University of Latvia

“Skrupulozās zemenītes” (LU)

- Valters Kalniņš
- Kristaps Štāls
- Matīss Kristiņš

Contents

1. C++

1.1. Optimizations

```
#pragma GCC optimize("Ofast, unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt,tune=native")
```

1.2. Hash function

```
static uint64_t splitmix64(uint64_t x)
{x+=0x9e3779b97f4a7c15;x=(x^(x>>30))*0xbf58476d1ce4e5b9;
x=(x^(x>>27))*0x94d049bb133111eb;
return x^(x>>31);}
struct custom_hash {size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
chrono::steady_clock::now().time_since_epoch().count();return
splitmix64(x+FIXED_RANDOM);}};
const long long mod=998244353;
//1000000007
long long modpow(long long n, long long m){long long
res=1;while(m){if(m&1)res=res*n%mod;n=n*n%mod;m>>=1;}return res;}
```

1.3. C++ random

```
mt19937
rng(chrono::steady_clock::now().time_since_epoch().count());
```

2. Algebra

$$\sum_{i=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

3. Number Theory

3.1. Rabin-Miller

```
using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if n is
probably prime, else returns false.
    if (n < 4)
        return n == 2 || n == 3;

    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }

    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}
```

3.2. Extended GCD

```
int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
```

3.3. Chinese Remainder Theorem

Notes:

- Assumes all modulo are pairwise coprime
- If not, splitting modulos using prime powers works

```
int mod_inv(int a, int mod){
    int x, y;
    int g = extGcd(a, mod, x, y);
    x = (x % mod + mod) % mod;
    return x;
```

```
}

pair<int, int> crt(vector<pair<int, int>> congurences){
    // {mod, remainder}
    int M = 1;
    for(auto c : congurences){
        M *= c.first;
    }
    int solution = 0;
    for(auto c : congurences) {
        int a_i = c.second;
        int m_i = M / c.first;
        int n_i = mod_inv(m_i, c.first);
        solution = (solution + a_i * m_i % M * n_i) % M;
    }
    return {M, solution};
}
```

3.4. Random usable primes

666240077 964865333 115091077 378347773 568491163 295451837
658540403 856004729 843998543 380557313

4. Combinatorics

4.1. Stars and bars

n balls, k boxes:

$$\binom{n+k-1}{k-1}$$

4.2. Vandermorde idenitity (and variants)

$\binom{m+n}{r} = \sum \binom{n}{k}\binom{n}{r-k}$

$\sum \binom{n}{x}\binom{m}{x} = \binom{n+m}{n}$

5. Data Structures

5.1. Treap

```
struct Node{
    int value, cnt, pri; Node *left, *right;
    Node(int p) : value(p), cnt(1), pri(gen()),
        left(NULL), right(NULL) {};
};
typedef Node* pnode;
int get(pnode q){if(!q) return 0; return q->cnt;}
void update_cnt(pnode &q){
    if(!q) return; q->cnt=get(q->left)+get(q->right)+1;
}
void merge(pnode &T, pnode lef, pnode rig){
    if(!lef){T=rig;return;} if(!rig){T=lef;return;}
    if(lef->pri>rig->pri){merge(lef->right,lef->right,rig);T=lef;
    }else{merge(rig->left, lef, rig->left); T = rig;}
    update_cnt(T);
}
void split(pnode cur, pnode &lef, pnode &rig, int key){
    if(!cur){lef=rig=NULL;return;} int id=get(cur->left)+1;
    if(id<=key){split(cur->right,cur->right,rig,key-id);lef=cur;}
    else {split(cur->left, lef, cur->left, key); rig = cur;}
    update_cnt(cur);
}
```

6. Algoritms

6.1. Kuhn's algorithm

```
// node matching indexed 1-n with 1-m
const int N = ansus;
vector<int> g[N];
int mt[N], ind[N];
bool used[N];
bool kuhn(int u)
{
    if(used[u])
        return 0;
    used[u]=1;
    for(auto v:g[u])
    {
        if(mt[v]==-1||kuhn(mt[v]))
        {
            mt[v]=u;
            ind[u]=v;
            return 1;
        }
    }
    return 0;
}
int main()
{
    for(int i = 0;i<m;i++)
        mt[i]=-1;
    for(int i = 0;i<n;i++)
        ind[i]=-1;
    for(int run = 1;run;)
    {
        run=0;
        for(int i = 0;i<n;i++)
            used[i]=0;
        for(int i = 0;i<n;i++)
            if(ind[i]==-1&&kuhn(i))
                run=1;
    }
    // ind[u] = -1, ja nav matchots, citadi ind[u] = indekss no
otras komponentes
}
```

