP in case that scenario 1, 2, or 3 realize. The ultimate purpose of problem (8.36–8.39r) is to compute trading decisions $P^{\text{D}}(1)$, $P^{\text{D}}(2)$, and $P^{\text{D}}(3)$ so that the expected profit of the VPP is maximized.

Max.

$$\mathbb{E}[\rho] = 20P^{\text{D}}(1) + 80P^{\text{D}}(2) + 45P^{\text{D}}(3) - 0.2\big[5\left(E_{G1}(1)^2 + E_{G1}(2)^2 + E_{G1}(3)^2\right)$$
$$+ 10\left(E_{G1}(1) + E_{G1}(2) + E_{G1}(3)\right) + 50\left(v_1(1) + v_1(2) + v_1(3)\right)$$

$$
\begin{aligned}
&+ C_1^{\mathrm{SU}}(1) + C_1^{\mathrm{SU}}(2) + C_1^{\mathrm{SU}}(3) \\
&+ 30\left(E_{L1}^2(1) + E_{L1}^2(2) + E_{L1}^2(3)\right) - 150\left(E_{L1}(1) + E_{L1}(2) + E_{L1}(3)\right) - 5\big] \\
&- 0.3\big[5\left(E_{G2}(1)^2 + E_{G2}(2)^2 + E_{G2}(3)^2\right) \\
&+ 10\left(E_{G2}(1) + E_{G2}(2) + E_{G2}(3)\right) + 50\left(v_2(1) + v_2(2) + v_2(3)\right) \\
&+ C_2^{\mathrm{SU}}(1) + C_2^{\mathrm{SU}}(2) + C_2^{\mathrm{SU}}(3) \\
&+ 30\left(E_{L2}^2(1) + E_{L2}^2(2) + E_{L2}^2(3)\right) - 150\left(E_{L2}(1) + E_{L2}(2) + E_{L2}(3)\right) - 5\big] \\
&- 0.5\big[5\left(E_{G3}(1)^2 + E_{G3}(2)^2 + E_{G3}(3)^2\right) \\
&+ 10\left(E_{G3}(1) + E_{G3}(2) + E_{G3}(3)\right) + 50\left(v_3(1) + v_3(2) + v_3(3)\right) \\
&+ C_3^{\mathrm{SU}}(1) + C_3^{\mathrm{SU}}(2) + C_3^{\mathrm{SU}}(3) \\
&+ 30\left(E_{L3}^2(1) + E_{L3}^2(2) + E_{L3}^2(3)\right) - 150\left(E_{L3}(1) + E_{L3}(2) + E_{L3}(3)\right) - 5\big]
\end{aligned}
\tag{8.36}
$$

s.t.

$$
\begin{aligned}
&E_{G1}(1) + 2.5 - P_{W1}^{\mathrm{curt}}(1) + P_{S1}^{d}(1) = E_{L1}(1) + P_{S1}^{c}(1) + P^{\mathrm{D}}(1), \\
&E_{G1}(2) + 4.0 - P_{W1}^{\mathrm{curt}}(2) + P_{S1}^{d}(2) = E_{L1}(2) + P_{S1}^{c}(2) + P^{\mathrm{D}}(2), \\
&E_{G1}(3) + 6.0 - P_{W1}^{\mathrm{curt}}(3) + P_{S1}^{d}(3) = E_{L1}(3) + P_{S1}^{c}(3) + P^{\mathrm{D}}(3),
\end{aligned}
\tag{8.37a}
$$

$$
\begin{aligned}
&v_1(1) \le P_{G1}(1) \le 5v_1(1), \;\; v_1(2) \le P_{G1}(2) \le 5v_1(2), \\
&v_1(3) \le P_{G1}(3) \le 5v_1(3),
\end{aligned}
\tag{8.37b}
$$

$$
\begin{aligned}
&C_1^{\mathrm{SU}}(1) \ge 10\left(v_1(1) - 1\right), \;\; C_1^{\mathrm{SU}}(2) \ge 10\left(v_1(2) - v_1(1)\right), \\
&C_1^{\mathrm{SU}}(3) \ge 10\left(v_1(3) - v_1(2)\right),
\end{aligned}
\tag{8.37c}
$$

$$
C_1^{\mathrm{SU}}(1), \;\; C_1^{\mathrm{SU}}(2), \;\; C_1^{\mathrm{SU}}(3) \ge 0,
\tag{8.37d}
$$

$$
P_{G1}(1) - 2 \le 2, \;\; P_{G1}(2) - P_{G1}(1) \le 2, \;\; P_{G1}(3) - P_{G1}(2) \le 2,
\tag{8.37e}
$$

$$
2 - P_{G1}(1) \le 2, \;\; P_{G1}(1) - P_{G1}(2) \le 2, \;\; P_{G1}(2) - P_{G1}(3) \le 2,
\tag{8.37f}
$$

$$
\begin{aligned}
&E_{G1}(1) = 1 + \frac{P_{G1}(1)}{2}, \;\; E_{G1}(2) = \frac{P_{G1}(1) + P_{G1}(2)}{2}, \\
&E_{G1}(3) = \frac{P_{G1}(2) + P_{G1}(3)}{2},
\end{aligned}
\tag{8.37g}
$$

$$
v_1(1), \;\; v_1(2), \;\; v_1(3) \in \{0, 1\},
\tag{8.37h}
$$

$$
0.5 \le P_{L1}(1) \le 2, \;\; 0.5 \le P_{L1}(2) \le 2, \;\; 0.5 \le P_{L1}(3) \le 2,
\tag{8.37i}
$$

$$
P_{L1}(1) - 1.5 \le 1, \;\; P_{L1}(2) - P_{L1}(1) \le 1, \;\; P_{L1}(3) - P_{L1}(2) \le 1,
\tag{8.37j}
$$

$$
1.5 - P_{L1}(1) \le 1, \;\; P_{L1}(1) - P_{L1}(2) \le 1, \;\; P_{L1}(2) - P_{L1}(3) \le 1,
\tag{8.37k}
$$

$$
E_{L1}(1) = \frac{1.5 + P_{L1}(1)}{2}, \;\; E_{L1}(2) = \frac{P_{L1}(1) + P_{L1}(2)}{2},
$$

$$E_{L1}(3) = \frac{P_{L1}(2) + P_{L1}(3)}{2}, \tag{8.37l}$$

$$E_{L1}(1) + E_{L1}(2) + E_{L1}(3) \geq 2.5, \tag{8.37m}$$

$$\begin{aligned} &E_{S1}(1) = 0.4 + 0.8P^{c}_{S1}(1) - 1.25P^{d}_{S1}(1),\\ &E_{S1}(2) = E_{S1}(1) + 0.8P^{c}_{S1}(2) - 1.25P^{d}_{S1}(2),\\ &E_{S1}(3) = E_{S1}(2) + 0.8P^{c}_{S1}(3) - 1.25P^{d}_{S1}(3), \end{aligned} \tag{8.37n}$$

$$0.2 \leq E_{S1}(1) \leq 1,\;\; 0.2 \leq E_{S1}(2) \leq 1,\;\; 0.2 \leq E_{S1}(3) \leq 1, \tag{8.37o}$$

$$0 \leq P^{c}_{S1}(1) \leq 0.3,\;\; 0 \leq P^{c}_{S1}(2) \leq 0.3,\;\; 0 \leq P^{c}_{S1}(3) \leq 0.3, \tag{8.37p}$$

$$0 \leq P^{d}_{S1}(1) \leq 0.5,\;\; 0 \leq P^{d}_{S1}(2) \leq 0.5,\;\; 0 \leq P^{d}_{S1}(3) \leq 0.5, \tag{8.37q}$$

$$0 \leq P^{\text{curt}}_{W1}(1) \leq 2.5,\;\; 0 \leq P^{\text{curt}}_{W1}(2) \leq 4.0,\;\; 0 \leq P^{\text{curt}}_{W1}(3) \leq 6.0, \tag{8.37r}$$

$$\begin{aligned} &E_{G2}(1) + 6.0 - P^{\text{curt}}_{W2}(1) + P^{d}_{S2}(1) = E_{L2}(1) + P^{c}_{S2}(1) + P^{\text{D}}(1),\\ &E_{G2}(2) + 4.0 - P^{\text{curt}}_{W2}(2) + P^{d}_{S2}(2) = E_{L2}(2) + P^{c}_{S2}(2) + P^{\text{D}}(2),\\ &E_{G2}(3) + 3.5 - P^{\text{curt}}_{W2}(3) + P^{d}_{S2}(3) = E_{L2}(3) + P^{c}_{S2}(3) + P^{\text{D}}(3), \end{aligned} \tag{8.38a}$$

$$\begin{aligned} &v_2(1) \leq P_{G2}(1) \leq 5v_2(1),\;\; v_2(2) \leq P_{G2}(2) \leq 5v_2(2),\\ &v_2(3) \leq P_{G2}(3) \leq 5v_2(3), \end{aligned} \tag{8.38b}$$

$$\begin{aligned} &C^{\text{SU}}_{2}(1) \geq 10\,(v_2(1) - 1),\;\; C^{\text{SU}}_{2}(2) \geq 10\,(v_2(2) - v_2(1)),\\ &C^{\text{SU}}_{2}(3) \geq 10\,(v_2(3) - v_2(2)), \end{aligned} \tag{8.38c}$$

$$C^{\text{SU}}_{2}(1),\;\; C^{\text{SU}}_{2}(2),\;\; C^{\text{SU}}_{2}(3) \geq 0, \tag{8.38d}$$

$$P_{G2}(1) - 2 \leq 2,\;\; P_{G2}(2) - P_{G2}(1) \leq 2,\;\; P_{G2}(3) - P_{G2}(2) \leq 2, \tag{8.38e}$$

$$2 - P_{G2}(1) \leq 2,\;\; P_{G2}(1) - P_{G2}(2) \leq 2,\;\; P_{G2}(2) - P_{G2}(3) \leq 2, \tag{8.38f}$$

$$\begin{aligned} &E_{G2}(1) = 1 + \frac{P_{G2}(1)}{2},\;\; E_{G2}(2) = \frac{P_{G2}(1) + P_{G2}(2)}{2},\\ &E_{G2}(3) = \frac{P_{G2}(2) + P_{G2}(3)}{2}, \end{aligned} \tag{8.38g}$$

$$v_2(1),\;\; v_2(2),\;\; v_2(3) \in \{0, 1\}, \tag{8.38h}$$

$$0.5 \leq P_{L2}(1) \leq 2,\;\; 0.5 \leq P_{L2}(2) \leq 2,\;\; 0.5 \leq P_{L2}(3) \leq 2, \tag{8.38i}$$

$$P_{L2}(1) - 1.5 \leq 1,\;\; P_{L2}(2) - P_{L2}(1) \leq 1,\;\; P_{L2}(3) - P_{L2}(2) \leq 1, \tag{8.38j}$$

$$1.5 - P_{L2}(1) \leq 1,\;\; P_{L2}(1) - P_{L2}(2) \leq 1,\;\; P_{L2}(2) - P_{L2}(3) \leq 1, \tag{8.38k}$$

$$\begin{aligned} &E_{L2}(1) = \frac{1.5 + P_{L2}(1)}{2},\;\; E_{L2}(2) = \frac{P_{L2}(1) + P_{L2}(2)}{2},\\ &E_{L2}(3) = \frac{P_{L2}(2) + P_{L2}(3)}{2}, \end{aligned} \tag{8.38l}$$

$$E_{L2}(1) + E_{L2}(2) + E_{L2}(3) \geq 2.5, \tag{8.38m}$$

$$E_{S2}(1) = 0.4 + 0.8P^c_{S2}(1) - 1.25P^d_{S2}(1),$$
$$E_{S2}(2) = E_{S2}(1) + 0.8P^c_{S2}(2) - 1.25P^d_{S2}(2),$$
$$E_{S2}(3) = E_{S2}(2) + 0.8P^c_{S2}(3) - 1.25P^d_{S2}(3), \tag{8.38n}$$

$$0.2 \leq E_{S2}(1) \leq 1, \;\; 0.2 \leq E_{S2}(2) \leq 1, \;\; 0.2 \leq E_{S2}(3) \leq 1, \tag{8.38o}$$

$$0 \leq P^c_{S2}(1) \leq 0.3, \;\; 0 \leq P^c_{S2}(2) \leq 0.3, \;\; 0 \leq P^c_{S2}(3) \leq 0.3, \tag{8.38p}$$

$$0 \leq P^d_{S2}(1) \leq 0.5, \;\; 0 \leq P^d_{S2}(2) \leq 0.5, \;\; 0 \leq P^d_{S2}(3) \leq 0.5, \tag{8.38q}$$

$$0 \leq P^{\text{curt}}_{W2}(1) \leq 6.0, \;\; 0 \leq P^{\text{curt}}_{W2}(2) \leq 4.0, \;\; 0 \leq P^{\text{curt}}_{W2}(3) \leq 3.5, \tag{8.38r}$$

$$E_{G3}(1) + 2.0 - P^{\text{curt}}_{W3}(1) + P^d_{S3}(1) = E_{L3}(1) + P^c_{S3}(1) + P^{\text{D}}(1),$$
$$E_{G3}(2) + 1.1 - P^{\text{curt}}_{W3}(2) + P^d_{S3}(2) = E_{L3}(2) + P^c_{S3}(2) + P^{\text{D}}(2),$$
$$E_{G3}(3) + 1.5 - P^{\text{curt}}_{W3}(3) + P^d_{S3}(3) = E_{L3}(3) + P^c_{S3}(3) + P^{\text{D}}(3), \tag{8.39a}$$

$$v_3(1) \leq P_{G3}(1) \leq 5v_3(1), \;\; v_3(2) \leq P_{G3}(2) \leq 5v_3(2),$$
$$v_3(3) \leq P_{G3}(3) \leq 5v_3(3), \tag{8.39b}$$

$$C^{\text{SU}}_3(1) \geq 10\,(v_3(1) - 1), \;\; C^{\text{SU}}_3(2) \geq 10\,(v_3(2) - v_3(1)),$$
$$C^{\text{SU}}_3(3) \geq 10\,(v_3(3) - v_3(2)), \tag{8.39c}$$

$$C^{\text{SU}}_3(1), \;\; C^{\text{SU}}_3(2), \;\; C^{\text{SU}}_3(3) \geq 0, \tag{8.39d}$$

$$P_{G3}(1) - 2 \leq 2, \;\; P_{G3}(2) - P_{G3}(1) \leq 2, \;\; P_{G3}(3) - P_{G3}(2) \leq 2, \tag{8.39e}$$

$$2 - P_{G3}(1) \leq 2, \;\; P_{G3}(1) - P_{G3}(2) \leq 2, \;\; P_{G3}(2) - P_{G3}(3) \leq 2, \tag{8.39f}$$

$$E_{G3}(1) = 1 + \frac{P_{G3}(1)}{2}, \;\; E_{G3}(2) = \frac{P_{G3}(1) + P_{G3}(2)}{2},$$
$$E_{G3}(3) = \frac{P_{G3}(2) + P_{G3}(3)}{2}, \tag{8.39g}$$

$$v_3(1), \;\; v_3(2), \;\; v_3(3) \in \{0, 1\}, \tag{8.39h}$$

$$0.5 \leq P_{L3}(1) \leq 2, \;\; 0.5 \leq P_{L3}(2) \leq 2, \;\; 0.5 \leq P_{L3}(3) \leq 2, \tag{8.39i}$$

$$P_{L3}(1) - 1.5 \leq 1, \;\; P_{L3}(2) - P_{L3}(1) \leq 1, \;\; P_{L3}(3) - P_{L3}(2) \leq 1, \tag{8.39j}$$

$$1.5 - P_{L3}(1) \leq 1, \;\; P_{L3}(1) - P_{L3}(2) \leq 1, \;\; P_{L3}(2) - P_{L3}(3) \leq 1, \tag{8.39k}$$

$$E_{L3}(1) = \frac{1.5 + P_{L3}(1)}{2}, \;\; E_{L3}(2) = \frac{P_{L3}(1) + P_{L3}(2)}{2},$$
$$E_{L3}(3) = \frac{P_{L3}(2) + P_{L3}(3)}{2}, \tag{8.39l}$$

$$E_{L3}(1) + E_{L3}(2) + E_{L3}(3) \geq 2.5, \tag{8.39m}$$

Table 8.7 VPP operation based on the stochastic-solution strategy. Powers in megawatts and energy in megawatt-hour

(a) Scenario 1, profit $\rho_1 = \$987.69$										
Period t	$P^{D*}(t)$	$v_1(t)$	$P_{G1}(t)$	$E_{G1}(t)$	$P_{L1}(t)$	$E_{L1}(t)$	$P^c_{S1}(t)$	$P^d_{S1}(t)$	$E_{S1}(t)$	$P^{curt}_{W1}(t)$
1	3.22	1	3.18	2.59	2.00	1.75	0.12	0	0.50	0
2	4.83	1	2.00	2.59	1.99	2.00	0	0.24	0.20	0
3	5.00	0	0	1.00	2.00	2.00	0	0	0.20	0
(b) Scenario 2, profit $\rho_2 = \$891.20$										
Period t	$P^{D*}(t)$	$v_2(t)$	$P_{G2}(t)$	$E_{G2}(t)$	$P_{L2}(t)$	$E_{L2}(t)$	$P^c_{S2}(t)$	$P^d_{S2}(t)$	$E_{S2}(t)$	$P^{curt}_{W2}(t)$
1	3.22	1	1.25	1.62	2.00	1.75	0.3	0	0.64	2.35
2	4.83	1	3.25	2.25	1.53	1.76	0	0.35	0.20	0
3	5.00	1	3.29	3.27	2.00	1.76	0	0	0.20	0
(c) Scenario 3, profit $\rho_3 = \$587.08$										
Period t	$P^{D*}(t)$	$v_3(t)$	$P_{G3}(t)$	$E_{G3}(t)$	$P_{L3}(t)$	$E_{L3}(t)$	$P^c_{S3}(t)$	$P^d_{S3}(t)$	$E_{S3}(t)$	$P^{curt}_{W3}(t)$
1	3.22	1	4.00	3.00	1.47	1.48	0.3	0	0.64	0
2	4.83	1	5.00	4.50	0.77	1.12	0	0.35	0.20	0
3	5.00	1	4.55	4.77	1.77	1.27	0	0	0.20	0

$$E_{S3}(1) = 0.4 + 0.8P^c_{S3}(1) - 1.25P^d_{S3}(1),$$

$$E_{S3}(2) = E_{S3}(1) + 0.8P^c_{S3}(2) - 1.25P^d_{S3}(2),$$

$$E_{S3}(3) = E_{S3}(2) + 0.8P^c_{S3}(3) - 1.25P^d_{S3}(3), \tag{8.39n}$$

$$0.2 \le E_{S3}(1) \le 1,\;\; 0.2 \le E_{S3}(2) \le 1,\;\; 0.2 \le E_{S3}(3) \le 1, \tag{8.39o}$$

$$0 \le P^c_{S3}(1) \le 0.3,\;\; 0 \le P^c_{S3}(2) \le 0.3,\;\; 0 \le P^c_{S3}(3) \le 0.3, \tag{8.39p}$$

$$0 \le P^d_{S3}(1) \le 0.5,\;\; 0 \le P^d_{S3}(2) \le 0.5,\;\; 0 \le P^d_{S3}(3) \le 0.5, \tag{8.39q}$$

$$0 \le P^{curt}_{W3}(1) \le 2.0,\;\; 0 \le P^{curt}_{W3}(2) \le 1.1,\;\; 0 \le P^{curt}_{W3}(3) \le 1.5, \tag{8.39r}$$

The solution to the maximization problem (8.36)–(8.39r), i.e., the so-called *stochastic trading strategy*, is $P^{D*}(1) = 3.22$ MW, $P^{D*}(2) = 4.83$ MW, and $P^{D*}(3) = 5.00$ MW, which yields an expected profit $\mathbb{E}_\omega[\rho] = \$758.44$. Besides, the operation of the VPP in scenarios 1, 2, and 3 derives directly from this very same problem. This information is gathered in Table 8.7. Observe that, compared to the robust solution in Table 8.6, the stochastic trading strategy manages to reduce the amount of PV production that is curtailed in scenarios 1 and 2, thus increasing the profit made by the VPP in both cases. In contrast, the stochastic solution shows a poorer performance in scenario 3, which calls for a higher production from the GT unit and a lower consumption from the gear factory than in the case of the robust strategy. Overall, the stochastic trading strategy leads to an expected profit that is $758.44 - 748.55 = \$9.89$—or 1.32 % in relative terms—higher than that linked to the robust solution.

For ease of comparison, Table 8.8 collates together the expected-value, robust and stochastic solutions, and the corresponding profits per scenario and in expectation. Note that the expected-value trading strategy suggests to sell more energy in the market, which would bring higher profits in scenario 1 and 2, where the PV production

Table 8.8 Expected-value, robust and stochastic trading strategies, and their associated profits per scenario and in expectation. Powers in megawatts and profit in dollars

Strategy	Trading decision $P^{D*}(t)$			Profit ρ_ω			$\mathbb{E}_\omega[\rho]$
	$P^{D*}(1)$	$P^{D*}(2)$	$P^{D*}(3)$	ρ_1	ρ_2	ρ_3	
Expected value	4.25	5.90	5.50	994.01	942.59	386.54	674.85
Robust	2.95	4.45	4.00	933.69	865.14	604.54	748.55
Stochastic	3.22	4.83	5.00	987.69	891.20	587.08	758.44

is abundant, while yielding a very low profit in scenario 3 due to the *unavoidable* use of the expensive and inefficient diesel generating set. On the other hand, the robust solution constitutes the trading strategy that places the lowest amount of energy in the market and consequently, it proves to be the strategy that performs best in the *worst-case* realization of PV power output, i.e., in scenario 3. Finally, the stochastic trading strategy can be seen as a compromise solution that performs relatively well in all the three scenarios and as a result, leads to the highest expected profit.

Nonetheless, the manager of the VPP is not 100 % enthusiastic about the stochastic solution for two reasons. First, it has become apparent to him that the stochastic programming model (8.36)–(8.39r) is much "bigger" than its expected-value and robust counterparts, as it includes as many blocks of equations of the type (8.37a)–(8.37r) as the number of considered scenarios. Second, what if the scenario that eventually realizes is not among those provided by the forecasters? To know the answer to this question, the reader is encouraged to solve Exercise 8.5 of this chapter.

The three offering models (8.21), (8.34), and (8.35) that we have presented so far to compute, respectively, the expected-value, robust, and stochastic trading strategies suffer from a basic flaw: They consider the market price $\lambda^D(t)$ as certain. This is, however, far from the truth. Electricity prices are actually unknown at the time producers and consumers submit their selling offers and purchasing bids to the market. As a result, trading decisions by a VPP are to be made with incomplete information on both the power output of its stochastic generating units and the market price. It follows thus that $\lambda^D(t)$ in (8.21), (8.34), and (8.35) should be treated in reality as a random variable and consistently denoted by $\widetilde{\lambda}^D(t)$. As in the case of the power produced by stochastic generating unit q, i.e., $\widetilde{P}_{Wq}$, the (conditional) information on the series of random variables $\{\widetilde{\lambda}^D(1), \widetilde{\lambda}^D(2), \ldots, \widetilde{\lambda}^D(T)\}$ can be alternatively provided in the form of point forecasts, forecast intervals, or scenarios.

In Example 8.5 we concluded that the operation of the components of a VPP was, to a large extent, driven by the market price. Therefore, it seems reasonable to expect that price uncertainty will have a significant adverse impact on the VPP profit. Fortunately, electricity markets allow producers and consumers to trade using, respectively, offer and bid *curves*. In particular, every producer is permitted to submit an *offer curve* for each time period t of the market horizon, expressing how much electricity it is willing to sell in the market at each price level. Most markets, though, require offer curves be nondecreasing.

Table 8.9 Scenario set modeling the uncertainty of both the market price $\widetilde{\lambda}^{\mathrm{D}}(t)$ and the PV power output $\widetilde{P}_W(t)$. Powers are in megawatts and prices in dollars per megawatt-hour

Period t	Scenario 1		Scenario 2		Scenario 3	
	$\lambda_1^{\mathrm{D}}(t)$	$P_{W1}(t)$	$\lambda_2^{\mathrm{D}}(t)$	$P_{W2}(t)$	$\lambda_3^{\mathrm{D}}(t)$	$P_{W3}(t)$
1	18	2.5	38	6.0	10	2.0
2	90	4.0	100	4.0	64	1.1
3	20	6.0	40	3.5	58	1.5
Probability π_ω	0.2		0.3		0.5	

Mathematically, the possibility of selling electricity using offer curves makes trading decision $P^{\mathrm{D}}(t)$ in offering models (8.21), (8.34), and (8.35) a function of the electricity price, i.e., $P^{\mathrm{D}}(t, \widetilde{\lambda}^{\mathrm{D}}(t))$. Such a function can be easily optimized if we consider a scenario-based modeling of random variable $\widetilde{\lambda}^{\mathrm{D}}(t)$, as this allows us to associate each plausible price outcome $\lambda_\omega^{\mathrm{D}}(t)$ with the corresponding traded quantity $P_\omega^{\mathrm{D}}(t)$ obtained from (8.21), (8.34), or (8.35). The nondecreasing condition of the offer curve can then be enforced by stating that, for any pair of price realizations $\lambda_\omega^{\mathrm{D}}(t)$ and $\lambda_{\omega'}^{\mathrm{D}}(t)$ such that $\lambda_\omega^{\mathrm{D}}(t) > \lambda_{\omega'}^{\mathrm{D}}(t)$, the relation $P_\omega^{\mathrm{D}}(t) \geq P_{\omega'}^{\mathrm{D}}(t)$ must hold true. Furthermore, the *nonanticipativity* of the offer must be enforced by requiring that, for any pair of scenarios $\lambda_\omega^{\mathrm{D}}(t)$ and $\lambda_{\omega'}^{\mathrm{D}}(t)$ such that $\lambda_\omega^{\mathrm{D}}(t) = \lambda_{\omega'}^{\mathrm{D}}(t)$, then $P_\omega^{\mathrm{D}}(t) = P_{\omega'}^{\mathrm{D}}(t)$, which guarantees that there is a unique decision for any price level. The resulting collection of optimal quantity-price pairs $\{(P_1^{\mathrm{D}}(t), \lambda_1^{\mathrm{D}}(t)), \ldots, (P_\omega^{\mathrm{D}}(t), \lambda_\omega^{\mathrm{D}}(t)), \ldots, (P_{N_\Omega}^{\mathrm{D}}(t), \lambda_{N_\Omega}^{\mathrm{D}}(t))\}$ defines the optimal offer curve for period t.

The following example illustrates how to build an offer curve using a scenario-based modeling of the market price. Further information on this can be found in [6].

Example 8.9 (Offer curves) Suppose that the scenario set in Table 8.1 (p. 255) has now been extended to include price uncertainty as indicated in Table 8.9. Note that the expected values of $\widetilde{\lambda}^{\mathrm{D}}(1)$, $\widetilde{\lambda}^{\mathrm{D}}(2)$, and $\widetilde{\lambda}^{\mathrm{D}}(3)$ are 20, 80, and \$45/MWh, respectively, which coincide with the series of perfectly known market prices used in the previous illustrative examples.

We show next how to obtain offer curves from stochastic programming model (8.36)–(8.39r) in Example 8.8. To this end, we need to make the following changes to this optimization problem:

1. Firstly, we make trading decisions dependent on the price scenario, i.e., $\{P^{\mathrm{D}}(1), P^{\mathrm{D}}(2), P^{\mathrm{D}}(3)\} \rightarrow \{P_\omega^{\mathrm{D}}(1), P_\omega^{\mathrm{D}}(2), P_\omega^{\mathrm{D}}(3)\}$, and rewrite the energy balance Eqs. (8.37a), (8.38a), and (8.39a) accordingly,

$$\begin{aligned}
&E_{G1}(1) + 2.5 - P_{W1}^{\mathrm{curt}}(1) + P_{S1}^{d}(1) = E_{L1}(1) + P_{S1}^{c}(1) + P_1^{\mathrm{D}}(1),\\
&E_{G1}(2) + 4.0 - P_{W1}^{\mathrm{curt}}(2) + P_{S1}^{d}(2) = E_{L1}(2) + P_{S1}^{c}(2) + P_1^{\mathrm{D}}(2),\\
&E_{G1}(3) + 6.0 - P_{W1}^{\mathrm{curt}}(3) + P_{S1}^{d}(3) = E_{L1}(3) + P_{S1}^{c}(3) + P_1^{\mathrm{D}}(3),
\end{aligned} \tag{8.40a}$$

$$E_{G2}(1) + 6.0 - P_{W2}^{\text{curt}}(1) + P_{S2}^{d}(1) = E_{L2}(1) + P_{S2}^{c}(1) + P_{2}^{\text{D}}(1),$$
$$E_{G2}(2) + 4.0 - P_{W2}^{\text{curt}}(2) + P_{S2}^{d}(2) = E_{L2}(2) + P_{S2}^{c}(2) + P_{2}^{\text{D}}(2),$$
$$E_{G2}(3) + 3.5 - P_{W2}^{\text{curt}}(3) + P_{S2}^{d}(3) = E_{L2}(3) + P_{S2}^{c}(3) + P_{2}^{\text{D}}(3), \tag{8.40b}$$

$$E_{G3}(1) + 2.0 - P_{W3}^{\text{curt}}(1) + P_{S3}^{d}(1) = E_{L3}(1) + P_{S3}^{c}(1) + P_{3}^{\text{D}}(1),$$
$$E_{G3}(2) + 1.1 - P_{W3}^{\text{curt}}(2) + P_{S3}^{d}(2) = E_{L3}(2) + P_{S3}^{c}(2) + P_{3}^{\text{D}}(2),$$
$$E_{G3}(3) + 1.5 - P_{W3}^{\text{curt}}(3) + P_{S3}^{d}(3) = E_{L3}(3) + P_{S3}^{c}(3) + P_{3}^{\text{D}}(3). \tag{8.40c}$$

2. Then, we add the following set of constraints to ensure that the resulting offer curves are nondecreasing,

$$P_{2}^{\text{D}}(1) \geq P_{1}^{\text{D}}(1) \geq P_{3}^{\text{D}}(1), \text{because } \lambda_{2}^{\text{D}}(1) > \lambda_{1}^{\text{D}}(1) > \lambda_{3}^{\text{D}}(1), \tag{8.41a}$$
$$P_{2}^{\text{D}}(2) \geq P_{1}^{\text{D}}(2) \geq P_{3}^{\text{D}}(2), \text{because } \lambda_{2}^{\text{D}}(2) > \lambda_{1}^{\text{D}}(2) > \lambda_{3}^{\text{D}}(2), \tag{8.41b}$$
$$P_{3}^{\text{D}}(3) \geq P_{2}^{\text{D}}(3) \geq P_{1}^{\text{D}}(3), \text{because } \lambda_{3}^{\text{D}}(3) > \lambda_{2}^{\text{D}}(3) > \lambda_{1}^{\text{D}}(3). \tag{8.41c}$$

3. Finally, we reformulate the objective function (8.36) as

$$\begin{aligned}
\text{Max. } \mathbb{E}[\rho] = 0.2\big[&18P_{1}^{\text{D}}(1) + 90P_{1}^{\text{D}}(2) + 20P_{1}^{\text{D}}(3) \\
&- 5\left(E_{G1}(1)^2 + E_{G1}(2)^2 + E_{G1}(3)^2\right) \\
&- 10\left(E_{G1}(1) + E_{G1}(2) + E_{G1}(3)\right) - 50\left(v_1(1) + v_1(2) + v_1(3)\right) \\
&- C_{1}^{\text{SU}}(1) - C_{1}^{\text{SU}}(2) - C_{1}^{\text{SU}}(3) \\
&- 30\left(E_{L1}^{2}(1) + E_{L1}^{2}(2) + E_{L1}^{2}(3)\right) + 150\big(E_{L1}(1) + E_{L1}(2) \\
&+ E_{L1}(3)\big) + 5\big] + 0.3\big[38P_{2}^{\text{D}}(1) + 100P_{2}^{\text{D}}(2) + 40P_{2}^{\text{D}}(3) \\
&- 5\left(E_{G2}(1)^2 + E_{G2}(2)^2 + E_{G2}(3)^2\right) - 10\big(E_{G2}(1) + E_{G2}(2) \\
&+ E_{G2}(3)\big) - 50\big(v_2(1) + v_2(2) + v_2(3)\big) - C_{2}^{\text{SU}}(1) - C_{2}^{\text{SU}}(2) \\
&- C_{2}^{\text{SU}}(3) - 30\big(E_{L2}^{2}(1) + E_{L2}^{2}(2) + E_{L2}^{2}(3)\big) + 150\big(E_{L2}(1) \\
&+ E_{L2}(2) + E_{L2}(3)\big) + 5\big] + 0.5\big[10P_{3}^{\text{D}}(1) + 64P_{3}^{\text{D}}(2) \\
&+ 58P_{3}^{\text{D}}(3) - 5\left(E_{G3}(1)^2 + E_{G3}(2)^2 + E_{G3}(3)^2\right) \\
&- 10\left(E_{G3}(1) + E_{G3}(2) + E_{G3}(3)\right) - 50\left(v_3(1) + v_3(2) + v_3(3)\right) \\
&- C_{3}^{\text{SU}}(1) - C_{3}^{\text{SU}}(2) - C_{3}^{\text{SU}}(3) \\
&- 30\left(E_{L3}^{2}(1) + E_{L3}^{2}(2) + E_{L3}^{2}(3)\right) + 150\big(E_{L3}(1) \\
&+ E_{L3}(2) + E_{L3}(3)\big) + 5\big].
\end{aligned} \tag{8.42}$$

Table 8.10 contains the optimal quantity-price points $\left(P_{\omega}^{\text{D}*}(t), \lambda_{\omega}^{\text{D}}(t)\right)$ that result from the thus modified expected-profit maximization problem. Now we can use these

Table 8.10 Optimal pairs $\left(P_{\omega}^{D*}(t), \lambda_{\omega}^{D}(t)\right)$ making up the offer curve of the VPP in the electricity market per time period t. Powers are in megawatts and prices in dollars per megawatt-hour

Period t	$\left(P_1^{D*}(t), \lambda_1^{D}(t)\right)$	$\left(P_2^{D*}(t), \lambda_2^{D}(t)\right)$	$\left(P_3^{D*}(t), \lambda_3^{D}(t)\right)$
1	(3.54, 18)	(7.30, 38)	(2.45, 10)
2	(5.81, 90)	(7.37, 100)	(3.60, 64)
3	(5.37, 20)	(5.49, 40)	(5.49, 58)

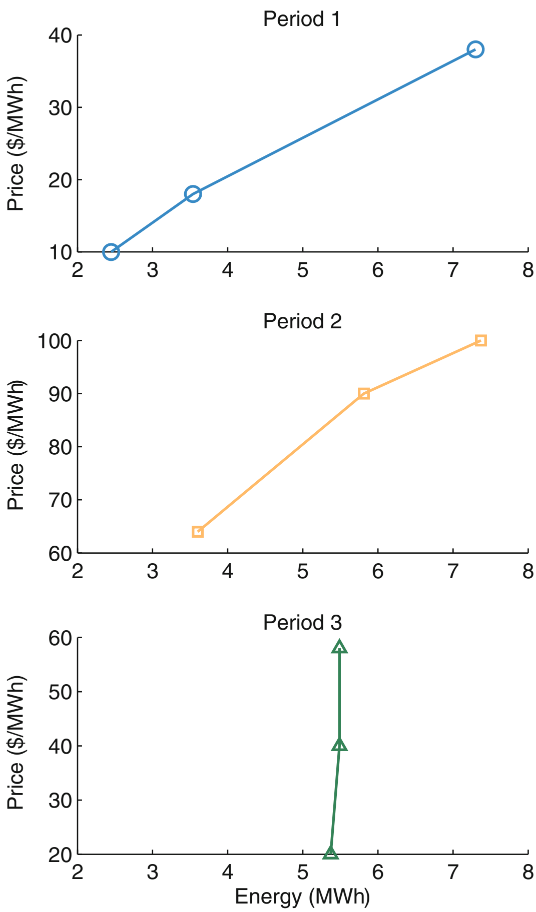

Fig. 8.7 Piecewise linear approximation of the optimal offer curves to be submitted by the virtual power plant in the electricity market for each time period $t = 1, 2$, and 3. These curves are required to be nondecreasing functions of the market price and indicate how much energy the VPP is willing to sell at each price level

points to reconstruct the offer curves using, for example, piecewise linear interpolations, as depicted in Fig. 8.7. It is worth noting that, compared to the other two, the offer curve to be submitted in period 3 is almost a vertical line. The reason for this is that, according to the probabilistic information gathered in Table 8.9, in period 3, the power production from the PV unit is *anticorrelated* with the market price. Indeed, observe that in this period, as opposed to what happens in the other two, the higher the electricity price, the lower the PV power output and hence, the lower the amount of energy the VPP can sell in the market. Notice that this kind of situations cannot be exploited by a nondecreasing offer curve, which boils down to a single quantity (vertical line in Fig. 8.7) as a result.

The implementation of offer curves in the stochastic offering model (8.36)–(8.39r) increases the associated expected profit of the VPP up to \$842.87.

We conclude this section by referring the interested reader to Chaps. 5 and 6 of [6] for a more thorough treatment and modeling of offer curves in an electricity market.

8.5 Extending the Offering Model of a Virtual Power Plant to a Multi-Market Framework

Up to now, we have regarded the electricity market as an infinite supplier/consumer with a constant generating cost/utility equal to the market price, λ^{D}, and whose production/consumption, P^{D}, must be decided prior to the operation of the VPP.

In reality, electricity markets are more complex systems that comprise several trading floors with different closure and lead times (futures market, day-ahead, intra-day, and real-time markets ...) and where various commodities are usually traded (e.g., energy and reserve) through diverse financial products (futures contracts, options, swaps, day-ahead contracts, etc.). On top of all this, the economic activity in the electricity market is strongly conditioned by the proper functioning and maintenance of the underlying electric infrastructure. References [5; 6; 19], among others, provides good overviews of the architecture of a typical electricity market.

From the point of view of a VPP, the availability of different trading arenas within an electricity market translates into more opportunities to increase its profit by means of a well-designed trading strategy. In Chaps. 3, 4, and 7 of this book, we highlighted two trading floors above all others: the day-ahead and the balancing markets. The former determines the daily operation of the power system and generally covers most of the energy volume that is traded throughout the day. The latter provides producers and consumers with the opportunity to sell and purchase electricity close to real time. Therefore, the balancing market can be used by the VPP as an additional resource to cope with the production uncertainty of its stochastic generating units. The following example serves to illustrate this point.

Example 8.10 (Multi-market framework) Let us look back at Example 8.5, where we introduce the problem of the manager of a VPP for the first time. He must decide how much energy to exchange with the market in each future time period prior to operating his VPP. This is so, because at that time he only considers the day-ahead market as trading floor. To make such decisions, he opts for the expected-value trading strategy. Later, in Example 8.6, he decides to test the resulting trading decisions $[P^{\mathrm{D}*}(1), P^{\mathrm{D}*}(2), P^{\mathrm{D}*}(3)] = [4.25, 5.90, 5.50]$ MW on the three scenarios of PV power output shown in Table 8.1 (p. 255). Then he concludes that in scenario 2, he would have to curtail PV production and, in scenario 3, he would have to use the expensive and old diesel generating set.

It is now time for us to bring the balancing market into the picture. We are going to repeat next the test made in Example 8.6, but, on this occasion, we will account for the possibility of trading in this other market. For this purpose, we define the positive variables $P_{\omega}^{B^+}(t)$ and $P_{\omega}^{B^-}(t)$ as the amounts of energy sold in and purchased from the balancing market, respectively, by the VPP in period t and scenario ω, and introduce them into the energy balance equations of the ω-parameterized optimization problem (8.26) (p. 262), i.e.,

$$
\begin{aligned}
&E_{G\omega}(1) + P_{W\omega}(1) + P_{S\omega}^{d}(1) + P_{\omega}^{B^-}(1) = E_{L\omega}(1) + P_{S\omega}^{c}(1) + P_{\omega}^{B^+}(1) + P^{D*}(1),\\
&E_{G\omega}(2) + P_{W\omega}(2) + P_{S\omega}^{d}(2) + P_{\omega}^{B^-}(2) = E_{L\omega}(2) + P_{S\omega}^{c}(2) + P_{\omega}^{B^+}(2) + P^{D*}(2),\\
&E_{G\omega}(3) + P_{W\omega}(3) + P_{S\omega}^{d}(3) + P_{\omega}^{B^-}(3) = E_{L\omega}(3) + P_{S\omega}^{c}(3) + P_{\omega}^{B^+}(3) + P^{D*}(3).
\end{aligned}
\tag{8.43a}
$$

Besides, let us assume that the VPP can sell energy in the balancing market at a price 20 % lower than the day-ahead price $\lambda^{D}(t)$ and purchase it at a price 10 % higher than $\lambda^{D}(t)$ (for precise details on the actual price formation process in the balancing market, the reader is referred to Chaps. 4 and 7). Then we just need to recast the objective function of problem (8.26) as follows:

$$
\begin{aligned}
\text{Max. } \rho_{\omega} = {} & 20P^{D*}(1) + 80P^{D*}(2) + 45P^{D*}(3)\\
& - \Big[5\left(E_{G\omega}(1)^2 + E_{G\omega}(2)^2 + E_{G\omega}(3)^2\right) + 10\big(E_{G\omega}(1) + E_{G\omega}(2)\\
& + E_{G\omega}(3)\big) + 50\,(v_{\omega}(1) + v_{\omega}(2) + v_{\omega}(3))\Big] - \big(C_{\omega}^{SU}(1) + C_{\omega}^{SU}(2)\\
& + C_{\omega}^{SU}(3)\big) - 30\left(E_{L\omega}(1)^2 + E_{L\omega}(2)^2 + E_{L\omega}(3)^2\right) + 150\,(E_{L\omega}(1)\\
& + E_{L\omega}(2) + E_{L\omega}(3)) + 5\\
& + 0.8\left(20P_{\omega}^{B+}(1) + 80P_{\omega}^{B+}(2) + 45P_{\omega}^{B+}(3)\right)\\
& - 1.1\left(20P_{\omega}^{B-}(1) + 80P_{\omega}^{B-}(2) + 45P_{\omega}^{B-}(3)\right).
\end{aligned}
\tag{8.44}
$$

Table 8.11 includes the solution to the so-modified profit-maximization problem (8.26) for each scenario $\omega = 1$, 2, and 3.

This solution describes how the VPP should be operated under each plausible realization of the PV power output, in case the manager of the VPP decides to sell 4.25, 5.90, 5.50 MWh in the day-ahead market, while making use of the balancing market to cope with the PV production uncertainty in real time. We can now compare Tables 8.11 and 8.3–8.5, and draw some relevant conclusions about the effect of the balancing market on the VPP operation. In particular:

1. There would be no longer any need to curtail the PV power production in scenario 2, as the potential energy surplus could now be sold in the balancing market, thus increasing the profit that the VPP would make under this scenario.
2. Likewise, the manager of the VPP would not have to resort anymore to the expensive and inefficient diesel generating set in scenario 3, as he could get some extra energy from the balancing market to cover any shortage of PV production

Table 8.11 VPP operation based on the expected-value solution strategy considering the balancing market. Powers in megawatts and energy in megawatt-hour

Scenario 1, profit $\rho_1 = \$1018.39$											
Period t	$P^{D*}(t)$	$v_1(t)$	$P_{G1}(t)$	$E_{G1}(t)$	$P_{L1}(t)$	$E_{L1}(t)$	$P^c_{S1}(t)$	$P^d_{S1}(t)$	$E_{S1}(t)$	$P_1^{B+}(t)$	$P_1^{B-}(t)$
1	4.25	1	4	3	1.92	1.71	0.30	0	0.64	0	0.76
2	5.90	1	2	3	1.00	1.46	0	0.35	0.20	0	0.01
3	5.50	0	0	1	2.00	1.50	0	0	0.20	0	0
Scenario 2, profit $\rho_2 = \$976.63$											
Period t	$P^{D*}(t)$	$v_2(t)$	$P_{G2}(t)$	$E_{G2}(t)$	$P_{L2}(t)$	$E_{L2}(t)$	$P^c_{S2}(t)$	$P^d_{S2}(t)$	$E_{S2}(t)$	$P_2^{B+}(t)$	$P_2^{B-}(t)$
1	4.25	1	2.67	2.34	2.00	1.75	0.3	0	0.64	2.04	0
2	5.90	1	4.65	3.66	1.30	1.65	0	0.35	0.20	0.46	0
3	5.50	1	2.65	3.65	2.00	1.65	0	0	0.20	0	0
Scenario 3, profit $\rho_3 = \$583.73$											
Period t	$P^{D*}(t)$	$v_3(t)$	$P_{G3}(t)$	$E_{G3}(t)$	$P_{L3}(t)$	$E_{L3}(t)$	$P^c_{S3}(t)$	$P^d_{S3}(t)$	$E_{S3}(t)$	$P_3^{B+}(t)$	$P_3^{B-}(t)$
1	4.25	1	4	3.0	1.93	1.72	0.3	0	0.64	0	1.27
2	5.90	1	5	4.5	0.97	1.45	0	0.35	0.20	0	1.40
3	5.50	1	3	4.0	1.97	1.47	0	0	0.20	0	1.47

at a more competitive price. Actually, replacing the diesel set with the balancing market results in a substantial increase of the VPP profit under this scenario.

3. Beyond its beneficial use to economically accommodate the uncertainty of the PV power output, the balancing market is a competitive source of energy that allows increasing the consumption of the gear factory and hence its utility, while decreasing the production of the GT unit and hence its generation cost.

Now that the balancing market has come into the picture, we must recompute the expected profit associated with the expected-value trading strategy. Specifically, the new calculation is as follows:

$$\mathbb{E}_\omega\{\rho\} = \sum_{\omega=1}^{N_\Omega} \pi_\omega \rho_\omega = 0.2 \times \$1018.39 + 0.3 \times \$976.63 \\ + 0.5 \times \$583.73 = \$788.53, \tag{8.45}$$

which yields an expected profit that is 16.85 % higher than in the case where the balancing market is disregarded.

Logically, the consideration of the balancing market is likely to have as well a positive effect on the robust and stochastic trading strategies, but this is something that the reader will have to check for him or herself by solving Exercise 8.10.

The reader should also notice that there are other important details that we have intentionally oversimplified. For instance, as we pointed out at the end of the preceding section, when it comes to trading electricity, market prices—both day-ahead and balancing—are uncertain. Reference [6] provides further insight into offering and bidding models for producers and consumers in a multi-market framework, including a more systematic treatment of uncertain prices.

