OPTI Assignment-1

Mayank Nagar
MDS202334

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Mathematical Optimization Model: Round Robin Match Scheduling

Sets

- T: Set of teams ($i, j \in T$)
- D_{day}: Set of days in the tournament in day D_{day}
- D_{night}: Set of days in the tournament in night D_{night}
- D: Set of days in the tournament ($d \in D = D_{day} \cup D_{night}$), $D = \{d_1, d_2, \ldots, d_n\}$
- S: Set of stadiums ($s \in S$)
- Z: Set of zones ($z \in Z$)
- S_z: Set of stadiums in zone z
- W: Set of weekdays ($w \in W$)
- M: Set of matches ($m \in M$)

Parameters

- $V_{s,d}$: Binary, 1 if stadium s is available on day d, 0 otherwise
- R_d: Binary, 1 if rain is forecasted on day d, 0 otherwise
- $I_{i,j}$: Importance score of a match between teams i and j (higher = more interesting)
- H_i: Home stadium of team i
- L_s: Capacity of stadium s
- C_{s_1,s_2}: Distance between stadium s_1 and s_2
- $G_{\min}$: Minimum gap between two matches for a team
- $G_{\max}$: Maximum gap between two matches for a team

- K_h: max consecutive home matches allowed
- K_a: max consecutive away matches allowed
- W_{weekend}: Set of weekend days
- $N_{\text{day/night}}$: Maximum allowed difference between day and night matches for a team

Decision Variables

- $x_{i,j,s,d}$: Binary, 1 if a match between team i and team j is scheduled in stadium s on day d, 0 otherwise
- $y_{i,j,s,d}$: Binary, 1 if match (i,j) is rescheduled due to rain in stadium s on day d, 0 otherwise
- $h_{i,d}$: Binary, 1 if i team plays a home match on day d, otherwise 0

$$h_{i,d} = \sum_{j \in T, j \neq i} \sum_{s \in H_i} x_{i,j,s,d} \quad i \in T, d \in D$$

- $a_{i,d}$: Binary, 1 if i team plays an away match on day d, otherwise 0

$$a_{i,d} = \sum_{j \in T, j \neq i} \sum_{s \in H_j} x_{i,j,s,d} \quad i \in T, d \in D$$

- $b_{i,d}$: Binary, 1 if team i plays a match on day d, 0 otherwise

$$b_{i,d} = \sum_{j \in T, i \neq j} \sum_{s \in S} x_{i,j,s,d} \quad i \in T, d \in D$$

- $z_{i,j,z}$: Binary, 1 if match (i,j) occurs in zone z, 0 otherwise

$$z_{i,j,z} = \sum_{s \in S_z} \sum_{d \in D} x_{i,j,s,d} \quad i,j \in T, i \neq j, z \in Z$$

- $\phi_{i,j}$: Binary, 1 if match (i,j) is scheduled on a weekend, 0 otherwise

$$\phi_{i,j} = \sum_{s \in S} \sum_{d \in W} x_{i,j,s,d} \quad i,j \in T$$

Objective Function

This objective function prioritizes the scheduling of interesting matches on weekends and in larger stadiums.

$$O_1 = \text{Maximize } \sum_{i,j \in T} \sum_{s \in S} \sum_{d \in D} \left(I_{i,j} \cdot x_{i,j,s,d} \cdot \left(1 + \phi_{i,j} - \frac{1}{L_s} \right) \right)$$

This objective function minimizes travel distances for teams during consecutive matches.

$$O_2 = \text{Minimize} \sum_{i \in T} \sum_{d_k, d_{k+1} \in D} (x_{i,j_1,s_1,d_k} \cdot x_{i,j_2,s_2,d_{k+1}}) \cdot C_{s_1,s_2}$$

This objective function is non-linear, therefore introducing a new binary variable, w such that

$$w_{i,s_1,s_2,d_k,d_{k+1}} = x_{i,j_1,s_1,d_k} \cdot x_{i,j_2,s_2,d_{k+1}}$$

Which can be expressed in the following constraints

$$w_{i,s_1,s_2,d_k,d_{k+1}} \leq x_{i,j_1,s_1,d_k}, \quad \forall i, j_1 \in T, s_1, s_2 \in S, d_k, d_{k+1} \in D$$

$$w_{i,s_1,s_2,d_k,d_{k+1}} \leq x_{i,j_2,s_2,d_{k+1}}, \quad \forall i, j_2 \in T, s_1, s_2 \in S, d_k, d_{k+1} \in D$$

$$w_{i,s_1,s_2,d_k,d_{k+1}} \geq x_{i,j_1,s_1,d_k} + x_{i,j_2,s_2,d_{k+1}} - 1, \quad \forall i \in T, j_1, j_2 \in T, s_1, s_2 \in S, d_k, d_{k+1} \in D$$

Hence, the linearized objective function

$$O_2 = \text{Minimize} \sum_{i \in T} \sum_{d_k, d_{k+1} \in D} w_{i,s_1,s_2,d_k,d_{k+1}} \cdot C_{s_1,s_2}$$

