An Adaptive Robust Optimization Model for Power Systems Planning With Operational Uncertainty

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***Abstract*—There is an increasing necessity for new long-term planning models to adequately assess the flexibility requirements of significant levels of short-term operational uncertainty in power systems with large shares of variable renewable energy. In this context, this paper proposes an adaptive robust optimization model for the generation and transmission expansion planning problem. The proposed model has a two-stage structure that separates investment and operational decisions, over a given planning horizon. The key attribute of this model is the representation of daily operational uncertainty through the concept of representative days and the design of uncertainty sets that determine load and renewable power over such days. This setup allows an effective representation of the flexibility requirements of a system with large shares of variable renewable energy, and the consideration of a broad range of operational conditions. To efficiently solve the problem, the column and constraint generation method is employed. Extensive computational experiments on a 20-bus and a 149-bus representation of the Chilean power system over a 20-year horizon show the computational efficiency of the proposed approach, and the advantages as compared to a deterministic model with representative days, due to an effective spatial placement of both variable resources and flexible resources.**

***Index Terms*—Generation expansion planning, renewable energy, robust optimization, transmission expansion planning.**

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I. Introduction

PLANNING power systems in the long term is a critical task for ensuring a reliable, sustainable, and cost-efficient electricity supply. The Generation and Transmission Expansion Planning (EP) problem provides a key support for this process. This problem aims to optimize the investments on the generation and transmission infrastructure of a power system over a given planning horizon [1]. This task has become particularly challenging in recent years, due to the significant levels of uncertainty introduced by large shares of variable renewable energy. In order to correctly asses such investment decisions, an effective representation of the system operation is required to properly account for the short-term uncertainty introduced by renewable resources. This paper proposes a new EP model to address this challenge.

The representation of the power system's operation within the Generation and Transmission EP problem has been extensively studied. Early EP models used the concept of the *load duration curve* and *load blocks*, which overlooked the chronology of demand and variable generation profiles in order to reduce the complexity of the problem [2]. This simplification becomes less useful as the penetration of variable renewable energy increases, where a low level of temporal detail may have a great impact in the resulting expansion plan [3]. Also, the use of load blocks is not compatible with generator ramps or commitment constraints, which may result in the underestimation of the flexibility requirements of a given power system.

Due to this challenge, recent models have developed new methods for an appropriate representation of the system's flexibility requirements, such as *system states*, *representative blocks* or *representative days*. In specific, the use of chronologically ordered time points has allowed the inclusion of ramp constraints and in some cases the inclusion of commitment constraints and reserve requirements in long term planning problems [4]–[6].

The treatment of uncertainty in planning problems has been extensively studied as well. It is possible to classify uncertainties by means of their time horizon into short-term and long-term uncertainties [7]. Short-term uncertainties may include operational parameters such as renewable resource availability, hourly load variations, and operational contingencies, among others, while long term uncertain parameters may include investment costs for different technologies, fuel prices, load growth, or even hydro inflows. In particular, large shares of variable renewable energy greatly increase short-term or operational uncertainty, and neglecting this uncertainty may lead to significant estimation errors in the system's costs and unprepared expansion plans [8], [9].

The scientific literature presents two main approaches to tackle uncertainty in expansion planning problems, namely, Stochastic Programming and Robust Optimization. On the one hand, most Stochastic Programming models use a discrete set of scenarios to represent the probability distribution of the uncertain parameters in the EP problem [10]–[12]. Stochastic Programming is applied mainly to long term uncertainties, due to the fact that including the required number of operational scenarios to represent short-term uncertainty may cause the problem to become computationally intractable, specially under long planning horizons.

On the other hand, Robust Optimization methods rely on the construction of uncertainty sets to explicitly represent potential uncertainty realizations, while preparing the system for every realization in such set. The combination of load blocks with Robust Optimization concepts has been used to model uncertainty in Generation EP [13], and also in Transmission EP [14]–[17]. Further, other authors have employed a set of operational time points in Robust Optimization models for the Transmission EP [7], [18], [19], and for the Generation and Transmission EP [20], [21]. Moreover, some works consider contingency and reliability constraints through the combination of system states and Robust Optimization concepts in Transmission EP [22], [23], and in Generation and Transmission EP [24], [25].

Despite recent progress, some important gaps remain in the EP literature. In fact, EP models usually consider various operational simplifications, such as the consideration of a single future target year as opposed to multi-period models, or the separate consideration of generation and transmission investment decisions, as opposed to their joint consideration. Further, none of the mentioned Robust Optimization works in this area consider a daily chronologically linked operation. This precludes the inclusion of various key operational aspects, including generator ramp constraints, which are an essential element of power system flexibility.

This paper aims to fill this gap through the development of an adaptive robust optimization model for the Generation and Transmission EP problem, considering the use of *uncertain representative days* (i.e., representative days with uncertainty sets) to represent short-term uncertainty and variability, allowing a chronological representation of the system operation, and thus enabling the inclusion of key operational aspects. Also, the use of uncertainty sets allows the consideration of a broad range of operating conditions without comprising computational tractability.

The main contributions of this paper can be summarized as follows:

1) Proposal of a novel two-stage adaptive robust optimization model for the Generation and Transmission Expansion Planning problem considering an explicit representation of short-term operational uncertainty and capturing the chronology in load and renewable profiles, resulting in a valuable support for the strategic placement of flexible and variable resources.
2) Proposal of a new approach for considering representative days in planning models based on specially designed uncertainty sets to represent load and renewable uncertainty. This allows the consideration of a broad range of operating conditions without comprising computational tractability, reducing the risk of inappropriately choosing specific load and renewable profiles. This is a key element of the proposed planning model and it can also be employed separately to provide useful insights about the future operation of the system under days with complicated realizations of load and renewable resources.
3) Assessment of the proposed model as compared to an approach based on a given set of deterministic representative days. The results show the effectiveness of the resulting expansion plan in handling operational uncertainty reliably and in a cost-effective way.

The remainder of this paper is organized as follows. The relevant mathematical models are presented in Section II. Section III describes the solution method for the proposed model. Section IV presents extensive computational experiments based on the Chilean power system. Section V concludes.