7. Flows

7.1. Dinitz

```
struct FlowEdge {
    int v, u;
    ll cap, flow = 0;
    FlowEdge(int v, int u, ll cap) : v(v), u(u), cap(cap) {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;
    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }
    void add_edge(int v, int u, ll cap) {
        edges.push_back(v, u, cap);
        edges.push_back(u, v, 0);
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }
    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }
    ll dfs(int v, ll pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++)
{
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap -
edges[id].flow < 1)
                continue;
            ll tr = dfs(u, min(pushed, edges[id].cap -
edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }
    ll flow() {
        ll f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
```

```
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (ll pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
};
```

7.2. Minimum-cost Max-Flow

```
struct Edge
{
    int from, to, capacity, cost;
};

vector<vector<int>> adj, cost, capacity;

const int INF = 1e9;

void shortest_paths(int n, int v0, vector<int>& d, vector<int>&
p) {
    d.assign(n, INF);
    d[v0] = 0;
    vector<bool> inq(n, false);
    queue<int> q;
    q.push(v0);
    p.assign(n, -1);

    while (!q.empty()) {
        int u = q.front();
        q.pop();
        inq[u] = false;
        for (int v : adj[u]) {
            if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
                d[v] = d[u] + cost[u][v];
                p[v] = u;
                if (!inq[v]) {
                    inq[v] = true;
                    q.push(v);
                }
            }
        }
    }
}

int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t)
{
    adj.assign(N, vector<int>());
    cost.assign(N, vector<int>(N, 0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to);
        adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost;
        cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity;
    }
    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p);
        if (d[t] == INF)
            break;

        // find max flow on that path
        int f = K - flow;
        int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]);
            cur = p[cur];
        }

        // apply flow
        flow += f;
        cost += f * d[t];
        cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f;
            capacity[cur][p[cur]] += f;
            cur = p[cur];
        }
    }

    if (flow < K)
        return -1;
    else
        return cost;
}
```

8. Strings

8.1. Manacher's algorithm longest palindromic substring

```
int manacher(string s){
    int n = s.size(); string p = "^#";
    rep(i,0,n) p += string(1, s[i]) + "#";
    p += "$"; n = p.size(); vector<int> lps(n, 0);
    int C=0, R=0, m=0;
    rep(i,1,n-1){
        int mirr = 2*C - i;
        if(i < R) lps[i] = min(R-i, lps[mirr]);
        while(p[i + 1 + lps[i]] == p[i - 1 - lps[i]]) lps[i]++;
        if(i + lps[i] > R){ C = i; R = i + lps[i]; }
        m = max(m, lps[i]);
    }
    return m;
}
```

8.2. Palindromic Tree (eertree)

```
struct eertree{

    int nex[N][AL];
    int ret[N];
    int par[N];
    int len[N];

    int id;
    void init(){
        len[0] = -1;
        ret[0] = 0;

        len[1] = 0;
        ret[1] = 0;

        id = 2;
    }
    string s;
    int n;
    void construct(string _s){
        s = _s;
        n = s.size();
        int las = 1;
        for(int i = 0 ; i < n; i ++ ){
            int cur = las;
            int l = s[i] - 'a' + 1;
            while(i - len[cur] - 1 < 0 || s[i] != s[i - len[cur]
- 1]){
                cur = ret[cur];
            }
            if(nex[cur][l] == 0){
                nex[cur][l] = id;
                len[id] = len[cur] + 2;
                par[id] = cur;
                if(cur == 0){
                    ret[id] = 1;
                }
                else{
                    int w = ret[cur];
                    while(i - len[w] - 1 < 0 || s[i] != s[i -
len[w] - 1]){
                        w = ret[w];
                    }
                    ret[id] = nex[w][l];
                }
                id ++ ;
            }
            las = nex[cur][l];
        }
    }

};
```