Final Objective function

$$\text{Maximize} \quad \lambda_1 O_1 - \lambda_2 O_2$$

Where values of λ_1 and λ_2 can be taken based on the importance of the objective functions

Constraints

1. Match Scheduling

Each pair of teams plays exactly twice (home and away):

$$\sum_{s \in S} \sum_{d \in D} x_{i,j,s,d} = 2, \quad \forall i, j \in T, i \neq j$$

Home and away distribution:

$$\sum_{s \in S, s \neq H_i} \sum_{d \in D} x_{i,j,s,d} = 1, \quad \forall i, j \in T, i \neq j$$

And

$$x_{ii,s,d} = 0, \quad \forall i \in T, s \in S, d \in D$$

2. Rain-Delayed Matches

If a match is abandoned due to rain, it must be rescheduled to the next day:

$$x_{i,j,s,d_k} \cdot R_{d_k} = y_{i,j,s,d_{k+1}}, \quad \forall i, j \in T, s \in S, d_k, d_{k+1} \in D$$

$\forall i, j \in T, s \in S, d_k, d_{k+1} \in D$ we have to ensure that:

1. No other matches of teams i and j are held on d_{k+1} day

$$y_{i,j,s,d_{k+1}} \leq 1 - \sum_{k \in T} \sum_{s \in S} x_{ik,s,d_{k+1}}$$

$$y_{i,j,s,d_{k+1}} \leq 1 - \sum_{k \in T} \sum_{s \in S} x_{jk,s,d_{k+1}}$$

2. Stadium s is available on day d_{k+1}

$$y_{i,j,s,d_{k+1}} \leq V_{s,d_{k+1}}$$

3. No overlapping of matches in stadium s

$$x_{i'j',s,d_{k+1}} + y_{i,j,s,d_{k+1}} \leq 1 \quad \forall i', j' \in T$$

3. Day and Night Match Balance

The number of day and night matches should be balanced across teams:

$$-N_{\text{day/night}} - \alpha \leq \sum_{d \in D_{\text{day}}} b_{i,d} - \sum_{d \in D_{\text{night}}} b_{i,d} \leq N_{\text{day/night}} + \alpha, \quad \forall i \in T$$

Where α is a small allowed deviation to ensure flexibility

4. Fairness Across Zones

Matches must be fairly distributed across zones:

$$\frac{\#M}{\#Z} - \beta \leq \sum_{j \in T} \sum_{z \in Z} z_{i,j,z} \leq \frac{\#M}{\#Z} + \beta, \quad \forall i \in T$$

Where β is a small allowed deviation to ensure flexibility.

5. Time Gaps Between Matches

The time gap between consecutive matches for a team must satisfy:

$$G_{\min} \leq d_2 - d_1 \leq G_{\max}, \quad \forall d_1, d_2 \in D, i \in T$$

where

$$\sum_{i \in T, i \neq j} \sum_{s \in S} x_{i,j,s,d_1} = 1, \sum_{i \in T, i \neq j} \sum_{s \in S} x_{i,j,s,d_2} = 1 \text{ and } d_1 < d_2 \quad j \in T$$

6. Home and away match distribution

To prevent long consecutive home or away streaks, enforce:

$$\sum_{d=d_i}^{d_{i+K_h}} h_{i,d} \leq K_h \quad \forall i \in T$$

$$\sum_{d=d_i}^{d_{i+K_a}} a_{i,d} \leq K_a \quad \forall i \in T$$

Ensure Even Distribution Over a Window

$$-\delta \leq \sum_{d=d_i}^{d_{i+k}} h_{i,d} - \sum_{d=d_i}^{d_{i+k}} a_{i,d} \leq \delta \quad \forall i \in T, d_i, d_{i+1}, \ldots, d_{i+k} \in D, k \in \{1, 2, \ldots, K_h\}$$

Where δ is a small tolerance factor to ensure that the number of home and away matches remains balanced over time.

7. Match Distribution

No two highly interesting matches occur at the same time. θ is a threshold for a match to be considered as an important match.

$$\sum_{s \in S} x_{i,j,s,d} + \sum_{s \in S} x_{kl,s,d} \leq 1, \quad \forall (i,j),(k,l) \in M, I_{i,j}, I_{kl} \geq \theta$$

Smaller stadiums get less interesting matches. τ is a threshold for a stadium to be considered as a smaller stadium.

$$I_{i,j} \cdot x_{i,j,s,d} \leq \tau \cdot L_s, \quad \forall i,j \in T, s \in S, d \in D$$

8. Stadium and City Events

No matches in unavailable stadiums or cities with big events:

$$x_{i,j,s,d} \leq V_{s,d}, \quad \forall i,j \in T, s \in S, d \in D$$

Assumptions

- A match lasts one day, and only one match is held in a stadium on a given day.
- Importance scores $I_{i,j}$ are pre-defined based on historical rivalries or fan demand.
- Availability $V_{s,d}$ and rain forecast R_d are known in advance.
- Travel distances between stadiums are symmetric and pre-calculated.