II. Mathematical Models

A. Deterministic Expansion Planning Model

EP models consist of selecting generation and transmission investments over a given planning horizon in order to minimize total system costs and satisfy various requirements. To compare the proposed model, we will begin presenting a deterministic expansion planning (DEP) model, that employs the concept of representative days. A planning horizon composed of several planning periods is considered, where the systems operation in each period is represented through the operation under a representative set of days (namely, representative days), each of which has a set of chronological time points in it. The formulation of the DEP model is as follows:

$$\begin{aligned}\min_{\boldsymbol{x}^g, \boldsymbol{x}^l, \boldsymbol{z}^l, \boldsymbol{p}^g, \boldsymbol{p}^{LS}, \boldsymbol{\theta}^b, \boldsymbol{f}^l} \quad & \sum_{e\in\mathcal{E}} \Bigg[\sum_{i\in\mathcal{G}^C} C^I_{ei}\, x^g_{ei} \\ & + \sum_{k\in\mathcal{L}^C} C^I_{ek}\, z^l_{ek} + \sum_{i\in\mathcal{G}^E} C^E_{ei}\, x^g_{ei} + \sum_{j\in\mathcal{L}^E} C^E_{ej}\, x^l_{ej} \\ & + \sum_{d\in\mathcal{D}} w_d \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{G}} C^g_{ei}\, p^g_{edti} + \sum_{b\in\mathcal{B}} C^{LS}_{eb}\, p^{LS}_{edtb} \right) \Bigg] \end{aligned} \tag{1a}$$

$$\text{s.t.} \quad 0 \le \sum_{e\in\mathcal{E}} x^g_{ei} \le \overline{x}^g_i \quad \forall\, i \in \mathcal{G} \tag{1b}$$

$$0 \le \sum_{e\in\mathcal{E}} x^l_{ej} \le \overline{x}^l_j \quad \forall\, j \in \mathcal{L}^E \tag{1c}$$

$$0 \le \sum_{e\in\mathcal{E}} z^l_{ek} \le 1 \quad \forall\, k \in \mathcal{L}^C \tag{1d}$$

$$z^l_{ek} \in [0,1] \quad \forall\, k \in \mathcal{L}^C, \forall\, e \in \mathcal{E} \tag{1e}$$

$$0 \leq \sum_{j\in\mathcal{L}^E} C^E_{ej}\, x^l_{ej} + \sum_{k\in\mathcal{L}^C} C^I_{ek}\, z^l_{ek} \leq \overline{I^L_e} \quad \forall\, e\in\mathcal{E} \quad (1f)$$

$$0 \leq \sum_{i\in\mathcal{G}^E} C^E_{ei}\, x^g_{ei} + \sum_{i\in\mathcal{G}^C} C^I_{ei}\, x^g_{ei} \leq \overline{I^G_e} \quad \forall\, e\in\mathcal{E} \quad (1g)$$

$$\Bigg\{ 0 \leq p^g_{edti} \leq \overline{p^g_{ei}} + \sum_{\bar{e}\leq e} x^g_{\bar{e},i} \quad \forall\, i\in\mathcal{G}\backslash\mathcal{G}^V \quad (1h)$$

$$0 \leq p^g_{edti} \leq \Big(\overline{p^g_{ei}} + \sum_{\bar{e}\leq e} x^g_{\bar{e}i}\Big)\, \alpha_{dti} \quad \forall\, i\in\mathcal{G}^V \quad (1i)$$

$$r^d_i \leq p^g_{edti} - p^g_{ed,t-1,i} \leq r^u_i \quad \forall\, i\in\mathcal{G},\, t>0 \quad (1j)$$

$$-\sum_{\bar{e}\leq e} z^l_{ek}\overline{f_{ek}} \leq f_{edtk} \leq \sum_{\bar{e}\leq e} z^l_{ek}\overline{f_{ek}} \quad \forall\, k\in\mathcal{L}^C \quad (1k)$$

$$-\overline{f_{ej}} - \sum_{\bar{e}\leq e} x^l_{ej} \leq f_{edtj} \leq \overline{f_{ej}} + \sum_{\bar{e}\leq e} x^l_{ej} \quad \forall\, j\in\mathcal{L}^E \quad (1l)$$

$$-M\left(1-\sum_{\bar{e}\leq e} z^l_{ej}\right) \leq b_j(\theta_{edt,j(s)}-\theta_{edt,j(r)}) - f_{edtj}$$
$$\leq M\left(1-\sum_{\bar{e}\leq e} z^l_{ek}\right) \quad \forall\, k\in\mathcal{L}^C \quad (1m)$$

$$b_j\left(\theta_{edt,j(s)} - \theta_{edt,j(r)}\right) = f_{edtj} \quad \forall\, j\in\mathcal{L}^E \quad (1n)$$

$$\sum_{i\in\mathcal{G}(b)} p^g_{edti} + \sum_{j|j(s)=b} f_{edt,j} - \sum_{j|j(r)=b} f_{edt,j}$$
$$= p^D_{edtb} - p^{LS}_{edtb} \quad \forall\, b\in\mathcal{B} \Bigg\},$$
$$\forall e\in\mathcal{E},\, \forall d\in\mathcal{D},\, \forall t\in\mathcal{T} \quad (1o)$$

Here, $\mathcal{E}, \mathcal{D}, \mathcal{T}, \mathcal{B}, \mathcal{G}, \mathcal{L}$ are the sets of planning periods, representative days, time points, buses, generators and transmission lines respectively. Sets $\mathcal{G}^E$, $\mathcal{G}^C$, $\mathcal{L}^E$ and $\mathcal{L}^C$ correspond to existing generators, candidate generators, existing lines and candidate lines, hence $\mathcal{G}^E \cup \mathcal{G}^C = \mathcal{G}$ and $\mathcal{L}^E \cup \mathcal{L}^C = \mathcal{L}$. $\mathcal{G}^V \subset \mathcal{G}$ is the set of variable generators such as solar or wind generators.

Decisions x^g_{ei}, x^l_{ej} and z^l_{ek} represent investments in the generation capacity of generator i, the transmission capacity of transmission line j, and the installation of a new candidate line k, for planning period e, respectively. The other decisions correspond to operational decisions for every time point t within every representative day d, in every planning period e, namely generation at each unit (p^g_{edti}), power flow through each line ($f_{edt,l}$), voltage at each bus (θ_{edtb}), and load shedding (p^{LS}_{edtb}).