In addition, as we explain in Chaps. 4 and 7 of this book, there exist various designs for the balancing market with different approaches to pricing electricity. Needless to say, the offering models presented in this chapter should then be tailored to the particular market design under consideration.

8.6 Summary and Conclusions

Nowadays, the international community is investing massively in research aimed at cutting down the consumption of fossil fuel in energy systems. To this end, future electric power systems will accommodate an increasing share of small-scale distributed energy sources such as wind turbines, photovoltaic cells, biomass plants, storage equipment, micro-power/heat plants, flexible consumers and/or electric vehicles. Faced with this new reality, researchers, industry practitioners, and governments must combine efforts to set up mechanisms that support the efficient utilization of the locally available energy sources. The primary aim of this chapter has been to point out some of the most important steps to be taken in order to navigate our way through this challenge, namely:

1. To group local resources (flexible consumers, storage systems, renewable generators, and others) in clusters—the so-called VPP—in order to facilitate their interaction with the bulk power system through the well-established wholesale electricity market.
2. To develop accurate mathematical models that allow predicting and simulating the operation of the components of a VPP. In this regard, the progress in forecasting methods capable of assessing the availability of renewable energy sources prove to be of particular importance.
3. To build decision support tools for the optimal involvement of the VPP in the wholesale energy market that can efficiently deal with the uncertainty associated with its stochastic generating units.
4. To make the most of the different types of commodities being traded in the electricity market (day-ahead and balancing energy, reserve capacity, etc.) to maximize the economic performance of the VPP, thus increasing its profitability and competitiveness.

There are, however, many other actions to be taken to turn the large-scale utilization of local energy resources into a reality. Indeed, due to the dispersed nature of these resources, there is only one infrastructure "branched" enough to reach all of them: the distribution grid. Consequently, the management of VPPs will also call, among other things, for:

1. The enhancement of the control and monitoring of the distribution network to guarantee the performance, reliability, and security of the electricity supply.
2. The modeling, design and test of advanced components acting actively in the grid such as generators, transformers, smart meters, cables, breakers, insulators, power electronics, and converters.

3. The development of procedures to identify weaknesses in the distribution grid and propose guidelines for its reinforcement and expansion.

Needless to say, all the previous measures and advances are subordinated to the establishment of system-wide and component-level information technology architectures that enable a reliable information structuring and exchange.

8.7 Further Reading

For a comprehensive and stimulating discussion on the benefits and challenges of distributed generation and its potential impact on future electricity markets, the reader is referred to [16] and [17], respectively. Likewise, [7], [8], and [20] offer insights into progress achieved and challenges ahead in the forthcoming transformation of electric energy systems.

Reference [1] provides a comprehensive mixed-integer linear model for the optimal operation of a thermal power plant, including a linear approximation of nonconvex cost functions. More information on how to properly characterize flexible loads can be found in [11; 12] and in Chap. 9 of this book. The concepts of point forecasts, forecast intervals and scenarios are formally introduced in Chap. 2 and a number of state-of-the-art techniques used to produce them is compiled in [9] for the case of wind power.

Robust optimization dates back to the work by Soyster [18] from the 1970s and very informative overviews of this field are presented in both [2] and [3]. The reader is also referred to Appendix D for a brief introduction to the topic. The book by Birge and Louveaux [4] constitutes a standard reference on stochastic programming and [6] describes a wide variety of its applications to the world of electricity markets, including offering models for consumers and conventional and stochastic producers. Furthermore, Appendix C provides a short introduction on stochastic programming.

Last, an example of a full-fledged offering model for a VPP in a multi-market framework for electricity can be found in [13; 14; 15].

Exercises

8.1 Consider that the minimum up and down times of the gas turbine in Example 8.1 are, actually, 5 h and 2 h, respectively. Besides, the GT unit has been on during 1 h prior to the beginning of the 3-h time horizon. Using the formulation of the minimum up-time and down-time constraints presented in Chap. 5 of this book, rewrite the set of equations defining the feasible operational region of the GT unit.

8.2 Given the new model for the gas turbine that results from the previous exercise and the probabilistic information contained in Table 8.1 (p. 255), redo Examples 8.5

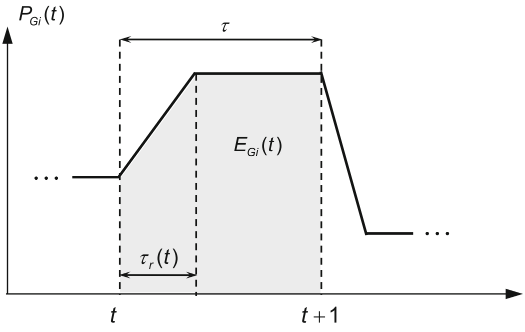

Fig. 8.8 Piecewise linear time trajectory followed by the power output $P_G(t)$ of the GT unit in Exercise 8.3. The shaded area represents the energy produced in the period in between time samples $t-1$ and t. The length of each period is τ and $\tau_r(t)$ represents the duration of the ramping excursion

and 8.6, that is, first compute the expected-value trading strategy and then evaluate it using the three PV-output scenarios included in the said table.

8.3 Write the equation establishing the equivalence between power and energy for a piecewise linear power-output trajectory according to which a dispatchable power plant uses all its ramping capability at the beginning of each time period to reach a steady generation level as quick as possible. For the sake of clarity, Fig. 8.8 provides an illustrative example of such a type of power trajectory. The reader may want to get inspiration from [21].

8.4 It turns out that the GT unit in Example 8.1 incurs additional costs when ramping. Knowing that the ramping costs are proportional to the amount of energy produced during the ramp duration, with a proportionality constant equal to \$50/MWh, recompute the expected-value solution strategy for the VPP in Example 8.5 in the following two cases:

1. The power output of the GT unit follows a linear time trajectory similar to that represented in Fig. 8.2 (p. 247).
2. The power output of the GT unit follows a piecewise linear trajectory analogous to that displayed in Fig. 8.8.

Comment on the differences, if any.

8.5 Some time after the electricity market has been closed, the manager of the VPP in Examples 8.5–8.6 is informed by forecasters that the forecasting model used to obtain the probabilistic information in Table 8.1 (p. 255) was badly calibrated.

Evaluate and compare the robust and stochastic trading strategies determined in Examples 8.7 and 8.8 in the following two cases:

1. The probabilities of occurrence of scenarios 1, 2, and 3 were wrong. Their correct values are 0.6, 0.3, and 0.1, in that order.
2. The PV-power-output values characterizing scenario 3 were wrong. This scenario is actually given by [0.8, 1.0, 1.5] MW.

8.6 The robust offering model (8.34) (p. 269) is inspired by the robust formulation proposed by Soyster in the early 1970 s (see reference [18]). This model, however,

Table 8.12 Exercise 8.8—Modeling of the PV power output in the form of point forecasts, intervals and scenarios. Powers are in megawatts

Period t	Point forecast	Interval	Scenario ω		
			1	2	3
1	1.600	[1.0, 4.0]	1.5	1.0	4.0
2	3.855	[0.5, 6.0]	0.5	6.0	2.7
3	3.600	[1.5, 4.5]	3.0	4.5	1.5
Probability π_ω	–	–	0.30	0.55	0.15

leads to trading strategies that are too conservative in the sense that we may give up too much of optimality in order to ensure feasibility even in the case where the worst possible outcomes of *all* the input random variables concur.

Carefully read the paper by Bertsimas and Sim [3] and then propose ways to relax and in general, control the degree of conservatism of the resulting trading strategy.

8.7 Beyond the grounds referred to in the previous exercise, the robust offering model (8.34) (p. 269) is too conservative, because it ignores the fact that the components of the VPP can be redispatched on a real-time basis as the power production from the stochastic generating units becomes known. This is, in contrast, accounted for in the stochastic offering model (8.35) by distinguishing between *here-and-now* and *wait-and-see* (recourse) decisions.

Use, as a reference, the robust formulation described in Chap. 3 for the market-clearing problem and propose a robust offering model alternative to (8.34) that takes into account the capability of the VPP components to adapt to the stochastic power production. The reader is also advised to consult reference [2].

8.8 Consider the VPP of Example 8.5 and the probabilistic information gathered in Table 8.8. Compute the expected-value, robust, and stochastic trading strategies and evaluate them using the three scenarios listed in the said table.

8.9 Consider the scenario representation of the market price given in Example 8.9. Extend the expected-value and robust offering models (8.21) and (8.34) to compute offer curves for the VPP in Example 8.5 following these two trading approaches.

8.10 Complete Example 8.10, where the balancing market is considered. Assess the effect of this market on the economic performance of the robust and stochastic trading strategies.
Tip: Take into account that the stochastic offering approach relies on explicitly modeling the operation of the VPP for each plausible realization of the PV power output. Therefore, unlike in the case of the expected-value solution strategy, the consideration of the balancing market in the stochastic offering model (8.35) may not only affect the VPP operation, but also the decision on the amount of energy to be sold in or purchased from the day-ahead market.

References

1. Arroyo, J., Conejo, A.: Optimal response of a thermal unit to an electricity spot market. IEEE Trans. Power Syst. **15**(3), 1098–1104 (2000)
2. Bertsimas, D., Brown, D., Caramanis, C.: Theory and applications of robust optimization. SIAM Rev. **53**(3), 464–501 (2011)
3. Bertsimas, D., Sim, M.: The price of robustness. Oper. Res. **52**(1), 35–53 (2004)
4. Birge, J.R., Louveaux, F.: Introduction to Stochastic Programming, 2nd edn. Springer Series in Operations Research and Financial Engineering. Springer, New York (2011)
5. Chao, H., Huntington, H.: Designing Competitive Electricity Markets. Fred Hillier's International Series in Operations Research & Management Science. Kluwer Academic Publishers, Boston, MA (1998)
6. Conejo, A.J., Carrión, M., Morales, J.: Decision Making Under Uncertainty in Electricity Markets. International Series in Operations Research & Management Science. Springer Verlag, New York (2010)
7. Djapic, P., Ramsay, C., Pudjianto, D., Strbac, G., Mutale, J., Jenkins, N., Allan, R.: Taking an active approach. IEEE Power Energy Mag. **5**(4), 68–77 (2007)
8. Fang, X., Misra, S., Xue, G., Yang, D.: Smart grid—The new and improved power grid: A survey. IEEE Commun. Surv. Tutor. **14**(4), 944–980 (2012)
9. Giebel, G., Brownsword, R., Kariniotakis, G., Denhard, M., Draxl, C.: The State-Of-The-Art in Short-Term Prediction of Wind Power. ANEMOS.plus (2011) Project funded by the European Commission under the 6th Framework Program, Priority 6.1: Sustainable Energy Systems
10. Hedegaard, K., Meibom, P.: Wind power impacts and electricity storage—A time scale perspective. Renew. Energy **37**(1), 318–324 (2012)
11. Karangelos, E., Bouffard, F.: Towards full integration of demand-side resources in joint forward energy/reserve electricity markets. IEEE Trans. Power Syst. **27**(1), 280–289 (2012)
12. Khodaei, A., Shahidehpour, M., Bahramirad, S.: SCUC with hourly demand response considering intertemporal load characteristics. IEEE Trans. Smart Grid **2**(3), 564–571 (2011)
13. Mashhour, E., Moghaddas-Tafreshi, S.: Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part I: Problem formulation. IEEE Trans. Power Syst. **26**(2), 949–956 (2011)
14. Mashhour, E., Moghaddas-Tafreshi, S.: Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part II: Numerical analysis. IEEE Trans. Power Syst. **26**(2), 957–964 (2011)
15. Pandžíc, H., Morales, J.M., Conejo, A.J., Kuzle, I.: Offering model for a virtual power plant based on stochastic programming. Appl. Energy **105**, 282–292 (2013)
16. Pepermans, G., Driesen, J., Haeseldonckx, D., Belmans, R., D'haeseleer, W.: Distributed generation: Definition, benefits and issues. Energy policy **33**(6), 787–798 (2005)
17. Schleicher-Tappeser, R.: How renewables will change electricity markets in the next five years. Energy Policy **48**, 64–75 (2012)
18. Soyster, A.: Convex programming with set-inclusive constraints and applications to inexact linear programming. Oper. Res. **21**(5), 1154–1157 (1973)
19. Stoft, S.: Power System Economics: Designing Markets for Electricity. John Wiley & Sons – IEEE Press, New York (2002)
20. Vittal, V.: The impact of renewable resources on the performance and reliability of the electricity grid. Bridge **23**(1), 5–12 (2010)
21. Wu, H., Shahidehpour, M., Khodayar, M.: Hourly demand response in day-ahead scheduling considering generating unit ramping cost. IEEE Trans. Power Syst. **28**(3), 2446–2454 (2013)

Chapter 9
Facilitating Renewable Integration by Demand Response

9.1 Introduction

Compared to conventional electricity sources, renewables have two important drawbacks: they are intermittent and stochastic. The former implies that renewable power output cannot be modulated by the producers throughout the day, as it depends on meteorological conditions. As a result of the latter feature, the level of power production is not known beforehand with certainty, but can only be forecast with a certain accuracy. In turn, this implies that corrective measures may have to be taken with short notice in order to ensure the safety of power system operation.

The deployment of increasing shares of renewable power into power systems requires a higher degree of flexibility, not only from the electricity suppliers from the demand side, as discussed in Chap. 5. For example, low capacity margin when demand peaks, and limited ramping capability from the supply side may jeopardize system security in case of unpredicted events, e.g., steep decreases in renewable power production. A power system equipped with demand response can better cope with such events by taking advantage of the flexibility of consumers by shifting or reducing their power consumption. Furthermore, demand response can support a reduction in systems costs, CO_2 emissions, and price volatility by shifting power consumption to periods characterized by low prices and high renewable power production.

In order to evolve from a setup where *supply follows demand* to one where *demand follows supply*, power systems must undergo drastic structural and operational changes. Indeed, bidirectional communication systems must be put in place to send appropriate signals to flexible consumers to exploit their flexibility nearly in real-time, and to monitor the status of the system. In turn, consumers must be equipped with "intelligent" appliances or appliance controllers, capable of optimizing consumption according to the owner's preferences and the market signals received.

This chapter deals with demand response under real-time dynamic pricing, and is structured as follows. Section 9.2 introduces the market framework for the involvement of consumers in demand response. Then, Sect. 9.3 models the optimization problem for consumers receiving deterministic real-time prices. Counterparts to

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this model considering the uncertainty in real-time market prices are presented in Sect. 9.4, making use of stochastic programming and robust optimization. Furthermore, this section proposes their use in combination with model predictive control (MPC). Tools to model and forecast the aggregate response of a group of consumers to dynamic prices are presented in Sect. 9.5. Then, Sect. 9.6 deals with the problem of determining the optimal real-time price sequence to make the most out of consumer flexibility. Finally, Sect. 9.7 concludes the chapter.

9.2 Market Framework for Controlling Flexible Demand

Situations where electricity retailers and consumers enter into contracts where the former provides electricity to the latter at a flat price are commonplace today. Under these conditions, small consumers are totally inflexible, as they have no incentive to modify their consumption patterns to better follow the supply.

In order to unlock the potential flexibility of consumers, a number of initiatives have been proposed or put in practice. According to [4], such initiatives can be classified into the following groups.

Dynamic Pricing (or Control-by-Price): In this setup, a market entity broadcasts a price for electricity to an elastic demand, and thus creates the conditions for exploiting its flexibility. The decision on energy consumption is ultimately left to the individual consumers, who must weigh cost savings against a potential loss of comfort. In practice, the decision may not be made by the consumer himself/herself, but rather by a computer or by intelligent appliances. We refer to Sect. 9.2.1 for further discussion on dynamic pricing.

Direct Control: The level of consumption is directly modified by a system or market entity by means of rationing or disconnection of certain consumers or appliances. For residential consumers, such appliances may include, e.g., air conditioning and water heating.

Interruptible Tariffs: Customers accept to reduce or shut down their consumption in return for an economic compensation. Initiatives of this type are popular among some large industrial consumers, who can shift energy-intensive operations in time.

Other Initiatives: This group refers to market-based initiatives where large consumers directly bid in energy markets, thus making their flexibility available either by specifying a price–demand curve or by offering flexibility in capacity markets.

Another type of demand response initiative, not mentioned in [4], but still of particular relevance, is the following:

Frequency-Based Control: In this framework, intelligent appliances automatically shut down or decrease their load when the system frequency drops below the reference level, and increase their consumption when the frequency rises. Note that a lower (higher) frequency than the reference one signals that total consumption exceeds (falls below) supply in the system. Hence, the control strategy sketched above contributes to restoring the power balance in the system. Note that the concept of frequency-based control of appliances is rather old—it was presented in [10] as early as in 1980. However, this concept is receiving increasing attention due to the technological development in the recent years, see for example [11].

In this chapter, we focus on dynamic pricing. From a market perspective, this setup is particularly relevant as it involves prices and rational consumers. Note that other demand response initiatives involving the direct participation of large consumers to the electricity market are dealt with in Chap. 5.

Dynamic pricing is particularly suitable to be studied with methods of optimization under uncertainty. Indeed, these techniques can be used to model both the utility functions for the consumers and their technical limitations. In the next section, a more detailed introduction to dynamic pricing is given.

9.2.1 Dynamic Pricing

The concept of influencing consumer preferences by a price signal is not new. Indeed, time-of-use prices have replaced fixed-price tariffs for small consumers in many countries during the last decades. In a time-of-use pricing scheme, electricity has a higher price during peak-demand hours than during off-peak hours of the day. On the contrary, consumption during the night hours is incentivized by lower prices than during diurnal hours.

Time-of-use tariffs can certainly play an important role in decreasing the total system costs by influencing consumer behavior, thus moving consumption to off-peak hours. However, their relevance is challenged as the penetration of renewables into power systems grows sufficiently large to be able to influence prices in the wholesale electricity markets. Time-of-use tariffs are *static*, i.e., they are fixed long time in advance, and therefore unable to adapt to the rapid fluctuations of renewables.

As stochastic production, in particular solar power, can lower market prices even during the peak hours of the day, the price signal must adapt dynamically on the basis of the forecast level of renewable output. Real-time dynamic pricing is meant to serve this purpose.

Figure 9.1 sketches an example of time-of-use and dynamic price signal. In the time-of-use case, the price is constant during hours of the same type (valley, off-peak or peak), and cannot adapt to the renewable power production. On the contrary, in the dynamic case the fluctuation of renewables are also taken into account, resulting in lower prices when the output from these stochastic sources is high.

The problem of determining the optimal price signal to be broadcast to consumers is of particular relevance for several entities that are interested in taking advantage of demand response. We describe them in the following list.

Transmission system operator (TSO) is interested in ensuring balance and reliability of the transmission grid. By appropriately choosing a dynamic price signal to be broadcast to consumers, the TSO can reduce system costs and increase the reliability of the power system, e.g., by shifting flexible consumption to periods characterized by high stochastic production.

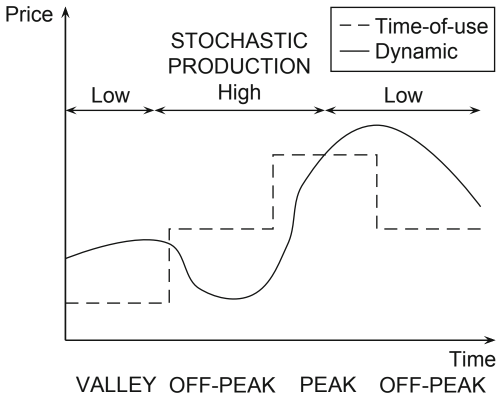

Fig. 9.1 Sketch of time-of-use and dynamic price. The dynamic price level adapts to both demand and stochastic power production

Distribution system operators (DSOs) have intents similar to the ones of the TSO. However, they operate on the local distribution grids, which may include small distributed generation units (e.g., solar) and are generally characterized by tight capacity constraints.

Electricity retailers act as buffers between consumers and the wholesale electricity market, assuming the risk of consumption deviations from schedule. Demand response through dynamic pricing offers retailers the possibility to simultaneously decrease their risk and costs on the market.

Power producers are, similarly to retailers, exposed to market prices and penalized for their real-time deviations from the production plans. If coupled with price-responsive demand, they have the possibility of improving their market performance.

Aggregators are entities that pool consumer resources, including their flexibility and possibly distributed production plants, optimize their management, and represent the consumers in the market. Such entities may be owned by the consumers.

Currently, the setup of demand response programs involving the activation of small residential consumers through the use of dynamic pricing is still under discussion. In particular, it is not clear yet which entity should be responsible for broadcasting the dynamic price signal. In any case, the determination of the optimal dynamic price sequence is a central problem of demand response. It is clear that knowledge of the response of the consumers to dynamic prices is needed to solve this problem. In Sects. 9.3 and 9.4 we present optimization models to describe the response of individual consumers to a dynamic price signal. Statistical tools to model the aggregate response of flexible consumers are introduced in Sect. 9.5

9.3 Modeling the Dynamics of Demand Response

Consumers exposed to real-time dynamic prices can reduce their cost for electricity procurement by modifying their behavior, possibly giving up part of their comfort. However, the following infrastructure needs to be in place for this to be possible:

- A communication system allowing the system operator or a market entity to broadcast real-time prices to the consumers. The communication system should possibly be bidirectional, so as to permit the system or market entity to monitor the consumption in real-time or close to real-time.
- The consumers are equipped with "intelligent" appliances that autonomously determine the optimal consumption schedule, given the broadcast real-time price sequence and the pre-specified consumer preferences. Such appliances may also broadcast the foreseen consumption level back to the entity setting the price. In alternative to this setup, where each appliance operates independently from the others, a unique optimization system may determine the optimal allocation of consumption between the different appliances installed in the household.

The very nature of the problem of the consumer exposed to dynamic prices requires that the optimization models employed be simple. This is both because the dynamic price sequence should be broadcast close to real-time, therefore requiring a quick solution, and because the optimization solvers may be embedded in consumer appliances, with the resulting limitations in terms of computational power. Notice, however, that the alternative setup involving a central computer directly controlling the appliances in the household allows more computational power than the former setup.

9.3.1 Deferrable Loads

Let us consider a consumer willing to perform a certain task requiring the use of electricity. The utility function $f_t(u_t)$ measures the benefit the consumer achieves by consuming the amount of energy u_t during time period t for performing the given task. For simplicity, we assume that the benefit for the consumer is constant throughout the day, i.e., independent of t, and a linear function of the consumption, u_t. As a result of these two assumptions, we can express consumer benefit as $f_t(u_t) = bu_t$. Note that assuming that the benefit is constant in time implies that the load is deferrable, since the task can be performed at any time during the day.

A simple optimization problem for the flexible consumer is the following:

$$\underset{u_t}{\text{Min.}} \quad \sum_{t=1}^{T} \left(\lambda_t^{\text{R}} u_t - f_t(u_t)\right) = \sum_{t=1}^{T} \left(\lambda_t^{\text{R}} - b\right) u_t \tag{9.1a}$$

$$\text{s.t.} \quad u_t - u_{t-1} \leq R^{\text{U}}, \qquad \forall t, \tag{9.1b}$$

$$u_t - u_{t-1} \geq -R^{\text{D}}, \qquad \forall t, \tag{9.1c}$$

$$u_t \leq \overline{U}^{\text{h}}, \qquad \forall t, \tag{9.1d}$$

$$\sum_{t=1}^{T} u_t \leq \overline{U}^{\text{d}}, \tag{9.1e}$$

Table 9.1 Parameters for deferrable load and price signal

(a) Load parameters

Parameter	Value	Unit
b	100	\$/MWh
$\overline{U}^{\mathrm{h}}$	3	kWh
$\underline{U}^{\mathrm{d}}$	6	kWh
$\overline{U}^{\mathrm{d}}$	8	kWh
R^{U}	1.5	kWh
R^{D}	1.5	kWh

(b) Prices

Time period	Price (\$/MWh)
1	120
2	75
3	110
4	60

$$\sum_{t=1}^{T} u_t \geq \underline{U}^{\mathrm{d}}, \tag{9.1f}$$

$$u_t \geq 0, \qquad \forall t. \tag{9.1g}$$

In the objective function (9.1a), λ_t^{R} is the real-time price of electricity at time period t, T is the length of the optimization horizon, and b is the per unit benefit of consumption. Therefore, such an objective function is the difference between the cost of purchasing energy and the benefit brought by such consumption. Considering this objective function and without any further constraints, consumption would only take place in periods when the real-time price is lower than the marginal benefit.

In order to model appliances where the consumption cannot change abruptly from a time period to the next, ramping constraints (9.1b) and (9.1c) are included. These inequalities guarantee that the change in consumption between two consecutive periods is within the ramping limits R^{U} and $-R^{\mathrm{D}}$, which bound consumption increase and decrease, respectively.

Besides, limits on consumption are imposed both for a single time period through constraint (9.1d) and for the total load in the considered horizon through inequalities (9.1e) and (9.1f). Notice that (9.1f) forces a minimum consumption level to be satisfied even though the real-time price is higher than the marginal benefit for the whole optimization horizon.

Finally, (9.1g) enforces the nonnegativity of consumption at any time.

Note that intertemporal constraints, such as the ramping constraints (9.1b) and (9.1c) and the bounds for total consumption (9.1e) and (9.1f) introduce dynamics in the system. In particular, the latter two constraints represent the fact that consumers have a limited flexibility as to what extent consumption can be postponed or increased.

Example 9.1 (Deferrable Load) Let us consider a deferrable load with the parameters listed in Table 9.1(a). The consumption schedule for time periods from $T = 1$ to $T = 4$ is to be optimized, assuming that the flexible consumer is subject to the price signal reported in Table 9.1(b).

It should be noticed that the price λ_t^{R} is lower than the consumer marginal benefit b during the second and the fourth time period. If there were no ramping constraints, the solution to the problem would be straightforward, i.e., there would be consumption

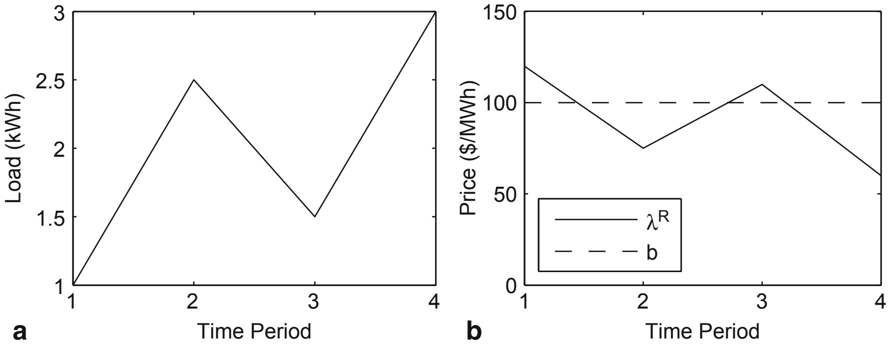

Fig. 9.2 Price and optimal consumption for a deferrable load. **a** Consumption. **b** Price

only during hours 2 and 4. Consumption at the maximum load level $\overline{U}^{h} = 3$ kWh during these hours would be sufficient to reach the minimum target $\underline{U}^{d} = 6$ kWh.

The inclusion of ramping limits $R^{U} = R^{D} = 1.5$ kWh results in a less trivial optimal consumption plan, the one illustrated in Fig. 9.2(a). In this case, the initial value for consumption, u_0, is set to 0. For comparison purposes, the price and consumer marginal benefit are plotted in Fig. 9.2(b). Observe that the consumption is not null during time periods 1 and 3, despite the fact that the high prices in these periods imply a negative utility. Intuitively, this happens because the difference between the price λ_t^{R} and the consumer marginal benefit b is larger during the low price periods 2 and 4 than during the high price periods 1 and 3. Therefore, a non-negative consumption during periods 1 and 3 is justified, because it is needed (due to the ramping constraints) to fully take advantage of the low prices at time periods 2 and 4. The highest load is placed at time period 4, i.e., when the price is lowest. On the other hand, consumption at time period 2 is not at the upper bound $\overline{U}^{h}$, because the aggregate consumption over the four time periods cannot exceed the total consumption limit $\overline{U}^{d}$. In fact, the final total consumption is $\sum_{t=1}^{4} u_t = 8$ kWh, i.e., equal to the upper bound $\overline{U}^{d}$ and 2 kWh larger than what it would be without ramping constraints.

9.3.2 Consumer Price Elasticity

Demand is said to have price elasticity when consumers are willing to purchase different amounts of energy at different price levels. Within a market framework, price elasticity is expressed through a demand function that maps price realizations to the corresponding amount of electricity to be purchased.

In this section, we model the consumer benefit function as a piecewise linear function. If the electricity consumption is divided into K blocks, we obtain a piecewise linear benefit function by assigning different coefficients b_k to each consumption block. The latter coefficients represent the marginal benefit of consumption for each block or, alternatively, the maximum price at which the consumer wishes to purchase the relative block of energy. Hence, a piecewise linear benefit function results in a marginal benefit function (or demand curve) that is constant by block. In practice, any nonlinear benefit function can be approximated by a piecewise linear one. The consumer optimization model with a piecewise linear benefit function is the following:

$$\underset{u_{tk}}{\text{Min.}} \quad \sum_{t=1}^{T}\sum_{k=1}^{K}\left(\lambda_t^{\mathrm{R}} - b_k\right) u_{tk} \tag{9.2a}$$

$$\text{s.t.} \quad u_{tk} - u_{(t-1)k} \leq R_k^{\mathrm{U}}, \qquad \forall t\ k, \tag{9.2b}$$

$$u_{tk} - u_{(t-1)k} \geq -R_k^{\mathrm{D}}, \qquad \forall t\ k, \tag{9.2c}$$

$$\sum_{k=1}^{K} u_{tk} \leq \overline{U}^{\mathrm{h}}, \qquad \forall t, \tag{9.2d}$$

$$\sum_{t=1}^{T} u_{tk} \leq \overline{U}_k^{\mathrm{d}}, \qquad \forall k, \tag{9.2e}$$

$$\sum_{t=1}^{T} u_{tk} \geq \underline{U}_k^{\mathrm{d}}, \qquad \forall k, \tag{9.2f}$$

$$u_{tk} \geq 0, \qquad \forall t\ k. \tag{9.2g}$$

Each block k in model (9.2) represents a certain appliance, whose use brings a marginal benefit b_k to the consumer. We assume that each appliance is characterized by technical limits that define, through constraints (9.2b) and (9.2c), how large deviations in consumption can be between consecutive time periods. Furthermore, upper and lower bounds (9.2e) and (9.2f) are enforced on the total consumption from each appliance. On the contrary, the upper bound to the consumption during a single time period (9.2d) is enforced for the sum of the consumption for each block. This is because the consumption for individual households is limited by the capacity of the distribution grid.

Example 9.2 Deferrable Load with Multiple Blocks The parameters in Example 9.1 are adapted here in order to include two blocks, and reported in Table 9.2. The maximum total load per time period is $\overline{U}^{\mathrm{h}} = 3$ kWh, i.e., the same value as in Example 9.1. The marginal benefit for the blocks are \$130/MWh and \$70/MWh, respectively. Ramping limits and bounds on total consumption for each block are half of the values for the single block in Example 9.1; this way, the aggregate consumption targets and ramping limits for the two examples are equal.

The optimal consumption schedule under the dynamic price in Table 9.1(b) is reported in Table 9.3. For Block 1, the marginal benefit is always higher than the electricity price. The problem is the same as for the single block in Example 9.1: How to allocate the maximum energy consumption $\overline{U}_1^{\mathrm{d}} = 4$ kWh at the lowest cost?

Table 9.2 Parameters for deferrable load with multiple blocks

Parameter	Value		Unit
	Block 1	Block 2	
b_k	130	70	$/MWh
$\underline{U}_k^{\mathrm{d}}$	3	3	kWh
$\overline{U}_k^{\mathrm{d}}$	4	4	kWh
R_k^{U}	0.75	0.75	kWh
R_k^{D}	0.75	0.75	kWh

Table 9.3 Optimal consumption schedule for deferrable load with blocks

Time period	Consumption (kWh)		
	Block 1	Block 2	Aggregate
1	0.50	0	0.50
2	1.25	0.75	2
3	0.75	0.75	1.50
4	1.50	1.50	3
Total	4	3	7

Consequently, the consumption plan for Block 1 is equal to the one shown in Fig. 9.2(a), scaled to half the capacity.

The marginal benefit of Block 2 is higher than the price only at time period 4. Since the consumption plan of Block 1 leaves 1.5 kWh consumption before reaching the capacity $\overline{U}^{\mathrm{h}}$ in this period, Block 2 needs to consume at other time periods to reach the minimum consumption target $\underline{U}_2^{\mathrm{d}} = 3$ kWh. The optimal consumption plan is then a result of the fact that Block 2 can only increase or decrease consumption by 0.75 kWh between different time periods.

Finally, we remark that, similarly to Example 9.1, the peaks in consumption are located at time periods 2 and 4, where prices are lower. However, the consumption level aggregated over all blocks is lower in the first two time periods than in the case of Example 9.1. This reflects the fact that consumption reaches the upper bound $\overline{U}_1^{\mathrm{d}}$ only for Block 1 (with highest marginal benefit). On the contrary, the lower marginal benefit for Block 2 forces its total consumption to its lower bound $\underline{U}_2^{\mathrm{d}}$.

9.3.3 *Consumption for Heating Purposes*

For many applications, electricity consumption is only deferrable up to a certain extent. In these cases, only small deviations from the consumption plan that guarantees the maximum benefit or comfort are tolerated. For example, this is the case of the load for heating purposes, which may be postponed for some time without large deviations in the indoor temperature.

A common way of modeling heating dynamics is by employing linear state–space models [7]. These models consider a number of states, x, which depends linearly on the value of the states at the previous time period and on the input, u, i.e.,

$$x_{t+1} = Ax_t + bu_t. \tag{9.3}$$

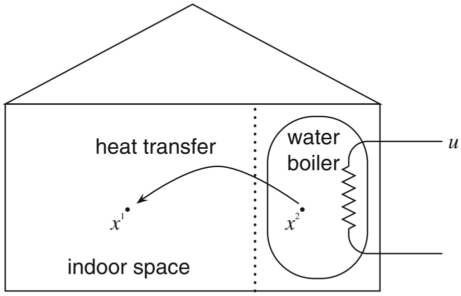

Fig. 9.3 Sketch of a residential heating system and the relative state–space variables

We assume here that the heating dynamics are approximated sufficiently well by employing two states x_t^1 and x_t^2, representing the indoor temperature and the temperature of water inside a boiler, respectively. The indoor temperature at the next time step, x_{t+1}^1, depends on the values of both states at the current time period, i.e.,

$$x_{t+1}^1 = a_{11}x_t^1 + a_{12}x_t^2. \tag{9.4}$$

The parameter a_{11} in the above expression indicates the thermal inertia of the building, i.e., how efficiently heat is retained within the indoor space. Instead, the parameter a_{12} models the heat transfer between the water boiler and the indoor space.

We assume that the water temperature, x_{t+1}^2, only depends on its value at the previous step, x_t^2, and on the electricity consumption, u_t, i.e.,

$$x_{t+1}^2 = a_{22}x_t^2 + b_2u_t. \tag{9.5}$$

Similarly to a_{11}, the parameter a_{22} represents the thermal inertia of the water boiler, while the b_2 models how the electricity consumption u is converted to heat.

Figure 9.3 sketches the setup of the model for the heating system, including the variables described above.

The comfort for the consumer is a function of the deviation of the indoor temperature from a given reference $\widehat{x}_t^1$. Typically, one would aim at minimizing the sum of the squared deviations from such a reference, i.e., consumer discomfort. Under this premise, the optimization model for the heating system is the following:

$$\underset{x_t^1, x_t^2, u_t}{\text{Min.}} \quad \sum_{t=1}^{T} \frac{c}{2}\left(x_t^1 - \widehat{x}_t^1\right)^2 + \lambda_t^{\mathrm{R}} u_t \tag{9.6a}$$

$$\text{s.t.} \quad x_{t+1}^1 = a_{11}x_t^1 + a_{12}x_t^2, \qquad \forall t < T, \tag{9.6b}$$

$$x_{t+1}^2 = a_{22}x_t^2 + b_2u_t, \qquad \forall t < T, \tag{9.6c}$$

$$u_t - u_{t-1} \le R^{\mathrm{U}}, \qquad \forall t, \tag{9.6d}$$

$$u_t - u_{t-1} \ge -R^{\mathrm{D}}, \qquad \forall t, \tag{9.6e}$$

$$u_t \le \overline{U}^{\mathrm{h}}, \qquad \forall t, \tag{9.6f}$$

$$u_t \geq 0, \qquad \forall t. \tag{9.6g}$$

The objective function weighs the cost of electricity against the discomfort (i.e., the sum of squared temperature deviations from the reference), scaled by a factor $c/2$. This parameter is to be tuned according to consumer preference: the higher the value of c, the lower the willingness to accept temperature deviations.

Constraints (9.6b) and (9.6c) are the state-update equations for the model for heating dynamics. Ramping limits, maximum load per time period as well as non-negativity of consumption are enforced by (9.6d), (9.6e), (9.6f), and (9.6g), respectively.

Example 9.3 (Deviation from Reference Temperature Under Fixed Price) Let us consider the following consumer model for optimizing the electricity consumption for heating.

$$\underset{x_t^1, x_t^2, u_t}{\text{Min.}} \quad \sum_{t=1}^{48} 0.001\left(x_t^1 - 22\right)^2 + \lambda_t^{\mathrm{R}} u_t \tag{9.7a}$$

$$\text{s.t.} \quad x_{t+1}^1 = 0.9 x_t^1 + 0.07 x_t^2, \qquad \forall t < 48, \tag{9.7b}$$

$$x_{t+1}^2 = 0.95 x_t^2 + 2u_t, \qquad \forall t < 48, \tag{9.7c}$$

$$u_t \leq 3, \qquad \forall t, \tag{9.7d}$$

$$u_t \geq 0, \qquad \forall t. \tag{9.7e}$$

The problem is a simplified instance of (9.6) with infinite ramping capability for consumption. At the initial time step ($t = 0$), the measured values for the states are $x_0^1 = 20\,^\circ\mathrm{C}$ (indoor temperature) and $x_0^2 = 29\,^\circ\mathrm{C}$ (temperature of water in the boiler).

The optimization horizon is $T = 48$. Note that problems of this type require the use of longer horizons, since they are characterized by the so-called *border effect*: consumption in the last time periods is discouraged, as the discomfort caused by temperature deviations is summed over fewer time periods than at the beginning of the horizon. Hence, a longer horizon is required to postpone this effect and guarantee sensible consumption values during the initial part of the schedule.

In two different runs of optimization model (9.7), we feed the constant price levels \$60/MWh and \$140/MWh for all the time periods in the horizon. The dynamics of the states and of the input are depicted in Fig. 9.4 for the whole optimization horizon. Notably, a higher price level induces a larger deviation from the reference temperature $\widehat{x}_t^1$. This is a consequence of the fact that objective function (9.7a) weighs the cost of electricity procurement with the discomfort caused by deviating from the reference temperature: the higher the price, the more the consumer is willing to deviate from the target.

We underline the temperature increase that can be observed for both states (at the end of the first day for the case of the higher price level, and roughly around the 30th hour with the lower price level), which is caused by the border effect. With the higher price level, consumption at the end of the first day in the horizon increases sensibly, and then ceases completely during the second day. Hence, the

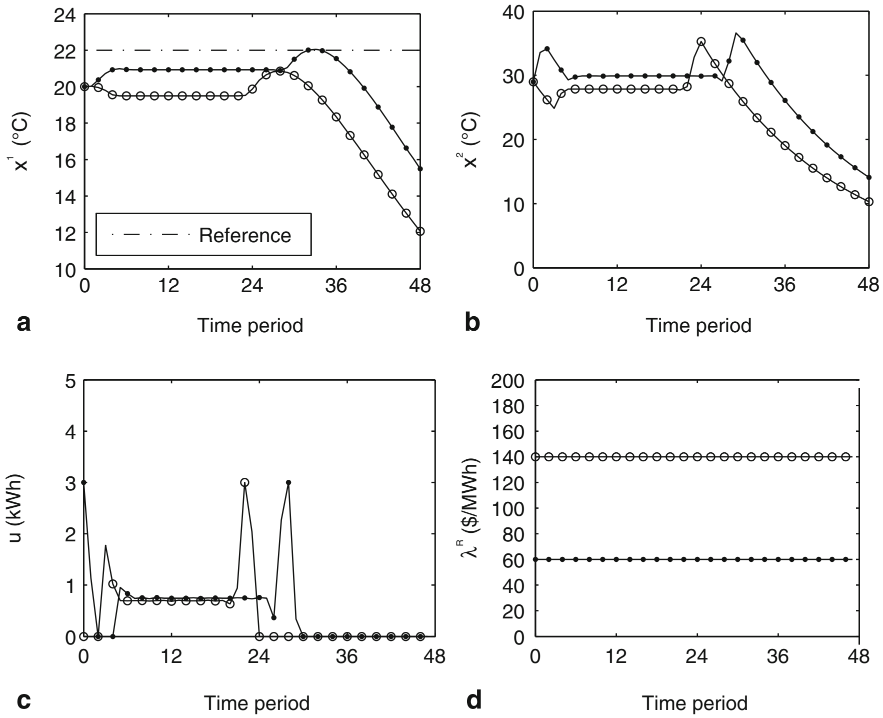

Fig. 9.4 Evolution of the system with different price levels. **a** Indoor temperature. **b** Temperature of water in the boiler. **c** Consumption. **d** Price

temperature of the states at first shoots up and then decreases slowly. In particular, the indoor temperature falls well below the reference temperature, as the constant price offsets the total discomfort, which involves a sum over a horizon of decreasing length. A similar behavior is noticeable for the lower price level, although the lower price delays the phenomenon. In practice, such a phenomenon would never actually take place, as one would constantly update the consumption schedule with a rolling horizon strategy. This issue is dealt with in more detail in Sect. 9.4.3.

Finally, we remark that consumption is lower with the higher price level. Indeed, it totals 19.86 kWh for the whole horizon in this case, which is remarkably lower than the 26.10 kWh consumed in the lower-price case.

Example 9.4 (Optimal Consumption for Heating Under Time-of-Use Price) The model of Example 9.3 is reconsidered here, this time employing a time-varying price. The employed price signal, which has a time-of-use nature, is defined in Table 9.4 and depicted in Fig. 9.5(d). Note that the price pattern repeats itself during the second part of the optimization horizon, i.e., in the time periods 25–48.

The optimal consumption is shown in Fig. 9.5(c). Remarkably, consumption takes place only during the hours characterized by low prices ($75 /MWh and $60 /MWh). The resulting evolution of the states is illustrated in Figs. 9.5(a) and 9.5(b). The

Table 9.4 Time-of-use price

Time period	1–6	7–12	13–18	19–24
Price ($/MWh)	120	75	110	60

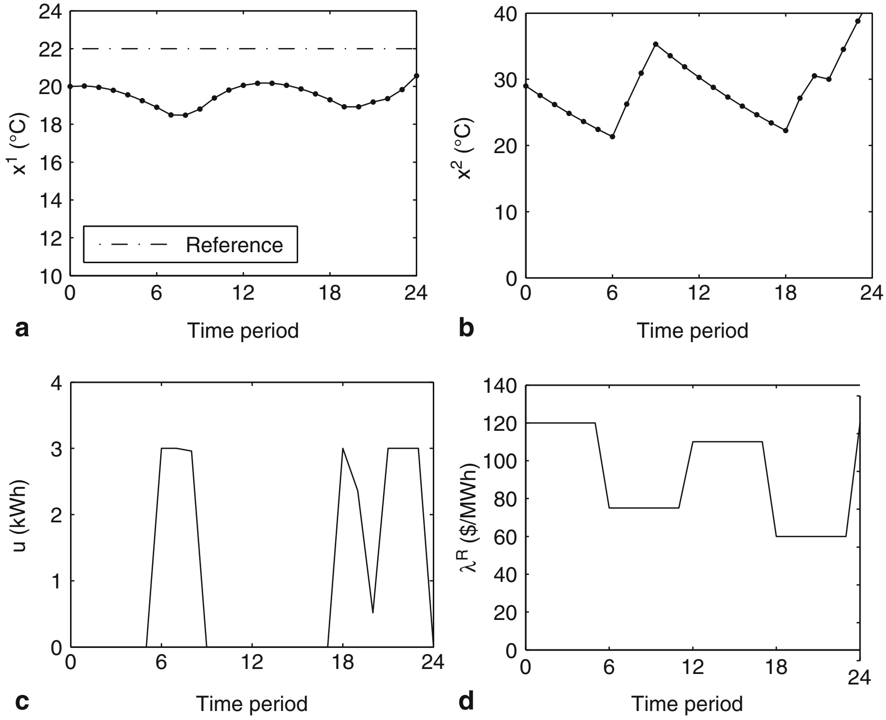

Fig. 9.5 Evolution of the system variables with time-of-use price. **a** Indoor temperature. **b** Temperature of water in the boiler. **c** Consumption. **d** Price

dynamic features of the system are notable in this example. The temperature of the water in the boiler, x_t^2, ramps up immediately as the consumption increases. Indeed, the consumption u_t is fed directly to this state in (9.7c). On the other hand, the indoor temperature, x_t^1, increases much more slowly following the rise in temperature x_t^2.

9.4 Solving Stochastic Consumer Problems

The optimization models presented in the previous section are developed under the premise that consumers know with certainty the current and the future prices during the considered horizon, i.e., they are deterministic models.

One of the basic assumption in demand response is that the current (real-time) electricity price is known with certainty by the consumer after being broadcast by a system or a market entity. However, the evolution of the price signal in the future is uncertain. The consumer might have an estimate of it, provided by the price broadcaster itself or determined by own forecasting models.

The uncertainty in the evolution of the electricity price in the future implies that the electricity procurement cost for the consumer (and therefore its objective function in an optimization framework) is also uncertain. Indeed, one can write the objective function as

$$\lambda_1^{\mathrm{R}} u_1 - f_1(x_1, u_1) + \sum_{t=2}^{T} \left\{ \widetilde{\lambda}_t^{\mathrm{R}} u_t - f_t(x_t, u_t) \right\}, \tag{9.8}$$

where $f_t(x_t, u_t)$ indicates the utility for the consumer, which may be dependent on the current consumption level as in Sects. 9.3.1 and 9.3.2 or on a state variable as in Sect. 9.3.3, or on a combination of both. The symbol $\sim$ denotes an uncertain parameter, in this case the future price.

In this section, we develop stochastic counterparts to the deterministic models presented in Sect. 9.3. The stochastic programming approach [2], see Appendix C, is presented in Sect. 9.4.1, where it is assumed that the price uncertainty is modeled by scenarios. Section 9.4.2 employs robust optimization models [1], see Appendix D, which deliver the optimal decision under the worst case realization of prices within an uncertainty set. Finally, Sect. 9.4.3 introduces the framework of MPC, which is suitable for use both in connection with stochastic programming and robust optimization models.

