OPTI Assignment-4
Timetable Scheduling

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1 Sets

- $S = \{1, 2, 3, 4\}$: Set of sections.
- $D = \{1, 2, 3, 4, 5\}$: Set of days in a week.
- $T = \{1, 2, \ldots, 8\}$: Set of timeslots per day.
- $J = \{1, 2, \ldots, 10\}$: Set of subjects.
- $J_{\text{sci}} \subset J$: Set of science subjects.
- $J_{\text{nonsci}} \subset J$: Set of non-science subjects.
- $L = \{1, 2, 3, 4\}$: Set of pre-lunch timeslots.

2 Parameters

- C_j: Total number of classes per week for subject j. For all $j \in J$, $C_j = 4$.
- C_j^{pr}: Number of practical classes for subject j. Equals 1 if $j \in J_{\text{sci}}$, otherwise 0.

3 Decision Variables

- $x_{s,d,t,j} \in \{0, 1\}$: Equals 1 if subject j is scheduled for section s on day d at timeslot t.
- $y^{\text{pr}}_{s,d,t,j} \in \{0, 1\}$: Equals 1 if a practical class for section s of subject j is scheduled on day d at timeslot t.
  Note that:

  1. Since there is no practical class for non-science subjects.

  $$y^{\text{pr}}_{s,d,t,j} = 0 \quad \forall s \in S, d \in D, t \in T, j \in J_{\text{nonsci}}$$

  2. If a practical class is scheduled, then there is a class. i.e. $y^{\text{pr}}_{s,d,t,j} = 1 \implies x_{s,d,t,j} = 1$. The following constraint can ensure this:

  $$y^{\text{pr}}_{s,d,t,j} \leq x_{s,d,t,j} \quad \forall s \in S, d \in D, t \in T, d \in J_{\text{sci}}$$

4 Objective Function

$$\text{Minimize} \quad \sum_{k=1}^{4} \left(\sum_{s \in S} \alpha_k V_{k,s} + \beta_k \sum_{s_1 < s_2} \delta_{k,s_1,s_2} \right)$$

Where

1. Penalizes practicals scheduled before lunch.

$$V_{1,s} = \sum_{d \in D, t \in \mathbf{L}, j \in J_{\text{sci}}} y^{\text{pr}}_{s,d,t,j}, \quad \forall s \in S$$

2. Penalizes consecutive practicals for a section.

$$V_{2,s} = \sum_{d} \sum_{t=1, t \neq 4}^{7} \sum_{j_1, j_2 \in J_{\text{sci}}} y^{\text{pr}}_{s,d,t,j_1} \cdot y^{\text{pr}}_{s,d,t+1,j_2}, \quad \forall s \in S$$

Since this constraint is non-linear, let's introduce an auxiliary binary variable:

$$w'_{s,d,t,j_1,j_2} = y^{\text{pr}}_{s,d,t,j_1} \cdot y^{\text{pr}}_{s,d,t+1,j_2} \quad \forall s \in S, d \in D, t \in T, j_1, j_2 \in J$$

Non-linear equations can be written as follows:

$$\begin{aligned} w'_{s,d,t,j_1,j_2} &\leq y^{\text{pr}}_{s,d,t,j_1} \\ w'_{s,d,t,j_1,j_2} &\leq y^{\text{pr}}_{s,d,t+1,j_2} \\ w'_{s,d,t,j_1,j_2} &\geq y^{\text{pr}}_{s,d,t,j_1} + y^{\text{pr}}_{s,d,t+1,j_2} - 1 \end{aligned}$$

Then the penalty equation becomes:

$$V_{2,s} = \sum_{d} \sum_{t=1, t \neq 4}^{7} \sum_{j_1, j_2 \in J_{\text{sci}}} w'_{s,d,t,j_1,j_2} \quad \forall s \in S$$

3. Penalizes consecutive theory classes of the same category (science or non-science).

$$V_{3,s} = \sum_{d} \sum_{t=1, t \neq 4}^{7} \sum_{j_1, j_2 \in J_c} x_{s,d,t,j_1} \cdot x_{s,d,t+1,j_2}, \quad J_c \in \{J_{\text{sci}}, J_{\text{nonsci}}\}, \forall s \in S$$

Since this constraint is non-linear, let's introduce an auxiliary binary variable:

$$w''_{s,d,t,j_1,j_2} = x_{s,d,t,j_1} \cdot x_{s,d,t+1,j_2} \quad \forall s \in S, d \in D, t \in T, j_1, j_2 \in J$$