8.3. Suffix Array

```
const int M = 26;

void count_sort(vector<int> &p, vector<int> &c)
{
    int n = p.size();
    vector<int> pos(M+1);
    for(auto x:c)
        pos[x+1]++;
```

```
    for(int i = 1;i<=M;i++)
        pos[i]+=pos[i-1];
    vector<int> p_new(n);
    for(int i = 0;i<n;i++)
        p_new[pos[c[p[i]]]++]=p[i];
    swap(p,p_new);
}
int main()
{
    fio
    //ifstream cin("in.in");
    int n, m;
    cin >> n >> m;
    vector<int> str(n);
    for(auto &x:str)
        cin >> x;
    str.pb(-1);
    n++;
    vector<int> p(n), c(n);
    {
        vector<pair<char,int> > ve(n);
        for(int i = 0;i<n;i++)
            ve[i]={str[i],i};
        sort(ve.begin(),ve.end());
        for(int i = 0;i<n;i++)
            p[i]=ve[i].se;
        for(int i = 1;i<n;i++)
            c[p[i]]=c[p[i-1]]+(ve[i].fi!=ve[i-1].fi);
    }
    for(int k = 0;(1<<k)<n;k++)
    {
        for(int i = 0;i<n;i++)
            p[i]=(p[i]-(1<<k)+n)%n;
        count_sort(p,c);
        vector<int> c_new(n);
        for(int i = 1;i<n;i++)
            c_new[p[i]]=c_new[p[i-1]]+(c[p[i]]!=c[p[i-1]]||
c[(p[i]+(1<<k))%n]!=c[(p[i-1]+(1<<k))%n]);
        swap(c,c_new);
    }
    vector<int> lcp(n);
    int k = 0;
    for(int i = 0;i<n-1;i++)
    {
        int j = p[c[i]-1];
        while(str[i+k]==str[j+k])
            k++;
        lcp[c[i]]=k;
        k=max(k-1,0);
    }
    return 0;
}
```

8.4. Suffix Array and LCP (MK)

```
vector<int> suffix_array(string s){
    int n = s.size();
    int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for(int i = 0 ; i < n; i ++ ){
        cnt[s[i]] ++ ;
    }
    for(int i = 1; i < cnt.size(); i ++ ){
        cnt[i] += cnt[i - 1];
    }
    for(int i = 0 ; i < n; i ++ ){
        cnt[s[i]] -- ;
        p[cnt[s[i]]]=i;
    } // order
    c[p[0]] = 0;
    int classes = 1;
    for(int i = 1; i < n; i ++ ){
        c[p[i]] = c[p[i - 1]];
        if(s[p[i]] != s[p[i - 1]]){
            classes ++  ;
        }
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++)
            cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) %
n]};
            pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 <<
h)) % n]};
            if (cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}

vector<int> lcp_construct(string s, vector<int> p){
    int n = s.size();
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++)
        rank[p[i]] = i;

    int k = 0;
    vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] == s[j+k])
            k++;
        lcp[rank[i]] = k;
        if (k)
            k--;
    }
    return lcp;
}

void baseline(string s){
    vector<int> suffix = suffix_array(s);
    suffix.erase(suffix.begin());
    s.pop_back();
    vector<int> lcp = lcp_construct(s, suffix);
}
```

8.5. Aho-Corasick

```
const int K = 26;

struct Vertex {
    int next[K];
    bool output = false;
    int p = -1;
    char pch;
    int link = -1;
    int go[K];

    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t(1);

void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].output = true;
}

int go(int v, char ch);

int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
```

```
    return t[v].link;
}

int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}
```

8.6. KMP

```
vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
```

8.7. Z-Function

```
vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for(int i = 1; i < n; i++) {
        if(i < r) {
            z[i] = min(r - i, z[i - l]);
        }
        while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}
```

9. Geometry

9.1. Point to Line

Line $(Ax + By + C = 0)$ and point $(x_0; y_0)$ distance is:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

9.2. Graham scan

```
struct pt {
    double x, y;
    bool operator == (pt const& t) const {
        return x == t.x && y == t.y;
    }
};

int orientation(pt a, pt b, pt c) {
    double v = a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) ==
0; }

void convex_hull(vector<pt>& a, bool include_collinear = false) {
    pt p0 = *min_element(a.begin(), a.end(), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x-a.x)*(p0.x-a.x) + (p0.y-a.y)*(p0.y-a.y)
                < (p0.x-b.x)*(p0.x-b.x) + (p0.y-b.y)*(p0.y-b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = (int)a.size()-1;
        while (i >= 0 && collinear(p0, a[i], a.back())) i--;
        reverse(a.begin()+i+1, a.end());
    }

    vector<pt> st;
    for (int i = 0; i < (int)a.size(); i++) {
        while (st.size() > 1 && !cw(st[st.size()-2], st.back(),
a[i], include_collinear))
            st.pop_back();
        st.push_back(a[i]);
    }

    if (include_collinear == false && st.size() == 2 && st[0] ==
st[1])
        st.pop_back();

    a = st;
}
```

