

- Valters Kalniņš
- Kristaps Štāls
- Matīss Kristiņš

Contents

1.	C++	1
1.1.	Optimizations	1
1.2.	Hash function	1
1.3.	Bitset	1
1.4.	C++ random	1
2.	Algebra	1
3.	Number Theory	1
3.1.	Rabin-Miller	1
3.2.	Extended GCD	2
3.3.	Chinese Remainder Theorem	2
3.4.	Random usable primes	2
3.5.	Euler's totient function	2
4.	Combinatorics	2
4.1.	Stars and bars	2
4.2.	Vandermonde identity (and variants)	2
5.	Data Structures	2
5.1.	Treap	2
6.	Graphs	2
6.1.	k-shortest path	2
7.	Algorithms	3
7.1.	Kuhn's algorithm	3
7.2.	Hopcroft-Karp (Max Matching)	3
7.3.	Hungarian Algorithm (Min cost, Max matching)	4
8.	Flows	4
8.1.	Dinitz	4
8.2.	Minimum-cost Max-Flow	5
9.	Strings	5
9.1.	Manacher's algorithm longest palindromic substring	5
9.2.	Palindromic Tree (eertree)	5
9.3.	Suffix Array	6
9.4.	Suffix Array and LCP (MK)	6
9.5.	Aho-Corasick	6
9.6.	KMP	7
9.7.	Z-Function	7
10.	Geometry	7
10.1.	Point to Line	7
10.2.	Graham scan	7
10.3.	Cross Product in 2D space	7
10.4.	Shoelace formula	7
10.5.	Online Convex Hull trick	7
10.6.	Maximum points in a circle of radius R	7
10.7.	Point in polygon	8
10.8.	Minkowski Sum	8
11.	Numerical	8
11.1.	FFT	8

11.2.	NTT	9
11.3.	Sum of n^k in $O(k^2)$	9
11.4.	Gauss method	9
11.5.	Berlekamp-Massey	9
12.	Our Geometry Template	10
12.1.	Point class	10
12.2.	Cross Product	10
12.3.	Circumcenter	10
12.4.	Line Distance	10
12.5.	Line Intersection	10
12.6.	Minimum-Enclosing Circle	10
12.7.	Polar-Sort	10
13.	General	10
13.1.	Simulated Annealing	10
13.2.	Expression Parsing (cp-algo)	11
14.	janY's 2D Geometry	11
14.1.	vec2	11
14.2.	2D Geometric Functions	12
14.3.	Halfplane Intersection	12
15.	janY's Algorithms	13
15.1.	Modulo	13
15.2.	Factorization	13
15.3.	Combinatorics	13
15.4.	Disjoint Set Union	14
15.5.	Merge Sort Tree	14
15.6.	Fenwick Tree	14
15.7.	Fenwick Tree (Range Updates)	15
15.8.	Kosaraju's Algorithm	15
15.9.	Range Minimum Query	15
15.10.	Polynomial Rolling Hash	16
15.11.	Matrix Template	16
15.12.	Convex Hull	16
15.13.	Prufer Codes	16
15.14.	Segment tree	17
15.15.	NTT	17
16.	Out of ideas?	18

1. C++

1.1. Optimizations

```
#pragma GCC optimize("Ofast, unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt,tune=native")
```

1.2. Hash function

```
static uint64_t splitmix64(uint64_t x)
{x+=0x9e377b97f4a7c15;x=(x^(x>>30))*0xbff58476d1ce4e5b9;
x=(x^(x>>27))*0x94d049bb133111eb;
return x^(x>>31);}
struct custom_hash {size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
        chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x+FIXED_RANDOM);}}
```

```
const long long mod=998244353;
//1000000007
long long modpow(long long n, long long m){long long
res=1;while(m&1)res=res*n%mod;n=n*n%mod;m>>=1;}return res;}
```

1.3. Bitset

```
bitset<10> bb("1010000000"); // reverse order constructor
cout << bb.count() << "\n"; // 2
cout << bb._Find_first() << "\n"; // 7
bb[0] = 1;
cout << bb._Find_first() << "\n"; // 0
```

1.4. C++ random

```
mt19937
rng(chrono::steady_clock::now().time_since_epoch().count());
```

2. Algebra

$$\sum_{i=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

3. Number Theory

3.1. Rabin-Miller

```
using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}
```

```
bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};
```

```
bool MillerRabin(u64 n, int iter=5) { // returns true if n is
    probably prime, else returns false.
    if (n < 4)
        return n == 2 || n == 3;
```

```

int s = 0;
u64 d = n - 1;
while ((d & 1) == 0) {
    d >>= 1;
    s++;
}

for (int i = 0; i < iter; i++) {
    int a = 2 + rand() % (n - 3);
    if (check_composite(n, a, d, s))
        return false;
}
return true;
}

```

3.2. Extended GCD

```

int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

```

3.3. Chinese Remainder Theorem

Notes:

- Assumes all modulo are pairwise coprime
- If not, splitting modulus using prime powers works

```

int mod_inv(int a, int mod){
    int x, y;
    int g = extGcd(a, mod, x, y);
    x = (x % mod + mod) % mod;
    return x;
}

pair<int, int> crt(vector<pair<int, int>> congruences){
    // {mod, remainder}
    int M = 1;
    for(auto c : congruences){
        M *= c.first;
    }
    int solution = 0;
    for(auto c : congruences) {
        int a_i = c.second;
        int m_i = M / c.first;
        int n_i = mod_inv(m_i, c.first);
        solution = (solution + a_i * m_i % M * n_i) % M;
    }
    return {M, solution};
}

```

3.4. Random usable primes

```

666240077 964865333 115091077 378347773 568491163 295451837
658540403 856004729 843998543 380557313

```

3.5. Euler's totient function

Useful stuff: $a^{\varphi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$

$$\sum_{i|n} \varphi(i) = n$$

```

phi[1] = 1;
for(ll i = 2; i <= n; i += ) {
    phi[i] = i;
}
for(ll i = 2; i <= n; i += ) {
    if(pr[i] == false) {
        for(ll j = i; j <= n; j += i) {
            phi[j] /= i;
            phi[j] *= (i - 1);
            pr[j] = true;
        }
    }
}

```

4. Combinatorics

4.1. Stars and bars

n balls, k boxes:

$$\binom{n+k-1}{k-1}$$

4.2. Vandermonde identity (and variants)

$$\binom{m+n}{r} = \sum \binom{n}{k} \binom{m}{r-k}$$

$$\sum \binom{n}{x} \binom{m}{x} = \binom{n+m}{n}$$

5. Data Structures

5.1. Treap

```

struct Node{
    int value, cnt, pri; Node *left, *right;
    Node(int p) : value(p), cnt(1), pri(gen()), left(NULL), right(NULL) {};
};

typedef Node* pnode;
int get(pnode q){if(!q) return 0; return q->cnt;}
void update_cnt(pnode &q){
    if(!q) return; q->cnt = get(q->left)+get(q->right)+1;
}
void merge(pnode &T, pnode lef, pnode rig){
    if(!lef){T=rig; return;} if(!rig){T=lef; return;}
    if(lef->pri>rig->pri){merge(lef->right, lef->right, rig); T=lef;}
    else{merge(rig->left, lef, rig->left); T = rig;}
    update_cnt(T);
}
void split(pnode cur, pnode &lef, pnode &rig, int key){
    if(!cur){lef=rig=NULL; return;} int id = get(cur->left)+1;
    if(id <= key){split(cur->right, cur->right, rig, key-id); lef=cur;}
}

```

```

else {split(cur->left, lef, cur->left, key); rig = cur;}
update_cnt(cur);
}

```

6. Graphs

6.1. k-shortest path

```

mt19937 mt(119);
template <typename T> using min_priority_queue =
priority_queue<T, vector<T>, greater<T>>;
template <typename T>
struct heap_node{
    array<heap_node*, 2> c;
    T key;
};

template <typename T>
heap_node<T>* insert(heap_node<T>* a, T new_key) {
    if(!a || new_key.first < a->key.first){
        heap_node<T>* n = new heap_node<T>;
        n->c = {a, nullptr};
        n->key = new_key;
        return n;
    }
    a = new heap_node<T>(*a);
    int z = mt() & 1;
    a->c[z] = insert(a->c[z], new_key);
    return a;
}

```

```

vector<ll> k_shortest_paths(int n, vector<pair<array<int, 2>, ll>> edges, int st, int en, int K){
    int M = edges.size();
    vector<vector<tuple<int, int, ll>>> radj(n);
    for(int e = 0; e < M; e ++ ){
        auto [x, l] = edges[e];
        auto [u, v] = x;
        radj[v].push_back({e, u, l});
    }
    vector<ll> dist(n, -1);
    vector<int> prvE(n, -1);
    vector<int> toposort;
    {
        min_priority_queue<pair<ll, int>> pq;
        pq.push({dist[en] = 0, en});
        while(!pq.empty()){
            ll d = pq.top().first;
            int cur = pq.top().second;
            pq.pop();
            if(d > dist[cur]) continue;
            toposort.push_back(cur);
            // for(auto [e, nxt, l] : radj[cur]){
            for(auto ee : radj[cur]){
                int e = get<0>(ee);
                int nxt = get<1>(ee);
                int l = get<2>(ee);
                if(dist[nxt] == -1 || d + l < dist[nxt]){
                    prvE[nxt] = e;
                    dist[nxt] = d + l;
                    pq.push({dist[nxt], nxt});
                }
            }
        }
    }
}