9.4.1 Stochastic Programming Approach

In a stochastic programming framework, one models uncertain parameters by employing a discrete number N_Ω of scenarios. A certain probability $\pi_\omega > 0$ is assigned to each scenario, so that $\sum_{t=1}^{N_\Omega} \pi_\omega = 1$.

Since the current value of the electricity price is broadcast to the consumer in real-time, this quantity is known with certainty. On the contrary, future prices are stochastic and modeled with a discrete number of trajectories $\lambda_{t\omega}^{\mathrm{R}}$, $t > 1$, $\omega = 1, 2, \ldots, N_\Omega$. An example of scenarios for the dynamic price is shown in Fig. 9.6.

The consumer problem in the stochastic programming framework boils down to

$$\text{Min. } \lambda_1^{\mathrm{R}} u_1 - f_1(x_1, u_1) + \sum_{\omega=1}^{N_\Omega} \pi_\omega \times \sum_{t=2}^{T} \left\{ \lambda_{t\omega}^{\mathrm{R}} u_{t\omega} - f_t(x_{t\omega}, u_{t\omega}) \right\} \tag{9.9a}$$

$$\text{s.t.} \quad g(x_\omega, u_\omega) \geq 0 \qquad \forall \omega, \tag{9.9b}$$

where $f_t(\cdot)$ is the consumer utility and $g(\cdot)$ defines the set of constraints. It should be noticed that the consumption, u_ω, and the states, x_ω, are scenario-dependent after the initial time period. This models the fact that consumers are allowed to modify their consumption plan in the subsequent time period, when new information is available

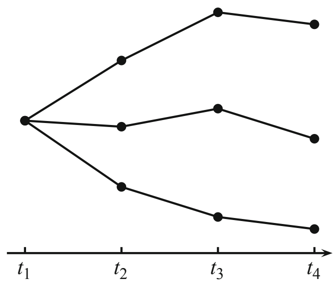

Fig. 9.6 Example of scenario fan for dynamic price

on the future price level. In other words, the consumption is a *recourse* decision (see Appendix C).

The summation over the scenario index ω in (9.9a) represents the expectation of the costs. This implies the assumption that the consumer is risk-neutral, since she/he aims at minimizing the expectation of the difference between cost and benefit, with no regards for whether the distribution entails large losses when the worst-case scenario(s) realize. Furthermore, we remark that constraints (9.9b) must be satisfied in any scenario.

Formulation (9.9) is rather general and can be applied to all the models presented in Sect. 9.3, provided that $f(\cdot)$ and $g(\cdot)$ are replaced with the relevant objective functions and constraints.

It should be noticed that the stochastic programming formulation preserves the linearity and convexity properties of its deterministic counterpart. However, there is an increase in the number of variables and constraints, which depends linearly on the number of scenarios N_Ω.

Example 9.5 (Deferrable Load with Stochastic Programming) We now consider the stochastic programming counterpart of the problem in Example 9.1, where the scenarios for prices are the ones given in Table 9.5.

The resulting optimization problem writes as follows.

$$\begin{aligned}
\text{Min.}\quad & (120-100)u_1 \\
& +0.5\times\{(105-100)u_{21}+(154-100)u_{31} \\
& +(84-100)u_{41}\}+0.5\times\{(45-100)u_{22} && \text{(9.10a)} \\
& +(66-100)u_{32}+(36-100)u_{42}\} \\
\text{s.t.}\quad & u_1-0\le 1.5, && \text{(9.10b)} \\
& u_{2\omega}-u_1\le 1.5, \qquad \omega=1,2, && \text{(9.10c)} \\
& u_{t\omega}-u_{(t-1)\omega}\le 1.5, \qquad t=3,4,\ \omega=1,2, && \text{(9.10d)} \\
& u_1-0\ge -1.5, && \text{(9.10e)} \\
& u_{2\omega}-u_1\ge -1.5, \qquad \omega=1,2, && \text{(9.10f)} \\
& u_{t\omega}-u_{(t-1)\omega}\ge -1.5, \qquad t=3,4,\ \omega=1,2, && \text{(9.10g)} \\
& u_1\le 3, && \text{(9.10h)}
\end{aligned}$$

Table 9.5 Price scenarios

Scenario	π_ω	λ_1^R	$\lambda_{2\omega}^R$	$\lambda_{3\omega}^R$	$\lambda_{4\omega}^R$
1	0.5	120	105	154	84
2	0.5		45	66	36

Table 9.6 Optimal consumption

Scenario	u_1	$u_{2\omega}$	$u_{3\omega}$	$u_{4\omega}$	Total
1	0.25	1.75	1.25	2.75	6
2		1.75	3	3	8

$$u_{t\omega} \leq 3, \qquad t = 2, 3, 4, \ \omega = 1, 2, \tag{9.10i}$$

$$u_1 + \sum_{t=2}^{4} u_{t\omega} \leq 8, \qquad \omega = 1, 2, \tag{9.10j}$$

$$u_1 + \sum_{t=2}^{4} u_{t\omega} \geq 6, \qquad \omega = 1, 2, \tag{9.10k}$$

$$u_1 \geq 0, \tag{9.10l}$$

$$u_{t\omega} \geq 0, \qquad t = 2, 3, 4, \ \omega = 1, 2. \tag{9.10m}$$

Note that the utility in each scenario is added after weighting it by its probability in the objective function. Furthermore, there is an instance of each constraint in the deterministic problem (9.1) for every scenario.

The optimal consumption is shown in Table 9.6, and illustrated in Fig. 9.7 along with the price scenarios. Observe that, in the first scenario, the electricity price is greater than the marginal benefit, exception made for the fourth time period, while in the second scenario it is lower than the marginal benefit, with the exception of the first time period. As a result, the total consumption is at the lower bound for the first scenario, and at the upper bound for the second one.

At this point, an important remark should be made regarding the modeling of the uncertainty in this example. The setup we considered involves a simplification: new branches of the price scenarios only generate at the second time period in Table 9.5. This results in scenario fans of the type sketched in Fig. 9.6. After the second time period, the price evolves along one of the two certain paths in Table 9.5, i.e., it stays high until the end of the period if it is high at time $t = 2$, and low if it is low at time $t = 2$. The optimal consumption plans in Table 9.6 are determined under this assumption that price trajectories are known with certainty after time $t = 2$. Therefore, they are suboptimal in reality, since at time $t = 2$ there is still uncertainty on the evolution of the price in the following time steps. In other words, the problem at hand is a problem of *multi-stage stochastic programming with recourse*. We refer the reader to Appendix C for an introduction to this type of problems, which are very challenging from a computational point of view as they involve *scenario trees*, where new realizations of the uncertainty branch out at every time period. In Sect. 9.4.3,

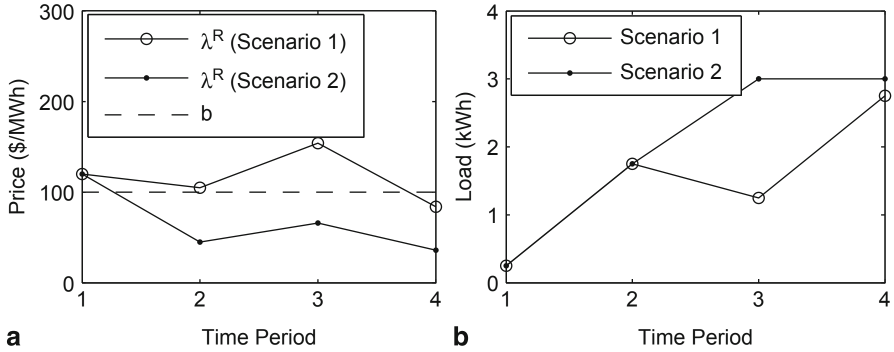

Fig. 9.7 Price and optimal consumption for a deferrable load with stochastic programming. **a** Price. **b** Consumption

we consider a receding-horizon approach to account for the multi-stage nature of the uncertainty without having to resort to computationally expensive scenario trees.

9.4.2 Robust Optimization Approach

In the robust optimization framework [1], stochastic parameters are characterized by the so-called *uncertainty sets*, which the uncertain parameters can take values in. The objective function is then optimized in the worst-case point of these sets.

To set up a robust optimization model, we make the assumption that the consumer price $\widetilde{\lambda}_t^{\mathrm{R}}$ takes values in the interval $[\bar{\lambda}_t^{\mathrm{R}} - \widehat{\lambda}_t^{\mathrm{R}},\ \bar{\lambda}_t^{\mathrm{R}} + \widehat{\lambda}_t^{\mathrm{R}}]$, where $\bar{\lambda}_t^{\mathrm{R}}$ is the "nominal" value for the price and $\widehat{\lambda}_t^{\mathrm{R}}$ is the maximum allowed deviation from this nominal value. Furthermore, the total deviation in the horizon is bounded so that the price cannot take an extreme value in more than Γ time periods. This results in the following deterministic set for the uncertain price sequence:

$$\Lambda = \left\{ \lambda_t^{\mathrm{R}} \in \mathbb{R}\ \forall t : \lambda_t^{\mathrm{R}} \in [\bar{\lambda}_t^{\mathrm{R}} - \widehat{\lambda}_t^{\mathrm{R}},\ \bar{\lambda}_t^{\mathrm{R}} + \widehat{\lambda}_t^{\mathrm{R}}]\ \forall t,\ \sum_{t=2}^{T} \frac{|\lambda_t^{\mathrm{R}} - \bar{\lambda}_t^{\mathrm{R}}|}{\widehat{\lambda}_t^{\mathrm{R}}} \leq \Gamma \right\}. \quad (9.11)$$

Notice that $0 \leq \Gamma \leq T-1$ must hold for (9.11) to be meaningful; however, Γ need not necessarily be an integer. Besides, since the dynamic price is known in real-time, we set $\widehat{\lambda}_1^{\mathrm{R}} = 0$. This way, the interval collapses into a single point at the first time period, implying that there is no uncertainty for the real-time price at this stage. An example of uncertainty set is sketched in Fig. 9.8, where the price has maximum deviation from the nominal value in at most three time periods ($\Gamma = 3$). Notice that the uncertainty set is actually a single point at t_1. The solid line represents a feasible realization of the price within the uncertainty set.

It is important to remark that the set Λ in (9.11) is a polytope, i.e., it is bounded and can be represented by a finite number of linear inequalities.

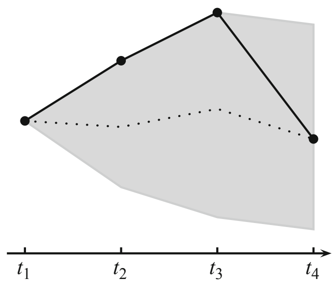

Fig. 9.8 Example of uncertainty set for prices with $\Gamma = 3$

The objective is to determine the consumption schedule for the next T time periods that ensures optimal benefit in the worst-case realization of the uncertain price in Λ. In the problems considered in this chapter, the dynamic price only appears in the objective function (9.1a), but not in the constraints. Therefore, we can formulate the problem as follows

$$\underset{x_t,u_t}{\text{Min.}} \quad \lambda_1^{\mathrm{R}} u_1 - f_1(x_1, u_1) - \sum_{t=2}^{T} f_t(x_t, u_t) + \underset{\lambda^{\mathrm{R}} \in \Lambda}{\text{Max}} \sum_{t=2}^{T} \lambda_t^{\mathrm{R}} u_t \tag{9.12a}$$

$$\text{s.t.} \quad g(x, u) \geq 0. \tag{9.12b}$$

The min–max structure of this problem is due to the fact that, while the outer minimization problem delivers the optimal consumption schedule, u, and the resulting states, x, the inner maximization problem enforces the realization of the uncertain price sequence, λ^{R}, resulting in the highest cost for the consumer. Notice that the consumption, u, enters the latter problem as a constant, since its optimal value is fixed by the outer minimization problem.

As a matter of fact, the worst-case realization of the uncertain price can only involve deviations in the upper halves of the intervals. Indeed, it is sufficient to notice that for any price sequence, one could simply replace the values of the price below the nominal value with the nominal value itself, without violating the constraints defining Λ and obtaining a higher cost for the consumer. Therefore, we can reformulate the inner problem equivalently as

$$\underset{z_t}{\text{Max.}} \quad \sum_{t=2}^{T} \left(\bar{\lambda}_t^{\mathrm{R}} + z_t \widehat{\lambda}_t^{\mathrm{R}} \right) u_t \tag{9.13a}$$

$$\text{s.t.} \quad z_t \leq 1 : \quad \xi_t, \qquad \forall t > 1, \tag{9.13b}$$

$$\sum_{t=2}^{T} z_t \leq \Gamma : \quad \beta, \tag{9.13c}$$

$$z_t \geq 0, \qquad \forall t > 1, \tag{9.13d}$$

where the symbols ξ_t and β after the colon indicate the dual variables of the corresponding constraint. We refer the reader to Appendix B for an introduction to duality.

The dual of problem (9.13) is the following

$$\begin{aligned}
\underset{\xi_t,\beta}{\text{Min.}} \quad & \sum_{t=2}^{T} \xi_t + \Gamma\beta && (9.14a)\\
\text{s.t.} \quad & \xi_t + \beta \geq \widehat{\lambda}_t^{\mathrm{R}} u_t, \qquad \forall t > 1, && (9.14b)\\
& \beta \geq 0,\ \xi_t \geq 0, \qquad \forall t > 1. && (9.14c)
\end{aligned}$$

Note that, since problem (9.13) is linear, this problem and its dual (9.14) obtain the same optimal value (see Appendix B).

Replacing (9.13) with the equivalent minimization problem (9.14) in (9.12), the following single-level optimization problem is obtained:

$$\begin{aligned}
\underset{x_t,u_t,\xi_t,\beta}{\text{Min.}} \quad & \lambda_1^{\mathrm{R}} u_1 - f_1(x_1,u_1) - \sum_{t=2}^{T} f_t(x_t,u_t) + \sum_{t=2}^{T} \xi_t + \Gamma\beta && (9.15a)\\
\text{s.t.} \quad & g(x,u) \geq 0, && (9.15b)\\
& \xi_t + \beta \geq \widehat{\lambda}_t^{\mathrm{R}} u_t, \qquad \forall t > 1, && (9.15c)\\
& \beta \geq 0,\ \xi_t \geq 0, \qquad \forall t > 1. && (9.15d)
\end{aligned}$$

Observe that this problem involves little additional complexity as compared to its deterministic counterpart. Indeed, the robust optimization problem (9.15) includes the additional T variables ξ_t and β, as well as the $T-1$ constraints (9.15c). Furthermore, the original (linear or convex) structure of the problem is preserved. This fact makes this problem particularly well suited to demand response applications, as it requires little computational effort.

In the next example, we formulate the problem presented in Example 9.1 in the framework of robust optimization.

Example 9.6 (Deferrable Load with Robust Optimization) The deferrable load with single block of Example 9.1 is here reconsidered in the framework of robust optimization. The consumer price is assumed to deviate at most by 40 % of the nominal values reported in Table 9.1(b), with the exception of the first time period, when the price is perfectly known to the consumer. This price band is represented in Fig 9.9(a) as a gray-shaded area. Furthermore, Γ is set to 2, thus allowing the price to reach the upper bound of the interval in no more than two time periods.

The optimal consumption schedule and the worst-case price are illustrated in Fig. 9.9, and reported in Table 9.7. As one expects, the peaks in consumption take place in the second and fourth time periods. Compared to its deterministic counterpart in Example 9.1, however, the robust optimization approach delivers a more conservative solution. Indeed, differently from the deterministic solution, the consumption in the considered horizon totals only $\sum_{t=1}^{4} u_t = 6$ kWh, i.e., equal to the lower bound $\underline{U}^{\mathrm{d}}$.

It should be remarked that there are several worst-case realizations of the price sequence for the same optimal consumption schedule. Table 9.7 and Fig. 9.9(a) only present one of them.

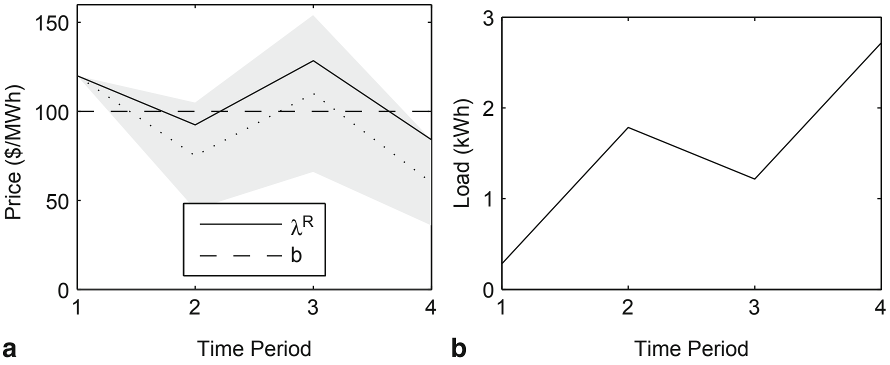

Fig. 9.9 Price and optimal consumption for a deferrable load with robust optimization. **a** Price. **b** Consumption

Table 9.7 Optimal consumption schedule for deferrable load with robust optimization

Time period	Consumption (kWh)	Price ($/MWh)
1	0.28	120.00
2	1.78	92.43
3	1.22	128.43
4	2.72	84.00
Total	6	–

Table 9.8 allows a comparison between the robust optimization approach and the stochastic programming one presented in Example 9.5. Let us indicate the stochastic programming solutions for the high-price and the low-price scenarios with SP_1 and SP_2, respectively. Table 9.8(a) displays the cost for electricity purchase, the utility and the objective function, which is equal to their difference, in the worst-case realization of the price on the rightmost column of Table 9.7. Notice that the stochastic solution SP_2 for the low-price realization performs rather poorly, resulting in a positive objective. In turn, the solution for the high-price realization obtains the same objective function value for this realization of the price as the robust optimization solution. However, we cannot conclude that the stochastic programming solution SP_1 achieves the same level of robustness as the robust optimization solution. Table 9.8(b) reports the consumer results for another worst-case realization of the price in the robust optimization approach, $\lambda^{R} = [120, 75, 154, 84]^{\top}$. Remarkably, the objective function

Table 9.8 Consumer results for the worst-case realization of the price $\lambda^{R} = [120, 92.43, 128.43, 84]^{\top}$ (left) and for the alternative worst-case price realization $\lambda^{R} = [120, 75, 154, 84]^{\top}$ (right). Values are expressed in $\$\times 10^{-3}$

(a) Worst-case price realization				(b) Alternative worst-case price realization			
Solution	Cost	Utility	Objective	Solution	Cost	Utility	Objective
SP_1	583.30	600	−16.70	SP_1	584.75	600	−15.25
SP_2	829.05	800	29.05	SP_2	875.25	800	75.25
RO	583.30	600	−16.70	RO	583.30	600	−16.70

Table 9.9 Sensitivity of the optimal schedule to Γ. Values are expressed in KWh

Time period	$\Gamma = 0.8$	$\Gamma = 1.2$	$\Gamma = 1.6$	$\Gamma = 2$
1	0.90	0.70	0.28	0.28
2	2.40	2.20	1.78	1.78
3	1.50	1.50	1.22	1.22
4	3.00	3.00	2.72	2.72
Total	7.80	7.40	6	6

for both the stochastic programming solutions SP_1 and SP_2 is worse than the one obtained with the robust optimization solution. Finally, we underline that the expected value of the objective function obtained with the robust solution, averaged over the two scenarios considered in Example 9.5, is $-\$135.41$, which is higher than (i.e., worse) the one obtained with the stochastic programming solution ($-\$174$).

Table 9.9 illustrates how the optimal consumption schedule changes as a result of different values of the level of conservatism Γ. Notice that increasing values of Γ result in decreasing total consumption over the horizon, reflecting the fact that the consumer expects higher real-time prices.

9.4.3 Model Predictive Control Framework for Flexible Consumers

The optimization models presented in the previous sections aim at determining the best future consumption pattern given the current real-time price and an estimate of its evolution given the information available at time t. However, the consumption pattern determined at time t becomes suboptimal as time goes owing to the following reasons:

- The estimate of the dynamic price obtained at time t may not be actual anymore, due to unforeseen system changes, e.g., an unexpected increase or decrease in the output of stochastic producers.
- Consumption needs may have changed as compared to the ones forecast at time t owing to, e.g., differences in the external temperature that require higher or lower electricity consumption to achieve the indoor temperature target.

For these reasons, it seems reasonable that the consumer decision on the load pattern is updated as new information becomes available.

A suitable framework to accommodate problems of this type is Model Predictive Control (MPC). MPC assumes that a model of the consumer dynamics is available. At time $t = 1$, an optimization problem under uncertainty is solved for the time horizon T, determining the optimal consumption for all the time periods included in the horizon, given the information available at the time. However, only the first value u_1 of the optimal consumption is implemented.

At the following time step $t = 2$, a new real-time price and a new estimate of the future prices become available. Furthermore, in some problems the consumer's

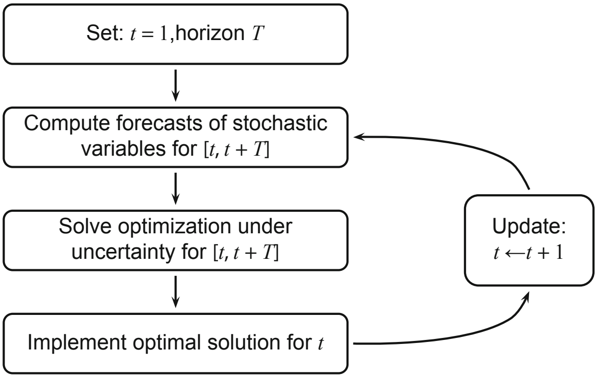

Fig. 9.10 Block diagram sketching the model predictive control setup

equipment can measure or estimate the value of the states of the system. For instance, in the heating problem (9.6), the value of the temperature states x_t^1 and x_t^2 can be measured. Notice that the actual value of the states may be uncertain in a practical situation due to disturbances in the model, e.g., external temperature or solar irradiance in the case of building dynamics.

Once all this information is updated, a new optimization is carried out for the time horizon between $t = 2$ and $T + 1$. Notice that the length of the optimization horizon is still equal to T, owing to the index increment of the last time period in the horizon. This strategy is often referred to as *receding horizon*. The block diagram in Fig. 9.10 sketches the steps in the implementation of MPC.

It is important to notice that MPC is relevant for consumer optimization problems involving uncertainty, employing both the stochastic programming and the robust optimization approach. Indeed, it is the continuous updating of the information, particularly regarding the electricity price and the system state, and of the consequent optimal strategy, that makes it possible to reap the benefits of real-time pricing.

In the next example, we apply the MPC setup to the determination of the optimal consumption plan for a deferrable load using stochastic programming. In order to do so, real-time price scenarios have to be updated and a new stochastic programming problem must be solved at every time period.

Example 9.7 (Deferrable Load with Stochastic Programming in an MPC Framework) Let us reconsider the problem in Example 9.5, where the optimal consumption for a deferrable load is determined using stochastic programming. In this example, we assume that price forecasts are updated at every time period, and therefore wish to exploit this information using the MPC approach. For the sake of simplicity, and for completeness of the analysis of the results, we do not consider a receding horizon. Instead, the look-ahead time decreases so that the final time period of the horizon is $T = 4$ at any time period.

The price forecasts used in this example, updated at every time period, are shown in Table 9.10. At the first time period, the high-price and the low-price scenarios are equally likely, as in Example 9.5. At time $t = 2$, the low price $\lambda_2^{\mathrm{R}} = \$45$/MWh realizes. At this point, the probability of the prices remaining low (0.7), i.e., scenario

Table 9.10 Price scenarios updated at every time period considered in the horizon. Prices in bold are actual realizations

Issuance time index	ω	p_ω	Price forecast ($/MWh) $\lambda^R_{1\omega}$	$\lambda^R_{2\omega}$	$\lambda^R_{3\omega}$	$\lambda^R_{4\omega}$
1	1	0.5	**120**	105	154	84
	2	0.5		45	66	36
2	1	0.3	–	**45**	154	84
	2	0.7	–		66	36
3	1	0.7	–	–	**154**	84
	2	0.3	–	–		36
4	1	1	–	–	–	**84**
	2		–	–	–	

Table 9.11 Optimal consumption plan as a result of the repeated optimization processes at every time period considered in the horizon. Values in bold font are implemented in the MPC strategy

Optimization time index	ω	Optimal consumption (kWh) $u_{1\omega}$	$u_{2\omega}$	$u_{3\omega}$	$u_{4\omega}$
1	1	**0.25**	1.75	1.25	2.75
	2		1.75	3	3
2	1	–	**1.75**	1.25	2.75
	2	–		3	3
3	1	–	–	**1.25**	2.75
	2	–	–		2.75
4	1	–	–	–	**2.75**
	2	–	–	–	

2, is higher than the probability (0.3) of switching to the high price scenario 1. However, the latter event realizes at time 3, resulting in the (certain) real-time price $\lambda^R_3 = \$154$/MWh. Again, the probability of the price remaining high at the next time period is higher than the probability of switching back to the low-price scenario (0.7 against 0.3). At the last time period $t = 4$, the high price $\lambda^R_4 = \$84$/MWh realizes.

Table 9.11 reports the consumption schedules obtained by solving a sequence of stochastic programming problems with the price scenario forecasts in Table 9.10 as inputs. Notice that only the first value of the optimal consumption schedule, indicated in bold font, is actually implemented at each time period.

In the case considered in Tables 9.10 and 9.11, the realization of the real-time price corresponds to the value in the original low-price scenario at time $t = 2$ and to the values in the high-price scenario at time $t = 3$ and $t = 4$. Notice that this is only one out of eight possible cases according to the considered scenarios. At any time period, we forecast that the price will either take a high or a low value with a certain probability. Since we are considering three stochastic stages (the real-time price at the first time stage is known with certainty), these successive bifurcations generate $2 \times 2 \times 2 = 8$ scenarios in total. In principle, we should have considered all these branches in the resulting scenario tree when solving the stochastic programming problem in Example 9.5, rather than the scenario fan of the type depicted in Fig. 9.6, where scenarios only branch out of the first node. However, observe that the growth in number of scenarios is exponential in the number of stages, which rapidly leads to intractability. Figure 9.11 illustrates the exponential growth of the number of scenarios in the number of time stages. We refer the reader to Appendix C for a discussion on *multi-stage stochastic programming with recourse*.

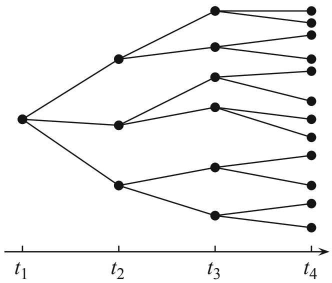

Fig. 9.11 Example of scenario tree for dynamic price

Table 9.12 Possible realizations of the price, their associated probability of occurrence and objective function for deterministic (Det.), stochastic programming (SP) and MPC approaches

		Actual objective function ($)		
Case	Probability	Det.	SP	MPC
High-high-high	0.245	65.50	37.25	37.25
High-high-low	0.105	−78.50	−94.75	−94.75
High-low-high	0.045	−66.50	−72.75	−136.25
High-low-low	0.105	−210.50	−204.75	−280.25
Low-low-low	0.245	−360.50	−385.25	−385.25
Low-low-high	0.105	−216.50	−241.25	−241.25
Low-high-low	0.045	−228.50	−121.25	−199.75
Low-high-high	0.105	−84.50	22.75	−67.75
Expectation	–	−147.50	−148.38	−172.20

Let us now take a look at the results obtained by solving the sequence of optimization problems required by the MPC approach for the eight possible realizations of the real-time price. Table 9.12 reports the actual objective function, i.e., obtained by summing over all the time periods the benefit of the decision actually implemented by the consumer, for each possible realization of the price. The results obtained by implementing the consumption schedule resulting from the deterministic problem in Example 9.1, the schedule from the stochastic programming problem in Example 9.5 and the MPC solution are reported. We remark that in the stochastic programming case, the consumption plan for the realization of the high-price scenario in Example 9.5 is implemented in the first four cases, while in the last four the schedule for the low-price scenario is implemented. Notice that the lower the values of the objective function, the better the result for the consumer. Furthermore, the probability associated with each realization of the real-time price is included in the table. For example, the probability associated with the *low-high-high* price scenario considered above is $0.5 \times 0.3 \times 0.7 = 0.105$, where 0.5 is the probability of any branch at $t = 2$, 0.3 is the probability of a price switch from low to high at $t = 3$, and 0.7 is the probability of the price remaining high at $t = 4$. The last row in the table contains the value of the actual objective function averaged over all the possible scenarios. As one can see, the MPC approach further enhances the value of the stochastic programming solution for the consumer, substantially increasing the benefit as compared to the deterministic approach.

Finally, we remark that the implementation of a multistage stochastic programming problem, where the uncertainty in the price is modeled with a scenario tree, would result in a solution yielding a benefit at least as high as the one of the MPC approaches employed here.

9.5 Forecasting the Potential for Demand Response

In an operational setup, it might be of interest to predict the aggregate potential for the consumers to adapt their load, rather than modeling directly and individually their elasticity on the basis of the expected use of electricity. Forecasts are not to be generated for each individual household, but instead at an aggregate level, at which demand response may provide services to the system (for instance, contributing to balancing the forecast errors of renewable energy generation). These forecasts may be used as input to approaches based on MPC or on some form of stochastic optimization, aimed at optimizing the response of electricity demand through a control signal, e.g., a dynamic price sequence.

Considering a control-by-price framework, where consumers adapt their load to the variations of a dynamic price, the interest is on predicting the *conditional dynamic elasticity* of electricity demand. Conceptually, it may be intuitive to split the consumption, u_t, into an inflexible and a flexible (or better say, responsive) component. In practice, though, metering may only report overall consumption measurements, thus not allowing a discrimination between these two components. Consequently, it might be harder to forecast the inflexible and the responsive parts of the electric demand individually than at an aggregate level.

Let us further clarify the concept of conditional dynamic elasticity. It is *conditional* because the potential to influence the responsive part of the load through a dynamic price is a function of external conditions. In the case of space heating, outdoor temperature directly affects the potential to adapt consumption to price variations. For example, extremely low external temperatures may reduce the willingness of consumers to lower their consumption for heating in case of high electricity prices. Similarly, for the case of electric vehicles the potential for demand response varies as a function of the time of the day, since this influences the number of plugged electric vehicles, and in turn the number of batteries made available for demand response services. In parallel, the consumer elasticity is *dynamic* as demand may not be deferred indefinitely. Batteries of electric vehicles must eventually be charged before driving, while households must be heated to keep indoor temperature at an acceptable level. Moreover, the elasticity of electricity consumption may evolve with time, owing to changes in consumption patterns, appliances and their functionalities, etc.

Numerous methodologies are available in the literature to model and predict the aggregate electricity consumption. For a recent overview of these approaches, see [6]. Originally, these methodologies were designed without distinguishing between inflexible and responsive parts of the load. This is since, historically, electricity demand was seen as nonresponsive, and mainly driven by external conditions, e.g., outdoor temperature, time of day, etc. In that general framework, consumption forecasts for lead time $t + k$ are issued at time t on the basis of functional expressions of the following form:

$$\widehat{u}_{t+k|t} = f(\bar{u}_t, \bar{x}_t), \tag{9.16}$$

that is, with future consumption being a function of its m most recent past values:

$$\bar{u}_t = \left[u_{t-1}, u_{t-2}, \ldots, u_{t-m}\right]^\top, \tag{9.17}$$

and of external conditions:

$$\bar{x}_t = \left[x_{t-1}, x_{t-2}, \ldots, x_{t-m}, \widehat{x}_{t+1|t}, \widehat{x}_{t+2|t}, \ldots, \widehat{x}_{t+k|t}\right]^\top. \tag{9.18}$$

Here the external conditions are summarized by the set of the m most recent past values and by the forecasts up to time $t + k$ for the relevant explanatory variables, e.g., outdoor temperature, solar irradiance, and time of the day.

Example 9.8 (Prediction of the Electric Consumption for a Group of Households) Let us consider a simple setup, where the average (over all households) consumption pattern for a group of households is temperature-driven and dependent upon activity (and thus, upon the time of the day). In this simple setup, consumption is basically seen as inflexible, i.e., not to be influenced by price variations.

An analyst and forecaster identifies the following model in order to predict how consumption at time $t + k$ is linked to the temperature forecast, $\widehat{x}_{t+k|t}$, and to the time of the day, h_t. The model involves the sum of two components. The first of them takes the form of a logistic regression on temperature forecasts, while the second, activity-based one is formed by a sum of kernel functions. This yields:

$$\widehat{u}^{\mathrm{I}}_{t+k|t} = \underbrace{3\left(1 - \frac{1}{1 + \exp\left(0.5(3 - \widehat{x}_{t+k|t})\right)}\right)}_{\text{temperature driven}} + \underbrace{2 + 0.8\ K(h_t, 7, 0.75) + 3\ K(h_t, 20, 1)}_{\text{activity related}}, \tag{9.19}$$

where

$$K(h_t, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(h_t - \mu)^2}{2\sigma^2}\right). \tag{9.20}$$

The superscript "I" on the left-hand side of (9.19) indicates that the consumption forecast is for an inflexible load (with respect to price variations).

These components are depicted in Fig. 9.12. The model for the temperature-driven consumption in Fig. 9.12(a) is decreasing with external temperature. Demand is equal to the nominal power consumption of the heating appliances for the low temperature values on the left-hand side of the figure. When the outdoor temperature is mild enough, i.e., on the right-hand side of Fig. 9.12(a), there is no need for heating, and hence the consumption drops to 0. Secondly, the activity-related component of the electricity consumption pattern peaks in morning and evening periods, as illustrated

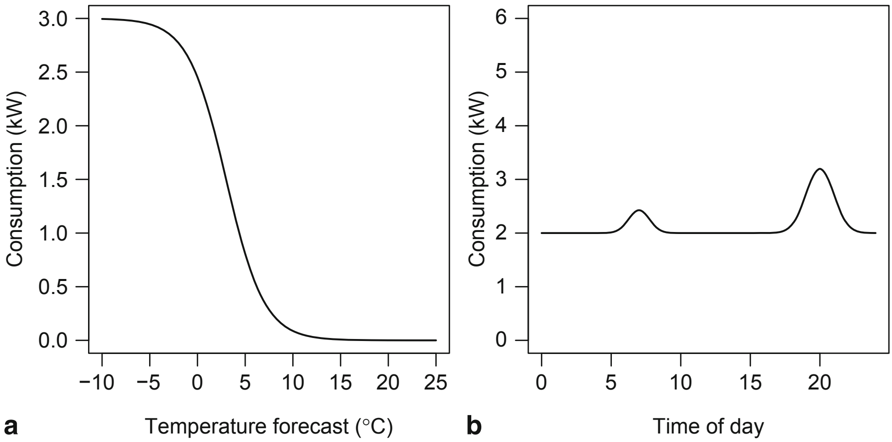

Fig. 9.12 Components of the model used for forecasting the electricity consumption for a group of households. **a** Regression on temperature forecasts. **b** Activity-based component

in Fig. 9.12(b). During these hours, consumers are at home, using light and electrical appliances.

Based on these models for the two components, forecasts for the electricity consumption of households may be readily obtained with temperature predictions and time of the day as inputs, after nonlinear transformations through the above models. Figure 9.13 illustrates two such forecasts issued for two different days, and hence with different predicted temperature evolutions. Both forecasts were issued at noon with hourly resolution and a 48-h horizon. Naturally, a higher electricity consumption is predicted in the case of lower temperature forecasts, which is a result of the fact that part of the load is a decreasing function of temperature.

Finally, note that if a dynamic price signal is used, a part of both types of consumption may become flexible, while a part may remain inflexible. This is since the indoor temperature ought to be kept above a certain threshold, while a minimum number of appliances are required to be on when the consumers are at home.

We now concentrate on the case where electricity consumption has a more flexible nature, i.e., it is price responsive. In a control-by-price framework, electricity consumption evolves dynamically as a function of electricity prices, as well as of external conditions. In this case, forecasts for consumption, issued at time t for time $t+k$, are based on functional expressions of the following type:

$$\widehat{u}_{t+k|t} = f(\bar{\lambda}_t^{\mathrm{R}}, \bar{u}_t, \bar{x}_t), \tag{9.21}$$

where

$$\bar{\lambda}_t^{\mathrm{R}} = [\lambda_{t-l_p}^{\mathrm{R}}, \ldots, \lambda_t^{\mathrm{R}}, \ldots, \lambda_{t+l_f}^{\mathrm{R}}]^\top \tag{9.22}$$

is a set of l_p past, the current and l_f future prices, while $\bar{x}_t$ is as in (9.18), i.e., it includes measured and predicted values for relevant explanatory variables. $\bar{u}_t$ was

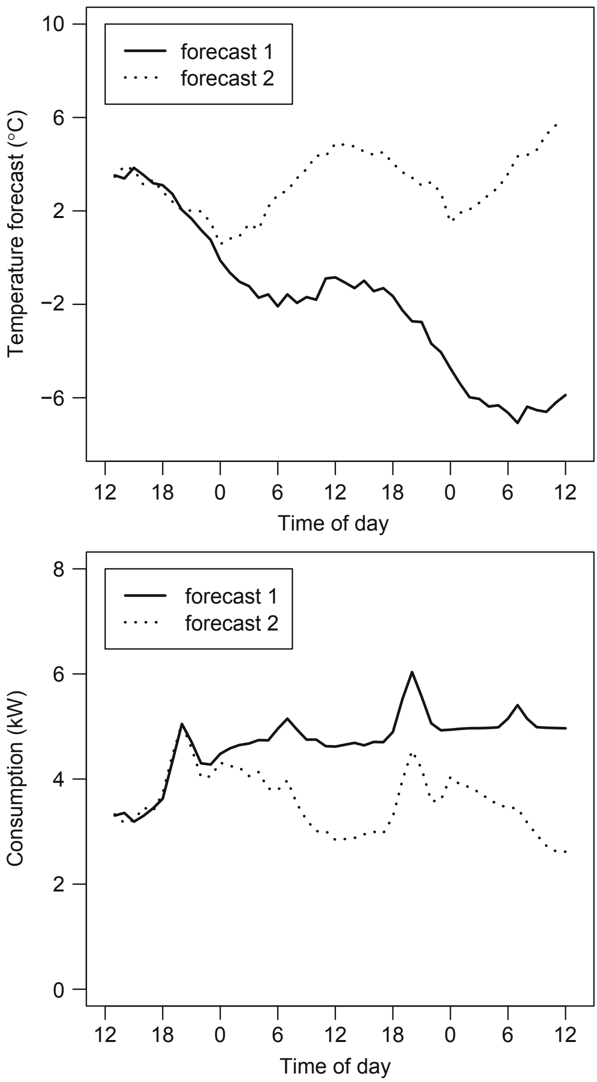

Fig. 9.13 Example with two forecasts for the electricity consumption of a group of households, corresponding to two different forecasts for the evolution of outdoor temperature. These were both issued at noon and for the following 48 h, with an hourly resolution

already defined in (9.17) as the set of past consumption values. In an MPC framework where electricity prices are issued in near real-time, only past price values are to be considered—translating to $l_f = 0$. However, in a more general stochastic optimization framework, where price sequences are communicated in advance for a given time period (say, the following day), both past and future prices influence demand response at any time t.

Example 9.9 (Prediction of the Electric Consumption for a Group of Households, Made Flexible Through Real-Time Pricing) Let us consider the setup of the previous

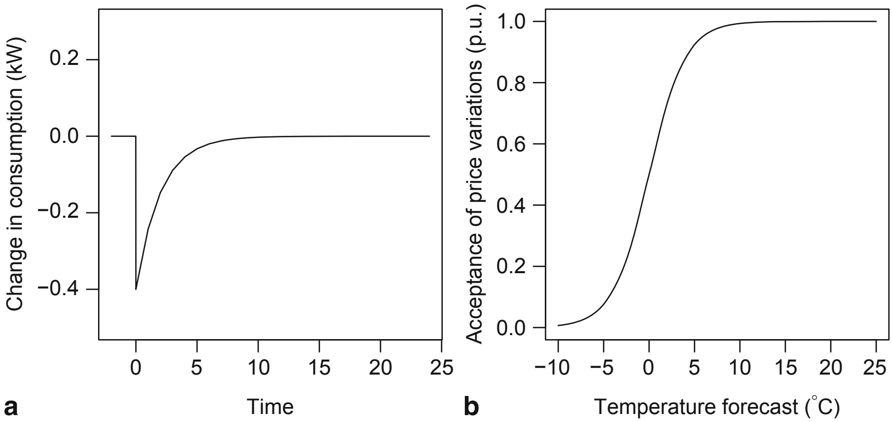

Fig. 9.14 Extra components for the model used to predict flexible consumption patterns as influenced by real-time prices and temperature-related acceptance of household inhabitants. **a** Finite Impulse Response (FIR). **b** Willingness to adapt consumption

example, though now assuming that the electricity consumption of these households is flexible, as enabled by a real-time pricing environment.

The analyst and forecaster extends the previous model, used to forecast electricity consumption, by adding an extra component describing how consumption evolves dynamically with price variations. Since prices are communicated in near real-time, the flexible part of the household consumption at a given time t can only be affected by previous and current deviations $\Delta\lambda_{t-i}$, $i \geq 0$, from the reference price (i.e., the price that is usually paid by the consumer). This can be conveniently written in the form of a finite impulse response (FIR) model, modeling the effect of price variations on the electricity consumption. However, the willingness of the consumers to adapt to price variations is to be seen as conditional, being in this case a function of the outdoor temperature. The observations above yield the following model:

$$\widehat{u}^{\mathrm{F}}_{t+k|t} = \widehat{u}^{\mathrm{I}}_{t+k|t} \left(1 + \underbrace{\frac{\tau}{1 + \exp\left(0.5(3 - \widehat{x}_{t+k|t})\right)}}_{\text{willingness to accept}} \underbrace{\sum_{i \geq 0} \exp\left(-i/2\right) \Delta\lambda_{t+k-i}}_{\text{FIR}} \right), \tag{9.23}$$

where $\widehat{u}^{\mathrm{I}}_{t+k|t}$ is obtained from (9.19). The superscript "F" on the left-hand side of (9.23) indicates that the consumption forecast is for a flexible (i.e., price-responsive) load. τ is an additional parameter controlling the share of the flexible part of total consumption.

These two extra components, which represent how the electricity consumption of this household is made additionally flexible through real-time pricing, are depicted in Fig. 9.14. The FIR part shows how a unit increase in the price at a given time, in the case of Fig. 9.14(a) of \$1 at time 0, induces changes in the consumption over the following time periods. The magnitude of the change is maximum at the time of the price change, when consumption drops by 0.4 kW per any \$ 1 increment in price. This impact slowly fades away as time passes. In parallel, the logistic function of Fig. 9.14(b) describes the willingness of consumers to adapt their consumption to changes in prices, as a function of outdoor temperature. This scales between 0 and 1, i.e., from not adapting at all to fully adapting to price variations. Indeed, when the temperature is very low, as on the left-hand side of Fig. 9.14(b), consumers may want to heat their house, no matter how high the price is. For milder temperatures, their willingness to adapt consumption to varying prices naturally increases. It is to be noted that these models are for the sake of illustration only: for instance, in a more advanced setup, the FIR component is likely to be nonlinear and a function of the consumption level itself.

For a given sequence of deviations from reference electricity prices, the way flexible consumption evolves can be predicted as a function of outdoor temperature and time of the day. As an extension to the example of Fig. 9.13, Fig. 9.15 illustrates how such a sequence of price variations impacts the previously issued consumption forecasts, when the potential price-related flexibility of the household was not accounted for. The parameter τ in (9.23) is set to 0.4. In this example, the real-time prices sent to the household are successively greater and lower than the reference price, hence inducing downward and upward pressure on the originally predicted consumption patterns, respectively. During the last part of the considered time period, there are significant differences between the two temperature forecasts in Fig. 9.13, as the first one is remarkably lower than the second one. As a result, limited flexibility in the household consumption can be observed in Fig. 9.15 for the first temperature forecast. In contrast, consumers are more willing to decrease their electricity demand in the case of the second forecast, even if they have a lower level of consumption, originally.

9.6 Bi-level Programming Models for Dynamic Pricing

The models presented in Sects. 9.3 and 9.4 deal with the optimization problems of consumers exposed to dynamic price signals. In this section, we consider the complementary problem of controlling demand response by choosing an appropriate price signal. We see this problem as a hierarchical optimization one, comprising the following problems.

1. The optimization problem of the price-setting entity, which decides the appropriate dynamic price signal *in anticipation* of the response of the consumers to this price.
2. The consumer problems, which decide on the consumption plan that maximizes their utility function once a dynamic price sequence is given.

The hierarchical structure of the problem is due to the fact that the decision on the price sequence at a given time period is fixed before the decision on the consumption is made. In view of this hierarchical relation, the price-setting problem can be called *upper-level*, while the problems of the consumers can be defined as *lower-level*.

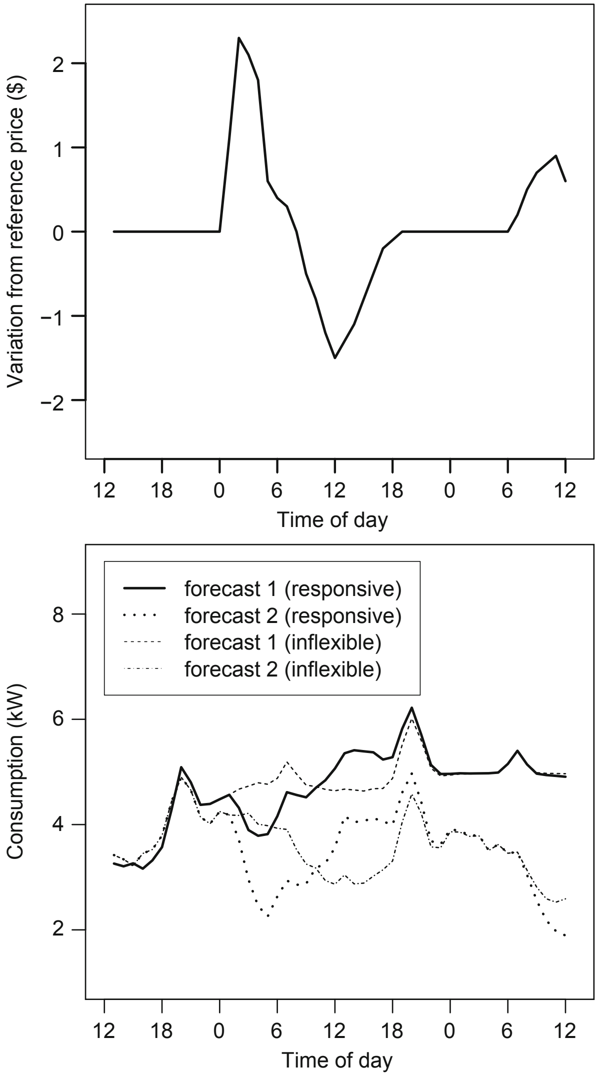

Fig. 9.15 Example with two forecasts for flexible electricity consumption of a household, as influenced by real-time pricing. These correspond to two different forecasts for the evolution of outdoor temperature and for a given sequence of variations from reference prices

In order to show how to model the problem as a bi-level program, we consider the case where the price-setting entity is an aggregator. This entity is coupled with stochastic production facilities and with consumers that are flexible in their consumption for heating. We refer to the aggregator with the name *energy management system (EMS)*, which highlights the fact that this entity is owned by the consumers that are coupled to it. In practice, this means that the aggregator aims at managing efficiently production and consumption within its area, rather than at making a profit. In turn, this implies that there is no competition at the aggregator level.

