Due on 21 January 2025

Done

Computer vision - Assignment 1

1. Prove geometrically that the projections of two parallel lines lying in some plane Φ appear to converge on a horizon line h formed by the intersection of the image plane Π with the plane parallel to Φ and passing through the pinhole.

2. Derive the perspective equation projections for a virtual image located at a distance d *in front* of the pinhole.

3. A human observes a structure of height 20 meters from a distance of 25 meters. Assuming that the distance between the lens and retina in the human eye is 17 mm, what will be the height of the retinal image?

4. Recall the following relation between the focal length f of a lens, the distance u between the object plane and the lens and the distance v between the image plane and the lens:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

   Let M denote the magnification factor i.e. the ratio of the size of the image to the size of the object. Then it can be shown that

$$f = \frac{uM}{M+1}.$$

   Suppose an object 20cm wide is to be imaged with a sensor of size 8.8×6.6 mm^2 from a distance of 0.3 m. What should be the required focal length?

5. What is the storage requirement for

   (a) a 1024×1024 binary image?

   (b) a 1024×1024 8-bit grayscale image?

   (c) a 1024×1024 32-bit colour image?

6. A picture of physical size 2.5 inches by 2 inches is scanned at 150 dpi. How many pixels would there be in the image?

7. Visit
   https://www.mentalfloss.com/article/514964/10-award-winning-optical-illusions-and-brain-puzzles

   Browse through the illusions and try to understand how each of them works. You can find other optical illusions online too.

   List 2 of your favourite illusions and explain in short why each of them works.

8. (No submission required for this exercise) Figure out the blind spot in your eye, i.e. the spot on the retina where the optic nerve leaves the eyeball. You can do it in the following way:

   (a) On a piece of paper, make a small dot with a black marker.

   (b) About six to eight inches to the right of the dot, make a small plus sign (+).

   (c) With your right eye closed, hold the paper about 20 inches away from you.

   (d) Focus on the plus sign with your left eye, and slowly bring the paper closer while still looking at the plus sign.

   At some point, the dot will vanish from your sight. This is the blind spot of your retina. If you close your left eye and look at the dot with your right eye, and repeat the process, the plus sign should disappear in the blind spot of your other eye.

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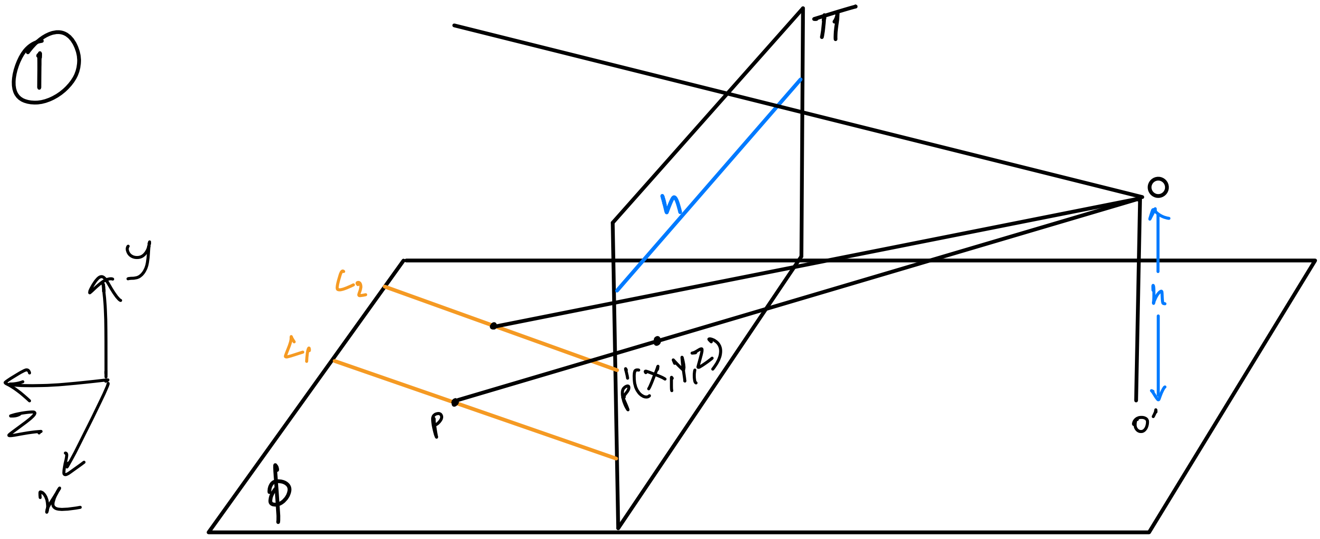
Assignment - 1