```

```

        pq.push({dist[nxt] = d + l, nxt});
    }
}
vector<vector<pair<ll, int>>> adj(n);
for(int e = 0; e < M; e ++){
    auto& [x, l] = edges[e];
    const auto& [u, v] = x;
    if(dist[v] == -1) continue;

    l += dist[v] - dist[u];

    if(e == prvE[u]) continue;
    adj[u].push_back({l, v});
}
for(int i = 0; i < n; i ++){
    sort(adj[i].begin(), adj[i].end());
    adj[i].push_back({-1, -1});
}
using iter_t = decltype(adj[0].begin());
using hnode = heap_node<pair<ll, iter_t>>;
vector<hnode*> node_roots(n, nullptr);
for(int cur : toposort){
    if(cur != en){
        int prv = edges[prvE[cur]].first[1];
        node_roots[cur] = node_roots[prv];
    } else {
        node_roots[cur] = nullptr;
    }
    const auto& [l, nxt] = adj[cur][0];
    if(nxt != -1){
        node_roots[cur] = insert(node_roots[cur], {l,
adj[cur].begin()});
    }
}
vector<pair<ll, int>> dummy_adj({{0, st}, {-1, -1}});
vector<ll> res; res.reserve(K);
min_priority_queue<tuple<ll, hnode*, iter_t>> q;
q.push({dist[st], nullptr, dummy_adj.begin()});
while(int(res.size()) < K && !q.empty()) {
    auto [l, start_heap, val_iter] = q.top(); q.pop();
    res.push_back(l);
    ll elen = val_iter->first;
    if(next(val_iter)->second != -1){
        q.push({l - elen + next(val_iter)->first, nullptr,
next(val_iter)});
    }
    if(start_heap){
        for(int z = 0; z < 2 ; z ++ ){
            auto nxt_start = start_heap->c[z];
            if(!nxt_start) continue;
            q.push({l - elen + nxt_start->key.first,
nxt_start, nxt_start->key.second});
        }
    }
    int nxt = val_iter->second;
    auto nxt_start = node_roots[nxt];
    if(nxt_start) {

```

```

        q.push({l + nxt_start->key.first, nxt_start,
nxt_start->key.second});
    }
}
return res;
}



## 7. Algorithms



### 7.1. Kuhn's algorithm



```

// node matching indexed 1-n with 1-m
const int N = ansus;
vector<int> g[N];
int mt[N], ind[N];
bool used[N];
bool kuhn(int u)
{
 if(used[u])
 return 0;
 used[u]=1;
 for(auto v:g[u])
 {
 if(mt[v]==-1||kuhn(mt[v]))
 {
 mt[v]=u;
 ind[u]=v;
 return 1;
 }
 }
 return 0;
}
int main()
{
 for(int i = 0;i<m;i++)
 mt[i]=-1;
 for(int i = 0;i<n;i++)
 ind[i]=-1;
 for(int run = 1;run;)
 {
 run=0;
 for(int i = 0;i<n;i++)
 used[i]=0;
 for(int i = 0;i<n;i++)
 if(ind[i]==-1&&kuhn(i))
 run=1;
 // ind[u] = -1, ja nav matchots, citadi ind[u] = indekss no
 otras komponentes
 }
}

```


```

7.2. Hopcroft-Karp (Max Matching)

```

// Hopcroft-Karp maximal matching in O(E*sqrt(V))

struct HopcroftKarp {
    int n, m;
    const int NIL = 0, INF = INT_MAX;
    vector<int> pair_u, pair_v, dist;
    vector<vector<int>> adj;

```

```

HopcroftKarp() {};
HopcroftKarp(int n, int m) : n(n), m(m) {
    adj.resize(n+1);
}
// 1-indexed
void add_edge(int u, int v) {adj[u].push_back(v);}

int calc() {
    pair_u.assign(n+1, NIL);
    pair_v.assign(m+1, NIL);
    dist.assign(n+1, 0);
    int ans = 0;
    while (bfs()) {
        for (int u = 1; u <= n; u++) {
            if (pair_u[u] == NIL && dfs(u)) ans++;
        }
    }
    return ans;
}

bool bfs() {
    queue<int> q;
    for (int u = 1; u <= n; u++) {
        dist[u] = INF;
        if (pair_u[u] != NIL) continue;
        q.push(u);
        dist[u] = 0;
    }
    dist[NIL] = INF;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        if (dist[u] >= dist[NIL]) continue;
        for (auto &v : adj[u]) {
            if (dist[pair_v[v]] != INF) continue;
            dist[pair_v[v]] = dist[u] + 1;
            q.push(pair_v[v]);
        }
    }
    return (dist[NIL] != INF);
}

bool dfs(int u) {
    if (u == NIL) return true;
    for (auto &v : adj[u]) {
        if (dist[pair_v[v]] != dist[u] + 1 || !
dfs(pair_v[v])) continue;
        pair_v[v] = u;
        pair_u[u] = v;
        return true;
    }
    dist[u] = INF;
    return false;
}
};


```

University of Latvia: Scrupulous strawberries

7.3. Hungarian Algorithm (Min cost, Max matching)

```
// Hungarian algorithm O(n^3) (but fast)
// 1-indexed
// It finds minimum cost maximum matching.
// For finding maximum cost maximum matching add -cost and return -matching()
// matching stored in l array. l[i] contains index of right side element that is match with the i-th left side element.
const int N = 1024;
struct Hungarian {
    ll c[N][N], fx[N], fy[N], d[N];
    int l[N], r[N], arg[N], trace[N];
    queue<int> q;
    int start, finish, n;
    const ll inf = 1e18;
    Hungarian() {}
    Hungarian(int n1, int n2) {init(n1, n2);}
    void init(int n1, int n2) {
        n = max(n1, n2);
        for (int i = 1; i <= n; ++i) {
            fy[i] = l[i] = r[i] = 0;
            for (int j = 1; j <= n; ++j) c[i][j] = inf;
        }
    }
    void add_edge(int u, int v, ll cost) {
        c[u][v] = min(c[u][v], cost);
    }
    inline ll getC(int u, int v) {
        return c[u][v] - fx[u] - fy[v];
    }
    void init_bfs() {
        while (!q.empty()) q.pop();
        q.push(start);
        for (int i = 0; i <= n; ++i) trace[i] = 0;
        for (int v = 1; v <= n; ++v) {
            d[v] = getC(start, v);
            arg[v] = start;
        }
        finish = 0;
    }
    void find_aug_path() {
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (int v = 1; v <= n; ++v) if (!trace[v]) {
                ll w = getC(u, v);
                if (!w) {
                    trace[v] = u;
                    if (!r[v]) {
                        finish = v;
                        return;
                    }
                    q.push(r[v]);
                }
                if (d[v] > w) {
                    d[v] = w;
                    arg[v] = u;
                }
            }
        }
    }
}
```

```

    }
    void subX_addY() {
        ll delta = inf;
        for (int v = 1; v <= n; ++v) if (trace[v] == 0 && d[v] < delta) {
            delta = d[v];
        }
        fx[start] += delta;
        for (int v = 1; v <= n; ++v) if (trace[v]) {
            int u = r[v];
            fy[v] -= delta;
            fx[u] += delta;
        } else d[v] -= delta;
        for (int v = 1; v <= n; ++v) if (!trace[v] && !d[v]) {
            trace[v] = arg[v];
            if (!r[v]) {
                finish = v; return;
            }
            q.push(r[v]);
        }
    }
    void enlarge() {
        do {
            int u = trace[finish];
            int nxt = l[u];
            l[u] = finish;
            r[finish] = u;
            finish = nxt;
        } while (finish);
    }
    ll maximum_matching() {
        for (int u = 1; u <= n; ++u) {
            fx[u] = c[u][1];
            for (int v = 1; v <= n; ++v) {
                fx[u] = min(fx[u], c[u][v]);
            }
        }
        for (int v = 1; v <= n; ++v) {
            fy[v] = c[1][v] - fx[1];
            for (int u = 1; u <= n; ++u) {
                fy[v] = min(fy[v], c[u][v] - fx[u]);
            }
        }
        for (int u = 1; u <= n; ++u) {
            start = u;
            init_bfs();
            while (!finish) {
                find_aug_path();
                if (!finish) subX_addY();
            }
            enlarge();
        }
        ll ans = 0;
        for (int i = 1; i <= n; ++i) {
            if (c[i][l[i]] != inf) ans += c[i][l[i]];
            else l[i] = 0;
        }
        return ans;
    }
}
```

8. Flows

8.1. Dinitz

```
struct FlowEdge {
    int v, u;
    ll cap, flow = 0;
    FlowEdge(int v, int u, ll cap) : v(v), u(u), cap(cap) {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;
    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }
    void add_edge(int v, int u, ll cap) {
        edges.push_back(v, u, cap);
        edges.push_back(u, v, 0);
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }
    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }
    ll dfs(int v, ll pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap -
                edges[id].flow < 1)
                continue;
```

```

        }
    }
}

int min_cost_flow(int n, vector<Edge> edges, int K, int s, int t)
{
    int m = 0;
    adj.resize(n);
    vector<Edge> edg;
    for(Edge e : edges){
        edg.push_back({e.from, e.to, e.capacity, e.cost});
        edg.push_back({e.to, e.from, 0, -e.cost});
        adj[e.from].push_back(m);
        adj[e.to].push_back(m + 1);
        m += 2;
    }
    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while(flow < K){
        shortest_paths(n, s, edg, d, p);
        if(d[t] == inf)
            break;
        int f = K - flow;
        int cur = t;
        while(cur != s){
            f = min(f, edg[p[cur]].capacity);
            cur = edg[p[cur]].from;
        }
        flow += f;
        cost += f * d[t];
        cur = t;
        while(cur != s){
            edg[p[cur]].capacity -= f;
            edg[p[cur]^1].capacity += f;
            cur = edg[p[cur]].from;
        }
        if(flow < K)
            return -inf;
        else
            return cost;
    }
}