Non-linear equations can be written as follows:

$$\begin{aligned} w''_{s,d,t,j_1,j_2} &\leq x_{s,d,t,j_1} \\ w''_{s,d,t,j_1,j_2} &\leq x_{s,d,t+1,j_2} \\ w''_{s,d,t,j_1,j_2} &\geq x_{s,d,t,j_1} + x_{s,d,t+1,j_2} - 1 \end{aligned}$$

Then the penalty equation becomes:

$$V_{3,s} = \sum_{d} \sum_{t=1, t \neq 4}^{7} \sum_{j_1, j_2 \in J_c} w''_{s,d,t,j_1,j_2}, \quad J_c \in \{J_{\text{sci}}, J_{\text{nonsci}}\}, \forall s \in S$$

4. Penalizes repetition of the same timeslot for a subject across days.

$$V_{4,s} = \sum_{j,t} \max\left(\sum_{d \in D} x_{s,d,t,j} - 1, 0 \right), \quad \forall s \in S$$

This is a nonlinear equation. To make it linear, let's introduce another variable $z_{s,j,t}$ such that,

$$z_{s,j,t} \geq \sum_{d \in D} x_{s,d,t,j} - 1 \quad \text{and} \quad z_{s,j,t} \geq 0, \quad \forall s \in S, j \in J, t \in T$$

Then V_4 can be written as follows:

$$V_{4,s} = \sum_{j,t} z_{s,j,t}, \quad \forall s \in S$$

Note: Since we are minimizing, V_4 therefore $z_{s,j,t} = \max\left(\sum_{d \in D} x_{s,d,t,j} - 1, 0\right)$.

5. Minimize δ_{k,s_1,s_2} to enforce fairness in penalties across sections.

α_k and β_k can be chosen based on the importance of the soft constraint and the fairness of the constraint, respectively.

5 Constraints

5.1 Only one class per slot per section

$$\sum_{j \in J} x_{s,d,t,j} = 1, \quad \forall s \in S, d \in D, t \in T$$

Explanation: Ensures that a section is assigned to only one class per timeslot.

5.2 Subject frequency

$$\sum_{d \in D} \sum_{t \in T} x_{s,d,t,j} = C_j, \quad \forall s \in S, j \in J$$

Explanation: Ensures that each subject appears exactly 4 times per week per section.

5.3 At most one class of the same subject per day

$$\sum_{t \in T} x_{s,d,t,j} \leq 1, \quad \forall s \in S, j \in J, d \in D$$

Explanation: Ensures a subject is not taught more than once in a day for any section.

5.4 Practical classes per subject

$$\sum_{d \in D} \sum_{t \in T} y^{\text{pr}}_{s,d,t,j} = C^{\text{pr}}_j, \quad \forall s \in S, j \in J_{\text{sci}}$$

Explanation: Ensures that every science subject has exactly one practical session per section per week.

5.5 At most one practical class per day per section

$$\sum_{j \in J_{\text{sci}}} \sum_{t \in T} y^{\text{pr}}_{s,d,t,j} \leq 1, \quad \forall s \in S, d \in D$$

Explanation: Prevents scheduling more than one practical class for a section on any given day.

5.6 No overlapping practicals of the same subject across sections

$$\sum_{s \in S} y^{\text{pr}}_{s,d,t,j} \leq 1, \quad \forall d \in D, t \in T, j \in J_{\text{sci}}$$

Explanation: Ensures only one practical class of a subject is conducted across the entire school at a time due to the shared lab.

5.7 Uniform penalty for all sections

$$|V_{k,s_1} - V_{k,s_2}| \leq \delta_{k,s_1,s_2}, \quad k \in \{1,2,3,4\}, s_1 \neq s_2, s_1, s_2 \in S$$

Explanation: This ensures that no single section is unfairly overloaded with violations of soft constraints. And soft constraints are equally applicable across all 4 sections.

Since the equation can be linearized as follows:

$$V_{k,s_1} - V_{k,s_2} \leq \delta_{k,s_1,s_2}, \quad \forall s_1 < s_2 \in S$$
$$V_{k,s_2} - V_{k,s_1} \leq \delta_{k,s_1,s_2}, \quad \forall s_1 < s_2 \in S$$

6 Assumptions

- Each subject is scheduled for exactly 4 periods per section per week.
- Science subjects include one practical class; others are theory only.
- Each section has one subject scheduled per timeslot.
- No teacher assignment or availability constraints are considered.
- Soft constraints are modelled via penalty terms in the objective.