OPTI Assignment-3
Office Space Planning

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1 Sets

- T : Set of teams
- D : Set of working days
- F : Set of floors
- S : Set of seats
- E_t : Set of employees in team $t \in T$

2 Parameters

- C_f : Capacity of floor $f \in F$ (adjusted for social distancing, e.g. $0.70 \times$ Original Capacity$_f$)
- FS_t : Number of fixed seats allocated to team $t \in T$
- MinTeamSize: Minimum number of employees per team per floor per day (5 employees)
- MaxLoading : Maximum loading percentage constraint
- $P_t(d)$: Collaboration requirement of team t on day $d \in D$
- α: Minimum proportion of target collaboration requirement that must be allocated (e.g., 60
- β: Maximum deviation allowed for collaboration shifts (1 day)
- L_t : Maximum number of different floors that team t can occupy on a given day

3 Decision Variables

- $x_{t,d,f}$: Number of employees from team $t \in T$ assigned to floor $f \in F$ on day $d \in D$
- $y_{t,d}$: Binary variable; 1 if team t is assigned to multiple floors on day d, 0 otherwise
- $z_{f,d}$: Binary variable; 1 if floor f is occupied on day d, 0 otherwise

$$z_{f,d} \geq \frac{\sum_{t \in T} x_{t,d,f}}{C_f}, \quad \forall f \in F, d \in D$$

$$z_{f,d} \leq \sum_{t \in T} x_{t,d,f} \quad \forall f \in F, d \in D$$

- $a_{t,d,f}$: Binary variable; 1 if team t is present on floor f on day d, 0 otherwise

$$a_{t,d,f} \geq \frac{x_{t,d,f}}{M}, \quad \forall t \in T, d \in D, f \in F$$

$$a_{t,d,f} \leq x_{t,d,f}, \quad \forall t \in T, d \in D, f \in F$$

Where M is the max team size.

$$y_{t,d} \geq \frac{\sum_{f \in F} a_{t,d,f} - 1}{|F| - 1}, \quad \forall t \in T, d \in D$$

$$y_{t,d} \leq \max\left(\sum_{f \in F} a_{t,d,f} - 1, 0\right), \quad \forall t \in T, d \in D$$

The second equation is a non-linear equation, this can be written to make it linear.
Let:

$$m_{t,d} \in \mathbb{Z}_{\geq 0} \quad \text{(auxiliary variable)}$$
$$w_{t,d} \in \{0, 1\} \quad \text{(binary switch)}$$

The constraints are:

$$\begin{aligned}
y_{t,d} &\leq m_{t,d}, \quad \forall t \in T, d \in D \\
m_{t,d} &\geq 0, \quad \forall t \in T, d \in D \\
m_{t,d} &\geq \sum_{f \in F} a_{t,d,f} - 1, \quad \forall t \in T, d \in D \\
m_{t,d} &\leq w_{t,d}, \quad \forall t \in T, d \in D \\
m_{t,d} &\leq \sum_{f \in F} a_{t,d,f} - 1 + (1 - w_{t,d}), \quad \forall t \in T, d \in D
\end{aligned}$$

Explanation:

- $\sum_{f \in F} a_{t,d,f} > 1 \implies$ team t is assigned more than one floor on the day d, this force $y_{t,d}$ to become 1.
- $\sum_{f \in F} a_{t,d,f} = 1$ or $0 \implies$ team t is assigned only one floor or no floor on the day d, this force $y_{t,d}$ to become 0.

4 Objective Function

$$\max \sum_{t \in T} \sum_{d \in D} \sum_{f \in F} x_{t,d,f} - \lambda_1 \sum_{t \in T} \sum_{d \in D} P_t(d).y_{t,d} - \lambda_2 \sum_{d \in D} \sum_{f \in F} z_{f,d} \tag{1}$$

where λ_1 and λ_2 are weights for penalizing multi-floor occupancy and minimizing the number of occupied floors. Teams with higher collaboration requirements are prioritized to be seated on a single floor by assigning a higher penalty when such teams are split across multiple floors..