```

8.2. Minimum-cost Max-Flow

```

struct Edge
{
    int from;
    int to;
    int capacity;
    int cost;
};

vector<vector<int>> adj, cost, capacity;
const int inf = (int)1e9;

void shortest_paths(int n, int v0, vector<Edge> &edges,
vector<int>& d, vector<int>& p){
    d.assign(n, inf);
    d[v0] = 0;
    vector<bool> inq(n, false);
    queue<int> q;
    q.push(v0);
    p.assign(n, -1);
    while(!q.empty()){
        int u = q.front();
        q.pop();
        inq[u] = false;
        for(int v : adj[u]){
            if(edges[v].capacity > 0 && d[edges[v].to] > d[u] + edges[v].cost){
                d[edges[v].to] = d[u] + edges[v].cost;
                p[edges[v].to] = v;
                if(!inq[edges[v].to])
                    inq[edges[v].to] = true;
                q.push(edges[v].to);
            }
        }
    }
}

```

```

        }
    }
}

int min_cost_flow(int n, vector<Edge> edges, int K, int s, int t)
{
    int m = 0;
    adj.resize(n);
    vector<Edge> edg;
    for(Edge e : edges){
        edg.push_back({e.from, e.to, e.capacity, e.cost});
        edg.push_back({e.to, e.from, 0, -e.cost});
        adj[e.from].push_back(m);
        adj[e.to].push_back(m + 1);
        m += 2;
    }
    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while(flow < K){
        shortest_paths(n, s, edg, d, p);
        if(d[t] == inf)
            break;
        int f = K - flow;
        int cur = t;
        while(cur != s){
            f = min(f, edg[p[cur]].capacity);
            cur = edg[p[cur]].from;
        }
        flow += f;
        cost += f * d[t];
        cur = t;
        while(cur != s){
            edg[p[cur]].capacity -= f;
            edg[p[cur]^1].capacity += f;
            cur = edg[p[cur]].from;
        }
        if(flow < K)
            return -inf;
        else
            return cost;
    }
}

```

9. Strings

9.1. Manacher's algorithm longest palindromic substring

```

int manacher(string s){
    int n = s.size(); string p = "^#";
    rep(i,0,n) p += string(1, s[i]) + "#";
    p += "$"; n = p.size(); vector<int> lps(n, 0);
    int C=0, R=0, m=0;
    rep(i,1,n-1){
        int mirr = 2*C - i;
        if(i < R) lps[i] = min(R-i, lps[mirr]);
        else lps[i] = 0;
        int j = i + lps[i];
        while(j < n && p[j] == p[i])
            lps[i]++;
        C = i;
        R = j;
    }
}

```

```

while(p[i + 1 + lps[i]] == p[i - 1 - lps[i]]) lps[i]++;
if(i + lps[i] > R){ C = i; R = i + lps[i]; }
m = max(m, lps[i]);
}
return m;
}

9.2. Palindromic Tree (eertree)
struct eertree{

    int nex[N][AL];
    int ret[N];
    int par[N];
    int len[N];

    int id;
    void init(){
        len[0] = -1;
        ret[0] = 0;
        len[1] = 0;
        ret[1] = 0;
        id = 2;
    }
    string s;
    int n;
    void construct(string _s){
        s = _s;
        n = s.size();
        int las = 1;
        for(int i = 0 ; i < n; i ++ ){
            int cur = las;
            int l = s[i] - 'a' + 1;
            while(i - len[cur] - 1 < 0 || s[i] != s[i - len[cur] - 1]){
                cur = ret[cur];
            }
            if(nex[cur][l] == 0){
                nex[cur][l] = id;
                len[id] = len[cur] + 2;
                par[id] = cur;
                if(cur == 0){
                    ret[id] = 1;
                }
                else{
                    int w = ret[cur];
                    while(i - len[w] - 1 < 0 || s[i] != s[i - len[w] - 1]){
                        w = ret[w];
                    }
                    ret[id] = nex[w][l];
                }
                id++;
            }
            las = nex[cur][l];
        }
    }
}

```

};

9.3. Suffix Array

```

const int M = 26;

void count_sort(vector<int> &p, vector<int> &c)
{
    int n = p.size();
    vector<int> pos(M+1);
    for(auto x:c)
        pos[x+1]++;
    for(int i = 1;i<=M;i++)
        pos[i]=pos[i-1];
    vector<int> p_new(n);
    for(int i = 0;i<n;i++)
        p_new[pos[c[p[i]]]]+=p[i];
    swap(p,p_new);
}

int main()
{
    fio
    //ifstream cin("in.in");
    int n, m;
    cin >> n >> m;
    vector<int> str(n);
    for(auto &x:str)
        cin >> x;
    str.pb(-1);
    n++;
    vector<int> p(n), c(n);
    {
        vector<pair<char,int> > ve(n);
        for(int i = 0;i<n;i++)
            ve[i]={str[i],i};
        sort(ve.begin(),ve.end());
        for(int i = 0;i<n;i++)
            p[i]=ve[i].se;
        for(int i = 1;i<n;i++)
            c[p[i]]=c[p[i-1]]+(ve[i].fi!=ve[i-1].fi);
    }
    for(int k = 0;(1<<k)<n;k++)
    {
        for(int i = 0;i<n;i++)
            p[i]=(p[i]-(1<<k)+n)%n;
        count_sort(p,c);
        vector<int> c_new(n);
        for(int i = 1;i<n;i++)
            c_new[p[i]]=c_new[p[i-1]]+(c[p[i]]!=c[p[i-1]])|
c[(p[i]+(1<<k))%n]!=c[(p[i-1]+(1<<k))%n];
        swap(c,c_new);
    }
    vector<int> lcp(n);
    int k = 0;
    for(int i = 0;i<n-1;i++)
    {
        int j = p[c[i]-1];
        while(str[i+k]==str[j+k])
    }

```

```

        k++;
        lcp[c[i]]=k;
        k=max(k-1,0);
    }
    return 0;
}

```

9.4. Suffix Array and LCP (MK)

```

vector<int> suffix_array(string s){
    int n = s.size();
    int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for(int i = 0 ; i < n; i ++ ){
        cnt[s[i]] += ;
    }
    for(int i = 1; i < cnt.size(); i ++ ){
        cnt[i] += cnt[i - 1];
    }
    for(int i = 0 ; i < n; i ++ ){
        cnt[s[i]] -- ;
        p[cnt[s[i]]]=i;
    }
    // order
    c[p[0]] = 0;
    int classes = 1;
    for(int i = 1; i < n; i ++ ){
        c[p[i]] = c[p[i - 1]];
        if(s[p[i]] != s[p[i - 1]]){
            classes += ;
        }
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++)
            cn[c[pn[i]]]++;
        for (int i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
            pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 << h)) % n]};
            if (cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}

```

```

}
vector<int> lcp_construct(string s, vector<int> p){
    int n = s.size();
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++)
        rank[p[i]] = i;

    int k = 0;
    vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] == s[j+k])
            k++;
        lcp[rank[i]] = k;
        if (k)
            k--;
    }
    return lcp;
}

void baseline(string s){
    vector<int> suffix = suffix_array(s);
    suffix.erase(suffix.begin());
    s.pop_back();
    vector<int> lcp = lcp_construct(s, suffix);
}

```

9.5. Aho-Corasick

```

const int K = 26;

struct Vertex {
    int next[K];
    bool output = false;
    int p = -1;
    char pch;
    int link = -1;
    int go[K];

    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t(1);

void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
    }
}

```

```

        v = t[v].next[c];
    }
    t[v].output = true;
}

int go(int v, char ch);

int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}

int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}

```

9.6. KMP

```

vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}

```

9.7. Z-Function

```

vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i < r) {
            z[i] = min(r - i, z[i - l]);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
}

```

```

        }
        return z;
    }
}

```

10. Geometry

10.1. Point to Line

Line ($Ax + By + C = 0$) and point $(x_0; y_0)$ distance is:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

10.2. Graham scan

```

struct pt {
    double x, y;
    bool operator == (pt const& t) const {
        return x == t.x && y == t.y;
    }
};

int orientation(pt a, pt b, pt c) {
    double v = a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& a, bool include_collinear = false) {
    pt p0 = *min_element(a.begin(), a.end(), [] (pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x-a.x)*(p0.x-a.x) + (p0.y-a.y)*(p0.y-a.y)
                < (p0.x-b.x)*(p0.x-b.x) + (p0.y-b.y)*(p0.y-b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = (int)a.size()-1;
        while (i >= 0 && collinear(p0, a[i], a.back())) i--;
        reverse(a.begin() + i + 1, a.end());
    }

    vector<pt> st;
    for (int i = 0; i < (int)a.size(); i++) {
        while (st.size() > 1 && !cw(st[st.size()-2], st.back(),
            a[i], include_collinear))
            st.pop_back();
        st.push_back(a[i]);
    }
}

```

```

        if (include_collinear == false && st.size() == 2 && st[0] ==
        st[1])
            st.pop_back();
    }
    a = st;
}

```

10.3. Cross Product in 2D space

$$\vec{a} \circ \vec{b} = a_x b_y - a_y b_x$$

10.4. Shoelace formula

$$A = \frac{1}{2} \sum_{i=1}^n x_i(y_{i+1} - y_{i-1}) \text{ (counter clock wise direction)}$$