Parameters C^I_{ei} and C^I_{ek} represent investment costs for candidate generator i and candidate line k. C^E_{ei} and C^E_{ej} represent expansion costs for existing generator i and existing line j for planning period e. C^g_{ei} and C^{LS}_{eb} represent respectively generation cost of generator i and load shedding penalty in bus b for planning period e. $\overline{x}^g_i$ and $\overline{x}^l_j$ represent upper bounds for the capacity of generator i and line j respectively. $\overline{I^G_e}$ and $\overline{I^L_e}$ are investment budgets for generation and transmission projects respectively, in expansion period e. $\overline{p^g_{ei}}$ represents the base capacity of generator i in expansion period e. α_{dti} is a capacity factor between 0 and 1, which represents the resource availability for variable generator i in time point t of representative day d. r^d_i and r^u_i are, respectively, the ramping down and up capacities of generator i between consecutive time points. $\overline{f_{ej}}$ and b_j are, respectively, the base flow capacity of transmission line j and its susceptance. p^D_{edtb} represents power demand at node b in a given time point. Finally, M is a large enough constant.

The objective function (1a) considers investment costs for candidate units and lines, expansion costs for existing units and lines, and generation and load shedding costs for every time point in every representative day. Parameter w_d represents the weight of representative day d, and it corresponds to the number of days in a planning period that such day represents.

Constraints can be divided into investment-related or operational. First, (1b)–(1g) are investment-related. Here, (1b) limits new capacity for generation units, (1c) limits new capacity for existing lines, (1d) ensures that a candidate line is only installed once during the planning horizon, (1e) restricts possible values for binary variable z^l, and (1f)–(1g) set investment budgets for generation units and transmission lines in each planning period.

Constraints (1h)–(1o) are related to the operation of the system. Here, (1h) limits the power output of conventional generators to the sum of their initial capacity and new capacity. For candidate units, the initial capacity is set to zero. Also, (1i) limits the power output of variable renewable generators to their resource availability, which is given by the product of the maximum capacity and the *capacity factor* α_{dti}. Note that α_{dti} is indexed by time point t in representative day d, and represents the renewable resource profile of variable generator i. Constraint (1j) limits the power output changes between two consecutive time points due to technical ramp limits. (1k) limits the power flow through candidate lines, and (1l) limits the power flow through previously existing lines. Eq. (1m) relates voltage angle variables to power flow variables through a DC power flow model for candidate lines, when such lines have been installed, using a Big-M model. Equivalently, (1n) determines a DC power flow model for existing lines. Finally, (1o) ensures the load balance in every time point.

Note that operational constraints are formulated in every planning period, for every time point of every representative day. The goal of using representative days is to provide a chronological, more detailed representation of the operation than other early EP approaches, while allowing the consideration of different operating conditions throughout a specific planning period. It has been shown that for power systems with large shares of renewable energy, the representative days approach is more adequate for an accurate representation of the system's requirements and operating conditions in comparison with load-block models, providing more efficient expansion plans [3], [11], [26].

Note also that the DEP model in (1) contains an implicit representation of operational uncertainty via representative days. In fact, this formulation is mathematically equivalent to a stochastic model where representative days are the scenarios, and their respective weight is the probability of occurrence of such scenarios. The goal is that the use of these representative days, typically selected by a clustering algorithm, will provide enough information for the model to consider different possible operational situations. Applications of this stochastic interpretation can be found in [7] and [20]. However, this approach may not completely capture intra-day flexibility requirements.

Due to such limitation of DEP, it is a common industry practice to consider a deterministic planning reserve margin (PRM) to prepare the system for errors in peak load forecasting or other unexpected events. PRM consists on a percentage capacity requirement above the system's peak or net peak load [27], [28]. In this paper we will consider the following equation for the inclusion of such margin:

$$\sum_{i\in\mathcal{G}} F_i \left(\overline{p^g_{ei}} + \sum_{\bar{e}\leq e} x^g_{\bar{e},i}\right) \geq \overline{p^{D_N}_e}\,(1+PRM) \tag{1p}$$

In (1p), $\overline{p^{D_N}_e}$ corresponds to the system's net peak load in planning period e. The factor F_i is included to indicate whether the generation unit i may or may not contribute with reserve capacity ($0 \leq F_i \leq 1$). A usual approach is to consider that non-dispatchable generators may not provide reserve capacity ($F_i = 0$).

B. Robust Expansion Planning Model

The presented DEP model considers a a set of deterministic representative days to prepare for the different conditions of load and renewable availability, and a planning reserve capacity margin to prepare against unexpected realizations of short-term uncertainty. This poses several challenges. On one hand, finding or sampling accurate load and renewable profiles that properly represent a whole year's operation is a complex task. Similarly to the scenario representation of a probability function, it becomes ideal to consider a large number of representative days or scenarios, however, this may cause the problem to lose computational tractability. On the other hand, the PRM presented in (1p) may be insufficient to capture key geographical and statistical aspects when planning for high load or renewable resource scarcity. An alternative approach to face these issues is the use of Robust Optimization techniques.

Motivated by this, the robust expansion planning (REP) model proposed in this paper is presented as follows,

$$\min_{\boldsymbol{x}\in X}\left(\boldsymbol{c}^\top\boldsymbol{x} + \max_{\boldsymbol{\xi}\in\Xi}\ \min_{\boldsymbol{y}\in Y(\boldsymbol{x},\boldsymbol{\xi})} \boldsymbol{b}^\top\boldsymbol{y}\right). \tag{2}$$

In this problem, $\boldsymbol{x} = (\boldsymbol{x}^g, \boldsymbol{x}^l, \boldsymbol{z}^l)$ and X represents DEP investment constraints (1b)–(1g). Vector $\boldsymbol{\xi}$ represents a realization of the short-term uncertainty in the operational constraints of the problem, and Ξ is the *uncertainty set* for such realizations. In this paper, we consider uncertainty in load and renewable capacity factors, hence $\boldsymbol{\xi} = (\boldsymbol{p}^D, \boldsymbol{\alpha})$. Also, we take $\Xi = \mathcal{U}\times\mathcal{A}$, where $\mathcal{U}$ is an uncertainty set for load ($\boldsymbol{p}^D$) and $\mathcal{A}$ is an uncertainty set for the capacity factors of the variable renewable units ($\boldsymbol{\alpha}$). Finally, $\boldsymbol{y} = (\boldsymbol{p}^g, \boldsymbol{p}^{LS}, \boldsymbol{f}^l, \boldsymbol{\theta})$ and $Y(\boldsymbol{x},\boldsymbol{\xi})$ represents DEP operational constraints (1h)–(1o) under given investment decisions ($\boldsymbol{x}$) as well as given loads and capacity factors of renewable units ($\boldsymbol{\xi}$) for all planning periods, representative days and time points.