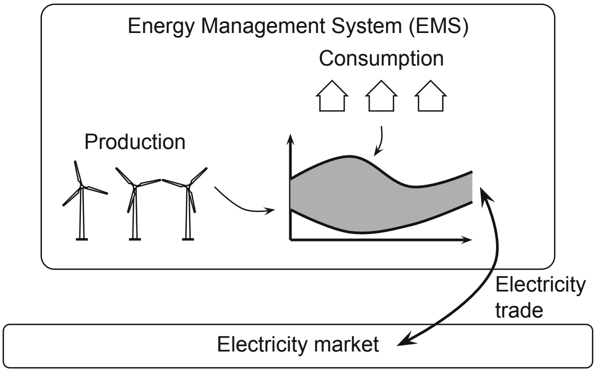

Fig. 9.16 Setup for the energy management system

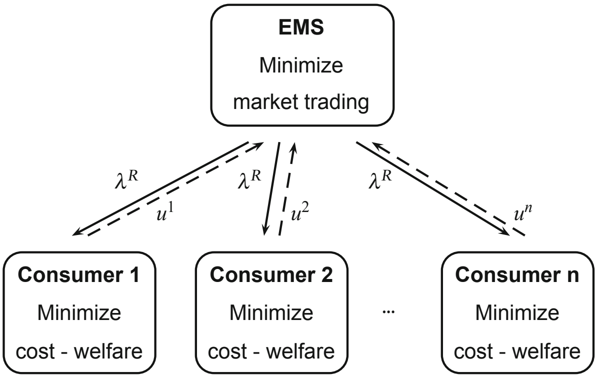

Fig. 9.17 Bi-level framework for energy management system problem

We assume that the EMS has the objective of minimizing the trade with an external market. Since it owns production facilities, such an objective directly translates into the minimization of the absolute value of the difference between the stochastic production and the load from the consumers. This setup is illustrated in Fig. 9.16. The gray-shaded area in the figure is the net difference between consumption and production within the area of the EMS, which is to be traded in an electricity market. In practice, the EMS aims at being self-sufficient as far as electricity is concerned.

In order to achieve the minimization of the electricity trading, the EMS broadcasts a *pseudo-price signal* according to which the consumers optimize their consumption schedule. It should be clarified that such pseudo-price signal is to be considered as a control signal, expressed in the same unit of a price ($/MWh). However, it is not to be intended as an electricity price in the sense implied by the marginal pricing theory [9], i.e., it is not the dual variable of a balancing condition for a commodity in a market-clearing problem.

The considered setup is sketched in Fig. 9.17. The dynamic (real-time) price signal is decided at the upper level and broadcast to the consumers (solid lines in the figure). In the lower level problem, the consumers decide on the optimal consumption level. The dashed lines in the figure indicate that the consumers do not broadcast their load back before the upper level problem is solved. However, their consumption can be inferred at the upper level if a model of the optimization problem of the consumer is available. If a bidirectional communication system is in place, the actual load for each consumer is known at the upper-level stage after the decision on the price signal. This can be used to refine the consumer models through statistical modeling.

The considered setup can be formulated as the following bi-level optimization model. For the sake of simplicity of the example, we only consider one consumer attached to the EMS. Nevertheless, there are no theoretical difficulties preventing an extension to the case of several consumers.

$$\text{Min.} \quad \sum_{t=1}^{T} |\triangle p_t| \tag{9.24a}$$

$$\text{s.t.} \quad \{u_t\} \in \arg\min \left\{ \sum_{\mathrm{t=1}}^{\mathrm{T}} \frac{\mathrm{c}}{2} \left(\mathrm{x}_\mathrm{t}^1 - \widehat{\mathrm{x}}_\mathrm{t}^1\right)^2 + \lambda_\mathrm{t}^\mathrm{R} \mathrm{u}_\mathrm{t}, \text{s.t. (9.6b)–(9.6g)} \right\} \tag{9.24b}$$

$$\triangle p_t = W_t - u_t, \qquad \forall t, \tag{9.24c}$$

$$\underline{\lambda}_t^{\mathrm{R}} \le \lambda_t^{\mathrm{R}} \le \overline{\lambda}_t^{\mathrm{R}}, \qquad \forall t. \tag{9.24d}$$

The net trading $\triangle p_t$ of the EMS with the external electricity market is defined in (9.24c) as the difference between renewable energy production (e.g., wind power generation), W_t, which is a parameter of the problem, and consumption, u_t, which is a decision variable of the lower-level problem. Notice that the net electricity trading can be either positive (sale) or negative (purchase). However, objective function (9.24a) aims at the minimization of its absolute value, summed over the considered horizon. Constraint (9.24b) enforces that the consumption is optimal for the consumer given the real-time price sequence, λ_t^{R}. Notice that the optimization problem within the *argmin* operator is equivalent to (9.6). Finally (9.24d) ensures that the real-time price stay within a predefined reasonable range.

Observe that the presence of the absolute value operator is a source of nonlinearity in (9.24). However, since such operator appears in the objective function of a minimization problem, it can be readily linearized. Indeed, the following formulation has the same optimal solution as (9.24).

$$\text{Min.} \quad \sum_{t=1}^{T} \left(\triangle p_t^+ + \triangle p_t^-\right) \tag{9.25a}$$

$$\text{s.t.} \quad u_t \in \arg\min \left\{ \sum_{\mathrm{t=1}}^{\mathrm{T}} \frac{\mathrm{c}}{2} \left(\mathrm{x}_\mathrm{t}^1 - \widehat{\mathrm{x}}_\mathrm{t}^1\right)^2 + \lambda_\mathrm{t}^\mathrm{R} \mathrm{u}_\mathrm{t}, \text{s.t. (9.6b)–(9.6g)} \right\}, \tag{9.25b}$$

$$\triangle p_t^+ - \triangle p_t^- = W_t - u_t, \qquad \forall t, \tag{9.25c}$$

$$\underline{\lambda}_t^{\mathrm{R}} \le \lambda_t^{\mathrm{R}} \le \overline{\lambda}_t^{\mathrm{R}}, \qquad \forall t, \tag{9.25d}$$

$$\triangle p_t^+, \ \triangle p_t^- \ge 0, \qquad \forall t. \tag{9.25e}$$

To prove the equivalence, it is sufficient to notice that imposing in (9.25) the additional constraint

$$\triangle p_t^+ \times \triangle p_t^- = 0, \qquad \forall t, \tag{9.26}$$

would imply that Δp_t^+ and Δp_t^- are equal to the positive and negative parts of Δp_t, respectively. As a consequence, the feasible space and the objective function of (9.25) would be equivalent to the ones of (9.24). However, it is not necessary to enforce (9.26), which is nonlinear, as such property holds at any optimal solution $(\Delta p_t^{+*}, \Delta p_t^{-*}, \lambda_t^{\mathrm{R}*}, u_t^*)$ of (9.25). By contradiction, if $\Delta p_t^{+*} > 0$ and $\Delta p_t^{-*} > 0$ held, one could replace Δp_t^{+*} and Δp_t^{-*} in the optimal solution with $\Delta p_t^{+*} - \min\{\Delta \mathrm{p}_\mathrm{t}^{+*}, \Delta \mathrm{p}_\mathrm{t}^{-*}\}$ and $\Delta p_t^{-*} - \min\{\Delta \mathrm{p}_\mathrm{t}^{+*}, \Delta \mathrm{p}_\mathrm{t}^{-*}\}$, respectively. This would render at least one between Δp_t^+ and Δp_t^- equal to 0, without violating (9.25c) or (9.25e) and obtaining a lower objective function. We remark that such a linearization technique works in more general cases where Δp_t^+ and Δp_t^- are penalized by different, though strictly positive, coefficients in the objective function.

Problem (9.25) cannot be tackled directly, due to the *argmin* operator in (9.25b), which embeds the lower-level optimization problem. However, we can replace this constraint with the set of Karush–Kuhn–Tucker conditions, which are necessary and sufficient for optimality, as the original problem (9.6) is convex (see Appendix B). The Lagrangian function for problem (9.6) is

$$
\begin{aligned}
&\mathcal{L}(x_t^1, x_t^2, u_t, \gamma_t^1, \gamma_t^2, \mu_t^+, \mu_t^-, \varepsilon_t^+, \varepsilon_t^-) \\
&\quad = \sum_{t=0}^{T} \frac{c}{2}\left(x_t^1 - \widehat{x}_t^1\right)^2 + \lambda_t^{\mathrm{R}} u_t \\
&\quad\quad + \sum_{t=0}^{T-1} \gamma_t^1 \left(x_{t+1}^1 - a_{11}x_t^1 - a_{12}x_t^2\right) + \gamma_t^2 \left(x_{t+1}^2 - a_{22}x_t^2 - b_2 u_t\right) \\
&\quad\quad + \sum_{t=0}^{T-1} \mu_t^+ \left(u_{t+1} - u_t - R^{\mathrm{U}}\right) + \mu_t^- \left(-u_{t+1} + u_t - R^{\mathrm{D}}\right) \\
&\quad\quad + \sum_{t=1}^{T} \varepsilon_t^+ \left(u_t - \overline{U}^{\mathrm{h}}\right) + \varepsilon_t^- \left(-u_t\right).
\end{aligned}
\tag{9.27}
$$

From the Lagrangian, we can derive the KKT conditions. These include the stationarity conditions

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial x_t^1} = c\left(x_t^1 - \widehat{x}_t^1\right) + \gamma_{t-1}^1 - a_{11}\gamma_t^1 = 0, \qquad \forall t, \tag{9.28a}
$$

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial x_t^2} = -a_{12}\gamma_t^1 + \gamma_{t-1}^2 - a_{22}\gamma_t^2 = 0, \qquad \forall t, \tag{9.28b}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}(\cdot)}{\partial u_t} = {} & \lambda_t^{\mathrm{R}} - b_2\gamma_t^2 + \mu_{t-1}^+ - \mu_t^+ \\
& -\mu_{t-1}^- + \mu_t^- + \varepsilon_t^+ - \varepsilon_t^- = 0, \qquad \forall t.
\end{aligned}
\tag{9.28c}
$$

Observe that care must be taken when writing the stationarity conditions for the terminal states and input, i.e., at $t = T$. This is because state-update and ramping constraints (and thus their associated dual variables) are not needed to link variables pertaining to time periods T and $T+1$. We refer the reader to Example 9.10, where stationarity conditions are determined.

Besides the stationarity conditions, KKTs include the following conditions:

$$\gamma_t^1 \text{free}, \qquad x_{t+1}^1 = a_{11}x_t^1 + a_{12}x_t^2, \qquad \forall t < T, \tag{9.29a}$$

$$\gamma_t^2 \text{free}, \qquad x_{t+1}^2 = a_{22}x_t^2 + b_2 u_t, \qquad \forall t < T, \tag{9.29b}$$

$$0 \le \mu_t^+ \perp R^{\mathrm{U}} - u_{t+1} + u_t \ge 0, \qquad \forall t < T, \tag{9.29c}$$

$$0 \le \mu_t^- \perp R^{\mathrm{D}} + u_{t+1} - u_t \ge 0, \qquad \forall t < T, \tag{9.29d}$$

$$0 \le \varepsilon_t^+ \perp \overline{U}^{\mathrm{h}} - u_t \ge 0, \qquad \forall t, \tag{9.29e}$$

$$0 \le \varepsilon_t^- \perp u_t \ge 0, \qquad \forall t, \tag{9.29f}$$

which include the non-negativity declarations of the dual variables associated with the inequality constraints of the primal problem (left-hand side), the constraints of the primal problem (right-hand side), linked by the complementarity slackness condition implied by the perpendicularity operator $\perp$. The latter conditions require that the dual variable be zero if the associated primal inequality is not binding and, viceversa, that the constraint be binding if the dual variable is different from zero.

Notice that the conditions including the perpendicularity operator $\perp$ are nonlinear, since they are enforcing that the product of the dual variable on the left-hand side with the constraint on the right-hand side be equal to 0. However, it is possible to linearize such conditions by employing binary variables as described in Appendix B.

The resulting formulation of the problem as a single-level mixed-integer linear program (MILP) is the following:

$$\text{Min.} \quad \sum_{t=1}^{T} \left(\Delta p_t^+ + \Delta p_t^-\right) \tag{9.30a}$$

$$\text{s.t.} \quad \Delta p_t^+ - \Delta p_t^- = W_t - u_t, \qquad \forall t, \tag{9.30b}$$

$$\underline{\lambda}_t^{\mathrm{R}} \le \lambda_t^{\mathrm{R}} \le \overline{\lambda}_t^{\mathrm{R}}, \qquad \forall t, \tag{9.30c}$$

$$\Delta p_t^+,\ \Delta p_t^- \ge 0, \qquad \forall t, \tag{9.30d}$$

$$c\left(x_t^1 - \widehat{x}_t^1\right) + \gamma_{t-1}^1 - a_{11}\gamma_t^1 = 0, \qquad \forall t, \tag{9.30e}$$

$$-a_{12}\gamma_t^1 + \gamma_{t-1}^2 - a_{22}\gamma_t^2 = 0, \qquad \forall t, \tag{9.30f}$$

$$\lambda_t^{\mathrm{R}} - b_2\gamma_t^2 + \mu_{t-1}^+ - \mu_t^+ - \mu_{t-1}^- + \mu_t^- + \varepsilon_t^+ - \varepsilon_t^- = 0, \qquad \forall t, \tag{9.30g}$$

$$x_{t+1}^1 = a_{11}x_t^1 + a_{12}x_t^2, \qquad \forall t < T, \tag{9.30h}$$

$$x_{t+1}^2 = a_{22}x_t^2 + b_2 u_t, \qquad \forall t < T, \tag{9.30i}$$

$$0 \le \mu_t^+ \le M i_t^+, \qquad \forall t < T, \tag{9.30j}$$

$$0 \le R^{\mathrm{U}} - u_{t+1} + u_t \le M\left(1 - i_t^+\right), \qquad \forall t < T, \tag{9.30k}$$

$$0 \le \mu_t^- \le M i_t^-, \qquad \forall t < T, \tag{9.30l}$$

$$0 \le R^{\mathrm{D}} + u_{t+1} - u_t \le M\left(1 - i_t^-\right), \qquad \forall t < T, \tag{9.30m}$$

$$0 \le \varepsilon_t^+ \le M j_t^+, \qquad \forall t < T, \tag{9.30n}$$

$$0 \le \overline{U}^{\mathrm{h}} - u_t \le M\left(1 - j_t^+\right), \qquad \forall t, \tag{9.30o}$$

$$0 \le \varepsilon_t^- \le M j_t^-, \qquad \forall t < T, \tag{9.30p}$$

$$0 \le u_t \le M\left(1 - j_t^-\right), \qquad \forall t, \tag{9.30q}$$

$$i_t^+,\ i_t^-,\ j_t^+,\ j_t^- \in \{0, 1\}, \qquad \forall t. \tag{9.30r}$$

In the next example, we consider the problem of an EMS including both wind power production facilities and a consumer that is flexible in heating consumption. The objective for the EMS is to minimize the amount of electricity exchanged with an external market.

Example 9.10 (Minimization of Electricity Trade for an Energy Management System) We consider the problem of an EMS including the flexible consumer of Examples 9.3 and 9.4, and a small stochastic production facility. After linearizing the complementarity conditions for the lower-level optimization problem, we obtain the following single-level MILP:

$$\text{Min.} \quad \sum_{t=1}^{T} \left(\Delta p_t^+ + \Delta p_t^-\right) \tag{9.31a}$$

$$\text{s.t.} \quad \Delta p_t^+ - \Delta p_t^- = W_t - u_t, \qquad \forall t, \tag{9.31b}$$

$$0 \le \lambda_t^{\mathrm{R}} \le 200, \qquad \forall t, \tag{9.31c}$$

$$\Delta p_t^+, \Delta p_t^- \ge 0, \qquad \forall t, \tag{9.31d}$$

$$0.001\left(x_t^1 - 22\right) + \gamma_{t-1}^1 - 0.9\gamma_t^1 = 0, \qquad \forall t < 48, \tag{9.31e}$$

$$0.001\left(x_{48}^1 - 22\right) + \gamma_{47}^1 = 0, \tag{9.31f}$$

$$-0.07\gamma_t^1 + \gamma_{t-1}^2 - 0.95\gamma_t^2 = 0, \qquad \forall t < 48, \tag{9.31g}$$

$$\gamma_{47}^2 = 0, \tag{9.31h}$$

$$\lambda_t^{\mathrm{R}} - 2\gamma_t^2 + \varepsilon_t^+ - \varepsilon_t^- = 0, \qquad \forall t, \tag{9.31i}$$

$$x_{t+1}^1 = 0.9x_t^1 + 0.07x_t^2, \qquad \forall t < 48, \tag{9.31j}$$

$$x_{t+1}^2 = 0.95x_t^2 + 2u_t, \qquad \forall t < 48, \tag{9.31k}$$

$$0 \le \varepsilon_t^+ \le 10 j_t^+, \qquad \forall t, \tag{9.31l}$$

$$0 \le 3 - u_t \le 10\left(1 - j_t^+\right), \qquad \forall t, \tag{9.31m}$$

$$0 \le \varepsilon_t^- \le 10 j_t^-, \qquad \forall t, \tag{9.31n}$$

$$0 \le u_t \le 10\left(1 - j_t^-\right), \qquad \forall t, \tag{9.31o}$$

$$j_t^+, j_t^- \in \{0, 1\}, \qquad \forall t. \tag{9.31p}$$

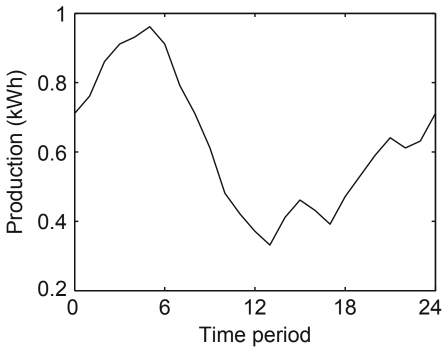

Fig. 9.18 Production forecast from wind farm owned by EMS

Table 9.13 Objective function value for EMS and results for the consumer in the fixed-price, fixed-load and dynamic price cases

Level	Quantity	Pricing strategy		
		Fixed price	Fixed load	Dynamic price
Upper	Objective (kWh)	23.75	7.86	0
Lower	Discomfort ($)	0.49	0.53	0.42
	Cost ($)	2.27	2.81	2.62
	Objective ($)	2.77	3.34	3.04

Observe that the stationarity condition with respect to the indoor temperature variable, x_t^1, have two different formulations: (9.31e) and (9.31f). The latter one is valid for $t = 48$, since the last state-update equation included in the model is the one linking x_{48}^1 with x_{47}^1 and x_{47}^2. Hence, no dual variable γ_{48}^1 is defined in the model. For the same reason, the stationarity condition with respect to the temperature of water in the boiler, x_t^2, appears in the two different formulations (9.31g), and (9.31h).

Furthermore, note that model (9.31) is simpler than (9.30), as it does not include ramping rates. If ramping rates were included, similar considerations as above would lead to different stationarity conditions with respect to the consumption, u_t, for $t < 48$ and at the last time period of the horizon $t = 48$.

The forecast output W_t from the wind farm owned by EMS is depicted in Fig. 9.18. For the sake of simplicity, we assume that the production forecast for the second day follows the same pattern.

Table 9.13 reports the results in terms of objective function of the lower- and upper-level problem in three different cases. In the first case, a fixed-price strategy is used by EMS, which broadcasts a constant price signal equal to \$ 100/MWh to the consumers. In the second case, the price is chosen so as to guarantee a constant consumption throughout the day equal to 0.65 kWh, i.e., equal to the average value of the wind power production, W_t. In the last case, the price is chosen so as to minimize the external trade of EMS, thus implementing model (9.31). Obviously, the latter case obtains the lowest objective function value, equal to 0, hence indicating that consumption follows exactly the forecast pattern of wind power production. In turn, the table shows that the results for the lower-level problem, i.e., the consumer problem, are not necessarily the lowest when EMS freely chooses the price signal.

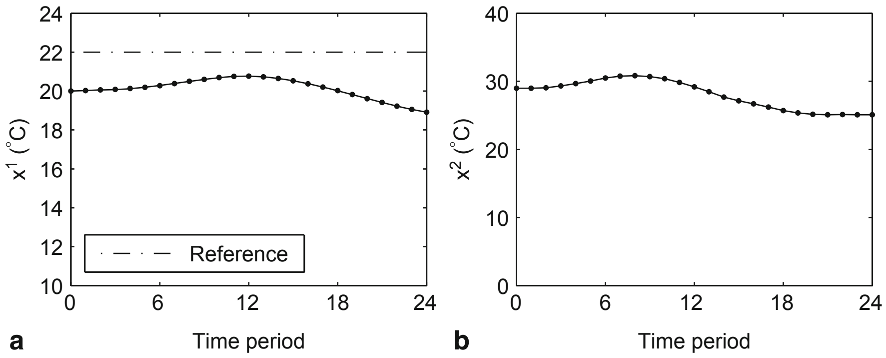

Fig. 9.19 Evolution of the states with dynamic price signal. **a** Indoor temperature. **b** Temperature of water in boiler

The evolution of the states for the lower-level problem with dynamic price signal is shown in Fig. 9.19. Remarkably, the peaks and valleys of the temperature evolution follow, with some delay, the peaks and valleys of forecast wind power production in Fig. 9.18.

9.7 Summary and Conclusions

Demand response is widely seen as one of the pillars of future energy systems where large shares of renewables are to be successfully integrated. This is because the activation of demand flexibility can help mitigate the drawbacks of most renewable sources for electricity generation, namely, the intermittency and the uncertainty of their output.

Currently, a number of demand response initiatives are being researched, implemented in demonstration studies, or operational at large scale. The target of these initiatives spans all the way from large-size industrial consumers to small residential ones, while their nature includes direct and indirect control, interruptible tariffs, and other initiatives based on direct market participation.

In this chapter, we first provide an introduction to demand response and a classification of the main initiatives proposed or implemented so far. Particular focus is placed on describing demand response programs based on dynamic pricing, which are designed to incentivize a more flexible residential consumption.

Optimization models are proposed to support the planning of the electrical load schedule of residential consumers subject to dynamic pricing. Such optimization models are then made stochastic, so as to account for the uncertainty that characterizes the future evolution of dynamic prices. In particular, both stochastic programming and robust optimization techniques are employed. Ideally, models of this type are to be incorporated into intelligent appliances or into automation systems in charge of managing the demand at the individual household level.

This chapter also provides an introduction to statistical methods that can be used to forecast the aggregate response from flexible consumers to time-varying electricity prices. These forecasting models will become of outmost importance in the future, since transmission and distribution system operators, market operators, electricity producers, and retailers will be called to make decisions in a completely new situation: a power system where a large share of the demand is responsive to time-varying prices.

Finally, the complementary problem of determining an appropriate dynamic-price sequence to be broadcast to the flexible demand is introduced. In particular, we show how bi-level programming can be employed to generate a dynamic price signal that maximizes the exploitation of renewable energy.

9.8 Further Reading

Readers interested in a comprehensive overview of the demand response initiatives undertaken or planned in Europe are referred to [12].

The models presented in this chapter are based on techniques of stochastic programming, robust optimization and complementarity modeling. We refer the reader to Appendix C for a short introduction to stochastic programming, while [2] is a textbook dealing with this topic at an introductory level. For an overview on robust optimization, we refer to Appendix D and [1]. A relevant textbook on applications of complementarity modeling to electricity markets, including bilevel models, is [5]. A very brief introduction can also be found in Appendix B.

The reader interested in applications of the model for deferrable loads presented in Sect. 9.3.1, employing robust optimization in combination with a realistic model for price forecasting, is referred to [3].

A method for modeling heat dynamics of buildings employing state–space models is presented in [8].

A different application of bi-level programming to model the problem of a profit maximizing electricity retailer coupled with consumers that are flexible in their consumption for heating is introduced in [13].

Exercises

9.1 Some electrical appliances cannot be switched on and off all the time. Once they are on, they must run for a certain time period to avoid getting damaged. Similarly, they cannot be switched on again right after they are turned off. Formulate two types of constraints for model (9.1) for a deferrable load. The first type of constraints enforces that when the device is on, it must satisfy a minimum power consumption requirement. Constraints of the second type enforce that the device must stay on for a certain number of periods after start-up, and off for a certain number of periods after shut-down. Hint: Constraints of this type are equivalent to the minimum up- and down-time constraints for a power plant, which are formulated in Sect. 5.3.4.

9.2 Implement the constraints formulated in the previous exercise in the deterministic model for deferrable load of Example 9.1. Reduce the values for minimum and the maximum daily consumption, $\underline{U}^{\mathrm{d}}$ and $\overline{U}^{\mathrm{d}}$, in Table 9.1(a), so that the newly introduced constraints are binding.

9.3 Write down the robust optimization formulation of the model for the flexible consumer with deferrable load in Example 9.6 as a *min–max* problem of the type (9.12), including all the actual values for the parameters used in the example. Recast the model as a single-level optimization problem of the type (9.15). Implement the model and solve it in your preferred optimization environment, and check your results with the ones of Example 9.6. Note that the GAMS code for the example is provided in Appendix E.

9.4 Electrical vehicles are equipped with batteries that, when the car is plugged in for charging, can provide energy storage services to the grid. Develop a deterministic mathematical model for an energy storage within the household considered in Example 9.1. You can also impose constraints enforcing that the battery is completely charged at a certain point, that the battery is off-line during certain time periods and that it is completely discharged when on again (i.e., before, during and after driving). Hint: You may get inspiration from the model for energy storage developed for the transmission system operator in Sect. 5.5.

9.5 Develop a stochastic version of your choice for the problem in the previous exercise, i.e., either using stochastic programming or robust optimization.

9.6 Develop a stochastic version of your choice for the problem of Exercise 9.1, i.e., either using stochastic programming or robust optimization.

9.7 Implement the model predictive control algorithm for the deferrable load model with robust optimization of Example 9.6. The length of the look-ahead horizon should decrease by one step every time an optimization problem is solved as in Example 9.7. Consider the high-price scenarios in Table 9.10 as the upper bounds of the price intervals at each time period, and the low-price ones as the lower bounds. Select a new actual realization of the price (i.e., change the bold numbers in Table 9.10) that is feasible for the uncertainty set defined in (9.11) where the nominal price vector is $\bar{\lambda}^{\mathrm{R}} = [120\ 75\ 110\ 60]^{\top}$ and the budget of uncertainty Γ is 2. In the sequence of optimization problems to be solved, Γ should be updated consistently with the chosen realization of the price, e.g., if the realized price is \$ 105/MWh at time 2, then Γ should be reduced to 1.

9.8 Consider the bi-level model for the determination of the optimal pricing for an energy management system, coupled with a consumer that is flexible in her/his consumption for heating, presented in Example 9.10. Extend the model so that it includes ramping constraints (9.30j)–(9.30m) in the lower-level problem for the consumer.

References

1. Bertsimas, D., Brown, D.B., Caramanis, C.: Theory and applications of robust optimization. SIAM Rev. **53**(3), 464–501 (2011)
2. Birge, J.R., Louveaux, F.: Introduction to Stochastic Programming, 2nd edn. Springer Series in Operations Research and Financial Engineering. Springer, New York (2011)
3. Conejo, A.J., Morales, J.M., Baringo, L.: Real-time demand response model. IEEE Trans. Smart Grid **1**(3), 236–242 (2010)
4. Federal Energy Regulatory Commission (FERC): A national assessment of demand response potential. Staff report, FERC (2009)
5. Gabriel, S.A., Conejo, A.J., Fuller, D.J., Hobbs, B.F., Ruiz, C.: Complementarity Modeling in Energy Markets. International Series in Operations Research and Management Science. Springer, New York (2013)
6. Hahn, H., Meyer-Nieber, S., Pickl, S.: Electric load forecasting methods: Tools for decision making. Eur. J. Oper. Res. **199**, 902–907 (2009)
7. Madsen, H.: Time Series Analysis. Texts in Statistical Science. Chapman & Hall/CRC, Boca Raton, FL (2008)
8. Madsen, H., Holst, J.: Estimation of continuous-time models for the heat dynamics of a building. Energy Build. **22**(1), 67–79 (1995)
9. Schweppe, F.C., Caramanis, M.C., Tabors, R.D., Bohn, R.E.: Spot Pricing of Electricity. International Series in Engineering and Computer Science. Kluwer Academic Publishers, Boston (1988)
10. Schweppe, F.C., Tabors, R.D., Kirtley, J.L., Outhred, H.R., Pickel, F.H., Cox, A.J.: Homeostatic utility control. IEEE Trans. Power Apparatus Syst. (3), 1151–1163 (1980)
11. Short, J.A., Infield, D.G., Freris, L.L.: Stabilization of grid frequency through dynamic demand control. IEEE Trans. Power Syst. **22**(3), 1284-1293 (2007)
12. Torriti, J., Hassan, M.G., Leach, M.: Demand response experience in Europe: policies, programmes and implementation. Energy **35**(4), 1575-1583 (2010)
13. Zugno, M., Morales, J.M., Pinson, P., Madsen, H.: A bilevel model for electricity retailers participation in a demand response market environment. Energy Econ. **36**, 182-197 (2013)

Appendix A
Random Variables and Stochastic Processes

In view of the modeling, forecasting, and forecast verification aspects covered throughout the various chapters of this book, it might be necessary to recall some of the necessary basics on random variables and stochastic processes here. It is assumed, however, that the readers of this book have some general understanding of probability theory and statistics. Instead, focus is given to the necessary basics and terminology when considering renewable energy generation and market quantities. Specifically, we shall focus on concepts like random variables, stochastic processes, time series, and how randomness is described by important concepts like probability densities, moments (mean and variance), and correlation structures; with a special focus on the autocorrelation.

The first section will center on *scalar (or univariate) random variables*, and first some important examples related to renewables and power systems will be mentioned. Section A.2 provides an overview of some of the most relevant probability densities for random variables related to this book.

Section A.3 will consider *multivariate random variables*. This gives us the possibility to introduce concepts like correlations and conditional expectations. Conditional expectations are important for forecasting, since it can be shown (see, e.g., [3]) that, assuming symmetric cost functions and densities, the optimal prediction is the conditional expectation. This section also introduces the multivariate normal density.

Later on, in Sect. A.4, *discrete time indexed random variables* (*stochastic processes*) will be introduced. The outcome of such stochastic processes is called *time series*. In general, it will be clear that stochastic processes can be handled like multivariate random variables. In fact, any stochastic process is completely characterized by the so-called family of multivariate densities, all related to vectorizations of the scalar random variables associated with the stochastic process at various fixed time points. Finally, correlation methods for measuring the serial dependence in time series are described in Sect. A.5.

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A.1 Random Variables

Random variables comprise a basis quantity that is used in all branches of probability and statistics. They are the basic description of events that are entailed by a certain level of uncertainty.

The outcome or value of a random variable can be a *real number (scalar)*, but one can consider arbitrary types such as *boolean*, *categorical*, *complex*, *vector*, *positive definite matrix*, and *time series*; just to mention some of the most relevant types.

After an introduction to random variables, this section will discuss how the likelihood of outcomes of random variables is described by probability density functions and its cumulative counterpart, namely, the cumulative distribution function. Later on, the moments of random variables are presented with a special focus on the expectation (mean) and variance. Finally, the concept of quantiles, which are often used for, e.g., probabilistic wind power forecasting, is briefly introduced, and ultimately, some concepts related to descriptive statistics are provided.

Definition A.1 (Random Variable). A random variable X is a variable whose value is subject to variations due to randomness. A random variable does not have a single, fixed value; rather, it can take on a set of possible different values, and each value is associated with a probability.

A random variable is said to be *discrete* if it can only take a number of countable values, i.e., if there exists a countable set $\mathcal{S} \in \mathbb{R}$ such that $P\{X \in \mathcal{S}\} = 1$. In contrast, it is said to be *continuous* if it can take an uncountable number of values, that is, if the set $\mathcal{S}$ such that $P\{X \in \mathcal{S}\} = 1$ is uncountable. $\mathcal{S}$ is called the *sample space*. Each value in the sample space is associated with a probability, which in the discrete case is given directly by the *probability mass function (PMF)*. In the continuous case, the corresponding function is called the *probability density function (PDF)*, however, the values of a PDF are not probabilities as such; the PDF must be integrated over an interval to yield a probability. The cumulated variant is in both cases called the *cumulative distribution function (CDF)*.

Let us illustrate these definitions based on examples inspired by renewable energy generation.

Example A.2 (Cutoff Event for a Wind Turbine). Wind turbines generate electricity for wind speeds up to the so-called cutoff speed. For higher wind speeds, turbines are shut down for security reasons in order to avoid structural damages. It commonly ranges between 25 and 30 ms^{-1} nowadays, depending on the turbine manufacturers and models. Consequently, when not having a perfect knowledge of the wind speed seen by a wind turbine, one can define a discrete random variable X such that

$$X = \begin{cases} 0, & \text{if the turbine is shut down} \\ 1, & \text{if the turbine is operating.} \end{cases}$$

In that case, X would actually be referred to as a binary random variable, since it can take two values only.

Example A.3 (Power Generation of a Wind Turbine). For wind speeds below the cutoff speed, a wind turbine can generate electric power between 0 and its nominal capacity P_n (typically between 2 and 5 MW for today's turbines) following its power curve. Having a limited knowledge of the wind conditions at the level of the turbine, one can then define a continuous random variable X such that X may take any value in the interval $[0, \mathrm{P}_n]$.

From now on, emphasis will be placed on continuous random variables, since they are the most relevant ones for the description of the types of uncertain processes considered in this book.

A.1.1 Probability Densities and Cumulative Distribution Functions

Describing discrete random variables is easier than describing continuous ones. For instance, for the case of cutoff events in Example A.2 above, the binary random variable X is fully characterized by the probabilities $\mathrm{P}\{X = 0\}$ and $\mathrm{P}\{X = 1\}$. This cannot be done for continuous random variables, because their support consists of an uncountable set. Actually, for continuous random variables, $\mathrm{P}\{X = x\} = 0, \forall x \in \mathcal{S}$. Therefore, in the continuous case, random variables are characterized instead by $\mathrm{P}\{X \leq x\}$, which leads to the definition of the *cumulative distribution function*.

Definition A.2 (Cumulative Distribution Function). The cumulative distribution function (abbreviated cdf or CDF) for a continuous random variable X is the function $F(x)$ such that

$$F(x) = \mathrm{P}\{X \leq x\}, \quad \forall x \in \mathcal{S}. \tag{A.1}$$

The CDF necessarily has the following properties:

(i) it is always positive, $F(x) \geq 0, \;\; \forall x \in \mathcal{S}$,
(ii) its left and right limits are 0 and 1, respectively,

$$\begin{aligned} \lim_{x \to \mathcal{S}^-} F(x) &= 0 \\ \lim_{x \to \mathcal{S}^+} F(x) &= 1, \end{aligned}$$

where $\mathcal{S}^-$ and $\mathcal{S}^+$ are the left and right bounds of the support $\mathcal{S}$ of X,
(iii) it is an increasing function, hence for all $x_1, x_2 \in \mathcal{S}$, $x_2 \geq x_1$ implies that $F(x_2) \geq F(x_1)$.

In direct relation with the CDF of a random variable, one may define its probability density function.

Definition A.3 (Probability Density Function). The probability density function $f(x)$ (abbreviated pdf or PDF) for a continuous random variable X corresponds to the derivative of its cumulative distribution function, i.e.,

$$f(x) = \frac{dF}{dx}, \ \ \forall x \in \mathcal{S}. \tag{A.2}$$

The PDF necessarily has the following properties:

(i) it is always positive, $f(x) \geq 0, \ \ \forall x \in \mathcal{S}$, since F is a nondecreasing function,
(ii) its left and right limits are 0, that is, $\lim_{x \to \mathcal{S}^-} f(x) = \lim_{x \to \mathcal{S}^+} f(x) = 0$ ($\mathcal{S}^-$ and $\mathcal{S}^+$ are again the left and right bounds of $\mathcal{S}$, the support of X),
(iii) it integrates to 1, $\int_{x \in \mathcal{S}} f(x)dx = 1$, given that $\mathrm{P}[x \in \mathcal{S}] = 1$.

A consequence of these definitions is that

$$\mathrm{P}[x_1 \leq X \leq x_2] = F(x_2) - F(x_1) = \int_{x_1}^{x_2} f(x)dx. \tag{A.3}$$

A.1.2 Expectations and Moments

For a discrete random variable X, the expectation (or the mean) is defined as

$$\mathrm{E}[X] = \sum_x x P\{X = x\},$$

i.e., an average of the possible values of X, each value being weighted by its probability.

For continuous variables, expectations are defined as integrals.

Definition A.4 (Expectation). The *expectation* (or *mean*) of a continuous variable X with probability density function f_X is

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx, \tag{A.4}$$

whenever this integral exists.

Note that we will write $f_X(x)$ and $F_X(x)$ instead of $f(x)$ and $F(x)$, respectively, whenever it is necessary to emphasize to which random variable the function belongs.

The expectation is also referred to as the *first moment*, because it is indeed the first moment of the area under f_X with respect to the line $x = 0$. It is, furthermore, a *linear operator*, i.e.,

$$\mathrm{E}[aX_1 + bX_2] = a\mathrm{E}[X_1] + b\mathrm{E}[X_2]. \tag{A.5}$$

If X is a random variable and g is a function, then $Y = g(X)$ is also a random variable, with the expectation

$$\mathrm{E}[Y] = \mathrm{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx. \tag{A.6}$$

This allows us to define *higher order moments*.

Definition A.5 (Moments). The *n'th moment* of X is

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_X(x)\, dx, \tag{A.7}$$

and the *n'th central moment* is defined as

$$E[(X - E[X])^n] = \int_{-\infty}^{\infty} (x - E[X])^n f_X(x)\, dx. \tag{A.8}$$

The second central moment is also called the *variance* of X. It is given by

$$\mathrm{Var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2. \tag{A.9}$$

For two random variables X_i and X_j, the central second-order moment is given by the *covariance* between X_i and X_j:

$$\begin{aligned} \mathrm{Cov}[X_i, X_j] &= E[(X_i - E[X_i])(X_j - E[X_j]) \\ &= E[X_i X_j] - E[X_i]E[X_j]. \end{aligned} \tag{A.10}$$

The covariance gives information about the simultaneous variation of two random variables and is useful for identifying interdependencies. Besides, this definition will be subsequently generalized to describe interdependencies in time-indexed sequences of random variables, i.e., in a stochastic process.

By using (A.10) and the fact that the expectation operator is linear, we obtain the following important *calculation rule for the covariance*:

$$\begin{aligned} \mathrm{Cov}[aX_1 + bX_2, cX_3 + dX_4] &= ac\mathrm{Cov}[X_1, X_3] + ad\mathrm{Cov}[X_1, X_4] \\ &\quad + bc\mathrm{Cov}[X_2, X_3] + bd\mathrm{Cov}[X_2, X_4], \end{aligned} \tag{A.11}$$

where $X_1, \ldots, X_4$ are random variables, and $a, \ldots, d$ are constants.

The *correlation* between two random variables, X_i and X_j, is a normalization of the covariance and can be written as

$$\rho_{ij} = \frac{\mathrm{Cov}[X_i, X_j]}{\sqrt{\mathrm{Var}[X_i]\mathrm{Var}[X_j]}} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}, \tag{A.12}$$

where $\sigma_{ij} = \mathrm{Cov}[X_i, X_j]$ and $\sigma_i = \sqrt{\mathrm{Var}[X_i]}$.

Example A.4 (Spatial Correlation of Wind Speed Data) Let us consider data for wind speed during the period from 1970 to 1999 measured every 6 h at a number of synoptic stations in Europe.

Using this data, we have estimated that the correlation between wind speed in the northern part of Germany and the wind speed in Denmark East is 0.64 for the same hour. This rather high correlation tells us that if the wind power generation in Denmark is high, then it is likely that the wind power generation in the northern part of Germany is also high at the same time.

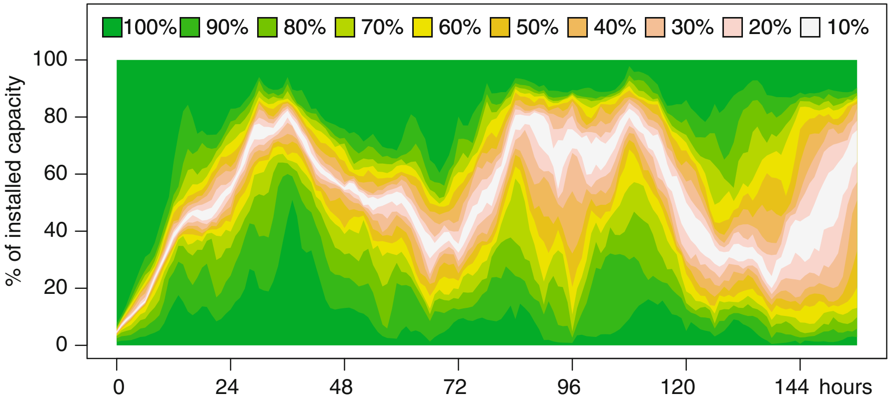


Fig. A.1 Probabilistic forecasts of future wind power generation using quantiles (deciles)

On the contrary, it is found that the correlation between the wind speed in the northern and southern parts of Spain for the same period in time is only 0.26. This indicates that, for Spain, the weather phenomena driving the wind speed are very different from the south to the north; maybe, in the southern part of Spain, wind speed is mainly caused by thermal differences, whereas for the northern part the main source of wind is frontal passages.

A.1.3 Description by Quantiles

This section briefly introduces the concept of *quantile*, which is of particular importance in the field of probabilistic forecasting.

Definition A.6 Provided that F_t is a strictly increasing function, the quantile $q_t^{(\alpha)}$ with proportion $\alpha \in [0, 1]$ of the random variable Y_t is uniquely defined as the value of y such that

$$\mathrm{P}\{Y_t \leq y\} = \alpha, \tag{A.13}$$

or equivalently as

$$q_t^{(\alpha)} = F_t^{-1}(\alpha). \tag{A.14}$$

Note that there exists a generalization for the definition of a quantile in the case where F_t is not a strictly increasing function. It is, though, not discussed here, for simplicity.

Quantiles are often used to describe the uncertainty of, e.g., a wind power forecast. This is illustrated in Fig. A.1, which shows a probabilistic forecast of the future wind power production by plotting the quantiles (deciles) of the future wind power generation up to 144 h ahead.

The forecasts in Fig. A.1 are generated by the WPPT approach as described in [5] and [6].

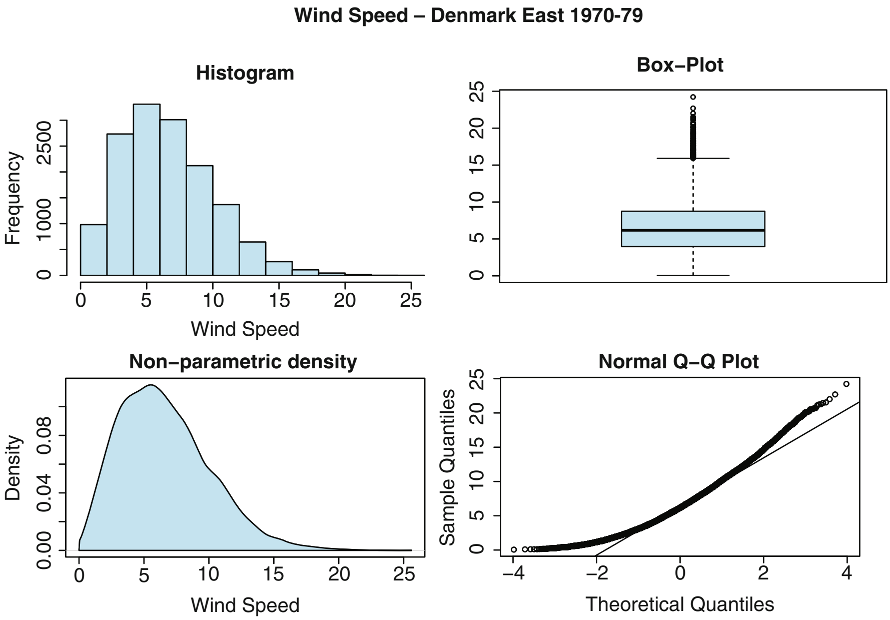


Fig. A.2 Some descriptive statistical analysis of the variations of wind speed in the eastern part of Denmark for the period from 1970 to 1979

A.1.4 Descriptive Statistics

So far, the discussion has focused on *probability theory*. We have, for example, assumed that the probability density function of a certain random variable is known. This is, however, seldom the case in practice, where such a function has to be inferred from a *set of data*.

Descriptive statistics is the discipline of quantitatively describing the most important features of the data. In this case, the *sample size* is important, since obviously more precise inference about, e.g., the mean value can be obtained from large sample sizes.

Univariate analysis involves methods for describing the distribution or variation of a single variable. This can be done graphically by means of, e.g., the *histogram* and the *box plot*. It can also be done quantitatively by a measure of the *central tendency* of the variable (including the *mean*, *median*, and *mode*), the dispersion of the data (through the *range* and *quantiles*, for example), and measures of spread (including the *variance* and *inter quantile range*, among others).

Bivariate analysis is relevant for samples with two or more variables and is used to describe the relationship between these variables. Important tools of bivariate analysis comprise *scatter plots* and quantitative measures of dependence like the *empirical correlation*, *Pearson's r*, *rank correlation*, and *empirical covariance*.

Example A.5 (Descriptive Statistics). Figure A.2 illustrates some of the possibilities for analyzing the wind speed data measured in the eastern part of Denmark from

1970 to 1979 (both years are included) using descriptive statistics. The upper left plot shows the histogram of wind speed for this period. A box plot of the same data is shown in the upper right part of the figure. The horizontal bar in the middle indicates the median of the wind speed data, and the bottom and the top of the box show the 25th and 75th percentiles, respectively, also referred to as the first and third quartiles and denoted by Q_1 and Q_3, in that order. The difference between these percentiles is known as *inter-quartile range* (IQR). The lowest and highest whiskers of the box plot are then computed as $Q1 - 1.5\,\mathrm{IQR}$ and $Q_3 + 1.5\,\mathrm{IQR}$ from the values of wind speed for the considered period, i.e., from 1970 to 1979. Finally, the points in the box plot are called *outliers*.