10.5. Online Convex Hull trick

```

// KTH notebook
struct Line {
    mutable ll k, m, p;
    bool operator <(const Line& o) const { return k < o.k; }
    bool operator <(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

```

10.6. Maximum points in a circle of radius R

```

typedef pair<double, bool> pdb;

#define START 0
#define END 1

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
}

```

```

PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
PT operator * (double c) const { return PT(x*c, y*c ); }
PT operator / (double c) const { return PT(x/c, y/c ); }
};

PT p[505];
double dist[505][505];
int n, m;

void calcDist()
{
    FOR(i,0,n)
    {
        FOR(j,i+1,n)
            dist[i][j]=dist[j][i]=sqrt((p[i].x-p[j].x)*(p[i].x-p[j].x)
                +(p[i].y-p[j].y)*(p[i].y-p[j].y));
    }
}
int intelInside(int point, double radius)
{
    vector<pdbs> ranges;
    FOR(j,0,n)
    {
        if(j==point || dist[j][point]>2*radius) continue;
        double a1=atan2(p[point].y-p[j].y,p[point].x-p[j].x);
        double a2=acos(dist[point][j]/(2*radius));
        ranges.pb({a1-a2,START});
        ranges.pb({a1+a2,END});
    }
    sort(ALL(ranges));
    int cnt=1, ret=cnt;
    for(auto it: ranges)
    {
        if(it.second) cnt--;
        else cnt++;
        ret=max(ret,cnt);
    }

    return ret;
}

int go(double r)
{
    int cnt=0;
    FOR(i,0,n)
    {
        cnt=max(cnt,intelInside(i,r));
    }
    return cnt;
}

10.7. Point in polygon
int sideOf(const PT &s, const PT &e, const PT &p)
{
    ll a = cross(e-s,p-s);
    return (a > 0) - (a < 0);
}

bool onSegment(const PT &s, const PT &e, const PT &p)

```

```

    {
        PT ds = p-s, de = p-e;
        return cross(ds,de) == 0 && dot(ds,de) <= 0;
    }

    /*
    Main routine
    Description: Determine whether a point t lies inside a given
    polygon (counter-clockwise order).
    The polygon must be such that every point on the circumference is
    visible from the first point in the vector.
    It returns 0 for points outside, 1 for points on the
    circumference, and 2 for points inside.
    */

    int insideHull2(const vector<PT> &H, int L, int R, const PT &p) {
        int len = R - L;
        if (len == 2) {
            int sa = sideOf(H[0], H[L], p);
            int sb = sideOf(H[L], H[L+1], p);
            int sc = sideOf(H[L+1], H[0], p);
            if (sa < 0 || sb < 0 || sc < 0) return 0;
            if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R ==
                (int)H.size()))
                return 1;
            return 2;
        }
        int mid = L + len / 2;
        if (sideOf(H[0], H[mid], p) >= 0)
            return insideHull2(H, mid, R, p);
        return insideHull2(H, L, mid+1, p);
    }

    int insideHull(const vector<PT> &hull, const PT &p) {
        if ((int)hull.size() < 3) return onSegment(hull[0],
hull.back(), p);
        else return insideHull2(hull, 1, (int)hull.size(), p);
    }
}

10.8. Minkowski Sum
struct pt{
    long long x, y;
    pt operator + (const pt & p) const {
        return pt{x + p.x, y + p.y};
    }
    pt operator - (const pt & p) const {
        return pt{x - p.x, y - p.y};
    }
    long long cross(const pt & p) const {
        return x * p.y - y * p.x;
    }
};

void reorder_polygon(vector<pt> & P){
    size_t pos = 0;
    for(size_t i = 1; i < P.size(); i++){
        if(P[i].y < P[pos].y || (P[i].y == P[pos].y && P[i].x <
P[pos].x))
            pos = i;
    }
}
```

```

    }

    rotate(P.begin(), P.begin() + pos, P.end());
}

vector<pt> minkowski(vector<pt> P, vector<pt> Q){
    // the first vertex must be the lowest
    reorder_polygon(P);
    reorder_polygon(Q);
    // we must ensure cyclic indexing
    P.push_back(P[0]);
    P.push_back(P[1]);
    Q.push_back(Q[0]);
    Q.push_back(Q[1]);
    // main part
    vector<pt> result;
    size_t i = 0, j = 0;
    while(i < P.size() - 2 || j < Q.size() - 2){
        result.push_back(P[i] + Q[j]);
        auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j]);
        if(cross >= 0 && i < P.size() - 2)
            ++i;
        if(cross <= 0 && j < Q.size() - 2)
            ++j;
    }
    return result;
}

11. Numerical
11.1. FFT
using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> & a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }
    if (invert) {
        for (cd & x : a)
            x /= n;
    }
}
```

```

}
vector<int> multiply(vector<int> const& a, vector<int> const& b)
{
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < a.size() + b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
    vector<int> result(n);
    for (int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    return result;
}

```

11.2. NTT

```

const ll mod = (119 << 23) + 1, root = 62; // 998244353
typedef vector<ll> vl;

int modpow(int n, int k);

void ntt(vl &a) {
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        for (int i=k;i<2*k;i++) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vl rev(n);
    for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) for (int j=0;j<k;j++) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
    vl conv(const vl &a, const vl &b) {
        if (a.empty() || b.empty()) return {};
        int s = a.size() + b.size() - 1, B = 32 - __builtin_clz(s),
            n = 1 << B;
        int inv = modpow(n, mod - 2);
        vl L(a), R(b), out(n);
        L.resize(n), R.resize(n);
        ntt(L), ntt(R);
        for (int i = 0; i < n; i++)
            out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
        ntt(out);
        return {out.begin(), out.begin() + s};
    }
}

```

11.3. Sum of n^k in $O(k^2)$

```

LL mod;
LL S[105][105];
void solve() {
    LL n, k;
    scanf("%lld %lld %lld", &n, &k, &mod);
    S[0][0] = 1 % mod;
    for (int i = 1; i <= k; i++) {
        for (int j = 1; j <= i; j++) {
            if (i == j) S[i][j] = 1 % mod;
            else S[i][j] = (j * S[i - 1][j] + S[i - 1][j - 1]) % mod;
        }
    }
    LL ans = 0;
    for (int i = 0; i <= k; i++) {
        LL fact = 1, z = i + 1;
        for (LL j = n - i + 1; j <= n + 1; j++) {
            LL mul = j;
            if (mul % z == 0) {
                mul /= z;
                z /= z;
            }
            fact = (fact * mul) % mod;
        }
        ans = (ans + S[k][i] * fact) % mod;
    }
    printf("%lld\n", ans);
}

```

11.4. Gauss method

```

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or a big number

int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
}

```

```

ans.assign (m, 0);
for (int i=0; i<n; ++i)
    if (where[i] != -1)
        ans[i] = a[where[i]][m] / a[where[i]][i];
for (int i=0; i<n; ++i)
    double sum = 0;
    for (int j=0; j<m; ++j)
        sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
        return 0;
}

for (int i=0; i<m; ++i)
    if (where[i] == -1)
        return INF;
return 1;
}

```

11.5. Berlekamp-Massey

```

template<typename T>
vector<T> berlekampMassey(const vector<T> &s) {
    vector<T> c;
    vector<T> oldC;
    int f = -1;
    for (int i=0; i<(int)s.size(); i++) {
        T delta = s[i];
        for (int j=1; j<=(int)c.size(); j++)
            delta -= c[j-1] * s[i-j];
        if (delta == 0)
            continue;
        if (f == -1) {

            c.resize(i + 1);
            mt19937 rng(222);
            for (T &x : c)
                x = rng();
            f = i;
        } else {
            vector<T> d = oldC;
            for (T &x : d)
                x = -x;
            d.insert(d.begin(), 1);
            T df1 = 0; // d(f + 1)
            for (int j=1; j<=(int)d.size(); j++)
                df1 += d[j-1] * s[f+1-j];
            T coef = delta / df1;
            for (T &x : d)
                x *= coef;
            vector<T> zeros(i - f - 1);
            zeros.insert(zeros.end(), d.begin(), d.end());
            d = zeros;
            vector<T> temp = c;
            c.resize(max(c.size(), d.size()));
            for (int j=0; j<(int)d.size(); j++)
                c[j] += d[j];
            if (i - (int) temp.size() > f - (int) oldC.size()) {
                oldC = temp;
                f = i;
            }
        }
    }
}

```

```

    }
}

return c;
}

```

12. Our Geometry Template

12.1. Point class

```

template<class T>
struct Point{
    T x;
    T y;

    Point operator+(const Point &o) const {
        return {x + o.x, y + o.y};
    }

    Point operator-(const Point &o) const {
        return {x - o.x, y - o.y};
    }

    Point operator*(T w) const {
        return {x * w, y * w};
    }

    Point operator/(T w) const {
        return {x / w, y / w};
    }

    Point perp() const {
        return Point{-y, x}; // rotates +90 degrees
    }

    bool operator<(Point &o){
        if(x == o.x) return y < o.y;
        else return x < o.x;
    }

    T cross(Point a) const {
        return x * a.y - y * a.x;
    }

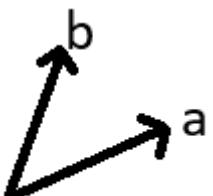
    T dist2() const {
        return x * x + y * y;
    }

    double dist() const {
        return sqrt(dist2());
    }

    T operator*(const Point &o) const {
        return x*o.x+y*o.y;
    }
};