The formulation in (2) consists of a two-stage robust optimization problem [7], [29]–[31], where the first stage considers investment decisions and the second stage is an operational problem where uncertainty is realized in a way that maximizes operational costs. The goal of this formulation is to immunize the system towards any possible realization within the uncertainty set Ξ, in other words, it explicitly protects the system against the worst-case realization of uncertain parameters in such set.

In summary, the proposed REP model employs *uncertain representative days*. This induces the consideration of a broader range of operational conditions within any given day, thus allowing a more thorough representation of the short-term operational dynamics within the planning problem. Further, this approach considerably reduces the risk of inappropriately selecting specific load or renewable profiles for representing the operation of the system throughout a whole year.

C. Uncertainty Sets for Load and Renewables

Uncertainty sets are a key concept in Robust Optimization. The idea is that the decisions selected by the model will be prepared to any realization of uncertainty within such set. There exists extensive discussion regarding various approaches on the construction of such sets (see e.g. [16], [24], [32], [33]). In this paper, we consider the following polyhedral uncertainty set for power demand:

$$\mathcal{U} = \Bigg\{\boldsymbol{p}^D : \sum_{b\in\mathcal{B}} \frac{\left|p^D_{edtb} - \tilde{p}^D_{edtb}\right|}{\hat{p}^D_{edtb}} \leq \Gamma_D\sqrt{|\mathcal{B}|} \tag{3a}$$

$$p^D_{edtb} \in [\tilde{p}^D_{edtb} - \Gamma_D\,\hat{p}^D_{edtb},\ \tilde{p}^D_{edtb} + \Gamma_D\,\hat{p}^D_{edtb}]$$

$$\forall e\in\mathcal{E}, d\in\mathcal{D}, t\in\mathcal{T}\Bigg\}, \tag{3b}$$

where Γ_D is a conservativeness parameter, which controls the size of the set. Also, $\tilde{p}^D_{edtb}$ and $\hat{p}^D_{edtb}$ are, respectively, a nominal value and a variability parameter for power load in period e, day d, time t and bus b. As an example, these parameters can be selected using the historical mean and historical standard deviation.

On the other hand, the uncertainty set for variable generation technologies is defined as $\mathcal{A}$. In this paper, we consider both wind and solar generation as uncertain generation technologies, and $\mathcal{A} = \mathcal{A}^{\text{Wind}} \times \mathcal{A}^{\text{Solar}}$, where each individual uncertainty set is described as follows:

$$\mathcal{A}^{\text{v}} = \Bigg\{\boldsymbol{\alpha} : \sum_{i\in\mathcal{G}^{\text{v}}} \frac{|\alpha_{dti} - \tilde{\alpha}_{dti}|}{\hat{\alpha}_{dti}} \leq \Gamma_{\text{v}}\sqrt{|\mathcal{G}^{\text{v}}|} \tag{4a}$$

$$\alpha_{dti} \in [\tilde{\alpha}_{dti} - \Gamma_{\text{v}}\ \hat{\alpha}_{dti}, \tilde{\alpha}_{dti} + \Gamma_{\text{v}}\,\hat{\alpha}_{dti}]$$

$$\forall e\in\mathcal{E}, d\in\mathcal{D}, t\in\mathcal{T}\Bigg\}. \tag{4b}$$

Here, $v \in \{\text{Wind}, \text{Solar}\}$, and $\mathcal{G}^{\mathrm{v}} \subset \mathcal{G}$ is the set of generators corresponding to variable technology v. The structure of the uncertainty set mirrors that of (3), with Γ_{v} as the conservativeness parameter that determines the size of each set.

Note that these equations consider every time point in every representative day during the planning period, hence, a realization of the uncertain vector $\boldsymbol{\xi}$ consists of $|\mathcal{E}| \times |\mathcal{D}|$ daily load profiles for each bus, and daily profiles for the capacity factors of each renewable generator. It is also important to note that uncertainty sets $\mathcal{U}$ and $\mathcal{A}$ contain an infinite number of load and renewable resource availability profiles. Recall from the description of (2) that the goal of that formulation is to protect the system against any possible realization within the defined uncertainty sets. By explicitly optimizing for the worst scenario within such sets, the problem remains tractable, while being able to consider a broad range of operational conditions, defined within the considered representative days.

Finally, note that by increasing the value of Γ, more extreme realizations of uncertain parameters are included in the uncertainty sets, hence, more conservative solutions will be obtained for any given EP problem. In this paper, empirical evidence regarding the impact of this parameter in the overall expansion plan is provided in Section IV-A. Also, Sections IV-B and IV-C provide insight on how such parameter impacts the operational performance of the obtained solutions. The election on the value of the conservativeness parameter for any application must be supported by similar experiments and simulation, and it will ultimately depend on reliability, environmental and economic criteria.

III. Solution Methods for Robust EP

Due to the structure of the REP model, a convenient reformulation is introduced using the auxiliary variable η:

$$\min_{\boldsymbol{x} \in X, \eta} \left(\boldsymbol{c}^\top \boldsymbol{x} + \eta\right) \tag{5a}$$

$$\text{s.t.} \quad \eta \geq \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \quad \forall \boldsymbol{\xi} \in \Xi. \tag{5b}$$

Here, η represents the worst-case operational cost in the objective function (5a). The inclusion of constraint (5b) ensures that this cost is that of the worst possible uncertainty realization within the set Ξ.

Note that given the continuous uncertainty set presented in Section II-C, the reformulation in (5) considers an infinite number of constraints, since there are infinite possible uncertainty realizations. An effective solution method has been developed for problems with this structure, namely, the column constraint and generation (CCG) method [29], [31].

A. Column and Constraint Generation Method

Let $\mathcal{S} \subset \Xi$, represent a subset containing a finite number of uncertainty realizations. Now, consider the following relaxation of the reformulation of REP (5):

$$\min_{\boldsymbol{x} \in X, \eta} \left(\boldsymbol{c}^\top \boldsymbol{x} + \eta\right) \tag{6a}$$

$$\text{s.t.} \quad \eta \geq \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \quad \forall \boldsymbol{\xi} \in \mathcal{S}. \tag{6b}$$

Algorithm 1: CCG Solution Method for REP.