The lower left plot of Fig. A.2 presents a non-parametric estimate of the probability density function using a kernel method (see [2]). Very often it is difficult to identify a known density function for the data and, in these cases, such non-parametric methods prove to be very useful. A simple test for normality (and, in many ways, the best) is the "quantile-quantile plot," which displays ranked values from the sample against a similar number of random numbers simulated using a normal distribution. If the sample is actually normally distributed, then the line of points will be straight. Departures from a normal distribution, as seen here, show up in various sorts of non-linear relations. Therefore, this plot highlights that the wind speed data under consideration are not normally distributed. This is not surprising, as it is well-known that wind speed is better described by a Weibull distribution [8].

Prior to investing in a new wind farm, the potential for wind power production is most often assessed using a statistical analysis of existing data from the candidate site or by correlating wind speed measurements from the actual site for a shorter period to wind speed data from a site which has a longer record of measured wind speed data. This analysis most often assumes stationarity, a concept which we shall study in detail in the section on stochastic processes later on. Let us now, as a simplistic approach, just compare the statistics for wind speed measurements for 1970–1979 and 1990–1999 for the eastern part of Denmark using the box plot. The results are shown in Fig. A.3 and seem to indicate that the wind speed has increased from the period 1970–1979 to the period 1990–1999 in the eastern part of Denmark.

A.2 Some Relevant Probability Distributions

There exists a large number of probability distributions for continuous variables, which are more or less relevant depending upon the considered application. A gentle introduction to probability distributions for general applications is, for instance, given in [1] and [4]. Let us provide here a brief overview of a set of probability distributions of particular relevance when it comes to the modeling of renewable energy generation and of market quantities (e.g., prices and energy quantities).

Uniform Distribution The Uniform distribution, denoted $\mathrm{U}[a, b]$, is one of the simplest probability distributions. It is fully characterized by the bounds a and b of

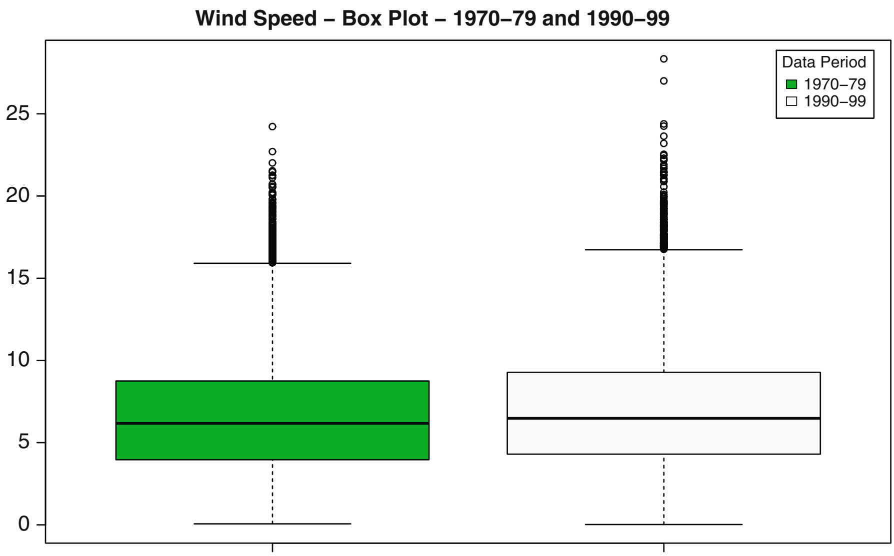


Fig. A.3 A comparison of wind speed measurements for the periods 1970–1979 and 1990–1999 for the eastern part of Denmark

its support. Stating that X is distributed U$[a, b]$, i.e.,

$$X \sim \mathrm{U}[a, b],$$

means that X has the same likelihood of taking any value in the interval $[a, b]$. Its PDF is then given as

$$f(x) = \begin{cases} (b-a)^{-1}, & x \in [a, b] \\ 0, & \text{otherwise}, \end{cases} \tag{A.15}$$

while the corresponding CDF writes

$$F(x) = \begin{cases} 0, & x < a \\ (x-a)(b-a)^{-1}, & x \in [a, b) \\ 1, & x \geq b. \end{cases} \tag{A.16}$$

The PDF and CDF of the Uniform distribution are depicted in Fig. A.4.

The uniform distribution has a number of interesting applications in the modeling of renewable energy generation and electricity markets. For instance, when issuing a probabilistic forecast for renewable energy generation, the most simple and naive prediction takes the form of a uniform distribution with its left bound a being 0 and its right bound b being the nominal capacity of the renewable energy portfolio.

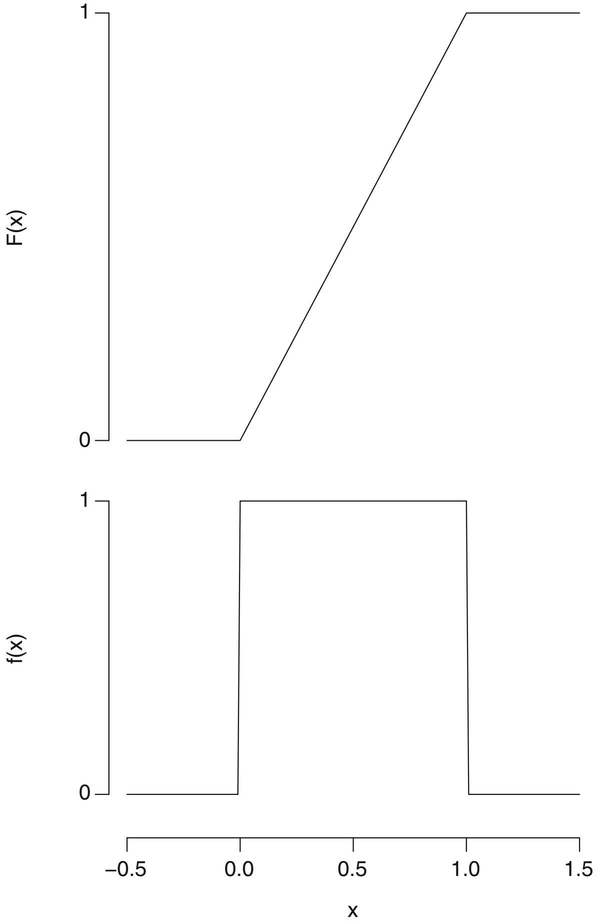


Fig. A.4 Probability density (*bottom*) and cumulative distribution (*top*) functions for the Uniform distribution U[0,1]

Gaussian Distribution The Gaussian (also called Normal) distribution is the most commonly used probability distribution owing to its nice analytical properties, among other reasons. Furthermore, the Central Limit Theorem (CLT) states that the mean of a large number of independent random variables, given some conditions, will be approximately normally distributed.

The Gaussian density is fully and uniquely characterized by its mean μ and its variance σ^2. Stating that X is distributed $\mathrm{N}(\mu, \sigma^2)$, i.e.,

$$X \sim \mathrm{N}(\mu, \sigma^2),$$

implies that X has a decreasing likelihood of taking values around its mean μ (which, besides, coincides with its mode and median). More specifically, the likelihood

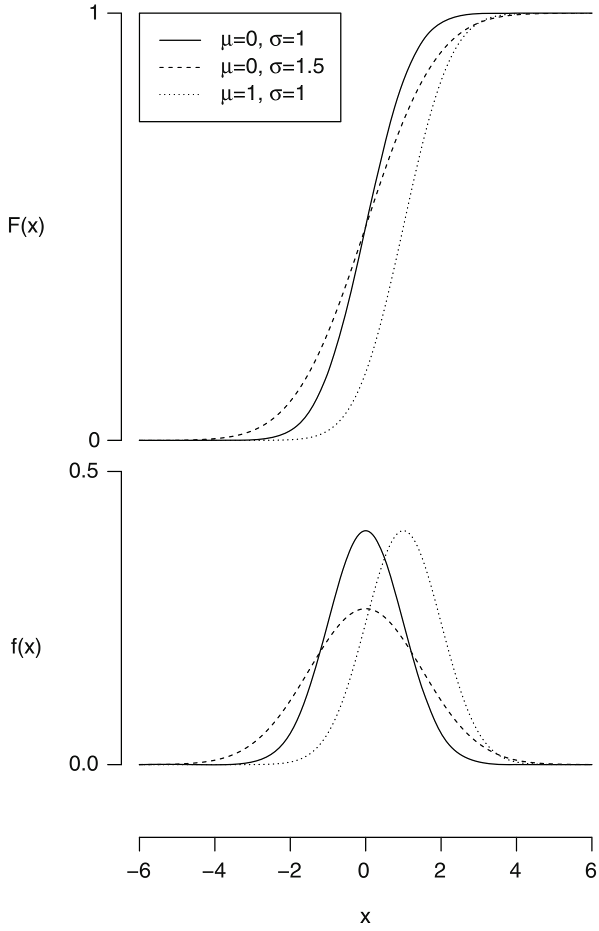


Fig. A.5 Probability density (*bottom*) and cumulative distribution (*top*) functions for the Gaussian distribution and for various values of its mean μ and standard deviation σ

decreases following a bell shape, as illustrated in Fig. A.5. Its PDF is then given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}, \tag{A.17}$$

which is naturally symmetric around μ. Moreover, the corresponding CDF writes

$$F(x) = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{(2)}}\right)\right\}, \tag{A.18}$$

where erf is the error function, defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right)dt. \tag{A.19}$$

The PDF and CDF of various Gaussian distributions are depicted in Fig. A.5.

Beta Distribution The Beta distribution is attractive for the modeling of bounded and nonlinear variables. This is so because this distribution is, by definition, bounded between 0 and 1, or any values a and b after appropriate scaling. It is fully characterized by its two shape parameters α and β, which are directly related to the mean μ and standard deviation σ of that distribution as follows:

$$\mu = \alpha(\alpha+\beta)^{-1} \tag{A.20}$$

$$\sigma = \alpha\beta(\alpha+\beta+1)^{-1}(\alpha+\beta)^{-2}. \tag{A.21}$$

Consider that a random variable X is distributed Beta(α, β), i.e.,

$$X \sim \text{Beta}(\alpha, \beta).$$

The PDF of X has then the following form:

$$f(x) = \frac{1}{\mathrm{B}(\alpha,\beta)} x^{(\alpha-1)}(1-x)^{(\beta-1)}, \tag{A.22}$$

where $\mathrm{B}(\alpha, \beta)$ is the Beta function, which serves as a normalizing constant for the PDF to integrate to 1. The Beta function is defined as

$$\mathrm{B}(\alpha,\beta) = \int_0^1 t^{(\alpha-1)}(1-t)^{(\beta-1)} dt, \tag{A.23}$$

while its incomplete counterpart is given by

$$\mathrm{B}(x;\alpha,\beta) = \int_0^x t^{(\alpha-1)}(1-t)^{(\beta-1)} dt. \tag{A.24}$$

The CDF of the Beta distribution is consequently obtained as

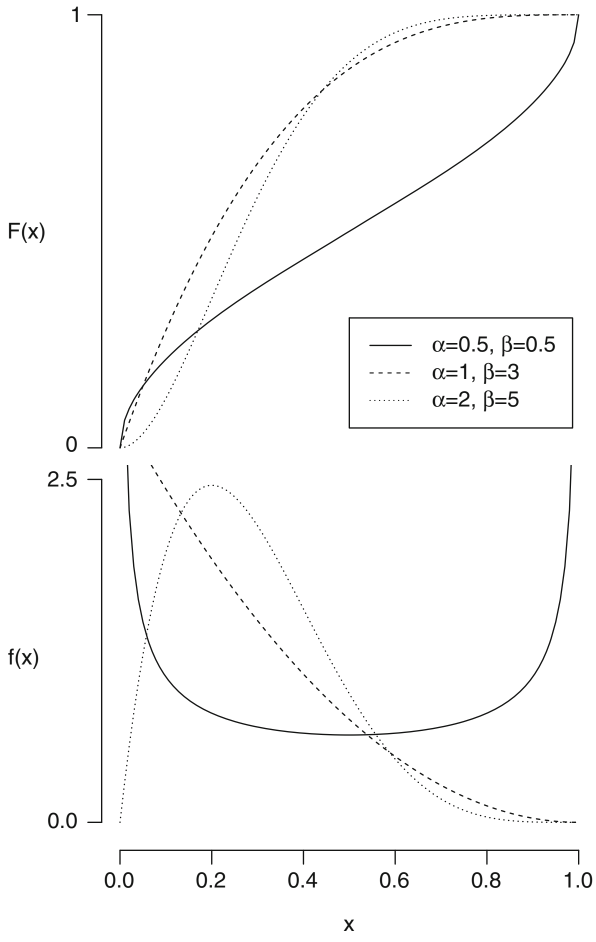
$$F(x) = \frac{\mathrm{B}(x;\alpha,\beta)}{\mathrm{B}(\alpha,\beta)}. \tag{A.25}$$

The PDF and CDF of the Beta distribution are depicted in Fig. A.6, for different values of α and β. Notice that different combinations of these two parameters may result in very different shapes, each characterized by a particular degree of skewness. For this reason, the Beta distribution is considered to be a quite flexible probability distribution for modeling data.

A.3 Multivariate Random Variables

The theory behind multivariate random variables is useful both for studying the relation between two (or more) random variables and for analyzing stochastic processes and time series. This is due to the fact that the theory for describing stochastic

Fig. A.6 Probability density (*bottom*) and cumulative distribution (*top*) functions for the Beta distribution and for various values of α and β



processes is basically the same as the theory for describing multivariate random variables.

An *n-dimensional random variable (random vector)* is a vector of n scalar random variables. The random vector is written as

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}. \tag{A.26}$$

Random vectors will also be referred to as *multivariate random variables*.

A.3.1 Joint Densities

The n-dimensional random variable X has the *joint distribution function*

$$F(x_1,\dots,x_n) = \mathrm{P}\{X_1 \le x_1,\dots,X_n \le x_n\}. \tag{A.27}$$

If X takes values on a continuous sample space, the *joint (probability) density function* is defined as

$$f(x_1,\dots,x_n) = \frac{\partial^n F(x_1,\dots,x_n)}{\partial x_1 \dots \partial x_n}, \tag{A.28}$$

and the relation between the distribution and density function is given by

$$F(x_1,\dots,x_n) = \int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_n} f(t_1,\dots,t_n)\, dt_1 \dots dt_n. \tag{A.29}$$

A multivariate random variable, X, is called *discrete* if it takes values on a discrete (countable) sample space. In this case, the joint density function (or *mass function*) is defined as

$$f(x_1,\dots,x_n) = P\{X_1 = x_1,\dots,X_n = x_n\}, \tag{A.30}$$

and the joint distribution and mass functions are related by

$$F(x_1,\dots,x_n) = \sum_{t_1 \le x_1} \cdots \sum_{t_n \le x_n} f(t_1,\dots,t_n). \tag{A.31}$$

A.3.2 Marginal Densities

For a sub-vector $S = (X_1,\dots,X_k)^T$ $(k < n)$ of the n-dimensional random vector X the *marginal density function* is

$$f_S(x_1,\dots,x_k) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1,\dots,x_n)\, dx_{k+1} \dots dx_n \tag{A.32}$$

in the continuous case, and

$$f_S(x_1,\dots,x_k) = \sum_{x_{k+1}} \cdots \sum_{x_n} f(x_1,\dots,x_n); \tag{A.33}$$

if X is discrete. In both cases the *marginal distribution function* is

$$F_S(x_1,\dots,x_k) = F(x_1,\dots,x_k,\infty,\dots,\infty). \tag{A.34}$$

A.3.3 Conditional Densities and Independence

The concept of forecasting is very important in power system operations and planning, and in electricity markets. When forecasting, we want to make a statement about future values of a certain time series given past observations. The full probabilistic information (*full probabilistic forecast*) about the value of a time series at a single future time instant is provided by the conditional density function, whereas *a point forecast* refers only to the conditional mean of the future value (see, for example, [3]).

Suppose that the continuous random variables X and Y have joint density $f_{X,Y}$. Based on the fact that $f_X(x)$ is positive for some values of x, we introduce next the concept of *conditional density function*.

Definition A.7 (Conditional Density). The *conditional density (function)* of random variable Y given $X = x$ is defined as

$$f_{Y|X=x}(y) = \frac{f_{X,Y}(x,y)}{f_X(x)}, \quad (f_X(x) > 0), \tag{A.35}$$

where $f_{X,Y}$ is the joint density function of X and Y. It should be noticed that both X and Y may be multivariate random variables.

The *conditional distribution function* is then found by integration or summation, as previously described in (A.29) and (A.31) for the continuous and discrete cases, respectively.

It follows from (A.35) that

$$f_{X,Y}(x,y) = f_{Y|X=x}(y) f_X(x), \tag{A.36}$$

and by interchanging X and Y on the right-hand side of (A.36), we obtain *Bayes' rule*:

$$f_{Y|X=x}(y) = \frac{f_{X|Y=y}(x) f_Y(y)}{f_X(x)}. \tag{A.37}$$

We are now in a position to introduce the concept of *independence*.

Definition A.8 (Independence). X and Y are *independent* if

$$f_{X,Y}(x,y) = f_X(x) f_Y(y), \tag{A.38}$$

which corresponds to

$$F_{X,Y}(x,y) = F_X(x) F_Y(y). \tag{A.39}$$

If X and Y are independent, it clearly holds that

$$f_{Y|X=x}(y) = f_Y(y). \tag{A.40}$$

Notice that *if two random variables are independent, then they are also uncorrelated*, while uncorrelated variables are not necessarily independent.

A.3.4 Conditional Expectations

The conditional expectation is the expected value of a random variable with respect to a conditional probability distribution, that is, given (or conditional on) values of another random variable. Under a symmetric cost function or the minimum variance criterion, the optimal forecast is the conditional mean, which is hence often used in optimal forecasts of load or wind power production.

Definition A.9 (Conditional Expectation). The *conditional expectation* of the random variable Y given $X = x$ is

$$\mathrm{E}[Y|X = x] = \int_{-\infty}^{\infty} y f_{Y|X=x}(y)\,dy. \tag{A.41}$$

Remark A.1. It is important to underline that $\mathrm{E}[Y|X]$ is actually a random variable, with its own distribution and, for instance, a mean value $\mathrm{E}[\mathrm{E}[Y|X]]$. However, for a given value of X, say $X = x$, we have that $\mathrm{E}[Y|X = x]$ is a number (for example, the point forecast of the future wind power output of a wind farm given a forecast of the wind speed).

A.3.5 Moments for Multivariate Random Variables

Often only the first moment (the mean value) and the second central moment (the variance) of a random vector are considered instead of the entire probability density function.

This leads us to extend the concepts of *expectation* and *covariance* to multivariate random variables.

Definition A.10 (Expectation of the Random Vector). The *expectation* (or the *mean value*) μ of the random vector X is

$$\mu = \mathrm{E}[X] = \begin{pmatrix} \mathrm{E}[X_1] \\ \mathrm{E}[X_2] \\ \vdots \\ \mathrm{E}[X_n] \end{pmatrix}. \tag{A.42}$$

Definition A.11 (Covariance). The *covariance (matrix)* Σ_X of X is given by

$$\begin{aligned} \Sigma_X &= \mathrm{Var}[X] \\ &= \mathrm{E}[(X-\mu)(X-\mu)^\top] \\ &= \begin{pmatrix} \mathrm{Var}[X_1] & \mathrm{Cov}[X_1, X_2] & \dots & \mathrm{Cov}[X_1, X_n] \\ \mathrm{Cov}[X_2, X_1] & \mathrm{Var}[X_2] & \dots & \mathrm{Cov}[X_2, X_n] \\ \vdots & & & \vdots \\ \mathrm{Cov}[X_n, X_1] & \mathrm{Cov}[X_n, X_2] & \dots & \mathrm{Var}[X_n] \end{pmatrix}. \end{aligned} \tag{A.43}$$

In some cases, the following notation is used instead:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2n} \\ \vdots & \vdots & & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 \end{pmatrix}. \tag{A.44}$$

Furthermore, for the variance, σ_i^2, the alternative notation σ_{ii} is sometimes employed.

We can now use the definition of the correlation coefficient (A.12) to introduce the correlation matrix of a random vector.

Definition A.12 (Correlation Matrix). The *correlation matrix* for X is

$$R = \rho = \begin{pmatrix} 1 & \rho_{12} & \dots & \rho_{1n} \\ \rho_{21} & 1 & \dots & \rho_{2n} \\ \vdots & \vdots & & \vdots \\ \rho_{n1} & \rho_{n2} & \dots & 1 \end{pmatrix}. \tag{A.45}$$

It can be easily shown that the covariance matrix Σ and the correlation matrix R are both (a) symmetric and (b) positive semi-definite (see, e.g., [3] and [7]).

The covariance between two multivariate random variables is also of particular interest and therefore, we present it below.

Definition A.13 The *covariance (matrix)* Σ_{XY} between a random vector X with dimension p and mean μ, and another random vector Y with dimension q and mean ν is computed as

$$\Sigma_{XY} = \mathrm{C}[X, Y] = \mathrm{E}\left[(X-\mu)(Y-\nu)^\top\right] \tag{A.46}$$

$$= \begin{pmatrix} \mathrm{Cov}[X_1, Y_1] & \dots & \mathrm{Cov}[X_1, Y_q] \\ \vdots & & \vdots \\ \mathrm{Cov}[X_p, Y_1] & \dots & \mathrm{Cov}[X_p, Y_q] \end{pmatrix}. \tag{A.47}$$

Clearly, we have that $\mathrm{C}[X, X] = \mathrm{Var}[X]$.

Let us now consider a generalization of the *covariance calculation rules* (A.11) to the multivariate case. Let X and Y be defined as in Definition (A.13), and let A and B be real matrices. Let U and V be random vectors of appropriate dimensions. Then

$$\mathrm{C}[A(X+U), B(Y+V) \quad = A\mathrm{C}[X, Y]B^\top + A\mathrm{C}[X, V]B^\top \tag{A.48}$$

$$+A\mathrm{C}[U, Y]B^\top + A\mathrm{C}[U, V]B^\top. \tag{A.49}$$

Important special cases are

$$\mathrm{Var}[AX] = A\mathrm{Var}[X]A^\top \tag{A.50}$$

$$\mathrm{C}[X+U, Y] = \mathrm{C}[X, Y] + \mathrm{C}[U, Y]. \tag{A.51}$$

A.3.6 The Multivariate Normal Distribution

In power engineering, the normal distribution, and the distributions derived from it, are of great use.

Let us therefore briefly introduce the multivariate normal probability density function (PDF). We assume that $X_1, X_2, \ldots, X_n$ are independent random variables with means $\mu_1, \mu_2, \ldots, \mu_n$, and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$. We write $X_i \sim \mathrm{N}(\mu_i, \sigma_i^2)$. Now, define the random vector $X = (X_1, X_2, \ldots, X_n)^\top$. As the random variables are independent, it follows from (A.38) that

$$\begin{aligned} f_X(x_1, \ldots, x_n) &= f_{X_1}(x_1) \ldots f_{X_n}(x_n) && \text{(A.52)} \\ &= \prod_{i=1}^{n} \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left[-\frac{(x_i - \mu_i)^2}{2\sigma_i^2}\right] && \text{(A.53)} \\ &= \frac{1}{\left(\prod_{i=1}^{n} \sigma_i\right)(2\pi)^{n/2}} \exp\left[-\tfrac{1}{2}\sum_{i=1}^{n}\left[\frac{x_i - \mu_i}{\sigma_i}\right]^2\right]. && \text{(A.54)} \end{aligned}$$

By introducing the mean vector $\mu = (\mu_1, \ldots, \mu_n)^\top$ and the covariance matrix $\Sigma_X = \mathrm{Var}[X] = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_n^2)$, this can be written as

$$f_X(x) = \frac{1}{(2\pi)^{n/2}\sqrt{\det \Sigma_X}} \exp\left[-\tfrac{1}{2}(x - \mu)^\top \Sigma_X^{-1}(x - \mu)\right]. \tag{A.55}$$

Note that the operator $\mathrm{diag}(\,\cdot\,)$ provides a square matrix with the elements of the specified vector on the main diagonal and $\det \Sigma_X$ denotes the determinant of the covariance matrix Σ_X.

The following definition generalizes (A.55) to the case where the covariance matrix is a full matrix.

Definition A.14 (The Multivariate Normal Distribution). The joint density function for the n-dimensional random variable X with mean μ and covariance Σ is

$$f_X(x) = \frac{1}{(2\pi)^{n/2}\sqrt{\det \Sigma}} \exp\left[-\tfrac{1}{2}(x - \mu)^\top \Sigma^{-1}(x - \mu)\right], \tag{A.56}$$

where $\Sigma > 0$. We write $X \sim \mathrm{N}(\mu, \Sigma)$.

If $X \sim \mathrm{N}(0, I)$, we say that X is *standardized normally distributed.*

A.4 Stochastic Processes

In general, a process $\{X_{z,t}\}$ is defined as the evolution of a variable X with time t and through space z. Such a process can be deterministic or stochastic. In the former case, even if chaotic, a perfect knowledge of the initial state and the governing equations of

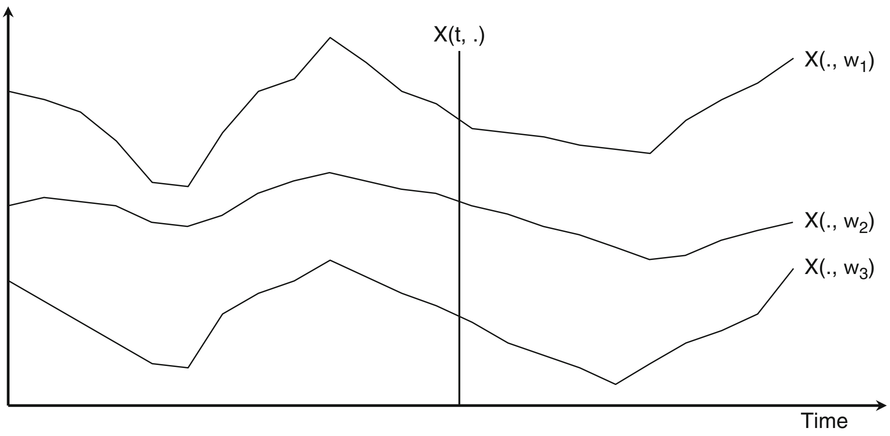


Fig. A.7 Realizations of a stochastic process. Each realization is a time series

this process permits to perfectly predict its future states. In contrast, for a stochastic one, even if the initial state of the system and its governing equations are known, several evolution paths are possible. Throughout the book, it will be assumed that the process of renewable energy generation, being from the wind, sun, or waves, is stochastic. Hence, it may not be possible to perfectly determine future values of that process; forecasts will always have a share of uncertainty. In the following, we will thus focus on stochastic processes.

A.4.1 Time Series and Stochastic Processes

A *time series* is a *realization* of a *stochastic process*. That is, the historical record (given as a time series) $\{x_t, t = 0, \pm 1, \ldots\}$ is assumed to be one realization out of an infinite number of possible realizations of the stochastic process $\{X_t, t = 0, \pm 1, \ldots\}$.

A stochastic process is defined as a family of random variables, $\{X(t)\}$ or $\{X_t\}$, where t belongs to a totally ordered *index set* T. In the first case, we are considering a process in *continuous time*, while in the latter, a process in *discrete time*. In this section, only processes in continuous time will be described for ease of notation, although the results and definitions presented next can be easily extended to the discrete case as well.

In essence, a stochastic process is a function having two arguments, $\{X(t, \omega),$ $t \in T, \omega \in \Omega\}$, where Ω is the *sample space* or the *ensemble* of the process. The stochastic process takes values on the *state space* S. This implies that Ω is the set of all the possible time series that can be generated by the process.

For a fixed t, we say that $X(t, \cdot)$ is a *random variable*, and for fixed $\omega \in \Omega$, we call it a *realization* of the process, i.e., a *time series* $X(\cdot, \omega)$; see Fig. A.7.

Table A.1 Classification of some stochastic processes

Time (Index set, T)	State space (S)	
	Discrete	Continuous
Discrete	Markov chain processes	Time series processes
Continous	Point processes	Stochastic differential equations.

Both the index set (usually interpreted as time) T and the state space S can be either discrete or continuous leading to the four different categories of stochastic processes included in Table A.1.

Due to the fact that a stochastic process is a family of random variables, its distributional specifications are almost equivalent to what we have seen for multidimensional random variables.

The function $f_{X(t_1),\ldots,X(t_n)}(x_1,\ldots,x_n)$ is called the n-dimensional probability distribution function for the process $\{X(t)\}$. The family of all these probability distribution functions, for all values of n, i.e., for $n = 1, 2, \ldots,$ and for all values of t, forms the *family of finite-dimensional probability distribution functions*, which fully characterizes the stochastic process $\{X(t)\}$.

In this appendix, we will limit ourselves to temporal processes. However, in some applications such as those involving variations in time and space of renewable power production, spatiotemporal processes are needed. These *spatiotemporal stochastic processes* can be equivalently seen, though, as a family of random variables indexed by time t and space z.

Rather than using the family of probability distribution functions, stochastic processes are typically described by their mean value and the so-called *covariance functions*, which are introduced in Sect. A.4.2. Subsequently, Sect. A.4.3. describes some characteristic types of processes, including the important family of stationary stochastic processes, which will be analyzed in more detail in Sect. A.5.

A.4.2 Mean Value and Covariance Functions

The concept of *moment*, first introduced in Sect. A.1.2 for a single random variable and subsequently extended in Sect. A.3.5. to random vectors, is now applied here to the case of stochastic processes.

We begin with the simplest moment, i.e., the mean value, which, for a stochastic process $\{X(t)\}$, is given by

$$\mu(t) = \mathrm{E}[X(t)] = \int_{-\infty}^{\infty} x f_{X(t)}(x)\, dx. \tag{A.57}$$

Notice that, for a stochastic process, the mean value, also known as *first moment*, is actually a function of t and accordingly, $\mu(t)$ in (A.57) is often referred to as the *mean value function*.

Similarly, we can define the *variance* (or the *variance function*) of the process, i.e.,

$$\sigma^2(t) = \mathrm{Var}[X(t)] = \mathrm{E}\left[(X(t) - \mu(t))^2\right]. \tag{A.58}$$

When it comes to stochastic processes, the concept of *autocovariance function* deserves special mention.

Definition A.15 (Autocovariance Function). The autocovariance function γ_{XX} of a stochastic process $\{X(t)\}$ is defined as

$$\begin{aligned}\gamma_{XX}(t_1, t_2) = \gamma(t_1, t_2) &= \mathrm{Cov}[X(t_1), X(t_2)] \\ &= \mathrm{E}[(X(t_1) - \mu(t_1))(X(t_2) - \mu(t_2))].\end{aligned} \tag{A.59}$$

Note that the variance of the process can be obtained as a special case of (A.59). Indeed, it holds that $\sigma^2(t) = \gamma(t, t)$.

The mean value and autocovariance functions constitute the *second-order moment representation* of the process.

Lastly, the *autocorrelation function* (ACF) is obtained by normalizing the autocovariance function with the variance function, i.e.,

$$\rho_{XX}(t_1, t_2) = \rho(t_1, t_2) = \frac{\gamma_{XX}(t_1, t_2)}{\sqrt{\sigma^2(t_1)\sigma^2(t_2)}}. \tag{A.60}$$

In a similar manner, we can define moments of higher order.

A.4.3 Characteristics for Stochastic Processes

In this section, we will introduce a number of useful characteristics for stochastic processes. This will pave the way for a more detailed study of stationary processes later on.

A.4.3.1 Stationary Processes

Definition A.16 (Strong Stationarity). A process $\{X(t)\}$ is said to be *strongly stationary* if all finite-dimensional distributions are invariant in time, i.e. for every n, any set $(t_1, t_2, \ldots, t_n)$, and for any h, it holds

$$f_{X(t_1),\ldots,X(t_n)}(x_1, \ldots, x_n) = f_{X(t_1+h),\ldots,X(t_n+h)}(x_1, \ldots, x_n). \tag{A.61}$$

Definition A.17 (Weak Stationarity). A process $\{X(t)\}$ is said to be *weakly stationary of order k* if all its first k moments are invariant in time. A weakly stationary process of order 2 is simply called *weakly stationary*.

Theorem A. 1. *A weakly stationary process is characterized by the fact that both the mean value and the variance are constant, while the autocovariance function depends only on the time difference, i.e., on* $(t_1 - t_2)$*, which is called the time lag.*

Proof. It follows directly from Definition A.17.

In the following, we will use the term *stationary* to denote a weakly stationary process.

A.4.3.2 Normal Processes

Definition A.18 (Normal Process). A process $\{X(t)\}$ is said to be a *normal process* (or, alternatively, a *Gaussian process*) if all the n-dimensional distribution functions $f_{X(t_1),\ldots,X(t_n)}(x_1, \ldots, x_n)$ for any n are (multidimensional) normal distributions.

A normal process is completely specified by its mean value function

$$\mu(t_i) = \mathrm{E}[X(t_i)], \tag{A.62}$$

and autocovariance function

$$\gamma(t_i, t_j) = \mathrm{Cov}[X(t_i), X(t_j)]. \tag{A.63}$$

By introducing the vector $\mu = (\mu(t_1), \mu(t_2), \ldots, \mu(t_n))^T$ and the variance matrix $\Sigma = \{\gamma(t_i, t_j)\}$, the joint distribution for $X = (X(t_1), X(t_2), \ldots, X(t_n))^\top$ is given by

$$f_X(x) = \frac{1}{(2\pi)^{n/2}\sqrt{\det \Sigma}} \exp\left(-\frac{1}{2}(x - \mu)^\top \Sigma^{-1}(x - \mu)\right), \tag{A.64}$$

where Σ is assumed to be regular. Notice the similarity with the multivariate normal density in (A.56). For normal processes, weak stationarity and strong stationarity are equivalent, since a normal process is completely characterized by its first two moments.

A.4.3.3 Markov Processes

Definition A.19 (Markov Process). A process $\{X(t)\}$ is called a *Markov process*, if for $t_1 < t_2 < \cdots < t_n$, the distribution of $X(t_n)$ given $(X(t_1), \ldots, X(t_{n-1}))$ is the same as the distribution of $X(t_n)$ given $X(t_{n-1})$. This implies that

$$\mathrm{P}\{X(t_n) \le x | X(t_{n-1}), \ldots, X(t_1)\} = \mathrm{P}\{X(t_n) \le x | X(t_{n-1})\}. \tag{A.65}$$

A Markov process is thus characterized by the fact that all the information about $X(t_n)$ from past observations of $\{X(t)\}$ is contained in the just previous observation $X(t_{n-1})$. Because only the most recent observation is needed, the process is also called a *first order Markov process*.

A.4.3.4 The White Noise Process

Definition A.20 (White Noise Process). A sequence $\{\varepsilon_t\}$ of uncorrelated identically distributed variables with $\mathrm{E}[\varepsilon_t] = 0$ and $\mathrm{Var}[\varepsilon_t] = \sigma^2$ is called a *white noise process*.

Example A.6 (*AR*(1) Process, Part I). Let $\{\varepsilon_t\}$ be a normally distributed white noise with $\mathrm{E}[\varepsilon_t] = 0$ and $\mathrm{Var}[\varepsilon_t] = \sigma^2$, and let $\{\varepsilon_t\}$ be the input to a dynamical system defined by the difference equation

$$Y_t = \phi Y_{t-1} + \varepsilon_t, \tag{A.66}$$

which defines a stochastic process $\{Y_t\}$. This process is called an Autoregressive process of first order, in short, AR(1). The name indicates that new values of Y are obtained by regression on old values of Y, and the order is given by the order of the difference equation.

By successively substituting $Y_{t-1} = \phi Y_{t-2} + \varepsilon_{t-1}$, $Y_{t-2} = \phi Y_{t-3} + \varepsilon_{t-2}, \ldots$ on the right-hand side of (A.66), it is seen that Y_t can be written as

$$Y_t = \varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \cdots + \phi^i\varepsilon_{t-i} + \ldots. \tag{A.67}$$

Moreover, by using the fact that the expectation operator is linear, one finds that

$$\mu_Y = \mathrm{E}[Y_t] = 0, \tag{A.68}$$

and

$$\sigma_Y^2 = \mathrm{Var}[Y_t] = (1 + \phi^2 + \phi^4 + \cdots + \phi^{2i} + \cdots)\sigma^2 = \frac{\sigma^2}{1-\phi^2}, \tag{A.69}$$

provided that $|\phi| < 1$. For $|\phi| \geq 1$, the variance is unbounded.

Finally, we have the covariance function

$$\begin{aligned}
\gamma(t_1, t_2) &= \mathrm{Cov}[Y_{t_1}, Y_{t_2}] \\
&= \mathrm{Cov}[\varepsilon_{t_1} + \phi\varepsilon_{t_1-1} + \cdots + \phi^{t_1-t_2}\varepsilon_{t_2} + \cdots, \varepsilon_{t_2} + \phi\varepsilon_{t_2-1} + \cdots] \\
&= \phi^{t_1-t_2}(1 + \phi^2 + \phi^4 + \ldots)\sigma^2 \\
&= \phi^{t_1-t_2}\sigma_Y^2,
\end{aligned}$$

for $t_1 > t_2$. Notice that $\mathrm{Cov}[\varepsilon_{t_i}, \varepsilon_{t_2} + \phi\varepsilon_{t_2-1} + \ldots] = 0$ for all $t_i > t_2$. Similarly, for $t_1 < t_2$, we obtain

$$\gamma(t_1, t_2) = \phi^{t_2-t_1}\sigma_Y^2.$$

Observe that $\gamma(t_1, t_2)$ depends only on the time lag $t_1 - t_2$, i.e.,

$$\gamma(t_1, t_2) = \gamma(t_1 - t_2) = \phi^{|t_1-t_2|}\sigma_Y^2. \tag{A.70}$$

Since the mean value and the variance are constant for $|\phi| < 1$, and the autocovariance function depends only on the time difference, $\{Y_t\}$ is a *weakly stationary* process for $|\phi| < 1$.

In addition, it is well known that a sum of normally distributed random variables is also normally distributed. Consequently, since ε_t is normally distributed, $\{Y_t\}$ is a *normal process*. The process (A.66) is thus *strongly stationary* for $|\phi| < 1$.

Last, from (A.66), it can be seen that

$$\mathrm{P}\{Y_t \le y | Y_{t-1}, Y_{t-2}, \dots\} = \mathrm{P}\{Y_t \le y | Y_{t-1}\}, \tag{A.71}$$

and therefore, $\{Y_t\}$ is a (first order) Markov process.

A.4.3.5 Deterministic Processes

Definition A.21 (Deterministic Processes). A process is said to be deterministic if it can be predicted *without uncertainty* from past observations.

Example A.7 The process $\{Y_t\}$ defined by

$$Y_t = \phi_1 Y_{t-1} - \phi_2 Y_{t-2} \quad (Y_0 = A_1; Y_1 = A_2), \tag{A.72}$$

where ϕ_1 and ϕ_2 are constants, is a deterministic process. Here A_1 and A_2 are random variables.

A.4.3.6 Purely Stochastic Processes

Definition A.22 (Purely Stochastic Process). A process is said to be (purely) stochastic, if it can be written in the form

$$X_t = \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \cdots$$

where $\{\varepsilon_t\}$ is a sequence of uncorrelated stochastic variables, with $\mathrm{E}[\varepsilon_t] = 0$, $\mathrm{Var}[\varepsilon_t] = \sigma^2$, and $\sum_{i=0}^{\infty} \psi_i^2 < \infty$.

A.5 Serial Dependence

We begin this section with the so-called autocovariance and autocorrelation functions, which are used to describe the serial dependence for a single time series, and conclude with the cross-correlation function, which serves to characterize the serial dependence between two time series.

A.5.1 Autocovariance and Autocorrelation Functions

For a stochastic process, $\{X(t)\}$, the *autocovariance function* is given by

$$\gamma_{XX}(t_1, t_2) = \gamma(t_1, t_2) = \mathrm{Cov}[X(t_1), X(t_2)], \tag{A.73}$$

where the variance of the process is $\sigma^2(t) = \gamma_{XX}(t, t)$.

Similarly, the *autocorrelation function* (ACF) is defined as

$$\rho_{XX}(t_1, t_2) = \rho(t_1, t_2) = \frac{\gamma_{XX}(t_1, t_2)}{\sqrt{\sigma^2(t_1)\sigma^2(t_2)}}. \tag{A.74}$$

For Stationary Processes

If the process is *stationary*, then (A.74) is only a function of the time lag $t_2 - t_1$. Denoting the time difference by τ, the *autocovariance* and *autocorrelation functions* for a stationary process $\{X(t)\}$ boil down to

$$\gamma_{XX}(\tau) = \mathrm{Cov}[X(t), X(t+\tau)], \tag{A.75}$$

and

$$\rho_{XX}(\tau) = \frac{\gamma_{XX}(\tau)}{\gamma_{XX}(0)} = \frac{\gamma_{XX}(\tau)}{\sigma_X^2}, \tag{A.76}$$

respectively, where $\sigma_X^2(t)$ is the variance of the process. Note that $\rho_{XX}(0) = 1$.

The following example illustrates how to compute the autocorrelation function for an AR(1) process.

Example A.8 (AR(1) Process, Part II). Consider the same stochastic process as in (A.66).

This process is stationary for $|\phi| < 1$. For $k > 0$, it holds

$$\begin{aligned}
\gamma(k) = \gamma(-k) &= \mathrm{Cov}[Y_t, Y_{t-k}] \\
&= \mathrm{Cov}[\phi Y_{t-1} + \varepsilon_t, Y_{t-k}] \\
&= \phi \mathrm{Cov}[Y_{t-1}, Y_{t-k}] \\
&= \phi\gamma(k-1) = \phi^2\gamma(k-2) = \dots
\end{aligned}$$

Therefore, we obtain that $\gamma(k) = \phi^k \gamma(0)$. Since $\gamma(k)$ is an even function, it follows that

$$\gamma(k) = \phi^{|k|}\gamma(0),$$

and thus, the autocorrelation function becomes

$$\rho(k) = \phi^{|k|}, \quad |\phi| < 1. \tag{A.77}$$

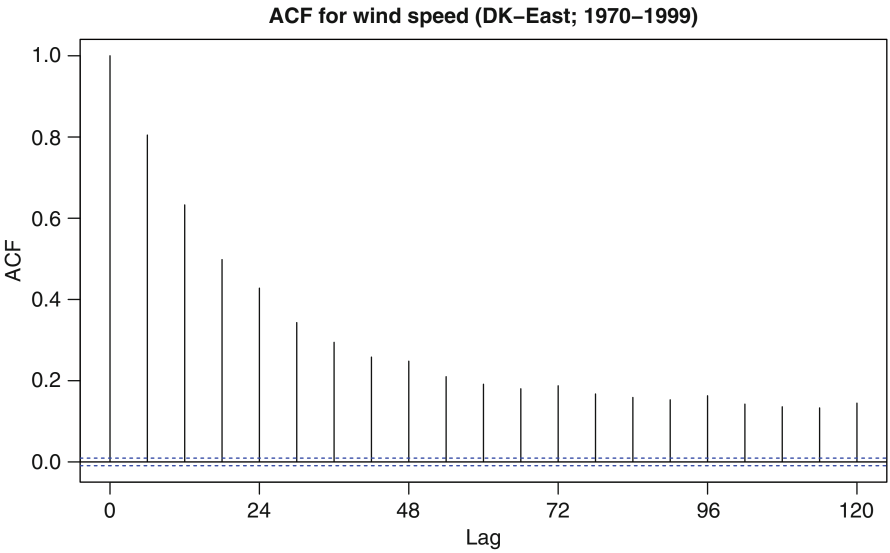


Fig. A.8 Autocorrelation function for wind speed in Denmark (East). Note that the x-axis represents hours

The argument k is often referred to as the *lag*, i.e., the time distance.

The coefficient ϕ determines the memory of the process. For ϕ close to 1, there is a long memory, while the memory is short for small values of ϕ. Finally, $\rho(k)$ will oscillate for $\phi < 0$.

Example A.9 (Autocorrelation Functions for Wind Speed in Denmark and Spain). Figures A.8 and A.9 show the autocorrelation function for wind speed in Denmark East and the central part of Spain, respectively. In both cases, the data correspond to wind speed observations collected every 6 h during the period from 1970 to 1999.

The autocorrelation functions turn out to be rather different. For Spain, a clear diurnal pattern is observed, whereas this is not the case for Denmark. On the other hand, we see that the hourly persistence is higher for the wind speed in Denmark than for the wind speed in Spain.

A similar analysis only focused on the northern region of Spain reveals a much lower diurnal variation of the wind speed in this part of the country, but a higher persistence. Indeed, for the wind speed in the northern part of Spain, the 6-h correlation is almost 0.8, whereas this value decreases to around 0.5 for the middle part of the country.

A.5.2 Cross-Covariance and Cross-Correlation Functions

Let us consider now the stochastic processes $\{X(t)\}$ and $\{Y(t)\}$. The dependence structure between these two stochastic processes is described by the *cross-covariance*

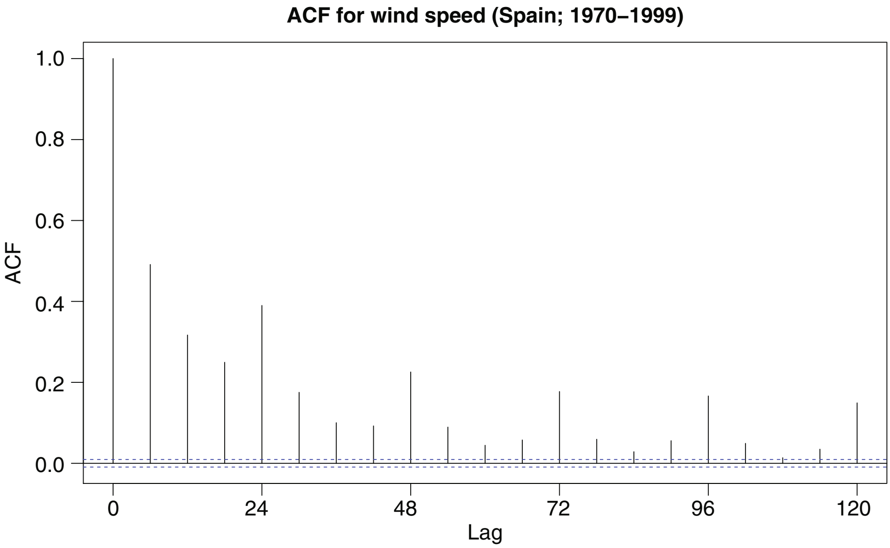


Fig. A.9 Autocorrelation function for wind speed in the central part of Spain. Note that the x-axis represents hours

function, which is determined as

$$\begin{aligned}\gamma_{XY}(t_1, t_2) &= \mathrm{Cov}[X(t_1), Y(t_2)] \\ &= \mathrm{E}[(X(t_1) - \mu_X(t_1))(Y(t_2) - \mu_Y(t_2)), \end{aligned} \quad \text{(A.78)}$$

where $\mu_X(t) = \mathrm{E}[X(t)]$ and $\mu_Y(t) = \mathrm{E}[Y(t)]$.