```

12.2. Cross Product



In this case $\vec{a} \times \vec{b} = a_x \cdot b_y - a_y \cdot b_x > 0$

12.3. Circumcenter

```

typedef Point<double> P;

double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist()/
        abs((B-A).cross(C-A))/2;
}

P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}

```

12.4. Line Distance

```

typedef Point<double> P;
double lineDist(const P& a, const P& b, const P& p) {
    return (double)(b-a).cross(p-a)/(b-a).dist();
}

```

12.5. Line Intersection

```

typedef Point<double> P;
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};
}

```

12.6. Minimum-Enclosing Circle

```

typedef Point<double> P;
pair<P, double> enclose(vector<P> ps) {
    shuffle(ps.begin(), ps.end(), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    int sz = (int)ps.size();
    for(int i = 0 ; i < sz; i ++ ){
        if((o - ps[i]).dist() > r * EPS){
            o = ps[i], r = 0;
            for(int j = 0 ; j < i; j ++ ){
                if((o - ps[j]).dist() > r * EPS){
                    o = (ps[i] + ps[j]) / 2;
                    r = (o - ps[i]).dist();
                    for(int k = 0 ; k < j ; k ++ ){
                        if((o - ps[k]).dist() > r * EPS){
                            o = ccCenter(ps[i], ps[j], ps[k]);
                            r = (o - ps[i]).dist();
                        }
                    }
                }
            }
        }
    }
    return {o, r};
}

```

12.7. Polar-Sort

```

sort(X.begin(), X.end(), [&](Point<int> a, Point<int> b){
    Point<int> origin{0, 0};
    bool ba = a < origin, bb = b < origin;
    if(ba != bb) {return ba < bb;}
    else return a.cross(b) > 0;
});

```

13. General

13.1. Simulated Annealing

```

const ld T = (ld)2000;
const ld alpha = 0.999999;
// (new_score - old_score) / (temperature_final) ~ 10 works well

const ld L = (ld)1e6;
ld small_rand(){
    return ((ld)gen(L))/L;
}

ld P(ld old, ld nw, ld temp){
    if(nw > old)
        return 1.0;
    return exp((nw-old)/temp);
}

{
    auto start = chrono::steady_clock::now();
    ld time_limit = 2000;
    ld temperature = T;
    ld max_score = -1;

    while(elapsed_time < time_limit){
        auto cur = chrono::steady_clock::now();
        elapsed_time =
chrono::duration_cast<chrono::milliseconds>(cur - start).count();
        temperature *= alpha;

        // try a neighboring state
        // ...
        // ...

        old_score = score(old_state);
        new_score = score(new_state);
        if(P(old_score, new_score, temperature) >= small_rand()){
            old_state = new_state;
            old_score = new_score;
        }
        if(old_score > max_score){
            max_score = old_score;
            max_state = old_state;
        }
    }
}

```

13.2. Expression Parsing (cp-algo):

```

bool delim(char c) {
    return c == ' ';
}

bool is_op(char c) {
    return c == '+' || c == '-' || c == '*' || c == '/';
}

int priority (char op) {
    if (op == '+' || op == '-')
        return 1;
    if (op == '*' || op == '/')
        return 2;
    return -1;
}

void process_op(stack<int>& st, char op) {
    int r = st.top(); st.pop();
    int l = st.top(); st.pop();
    switch (op) {
        case '+': st.push(l + r); break;
        case '-': st.push(l - r); break;
        case '*': st.push(l * r); break;
        case '/': st.push(l / r); break;
    }
}

int evaluate(string& s) {
    stack<int> st;
    stack<char> op;
    for (int i = 0; i < (int)s.size(); i++) {
        if (delim(s[i]))
            continue;

        if (s[i] == '(') {
            op.push('(');
        } else if (s[i] == ')') {
            while (op.top() != '(')
                process_op(st, op.top());
            op.pop();
        }
        op.pop();
    } else if (is_op(s[i])) {
        char cur_op = s[i];
        while (!op.empty() && priority(op.top()) >=
priority(cur_op)) {
            process_op(st, op.top());
            op.pop();
        }
        op.push(cur_op);
    } else {
        int number = 0;
        while (i < (int)s.size() && isalnum(s[i]))
            number = number * 10 + s[i++]- '0';
        --i;
        st.push(number);
    }
}

```

```

while (!op.empty()) {
    process_op(st, op.top());
    op.pop();
}
return st.top();

}

14. janY's 2D Geometry

14.1. vec2
typedef long double ld;

const ld eps = 1e-9, inf = 1e9;

template<typename V>
struct vec2 {
    V x, y;

    vec2(): x(0), y(0) {}
    vec2(V x, V y): x(x), y(y) {}
    // optional conversion constructor
    template<typename U> vec2(const vec2<U>& other) : x(other.x),
y(other.y) {}

    // optional
    V length() {return sqrt(x * x + y * y);}

    V dot(vec2<V> other) {return x*other.x + y*other.y;}

    // a.cross(b)>0: b is to the left of a
    // a.cross(b)<0: b is to the right of a
    // a.cross(b)=0: vectors are collinear (same or opposite
direction)
    V cross(vec2<V> other) {return x*other.y - y*other.x;}

    // optional
    void reduce(bool pos_x = false) { // only for integer
        V g = __gcd(x, y);
        if (g == 0) return;
        if (g < 0) g = -g;
        x /= g;
        y /= g;
        // ensure canonical representation for direction
        if (pos_x && (x < 0 || (x == 0 && y < 0))) {
            x = -x;
            y = -y;
        }
    }

    // optional (need .length())
    void normalize() { // only for floating point
        V len = this->length();
        if (len == 0) return;
        x /= len;
        y /= len;
    }

    // optional, rotate angle radians around (0, 0).
}


```

```

void rotate(ld angle) { // only for floating point
    ld sin_angle = sin(angle);
    ld cos_angle = cos(angle);
    V new_x = x*cos_angle - y*sin_angle;
    y = x*sin_angle + y*cos_angle;
    x = new_x;
}

// optional, returns new vec2 rotated angle radians around
(0, 0).
vec2 rotated(ld angle) {
    vec2<V> this = *this;
    this.rotate(angle);
    return this;
}

// optional
void rotate90() {V new_x = -y; y = x; x = new_x;}

// optional, returns angle between two directions, always
positive
ld angle_to(vec2<V> w) {
    ld cos_theta = (dot(w) / w.length()) / length();
    return acos(max((ld)-1.0, min((ld)1.0, cos_theta)));
}

//optional addition/subtraction
vec2 operator+(const vec2<V>& other) {return vec2(x +
other.x, y + other.y);}
vec2 operator-(const vec2<V>& other) {return vec2(x -
other.x, y - other.y);}

// optional scalar multiplication
vec2 operator*(const V& k) {return vec2(x * k, y * k);}
vec2& operator*=(const V& k) {x *= k; y *= k; return *this;}

// optional scalar division
vec2 operator/(const V& k) {return vec2(x / k, y / k);}
vec2& operator/=(const V& k) {x /= k; y /= k; return *this;}

// optional equality operators
bool operator==(const vec2<V>& other) {return x == other.x &&
y == other.y;}
bool operator!=(const vec2<V>& other) {return !(*this ==
other);}

// optional nice cout
template<typename V>
ostream& operator<<(ostream& os, const vec2<V>& v) {
    return os << "(" << v.x << " " << v.y << ")";
}

// optional nice cin
template<typename V>
istream& operator>>(istream& is, vec2<V>& v) {
    return is >> v.x >> v.y;
}

```

14.2. 2D Geometric Functions

```
// line line intersection
// returns true if exists, stores result in out
bool line_intersection(vec2<ld> p1, vec2<ld> d1, vec2<ld> p2,
vec2<ld> d2, vec2<ld> &out) {
    ld cross_d = d1.cross(d2);
    if (abs(cross_d) < 1e-10) return false;
    vec2<ld> r = p2 - p1;
    ld t1 = r.cross(d2) / cross_d;
    out = p1 + d1 * t1;
    return true;
}

// circle circle intersection
// returns true if exists, stores result in out
bool circle_circle(vec2<ld> c1, ld r1, vec2<ld> c2, ld r2,
pair<vec2<ld>, vec2<ld>> &out) {
    ld d = (c2-c1).length();
    ld co = (d*d + r1*r1 - r2*r2)/(2*d*r1);
    if (abs(co) > 1) return false;
    ld alpha = acos(co);
    vec2<ld> rad = (c2-c1)/d*r1; // vector C1C2 resized to have
length r1
    out = {c1 + rad.rotated(-alpha), c1 + rad.rotated(alpha)};
    return true;
}

// quadratic formula a*x^2 + b*x + c = 0
// returns root count and stores result in out
int quad_roots(ld a, ld b, ld c, pair<ld,ld> &out) {
    assert(a != 0);
    ld disc = b*b - 4*a*c;
    if (disc < 0) return 0;
    ld sum = (b >= 0) ? -b+sqrt(disc) : -b-sqrt(disc);
    out = {sum/(2*a), sum == 0 ? 0 : (2*c)/sum};
    return 1 + (disc > 0);
}

struct StableSum {
    int cnt = 0;
    vector<ld> v, pref{0};
    void operator+=(ld a) {
        assert(a >= 0);
        int s = ++cnt;
        while (s % 2 == 0) {
            a += v.back();
            v.pop_back(), pref.pop_back();
            s /= 2;
        }
        v.push_back(a);
        pref.push_back(pref.back() + a);
    }
    ld val() {return pref.back();}
};

// sorts starting from (1, 0) inclusive going clockwise.
template<typename T> bool half(const vec2<T> &p) {
    return (p.y < 0 || (p.y == 0 && p.x < 0));
}
```

```
template<typename T> void polar_sort(vector<vec2<T>> &v) {
    sort(v.begin(), v.end(), [](vec2<T> v, vec2<T> w) {
        return make_tuple(half(v), 0) < make_tuple(half(w),
v.cross(w));
    });
}
```