5 Constraints

5.1 Capacity Constraint

$$\sum_{t \in T} x_{t,d,f} \leq C_f, \quad \forall f \in F, d \in D \tag{2}$$

Explanation: This ensures that the total number of employees assigned to a floor does not exceed its adjusted capacity.

5.2 Fixed Seat Allocation

$$\sum_{d \in D} \sum_{f \in F} x_{t,d,f} \geq FS_t, \quad \forall t \in T \tag{3}$$

Explanation: Ensures that teams with fixed seats are always allocated at least their predefined number of seats.

5.3 Collaboration Pattern and Shift Constraint

$$\sum_{d' \in \{d-1,\, d,\, d+1\}} \delta_{t,d'} = 1, \quad \forall t \in T,\, d \in D \tag{4}$$

$$\sum_{f \in F} x_{t,d',f} \geq \alpha \cdot P_t(d) \cdot \delta_{t,d'}, \quad \forall t \in T,\, d \in D,\, d' \in \{d-1,\, d,\, d+1\} \tag{5}$$

Explanation:

- Guarantees that a minimum proportion (e.g., $\alpha = 60\%$) of the collaboration requirement is met for each team on its designated collaboration days.
- Allows shifting collaboration to an adjacent day within a given threshold to optimize seat allocation.

5.4 Fair Share Allocation of Floating Seats Across Teams

$$(1-\epsilon)\frac{S_{t_1}}{|E_{t_1}|} \leq \frac{S_{t_2}}{|E_{t_2}|} \leq (1+\epsilon)\frac{S_{t_1}}{|E_{t_1}|} \quad \forall t_1, t_2 \in T \tag{6}$$

where $S_t = \sum_{f \in F, d \in D} x_{t,d,f} - FS_t$ is remaining seats after assigning fixed seats assigned to team t. And ϵ is small slack (like 10%)
Explanation: If teams have different sizes, we can enforce that the allocated seats are proportional to team sizes.

5.5 Fair Share Allocation Across Days

$$\sum_{t \in T} \sum_{f \in F} x_{t,d,f} \geq \text{MinLoad}, \quad \forall d \in D \tag{7}$$

$$\sum_{t \in T} \sum_{f \in F} x_{t,d,f} \leq \text{MaxLoad}, \quad \forall d \in D \tag{8}$$

$$\text{MaxLoad} \leq (1 + \text{MaxLoading}) \cdot \text{MinLoad} \tag{9}$$

Explanation: Ensures that the daily employee count does not vary too much across days, maintaining fairness in seat allocation.

5.6 Team Floor Allocation Consistency

$$\sum_{f \in F} a_{t,d,f} \leq L_t, \quad \forall t \in T, d \in D \tag{10}$$

Explanation: The total number of distinct floors used by a team cannot exceed L_t.

5.7 Minimum Team Allocation Per Floor

$$x_{t,d,f} \geq \text{MinTeamSize}.a_{t,d,f}, \quad \forall t \in T, d \in D, f \in F \tag{11}$$

$$x_{t,d,f} \geq M.a_{t,d,f}, \quad \forall t \in T, d \in D, f \in F \tag{12}$$

where M is max team size.
Explanation: Ensures that if a team occupies a floor, at least a minimum number of employees (e.g., 5) must be present to maintain efficiency.

5.8 Night Shift Exclusion

$$\text{(Night shift employees do not impact seat allocation calculations)} \tag{13}$$

Night shift employees are excluded since they can use the same seats vacated by day shift employees.

5.9 Minimizing Number of Floors Occupied on Low-Demand Days

$$\sum_{f \in F} z_{f,d} \leq \text{TargetFloors}_d, \quad \forall d \in D \tag{14}$$

Explanation: On holidays or low-occupancy days, fewer floors (lower $TargetFloors_d$) should be used to minimize operational costs.

6 Assumptions

- Each employee can only be assigned to one floor per day.
- Collaboration requirements are flexible within constraints.
- Night shift employees do not impact seating constraints.