Please formulate the mathematical model for the following business problem. The mathematical model would consist of:

- *Sets*
- *Parameters*
- *Decision Variables*
- *Objective function*
- *Constraints*
- *Necessary assumptions*

You can write the mathematical model in MS Word, MS Powerpoint, or any other medium of your choice.

Alternatively, you can write the mathematical model on paper and upload the scanned image.

Please make any assumptions which you feel are necessary for solving the problem.

Please explain clearly the meaning of each set, parameter, variable, constraint, objective.

Also if you think there is any rule/metric which the model should try to attain, but is not mentioned in the document, please feel free to add that.

Deadline: Please submit the assignment by 2nd March 11 PM.

Note: This assignment has 15% weightage

Business problem:

Optimally refilling ATMs: Deliver the right cash, at the right time, to the right ATMs

Objectives

- Minimize number of vehicles used
- Minimize inventory holding cost: Minimize cash that can be stored in an ATM. This if for security reasons as well as to ensure that too much cash is not lying idle. This threshold would obviously be higher in ATMs in areas with more footfall (eg: malls)
- Minimize number of visits to an ATM - Tradeoff between the number of visits and minimizing cash kept in the ATM. Assuming demand of 50000 for next 3 days and ATM capacity > 150000
    - Minimizing number of visits would lead to a single refill of 150000 on day 1
    - Minimizing cash kept in the ATM would lead to one refill of 50000 on day 1, one refill of 50000 on day 2, one refill of 50000 on day 3
- Workload balance across vehicles: both in terms of #ATMs visited, and cash deposited (for security)

Constraints

- There are vehicles of different capacities and different costs
- On a day, a vehicle can visit multiple ATMs. But one ATM can be refilled by max one vehicle.
- It does not matter at what time of the day a vehicle is refilling a particular ATM machine
- For security reasons, the ATMs visited by the vehicle should be as different as possible across different days. If a vehicle visits an ATM today, then the vehicle cannot visit the same ATM before today+x days (even if that results in lower cost).
- For security reasons, maximum cash that can be carried by a vehicle at any point of time. This means that even if the vehicle has capacity to carry say 20 Lakhs cash at one time, we will not let the vehicle carry more than say 15 Lakhs.
- Minimum and maximum cash that an ATM must have
- Denomination: Based on the denominations which the ATM can handle/require, the vehicle must refill accordingly.
    - This is a soft constraint i.e. satisfying the denomination requirement is a lower priority than meeting demand requirement Example: If an ATM has requirement of 50K cash, out of which 45K needs to be in 500 denomination, and the rest in 200/100. However, the vehicle supplying to the ATM has run out of 500 notes, then it can refill only 500 notes amounting to 50K.
    - Refilling with lower denomination is more preferred than refilling with higher denomination
- Cash requirement of every ATM must be met on time, even if that requires hiring vehicles on rent for the days with high demand.
- Avg time to refill:
    - On weekdays, refilling every ATM requires 30 min (including travel time, driver's lunch time etc.) and number of working hours in a day = 10 => 20 ATMs can be visited by a vehicle every day.
    - On weekends, this time reduces to 20 min because of lesser traffic => 30 ATMs can be visited by a vehicle every day.

  What tradeoff would this lead to?
- Maximum number of ATMs that a vehicle can visit in a day
- Cash deposit at a particular ATM would be 0 or >= 50K, and it would be in multiples of 10K

Assumptions

Planning horizon: 1 week

Vehicles cater to the needs of only one bank

Refilling vehicles in between ATM refilling is not allowed. Assume that the vehicle is refilled only at the beginning of the day.

Output

Which ATMs would be visited by which vehicle on a particular day and how much cash would be loaded

We are not looking at scheduling aspects in this assignment i.e. we are not deciding the exact route of the vehicles on a particular day