$k \leftarrow 0$, $S \leftarrow \emptyset$
repeat
  $(\boldsymbol{x}, \eta) \leftarrow$ optimal solution of the master problem (6)
  Evaluate $f(\boldsymbol{x})$: $\boldsymbol{\xi}^*_{k+1} \leftarrow$ optimal solution of (7)
  $S \leftarrow S \cup \{\boldsymbol{\xi}^*_{k+1}\}$
  $k \leftarrow k+1$
until $f(\boldsymbol{x}) \leq \eta$

Here, (6b) consists of a finite subset of the infinite number of constraints in (5b). This problem yields a first stage solution, $\boldsymbol{x}^*$, which consists of an expansion plan prepared to deal with the worst realization of uncertainty within the subset $\mathcal{S}$.

The CCG method consists of the sequential addition of uncertainty realizations, $\boldsymbol{\xi}^*$, to the subset $\mathcal{S}$, until the optimal solution of the relaxation presented in (6) remains invariant. Here, $\boldsymbol{\xi}^*$ corresponds to the uncertainty realization within Ξ which maximizes operational costs under a given first stage solution, $\boldsymbol{x}^*$, and it can be found solving the following optimization subproblem:

$$\max_{\boldsymbol{\xi} \in \Xi} \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y}. \tag{7}$$

Let $f(\boldsymbol{x})$ be the objective value of problem (7), or in other words, the operational cost for the worst realization of uncertainty for fixed first stage investment decisions $\boldsymbol{x}$). With this, the CCG method is formally presented in Algorithm 1.

It has been shown that the CCG algorithm has finite convergence for polyhedral Ξ and $Y(\boldsymbol{x}, \boldsymbol{\xi})$ [29], which is the case in this paper.

It remains to discuss how to solve the operational subproblem (7). Since $Y(\boldsymbol{x}, \boldsymbol{\xi})$ is polyhedral, we can take the dual in the inner min problem to obtain an overall bilinear problem, given that Ξ is also polyhedral. This bilinear problem can be approximately solved using an *alternating direction method*. The idea is to fix $\boldsymbol{\xi}$ and solve over the dual variables, then fix the dual variables and solve over $\boldsymbol{\xi}$, and so on. See [32] for details.

IV. Computational Experiments

This Section presents extensive computational experiments comparing the DEP and REP models with the purpose of understanding how the proposed model can provide support within the EP process, by finding expansion plans that are effectively prepared for significant operational uncertainty.

In Sections IV-A–IV-C we employ a test case that consists of a 20-bus representation of the Chilean Power System. This test case has 24 transmission corridors, 136 existing generation units and 178 candidate generation units, and is based on [34]. For Section IV-D we employ a larger test case that consists of a 149-bus representation of the same system. Load and renewable capacity factor profiles for the 365 days of the base year (2018), load growth for each planning period, and generation and investment costs are also based on [34].

A 20-year planning horizon was considered, divided into five planning periods of 4 years each, considering a yearly 6% discount rate in the objective function of DEP and REP.

In order to determine the operating conditions for the case study, the 365 days of data for the base year were grouped into

TABLE I
COST DECOMPOSITION FOR DEP AND REP FOR THE 20-BUS CASE STUDY

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Gen Inv. (MM $)	11,107	11,272	11,616	12,178
Line Inv. (MM $)	5.6443	5.8749	6.1317	6.4246
Inv. Cost (MM $)	11,112	11,278	11,622	12,184
Op. Cost (MM $)	17,132	17,073	16,852	16,517
Total Cost (MM $)	28,360	28,467	28,591	28,818
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Gen Inv. (MM $)	11,546	11,970	12,343	12,927
Line Inv. (MM $)	5.2057	5.9421	6.0166	5.9851
Inv. Cost (MM $)	11,551	11,976	12,349	12,933
Worst Op. Cost (MM $)	18,907	19,860	20,910	21,671
Worst Total Cost (MM $)	30,487	31,860	33,286	34,629

three clusters using a *hierarchical clustering* technique [35]. Each of these clusters determines a deterministic representative day in DEP, and an uncertain representative day in REP. The weight of these representative days was set based on the number of elements grouped on each cluster times the number of years in each expansion period.

As discussed before, the construction of representative days for DEP requires the election of individual load and renewable profiles from these clusters, while REP requires the construction of uncertainty sets, which employ nominal and variation profiles. For DEP, the medioid of each cluster was chosen to provide the load and capacity factors for each representative day. On the other hand, in REP, for each representative day uncertainty sets were built considering the hourly average and standard deviation as nominal and variation parameters for load and capacity factor profiles. For analysis purposes, throughout the experiments the value of the conservativeness parameters Γ_D and Γ_v are considered to be the same for each instance of REP, and equal to a global conservativeness parameter Γ.

All models were programmed on Pyomo, a Python-based, open-source optimization modelling language, using Gurobi as solver. For all cases, we considered a convergence tolerance of 0.1%. All experiments were implemented in a Dell PowerEdge R360 server with an Intel Xeon CPU E5-2630 v4 processor running at 2.20GHz, and 64 GB of RAM, using a maximum of 20 cores.

A. Expansion Plans and Computational Aspects

This part studies the solution of the proposed REP model, and its comparison to DEP for various levels of PRM and various levels of Γ, respectively.

First, Table I presents the cost structure for the solutions obtained by the DEP and REP models. From this table, it can be observed that investment cost and investment on generation capacity increases for more conservative solutions (i.e. as PRM or Γ increase). However, investment on transmission capacity does not follow this trend in the REP model, decreasing when Γ shifts from 1.0 to 1.25. This is due to the fact that transmission investments get postponed due to increased generation reserve capacities. Note that operational costs in the objective function