14.3. Halfplane Intersection

```
// Basic half-plane struct.
struct Halfplane {
    // 'p' is a passing point of the line and 'pq' is the
direction vector of the line.
    vec2<ld> p, pq;
    ld angle;

    Halfplane() {}
    Halfplane(vec2<ld> a, vec2<ld> b) : p(a), pq(b-a) {
        angle = atan2l(pq.y, pq.x);
    }

    // Check if point 'r' is outside this half-plane.
    // Every half-plane allows the region to the LEFT of its
line.
    bool out(vec2<ld> r) {
        return pq.cross(r - p) < -eps;
    }

    // Comparator for sorting.
    bool operator<(const Halfplane& e) const {
        return angle < e.angle;
    }

    // Intersection point of the lines of two half-planes. It is
assumed they're never parallel.
    vec2<ld> inter(Halfplane& t) {
        ld alpha = (t.p - p).cross(t.pq) / pq.cross(t.pq);
        return p + (pq * alpha);
    }
};

vector<vec2<ld>> hp_intersect(vector<Halfplane>& H) {
    vec2<ld> box[4] = { // Bounding box in CCW order
        vec2<ld>(inf, inf),
        vec2<ld>(-inf, inf),
        vec2<ld>(-inf, -inf),
        vec2<ld>(inf, -inf)
    };

    for(int i = 0; i<4; i++) { // Add bounding box half-planes.
        Halfplane aux(box[i], box[(i+1) % 4]);
        H.push_back(aux);
    }

    // Sort by angle and start algorithm
    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); i++) {

        // Remove from the back of the deque while last half-
plane is redundant
        while (len > 1 && H[i].out(dq[len-1].inter(dq[len-2]))) {
            dq.pop_back();
            --len;
        }

        // Remove from the front of the deque while first half-
plane is redundant
        while (len > 1 && H[i].out(dq[0].inter(dq[1]))) {
            dq.pop_front();
            --len;
        }

        // Special case check: Parallel half-planes
        if (len > 0 && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps)
        {
            // Opposite parallel half-planes that ended up
checked against each other.
            if (H[i].pq.dot(dq[len-1].pq) < 0.0)
                return vector<vec2<ld>>();

            // Same direction half-plane: keep only the leftmost
half-plane.
            if (H[i].out(dq[len-1].p)) {
                dq.pop_back();
                --len;
            }
            else continue;
        }

        // Add new half-plane
        dq.push_back(H[i]);
        ++len;
    }

    // Final cleanup: Check half-planes at the front against the
back and vice-versa
    while (len > 2 && dq[0].out(dq[len-1].inter(dq[len-2]))) {
        dq.pop_back();
        --len;
    }

    while (len > 2 && dq[len-1].out(dq[0].inter(dq[1]))) {
        dq.pop_front();
        --len;
    }

    // Report empty intersection if necessary
    if (len < 3) return vector<vec2<ld>>();

    // Reconstruct the convex polygon from the remaining half-
planes.
    vector<vec2<ld>> ret(len);
    for(int i = 0; i+1 < len; i++) {
        ret[i] = dq[i].inter(dq[i+1]);
    }
    ret.back() = dq[len-1].inter(dq[0]);
}
```

```

// Check if area is non-zero
ld area = 0;
for(int i = 0; i < ret.size(); i++) {
    int nxt = i+1;
    if (nxt == ret.size()) nxt = 0;
    area += ret[i].x*ret[nxt].y - ret[i].y*ret[nxt].x;
}
if (abs(area)/2.0 < eps) return vector<vec2<ld>>();

return ret;
}



## 15. janY's Algorithms



### 15.1. Modulo



```

int mod;
int mod_f(long long a){
 return ((a%mod)+mod)%mod;
}

int m_add(long long a, long long b){
 return mod_f((long long)mod_f(a) + mod_f(b));
}

int m_mult(long long a, long long b){
 return mod_f((long long)mod_f(a) * mod_f(b));
}

// C function for extended Euclidean Algorithm (used to
// find modular inverse.
int gcdExt(int a, int b, int *x, int *y) {
 // Base Case
 if (a == 0){
 *x = 0, *y = 1;
 return b;
 }

 int x1, y1; // To store results of recursive call
 int gcd = gcdExt(b%a, a, &x1, &y1);

 // Update x and y using results of recursive
 // call
 *x = y1 - (b/a) * x1;
 *y = x1;
 return gcd;
}

// Function to find modulo inverse of b. It returns
// -1 when inverse doesn't
int modInverse(int b, int m) {
 int x, y; // used in extended GCD algorithm
 int g = gcdExt(b, m, &x, &y);

 // Return -1 if b and m are not co-prime
 if (g != 1) return -1;

 // m is added to handle negative x
 return (x%m + m) % m;
}

```


```

```

// Function to compute a/b under modulo m
int m_divide(long long a, long long b) {
    a = a % mod;
    int inv = modInverse(b, mod);
    if (inv == -1)
        return -1;
    else
        return (inv * a) % mod;
}

// exponent function (with mod)
int m_pow(long long base, long long exp) {
    base %= mod;
    int result = 1;
    while (exp > 0) {
        if (exp & 1) result = ((long long)result * base) % mod;
        base = ((long long)base * base) % mod;
        exp >>= 1;
    }
    return result;
}

```

15.2. Factorization

```

vector<int> getPrimes(int n)
{
    vector<int> res;
    bool prime[n + 1];
    memset(prime, true, sizeof(prime));
    for (ll p = 2; p * p <= n; p++) {
        if (prime[p] == true) {
            for (ll i = p * p; i <= n; i += p){
                prime[i] = false;
            }
        }
        for (int p = 2; p <= n; p++){
            if (prime[p]){
                res.push_back(p);
            }
        }
        return res;
    }

    // only prime factors
    vector<int> primes;
    vector<int> get_prime_factors(long long n){
        vector<int> factors;
        if (n == 1) return factors;
        for (auto &i : primes){
            if (i*i > n) break;
            if (n%i==0){
                factors.push_back(i);
                while (n%i==0) n/=i;
            }
        }
        if (n != 1) factors.push_back(n);
        return factors;
    }
}

```

```

map<int,int> get_prime_factors(long long n){
    map<int,int> factors;
    if (n == 1) return factors;
    for (auto &i : primes){
        if (i*i > n) break;
        while (n%i==0) {
            factors[i]++;
            n/=i;
        }
    }
    if (n != 1) factors[n]++;
    return factors;
}

vector<int> get_factors(long long n){
    vector<int> factors;
    for (int i = 1; i*i <= n; i++){
        if (n%i==0){
            factors.push_back(i);
            if (i*i != n) factors.push_back(n/i);
        }
    }
    //factors.push_back(n);
    return factors;
}



### 15.3. Combinatorics


long long nPr(long long n, long long r){
    if (r > n) return 0;
    if (n-r < r) r = n-r;
    long long count = r;
    long long result = 1;
    while (count > 0){
        //result = m_mult(result, n);
        result = result * n;
        n--;
        count--;
    }
    return result;
}

// slow nCr
long long nCr(long long n, long long r){
    if (r > n) return 0;
    if (n-r < r) r = n-r;
    long long count = r;
    long long result = 1;
    while (count > 0){
        //result = m_mult(result, n);
        result = result * n;
        n--;
        count--;
    }
    long long num = 1;
    while (num <= r){
        //result = m_divide(result, num);
        result = result / num;
        num++;
    }
}

```

```

    }
    return result;
}

// fast nCr (REQ modulo m_mult, modInverse)
int MAX_CHOOSE = 3e5;
vector<long long> inverse_fact(MAX_CHOOSE+5);
vector<long long> fact(MAX_CHOOSE+5);

long long fast_nCr(long long n, long long r) {
    if (n < r || r < 0) return 0;
    return (((fact[n] * inverse_fact[r]) % mod) * inverse_fact[n-r]) % mod;
}

void precalc_fact(int n){
    fact[0] = fact[1] = 1;
    for (long long i = 2; i <= n; i++){
        fact[i] = (fact[i-1]*i) % mod;
    }
    inverse_fact[0] = inverse_fact[1] = 1;
    for (long long i = 2; i <= n; i++){
        inverse_fact[i] = (modInverse(i, mod) *
inverse_fact[i-1]) % mod;
    }
}

```

15.4. Disjoint Set Union

```

struct disjSet { // Disjoint set
    int *rank, *parent, n;
    disjSet() {}
    disjSet(int n) {init(n);}
    void init(int n){
        rank = new int[n];
        parent = new int[n];
        this->n = n;
        for (int i = 0; i < n; i++) {
            rank[i] = 0;
            parent[i] = i;
        }
    }
    int find(int a) {
        if (parent[a] != a){
            //return find(parent[a]); // no path compression
            parent[a] = find(parent[a]); // path compression
        }
        return parent[a];
    }
    void Union(int a, int b) {
        int a_set = find(a);
        int b_set = find(b);
        if (a_set == b_set) return;
        if (rank[a_set] < rank[b_set]) {
            update_union(a_set, b_set);
        } else if (rank[a_set] > rank[b_set]) {
            update_union(b_set, a_set);
        } else {
            update_union(b_set, a_set);
            rank[a_set] = rank[a_set] + 1;
        }
    }
}