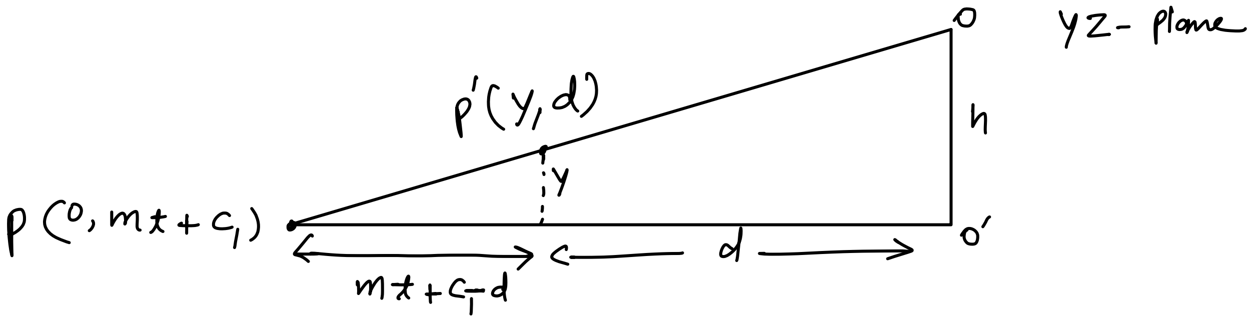
①



Assume two lines on the plane ϕ

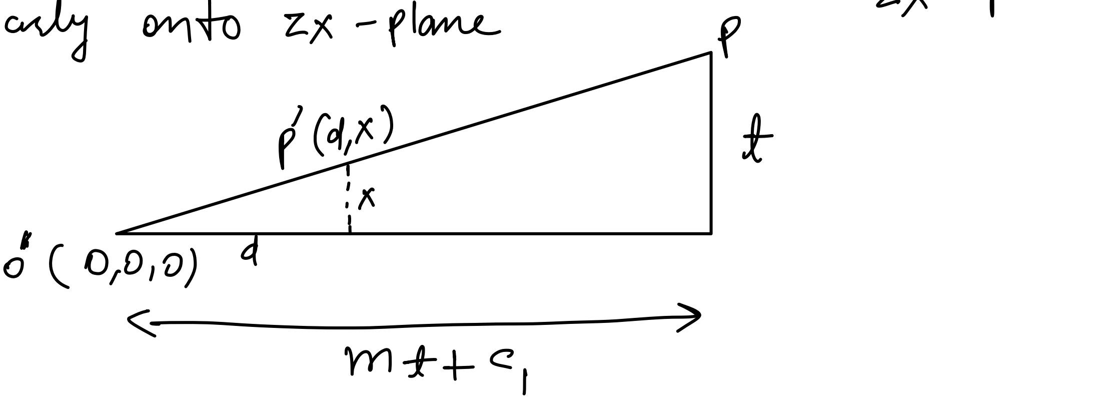
$L_1 : (t, 0, mt + c_1) \quad ; \quad L_2 : (t, 0, mt + c_2) \quad , \quad t \in \mathbb{R}$

Looking at the projection of $\Delta POO'$ onto YZ-plane



$$Y = h \frac{mt + c_1 - d}{mt + c_1} \quad (\because \text{property of similar triangles})$$

Similarly onto ZX-plane



$$x = \frac{dt}{mt + c_1} \quad (\because \text{property of similar triangles})$$

Hence the equation of the projections of line L_1 on π plane

$$L_1' : (X,Y,Z) = \left(\frac{dt}{mt+c_1} , h \frac{mt+c_1-d}{mt+c_1} , d \right)$$

Similarly, projection of L_2 line

$$L_2' : (X,Y,Z) = \left(\frac{dt}{mt+c_2} , h \frac{mt+c_2-d}{mt+c_2} , d \right)$$

when, the point on line L_1 tends to ∞, $t \to \infty$
then the co-ordinates of L_1'