Similarly to (A.74), we define the *cross-correlation function* as follows:

$$\rho_{XY}(t_1, t_2) = \frac{\gamma_{XY}(t_1, t_2)}{\sqrt{\sigma_X^2(t_1)\sigma_Y^2(t_2)}}, \quad \text{(A.79)}$$

where $\sigma_X^2(t) = \mathrm{Var}[X(t)]$ and $\sigma_Y^2(t) = \mathrm{Var}[Y(t)]$.

For Stationary Processes

If the stochastic processes $\{X(t)\}$ and $\{Y(t)\}$ are stationary[1], the cross-covariance and cross-correlation functions, (A.78) and (A.79), become

$$\gamma_{XY}(\tau) = \mathrm{Cov}[X(t), Y(t+\tau)] \text{ and} \quad \text{(A.80)}$$

$$\rho_{XY}(\tau) = \frac{\gamma_{XY}(\tau)}{\sqrt{\gamma_{XX}(0)\gamma_{YY}(0)}} = \frac{\gamma_{XY}(\tau)}{\sigma_X \sigma_Y}, \quad \text{(A.81)}$$

respectively.

[1] Actually, the bivariate process $(X(t), Y(t))^T$ must be stationary.

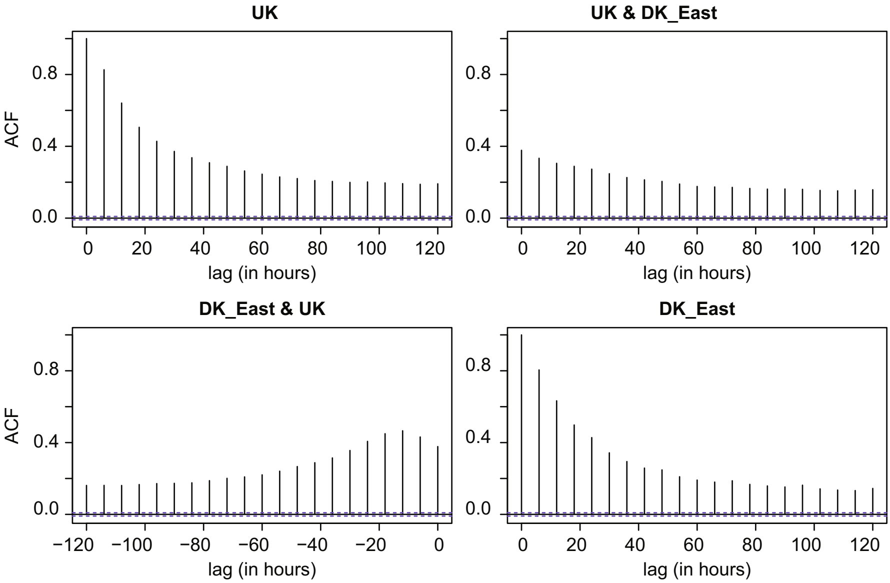


Fig. A.10 Autocorrelation functions for UK and eastern Denmark (in the diagonal) and the cross-correlation function between UK and eastern Denmark (outside the diagonal). Note that the x-axes represent hours

Example A.10 (Auto-Correlation and Cross-Correlation Functions for Wind Speed in UK and Eastern Denmark)
Figure A.10 shows the auto-correlation and cross-correlation functions for wind speed in UK and Denmark (East) for the period from 1970 to 1999. It is seen that the autocorrelation functions for UK and Denmark are very similar—and rather different from the autocorrelation function shown previously in Fig. A.9 for the wind speed in Spain. The figure also shows the cross-correlation function between the wind speed in UK and Denmark (East). Since the maximum of the cross-correlation function is apparently around the lags corresponding to -18 and -12 h, it is concluded that the phase shift between UK and Denmark for the wind speed is on average between 12 and 18 h. The cross correlation is rather high, indicating that it would probably be beneficial to exploit this cross correlation in any tool for wind power forecasting in Denmark.

Similar analyses using cross-correlation functions have shown that the cross correlation between Germany and Greece is negative, which indicates that if the wind speed is high in Germany, then the wind speed is low in Greece.

References

1. Bury, K.: Statistical Distributions in Engineering. Cambridge University Press, New York (1999)
2. Hastie, T., Tibshirani, R., Friedman, J.: The Elements of Statistical Learning. Springer, New York (2001)

3. Madsen, H.: Time Series Analysis. Chapman & Hall/CRC, Boca Raton (2008)
4. Madsen, H., Thyregod, P.: Introduction to General and Generalized Linear Models. Chapman & Hall/CRC, Boca Raton (2011)
5. Nielsen, H.A., Madsen, H., Nielsen, T.S.: Using quantile regression to extend an existing wind power forecasting system with probabilistic forecasts. Wind Energy **9**(1-2), 95–108 (2006)
6. Pinson, P., Nielsen, H.A., Møller, J.K., Madsen, H., Kariniotakis, G.N.: Non-parametric probabilistic forecasts of wind power: Required properties and evaluation. Wind Energy **10**(6), 497–516 (2007)
7. Rao, C.R.: Linear Statistical Inference and its Applications. Wiley, New York (1973)
8. Zaharim, A., Najid, S., Razali, A., Sopian, K.: The suitability of statistical distribution in fitting wind speed data. WSEAS Trans. Math. **7**(12), 718–727 (2008)

Appendix B
Basics of Optimization

In this appendix, we review some basics of optimization. In Sect. B.1, we introduce the mathematical formulation for both general and linear optimization problems. Duality theory in linear programming is briefly presented in Sect. B.2. The Karush–Kuhn–Tucker (KKT) optimality conditions are presented in Sect. B.3. Finally, Mathematical Programs with Equilibrium Constraints (MPECs) are introduced in Sect. B.4.

B.1 Formulation of an Optimization Problem

The general mathematical formulation of an optimization problem is:

$$\underset{x}{\text{Min.}}\, f(x) \tag{B.1a}$$

$$\text{s.t. } h(x) = 0, \tag{B.1b}$$

$$g(x) \leq 0. \tag{B.1c}$$

Problem (B.1) includes the following elements:

- $x \in \mathbb{R}^n$ is a vector including the n decision variables.
- $f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function of the optimization problem. It maps values of the decision vector x to a real value representing the desirability of this solution to the decision-maker. Typically the objective function represents a cost in minimization problems or a benefit in maximization ones.
- $h(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $g(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^l$ are vector-valued functions of the decision vector x. They define m equality and l inequality constraints through (B.1b) and (B.1c), respectively. Note that we assume that the zero-valued vectors on the right-hand side of (B.1b) and (B.1c) are properly sized to match the dimension of the vectors on the left-hand side.

The joint enforcement of equalities (B.1b) and inequalities (B.1c) defines the feasibility region of the optimization problem. A decision x is called *feasible* if it satisfies (B.1b)–(B.1c).

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The aim of problem (B.1) is to determine, among the set of feasible decisions, the one that yields the lowest value of the objective function (B.1a).

The simplest instance of an optimization problem is a linear programming problem (LP). This is obtained when the functions $f(\cdot)$, $h(\cdot)$, and $g(\cdot)$ in (B.1) are linear. We can formulate a linear programming problem as:

$$\underset{x}{\text{Min.}}\ c^\top x \tag{B.2a}$$

$$\text{s.t. } A_\text{E} x = b_\text{E}, \tag{B.2b}$$

$$A_\text{I} x \geq b_\text{I}. \tag{B.2c}$$

Note that the general functions $f(\cdot)$, $h(\cdot)$, and $g(\cdot)$ are replaced by affine expressions involving the following vectors and matrices:

- $c \in \mathbb{R}^n$ is the cost coefficient of the decision vector x.
- $A_\text{E} \in \mathbb{R}^{m \times n}$, and $b_\text{E} \in \mathbb{R}^m$ define the m equality constraints (B.2b).
- $A_I \in \mathbb{R}^{l \times n}$ and $b_I \in \mathbb{R}^l$ define the l linear inequality constraints (B.2c). Note that the sign of the constraints is changed with respect to (B.1c). This is to simplify the representation of the dual problem in the next section.

LPs model a wide variety of real-world problems, also within the area of electricity markets. Very large LPs can be solved using commercially available software.

B.2 Duality in Linear Programming

Let us associate the vector $\lambda \in \mathbb{R}^m$ to the equalities (B.2b) and the vector $\mu \in \mathbb{R}^l$ to the inequalities (B.2c). The following linear maximization problem is the *dual* of LP (B.2), which is referred to as the *primal* problem:

$$\underset{\lambda,\mu}{\text{Max.}}\ b_\text{E}^\top \lambda + b_\text{I}^\top \mu \tag{B.3a}$$

$$\text{s.t. } A_\text{E}^\top \lambda + A_\text{I}^\top \mu = c, \tag{B.3b}$$

$$\mu \geq 0. \tag{B.3c}$$

The dual problem (B.3) can be considered a transposed version of the primal problem (B.2). Indeed, the following relationships hold:

- While the primal problem (B.2) has n decision variables and $m + l$ constraints, the dual problem has $m + l$ decision variables (λ and μ) and n constraints.
- The constraints (B.3b) of the dual problem involve the transposed of the matrices A_E and A_I defining the constraints (B.2b)–(B.2c) of the primal problem.
- The constant vectors b_E and b_I on the right-hand side of the primal constraints (B.2b)–(B.2c) form the cost coefficients of the dual linear objective function (B.3a). Viceversa, the cost coefficient vectors c of the primal objective function (B.2a) appear on the right-hand side of the dual constraints (B.3b).

Table B.1 Relationships between direction of the optimization problem, sign of the constraints, and bounds on the optimization variables in the primal and dual problems

Problem	Primal		Dual
Objective	Minimization	Objective	Maximization
	≥ 0		≥ 0
Constraint type	$= 0$	Variable bound	free
	≤ 0		≤ 0
	≥ 0		≤ 0
Variable bound	free	Constraint type	$= 0$
	≤ 0		≥ 0

The direction of optimization (minimization or maximization), the sign of the constraints ($\geq$, $=$, or $\leq$) and the bounds on the variables ($\geq$ 0, free, or $\leq$ 0) for the primal and the dual problem are linked. Specifically, the direction of the dual optimization problem is opposite to the one of the primal one. Furthermore, the signs of the primal constraints set the bounds on the associated dual variables and, conversely, the bounds on the primal variables set the signs of the dual constraints. Table B.1 includes all the possible combinations of constraint types and variable bounds for a primal minimization problem, and the corresponding ones for the associated dual maximization problem. Finally, note that the dual of the dual problem is the primal problem, see [2]. This implies that the headers in Table B.1 can be swapped, so that the right column pertains to a primal maximization problem and the left one to its dual minimization one.

The objective function values of the primal and dual problems are related to each other through the so-called *weak* and *strong* duality theorems. Such theorems are of particular importance. In what follows, we shall present them without proof. The interested reader is referred to [5] for further details and proofs.

Theorem B.1 (Weak Duality) *If* x *is feasible for* (B.2), *and* λ, μ *are feasible for* (B.3), *then* $c^\top x \geq b_{\mathrm{E}}^\top \lambda + b_{\mathrm{I}}^\top \mu$.

Theorem B.2 (Strong Duality) *If the primal problem has a finite optimal solution* x^*, *so does the dual problem and at optimality it holds that* $c^\top x^* = b_{\mathrm{E}}^\top \lambda^* + b_{\mathrm{I}}^\top \mu^*$.

Since the dual of the dual problem is again the primal problem, the converse of the previous theorems holds trivially.

Note that the dual variables λ and μ have an important economic interpretation, as they are marginal costs. Indeed, they represent the per-unit change (increase) in the optimal value of the objective function (B.2a) if the right-hand side of the associated constraint is increased marginally. Naturally, $\mu \geq 0$. Indeed, a marginal increase of any element of the vector b_{I} would result in a smaller feasible space for (B.2), and hence in a larger, i.e., worse, optimal value of the objective function.

B.3 Karush–Kuhn–Tucker Conditions

In this appendix, we only deal with KKT optimality conditions for convex problems, and refer to [1] for a general introduction to duality theory in nonlinear programming.

Let us consider the general formulation (B.1), and suppose that $f(\cdot)$, $g(\cdot)$ are continuously differentiable and convex, and $h(\cdot)$ is affine. Furthermore, we assume that a constraint qualification holds. For example, we may require that $g(\cdot)$ be affine (linearity constraint qualification). Another common constraint qualification requires linear independence of the gradients of active inequality constraints and of equality constraints. We refer the reader to specialized books on optimization, for instance [1], for a detailed treatment of constraint qualifications.

We can define the Lagrangian function for problem (B.1) as follows:

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^\top h(x) + \mu^\top g(x). \tag{B.4}$$

Under the assumptions above, the following KKT conditions are necessary and sufficient for optimality for problem (B.1):

$$\nabla_x f(x) + \lambda^\top \nabla_x h(x) + \mu^\top \nabla_x g(x) = 0, \tag{B.5a}$$

$$h(x) = 0, \tag{B.5b}$$

$$g(x) \leq 0, \tag{B.5c}$$

$$\mu \geq 0, \tag{B.5d}$$

$$\mu^\top g(x) = 0. \tag{B.5e}$$

Equations (B.5a) are stationarity conditions. Constraints (B.5b) and (B.5c) enforce feasibility of the primal problem, while (B.5d) is a feasibility condition of the dual problem. Finally, (B.5e) enforces complementary slackness. Note that in view of (B.5c) and (B.5d), the scalar product on the left-hand side of (B.5e) is actually the sum of non-positive terms only. As a result, (B.5e) implies that the element-by-element product between μ_i and $g_i(x)$ is equal to 0.

Note that constraint qualifications are needed for ensuring that KKT conditions are necessary for optimality, while convexity is needed to ensure their sufficiency.

The dual vectors λ and μ retain the interpretation of marginal costs discussed in Sect. B.2.

Finally, the notation for constraints (B.5c)–(B.5e) can be compacted into the following nonlinear constraint:

$$0 \geq g(x) \perp \mu \geq 0, \tag{B.6}$$

where the $\perp$ (*perpendicular*) operator enforces the perpendicularity condition between the vectors on the left- and right-hand sides, i.e., that their element-by-element product is equal to zero.

B.4 Mathematical Programs with Equilibrium Constraints

MPECs is a relatively recent area of optimization, which has been applied to study electricity markets with increasing success in the recent years. In this section, we briefly introduce the concept of MPEC and present how it can be used to model bilevel programs, i.e., optimization problems constrained by other optimization problems. The reader is referred to [4] and [6] for an in-depth treatment of the subject.

The general formulation of a bilevel optimization problem is the following:

$$\underset{x,y}{\text{Min.}}\; f^{\mathrm{U}}(x, y) \tag{B.7a}$$

$$\text{s.t. } g^{\mathrm{U}}(x, y) \leq 0, \tag{B.7b}$$

$$h^{\mathrm{U}}(x, y) = 0, \tag{B.7c}$$

$$y \in \underset{z}{\text{argmin}} \left\{ f^{\mathrm{L}}(x, z) \text{ s.t. } h^{\mathrm{L}}(x, z) = 0, g^{\mathrm{L}}(x, z) \leq 0 \right\}. \tag{B.7d}$$

The fundamental difference between the MPEC (B.7) and the general optimization problem (B.1) is the enforcement of conditions (B.7d). These conditions ensure that at any feasible point (x, y) of problem (B.7), the choice of variable y is optimal for the minimization problem within the braces in (B.7d).

Note that formulation (B.7) includes two optimization problems: an upper-level one that aims at the minimization of $f^{\mathrm{U}}(\,\cdot\,)$, and a lower-level one consisting in the minimization of $f^{\mathrm{L}}(\,\cdot\,)$.

The two problems are interdependent, since in general the upper-level objective function (B.7a) and constraints (B.7b)–(B.7c) depend on the lower-level decision variables y. Conversely, the objective function and the constraints of the lower-level problem (B.7d) depend on the upper-level variable vector x.

There is a hierarchical relationship between the two problems. Indeed, the lower-level problem is solved assuming that the upper-level decision x is fixed. On the contrary, the upper-level problem is solved accounting for the response of the lower-level problem to decision vector x.

Moreover, it should be emphasized that model (B.7) can accommodate several lower-level optimization problems, simply by concatenating multiple optimality conditions of the type of (B.7d).

Under the assumption that KKT conditions are necessary and sufficient for optimality in the lower-level problem, we can employ them to replace condition (B.7d). This results in the following formulation for the bilevel problem:

$$\underset{x,y,\lambda,\mu}{\text{Min.}}\; f^{\mathrm{U}}(x, y) \tag{B.8a}$$

$$\text{s.t. } h^{\mathrm{U}}(x, y) = 0, \tag{B.8b}$$

$$g^{\mathrm{U}}(x, y) \leq 0, \tag{B.8c}$$

$$\nabla_y f^{\mathrm{L}}(x, y) + \lambda^{\top} \nabla_y h^{\mathrm{L}}(x, y) + \mu^{\top} \nabla_y g^{\mathrm{L}}(x) = 0, \tag{B.8d}$$

$$h^{\mathrm{L}}(x, y) = 0, \tag{B.8e}$$

$$g^{\mathrm{L}}(x, y) \leq 0, \tag{B.8f}$$

$$\mu \geq 0, \tag{B.8g}$$

$$\mu^{\top} g^{\mathrm{L}}(x, y) = 0, \tag{B.8h}$$

where λ and μ represent the dual variables associated to constraints $h^{\mathrm{L}}(x, z) = 0$ and $g^{\mathrm{L}}(x, z) \leq 0$, respectively, in the lower-level problem (B.7d).

The advantage of formulation (B.8) is the replacement of the nested lower-level problem with the set (B.8d)–(B.8h) of equations and inequalities, which results in a single-level optimization problem that fits the general formulation (B.1). However, note that solving the single-level program (B.8) is far from trivial. Indeed, KKT conditions are in general nonlinear and non convex, as they involve cross products between variables in the complementarity condition (B.8h).

A number of approaches for solving MPECs have been proposed in the literature. Among these, the method presented in [3] deserves to be mentioned because of its simplicity and its wide use in the literature on MPEC. This approach is based on the so-called *big M* reformulation of the complementarity conditions (B.8h) employing binary variables. In practice, we can replace the conditions:

$$\mu_i g_i^{\mathrm{L}}(x, y) = 0, \qquad \forall i, \tag{B.9}$$

with the following ones:

$$g_i^{\mathrm{L}}(x, y) \geq -z_i M_{1i}, \qquad \forall i, \tag{B.10a}$$

$$\mu_i \leq (1 - z_i) M_{2i}, \qquad \forall i, \tag{B.10b}$$

$$z_i \in \{0, 1\}, \qquad \forall i. \tag{B.10c}$$

The use of binary variable z_i forces one of the right-hand sides of (B.10a) and (B.10b) to be equal to 0. In combination with (B.8f) and (B.8g), this implies that $g_i^{\mathrm{L}}(x, y)$ and/or μ_i must be equal to 0, as required by (B.8h).

For reformulation (B.10) to be valid within a bilevel problem, the constants M_{1i} and M_{2i} must be large enough so as not to leave the optimal solution out of the feasible space of (B.10). In practice, the choice of the *big M* constants is a rather challenging issue, as too large values for the constants result in computational inefficiencies in the solution of the resulting mixed-integer optimization problems.

As a final remark, it should be pointed out that if the feasible space of the lower-level problem(s) is defined by affine equality and inequality constraints, and if the partial derivatives of its objective function with respect to the lower-level decision variables are also affine, reformulation (B.10) of the complementarity conditions results in a mixed-integer linear program (MILP). Problems of this type can be efficiently tackled by specialized software.

We refer the interested reader to [4] and [6] for a more detailed presentation of the MPEC framework and of alternative solution techniques.

References

1. Bazaraa, M.S., Sherali, H.D., Shetty, C.M.: Nonlinear Programming: Theory and Algorithms, 3rd edn. Wiley, Hoboken, NJ (2013)

2. Castillo, E., Conejo, A.J., Pedregal, P., Garcia, R., Alguacil, N.: Building and Solving Mathematical Programming Models in Engineering and Science. Pure and Applied Mathematics. Wiley, New York (2001)
3. Fortuny-Amat, J., McCarl, B.: A representation and economic interpretation of a two-level programming problem. J. Oper. Res. Soc. **32**(9), 783–792 (1981)
4. Gabriel, S.A., Conejo, A.J., Fuller, J.D., Hobbs, B.F., Ruiz, C.: Complementarity Modeling in Energy Markets. International Series in Operations Research & Management Science. Springer, New York (2013)
5. Luenberger, D., Ye, Y.: Linear and Nonlinear Programming, 3rd edn. International Series in Operations Research & Management Science. Springer, New York (2008)
6. Luo, Z.Q., Pang, J.S., Ralph, D.: Mathematical Programs with Equilibrium Constraints. Cambridge University Press, Cambridge, UK (1996)

Appendix C
Introduction to Stochastic Programming

Throughout this book, stochastic programming is, on repeated occasions, used to build decision-making models for the efficient management of renewable energy sources in electricity markets. The main objective of this appendix is threefold: To acquaint the reader with the basic concepts and terms that compose the working language of stochastic programming, to outline the family of decision-making problems that stochastic programming is particularly suitable for, and to provide straightforward measures to identify instances where the use of stochastic programming qualifies to be advantageous. This appendix is meant to be comprehensive enough for the reader to get a handle on the various stochastic programming models described in this book, but is far from covering all the many and singular aspects of stochastic programming. The reader intrigued by the fascinating and constantly evolving world of stochastic programming is encouraged to visit the *Stochastic Programming Community Home Page* (http://www.stoprog.org/).

C.1 Decision Making Under Uncertainty

Most decision-making problems are subject to uncertainty, due to the inherent randomness of natural phenomena conditioning our choices (e.g., the weather) or, more generally, to the inaccurate knowledge of input information. Decision makers are, therefore, eager for methods and tools that lead them to solutions less sensitive to environmental influences or imprecise data, while simultaneously reducing cost, increasing profit, or improving reliability.

Many decision-making processes can be posed as optimization problems, where a set of control variables (the decisions) is to be determined to optimize a certain objective, say minimize cost or maximize profit. The range of possible values the control variables can take on is typically limited by economic, technical, physical, or environmental considerations.

Without loss of generality, let us consider the following linear optimization problem:

$$\underset{x}{\text{Max.}} \quad c^{\top} x \tag{C.1}$$

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$$\text{s.t.} \quad Ax \leq b, \tag{C.2}$$

$$x \geq 0, \tag{C.3}$$

where x is the decision variable vector, and matrix A and vectors b and c are input data. If parameters A, b, and c are *perfectly* known, solution algorithms for linear optimization problems (e.g., the famous simplex method) can be used to find the *best* value of the decision variable vector x. This value is then implemented in practice and the objective $c^\top x$ is consequently maximized.

Let us suppose now that some of these input data are contingent on the realization λ_ω of a certain random vector λ. Mathematically, one can write $A_\omega = A(\lambda_\omega)$, $b_\omega = b(\lambda_\omega)$, and $c_\omega = c(\lambda_\omega)$. If the decision-making process is such that the decision variable vector x needs to be determined before the realization λ_ω of the random parameter vector λ, determining the optimal solution to problem (C.1)–(C.3) becomes more intricate in the following three aspects:

Feasibility. How can the decision maker be sure that constraint (C.2) will be satisfied now that matrix A and vector b are not completely known in advance? Clearly, the issue of how to guarantee the feasibility of decision vector x becomes remarkably more involved when optimizing under uncertainty. As a matter of fact, the way feasibility is ensured in decision making under uncertainty is directly linked to the considered modeling approach. For example, while stochastic programming looks at solutions that are feasible for all (or almost all) plausible realizations of random vector λ, robust optimization leads to decisions that are feasible just for those realizations λ_ω within a prespecified set.

Optimality. As cost vector c is assumed to be uncertain, objective function (C.1) is not a real-valued function anymore, but a family of random variables $f(x, \omega) = c_\omega^\top x$. That is, for each feasible decision vector x, $f(x, \omega)$ can be regarded as the value of random variable $f(x, \cdot)$ at the argument ω. Therefore, optimizing under uncertainty translates into finding the decision vector x that yields the *best* random variable $f(x, \omega)$. For this purpose, we need to specify a criterion to discriminate the goodness of each member of the family. For instance, in stochastic programming, it is a common practice to rank random variables $f(x, \omega)$ according to their expectations and pick the biggest (in a maximization problem). In robust optimization, on the contrary, random variables $f(x, \omega)$ are ranked by their worst possible outcome.

Solution algorithm. Optimization problem (C.1)–(C.3) need to be recast so that solution algorithms for linear programming problems (e.g., the simplex method) can be used to obtain the optimal value of decision vector x in the terms previously discussed. If, for example, the random parameter vector λ affecting problem (C.1)–(C.3) is continuous, a discrete approximation of this random vector is usually required in stochastic programming to make such a problem solvable. Random vector λ is then modeled as a set Ω of plausible outcomes or scenarios ω, where each $\omega \in \Omega$ has an associated probability of occurrence π_ω such that $\sum_{\omega \in \Omega} \pi_\omega = 1$.

Stochastic programming provides us with the concepts and tools required to deal with the implications of having uncertain data in an optimization problem for decision making. In the next section, we focus on the family of optimization problems with recourse decisions, which encompass the decision-making problems dealt with in this book and for which stochastic programming is particularly suitable.

C.2 Stochastic Programming Problems with Recourse

The balancing market-clearing problem introduced in Chap. 4 or the trading problem of a virtual power plant (VPP) described in Chap. 8 are both *recourse problems*, in which some decisions, e.g., the deployment of reserves in the former case or the real-time operation of the VPP components in the latter, can be made after uncertainty is revealed [2]. These decisions are referred to as *recourse actions*.

Stochastic programming problems with recourse are classified by their number of *stages*. Each stage represents a point in time where decisions are made. The number of stages of a recourse problem is dependent on how decisions are sequentially made in relation to how the uncertain input information is disclosed over time. In particular, all decisions that are made based on identical information on uncertain parameters are grouped in the same stage [5].

The simplest recourse problem is the two-stage stochastic programming problem, in which decisions are divided into two groups, namely:

- Decisions that have to be made before the realization of uncertain parameters. These decisions are known as *first-stage* or *here-and-now* decisions and do not depend on the realization of the random parameters.
- Decisions that are made after the actual values of uncertain parameters are disclosed. These decisions are called *second-stage*, *wait-and-see*, or *recourse* decisions and are dependent on each plausible value of the random parameters. The term *recourse* points to the fact that second-stage decisions enable the decision maker to adapt to the actual outcomes of the random events.

Schematically, the sequence of decisions and events in a two-stage stochastic programming problem can be expressed as follows:

1. First-stage decisions, x, are made.
2. The actual outcome λ_ω of the random parameter vector λ is realized.
3. Second-stage decisions, $y(x, \omega)$, are made accordingly.

Note that we intentionally write $y(x, \omega)$ to stress that wait-and-see decisions differ as functions of the outcome of the random event and of the first-stage decisions.

If this decision sequence is repeated more than once, then we have a multistage stochastic programming problem (with a number of stages greater than two). The sequence of decisions and events in a multistage decision-making process with s stages can be outlined as follows:

1. Decisions x^1 are made.
2. Uncertain parameter vector λ^1 is realized as $\lambda^1_{\omega^1}$.
3. Decisions $x^2(x^1, \omega^1)$ are made.
4. Uncertain parameter vector λ^2 is realized as $\lambda^2_{\omega^2}$.

5. Decisions $x^3(x^1, \omega^1, x^2, \omega^2)$ are made.

   ...

2s-2. Uncertain parameter vector λ^{s-1} is realized as $\lambda^{s-1}_{\omega^{s-1}}$.
2s-1. Decisions $x^s(x^1, \omega^1, \ldots, x^{s-1}, \omega^{s-1})$ are made.

In all stochastic programming problems, and especially in the multistage ones, it is necessary to enforce the *nonanticipatory* character of information [2]. That is, if the outcomes of the random events are identical up to stage k, then the values taken by decision variables must be identical up to that stage as well.

The *nonanticipativity* of decisions is implicitly accounted for in the previous decision sequence. Specifically, observe that decisions x^1 are independent of the future realization of uncertain parameters $\{\lambda^1, \ldots, \lambda^{s-1}\}$. In the next stage, however, decisions x^2 are contingent on the outcome of random vector λ^1, but they are unique for all random parameters that are realized in the future, namely, $\{\lambda^2, \ldots, \lambda^{s-1}\}$. Analogous considerations hold for the rest of stages.

Mathematically, a two-stage stochastic LP problem can be formulated as

$$\begin{aligned} \underset{x}{\text{Max.}} \;\; & c^\top x + \mathrm{E}_\omega\{Q(x,\omega)\} \\ \text{s.t.} \;\; & Ax \le b, \\ & x \ge 0, \end{aligned} \tag{C.4}$$

being

$$Q(x,\omega) = \left\{ \begin{aligned} \underset{y_\omega}{\text{Max.}} \;\; & q_\omega^\top y_\omega \\ \text{s.t.} \;\; & W_\omega y_\omega \le h_\omega - T_\omega x, \\ & y_\omega \ge 0 \end{aligned} \right\}, \quad \forall \omega \in \Omega, \tag{C.5}$$

where x and y_ω are the first-stage and second-stage decision variable vector, respectively, c, q_ω, b, h_ω, A, T_ω, and W_ω are known vectors and matrices of appropriate size, and E_ω is the expectation operator over the outcome index ω. Note that, in problem (C.4)–(C.5), the mathematical expectation is used as the criterion for ranking the objective function random variables $\{f(x,\omega) = c^\top x + Q(x,\omega), \forall x : \; Ax \le 0, \; x \ge 0\}$. Moreover, observe that the uncertainty involved in problem (C.4)–(C.5) is assumed to be properly represented by means of a finite set Ω of scenarios, realizations, or outcomes ω.

Subproblem (C.5), wherein decisions y_ω are made after uncertainty is disclosed, is named *recourse problem*.

Under rather general assumptions [2], the two-stage stochastic programming problem (C.4)–(C.5) can be equivalently expressed as

$$\begin{aligned} \underset{x, y_\omega}{\text{Max.}} \;\; & c^\top x + \sum_{\omega \in \Omega} \pi_\omega q_\omega^\top y_\omega \\ \text{s.t.} \;\; & Ax \le b, \end{aligned} \tag{C.6}$$

$$
\begin{aligned}
W_\omega y_\omega &\leq h_\omega - T_\omega x, \quad \forall \omega \in \Omega, \\
x &\geq 0, \\
y_\omega &\geq 0, \quad \forall \omega \in \Omega,
\end{aligned}
$$

where π_ω is the probability of occurrence of scenario ω.

Formulation (C.6) is known as the *deterministic equivalent problem* of the stochastic programming problem (C.4)–(C.5) and can be directly processed by off-the-shelf optimization software for linear programs.

Last, both the stochastic problem (C.4)–(C.5) and its associated deterministic equivalent problem (C.6) can be easily extended to include a number of stages greater than two [2, 3].

C.3 Estimating the Value of Information and Stochastic Programming

When dealing with decision-making problems subject to uncertain input data, it is always a good practice to answer the following two questions:

- How much do we gain by improving the forecasting techniques to better foretell the future?
- Why to use a stochastic approach to tackle the decision-making problem instead of a deterministic one? Deterministic problems are usually fabricated out of stochastic models by replacing the uncertain parameters by their expected values. Therefore, deterministic problems are simpler and easier to solve than stochastic ones.

Two metrics are usually computed to respond to these questions, namely, the expected value of perfect information (EVPI) and the Value of Stochastic Solution (VSS), respectively. These measures, which are widely used in stochastic programming [2], are explained below.

C.3.1 The Expected Value of Perfect Information

The expected value of perfect information is an index that quantifies how much a decision maker is *willing to pay* for obtaining perfect information about the future.

The EVPI is computed as follows:

Step 1: Stochastic programming problem (C.6) is solved, yielding an optimal objective function value equal to z^{s*}.

Step 2: The problem that results from (C.6) if the nonanticipativity of decisions is relaxed, i.e., if all variables are defined as scenario-dependent, is solved.

This problem, usually referred to as the *wait-and-see* problem, can be stated as

$$\begin{aligned} \underset{x_\omega, y_\omega}{\text{Max.}} \quad & z^{\text{ws}} = \sum_{\omega \in \Omega} \pi_\omega \left(c^\top x_\omega + q_\omega^\top y_\omega \right) \\ \text{s.t.} \quad & A x_\omega \le b, \quad \forall \omega \in \Omega, \\ & W_\omega y_\omega \le h_\omega - T_\omega x_\omega, \quad \forall \omega \in \Omega, \\ & x_\omega \ge 0, \quad \forall \omega \in \Omega, \\ & y_\omega \ge 0, \quad \forall \omega \in \Omega. \end{aligned} \tag{C.7}$$

It is said that decisions in problem (C.7) are made with perfect information. This problem can be decomposed by scenario, which makes it easier to solve.

Step 3: Let us denote by $z^{\text{ws}*}$ the optimal objective function value of problem (C.7), where "ws" stands for "wait-and-see". The EVPI is given by

$$\text{EVPI} = z^{\text{ws}*} - z^{\text{s}*}. \tag{C.8}$$

C.3.2 The Value of Stochastic Solution

The VSS quantifies the economic advantage of using a stochastic programming approach over a deterministic one to cope with decision-making problems under uncertainty. The VSS for a two-stage stochastic programming problem is calculated in four steps:

Step 1: Stochastic programming problem (C.6) is solved, yielding an optimal objective function value denoted by $z^{\text{s}*}$. Superscript s stands for "stochastic".

Step 2: The deterministic problem that results from replacing the uncertain parameters in (C.6) by their expected values is solved. To be more precise, this problem can be formulated as follows:

$$\begin{aligned} \underset{x^{\text{d}}, \widehat{y}}{\text{Max.}} \quad & c^\top x^{\text{d}} + \widehat{q}^\top \widehat{y} \\ \text{s.t.} \quad & A x^{\text{d}} \le b, \\ & \widehat{W} \widehat{y} \le \widehat{h} - \widehat{T} x^{\text{d}}, \\ & x^{\text{d}} \ge 0, \\ & \widehat{y} \ge 0, \end{aligned} \tag{C.9}$$

where $\widehat{q} = \text{E}_\omega \{q_\omega\}$, $\widehat{W} = \text{E}_\omega \{W_\omega\}$, $\widehat{h} = \text{E}_\omega \{h_\omega\}$, and $\widehat{T} = \text{E}_\omega \{T_\omega\}$. Let us represent the optimal decision vector of problem (C.9) as $x^{\text{d}*}$, where superscript d stands for "deterministic".

Step 3: The original stochastic programming problem (C.6) is then solved fixing the value of the first-stage variable vector (x) to that obtained in the previous step (x^{d*}). The resulting problem can be formulated as

$$\begin{aligned} \underset{y_\omega}{\text{Max.}} \quad & z^{\mathrm{d}} = c^\top x^{\mathrm{d}*} + \sum_{\omega\in\Omega} \pi_\omega q_\omega^\top y_\omega \\ \text{s.t.} \quad & W_\omega y_\omega \le h_\omega - T_\omega x^{\mathrm{d}*}, \quad \forall \omega \in \Omega, \\ & y_\omega \ge 0, \quad \forall \omega \in \Omega, \end{aligned} \tag{C.10}$$

which provides an optimal objective function value denoted by z^{d*}. Note that problem (C.10) decomposes by scenario and as such, is generally easy to solve.

Step 4: Finally, the Value of the Stochastic Solution is computed as

$$\mathrm{VSS} = z^{\mathrm{s}*} - z^{\mathrm{d}*}. \tag{C.11}$$

In reference [4], some criteria to properly extend the application of the VSS to multi-stage stochastic programming problems are provided.

Furthermore, even though the Value of the Stochastic Solution is traditionally defined with respect to x^{d*}, i.e., the decision vector obtained from the expected-value problem (C.9), the same idea can be applied to compare the stochastic solution with any other decision. For example, suppose that x^{r*} is the decision resulting from solving the optimization problem under uncertainty using a robust optimization approach, see Appendix D. We then solve

$$\begin{aligned} \underset{y_\omega}{\text{Max.}} \quad & z^{\mathrm{r}} = c^\top x^{\mathrm{r}*} + \sum_{\omega\in\Omega} \pi_\omega q_\omega^\top y_\omega \\ \text{s.t.} \quad & W_\omega y_\omega \le h_\omega - T_\omega x^{\mathrm{r}*}, \quad \forall \omega \in \Omega, \\ & y_\omega \ge 0, \quad \forall \omega \in \Omega, \end{aligned} \tag{C.12}$$

which yields the objective function value z^{r*}. We can now compute the VSS *with respect to the robust solution*, VSS^r, as

$$\mathrm{VSS}^r = z^{\mathrm{s}*} - z^{\mathrm{r}*}. \tag{C.13}$$

Notice that, in Chap. 8 of this book, we use an analysis of this kind to evaluate and compare the different trading strategies of a VPP in an electricity market.

C.4 Risk Management

Defining

$$f(x,\omega) = c^\top x + \max_{y_\omega} \left\{ q_\omega^\top y_\omega : W_\omega y_\omega \le h_\omega - T_\omega x, y_\omega \ge 0 \right\}, \tag{C.14}$$

problem (C.6) can be equivalently expressed in compact formulation as

$$\begin{aligned} \underset{x}{\text{Max.}} \quad & \mathrm{E}_{\omega}\left\{f(x, \omega)\right\} \\ \text{s.t.} \quad & Ax \leq \mathrm{b}, \\ & x \geq 0. \end{aligned} \tag{C.15}$$

The objective of problem (C.15) is to maximize the expected value of function $f(x, \omega)$, which may correspond, for instance, to the profit achieved by a wind power producer participating in an electricity market, as in Sect. 3. Notwithstanding the fact that ranking a random variable by its expected value is advantageous in many aspects, its main drawback is that the remaining characteristics of the distribution associated with the random variable are disregarded, particularly those providing information about how *dispersed* its possible outcomes are. For example, a random variable representing a profit distribution with an expected value that is acceptable to the decision maker could also exhibit a nonnegligible probability of experiencing negative profits (losses).

The notion of *risk* stems from the possibility of obtaining a decision vector x leading to a profit distribution with undesirable properties, e.g., a high probability of low profit. In order to control the variability of profit $f(x, \omega)$, a *risk measure* is included in the formulation of the decision-making problem. This risk measure is a function $r_{\omega}\left\{f(x, \omega)\right\}$ that assigns to a given random variable representing profit $\left\{f(x, \omega), \forall \omega \in \Omega\right\}$ a real number characterizing the risk associated with that profit. The risk function $r_{\omega}\left\{f(x, \omega)\right\}$ can be incorporated into problem (C.15) as follows:

$$\begin{aligned} \underset{x}{\text{Max.}} \quad & (1-k)\mathrm{E}_{\omega}\left\{f(x, \omega)\right\} - k r_{\omega}\left\{f(x, \omega)\right\} \\ \text{s.t.} \quad & Ax \leq \mathrm{b}, \\ & x \geq 0, \end{aligned} \tag{C.16}$$

where parameter $k \in [0, 1]$ is a weighting factor used to resolve the trade-off between expected profit and risk aversion. If $k = 0$, the risk term in the objective function of (C.16) vanishes and the resulting problem boils down to (C.15), which is known as the *risk-neutral problem*. As k approaches 1, the expected profit becomes less important in relation to the risk term and consequently, the decision maker becomes more risk averse.

The optimal solution obtained from problem (C.16) can be expressed as a function of parameter k, and thus represented in the so-called *efficient frontier* [6]. The efficient frontier is a curve made up of pairs (expected profit, risk) in such a way that it is impossible to find a set of decision variables yielding greater expected profit and less risk than those of any point in the curve simultaneously. This way, a solution with greater expected profit than that of an efficient point can only be obtained at the cost of experiencing a higher risk, and viceversa. Figure C.1 illustrates an example of efficient frontier for problem (C.16). Note that small values of k provide solutions with high expected profit and also high risk. In contrast, values of k close to 1

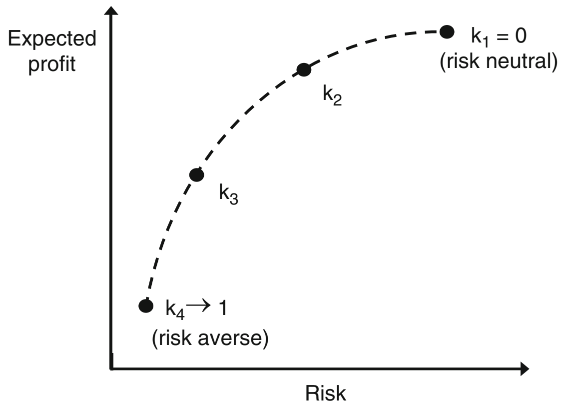

Fig. C.1 Example of efficient frontier

yield solutions with smaller expected profit and smaller risk. Efficient frontiers are therefore useful for decision makers to face the trade-off between expected profit and risk.

Several risk measures have been analyzed in the technical literature, namely, the variance, the shortfall probability, the Value-at-Risk (VaR), and the Conditional Value-at-Risk (CVaR) [1, 6–10]. Among all of them, the CVaR is deemed superior because:

1. It can be included in problem (C.15) using a linear formulation.
2. It is able to quantify "fat tails" in probability distributions.
3. It is a coherent risk measure, i.e., it exhibits good properties such as translation invariance, subadditivity, positive homogeneity, and monotonicity [1–8].

For a given $\alpha \in [0, 1)$, the Conditional Value-at-Risk , $\mathrm{CVaR}_{1-\alpha}$, is defined as the expected value of the profit smaller than the $(1-\alpha)$-quantile of the profit distribution. This quantile is usually referred to as the Value-at-Risk at confidence level α, $\mathrm{VaR}_{1-\alpha}$, and can be mathematically expressed as

$$\mathrm{VaR}_{1-\alpha}(x) = \max\left\{\zeta : P(\omega | f(x,\omega) < \zeta) \leq 1-\alpha\right\}, \ \forall \alpha \in [0, 1), \tag{C.17}$$

where P stands for "probability".

In turn, the $\mathrm{CVaR}_{1-\alpha}(x)$ for a discrete distribution is defined as [10],

$$\mathrm{CVaR}_{1-\alpha}(x) = \max_{\zeta}\left\{\zeta - \frac{1}{1-\alpha}\mathrm{E}_{\omega}\left\{\max\{\zeta - f(x,\omega), 0\}\right\}\right\}, \forall \alpha \in [0, 1). \tag{C.18}$$

If all profit scenarios are equiprobable, $\mathrm{CVaR}_{1-\alpha}(x)$ is computed as the expected profit of the $(1-\alpha) \times 100\%$ worst scenarios. For example, $\mathrm{CVaR}_{0.05}$ represents the expected value of the profit associated with the 5% of the scenarios with the worst outcomes.

The CVaR can be embedded into problem (C.15) as follows:

$$\begin{aligned} \underset{x}{\text{Max.}} \quad & (1-k)\mathrm{E}_\omega\{f(x,\omega)\} + k\ \mathrm{CVaR}_{1-\alpha}(x) \\ \text{s.t.} \quad & \mathrm{CVaR}_{1-\alpha}(\mathrm{x}) = \zeta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta_\omega, \\ & \zeta - f(x,\omega) \le \eta_\omega, \quad \forall\omega\in\Omega, \\ & \eta_\omega \ge 0, \quad \forall\omega\in\Omega, \\ & Ax \le b, \\ & x \ge 0. \end{aligned} \tag{C.19}$$

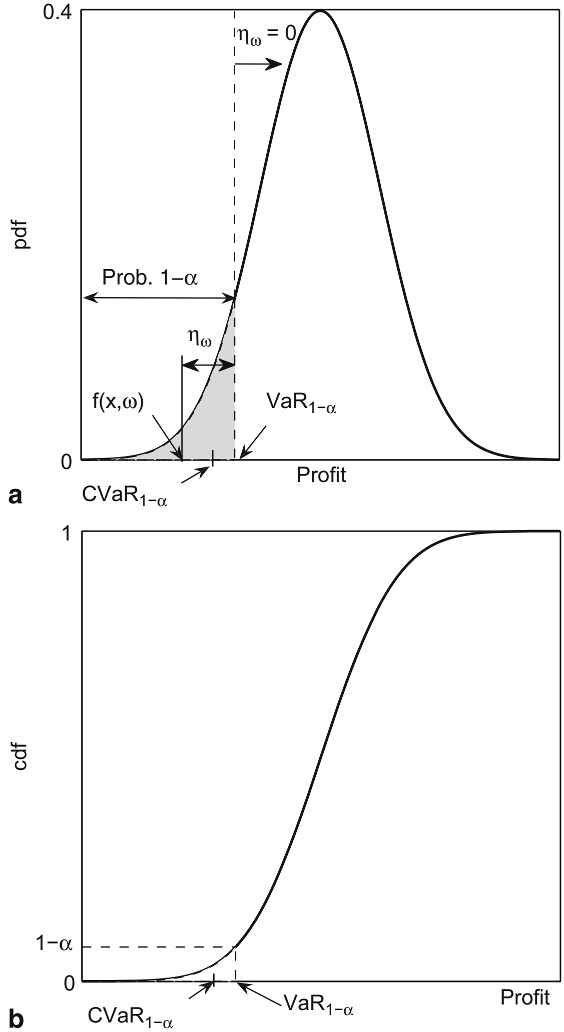

Fig. C.2 Illustration of the concepts of Value-at-Risk and Conditional Value-at-Risk. **a** PDF. **b** CDF

Notice that the CVaR is included in the objective function of problem (C.19) with a positive sign, unlike the general risk measure introduced in (C.16), because here minimizing risk implies maximizing the expected profit of the worst scenarios. It should also be mentioned that, at the optimum, variable ζ in problem (C.19) coincides with $\mathrm{VaR}_{1-\alpha}$, i.e., $\zeta^* = \mathrm{VaR}_{1-\alpha}(x^*)$.

Last, Fig. C.2 illustrates the concepts of VaR and CVaR on both the probability density function (pdf) and the cumulative distribution function (cdf) of a certain continuous random variable representing profit.