9.3. Cross Product in 2D space

$\vec{a} \circ \vec{b} = a_x b_y - a_y b_x$

9.4. Shoelace formula

$A = \frac{1}{2} \sum_{i=1}^{n} x_i (y_{i+1} - y_{i-1})$ (counter clock wise direction)

9.5. Online Convex Hull trick

```
// KTH notebook
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

9.6. Maximum points in a circle of radius R

```
typedef pair<double,bool> pdb;

#define START 0
#define END 1

struct PT
{
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

PT p[505];
double dist[505][505];
int n, m;

void calcDist()
{
  FOR(i,0,n)
  {
```

```
    FOR(j,i+1,n)
      dist[i][j]=dist[j][i]=sqrt((p[i].x-p[j].x)*(p[i].x-p[j].x)
        +(p[i].y-p[j].y)*(p[i].y-p[j].y));
  }
}
int intelInside(int point, double radius)
{
  vector<pdb> ranges;
  FOR(j,0,n)
  {
    if(j==point || dist[j][point]>2*radius) continue;
    double a1=atan2(p[point].y-p[j].y,p[point].x-p[j].x);
    double a2=acos(dist[point][j]/(2*radius));
    ranges.pb({a1-a2,START});
    ranges.pb({a1+a2,END});
  }
  sort(ALL(ranges));
  int cnt=1, ret=cnt;
  for(auto it: ranges)
  {
    if(it.second) cnt--;
    else cnt++;
    ret=max(ret,cnt);
  }

  return ret;
}

int go(double r)
{
  int cnt=0;
  FOR(i,0,n)
  {
    cnt=max(cnt,intelInside(i,r));
  }
  return cnt;
}
```

9.7. Point in polygon

```
int sideOf(const PT &s, const PT &e, const PT &p)
{
  ll a = cross(e-s,p-s);
  return (a > 0) - (a < 0);
}

bool onSegment(const PT &s, const PT &e, const PT &p)
{
  PT ds = p-s, de = p-e;
  return cross(ds,de) == 0 && dot(ds,de) <= 0;
}

/*
Main routine
Description: Determine whether a point t lies inside a given
polygon (counter-clockwise order).
The polygon must be such that every point on the circumference is
visible from the first point in the vector.
It returns 0 for points outside, 1 for points on the
circumference, and 2 for points inside.
*/

int insideHull2(const vector<PT> &H, int L, int R, const PT &p) {
  int len = R - L;
  if (len == 2) {
    int sa = sideOf(H[0], H[L], p);
    int sb = sideOf(H[L], H[L+1], p);
    int sc = sideOf(H[L+1], H[0], p);
    if (sa < 0 || sb < 0 || sc < 0) return 0;
    if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R ==
(int)H.size()))
      return 1;
    return 2;
  }
  int mid = L + len / 2;
  if (sideOf(H[0], H[mid], p) >= 0)
    return insideHull2(H, mid, R, p);
  return insideHull2(H, L, mid+1, p);
}

int insideHull(const vector<PT> &hull, const PT &p) {
  if ((int)hull.size() < 3) return onSegment(hull[0],
hull.back(), p);
  else return insideHull2(hull, 1, (int)hull.size(), p);
}
```

10. Numerical

10.1. FFT

```
using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> & a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <<= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }
    if (invert) {
        for (cd & x : a)
            x /= n;
    }
}
vector<int> multiply(vector<int> const& a, vector<int> const& b)
{
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < a.size() + b.size())
        n <<= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
    vector<int> result(n);
    for (int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    return result;
}
```

10.2. NTT

```
const ll mod = (119 << 23) + 1, root = 62; // 998244353
typedef vector<ll> vl;

int modpow(int n, int k);

void ntt(vl &a) {
  int n = a.size(), L = 31 - __builtin_clz(n);
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    for(int i=k;i<2*k;i++) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  vl rev(n);
  for(int i = 0 ; i < n; i ++ ) rev[i] = (rev[i / 2] | (i & 1) <<
L) / 2;
  for(int i = 0 ; i < n; i ++ ) if (i < rev[i]) swap(a[i],
a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) for(int j=0;j<k;j++) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }
}
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = a.size() + b.size() - 1, B = 32 - __builtin_clz(s),
      n = 1 << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  for(int i = 0 ; i < n;i ++ )
    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