$$X = \lim_{t \to \infty} \frac{dt}{mt+c_1}$$

$$= \lim_{t \to \infty} \frac{d}{m + c_1/t}$$

$$= d/m$$

$$Y = \lim_{t \to \infty} h \frac{mt+c_1-d}{mt+c_1}$$

$$= \lim_{t \to \infty} h \frac{m + c_1/t - d/t}{m + c_1/t}$$

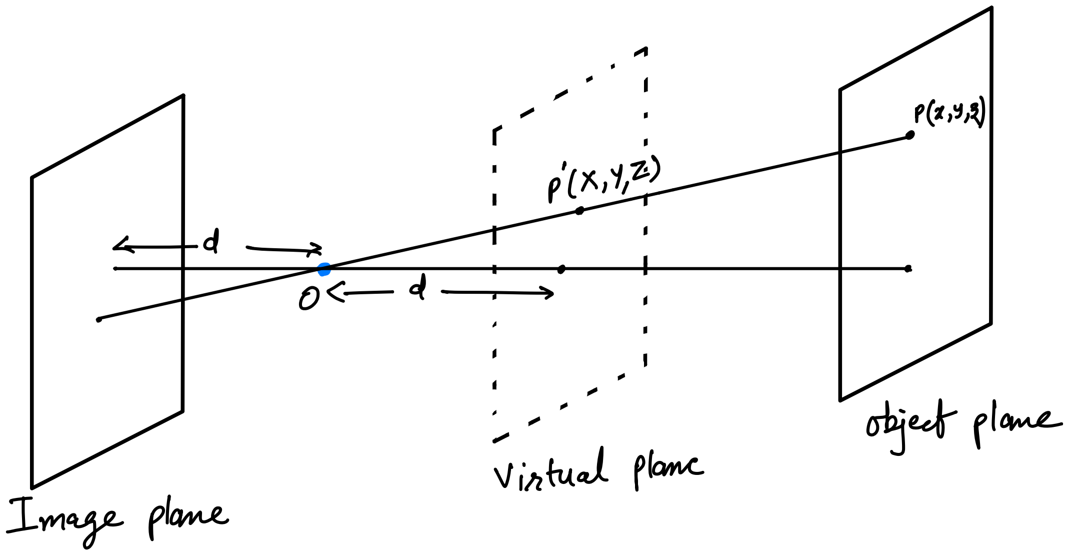
$$= h$$

$$Z = d$$

Similarly for line L_2', $X = d/m$, $Y = h$, $Z = d$

Since for both lines L_1' and L_2', the co-ordinates are same.
Therefore both lines appears to converge on a single point
$(d/m, h, d)$
And for all parallel lines ($m \in \mathbb{R}$), the projection's co-ordinate
$Y = h$, $Z = d$ $\Rightarrow$ These forms a line called horizon on plane
π with same height as pin hole.

(2)



Suppose a point $P(x,y,z)$ on image and relative point $P'(X,Y,Z)$ on virtual plane.

Since OP and OP' are colinear.

$$OP' = \lambda OP$$

In co-ordinates

$$X = \lambda x$$

$$Y = \lambda y$$

$$Z = \lambda z$$

Here $Z = d$

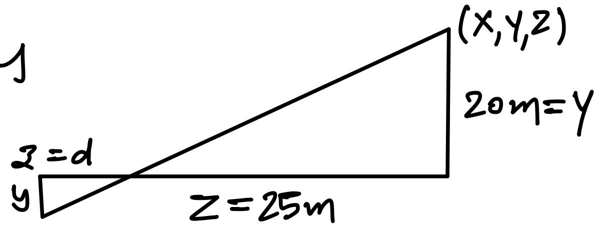
$$\Rightarrow d = \lambda z$$

$$\lambda = \frac{d}{z}$$

Hence $\boxed{X = \frac{x}{z} d, \; Y = \frac{y}{z} d, \; Z = d}$

Perspective equations for virtual plane.

③ Using persepective equations



$$y = d\,\frac{Y}{Z}$$

Here $d = 17\text{ mm}$, $Y = 20\text{m}$, $Z = 25\text{m}$

$$y = 17\text{ mm}\ \frac{20\text{m}}{25\text{m}}$$

$$y = 13.6\text{ mm}$$

④ Since $$M = \frac{\text{Size of Image}}{\text{Size of object}}$$

$$M = \frac{8.8\text{ mm}}{20\text{ cm}}$$

$$M = \frac{8.8}{200}$$

$$M = \frac{11}{250}$$

Given $u = 0.3\text{m} = 300\text{ mm}$, Now putting values in given formula

$$f = \frac{300 \times 11/250}{11/250 + 1}$$

$$= \frac{3300}{261}$$

$$= 12.64\text{ mm}$$

5

(a) Storage required for a 1024×1024 binary image.

For each pixel 1 bit is enough for storing the value 0 or 1.

Total number of pixels $= 1024 \times 1024$
$= 2^{20}$

Since 8 bits = 1 bytes

Total storage required $= \frac{2^{20}}{2^{3}} = 2^{17}$ bytes

$= \frac{2^{17}}{2^{10}}$ KB

$= 128$ KB

(b) For 8-bit grayscale image. 8 bits or 1 byte storage is required for each pixel.

Hence the total storage required =

Total number of pixels $= 2^{20}$ bytes

$= 1$ MB

(c) For 32-bit colour image, 32 bits or $\frac{32}{8} = 4$ bytes storage is required for each pixel.

Hence the total storage required $= 2^{20} \times 4$

$= 4$ MB

⑥ Since the length of the picture is 2.5 inches

Therefore the number of pixels in length of digital image $= 2.5 \times 150$

$= 375$

Similarly, the number of pixels in width of digital image $= 2 \times 150$

$= 300$

Hence, the total number of pixels $= 375 \times 300$

$= 1,12,500$

⑦ i) "PULSATING HEART", GIANNI SARCONE, COURTNEY SMITH, AND MARIE-JO WAEBER.

This art inspired an illusion of pulsating like a heart beat. The parallel Red lines and white background with right amount contrast tricks our visual system's motion sensitive neurons into signaling motion.

ii) "FLOATING STAR" JOSEP HAUTMAN/KAIA NAO

In this illusion, five pointed star appears to rotate clockwise. Here the dark blue jigsaw pieces have white and black borders against a lightly background. Carefully arranged transitions between white, light-Coloured, black and dark-Coloured regions fools the neurons into responding as if the star is moving.