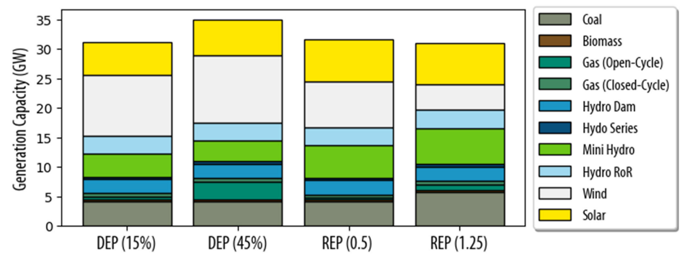
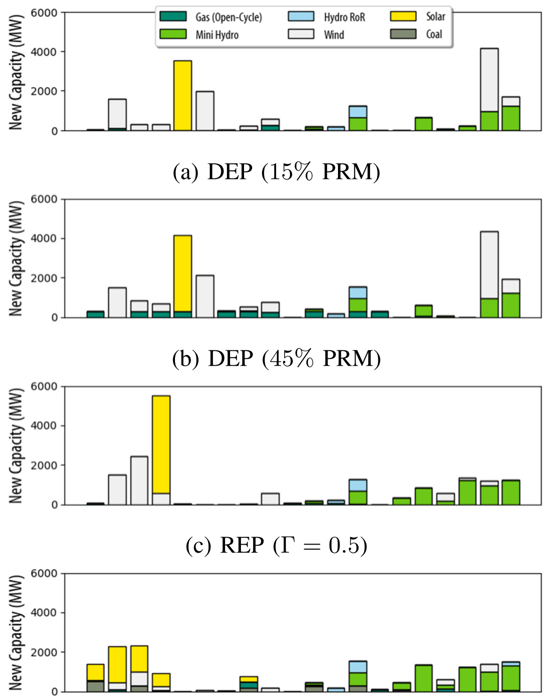


Fig. 1. Generation capacity mix under DEP and REP in the last planning period for the 20-bus case study.

are not comparable between DEP and REP models, since this value is an estimation of real operation in DEP, whereas in REP it represents the worst operational cost as determined by the uncertainty set. However, the observed trends provide insight on how these models make investment decisions based on fundamentally different understandings of conservatism. For DEP, as PRM increases, the estimated operational cost decreases, as the deterministic representative days become easier to handle due to increased reserve capacity. In REP however this trend is inverted: as Γ increases, the estimated operational cost increases, as uncertain representative days become more challenging and the system is confronted with more extreme operational conditions.

Fig. 1 presents the final generation capacity mix for a non-conservative and conservative solution under both the DEP model (15% and 45% PRM) and the REP model ($\Gamma = 0.50$ and $\Gamma = 1.25$). For the DEP model, a low PRM results in a highly renewable mix. When shifting towards a higher PRM, the investment in variable renewable capacity increases (10.4 GW vs 11.5 GW in wind, 5.7 GW vs 6.0 GW in solar), together with an increased investment in firm generation capacity, particularly in gas generation technology (1.0 GW vs 3.3 GW). This is due to the fact that renewable generation reduces net load, thus lowering reserve requirements, and also gas generation units present the cheapest capacity investment, thus these units are the most cost-effective way of satisfying the PRM requirement. Regarding the REP model, $\Gamma = 0.5$ provides a highly renewable mix as well, but considering more investments on solar (7.1 GW) and less investments on wind (7.9 GW) than both DEP models. This is due to the fact that wind generation gets penalized, even in the non-conservative solution of the REP model, because it presents a higher variability through the year than solar generation. This is also observed when comparing the conservative REP solution ($\Gamma = 1.25$) with the non conservative REP solution ($\Gamma = 0.5$), where wind is further penalized (4.3 GW vs 7.8 GW), in addition to increased investments in reserve capacity through coal (5.7 GW vs 4.1 GW) and total gas (1.5 GW vs 0.8 GW) generation capacity.

Fig. 2 presents the spatial placements of generation capacity investments, for the most conservative and non-conservative solutions of DEP and REP. In this Figure, the x-axis spans the different buses of the Chilean power system from north to south. From Fig. 2(a) and (b) it can be observed that both the conservative and non conservative DEP solutions present an extremely



(a) DEP (15% PRM)

(b) DEP (45% PRM)

(c) REP ($\Gamma = 0.5$)

(d) REP ($\Gamma = 1.25$)

Fig. 2. Spatial distribution of generation capacity investments under DEP and REP (from north to south) for the 20-bus case study.

TABLE II
COMPUTATIONAL PERFORMANCE OF THE PROPOSED MODEL FOR THE 20-BUS CASE STUDY

REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Running time (h)	2.65	6.20	5.03	4.53
Master iterations	6	7	6	6

similar placement of renewable resources. The main difference between these expansion plans lies in an increased investment in gas generation throughout the power system. As mentioned above, this investment is induced by the fact that, for this case study, gas technology is the cheapest capacity investment and thus it is the most cost-efficient way of meeting PRM requirements. In contrast, from Fig. 2(c) and (d) it can be observed that for REP: i) There is fewer wind investments under the conservative solution, due to its higher variability (as discussed above), and ii) Under the conservative solution, the associated risk of uncertain generation is diversified by spreading solar investments across several buses in the system. Also, the additional firm generation capacity of the conserative REP solution is installed in a different form as compared to the conservative DEP solution, in terms of both spatial location and type of generation.

Finally, Table II presents the running time and number of master iterations needed until convergence of Algorithm 1 for the proposed REP model, under different values of Γ. We can observe that the running times and the number of master iterations do not vary widely for different values of Γ. Note that the running time is rather long but remains adequate for planning purposes. In comparison, DEP is solved within minutes.

From the above results, we can observe a different approach towards understanding flexibility requirements from the DEP and REP models, including reserve capacity. This is explained by the fundamental difference in their representation of operational uncertainty. In particular, we observed a diversified spatial distribution of both variable renewable and firm generation capacities under the conservative REP model. In what follows, we provide experimental evidence of the impact of this modelling approach in the system's future operation.

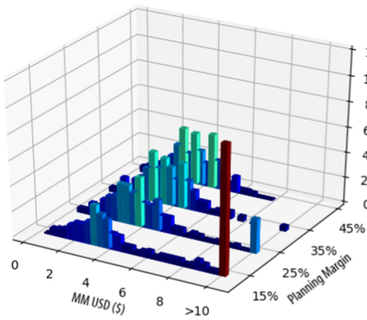
TABLE III
DEP AND REP PERFORMANCE OVER THE PLANNING HORIZON UNDER 365 DAYS OF OPERATION IN EACH PLANNING PERIOD FOR THE 20-BUS CASE STUDY

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Total Cost (MM\$)	31,364	29,132	28,708	28,819
Inv. Cost (MM \$)	11,112	11,278	11,622	12,184
Op. Cost (MM\$)	20,251	17,854	17,086	16,634
LS Frequency (Days %)	12.1	4.77	1.59	0.05
Total LS (GWh)	1,532	321.1	44.54	0.467
LS Cost (MM\$)	6,067	1,272	176.4	1.849
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Total Cost (MM\$)	28,621	28,571	28,822	28,920
Inv. Cost (MM \$)	11,551	11,976	12,349	12,933
Op. Cost (MM\$)	17,070	16,596	16,473	15,988
LS Frequency (Days %)	5.10	0.60	0.38	0.05
Total LS (GWh)	176.3	6.064	5.438	0.558
LS Cost (MM\$)	698.1	24.02	21.54	2.209