```

```

    }
    // change merge behaviour here
    void update_union(int a, int b){ // merge a into b
        parent[a] = b;
    }
};



#### 15.5. Merge Sort Tree


struct MergeSortTree {
    int size;
    vector<vector<ll>> values;

    void init(int n){
        size = 1;
        while (size < n){
            size *= 2;
        }
        values.resize(size*2, v(0));
    }

    void build(vl &arr, int x, int lx, int rx){
        if (rx - lx == 1){
            if (lx < arr.size()){
                values[x].pb(arr[lx]);
            } else {
                values[x].pb(-1);
            }
            return;
        }
        int m = (lx+rx)/2;
        build(arr, 2 * x + 1, lx, m);
        build(arr, 2 * x + 2, m, rx);

        int i = 0;
        int j = 0;
        int asize = values[2*x+1].size();
        while (i < asize && j < asize){
            if (values[2*x+1][i] < values[2*x+2][j]){
                values[x].pb(values[2*x+1][i]);
                i++;
            } else {
                values[x].pb(values[2*x+2][j]);
                j++;
            }
        }
        while (i < asize) {
            values[x].pb(values[2*x+1][i]);
            i++;
        }
        while (j < asize){
            values[x].pb(values[2*x+2][j]);
            j++;
        }
    }

    void build(vl &arr){
        build(arr, 0, 0, size);
    }
}

```

```

int calc(int l, int r, int x, int lx, int rx, int k){
    if (lx >= r || rx <= l) return 0;

    if (lx >= l && rx <= r) { // CHANGE HEURISTIC HERE
        (elements strictly less than k currently)
        int lft = -1;
        int rght = values[x].size();
        while (rght - lft > 1){
            int mid = (lft+rght)/2;
            if (values[x][mid] < k){
                lft = mid;
            } else {
                rght = mid;
            }
        }
        return lft+1;
    }

    int m = (lx+rx)/2;
    int values1 = calc(l, r, 2*x+1, lx, m, k);
    int values2 = calc(l, r, 2*x+2, m, rx, k);
    return values1 + values2;
}

int calc(int l, int r, int k){
    return calc(l, r, 0, 0, size, k);
}

```

15.6. Fenwick Tree

```

struct fenwick { // point update (delta), range sum
    ll* bit;
    int fsize;

    fenwick(){}
    fenwick(int n){ init(n); }
    ~fenwick(){ delete[] bit; }

    void init(int n){
        bit = new ll[n+1];
        fsize = n;
        for (int i = 1; i <= n; i++){
            bit[i] = 0;
        }
    }

    int lsb(int x){ // Least significant bit
        return x&(-x);
    }

    ll query(int v){
        ll sum = 0;
        while (v > 0){
            sum += bit[v];
            v -= lsb(v);
        }
        return sum;
    }
}

```

```

void add(int v, int delta){
    v++; // because 1 indexed
    while (v <= fsize){
        bit[v] += delta;
        v += lsb(v);
    }
}

void build(vector<ll> &inp){
    for (int i = 1; i <= inp.size(); i++){
        bit[i] = inp[i-1];
    }
    for (int i = 1; i <= inp.size(); i++){
        int p = i + lsb(i);
        if (p <= fsize){
            bit[p] += bit[i];
        }
    }
}

ll calc(int l, int r){ // sum from l to r inclusive (of the
original array)
    return query(r+1) - query(l);
}
};

```

15.7. Fenwick Tree (Range Updates)

```
struct fenwick { // range update
```

```

    ll* bit1;
    ll* bit2;
    int fsize;

    fenwick(){}
    fenwick(int n){
        init(n);
    }

    ~fenwick(){
        delete bit1;
        delete bit2;
    }

    void init(int n){
        bit1 = new ll[n+1];
        bit2 = new ll[n+1];
        fsize = n;
        for (int i = 1; i <= n; i++){
            bit1[i] = 0;
            bit2[i] = 0;
        }
    }

    ll getSum(ll BITree[], int index){
        ll sum = 0;
        index++;
        while (index > 0) {
            sum += BITree[index];
            index -= index & (-index);
        }
    }
};
```

```

        return sum;
    }

    void updateBIT(ll BITree[], int index, ll val){
        index++;
        while (index <= fsize) {
            BITree[index] += val;
            index += index & (-index);
        }
    }

    ll sum(ll x){
        return (getSum(bit1, x) * x) - getSum(bit2, x);
    }

    void add(ll l, ll r, ll val){ // add val to range l:r
INCLUSIVE
        updateBIT(bit1, l, val);
        updateBIT(bit1, r + 1, -val);
        updateBIT(bit2, l, val * (l - 1));
        updateBIT(bit2, r + 1, -val * r);
    }

    ll calc(ll l, ll r){ // sum on range l:r INCLUSIVE
        return sum(r) - sum(l - 1);
    }
};
```

15.8. Kosaraju's Algorithm

```

// kosaraju's algorithm - find strongly connected components
(SCC) in a directed graph O(V+E)
// V - vertex count, E - edge count
// SCC - every vertex in a component has a path to every other
vertex in the same component
// add directed edges, run .work(), answer stored in scc
struct kosaraju {
    vector<vector<int>> g, gT, scc;
    stack<int> stck;
    vector<int> visited;
    int siz;

    kosaraju() {}
    kosaraju(int siz){
        init(siz);
    }

    void init(int siz){
        this->siz = siz;
        g.assign(siz, vector<int>(0));
        gT.assign(siz, vector<int>(0));
    }

    void add_edge(int u, int v){ // directed edge from u to v
        g[u].push_back(v);
        gT[v].push_back(u);
    }

    void dfs(int loc){
        if (visited[loc]) return;
```

```

        visited[loc] = 1;
        for (auto &i : g[loc]) dfs(i);
        stck.push(loc);
    }

    void dfsT(int loc){
        if (visited[loc]) return;
        visited[loc] = 1;
        scc.back().push_back(loc);
        for (auto &i : gT[loc]) dfsT(i);
    }

    void work(){ // call after adding all the edges to get scc
        scc.clear();
        visited.assign(siz, 0);
        for (int i = 0; i < siz; i++) dfs(i);
        visited.assign(siz, 0);
        while (stck.size()){
            int top = stck.top();
            stck.pop();
            if (visited[top] == 0){
                scc.pb(vi(0));
                dfsT(top);
            }
        }
    }
};
```

15.9. Range Minimum Query

```

typedef int typ;
struct RMQ {
    vector<vector<typ>> arr;
    bool do_min_query = 1; // 0 = max query; 1 = min query

    RMQ(){}
    RMQ(vector<typ> &a){
        build(a);
    }

    RMQ(vector<typ> &a, bool is_min) {
        do_min_query = is_min;
        build(a);
    }

    void build(vector<typ> &a){
        arr.pb(a);
        int len = 2, at = 1;
        while (len <= (int)a.size()){
            arr.pb(vector<typ>(0));
            int pos = 0;
            while (pos+len <= (int)a.size()){
                typ val = max(arr[at-1][pos], arr[at-1]
[pos+len/2]);
                if (do_min_query) val = min(arr[at-1][pos],
arr[at-1][pos+len/2]);
                arr[at].pb(val);
                pos++;
            }
            len *= 2; at++;
        }
    }
};
```

```

    }

    typ calc(int l, int r){ // 0 indexed, [l; r] inclusive
        int dist = r-l+1;
        int bigbit = 31-_builtin_clz(dist);
        typ ret = max(arr[bigbit][l], arr[bigbit][r-(1<<bigbit)+1]);
        if (do_min_query) ret = min(arr[bigbit][l], arr[bigbit][r-(1<<bigbit)+1]);
        return ret;
    }
};

```

15.10. Polynomial Rolling Hash

```

int gcdExt(int a, int b, int *x, int *y) {
    if (a == 0){
        *x = 0, *y = 1;
        return b;
    }
    int x1, y1;
    int gcd = gcdExt(b%a, a, &x1, &y1);
    *x = y1 - (b/a) * x1;
    *y = x1;
    return gcd;
}

```

```

int modInverse(int b, int m) {
    int x, y;
    int g = gcdExt(b, m, &x, &y);
    if (g != 1) return -1;
    return (x%m + m) % m;
}

```

```

int m_divide(long long a, long long b, long long md) {
    a = a % md;
    int inv = modInverse(b, md);
    if (inv == -1)
        return -1;
    else
        return (inv * a) % md;
}

```

```

struct poly_hash {
    const vector<ll> mods = {(ll)1e9+9, (ll)1e9+7};
    const vector<ll> p = {59, 61};
    vector<vector<ll>> pref_hash, powers, divs;

    void work(string &s){
        // precalc powers
        powers.clear();
        powers.resize(2);
        divs.clear();
        divs.resize(2);
        for (int i = 0; i < 2; i++){
            powers[i].push_back(1);
            divs[i].push_back(1);
            for (int j = 0; j < s.size(); j++){
                powers[i].push_back((powers[i].back()*p[i])%mods[i]);