References

1. Artzner, P., Delbaen, F., Eber, J.M., Heath, D.: Coherent measures of risk. Math. Financ. **9**(3), 203–228 (1999)
2. Birge, J.R., Louveaux, F.: Introduction to Stochastic Programming, 2nd edn. Springer, New York (2011)
3. Conejo, A.J., Carrión, M., Morales, J.: Decision Making Under Uncertainty in Electricity Markets. International Series in Operations Research & Management Science. Springer Verlag, New York (2010)
4. Escudero, L.F., Garín, A., Merino, M., Pérez, G.: The value of the stochastic solution in multistage problems. Top **15**(1), 48–64 (2007)
5. Gassmann, H., Prékopa, A.: On stages and consistency checks in stochastic programming. Oper. Res. Lett. **33**(2), 171–175 (2005)
6. Markowitz, H.M.: Portfolio Selection: Efficient Diversification of Investments. Wiley, New York (1959)
7. Pflug, G.: Some remarks on the value-at-risk and the conditional value-at-risk. In: S. Uryasev (ed.) Probabilistic Constrained Optimization: Methodology and Applications. Kluwer Academic Publishers, Norwell, MA (2000)
8. Rockafellar, R.T.: Coherent approaches to risk in optimization under uncertainty. In: Tutorials in Operations Research. OR Tools and Applications: Glimpses of Future Technologies. INFORMS (2007)
9. Rockafellar, R.T., Uryasev, S.: Optimization of conditional value-at-risk. J. Risk **2**(3), 21–41 (2000)
10. Rockafellar, R.T., Uryasev, S.: Conditional value-at-risk for general loss distributions. J. Bank Financ. **26**(7), 1443–1471 (2002)

Appendix D
Introduction to Robust Optimization

This appendix provides a brief, conceptual, and rather terse introduction to robust optimization. It considers both robust optimization problems with and without recourse. It explains the rationale underlying robust optimization models, but it does not describe specific instances of robust optimization problems.

D.1 Introduction

Robust optimization addresses optimization problems with uncertain parameters that are not described using probability distributions but *uncertainty sets* [1]. An uncertainty set is used to characterize the possible outcomes of the uncertain parameters and, as its name indicates, it has a set structure. A robust optimization problem seeks to determine a solution to an optimization problem that is feasible for any realization of the uncertain parameters within the uncertainty set, and optimal for the worst-case realization of these uncertain parameters.

A key element to formulate a robust optimization problem is the adequate definition of the uncertainty set, which should describe in detail the uncertain phenomena (physical or otherwise) represented by the uncertain parameters. Naive definitions of the uncertainty set may lead to either highly *conservative* or highly *risky* solutions. However, complex uncertainty sets may damage the tractability that characterizes robust optimization problems with polyhedral or elliptical uncertainty sets.

A general overview of robust optimization is available in the tutorial reference [2].

D.2 Why Robust Optimization?

Uncertain parameters within an optimization problem have been traditionally characterized using distribution functions.

In turn, these distribution functions are used to generate scenarios describing most plausible realizations of the uncertain parameters.

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Then, these scenarios allow formulating a stochastic programming problem that is used for making informed decisions under uncertainty.

The earlier approach exhibits two drawbacks:

1. Some uncertain parameters are difficult to characterize using distribution functions; for example, the offering behavior of rival producers.
2. Generally, the number of scenarios needed to describe the most plausible outcomes of the uncertain parameters is generally very large, leading to large-scale optimization problems that may become difficult to solve or intractable.

At the cost of reduced flexibility, robust optimization provides an alternative and compact manner to describe uncertain parameters. Such approach results in models that are generally more computationally efficient than their stochastic programming counterparts, while preserving sufficient modeling flexibility.

D.3 Robust Optimization Without Recourse

A robust optimization problem without recourse has the general form [7],

$$\begin{aligned} &\min_x \max_u \{f(x,u),\ u \in \mathcal{U}\} \\ &\text{s.t. } g_i(x,u) \le 0, \qquad \forall i, \forall u \in \mathcal{U}, \\ &\quad x \in \mathcal{X}. \end{aligned} \tag{D.1}$$

Optimization variable vector $x \in \mathcal{X}$, which may include both continuous and discrete variables, represents decisions to be made prior to the realization of the uncertain parameter vector u, which realizes within the uncertainty set $\mathcal{U}$. Note that problem (D.1) is forced to be feasible for all realizations of the uncertain parameter u. Since no decision (reaction) is possible once the uncertain parameters realize, we say that robust optimization problem (D.1) has no recourse.

Real-valued function f represents the objective to be minimized (or maximized), while functions $g_i, \forall i$ represent the constraints of the optimization problem.

The objective of problem (D.1) is to determine the decision vector x that ensures feasibility for all realizations of the parameters in vector u, and optimizes the objective function in the worst-case realization of these parameters.

Problem (D.1) can be recast as

$$\begin{aligned} &\min_{x,z} z \\ &\text{s.t. } \max_u \{f(x,u),\ u \in \mathcal{U}\} \le z, \\ &\quad \max_u \{g_i(x,u),\ u \in \mathcal{U}\} \le 0, \qquad \forall i, \\ &\quad x \in \mathcal{X}. \end{aligned} \tag{D.2}$$

Problem (D.2) is equivalent to problem (D.1) due to the combined effect of the following three actions:

1. Unconstrained real-valued variable z (a proxy for the objective function) is minimized subject to:
2. The maximization over u of the objective function f that must remain below z, and
3. Satisfying all constraints $g_i, \forall i$, and for all values of the uncertain parameter vector u.

In other words, the above problem selects values for the components of vector $u \in \mathcal{U}$ that push the objective function f and the constraint functions $g_i, \forall i$ to a maximum, and simultaneously picks up values for the components of the decision vector x that push the objective function f to a minimum and ensure feasibility.

How to solve problem (D.2)?: If the lower-level problems, the u-problems, are convex, they can be generally replaced by their dual problems and merged with the upper-level problem, the x-problem, transforming min-max bilevel problem (D.1) into a single-level optimization problem. This is the most common manner to solve problem (D.1), and the one used in [1]. Example D.1 illustrates this solution technique.

Example D.1 (Robust optimization example without recourse) We consider the following minimization problem in the variables x_1 and x_2

$$\min_{x_1,x_2} \max_{u_1,u_2} \left\{ \frac{2-u_1}{4} x_1 + (2-u_2)\, x_2,\ (u_1,u_2) \in \mathcal{U} \right\} \tag{D.3a}$$

$$\text{s.t. } u_1 x_1 + 2u_2 x_2 \geq 3, \qquad \forall (u_1,u_2) \in \mathcal{U}, \tag{D.3b}$$

$$x_1, x_2 \geq 0, \tag{D.3c}$$

in which the coefficients u_1 and u_2 appearing in the objective function and in the inequality constraint are uncertain and can take values in the following uncertainty set

$$\mathcal{U} = \{(u_1,u_2) \mid u_1,u_2 \geq 0,\ u_1 + u_2 \geq 2\}\,. \tag{D.4}$$

The optimal solution to problem (D.3) must be feasible for any realization of (u_1,u_2) within the set $\mathcal{U}$, as required by (D.3b). Moreover, the *max* operator in (D.3a) enforces that the solution be optimal in the worst-case realization of the uncertain parameters.

Problem (D.3) can be recast in a formulation similar to the one in (D.2), resulting in a minimization problem constrained by two maximization problems. Note that in order to get the following formulation, we must bring the terms not depending on u_1 and u_2 out of the *max* operator in (D.3a) and change the sign of inequality (D.3b):

$$\min_{x_1,x_2,z} z \tag{D.5a}$$

$$\text{s.t. } \frac{x_1}{2} + 2x_2 + \max_{u_1,u_2} \left\{ -\frac{1}{4} u_1 x_1 - u_2 x_2,\ (u_1,u_2) \in \mathcal{U} \right\} \leq z, \tag{D.5b}$$

$$\max_{u_1,u_2} \{-u_1x_1 - 2u_2x_2,\ (u_1,u_2) \in \mathcal{U}\} \leq -3, \tag{D.5c}$$

$$x_1, x_2 \geq 0. \tag{D.5d}$$

Let us begin by considering (D.5b). This inequality implies that $z - x_1/2 - 2x_2$ is not lower than the objective function value of the following optimization problem

$$\max_{u_1,u_2} \ -\frac{x_1}{4}u_1 - x_2u_2 \tag{D.6a}$$

$$\text{s.t. } u_1 + u_2 \geq 2, \quad : \mu, \tag{D.6b}$$

$$u_1, u_2 \geq 0. \tag{D.6c}$$

Note that this problem is bounded as a result of (D.3c). The dual of (D.6) is the following minimization problem

$$\min_{\mu} 2\mu \tag{D.7a}$$

$$\text{s.t. } \mu \geq -\frac{x_1}{4}, \tag{D.7b}$$

$$\mu \geq -x_2, \tag{D.7c}$$

$$\mu \leq 0. \tag{D.7d}$$

Considering the weak and strong duality theorems [6], note that for any (u_1, u_2) and μ feasible for the primal and dual problems (D.6) and (D.7), respectively, it holds that $-x_1u_1/4 - x_2u_2 \leq 2\mu$, and that, at the optimum, the terms on both sides of the inequality are equal. This implies that we can replace (D.5b) in the optimization problem (D.3) with the following set of constraints

$$\frac{x_1}{2} + 2x_2 + 2\mu \leq z, \tag{D.8a}$$

$$\mu \geq -\frac{x_1}{4}, \tag{D.8b}$$

$$\mu \geq -x_2, \tag{D.8c}$$

$$\mu \leq 0. \tag{D.8d}$$

Let us now consider constraint (D.5c). According to this constraint, the optimal objective function value of the following maximization problem must be lower than or equal to -3

$$\max_{u_1,u_2} \ -u_1x_1 - 2u_2x_2 \tag{D.9a}$$

$$\text{s.t. } u_1 + u_2 \geq 2, \quad : \lambda, \tag{D.9b}$$

$$u_1, u_2 \geq 0. \tag{D.9c}$$

The dual of problem (D.9) is

$$\min_{\lambda} 2\lambda \tag{D.10a}$$

$$\text{s.t. } \lambda \geq -x_1, \tag{D.10b}$$

$$\lambda \geq -2x_2, \tag{D.10c}$$

$$\lambda \leq 0. \tag{D.10d}$$

By duality theory, for any feasible λ, u_1, and u_2, it holds that $-u_1x_1 - 2u_2x_2 \leq 2\lambda$ and, at the optimum, the terms on both sides of the inequality are equal. Therefore, we can replace Eq. (D.5c) with the following constraints

$$2\lambda \leq -3, \tag{D.11a}$$

$$\lambda \geq -x_1, \tag{D.11b}$$

$$\lambda \geq -2x_2, \tag{D.11c}$$

$$\lambda \leq 0. \tag{D.11d}$$

As a result of the observations above, we can reformulate problem (D.3) as follows:

$$\min_{x_1,x_2,z,\mu,\lambda} z \tag{D.12a}$$

$$\text{s.t. } \frac{x_1}{2} + 2x_2 + 2\mu \leq z, \tag{D.12b}$$

$$\mu \geq -\frac{x_1}{4}, \tag{D.12c}$$

$$\mu \geq -x_2, \tag{D.12d}$$

$$2\lambda \leq -3, \tag{D.12e}$$

$$\lambda \geq -x_1, \tag{D.12f}$$

$$\lambda \geq -2x_2, \tag{D.12g}$$

$$x_1, x_2 \geq 0, \lambda, \mu \leq 0. \tag{D.12h}$$

Solving the above linear program, it is easy to verify that the minimum worst-case cost is $z = 1.5$, which is attained at the following values of the decision variables

$$\frac{3}{2} \leq x_1 \leq 3, \tag{D.13}$$

$$x_2 = \frac{3}{4}. \tag{D.14}$$

We can now interpret the results looking back at the original problem formulation (D.3). In principle, either u_1 or u_2 can be zero, but not both. Since the realization of these parameters is not known when making the decision on x_1 and x_2, we must ensure that (D.3b) is feasible. The lower bound in (D.13) and (D.14) guarantee that this constraint is feasible regardless of whether $u_1 = 0$ (and $u_2 = 2$) or $u_2 = 0$ (and $u_1 = 2$).

Because the coefficient of x_1 in the objective function (D.3a) is divided by 4, it makes intuitive sense that the worst-case cost is obtained when $u_1 = 2, u_2 = 0$, since that way a larger cost on x_2 can be imposed. For this realization of the parameters, the optimal choice $x_2 = 3/4$ yields a cost equal to 1.5. Notice that $u_1 = 2, u_2 = 0$ is the worst-case realization of the parameters as long as $3/2 \leq x_1 \leq 3$. For $x_1 = 3$, the parameter values $u_1 = 0, u_2 = 2$ yield the same worst-case cost as the former parameter realization. Notice, however, that increasing x_1 beyond the value 3/2 implies no improvement in the worst-case cost. In fact, it would leave the objective function unchanged in the worst-case realization of the stochastic parameters, while worsening it for other realizations of the uncertainty.

Further details on robust optimization without recourse are provided in [2] and relevant examples pertaining to linear and mixed-integer linear programming problems are analyzed in [4].

D.4 Robust Optimization with Recourse

A robust optimization problem with recourse has the general form,

$$
\begin{aligned}
\min_{x}\ & \max_{u}\ \min_{y}\ f(x,u,y) \\
& \qquad\quad \text{s.t. } h^{\mathrm{R}}(x,u,y) = 0, \\
& \qquad\qquad\ \ g^{\mathrm{R}}(x,u,y) \leq 0, \\
& \qquad\qquad\ \ y \in \mathcal{Y}, \\
& \quad \text{s.t. } u \in \mathcal{U}, \\
\text{s.t. } & h^{\mathrm{P}}(x) = 0, \\
& g^{\mathrm{P}}(x) \leq 0, \\
& x \in \mathcal{X}.
\end{aligned}
\tag{D.15}
$$

Optimization variable vector $x \in \mathcal{X}$, which may include both continuous and discrete variables, represents decisions to be made prior to the realization of the uncertain parameter vector $u \in \mathcal{U}$. Optimization variable vector $y \in \mathcal{Y}$, which may also include both continuous and discrete variables, represents the recourse (reaction) decisions to be made once the uncertain parameter vector u realizes.

Real valued function f represents the objective to be minimized (or maximized), vector functions h^{P} and g^{P} represent the constraints at the *planning* level, i.e., prior to the realization of the uncertain parameter vector u, and vector functions h^{R} and g^{R} represent the constraints at the *recourse* level, i.e., after the realization of the uncertain parameter vector u.

The objective of problem (D.15) is to make the best decisions represented by variable vector x for the worst realization of parameters in vector u and considering the recourse (reaction) decisions described by variable vector y.

How to solve problem (D.15)?: If the right-hand-side problem, the y-problem, is convex, it can be replaced by its dual and merged with the middle u-problem rendering it a conventional single-level maximization problem. Overall, the resulting problem is a min-max problem that in some cases can be solved using decomposition [5]. This is the solution technique used in [3]. Example D.2 illustrates how to solve a problem of robust optimization with recourse.

Example D.2 (Robust optimization example with recourse). Let us consider the following example of a robust optimization problem with recourse.

$$
\begin{aligned}
\min_{x_1,x_2} \; & 30x_1 + 150x_2 + \max_{u_1,u_2} \min_{y_1,y_2} \; -u_1y_1 - u_2y_2 \\
& \qquad\qquad \text{s.t. } y_1 \le 100x_1, \\
& \qquad\qquad\quad\;\; y_2 \le 100x_2, \\
& \qquad\qquad\quad\;\; y_1 + y_2 = 100, \\
& \qquad\qquad\quad\;\; y_1, y_2 \ge 0, \\
& \qquad \text{s.t. } 2u_1 + u_2 \ge 16, \\
& \qquad\quad\;\; u_1 \ge 3, \\
& \qquad\quad\;\; u_2 \ge 5, \\
\text{s.t. } & x_1, x_2 \in \{0,1\}\,.
\end{aligned} \tag{D.16}
$$

In the above problem, x_1 and x_2 are first-stage binary variables. These variables can be interpreted as decisions on building certain facilities: if x_1 (or x_2) takes the value 1, it allows producing a quantity y_1 (or y_2) of a commodity up to maximum of 100. Overall, a demand of 100 must be satisfied for this commodity. Hence, in principle, building a single plant is sufficient. The recourse decisions on y_1 and y_2 are made after the inner maximization problem picks the worst-case realization of the uncertain parameters u_1 and u_2, which can be interpreted as the cost of production for the plants minus the price of the commodity. Therefore, the innermost problem aims at minimizing the minus-profit for operating the plants, i.e., at maximizing the profit.

To solve the problem, we employ a simple strategy based on the enumeration of all the possible combinations of the first-stage variables. The innermost *max-min* problem can be cast into a single-level maximization problem by noticing that the y-problem

$$\min_{y_1,y_2} \; -u_1y_1 - u_2y_2 \tag{D.17a}$$

$$\text{s.t. } y_1 \le 100x_1, \qquad : \mu_1, \tag{D.17b}$$

$$y_2 \le 100x_2, \qquad : \mu_2, \tag{D.17c}$$

$$y_1 + y_2 = 100, \qquad : \lambda, \tag{D.17d}$$

$$y_1, y_2 \ge 0, \tag{D.17e}$$

is equivalent to its dual

$$\max_{\mu_1,\mu_2,\lambda} \qquad 100x_1\mu_1 + 100x_2\mu_2 + 100\lambda \tag{D.18a}$$

$$\text{s.t.} \quad \mu_1 + \lambda \le -u_1, \tag{D.18b}$$

$$\mu_2 + \lambda \le -u_2, \tag{D.18c}$$

$$\mu_1, \mu_2 \le 0. \tag{D.18d}$$

Therefore, for any combination of x_1 and x_2, we are interested in solving the following problem

$$\max_{u_1,u_2,\mu_1,\mu_2,\lambda} \quad 100\,(x_1\mu_1 + x_2\mu_2 + \lambda) \tag{D.19a}$$

$$\text{s.t.} \quad \mu_1 + \lambda \le -u_1, \tag{D.19b}$$

$$\mu_2 + \lambda \le -u_2, \tag{D.19c}$$

$$\mu_1, \mu_2 \le 0, \tag{D.19d}$$

$$2u_1 + u_2 \ge 16, \tag{D.19e}$$

$$u_1 \ge 3, \tag{D.19f}$$

$$u_2 \ge 5. \tag{D.19g}$$

The following four cases can occur:

1. No plants are built, i.e., $x_1 = x_2 = 0$. This case is infeasible since $y_1 = y_2 = 0$, and therefore the demand is not satisfied.
2. Only plant 1 is built, i.e., $x_1 = 1$ and $x_2 = 0$, resulting in a construction cost of 30. Since μ_2 has no impact on the objective function, it can be made arbitrarily small, therefore (D.19c) can be made feasible regardless of the values of the other variables. On the contrary, we wish to make $\mu_1 + \lambda$ as large as possible, i.e., equal to $-u_1$. The highest value of $-u_1$ is obtained when $u_1 = 3$ and $u_2 \ge 10$. This delivers a profit of 300 for operating the facility. Looking at the complete problem formulation (D.16), it is clear that, if only plant 1 is built, the worst-case realization of the uncertainty occurs if u_1 is lowest. The total profit in the worst-case is therefore equal to $300 - 30 = 270$.
3. Only plant 2 is built, i.e., $x_1 = 0$ and $x_2 = 1$, the building cost is 150. This case is specular to the earlier one. Since the worst-case realization of the uncertainty occurs if $u_2 = 5$, the operating revenues are equal to 500. Hence, the net profit in this case is equal to 350.
4. If both plants are built, i.e., $x_1 = x_2 = 1$, the construction cost totals $30 + 150 = 180$. Solving (D.19) delivers $u_1 = u_2 = 5.33$. Notice that with this realization of the uncertainty, producing with plant 1 yields the same operating revenue as producing with plant 2, equal to 533. The net profit in this case is equal to $533 - 180 = 353$.

Hence, the last case is the optimal robust solution.

Further details on robust optimization with recourse are provided in [2] and a relevant electric energy example is analyzed in [3].

References

1. Bertsimas, D., Brown, D.B.: Constructing uncertainty sets for robust linear optimization. Oper. Res. **57**(6), 1483–1495 (2009)
2. Bertsimas, D., Brown, D.B., Caramanis, C.: Theory and applications of robust optimization. SIAM Rev. **53**(3), 464–501 (2011)
3. Bertsimas, D., Litvinov, E., Sun, X.A., Zhao, J., Zheng, T.: Adaptive robust optimization for the security constrained unit commitment problem. IEEE Trans. Power Syst. **28**(1), 52–63 (2013)
4. Bertsimas, D., Sim, M.: Robust discrete optimization and network flows. Math. Program. **98**(1–3), 49–71 (2003)
5. Conejo, A., Castillo, E., Minguez, R., Garcia-Bertrand, R.: Decomposition Techniques in Mathematical Programming: Engineering and Science Applications. Springer (2006)
6. Luenberger, D.G., Ye, Y.: Linear and Nonlinear Programming, 3rd edn. International Series in Operations Research & Management Science. Springer, New York (2008)
7. Soyster, A.: Convex programming with set-inclusive constraints and applications to inexact linear programming. Oper. Res. **21**(5), 1154–1157 (1973)

Appendix E
GAMS Codes

A number of decision-making models for market/system operators, managers of VPPs, and flexible consumers are introduced and discussed throughout this book. For the reader to become familiar with the implementation of these models, this appendix provides GAMS codes for some of the optimization problems in this book. Further information on GAMS can be found in [1, 2].

In particular, the examples that are included in this appendix are the following:

Section E.1: Example on co-optimization of energy and reserve (Sect. 3.2.1).
Section E.2: Example on estimating reserve requirements (Sect. 3.2.2).
Section E.3: Examples on the stochastic and robust energy-reserve dispatch models (Sects. 3.2.3 and 3.4).
Section E.4: Example on balancing auction with excess production and network congestion (Sect. 4.6).
Section E.5: Example on system inflexibility due to minimum up- and down-time constraints (Sect. 5.3.4).
Section E.6: Example on system inflexibility due to limited hydro energy availability (Sect. 5.3.5).
Section E.7: Example on the benefits of an adjustment market (Sect. 7.6).
Section E.8: Example on the VPP robust trading strategy (Sect. 8.4).
Section E.9: Example on the VPP stochastic trading strategy (Sect. 8.4).
Section E.10: Example on the VPP stochastic trading strategy based on offering curves (Sect. 8.4).
Section E.11: Example on demand-side management using stochastic programming (Sect. 9.4.1).
Section E.12: Example on demand-side management employing robust optimization (Sect. 9.4.2).

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E.1 GAMS Code for the Example on Co-optimization of Energy and Reserve in Sect. 3.2.1

```
$title CO-OPTIMIZATION OF ENERGY & RESERVE

Sets
i           'Power producers' /A, B/ ;

Scalar
DE          'System demand' /130/
DR          'Reserve requirement'/20/;

Parameters
CE (i)      'Marginal cost of energy production'
/
A          10
B          30/

CR(i)       'Reserve capacity offer costs'
/
A          0
B          25/

Pmax(i)    'Capacity limit'
/
A          100
B          100/;

Variables
TC         'Total system operation costs';

Positive variables
P(i)       'Energy dispatch'
R(i)       'Reserve capacity dispatch';

Equations
Obj      'Objective function'
PB       'Power balance equation'
RR       'Reserve requirement constraint'
CL(i)    'Capacity limits';

Obj..      TC =e= sum(i, CE(i) * P(i) + CR(i) * R(i));
PB..       sum(i, P(i)) =e= DE;
RR..       sum(i, R(i)) =e= DR;
CL(i)..    P(i) + R(i) =l= Pmax(i);

Model IE /all/;
Solve IE minimizing TC using lp;
```

E.2 GAMS Code for the Example on Estimating Reserve Requirements in Sect. 3.2.2

```
$title ESTIMATING RESERVE REQUIREMENTS

Sets
i           'Power producers'  /A, B/;

Scalar
DE          'System demand'  /130/
DI          'Demand imbalance'  /20/
Vlol        'Value of lost load'  /1000/
prob        'Probability of occurrence of demand imbalance'  /0.05/;

Parameters
CE(i)       'Marginal cost of energy production'
/
A          10
B          30/

CR(i)       'Reserve capacity offer costs'
/
A          0
B          25/

Pmax(i)     'Capacity limit'
/
A          100
B          100/;

Variables
TC          'Total system operation costs';

Positive variables
P(i)        'Energy dispatch'
R(i)        'Reserve capacity dispatch'
res(i)      'Deployed reserve'
Lshed       'Involuntarily curtailed load';

Lshed.up = DI;

Equations

Obj           'Objective function'
PBF           'Power balance equation at the forward stage'
PBR           'Power balance equation at the real-time stage'
lim_res(i)    'Limit of reserve deployment'
CL(i)         'Capacity limits';
```

```
Obj..              TC =e= sum(i, CE(i) * P(i) + CR(i) * R(i)) +
    prob * sum(i, CE(i) * res(i)) + prob * Vlol * Lshed;
PBF..              sum(i, P(i)) =e= DE;
PBR..              sum(i, res(i)) + Lshed =e= DI;
lim_res(i)..       res(i) =l= R(i);
CL(i)..            P(i) + R(i) =l= Pmax(i);

Model IE /all/;
Solve IE minimizing TC using lp;
```

E.3 GAMS Code for the Examples on the Stochastic and Robust Energy-Reserve Dispatch Models in Sects. 3.2.3 and 3.4

```
$title STOCHASTIC & ROBUST ENERGY-RESERVE DISPATCH: A COMPARISON

SETS
k        'Producers'                                 /k1,k2,k3/
n        'Nodes'                                     /n1,n2/
sc       'Complete set of scenarios'                 /s1*s100/
s(sc)    'Set of scenarios actually employed' ;

s(sc) = no;

ALIAS(m,n);

PARAMETERS
C(k)     'Production cost per unit'       /k1 10, k2 30, k3 35/
Cu(k)    'Upward reserve cost per unit'   /k1 16, k2 13, k3 10/
Cd(k)    'Downward reserve cost per unit' /k1 15, k2 12, k3 9/
Pmax(k)  'Maximum production'             /k1 50, k2 110, k3 100/
L(n)     'Load'                           /n1 40, n2 100/
pi(sc)   'Scenario probability'
w(n,sc)  'Wind power production scenario'
wfc(n)   'Conditional mean forecast for wind power production' ;

TABLE MG(k,n)    '1/0 matrix for nodal location of generators'
         n1      n2
k1        1       0
k2        1       0
k3        0       1;

TABLE MN(n,n)    '1/0 matrix for adjacent nodes'
         n1      n2
n1        0       1
n2        1       0;

TABLE X(n,n)     'Reactance of transmission line'
        n1      n2
n1      0       0.13
n2      0.13    0;

TABLE Tmax(n,n) 'Transmission capacity'
```

```
        n1      n2
n1       0       100
n2      100      0;

SCALARS vLOL  'Value of lost load'      /200/;

POSITIVE VARIABLES
P(k)          'Energy dispatch'
Ru(k)         'Upward reserve dispatch'
Rd(k)         'Downward reserve dispatch'
Pu(k,sc)      'Upward energy redispatch'
Pd(k,sc)      'Downward energy redispatch'
Wsc(n)        'Wind energy dispatch'
Lsh(n,sc)     'Load shedding'
Wsp(n,sc)     'Wind spillage' ;

VARIABLES
z             'Objective function'
del0(n)       'Forward stage voltage angles'
del(n,sc)     'Balancing stage voltage angles'
beta          'Auxiliary variable equal to worst-case balancing market cost' ;

* Setting the reference node
del0.fx('n1') = 0;
del.fx('n1',s) = 0;

EQUATIONS
EC            'Expected cost (SP only)'
WCC           'Worst-case cost (RO only)'
auxBC(sc)     'Auxiliary variable beta not lower than balancing market cost (RO only)'
WscFX(n)      'Fix wind schedule to conditional mean (not used in the "original" version of SP)'
bal0(n)       'Power balance equation at the forward stage'
TC0(n,n)      'Transmission capacity at the forward stage'
PRu(k)        'Dispatch and reserve upper limit'
PRl(k)        'Dispatch and reserve lower limit'
bal(n,sc)     'Power balance equation at the balancing stage'
TC(n,n,sc)    'Transmission capacity at the balancing stage'
Puu(k,sc)     'Upward energy redispatch upper limit'
Pdu(k,sc)     'Downward energy redispatch upper limit'
Wspu(n,sc)    'Wind power spillage upper limit'
Lshu(n,sc)    'Load shedding upper limit' ;

EC .. z =E= sum(k, C(k)*P(k) + Cu(k)*Ru(k) + Cd(k)*Rd(k) )
          + sum(s, pi(s)*(sum(k, C(k)*(Pu(k,s) - Pd(k,s)) ) + vLOL*sum(n, Lsh(n,s) )) );
WCC .. z =E= sum(k, C(k)*P(k) + Cu(k)*Ru(k) + Cd(k)*Rd(k) ) + beta;
auxBC(s) .. beta =G= sum(k, C(k)*(Pu(k,s) - Pd(k,s)) ) + vLOL*sum(n, Lsh(n,s) );
WscFX(n) .. Wsc(n) =E= wfc(n);
```

```
bal0(n) ..  sum(k$MG(k,n), P(k)) + Wsc(n) - L(n) =E= sum(m$(MN(n,m) EQ 1), (del0(n) - del0(m))/X(n,m) );
TC0(n,m)$MN(n,m) .. (del0(n) - del0(m))/X(n,m) =L= Tmax(n,m);
PRu(k) .. P(k) + Ru(k) =L= Pmax(k);
PRl(k) .. P(k) - Rd(k) =G= 0;
bal(n,s) .. sum(k$MG(k,n), Pu(k,s) - Pd(k,s)) + Lsh(n,s) + w(n,s) - Wsc(n) - Wsp(n,s) =E=
             sum(m$MN(n,m), (del(n,s) - del0(n) - del(m,s) + del0(m))/X(n,m) );
TC(n,m,s)$MN(n,m) .. (del(n,s) - del(m,s))/X(n,m) =L= Tmax(n,m);
Puu(k,s) .. Pu(k,s) =L= Ru(k);
Pdu(k,s) .. Pd(k,s) =L= Rd(k);
Wspu(n,s) .. Wsp(n,s) =L= w(n,s);
Lshu(n,s) .. Lsh(n,s) =L= L(n);

* Stochastic programming model with dispatch of wind
* power as a variable
MODEL DispSP /EC, bal0, TC0, PRu, PRl, bal, TC, Puu, Pdu, Wspu, Lshu/;

* Alternative stochastic programming model with dispatch
* of wind equal to conditional mean forecast
MODEL DispSPvar /EC, WscFX, bal0, TC0, PRu, PRl, bal, TC, Puu, Pdu, Wspu, Lshu/;

* Robust optimization model
MODEL DispRO /WCC, auxBC, WscFX, bal0, TC0, PRu, PRl, bal, TC, Puu, Pdu, Wspu, Lshu/;

********** Example 3.3 **********
* Only 2 scenarios for wind power production are needed
s('s1') = yes;
s('s2') = yes;

TABLE w(n,sc)   'Wind power production'
         s1      s2
n1       50      10
n2        0       0;

pi('s1') = 0.6;
pi('s2') = 0.4;

* Setting the conditional mean forecast of wind power production
wfc(n) = sum(s, pi(s)*w(n,s) );

* Solve the stochastic programming problem
SOLVE DispSP MINIMIZING z USING LP;

SET
mt                 'Solution method'        /SP,SPvar,RO/;

PARAMETERS
EDC(mt)            'Energy dispatch cost'
RC(mt)             'Reserve cost'
```

```
ERDC(sc,mt)      'Energy redispatch cost'
ERDCexp(mt)      'Expected energy redispatch cost'
ERDCwcc(mt)      'Worst-case energy redispatch cost'
TCexp(mt)        'Total expected cost'
TCwcc(mt)        'Total worst-case cost' ;

EDC('SP') = sum(k, C(k)*P.l(k) );
RC('SP') = sum(k, Cu(k)*Ru.l(k) + Cd(k)*Rd.l(k) );
ERDC(s,'SP') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*sum(n
    , Lsh.l(n,s) );

DISPLAY z.l, P.l, Ru.l, Rd.l, Wsc.l, del0.l, Pu.l, Pd.l, del.l,
    Wsp.l, Lsh.l;

* Solve the alternative stochastic programming problem
 SOLVE DispSPvar MINIMIZING z USING LP;

EDC('SPvar') = sum(k, C(k)*P.l(k) );
RC('SPvar') = sum(k, Cu(k)*Ru.l(k) + Cd(k)*Rd.l(k) );
ERDC(s,'SPvar') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*
    sum(n, Lsh.l(n,s) );

DISPLAY z.l, P.l, Ru.l, Rd.l, Wsc.l, del0.l, Pu.l, Pd.l, del.l,
    Wsp.l, Lsh.l;

* Solve the robust optimization problem
SOLVE DispRO MINIMIZING z USING LP;

* To determine the correct redispatch costs, we solve the
* stochastic programming model fixing the dispatch obtained
* from the robust optimization model. This is needed because
* the energy redispatch for the scenarios other than the
* worst-case one is not optimized by the robust optimization
* model
P.fx(k) = P.l(k);
Ru.fx(k) = Ru.l(k);
Rd.fx(k) = Rd.l(k);
del0.fx('n2') = del0.l('n2');

SOLVE DispSPvar MINIMIZING z USING LP;

EDC('RO') = sum(k, C(k)*P.l(k) );
RC('RO') = sum(k, Cu(k)*Ru.l(k) + Cd(k)*Rd.l(k) );
ERDC(s,'RO') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*sum(n
    , Lsh.l(n,s) );

DISPLAY z.l, beta.l, P.l, Ru.l, Rd.l, Wsc.l, del0.l, Pu.l, Pd.l,
    del.l, Wsp.l, Lsh.l;

* Determine expected and worst-case costs for the three
* approaches
ERDCexp(mt) = sum(s, pi(s)*ERDC(s,mt) );
ERDCwcc(mt) = smax(s, ERDC(s,mt) );
TCexp(mt) = EDC(mt) + RC(mt) + ERDCexp(mt);
```

```
TCwcc(mt) = EDC(mt) + RC(mt) + ERDCwcc(mt);

* Compare the results from the three approaches
DISPLAY EDC, RC, ERDC, ERDCexp, ERDCwcc, TCexp, TCwcc;

********** Examples 3.6 - 3.7 **********
* Free all previously fixed variables
P.lo(k)= 0;                 P.up(k)  = Pmax(k);
Ru.lo(k)= 0;                Ru.up(k)  = Pmax(k);
Rd.lo(k)=0;                 Rd.up(k)  = Pmax(k);
del0.lo('n2')= -13;         del0.up('n2') = 13;

* 100 scenarios for wind power production are needed
s(sc) = yes;

* Import 100 scenarios of wind power deviation from csv file
TABLE
Del_w_sc(n,sc) 'Deviation of wind power production from forecast'
$ondelim
$include scenarios.csv
$offdelim;

* Define conditional mean forecast for wind power production
* at each node
wfc('n1')=15;              wfc('n2') = 30;

* Set wind power production scenarios and probability
w(n,s) = wfc(n) + Del_w_sc(n,s);
pi(s) = 1/card(s);

* Solve the stochastic programming problem
SOLVE DispSP MINIMIZING z USING LP;

EDC('SP') = sum(k, C(k)*P.l(k) );
RC('SP') = sum(k, Cu(k)*Ru.l(k) + Cd(k)*Rd.l(k) );
ERDC(s,'SP') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*sum(n
    , Lsh.l(n,s) );
ERDCexp('SP') = sum(s, pi(s)*ERDC(s,'SP') );
TCexp('SP') = EDC('SP') + RC('SP') + ERDCexp('SP');

DISPLAY z.l, P.l, Ru.l, Rd.l, Wsc.l, del0.l, Pu.l, Pd.l, del.l,
    Wsp.l, Lsh.l;

* Store the solution to determine the worst-case cost later on
PARAMETERS
PSol(k,mt)          'Energy dispatch'
WscSol(n,mt)        'Wind energy dispatch'
RuSol(k,mt)         'Upward reserve dispatch'
RdSol(k,mt)         'Downward reserve dispatch'
del0Sol(n,mt)       'Forward stage voltage angles' ;

PSol(k,'SP') = P.l(k);
WscSol(n,'SP') = Wsc.l(n);
RuSol(k,'SP') = Ru.l(k);
```

```
RdSol(k,'SP') = Rd.l(k);
del0Sol(n,'SP') = del0.l(n);

* Solve the alternative stochastic programming problem
SOLVE DispSPvar MINIMIZING z USING LP;

EDC('SPvar') = sum(k, C(k)*P.l(k) );
RC('SPvar') = sum(k, Cu(k)*Ru.l(k) + Cd(k)*Rd.l(k) );
ERDC(s,'SPvar') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*
    sum(n, Lsh.l(n,s) );
ERDCexp('SPvar') = sum(s, pi(s)*ERDC(s,'SPvar') );
TCexp('SPvar') = EDC('SPvar') + RC('SPvar') + ERDCexp('SPvar');

DISPLAY z.l, P.l, Ru.l, Rd.l, Wsc.l, del0.l, Pu.l, Pd.l, del.l,
    Wsp.l, Lsh.l;

PSol(k,'SPvar') = P.l(k);
RuSol(k,'SPvar') = Ru.l(k);
RdSol(k,'SPvar') = Rd.l(k);
del0Sol(n,'SPvar') = del0.l(n);

* Only 8 indices are needed for the vertices of the polyhedral
* uncertainty set
s(sc) = no;
s('s1') = yes; s('s2') = yes; s('s3') = yes; s('s4') = yes;
s('s5') = yes; s('s6') = yes; s('s7') = yes; s('s8') = yes;

* Define the vertices of the polyhedral uncertainty set
TABLE Del_w_vx(n,sc)
    s1      s2      s3      s4      s5      s6      s7      s8
n1  10      4       -4      -10     -10     -4      4       10
n2  8       20      20      8       -8      -20     -20     -8;

* Set vertices of wind power production
w(n,s) = wfc(n) + Del_w_vx(n,s);

* Solve the robust optimization problem
SOLVE DispRO MINIMIZING z USING LP;

EDC('RO') = sum(k, C(k)*P.l(k) );
RC('RO') = sum(k, Cu(k)*Ru.l(k) + Cd(k)*Rd.l(k) );
ERDC(s,'RO') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*sum(n
    , Lsh.l(n,s) );
ERDCwcc('RO') = smax(s, ERDC(s,'RO') );
TCwcc('RO') = EDC('RO') + RC('RO') + ERDCwcc('RO');

DISPLAY z.l, beta.l, P.l, Ru.l, Rd.l, Wsc.l, del0.l, Pu.l, Pd.l,
    del.l, Wsp.l, Lsh.l;

PSol(k,'RO') = P.l(k);
RuSol(k,'RO') = Ru.l(k);
RdSol(k,'RO') = Rd.l(k);
del0Sol(n,'RO') = del0.l(n);
```

```
* Determine the worst-case cost for SP solution among the
    vertices
P.fx(k) = PSol(k,'SP');
Ru.fx(k) = RuSol(k,'SP');
Rd.fx(k) = RdSol(k,'SP');
del0.fx(n) = del0Sol(n,'SP');

* Scheduled wind power should also be set to the solution
wfc(n) = WscSol(n,'SP');

SOLVE DispRO MINIMIZING z USING LP;
ERDC(s,'SP') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*sum(n
    , Lsh.l(n,s) );
ERDCwcc('SP') = smax(s, ERDC(s,'SP') );
TCwcc('SP') = EDC('SP') + RC('SP') + ERDCwcc('SP');

* Determine the worst-case cost for SPvar solution among the
* vertices
P.fx(k) = PSol(k,'SPvar');
Ru.fx(k) = RuSol(k,'SPvar');
Rd.fx(k) = RdSol(k,'SPvar');
del0.fx(n) = del0Sol(n,'SPvar');

* Redefine conditional mean forecast for wind power production
* at each node
wfc('n1')= 15;          wfc('n2') = 30;

SOLVE DispRO MINIMIZING z USING LP;
ERDC(s,'SPvar') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*
    sum(n, Lsh.l(n,s) );
ERDCwcc('SPvar') = smax(s, ERDC(s,'SPvar') );
TCwcc('SPvar') = EDC('SPvar') + RC('SPvar') + ERDCwcc('SPvar');

* Determine the expected cost for RO solution over the
* 100 scenarios
s(sc) = yes;

P.fx(k) = PSol(k,'RO');
Ru.fx(k) = RuSol(k,'RO');
Rd.fx(k) = RdSol(k,'RO');
del0.fx(n) = del0Sol(n,'RO');

* Set the scenarios
w(n,s) = wfc(n) + Del_w_sc(n,s);
pi(s) = 1/card(s);

SOLVE DispSPvar MINIMIZING z USING LP;

ERDC(s,'RO') = sum(k, C(k)*(Pu.l(k,s) - Pd.l(k,s)) ) + vLOL*sum(n
    , Lsh.l(n,s) );
ERDCexp('RO') = sum(s, pi(s)*ERDC(s,'RO') );
TCexp('RO') = EDC('RO') + RC('RO') + ERDCexp('RO');

* Compare the results from the three approaches

DISPLAY EDC, RC, ERDC, ERDCexp, ERDCwcc, TCexp, TCwcc;
```

E.4 GAMS Code for the Example on Balancing Auction with Excess Production and Network Congestion in Sect. 4.6

```
$title BALANCING AUCTION

SETS
j        'demand'                       /j1,j2/
i        'non-dispatchable producer'    /i1,i2/
k        'dispatchable producer'        /k1,k2,k3/;

PARAMETERS
l_da(j) 'load day-ahead'                /j1 40, j2 100/
w_da(i) 'production day-ahead'          /i1 40, i2 30/
C_da(k) 'bid offer day-ahead'           /k1 50, k2 100, k3 150/
p_da(k) 'bid price day-ahead'           /k1 10, k2 20, k3 35/;

SCALAR
C        'transmission capacity'        /90/;

POSITIVE VARIABLES
x_da(k) 'schedule day-ahead';

VARIABLES
f        'flow from n1 to n2'
z        'system cost';

EQUATIONS
bal_da1     'node 1 balance'
bal_da2     'node 2 balance'
Pcap_da(k) 'day-ahead production capacity'
Tcap_p      'day-ahead transmission capacity (positive)'
Tcap_n      'day-ahead transmission capacity (negative)'
obj         'objective function';

bal_da1.. l_da('j1')+ f =E= w_da('i1') + x_da('k1') + x_da('k2');
bal_da2.. l_da('j2') =E= w_da('i2') + x_da('k3') + f;
Pcap_da(k).. x_da(k) =L= C_da(k);
Tcap_p.. f =L= C;
Tcap_n.. f =G= -C;
obj.. z =E= sum(k, p_da(k)*x_da(k));

MODEL DAMC /all/;

SOLVE DAMC MINIMIZING z USING LP;

DISPLAY z.l, x_da.l, f.l, bal_da1.m, bal_da2.m;
```

```
x_da.fx(k) = x_da.l(k);

PARAMETERS
l_rt(j)     'load real-time'             /j1 22, j2 120/
w_rt(i)     'production real-time'       /i1 65, i2 25/
Cu_rt(k)    'bid offer up real-time'     /k1 0, k2 10, k3 50/
pu_rt(k)    'bid price up real-time'     /k1 20, k2 30, k3 50/
Cd_rt(k)    'bid offer down real-time'   /k1 40, k2 10, k3 0/
pd_rt(k)    'bid price down real-time'   /k1 8, k2 15, k3 0/;

POSITIVE VARIABLES
xu_rt(k)    'scheduled up-regulation'
xd_rt(k)    'scheduled down-regulation';

EQUATIONS
bal_rt1      'node 1 balance'
bal_rt2      'node 2 balance'
Pcap_rtU(k) 'real-time up-regulation capacity'
Pcap_rtD(k) 'real-time down-regulation capacity'
obj_rt       'objective function';

bal_rt1.. l_rt('j1') + xd_rt('k1') + xd_rt('k2') + f =E= w_rt('i1') + x_da('k1') + x_da('k2') + xu_rt('k2');
bal_rt2.. l_rt('j2') + xd_rt('k3') =E= w_rt('i2') + x_da('k3') + xu_rt('k3') + f;
Pcap_rtU(k).. xu_rt(k) =L= Cu_rt(k);
Pcap_rtD(k).. xd_rt(k) =L= Cd_rt(k);
obj_rt.. z =E= sum(k, pu_rt(k)*xu_rt(k) - pd_rt(k)*xd_rt(k));

MODEL BMC /bal_rt1,bal_rt2,Pcap_rtU,Pcap_rtD,Tcap_p,Tcap_n,obj_rt/

SOLVE BMC MINIMIZING z USING LP;

DISPLAY z.l, x_da.l, xu_rt.l, xd_rt.l, f.l, bal_rt1.m, bal_rt2.m;
```