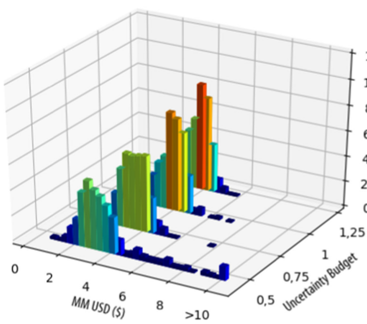
B. Operational Performance of DEP and REP

This part studies the operational performance of the expansion plans obtained by DEP and REP over the planning horizon, simulating the operation of the system under 365 days for each planning period. To perform this analysis, real data for 365 days in the base year, in combination with a load growth factor, were used to generate daily operational profiles for both load and renewable capacity factors for a total of 365 future days in each planning period. Using this data, an economic dispatch problem was solved for every day over the whole planning horizon, for a total of 1825 days of operation (365 days multiplied by 5 planning periods). The costs are extrapolated to represent the fact that each planning period has a duration of 4 years. Note that this experimental setup allows the evaluation of the operational performance and the total costs of the expansion plans obtained by DEP and REP over the planning horizon, for a particular realization of the uncertain parameters (based on historic data). In fact, it shows the operational outcomes of the planned systems if the future uncertain parameters were realized following the true data from the base year (which was used to define the representative days in DEP and REP).

Table III presents the total cost, the investment cost, the operational cost, the frequency of load shedding (LS), the total LS, and the cost associated with LS, for DEP and REP under various



(a) DEP



(b) REP

Fig. 3. Histogram for the daily operational cost of DEP and REP under 365 days of operation in the last planning period for the 20-bus case study.

values of PRM and the conservativeness parameter Γ, respectively. The total cost is calculated as the sum of the investment cost and the operational cost. The investment cost is obtained directly from the solutions of DEP and REP, respectively, and the operational cost and other operational metrics are based on the simulation of the obtained expansion plans over 365 days for every planning period, as explained above, thus allowing direct comparison of the performance between the solutions obtained from both models. With this, note that the lowest total cost is achieved by the REP model considering $\Gamma = 0.75$. The DEP model achieves its lower total cost considering a 35% PRM, but it incurs in higher LS frequency and total LS. Also, note the decrease for in operational cost, LS frequency and total LS, and the increase in the investment cost for both models, as the solutions become more conservative (as PRM and Γ increase).

It is also interesting to look more closely at the final planning period, where the largest shares of variable renewable energy are achieved. Fig. 3 presents a visual comparison of the daily cost histograms for the 365 days of operation in the last planning period under DEP and REP, under different levels of conservatism. The heights and colors of the bars represent the frequency for each bin in the histogram. Note that the last bin includes outlier values for scale. From Fig. 3(a), it can be observed that as PRM increases, the DEP model manages to eliminate the days with operational costs above 8 MM $ due to the increased investments in additional generation capacity. However, the daily operational cost remains highly variable throughout the year, specially in comparison to the REP model, as shown in Fig. 3(b). In fact, the proposed REP model presents a considerably more stable operational cost throughout the year.

TABLE IV
DEP AND REP PERFORMANCE FOR THE DAYS WITH HIGHER COSTS IN THE FINAL PLANNING PERIOD FOR THE 20-BUS CASE STUDY

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Max Daily Cost (MM$)	85.39	41.08	17.75	7.687
Max 36 Avg. Cost (MM $)	35.04	15.01	7.421	6.011
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Max Daily Cost (MM$)	22.11	7.644	7.122	5.864
Max 36 Avg. Cost (MM $)	9.199	4.816	5.131	4.717

To further understand the variability of the operational costs achieved by DEP and REP, Table IV presents the maximum daily operational cost and the average of the 36 days with the highest operational cost, over the 365 days of operation in the final planning periods. It can be observed that all the expansion plans obtained under DEP incur in very high operational costs under such worst 36 days. Only the most conservative solution, with a PRM of 45%, manages to reduce the max daily cost value below the 8 MM $ threshold, but still maintaining a high average cost under the worst 36 days. When compared to the proposed REP model, only the least conservative solution ($\Gamma = 0.5$) incurs in higher costs than the most conservative DEP solution; in fact, every other REP solution performs better under both metrics. These results further support the notion that the proposed REP model is significantly better prepared for high levels of operational uncertainty.

Finally, if we compare the best DEP and REP solutions in terms of total cost (considering a 35% PRM and $\Gamma = 0.75$, respectively), we can observe that REP achieves a 0.86% lower total cost. Moreover, regarding metrics related with reliability (LS frequency, total LS) and cost stability in the final planning period, a significant difference can be observed in favour of the REP model.

C. *Operational Performance Under a Worst-Case Day*

In Section IV-B we studied the operational performance of the expansion plans obtained from the DEP and REP models over the planning horizon. We are now interested in studying the operational performance of DEP and REP under a "worst-case day". To perform this analysis, an uncertainty set was built using the hourly average and standard deviation through the 365 days of data as the nominal and variability parameters for this set. Then, the *min-max* operational subproblem (7) was solved under the various DEP and REP expansion plans, considering different levels of conservatism. The conservativeness parameter for this operational subproblem was set to $\Gamma = 1$. Then, the operation of each plan under the obtained worst-case day was analyzed. Note that this experimental setup allows to individually find the worst daily realization (in terms of operational cost) of uncertain parameters for each tested expansion plan, and evaluate the operational performance of these models under such realization. This provides a notion of how much operational

TABLE V
OPERATIONAL PERFORMANCE OF DEP AND REP UNDER A NOMINAL AND A WORST-CASE DAY IN THE LAST PLANNING PERIOD FOR THE 20-BUS CASE STUDY

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Nominal Op. Cost (MM $)	3.212	3.071	2.997	2.935
Worst Op. Cost (MM $)	114.9	58.95	18.17	8.848
Worst Gen Cost (MM $)	6.812	8.145	8.768	8.848
Worst LS Cost (MM $)	108.1	50.80	9.404	0.0
Worst LS (Load %)	5.6	2.6	0.5	0.0
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Nominal Op. Cost (MM $)	2.938	3.308	3.769	3.560
Worst Op. Cost (MM $)	27.76	5.498	5.565	5.335
Worst Gen Cost (MM $)	5.657	5.498	5.565	5.335
Worst LS Cost (MM $)	22.11	0.0	0.0	0.0
Worst LS (Load %)	1.1	0.0	0.0	0.0

costs could vary under unexpected and complicated realizations of operational uncertainty.

