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Introduction to Matrix Theory

Unit – VII

Bridge Course - MFCA

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Syllabus

Unit 7: Introduction to Matrix Theory

Definition a matrix as rectangular arrangement, Types of matrices - column matrix, row matrix, rectangular matrix, square matrix, zero matrix, diagonal matrix, scalar matrix and unit matrix, Algebra of matrices - Equality of matrices, Addition, multiplication, scalar multiplication of matrices, Transpose of a matrix, Introduction to elementary operations and finding inverse of a matrix using elementary operations.

Definition: A matrix is an arrangement of mn numbers along m rows and n columns.

For example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ -1 & 2 & 3 \end{bmatrix}$

A matrix with m rows and n columns is called a $m \times n$ matrix. A matrix with m rows and n columns is said to be of order $m \times n$.

A is of order 2×2

B is of order 3×3

Different types of matrices

Row matrix: A matrix is called a row matrix if it has only one row. If a row matrix has n columns, then its order is $1 \times n$.

Example: $A = \begin{bmatrix} 1 & 5 & 7 & 8 \end{bmatrix}$ 1×4

Column matrix: A matrix is called a column matrix if it has only one column. If a column matrix has n rows then its order is $n \times 1$.

Example: $A = \begin{bmatrix} 1 \\ 7 \\ 8 \\ -6 \end{bmatrix}$

Square and rectangular matrices: A $m \times n$ matrix is called a square matrix if $m = n$. Otherwise, it is a rectangular matrix.

Example: $A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 9 & 5 & 3 & 2 \\ -1 & 7 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ is a square matrix and

$B = \begin{bmatrix} 1 & 4 & 4 & 2 \\ 3 & 7 & 8 & 8 \\ 1 & -6 & 12 & 1 \end{bmatrix}$ is not a square matrix, it is a rectangular matrix.

Null (Zero) matrix: A matrix in which all elements are zeroes is called a null matrix.

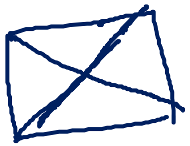
Example $N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Diagonal matrix: A square matrix in which all non-diagonal elements are zeroes is called a diagonal matrix.

Top Left Corner

Principal / leading diagonal

Example $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$



Bottom right corner

Scalar matrix: A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.

Example $S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ diagonal matrix $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

Scalar

Identity matrix: A scalar matrix in which all diagonal elements are equal to 1 is called an identity matrix.

Example $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Equality of matrices: Two matrices are said to be equal if their corresponding elements are equal.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ -1 & 2 & 3 \end{bmatrix} B = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ -1 & 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & -3 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{bmatrix}$

We note that $A = B$, whereas $A \neq C$ and $B \neq C$.

Transpose of a matrix: A matrix obtained by transposing rows into columns and columns into rows is called transpose of a matrix. If A is a matrix of order $m \times n$, then the transpose of A is denoted by A' or A^T and it will be of order $n \times m$.

Let $A = \begin{bmatrix} 1 & 4 & 4 & 2 \\ 3 & 7 & 8 & 8 \\ 1 & -6 & 12 & 1 \end{bmatrix}$ Then $A^T = \begin{bmatrix} 1 & 3 & 1 \\ 4 & 7 & -6 \\ 4 & 8 & 12 \\ 2 & 8 & 1 \end{bmatrix}$

Matrix addition: If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of same order say, $m \times n$, then their sum $A + B$ is a matrix obtained by adding corresponding element of A and B. That is $C = [c_{ij}] = [a_{ij}] + [b_{ij}]$

Symmetric matrix: A matrix A is called a symmetric matrix if $A^T = A$

Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 9 \\ 3 & 9 & 3 \end{bmatrix} A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 9 \\ 3 & 9 & 3 \end{bmatrix}$

Here A is a symmetric matrix.

Skew-symmetric matrix: A matrix A is called a symmetric matrix if $A^T = -A$

Example: $A = \begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix} A^T = -\begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}$

Problems

1) Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following:

(i) $A + B$ (ii) $A - B$ (iii) $3A - C$

(iv) AB (v) BA

i) $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

ii) $A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

iii) $3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

iv) $AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2\times 1 + 4\times -2 & 2\times 3 + 4\times 5 \\ 3\times 1 + 2\times -2 & 3\times 3 + 2\times 5 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

v) $BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times 2 + 3\times 3 & 1\times 4 + 3\times 2 \\ -2\times 2 + 5\times 3 & -2\times 4 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\cos^2\theta + \sin^2\theta = 1$

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Compute the following:

$m \times n \quad n \times p \; : \; m \times p$

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & -ab+ba \\ -ba+ab & b^2+a^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & b^2+a^2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times2 & 1\times3 & 1\times4 \\ 2\times2 & 2\times3 & 2\times4 \\ 3\times2 & 3\times3 & 3\times4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

$3\times1 \quad 1\times3$

3×3

1

2

1 2 3 4

5

6

(iii) $\underset{2\times 2}{\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}} = \begin{bmatrix} 1\times1-2\times2 & 1\times2-2\times3 & 1\times3-2\times1 \\ 2\times1+3\times2 & 2\times2+3\times3 & 2\times3+3\times1 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute $3A - 5B$.

Sol^n :

$$3A - 5B = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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Find X and Y, if $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

Sol^n :

$$X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

Add

$$2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$\Rightarrow \quad X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

But $\quad X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow \quad Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

$4\cancel{x} - 9x$

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$2 - \frac{4}{5} = \frac{10-4}{5} = 6/5$

$3 + \frac{24}{5} = \frac{15+24}{5} = 39/5$

Find X and Y, if $2X+3Y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}$

Sol^n:

$2X+3Y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix} \times 2$

$3X+2Y=\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix} \times 3$

Subtract (−) (−) (−)

$-5X=\begin{bmatrix}4 & 6\\8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6\\-3 & 15\end{bmatrix}$

$\Rightarrow -5X=\begin{bmatrix}-2 & 12\\11 & -15\end{bmatrix} \Rightarrow X=\begin{bmatrix}2/5 & -12/5\\-11/5 & 3\end{bmatrix}$

Consider $2X+3Y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}4/5 & -24/5\\-22/5 & 6\end{bmatrix}+3Y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$

$\Rightarrow 3Y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}-\begin{bmatrix}4/5 & -24/5\\-22/5 & 6\end{bmatrix}$

$\Rightarrow 3Y=\begin{bmatrix}6/5 & 39/5\\42/5 & -6\end{bmatrix} \Rightarrow Y=\begin{bmatrix}2/5 & 13/5\\14/5 & -2\end{bmatrix}$

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If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$, (ii) $(A - B)' = A' - B'$

Soln: i) $A + B = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix} \Rightarrow (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

$RHS = A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

$\Rightarrow (A+B)^T = A^T + B^T$

Elementary Transformations

In many matrix computations we perform the following operations:

1. We interchange two rows or two columns.
2. We multiply all elements of a row (or a column) by a non-zero constant.
3. To k times the elements of a row (or a column) we add p times the corresponding elements of another row (or column), where p and k are non-zero constants.

These operations are called elementary operations or elementary transformations.

$3R_1 + 4R_2$ $\qquad$ $\frac{1}{3}R_1 - 100R_3$

Q & A

THANK YOU