E.5 GAMS Code for the Example in Sect. 5.3.4 on System Inflexibility Due to Minimum Up- and Down-Time

```
$title INFLEXIBILITY DUE TO MUT AND MDT

SETS
        t           'time periods'            /t1 * t3/
        i           'nodes of the system'     /i1, i2/
        k           'production blocks'       /k1 * k3/
        w           'scenarios'               /w1, w2/;

ALIAS(i,j);
```

```
ALIAS(t,tau);

SCALARS
          Vlol            'value of lost load'    /200/
          Vsp             'value of spilled wind' /0/;

PARAMETERS
c(t,k)              'per unit cost of dispatch'
C_up(t,k)           'per unit cost of up-regulation reserve'
C_dw(t,k)           'per unit cost of down-regulation reserve'
b_up(t,k)           'per unit cost of up-regulation'
b_dw(t,k)           'per unit benefit of down-regulation'
Pmin(t,k)           'minimum production capacity'
Pmax(t,k)           'maximum production capacity'
TU(k)               'minimum up time'
TD(k)               'minimum down time'
TU0(k)              'initial minimum up time'
TD0(k)              'initial minimum down time'
v0(k)               'initial on/off status';

c(t,'k1') = 10; c(t,'k2') = 30; c(t,'k3') = 35;
b_up(t,k) = c(t,k); b_dw(t,k) = c(t,k);
C_up(t,'k1') = 16; C_up(t, 'k2') = 13; C_up(t,  'k3') = 10;
C_dw(t,'k1') = 15; C_dw(t, 'k2') = 12; C_dw(t,  'k3') = 9;
Pmin(t,'k1') = 10; Pmin(t, 'k2') = 10; Pmin(t,  'k3') = 5;
Pmax(t,'k1') = 50; Pmax(t, 'k2') = 110; Pmax(t,  'k3') = 100;
TU(k) = 2; TD(k) = 2; TU0(k) = 0; TD0(k) = 0; v0(k) = 1;

PARAMETERS
p(w)                'scenario probability'
/w1       0.6
 w2       0.4/

wp(t,i,w)           'wind power production forecast'
/
t1.i1.w1      50
t2.i1.w1      65
t3.i1.w1      35
t1.i2.w1       0
t2.i2.w1       0
t3.i2.w1       0
t1.i1.w2      10
t2.i1.w2      30
t3.i1.w2      15
t1.i2.w2       0
t2.i2.w2       0
t3.i2.w2       0
/;

TABLE d(t,i)              'demand'
          i1       i2
t1        40       100
t2        25        80
t3        45        95
```

```
TABLE Wsmax(t,i)        'maximum dispatch of wind power'
            i1      i2
t1          50      0
t2          65      0
t3          35      0

TABLE  MG(i,k)          '1 if production block k is located at bus i'
            k1      k2      k3
i1          1       1       0
i2          0       0       1

TABLE MN(i,i)           '1 if nodes are connected'
            i1      i2
i1          0       1
i2          1       0

TABLE b(i,i)            'per unit susceptance of line'
            i1              i2
i1          0               7.692308
i2          7.692308        0

TABLE Tmax(i,i)         'transmission capacity'
            i1      i2
i1          0       100
i2          100       0;

POSITIVE VARIABLES
x(t,k)              'day-ahead dispatch'
ws(t,i)             'day-ahead wind power dispatch'
r_up(t,k)           'reserve for up-regulation'
r_dw(t,k)           'reserve for down-regulation'
y_up(t,k,w)         'up-regulation redispatch'
y_dw(t,k,w)         'down-regulation redispatch'
l_sh(t,i,w)         'load shedding'
w_sp(t,i,w)         'wind spillage' ;

VARIABLES
z                   'objective function'
delta0(t,i)         'day-ahead voltage'
delta(t,i,w)        'balancing market voltage' ;

* Setting bus i1 as the reference bus
delta0.fx(t,'i1') = 0;
delta.fx(t,'i1',w) = 0;

BINARY VARIABLES
v(t,k)              'on/off status of unit'
su(t,k)             'start-up of unit'
sd(t,k)             'shut-down of unit' ;

EQUATIONS
obj                 'dispatch costs'
```

```
bal0(t,i)          'day-ahead power balance'
cap_ws0(t,i)       'upper bound for wind schedule'
cap_up0(t,k)       'day-ahead capacity upper bound'
cap_dw0(t,k)       'day-ahead capacity lower bound'
trn0(t,i,j)        'day-ahead transmission capacity'
bal(t,i,w)         'balancing market power balance'
rcap_up(t,k,w)     'regulation capacity upper bound'
rcap_dw(t,k,w)     'regulation capacity lower bound'
cap_sh(t,i,w)      'load-shedding upper bound'
cap_sp(t,i,w)      'wind-spillage upper bound'
trn(t,i,j,w)       'balancing market transmission capacity'
status(t,k)        'binary relations for unit status'
TUC(t,k)           'up time constraint'
TDC(t,k)           'down time constraint'
TUC0(k)            'initial up time constraint'
TDC0(k)            'initial down time constraint';

obj.. z =E= sum((t,k), c(t,k)*x(t,k)
  + C_up(t,k)*r_up(t,k) + C_dw(t,k)*r_dw(t,k)
  + sum(w, p(w)*(b_up(t,k)*y_up(t,k,w) - b_dw(t,k)*y_dw(t,k,w))))
  + sum((t,i,w), p(w)*(Vlol*l_sh(t,i,w) + Vsp*w_sp(t,i,w)));
bal0(t,i).. sum(k, MG(i,k)*x(t,k)) + ws(t,i)
          - sum(j$(MN(i,j) EQ 1), b(i,j)
          *(delta0(t,i) - delta0(t,j))) =E= d(t,i);
cap_up0(t,k).. x(t,k) + r_up(t,k) =L= Pmax(t,k)*v(t,k);
cap_dw0(t,k).. x(t,k) - r_dw(t,k) =G= Pmin(t,k)*v(t,k);
cap_ws0(t,i).. ws(t,i) =L= Wsmax(t,i);
trn0(t,i,j)$(MN(i,j) EQ 1)..
       b(i,j)*(delta0(t,i) - delta0(t,j)) =L= Tmax(i,j);
bal(t,i,w).. sum(k, MG(i,k)*(y_up(t,k,w) - y_dw(t,k,w)))
+ l_sh(t,i,w) + wp(t,i,w) - ws(t,i) - w_sp(t,i,w)
- sum(j$(MN(i,j) EQ 1), b(i,j)*(delta(t,i,w) - delta(t,j,w)))=E=
- sum(j$(MN(i,j) EQ 1), b(i,j)*(delta0(t,i) - delta0(t,j)));
rcap_up(t,k,w).. y_up(t,k,w) =L= r_up(t,k);
rcap_dw(t,k,w).. y_dw(t,k,w) =L= r_dw(t,k);
cap_sh(t,i,w).. l_sh(t,i,w) =L= d(t,i);
cap_sp(t,i,w).. w_sp(t,i,w) =L= wp(t,i,w);
trn(t,i,j,w)$(MN(i,j) EQ 1)..
      b(i,j)*(delta(t,i,w) - delta(t,j,w)) =L= Tmax(i,j);
status(t,k).. v(t-1,k)$(ord(t) > 1) + v0(k)$(ord(t) = 1)
               - v(t,k) + su(t,k) - sd(t,k) =E= 0;
TUC(t,k).. sum(tau$(ord(tau) ge ord(t) and ord(tau) < ord(t) + TU
    (k)),
             v(tau,k) - su(t,k)) =G= 0;
TDC(t,k).. sum(tau$(ord(tau) ge ord(t) and ord(tau) < ord(t) + TD
    (k)),
             1 - v(tau,k) - sd(t,k)) =G= 0;
TUC0(k).. sum(t$(ord(t) le TU0(k)), v(t,k)) =E= TU0(k);
TDC0(k).. sum(t$(ord(t) le TD0(k)), v(t,k)) =E= 0;

MODEL SP /all/;

SP.optcr = 0;
SP.iterlim = 10000;
```

```
SOLVE SP MINIMIZING z USING MIP;

SCALARS
          DCost                    'dispatch cost'
          RCost                    'reserve cost'
          RDCost                   'expected redispatch cost'
          LSCost                   'expected load shedding cost';

DCost = sum((t,k), c(t,k)*x.l(t,k));
RCost = sum((t,k), C_up(t,k)*r_up.l(t,k)+ C_dw(t,k)*r_dw.l(t,k));
RDCost = sum((t,k), sum(w, p(w)*(b_up(t,k)*y_up.l(t,k,w)
                   - b_dw(t,k)*y_dw.l(t,k,w))));
LSCost = sum((t,i,w), p(w)*Vlol*l_sh.l(t,i,w));

DISPLAY z.l, v.l, su.l, sd.l, x.l, ws.l, r_up.l, r_dw.l,
        y_up.l, y_dw.l, p, DCost, RCost, RDCost, LSCost;
```

E.6 GAMS Code for the Example in Sect. 5.3.5 on System Inflexibility Due to Limited Hydro Energy Availability

```
$TITLE INFLEXIBILITY DUE TO LIMITED HYDRO ENERGY AVAILABIL ITY

SETS
         t              'time periods'          /t1*t3/
         i              'nodes of the system'   /i1,i2/
         k              'production blocks'     /k1*k3/
         h              'hydro power plant'     /h1/
         w              'scenarios'             /w1,w2/;

ALIAS(i,j);
ALIAS(t,tau);

SCALARS
Vlol        'value of lost load'    /200/
Vsp         'value of spilled wind' /0/;

PARAMETERS
c(t,k)      'per unit cost of dispatch'
C_up(t,k)   'per unit cost of up-regulation reserve'
C_dw(t,k)   'per unit cost of down-regulation reserve'
Ch_up(t,h)  'per unit cost of up-regulation reserve from hydro'
Ch_dw(t,h)  'per unit cost of down-regulation reserve from hydro'
Pmin(t,k)   'minimum production capacity'
Pmax(t,k)   'maximum production capacity'
Phmax(t,h)  'maximum hydro production capacity (power)'
Emax(h)     'maximum hydro production capacity (energy)'
b_up(t,k)   'per unit cost of up-regulation'
b_dw(t,k)   'per unit benefit of down-regulation' ;
```

```
c(t,'k1') = 10; c(t, 'k2') = 30; c(t, 'k3') = 35;
b_up(t,k) = c(t,k); b_dw(t,k) = c(t,k);
C_up(t,'k1') = 16; C_up(t, 'k2') = 13; C_up(t, 'k3') = 10;
C_dw(t,'k1') = 15; C_dw(t, 'k2') = 12; C_dw(t, 'k3') = 9;
Pmin(t,k) = 0;
Pmax(t,'k1') = 50; Pmax(t, 'k2') = 110; Pmax(t, 'k3') = 100;
Phmax(t,h) = 50;
Emax(h) = 60;
Ch_up(t,h) = 5; Ch_dw(t,h) = 5;

PARAMETERS
p(w)                'scenario probability'
/w1       0.6
 w2       0.4/

wp(t,i,w)           'wind power production forecast'
/
t1.i1.w1      50
t2.i1.w1      65
t3.i1.w1      35
t1.i2.w1       0
t2.i2.w1       0
t3.i2.w1       0
t1.i1.w2      10
t2.i1.w2      30
t3.i1.w2      15
t1.i2.w2       0
t2.i2.w2       0
t3.i2.w2       0
/;

TABLE  d(t,i)          'demand'
           i1       i2
t1         40       100
t2         25        80
t3         45        95

TABLE  Wsmax(t,i)      'maximum dispatch of wind power'
           i1       i2
t1         50       0
t2         65       0
t3         35       0

TABLE  MG(i,k)         '1 if production block k is located at bus i'
           k1       k2       k3
i1         1        1        0
i2         0        0        1

TABLE  MN(i,i)         '1 if nodes are connected'
           i1       i2
i1         0        1
i2         1        0
```

```
TABLE  MH(i,h)        '1 if hydro producer h is located ad bus i'
           h1
i1         0
i2         1

TABLE  b(i,i)         'per unit susceptance of line'
           i1                 i2
i1         0                  7.692308
i2         7.692308           0

TABLE  Tmax(i,i)      'transmission capacity'
           i1        i2
i1         0         100
i2         100         0;

POSITIVE VARIABLES
x(t,k)              'day-ahead dispatch'
xh(t,h)             'day-ahead hydro power dispatch'
ws(t,i)             'day-ahead wind power dispatch'
r_up(t,k)           'reserve for up-regulation'
r_dw(t,k)           'reserve for down-regulation'
rh_up(t,h)          'hydro reserve for up-regulation'
rh_dw(t,h)          'hydro reserve for down-regulation'
y_up(t,k,w)         'up-regulation redispatch'
y_dw(t,k,w)         'down-regulation redispatch'
yh_up(t,h,w)        'up-regulation hydro redispatch'
yh_dw(t,h,w)        'down-regulation hydro redispatch'
l_sh(t,i,w)         'load shedding'
w_sp(t,i,w)         'wind spillage'  ;

VARIABLES
z                   'objective function'
delta0(t,i)         'day-ahead voltage'
delta(t,i,w)        'balancing market voltage'   ;

* Setting bus i1 as the reference bus
delta0.fx(t,'i1') = 0;
delta.fx(t,'i1',w) = 0;

EQUATIONS
obj                 'dispatch costs'
bal0(t,i)           'day-ahead power balance'
cap_up0(t,k)        'day-ahead capacity upper bound'
cap_dw0(t,k)        'day-ahead capacity lower bound'
Hcap_up0(t,h)       'upper bound for hydro schedule (power)'
Hcap_dw0(t,h)       'lower bound for hydro schedule (power)'
Hcap_E0(h)          'upper bound for hydro schedule (energy)'
cap_ws0(t,i)        'upper bound for wind schedule'
trn0(t,i,j)         'day-ahead transmission capacity'
bal(t,i,w)          'balancing market power balance'
Rcap_up(t,k,w)      'regulation capacity upper bound'
Rcap_dw(t,k,w)      'regulation capacity lower bound'
HRcap_up(t,h,w)     'hydro regulation capacity upper bound'
HRcap_dw(t,h,w)     'hydro regulation capacity lower bound'
```

```
Hcap_E(h,w)       'hydro capacity bound (energy)'
cap_sh(t,i,w)     'load-shedding upper bound'
cap_sp(t,i,w)     'wind-spillage upper bound'
trn(t,i,j,w)      'balancing market transmission capacity';

obj.. z =E= sum((t,k), c(t,k)*x(t,k) + C_up(t,k)*r_up(t,k)
      + C_dw(t,k)*r_dw(t,k)
      + sum(w, p(w)*(b_up(t,k)*y_up(t,k,w) - b_dw(t,k)*y_dw(t,k,w
         ))))
      + sum((t,h), Ch_up(t,h)*rh_up(t,h) + Ch_dw(t,h)*rh_dw(t,h))
      + sum((t,i,w), p(w)*(Vlol*l_sh(t,i,w) + Vsp*w_sp(t,i,w)));
bal0(t,i).. sum(k, MG(i,k)*x(t,k)) + sum(h, MH(i,h)*xh(t,h))
             + ws(t,i) - sum(j$(MN(i,j) EQ 1), b(i,j)
             *(delta0(t,i) - delta0(t,j))) =E= d(t,i);
cap_up0(t,k).. x(t,k) + r_up(t,k) =L= Pmax(t,k);
cap_dw0(t,k).. x(t,k) - r_dw(t,k) =G= Pmin(t,k);
Hcap_up0(t,h).. xh(t,h) + rh_up(t,h) =L= Phmax(t,h);
Hcap_dw0(t,h).. xh(t,h) - rh_dw(t,h) =G= 0;
Hcap_E0(h).. sum(t, xh(t,h)) =L= Emax(h);
cap_ws0(t,i).. ws(t,i) =L= Wsmax(t,i);
trn0(t,i,j)$(MN(i,j) EQ 1).. b(i,j)*(delta0(t,i) - delta0(t,j))
                             =L= Tmax(i,j);
bal(t,i,w).. sum(k, MG(i,k)*(y_up(t,k,w) - y_dw(t,k,w)))
   + sum(h, MH(i,h)*(yh_up(t,h,w) - yh_dw(t,h,w)))
   + l_sh(t,i,w) + wp(t,i,w) - ws(t,i) - w_sp(t,i,w)
   - sum(j$(MN(i,j) EQ 1), b(i,j)*(delta(t,i,w) - delta(t,j,w)))
     =E=
   - sum(j$(MN(i,j) EQ 1), b(i,j)*(delta0(t,i) - delta0(t,j)));
Rcap_up(t,k,w).. y_up(t,k,w) =L= r_up(t,k);
Rcap_dw(t,k,w).. y_dw(t,k,w) =L= r_dw(t,k);
HRcap_up(t,h,w).. yh_up(t,h,w) =L= rh_up(t,h);
HRcap_dw(t,h,w).. yh_dw(t,h,w) =L= rh_dw(t,h);
Hcap_E(h,w).. sum(t, xh(t,h) + yh_up(t,h,w) - yh_dw(t,h,w)) =L=
     Emax(h);
cap_sh(t,i,w).. l_sh(t,i,w) =L= d(t,i);
cap_sp(t,i,w).. w_sp(t,i,w) =L= wp(t,i,w);
trn(t,i,j,w)$(MN(i,j) EQ 1)..
   b(i,j)*(delta(t,i,w) - delta(t,j,w)) =L= Tmax(i,j);

MODEL SP /all/;

SP.optcr = 0;
SP.iterlim = 10000;

SOLVE SP MINIMIZING z USING MIP;

SCALARS
          DCost                   'dispatch cost'
          RCost                   'reserve cost'
          RDCost                  'expected redispatch cost'
          LSCost                  'expected load shedding cost';
```

```
DCost = sum((t,k), c(t,k)*x.l(t,k));
RCost = sum((t,k), C_up(t,k)*r_up.l(t,k) + C_dw(t,k)*r_dw.l(t,k))
+ sum((t,h), Ch_up(t,h)*rh_up.l(t,h) + Ch_dw(t,h)*rh_dw.l(t,h));
RDCost = sum((t,k), sum(w, p(w)*(b_up(t,k)*y_up.l(t,k,w)
                                  - b_dw(t,k)*y_dw.l(t,k,w))));
LSCost = sum((t,i,w), p(w)*Vlol*l_sh.l(t,i,w));

DISPLAY z.l, x.l, xh.l, ws.l, r_up.l, r_dw.l, rh_up.l, rh_dw.l,
y_up.l, y_dw.l, yh_up.l, yh_dw.l, p, DCost, RCost, RDCost,
LSCost;

DISPLAY bal0.m, bal.m;
```

E.7 GAMS Code for the Example on the Benefits of an Adjustment Market in Sect. 7.6

We present first the code to compute the optimal offer by the stochastic producer in the day-ahead market if the adjustment market is not taken into account.

```
$title OPTIMAL OFFER IN THE DAY-AHEAD MARKET WITHOUT ADJUSTMENT
    MARKET (FIRST PART)

SET
w         Scenarios /w1 * w4/;

SCALARS
imbUP     Imbalance penalty(upward)      /9/
imbDW     Imbalance penalty(downward)    /4/
Emax      Producer capacity              /100/;

PARAMETERS

e(w)      Production in scenario w
/w1       100
 w2       50
 w3       40
 w4       0/

pi(w)     Scenario probability
/w1       0.3
 w2       0.1
 w3       0.3
 w4       0.3/;

VARIABLES

EOL       Expecte dopportunity loss
eD        Optimal offer in the day-ahead market
eUP(w)    Upward regulation
eDW(w)    Downward regulation

POSITIVE VARIABLES eD, eDW(w);
```

```
NEGATIVE VARIABLE   eUP(w);

eD.up = Emax;

EQUATIONS
OF        Objective function
R(w)      Regulation needs;

OF.. EOL =e= sum(w, pi(w) * (-imbUP * eUP(w) + imbDW * eDW(w)));
R(w)..  eUP(w) + eDW(w) =e= e(w)-eD;

MODEL onlyDA /ALL/;
OPTION iterlim = 1e8;
OPTION reslim = 1e10;
OPTION lp = cplex;

OnlyDA.optcr=0;

SOLVE OnlyDA USING lp MINIMIZING EOL;

display eD.l, EOL.l
```

We now provide the code to determine the optimal offer by the stochastic producer in the day-ahead market if the adjustment market is considered.

```
$title OPTIMAL OFFER IN THE DAY-AHEAD MARKET WHEN AN ADJUSTMENT
    MARKET IS AVAILABLE (SECOND PART).

SET
w         'Scenarios'  /w1 * w4/;

SCALARS
lambdaD      'Day-ahead market price'          /20/
lambdaA      'Adjustment market price'         /19/
lambdaUP     'Balancing price (upward)'        /29/
lambdaDW     'Balancing price (downward)'      /16/
Emax         'Producer capacity'               /100/;

PARAMETERS

e(w)      'Production in scenario w'
/w1       100
 w2       50
 w3       40
 w4       0/
```

```
pi(w)     'Scenario probability'
/w1        0.3
 w2        0.1
 w3        0.3
 w4        0.3/;

VARIABLES

EP        'Expected profit'
eD        'Optimal offer in the day-ahead market'
eUP(w)    'Upward regulation'
eDW(w)    'Downward regulation'
eA(w)     'Optimal offer in the adjustment market';

POSITIVE VARIABLES eD, eDW(w);

NEGATIVE VARIABLE  eUP(w);

eD.up = Emax;

EQUATIONS

OF          'Objective function'
R(w)        'Regulation needs'
CapUP(w)    'Upper bound on the energy traded in both markets'
CapDW(w)    'Lower bound on the energy traded in both markets'
NA1         'Non-anticipativity constraint'
NA2         'Non-anticipativity constraint'
* Add constraint below to supress the certainty gain effect of the adjustment market
* NA3        Non-anticipativity constraint;

OF.. EP =e= lambdaD * eD + sum(w, pi(w) * (lambdaA * eA(w)
          + lambdaUP * eUP(w) + lambdaDW * eDW(w)));
R(w)..  eUP(w) + eDW(w) =e= e(w)-eD-eA(w);
CapUP(w).. eD + eA(w) =l= Emax;
CapDW(w).. eD + eA(w) =g= 0;
NA1.. eA('w1') =e= eA('w2');
NA2.. eA('w3') =e= eA('w4');
* Add constraint below to supress the certainty gain effect of the adjustment market
* NA3.. eA('w2') =e= eA('w3');

MODEL withAM /ALL/;
OPTION iterlim = 1e8;
OPTION reslim = 1e10;
OPTION lp = cplex;

withAM.optcr=0;

SOLVE withAM USING lp MAXIMIZING EP;

display eD.l, eA.l, EP.l
```

E.8 GAMS Code for the Example on the VPP Robust Trading Strategy in Sect. 8.4

```
$title ROBUST TRADING STRATEGY.

Sets
t          'Time periods' /t1, t2, t3/;

Scalar
PGmax      'GT capacity' /5/
PGmin      'GT minimum power output' /1/
PGramp     'GT ramp rate' /2/
PG0        'GT initial power output' /2/
V0         'GT initial status' /1/
aG         'Quadratic coefficient of GT cost function' /5/
bG         'Linear coeffcient of GT cost function'     /10/
cG         'No-load cost' /50/
SU         'Start-up cost' /10/

PLmax      'Gear factory maximum power consumption' /2/
PLmin      'Gear factory minimum power consumption' /0.5/
PLramp     'Gear factory ramp rate' /1/
PL0        'Gear factory initial power consumption' /1.5/
MinCon     'Total minimum energy consumption' /2.5/
aL         'Quadratic coefficient of GF utility function' /-30/
bL         'Linear coefficient of GF utility function'     /150/
cL         'Constant term of GF utility function' /5/

ESmax      'PSP equivalent energy capacity' /1/
ESmin      'PSP minimum storage level' /0.2/
ES0        'PSP initial storage level' /0.4/
PScmax     'PSP charging power limit' /0.3/
PSdmax     'PSP discharging power limit' /0.5/
eff        'PSP efficiency' /0.80/
Cdiesel    'Generating cost of the diesel set' /200/
MaxDiesel  'Capacity of the diesel set' /0.5/;

Parameters

Price(t)   'Market price'
/
t1         20
t2         80
t3         45/

PVoutput(t)   'Worst-case PV power output realization'
/
t1         2.0
t2         1.1
t3         1.5/;
```

```
Variables

rho       'Profit'
PD(t)     'Energy sold in (>0) or bought from (<0) the market';

Positive variables

CSU(t)         'GT start-up cost'
PG(t)          'GT power output'
EG(t)          'GT energy production'

PL(t)          'Gear factory load consumption'
EL(t)          'Gear factory energy consumption'

ES(t)          'PSP storage level'
PSc(t)         'PSP power charge'
PSd(t)         'PSP power discharge'
EGdiesel(t)    'Electricity generated by the diesel set'

PWcurt(t)      'PV generation curtailment';

Binary variables

v(t)       'On-off status of the GT unit';

PL.up(t) = PLmax;
PL.lo(t) = PLmin;
PSc.up(t) = PScmax;
PSd.up(t) = PSdmax;
ES.up(t) = ESmax;
ES.lo(t) = ESmin;
EGdiesel.up(t) = MaxDiesel;
PWcurt.up(t) = PVoutput(t);

Equations

Obj        'Objective function'

EB(t)      'Energy balance'

GLup(t)    'GT capacity limit'
GLlo(t)    'GT minimum power output'
SUC0       'Start-up cost definition (t = 1)'
SUC(t)     'Start-up cost definition'
GRL0up     'GT ramping limits (t = 1 upward)'
GRLup(t)   'GT ramping limits (upward)'
GRL0dn     'GT ramping limits (t = 1 downward)'
GRLdn(t)   'GT ramping limits (downward)'
GPE0       'GT power-energy equivalence (t = 1)'
GPE(t)     'GT power-energy equivalence'

LRLup0     'Gear factory ramping limits (t = 1 upward)'
LRLup(t)   'Gear factory ramping limits (upward)'
LRLdn0     'Gear factory ramping limits (t = 1 downward)'
```

```
LRLdn(t) 'Gear factory ramping limits (downward)'
LPE0     'Gear factory power equivalence (t = 1)'
LPE(t)   'Gear factory power equivalence'
MEC      'Gear factory minimum energy consumption'

STE0     'PSP state-transition function (t = 1)'
STE(t)   'PSP state-transition function';

Obj..    rho =e= sum(t, Price(t)*PD(t)- aG*EG(t)*EG(t)- bG*EG(t)
             - cG*v(t) + aL*EL(t)*EL(t) + bL*EL(t) + cL
             - CSU(t) - Cdiesel * EGdiesel(t));
EB(t)..  EG(t) + EGdiesel(t) + PVoutput(t)- PWcurt(t)
       + PSd(t) =e= EL(t) + PSc(t) + PD(t);

GLup(t).. PG(t) =l= PGmax * v(t);
GLlo(t).. PG(t) =g= PGmin * v(t);
SUC0.. CSU('t1') =g= SU*(v('t1')-V0);
SUC(t)$(ord(t) GT 1)..  CSU(t) =g= SU*(v(t)-v(t-1));
GRL0up..  PG('t1') - PG0 =l= PGramp;
GRLup(t)$(ord(t) GT 1).. PG(t) - PG(t-1) =l= PGramp;
GRL0dn..  PG0 - PG('t1') =l= PGramp;
GRLdn(t)$(ord(t) GT 1).. PG(t-1) - PG(t) =l= PGramp;
GPE0.. (PG0 + PG('t1'))/2 =e= EG('t1');
GPE(t)$(ord(t) GT 1).. (PG(t-1) + PG(t))/2 =e= EG(t);

LRLup0.. PL('t1') - PL0 =l= PLramp;
LRLup(t)$(ord(t) GT 1).. PL(t) - PL(t-1) =l= PLramp;
LRLdn0.. PL0 - PL('t1') =l= PLramp;
LRLdn(t)$(ord(t) GT 1).. PL(t-1) - PL(t) =l= PLramp;
LPE0.. (PL0 + PL('t1'))/2 =e= EL('t1');
LPE(t)$(ord(t) GT 1).. (PL(t-1) + PL(t))/2 =e= EL(t);
MEC.. sum(t, EL(t)) =g= MinCon;

STE0.. ES('t1') =e= ES0 + eff * PSc('t1') - (1/eff) * PSd('t1');
STE(t)$(ord(t) GT 1).. ES(t) =e= ES(t-1) + eff * PSc(t)
                       - (1/eff) * PSd(t);

Model RobStrategy /all/;
OPTION iterlim = 1e8;
OPTION reslim = 1e10;
OPTION miqcp = cplex;

RobStrategy.optcr=0;

Solve RobStrategy maximizing rho using miqcp;
```

E.9 GAMS Code for the Example on the VPP Stochastic Trading Strategy in Sect. 8.4

```
$title STOCHASTIC TRADING STRATEGY.
Sets
t         'Time periods' /t1, t2, t3/
w         'Scenario index' /w1, w2, w3/;

Scalar
PGmax      'GT capacity' /5/
PGmin      'GT minimum power output' /1/
PGramp     'GT ramp rate' /2/
PG0        'GT initial power output' /2/
V0         'GT initial status' /1/
aG         'Quadratic coefficient of GT cost function' /5/
bG         'Linear coefficient of GT cost function'     /10/
cG         'No-load cost' /50/
SU         'Start-up cost' /10/

PLmax      'Gear factory maximum power consumption' /2/
PLmin      'Gear factory minimum power consumption' /0.5/
PLramp     'Gear factory ramp rate' /1/
PL0        'Gear factory initial power consumption' /1.5/
MinCon     'Total minimum energy consumption' /2.5/
aL         'Quadratic coefficient of GF utility function' /-30/
bL         'Linear coeffcient of GF utility function'     /150/
cL         'Constant term of gear factory utility function' /5/

ESmax      'PSP equivalent energy capacity' /1/
ESmin      'PSP minimum storage level' /0.2/
ES0        'PSP initial storage level' /0.4/
PScmax     'PSP charging power limit' /0.3/
PSdmax     'PSP discharging power limit' /0.5/
eff        'PSP efficiency' /0.80/
Cdiesel    'Generating cost of the diesel set' /200/
MaxDiesel  'Capacity of the diesel set' /0.5/;

Parameters

Price(t)       'Market price'
/
t1          20
t2          80
t3          45/

pi(w)     'Scenario probability'
/
w1         0.2
w2         0.3
w3         0.5/;

TABLE PVoutput(t,w)  'PV power output scenarios'

            w1      w2       w3
t1          2.5     6.0      2.0
t2          4.0     4.0      1.1
```

```
t3          6.0     3.5      1.5;

Variables

rho       'Profit'
PD(t)     'Energy sold in (>0) or bought from (<0) the market';

Positive variables

CSU(t,w)         'GT start-up cost'
PG(t,w)          'GT power output'
EG(t,w)          'GT energy production'

PL(t,w)          'Gear factory load consumption'
EL(t,w)          'Gear factory energy consumption'

ES(t,w)          'PSP storage level'
PSc(t,w)         'PSP power charge'
PSd(t,w)         'PSP power discharge'
EGdiesel(t,w)    'Electricity generated by the diesel set'

PWcurt(t,w)      'PV generation curtailment';

Binary variables

v(t,w)       'On-off status of the GT unit';

PL.up(t,w)  = PLmax;
PL.lo(t,w)  = PLmin;
PSc.up(t,w)  = PScmax;
PSd.up(t,w)  = PSdmax;
ES.up(t,w)  = ESmax;
ES.lo(t,w)  = ESmin;
EGdiesel.up(t,w)  = MaxDiesel;
PWcurt.up(t,w)  = PVoutput(t,w);

Equations

Obj          'Objective function'

EB(t,w)      'Energy balance'

GLup(t,w)    'GT capacity limit'
GLlo(t,w)    'GT minimum power output'
SUC0(w)      'Start-up cost definition (t = 1)'
SUC(t,w)     'Start-up cost definition'
GRL0up(w)    'GT ramping limits (t = 1 upward)'
GRLup(t,w)   'GT ramping limits (upward)'
GRL0dn(w)    'GT ramping limits (t = 1 downward)'
GRLdn(t,w)   'GT ramping limits (downward)'
GPE0(w)      'GT power-energy equivalence (t = 1)'
GPE(t,w)     'GT power-energy equivalence'

LRLup0(w)    'Gear factory ramping limits (t = 1 upward)'
```

```
LRLup(t,w) 'Gear factory ramping limits (upward)'
LRLdn0(w)  'Gear factory ramping limits (t = 1 downward)'
LRLdn(t,w) 'Gear factory ramping limits (downward)'
LPE0(w)    'Gear factory power equivalence (t = 1)'
LPE(t,w)   'Gear factory power equivalence'
MEC(w)     'Gear factory minimum energy consumption'

STE0(w)    'PSP state-transition function (t = 1)'
STE(t,w)   'PSP state-transition function';

Obj.. rho =e= sum(t, Price(t)*PD(t)) +sum(w,
        pi(w)*sum(t,-aG*EG(t,w)*EG(t,w) - bG*EG(t,w) - cG*v(t,w)
      + aL*EL(t,w)*EL(t,w) + bL*EL(t,w) + cL - CSU(t,w)
      - Cdiesel*EGdiesel(t,w)));

EB(t,w).. EG(t,w) + EGdiesel(t,w) + PVoutput(t,w) - PWcurt(t,w)
        + PSd(t,w) =e= EL(t,w) + PSc(t,w) + PD(t);

GLup(t,w).. PG(t,w) =l= PGmax * v(t,w);
GLlo(t,w).. PG(t,w) =g= PGmin * v(t,w);
SUC0(w).. CSU('t1',w) =g= SU*(v('t1',w)-V0);
SUC(t,w)$(ord(t)GT1)..  CSU(t,w) =g= SU*(v(t,w)-v(t-1,w));
GRL0up(w)..  PG('t1',w) - PG0 =l= PGramp;
GRLup(t,w)$(ord(t)GT1).. PG(t,w) - PG(t-1,w) =l= PGramp;
GRL0dn(w)..  PG0 - PG('t1',w) =l= PGramp;
GRLdn(t,w)$(ord(t)GT1).. PG(t-1,w) - PG(t,w) =l= PGramp;
GPE0(w).. (PG0 + PG('t1',w))/2 =e= EG('t1',w);
GPE(t,w)$(ord(t)GT1).. (PG(t-1,w) + PG(t,w))/2 =e= EG(t,w);

LRLup0(w).. PL('t1',w) - PL0 =l= PLramp;
LRLup(t,w)$(ord(t)GT1).. PL(t,w) - PL(t-1,w) =l= PLramp;
LRLdn0(w).. PL0 - PL('t1',w) =l= PLramp;
LRLdn(t,w)$(ord(t)GT1).. PL(t-1,w) - PL(t,w) =l= PLramp;
LPE0(w).. (PL0 + PL('t1',w))/2 =e= EL('t1',w);
LPE(t,w)$(ord(t) GT 1).. (PL(t-1,w) + PL(t,w))/2 =e= EL(t,w);
MEC(w).. sum(t, EL(t,w)) =g= MinCon;

STE0(w).. ES('t1',w) =e= ES0 + eff * PSc('t1',w)
                       - (1/eff)* PSd('t1',w);
STE(t,w)$(ord(t) GT 1).. ES(t,w) =e= ES(t-1,w) + eff * PSc(t,w)
                         - (1/eff) * PSd(t,w);

Model StochStrategy /all/;

OPTION iterlim = 1e8;
OPTION reslim = 1e10;
OPTION miqcp = cplex;

StochStrategy.optcr=0;

Solve StochStrategy maximizing rho using miqcp;
```

E.10 GAMS Code for the Example on the VPP Stochastic Trading Strategy Based on Offer Curves in Sect. 8.4

```
$title STOCHASTIC TRADING STRATEGY BASED ON OFFER CURVES.
Sets
t         'Time periods'  /t1, t2, t3/
w         'Scenario index'  /w1, w2, w3/;

alias(w, waux);

Scalar

PGmax       'GT capacity'  /5/
PGmin       'GT minimum power output'  /1/
PGramp      'GT ramp rate'  /2/
PG0         'GT initial power output'  /2/
V0          'GT initial status'  /1/
aG          'Quadratic coefficient of GT cost function'  /5/
bG          'Linear coefficient of GT cost function'  /10/
cG          'No-load cost'  /50/
SU          'Start-up cost'  /10/

PLmax       'Gear factory maximum power consumption'  /2/
PLmin       'Gear factory minimum power consumption'  /0.5/
PLramp      'Gear factory ramp rate'  /1/
PL0         'Gear factory initial power consumption'  /1.5/
MinCon      'Total minimum energy consumption'  /2.5/
aL          'Quadratic coefficient of GF utility function'  /-30/
bL          'Linear coefficient of GF utility function'  /150/
cL          'Constant term of GF utility function'  /5/

ESmax       'PSP equivalent energy capacity'  /1/
ESmin       'PSP minimum storage level'  /0.2/
ES0         'PSP initial storage level'  /0.4/
PScmax      'PSP charging power limit'  /0.3/
PSdmax      'PSP discharging power limit'  /0.5/
eff         'PSP efficiency'  /0.80/
Cdiesel     'Generating cost of the diesel set'  /200/
MaxDiesel   'Capacity of the diesel set'  /0.5/;

Parameters

pi(w)     'Scenario probability'
/
w1        0.2
w2        0.3
w3        0.5/;

TABLE Price(t,w)      'Market price'

           w1      w2       w3
t1         18      38       10
```

```
t2          90      100      64
t3          20      40       58;

TABLE PVoutput(t,w)   'PV power output scenarios'

            w1      w2       w3
t1          2.5     6.0      2.0
t2          4.0     4.0      1.1
t3          6.0     3.5      1.5;

Variables

rho         'Profit'
PD(t,W)     'Energy sold in (>0) or bought from (<0) the market';

Positive variables

CSU(t,w)         'GT start-up cost'
PG(t,w)          'GT power output'
EG(t,w)          'GT energy production'

PL(t,w)          'Gear factory load consumption'
EL(t,w)          'Gear factory energy consumption'

ES(t,w)          'PSP storage level'
PSc(t,w)         'PSP power charge'
PSd(t,w)         'PSP power discharge'
EGdiesel(t,w)    'Electricity generated by the diesel set'

PWcurt(t,w)      'PV generation curtailment';

Binary variables

v(t,w)       'On-off status of the GT unit';

PL.up(t,w)  = PLmax;
PL.lo(t,w)  = PLmin;
PSc.up(t,w)  = PScmax;
PSd.up(t,w)  = PSdmax;
ES.up(t,w)  = ESmax;
ES.lo(t,w)  = ESmin;
EGdiesel.up(t,w)  = MaxDiesel;
PWcurt.up(t,w)  = PVoutput(t,w);

Equations

Obj                  'Objective function'

EB(t,w)              'Energy balance'

GLup(t,w)            'GT capacity limit'
GLlo(t,w)            'GT minimum power output'
SUC0(w)              'Start-up cost definition (t = 1)'
SUC(t,w)             'Start-up cost definition'
```

```
GRL0up(w)          'GT ramping limits (t = 1 upward)'
GRLup(t,w)         'GT ramping limits (upward)'
GRL0dn(w)          'GT ramping limits (t = 1 downward)'
GRLdn(t,w)         'GT ramping limits (downward)'
GPE0(w)            'GT power-energy equivalence (t = 1)'
GPE(t,w)           'GT power-energy equivalence'

LRLup0(w)          'Gear factory ramping limits (t = 1 upward)'
LRLup(t,w)         'Gear factory ramping limits (upward)'
LRLdn0(w)          'Gear factory ramping limits (t = 1 downward)'
LRLdn(t,w)         'Gear factory ramping limits (downward)'
LPE0(w)            'Gear factory power equivalence (t = 1)'
LPE(t,w)           'Gear factory power equivalence'
MEC(w)             'Gear factory minimum energy consumption'

STE0(w)            'PSP state-transition function (t = 1)'
STE(t,w)           'PSP state-transition function'

Inc(t,w, waux)     'Offering curves monotonically increasing';

Obj.. rho =e= sum(w, pi(w)* sum(t,Price(t,w) *PD(t,w)
             - aG*EG(t,w)*EG(t,w) - bG*EG(t,w)
             - cG*v(t,w) + aL*EL(t,w)*EL(t,w) + bL*EL(t,w)
             + cL - CSU(t,w) - Cdiesel*EGdiesel(t,w)));

EB(t,w).. EG(t,w) + EGdiesel(t,w) + PVoutput(t,w) - PWcurt(t,w)
                + PSd(t,w) =e= EL(t,w) + PSc(t,w) + PD(t,w);

GLup(t,w).. PG(t,w) =l= PGmax * v(t,w);
GLlo(t,w).. PG(t,w) =g= PGmin * v(t,w);
SUC0(w).. CSU('t1',w) =g= SU*(v('t1',w)-V0);
SUC(t,w)$(ord(t) GT 1)..  CSU(t,w) =g= SU*(v(t,w)-v(t-1,w));
GRL0up(w)..  PG('t1',w) - PG0 =l= PGramp;
GRLup(t,w)$(ord(t) GT 1).. PG(t,w) - PG(t-1,w) =l= PGramp;
GRL0dn(w)..  PG0 - PG('t1',w) =l= PGramp;
GRLdn(t,w)$(ord(t) GT 1).. PG(t-1,w) - PG(t,w) =l= PGramp;
GPE0(w).. (PG0 + PG('t1',w))/2 =e= EG('t1',w);
GPE(t,w)$(ord(t) GT 1).. (PG(t-1,w) + PG(t,w))/2 =e= EG(t,w);

LRLup0(w).. PL('t1',w) - PL0 =l= PLramp;
LRLup(t,w)$(ord(t) GT 1).. PL(t,w) - PL(t-1,w) =l= PLramp;
LRLdn0(w).. PL0 - PL('t1',w) =l= PLramp;
LRLdn(t,w)$(ord(t) GT 1).. PL(t-1,w) - PL(t,w) =l= PLramp;
LPE0(w).. (PL0 + PL('t1',w))/2 =e= EL('t1',w);
LPE(t,w)$(ord(t) GT 1).. (PL(t-1,w) + PL(t,w))/2 =e= EL(t,w);
MEC(w).. sum(t, EL(t,w)) =g= MinCon;

STE0(w).. ES('t1',w) =e= ES0 + eff * PSc('t1',w) - (1/eff) * PSd(
     't1',w);
STE(t,w)$(ord(t) GT 1).. ES(t,w) =e= ES(t-1,w) + eff * PSc(t,w)
                                - (1/eff) * PSd(t,w);
```

```
Inc(t,w, waux)$((Price(t,w) GE Price(t,waux)) and (ord(w) GT ord(
    waux)))..
PD(t,w) =g= PD(t,waux);

Model StochCurves /all/;
OPTION iterlim = 1e8;
OPTION reslim = 1e10;
OPTION miqcp = cplex;

StochCurves.optcr=0;

Solve StochCurves maximizing rho using miqcp;
```

E.11 GAMS Code for the Example in Sect. 9.4.1 on Solving the Consumer Problem Using Stochastic Programming

```
$TITLE DEFERRABLE LOAD PROBLEM BASED ON STOCHASTIC PROGRAMMING

SET
         t          'time period'              /t1 * t4/
         k          'demand block'             /k1 * k1/
         w          'scenario'                 /w1 * w2/;

SCALARS
         Hmax       'maximum hourly consumption'     /3/;
TABLE    pDA(t,w)   'price of electricity'
         w1      w2
t1       120     120
t2       105      45
t3       154      66
t4        84      36;

PARAMETERS
p(w)            'probability of scenario w'
/w1      0.5
 w2      0.5/

b(k)            'utility of block k (time-dependent in exercises)'
/k1      100/

Dmin(k)         'minimum consumption from block k'
/k1      6/

Dmax(k)         'maximum consumption from block k'
/k1      8/

RU(k)           'ramp-up limit'
/k1      1.5/

RD(k)           'ramp-down limit'
/k1      1.5/

u0(k)           'initial consumption for block k'
/k1      0/;
```

```
POSITIVE VARIABLE
u(t,k,w)          'consumption for block k, time t, scenario w';

VARIABLE
z                 'objective value';
EQUATIONS
obj               'objective function'
nant(t,k,w)       'non-anticipativity'
RUL(t,k,w)        'ramp-up limit'
RDL(t,k,w)        'ramp-down limit'
HL(t,w)           'hourly consumption limit'
DLL(k,w)          'daily consumption lower limit'
DUL(k,w)          'daily consumption upper limit';

obj .. z =E= sum((t,k,w), p(w)*(pDA(t,w) - b(k))*u(t,k,w));
nant(t,k,w)$(ord(w)>1).. u('t1',k,w) =E= u('t1',k,w-1);
RUL(t,k,w)$(ord(t)>1)..
u(t,k,w) - u(t-1,k,w)$(ord(t)>1) - u0(k)$(ord(t)=1) =L= RU(k);
RDL(t,k,w)$(ord(t)>1)..
u(t,k,w) - u(t-1,k,w)$(ord(t)>1) - u0(k)$(ord(t)=1) =G= - RD(k);
HL(t,w).. sum(k, u(t,k,w) ) =L= Hmax;
DLL(k,w).. sum(t, u(t,k,w) ) =G= Dmin(k);
DUL(k,w).. sum(t, u(t,k,w) ) =L= Dmax(k);

MODEL DR /obj, nant, RUL, RDL, HL, DLL, DUL/;

SOLVE DR MINIMIZING z USING LP;

DISPLAY z.l, u.l;
```

E.12 GAMS Code for the Example in Sect. 9.4.2 on Solving the Consumer Problem Using Robust Optimization

```
$TITLE DEFERRABLE LOAD PROBLEM BASED ON ROBUST OPTIMIZATION

SET
         t               'time period'            /t1 * t4/
         k               'demand block'           /k1/;

SCALARS
         Hmax            'maximum hourly consumption'     /3/
         Gamma           'uncertainty budget'             /2/;
PARAMETERS
pDA(t)         'price of electricity'
/t1       120
 t2        75
 t3       110
 t4        60/

pDAdv(t)       'maximum deviation of electricity price'
/t1         0
 t2        30
 t3        44
 t4        24/
b(k) 'utility of block k (time-dependent in exercises)'
```

```
/k1       100/

Dmin(k) 'minimum consumption from block k'
/k1       6/

Dmax(k) 'maximum consumption from block k'
/k1       8/

RU(k) 'ramp-up limit'
/k1       1.5/

RD(k) 'ramp-down limit'
/k1       1.5/

u0(k) 'initial consumption for block k'
/k1       0/;

POSITIVE VARIABLE
u(t,k)    'consumption for block k, time t, scenario w'
xi(t)     'dual variable for upper bound of relative deviation being 1'
beta      'dual variable for uncertainty budget constraint';

VARIABLE
z         'objective value';

EQUATIONS
obj       'objective function'
RUL(t,k)  'ramp-up limit'
RDL(t,k)  'ramp-down limit'
HL(t)     'hourly consumption limit'
DLL(k)    'daily consumption lower limit'
DUL(k)    'daily consumption upper limit'
DC(t)     'constraint of the dual inner maximization problem'
obj.. z =E= sum((t,k), (pDA(t) - b(k))*u(t,k))
                       + sum(t$(ord(t)>1), xi(t)) + Gamma*beta;
RUL(t,k)$(ord(t)>1)..
u(t,k) - u(t-1,k)$(ord(t)>1) - u0(k)$(ord(t)=1) =L= RU(k);
RDL(t,k)$(ord(t)>1)..
u(t,k) - u(t-1,k)$(ord(t)>1) - u0(k)$(ord(t)=1) =G= - RD(k);
HL(t).. sum(k, u(t,k)) =L= Hmax;
DLL(k).. sum(t, u(t,k)) =G= Dmin(k);
DUL(k).. sum(t, u(t,k)) =L= Dmax(k);
DC(t)$(ord(t)>1).. xi(t) + beta =G= pDAdv(t)*sum(k, u(t,k));

MODEL DR /obj, RUL, RDL, HL, DLL, DUL, DC/;

SOLVE DR MINIMIZING z USING LP;

DISPLAY z.l, u.l, xi.l, beta.l, DC.m;
```

References

1. Rosenthal, R.E.: GAMS — A User's Guide. GAMS Development Corporation, Washington, DC, USA (2013)
2. The GAMS Development Corporation Website: http://www.gams.com (2013)

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J. M. Morales et al., *Integrating Renewables in Electricity Markets,*
International Series in Operations Research & Management Science 205,
DOI 10.1007/978-1-4614-9411-9,