Table V presents the cost decomposition and load shedding (LS) for the operation of DEP and REP under the nominal day (based on the nominal parameters of the uncertainty set) and under the worst-case day, in the final planning period. It can be observed that under the nominal values of uncertain parameters, the DEP model has a similar performance as compared to the proposed REP model. However, under the worst-case day, the difference between DEP and REP becomes critical. Only the most conservative solution of DEP, considering a 45% PRM, manages to reduce LS to zero, whereas the only expansion plan obtained through the REP model that incurs in LS is the least conservative one ($\Gamma = 0.5$). Every other solution obtained through the proposed REP approach manages to avoid LS under a worst-case day. Further, the expansion plans provided by the REP model with $\Gamma \geq 0.75$ are able to maintain operational costs under 6 MM $, whereas the best DEP model solution, even though managing to avoid LS, still incurs in a considerably higher generation cost, over 8 MM $. This is due to the conceptual differences in the modelling approach, specifically, the fact that the proposed REP model estimates reserve capacity requirements by optimizing real operation under inconvenient realizations of uncertainty and adjusting investment decisions under such scenarios.

In conclusion, through the previous experiments we have observed that there are various benefits from the REP model. First, investments consider key spatial and statistical aspects of the EP planning problem. Secondly, for a particular realization of uncertain parameters directly based on 365 days of data, the proposed REP model achieves lower total costs, higher cost stability and performs better under reliability metrics. Finally, under a worst-case day, REP manages to avoid LS while keeping operational costs at a reasonable range, while DEP pays a much higher price for reliability.

D. Applicability on a Larger Test Case

In Sections IV-A–IV-C we provided a comparative analysis between the DEP and REP models, including investment and

TABLE VI
COST DECOMPOSITION FOR REP FOR THE 149-BUS CASE STUDY

REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Gen Inv. (MM $)	11,382	11,834	11,966	12,342
Line Inv. (MM $)	24.480	25.006	25.215	25.072
Inv. Cost (MM $)	11,406	11,859	11,991	12,367
Op. Cost (MM $)	18,734	19,499	20,492	21,407
Total Cost (MM $)	30,169	31,382	32,505	33,799

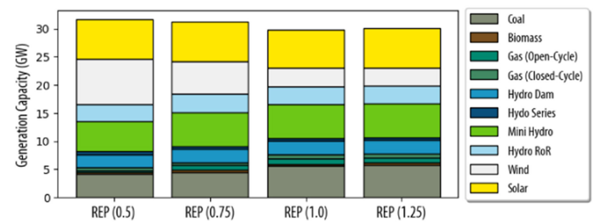


Fig. 4. Generation capacity mix obtained by REP in the last planning period for the 149-bus case study.

operational aspects, under normal and extreme operating conditions. The results have shown several benefits from the REP model. We are now interested on validating the applicability of such model in a larger system. To perform this validation, the previous case study was extended to a 149-bus representation of the Chilean Power System, considering 154 transmission lines, based on the transmission network used by the Chilean System Operator to operate the system up to 220 kV [36]. The same generation units, existing and candidate, together with operational profiles and clusters for load and renewables were based on previous experiments, however they were spatially disaggregated for a better representation of the power system. Using this data, the REP model was solved for various levels of Γ.

Table VI presents the cost structure for the solutions obtained by the various instances of the REP model. From this table, it can be observed that as the conservatism paramater Γ increases its value, both investment and operational costs increase, as the uncertain representative days consider more challenging operational conditions, which leads to aditional investment, specially in flexible generation resources.

This result is further supported by Fig. 4, which shows the final generation capacity mix at the end of the planning horizon. From this figure it can be observed that as Γ increases its value, investment in wind generation decreases (8.1 GW for $\Gamma = 0.5$ to 3.2 GW for $\Gamma = 1.25$) due to its higher variability. Also, an increased investment in firm capacity is observed, specially in coal (4.2 GW for $\Gamma = 0.5$ to 5.8 GW for $\Gamma = 1.25$) and gas (0.77 GW for $\Gamma = 0.5$ to 1.62 GW for $\Gamma = 1.25$) generation technologies. Finally, it is important to note that the total generation capacity initially decreases with higher values of Γ, reaching a minimum value for $\Gamma = 1.0$, and finally showing an increase for the most conservative solution. This is due to the fact that for the first increments in Γ, as flexible capacity increases its share on the generation mix, less total generation capacity is required. However for the final increment in Γ, challenging

TABLE VII
COMPUTATIONAL PERFORMANCE OF REP FOR THE 149-BUS CASE STUDY

REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Running time (h)	8.87	9.41	7.62	2.96
Master iterations	6	7	6	6

operational conditions from load become more significant, and in order to achieve the optimal operational and investment cost through the planning horizon, it becomes convenient to invest further on additional capacity.

Finally, Table VII presents the running time and number of master iterations needed until convergence of Algorithm 1 for the various instances of the REP model in this case study. We can observe that these remain similar in magnitude for the 149-bus system, as compared with the smaller test case using a 20-bus system in Table II, which highlights the convenient applicability of REP on larger power systems.

V. CONCLUSION

This paper has presented an adaptive robust optimization model for the Generation and Transmission Expansion Planning problem, considering the use of *uncertain representative days* as an effective representation of operational uncertainty and flexibility requirements in systems with large shares of variable renewable energy. This problem was efficiently solved using the CCG method. Computational experiments on a 20-bus and a 149-bus representation of the Chilean power system show the effectiveness of the proposed REP model under different realizations of short-term uncertainty. An innovative aspect of the proposed modeling framework is the joint consideration of chronology and uncertainty in load and renewable profiles, which results on an endogenous comprehension of both flexibility and reserve capacity requirements. Also, by employing uncertainty sets for representative days, the risk of inappropriately choosing specific load and renewable profiles is reduced, as a broader range of operational conditions may be considered in the uncertainty set.

Finally, through a comparative analysis of the operational performance of the Chilean power system, we provide experimental evidence of the advantages, both in costs and security of energy supply, of the proposed REP model, as compared to a deterministic approach based on representative days with a planning reserve margin, due to an effective spatial placement of variable and flexible resources.

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