```

```

                divs[i].push_back(m_divide(divs[i].back(), p[i],
                                             mods[i]));
            }
        }

        pref_hash.clear();
        pref_hash.resize(2);
        for (int i = 0; i < 2; i++){
            pref_hash[i].push_back(0);
            for (int j = 0; j < s.size(); j++){
                int val = s[j]-'a'+1; // a -> 1, b -> 2...
                pref_hash[i].push_back((pref_hash[i].back()+val*powers[i]
                                         [j])%mods[i]);
            }
        }

        pair<ll, ll> calc(int l, int r){ // [l; r] inclusive!
            ll hash1 = ((pref_hash[0][r+1]-pref_hash[0]
                         [l]+mods[0])*divs[0][l])%mods[0];
            ll hash2 = ((pref_hash[1][r+1]-pref_hash[1]
                         [l]+mods[1])*divs[1][l])%mods[1];
            return {hash1, hash2};
        }
};

15.11. Matrix Template
template<typename V> struct Matrix {
    int rows, cols;
    vector<vector<V>> mat;

    Matrix(){}
    Matrix(int row, int col){
        resize(row, col);
    }

    void resize(int row, int col){
        mat.assign(row, vector<V>(col));
        rows = row;
        cols = col;
    }

    Matrix<V> operator* (Matrix<V> const& oth){
        assert(cols == oth.rows);
        Matrix<V> res(rows, oth.cols);
        for (int i = 0; i < rows; i++){
            for (int j = 0; j < oth.cols; j++){
                for (int z = 0; z < cols; z++){
                    res.mat[i][j] += mat[i][z]*oth.mat[z][j];
                    res.mat[i][j] %= mod; // if need mod
                }
            }
        }
        return res;
    }

    Matrix<V> exp(ll pow){ // A^(pow)
        assert(rows == cols);
        Matrix<V> res(rows, cols);

```

```

        for (int i = 0; i < rows; i++) res.mat[i][i] = 1;
        Matrix<V> mult = *this;
        while (pow){
            if (pow&1) res = res*mult;
            mult = mult*mult;
            pow /= 2;
        }
        return res;
    }

    void print(){
        int i, j;
        fo(i, rows) {
            fo(j, cols) cout << mat[i][j] << " ";
            cout << "\n";
        }
        cout << "\n";
    }
};

```

15.12. Convex Hull

```

typedef pair<ll, ll> P;
ll cross(P a, P b) {return a.fi*b.se - a.se*b.fi;}
ll cross_from(P start, P a, P b) {
    a.fi -= start.fi;
    a.se -= start.se;
    b.fi -= start.fi;
    b.se -= start.se;
    return cross(a, b);
}
vector<P> convexHull(vector<P> pnts) {
    if (pnts.size() <= 1) return pnts;
    sort(pnts.begin(), pnts.end());
    vector<P> h(pnts.size()+1);
    int s = 0, t = 0;
    for (int it = 2; it--; s = --t, reverse(pnts.begin(),
                                              pnts.end()))
        for (P p : pnts) {
            while (t >= s + 2 && cross_from(h[t-2], h[t-1], p) <= 0)
                t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}

```

15.13. Prüfer Codes

```

// there are n^(n-2) different trees (labeled trees)

// multinomial coefficients: n choose n1, n2, n3, ..., nk - number
// of ways to partition n elements into k distinct groups of sizes
// n1, n2, n3, ..., nk.
// n! / (n1! * n2! * ... * nk!)

// multinomial theorem:
// (x1 + x2 + ... + xm)^p = sum[ci>=0 & sum ci = p] (x1^c1 * x2^c2
// * ... * xm^cm * (p choose c1, c2, ..., ck))

// 0 indexed
vector<vector<int>> prüfer_decode(vector<int> &code) {

```

```

vector<vector<int>> g(code.size() + 2);
vector<int> degrees(g.size(), 1);
for (auto &i : code) degrees[i]++;
int nxt = g.size(), ptr = 0;
for (int i = 0; i < code.size(); i++) {
    if (nxt > ptr) {
        while (degrees[ptr] != 1) ptr++;
        nxt = ptr;
    }
    g[nxt].push_back(code[i]);
    g[code[i]].push_back(nxt);
    degrees[nxt] = 0;
    if (--degrees[code[i]] == 1) nxt = code[i];
    else nxt = g.size();
}
if (nxt > ptr) while (degrees[ptr] != 1) ptr++;
else ptr = nxt;
g[g.size() - 1].push_back(ptr);
g[ptr].push_back(g.size() - 1);
return g;
}

int *prufer_parent;
void dfs(int at, int from, vector<vector<int>> &g) {
    prufer_parent[at] = from;
    for (auto &i : g[at]) {
        if (i == from) continue;
        dfs(i, at, g);
    }
}
// 0 indexed
vector<int> prufer_encode(vector<vector<int>> &g) {
    if (g.size() <= 2) return vector<int>();
    prufer_parent = new int[g.size()];
    dfs(g.size() - 1, g.size() - 1, g); // will never be removed
    int *degrees = new int[g.size()];
    for (int i = 0; i < g.size(); i++) degrees[i] = g[i].size();
    int nxt = g.size(), ptr = 0;
    vector<int> code(g.size() - 2);
    for (int i = 0; i < g.size() - 2; i++) {
        if (nxt > ptr) {
            while (degrees[ptr] != 1) ptr++;
            nxt = ptr;
        }
        code[i] = prufer_parent[nxt];
        degrees[nxt] = 0;
        if (--degrees[prufer_parent[nxt]] == 1) nxt =
            prufer_parent[nxt];
        else nxt = g.size();
    }
    return code;
}

```

15.14. Segment tree

```

struct get_item { ll sum; };
struct set_item { ll val; };
struct segtree {
    int size;
    get_item *values;
}

```

```

get_item NEUTRAL_ELEMENT = {0}; // dont forget to use brain
here please

get_item merge(get_item &a, get_item &b){
    return {a.sum + b.sum};
}
void apply(get_item &x, set_item &y){
    x.sum = y.val;
}

void init(int n){
    size = 1;
    while (size < n) size *= 2;
    values = new get_item[size * 2];
    fill_n(values, size * 2, NEUTRAL_ELEMENT);
}

void upd(int i, set_item &v, int x, int lx, int rx){
    if (rx - lx == 1){
        apply(values[x], v);
        return;
    }
    int m = (lx + rx) / 2;
    if (i < m) upd(i, v, 2 * x + 1, lx, m);
    else upd(i, v, 2 * x + 2, m, rx);
    values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
}
void upd(int i, set_item v){
    upd(i, v, 0, 0, size);
}

get_item calc(int l, int r, int x, int lx, int rx){
    if (lx >= r || rx <= l) return NEUTRAL_ELEMENT;
    if (lx >= l && rx <= r) return values[x];
    int m = (lx + rx) / 2;
    get_item v1 = calc(l, r, 2 * x + 1, lx, m);
    get_item v2 = calc(l, r, 2 * x + 2, m, rx);
    return merge(v1, v2);
}
// INCLUSIVE
get_item calc(int l, int r){
    return calc(l, r + 1, 0, 0, size);
}

void build(vector<set_item> &arr, int x, int lx, int rx){ // optional
    if (rx - lx == 1){
        if (lx < arr.size()) apply(values[x], arr[lx]);
        return;
    }
    int m = (lx + rx) / 2;
    build(arr, 2 * x + 1, lx, m);
    build(arr, 2 * x + 2, m, rx);
    values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
}
void build(vector<set_item> &arr){
    build(arr, 0, 0, size);
}

```

15.15. NTT

```

int gcdExt(int a, int b, int *x, int *y) {
    // Base Case
    if (a == 0){
        *x = 0, *y = 1;
        return b;
    }

    int x1, y1; // To store results of recursive call
    int gcd = gcdExt(b % a, a, &x1, &y1);

    // Update x and y using results of recursive
    // call
    *x = y1 - (b / a) * x1;
    *y = x1;
    return gcd;
}

int modInverse(int b, int m) {
    int x, y;
    int g = gcdExt(b, m, &x, &y);
    if (g != 1) return -1;
    return (x % m + m) % m;
}

const int mod = 998244353;
const int root = 15311432;
const int root_1 = modInverse(root, mod);
const int root_pw = 1 << 23;

void fft(vector<int> &a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
    }

    if (i < j)
        swap(a[i], a[j]);
}

for (int len = 2; len <= n; len <= 1) {
    int wlen = invert ? root_1 : root;

    for (int i = len; i < root_pw; i <= 1)
        wlen = (int)(1LL * wlen * wlen % mod);

    for (int i = 0; i < n; i += len) {
        int w = 1;
        for (int j = 0; j < len / 2; j++) {
            int u = a[i + j], v = (int)(1LL * a[i + j + len / 2] * w
                % mod);
            a[i + j] = u + v < mod ? u + v : u + v - mod;
            a[i + j + len / 2] = u - v >= 0 ? u - v : u - v + mod;
            w = (int)(1LL * w * wlen % mod);
        }
    }
}

```

```

}

if (invert) {
    int n_1 = modInverse(n, mod);
    for (int & x : a)
        x = (int)(1LL * x * n_1 % mod);
}
}

// polynomial multiplication
vector<int> multiply(vector<int> const& a, vector<int> const& b)
{
    vector<int> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < a.size() + b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    if (a == b) {
        fb = fa;
    } else {
        fft(fb, false);
    }

    for (int i = 0; i < n; i++) {
        fa[i] = ((ll)fa[i]*fb[i])%mod;
    }
    fft(fa, true);

    vector<int> result(a.size() + b.size() - 1);
    for (int i = 0; i < result.size(); i++)
        result[i] = fa[i];
    return result;
}

vi powz(vi a, ll p){
    vi base = a;
    vi ans(1, 1);
    while (p) {
        if (p&1) ans = multiply(ans, base);
        base = multiply(base, base);
        p >>= 1;
    }
    return ans;
}

```

16. Out of ideas?

1. $\text{opt}(i) \leq \text{opt}(i + 1)$
2. Divide & Conquer on queries. Object A_i exists for query range $[L_i; R_i]$ - construct a segtree assuming A_i can be rerolled.