10.3. Sum of n^k in $O(k^2)$

```
LL mod;
LL S[105][105];
void solve() {
    LL n, k;
    scanf("%lld %lld %lld", &n, &k, &mod);
    S[0][0] = 1 % mod;
    for (int i = 1; i <= k; i++) {
        for (int j = 1; j <= i; j++) {
            if (i == j) S[i][j] = 1 % mod;
            else S[i][j] = (j * S[i - 1][j] + S[i - 1][j - 1]) %
mod;
        }
    }
    LL ans = 0;
    for (int i = 0; i <= k; i++) {
        LL fact = 1, z = i + 1;
        for (LL j = n - i + 1; j <= n + 1; j++) {
            LL mul = j;
            if (mul % z == 0) {
                mul /= z;
                z /= z;
            }
            fact = (fact * mul) % mod;
        }
        ans = (ans + S[k][i] * fact) % mod;
    }
    printf("%lld\n", ans);
}
```

10.4. Gauss method

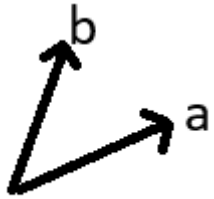
```
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or
a big number

int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}
```

11. Our Geometry Template

11.1. Point class

```
template<class T>
struct Point{
    T x;
    T y;
    Point operator+(const Point &o) const {
        return {x + o.x, y + o.y};
    }
    Point operator-(const Point &o) const {
        return {x - o.x, y - o.y};
    }
    Point operator*(T w) const {
        return {x * w, y * w};
    }
    Point operator/(T w) const {
        return {x / w, y / w};
    }
    Point perp() const {
        return Point{-y, x}; // rotates +90 degrees
    }
    bool operator<(Point &o){
        if(x == o.x) return y < o.y;
        else return x < o.x;
    }
    T cross(Point a) const {
        return x * a.y - y * a.x;
    }
    T dist2() const {
        return x * x + y * y;
    }
    double dist() const {
        return sqrt(dist2());
    }
    T operator*(const Point &o) const {
        return x*o.x+y*o.y;
    }
};
```

11.2. Cross Product



In this case $\vec{a} \times \vec{b} = a_x \cdot b_y - a_y \cdot b_x > 0$

11.3. Circumcenter

```
typedef Point<double> P;

double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist()/
    abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

11.4. Minimum-Enclosing Circle

```
typedef Point<double> P;

pair<P, double> enclose(vector<P> ps) {
    shuffle(ps.begin(), ps.end(), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    int sz = (int)ps.size();
    for(int i = 0 ; i < sz; i ++ ){
        if((o - ps[i]).dist() > r * EPS){
            o = ps[i], r =0;
            for(int j = 0 ; j < i; j ++ ){
                if((o - ps[j]).dist() > r * EPS){
                    o = (ps[i] + ps[j]) / 2;
                    r = (o - ps[i]).dist();
                    for(int k = 0 ; k < j ; k ++ ){
                        if((o - ps[k]).dist() > r * EPS){
                            o = ccCenter(ps[i], ps[j], ps[k]);
                            r = (o - ps[i]).dist();
                        }
                    }
                }
            }
        }
    }
    return {o, r};
}
```

11.5. Polar-Sort

```
sort(X.begin(), X.end(), [&](Point<int> a, Point<int> b){
    Point<int> origin{0, 0};
    bool ba = a < origin, bb = b < origin;
```

```
    if(ba != bb) {return ba < bb;}
    else return a.cross(b) > 0;
});
```

12. General

12.1. Simulated Annealing

```
const ld T = (ld)2000;
const ld alpha = 0.999999;
// (new_score - old_score) / (temperature_final) ~ 10 works well

const ld L = (ld)1e6;
ld small_rand(){
  return ((ld)gen(L))/L;
}

ld P(ld old, ld nw, ld temp){
  if(nw > old)
    return 1.0;
  return exp((nw-old)/temp);
}

{
  auto start = chrono::steady_clock::now();
  ld time_limit = 2000;
  ld temperature = T;
  ld max_score = -1;

  while(elapsed_time < time_limit){
    auto cur = chrono::steady_clock::now();
    elapsed_time =
chrono::duration_cast<chrono::milliseconds>(cur - start).count();
    temperature *= alpha;

    // try a neighboring state
    // ....
    // ....

    old_score = score(old_state);
    new_score = score(new_state);
    if(P(old_score, new_score, temperature) >= small_rand()){
      old_state = new_state;
      old_score = new_score;
    }
    if(old_score > max_score){
      max_score = old_score;
      max_state = old_state;
    }
  }
}
```

13. Out of ideas?

1. $\text{opt}(i) \leq \text{opt}(i+1)$