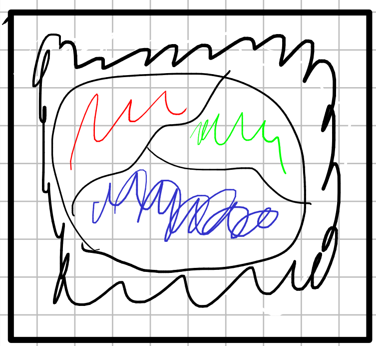
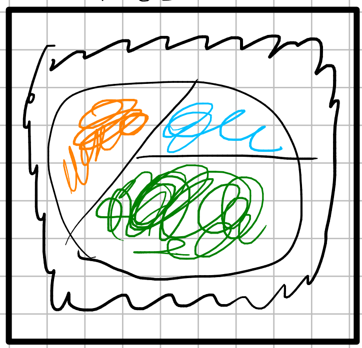
Ground truth



Pred



$\mathcal{O} = \{o_1, \dots o_N\}$

$A = \{A_\alpha\} \quad A_{\alpha_1} \cap A_{\alpha_2} = \varnothing$

$\bigsqcup_\alpha A_\alpha = \mathcal{O} \quad A_{bg}$

$B = \{B_\alpha\} \quad B_{\alpha_1} \cap B_{\alpha_2} = \varnothing$

$\bigsqcup_\alpha B_\alpha = \mathcal{O} \quad B_{bg}$

$A: \mathcal{O} \to \{1, \dots, k\}$

$B: \mathcal{O} \to \{1 \dots s\}$

$n_{ij} = \sum_{m=1}^{N} \mathbb{1}(A(o_m) = i, B(o_m) = j)$

$n_{i\cdot} = \sum_{j=1}^{s} n_{ij}$

$n_{\cdot j} = \sum_{i=1}^{k} n_{ij}$

$N = \sum_{ij} n_{ij}$

	B_1	B_2	$\dots$	B_s
A_1	n_{11}	n_{12}		
A_2				
$\vdots$				
A_k				n_{ks}

$\xi_i = (A_i, B_i) \qquad \xi_i - \text{iid}$

$n_{ij} = \sum_{m=1}^{N} \mathbb{1}(A_m = i, B_m = j)$

$H_0: \; F_\xi(a, b) = F_A(a) F_B(b)$

$\chi^2 = N\left(\sum_{i,j} \frac{n_{ij}^2}{n_{i\cdot} n_{\cdot j}} - 1\right) \longrightarrow \chi^2_{(s-1)(k-1)}$

- $\frac{\chi^2}{N} \qquad 0 \quad +\infty$

-

	B_1	B_2
A_1	a	b
A_2	c	d

$N/2$	0
0	$N/2$

Yule

$Q = \frac{ad - bc}{ad + bc} \qquad [-1; 1]$

$Q = \frac{\frac{N^2}{4} - 0}{\frac{N^2}{4} + 0} = 1.$

$\cos$

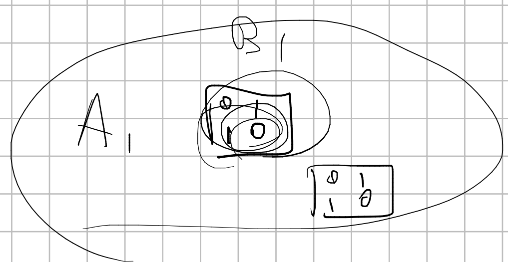
$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

Коэффициент динамики

Complex analysis

- $C = \sqrt{\frac{\chi^2/N}{1 - \chi^2/N}}$

GRI



$\circ \in A$

$$\begin{array}{c|ccc|c}
 & B_1 & B_2 & B_3 & \\
\hline
A_1 & n_{11} & & & n_{1\cdot} \\
A_2 & & & & n_{2\cdot} \\
A_3 & & & n_{33} & n_{3\cdot} \\
\hline
 & n_{\cdot 1} & n_{\cdot 2} & n_{\cdot 3} &
\end{array}$$

1) $?A$

2) $!A$

$$n_{\cdot m} = \max \{ n_{\cdot i} \}$$

$$n_{m\cdot} = \max \{ n_{i\cdot} \}$$

$$n_{im} = \max_j \{ n_{ij} \}$$

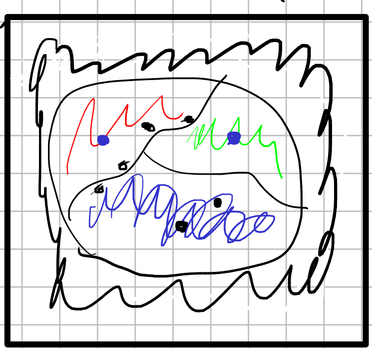
$$n_{mj} = \max_i \{ n_{ij} \}$$

$$\lambda_B = \frac{\sum_i n_{im} - n_{\cdot m}}{1 - n_{\cdot m}} = \frac{(1)-(2)}{(1)} < 1$$

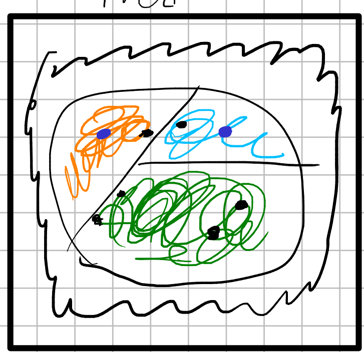
$\lambda = 1 \qquad (2) = 0$

Rand Index

Ground truth



Pred



(a, b)

R_{11} 1) $\exists k: a \in A_k \quad b \in A_k$
$\exists l: a \in B_l \quad b \in B_l$ } успех

R_{00} 2) $\exists k_1, k_2: \quad a \in A_{k_1}, b \in A_{k_2} \; k_1 \neq k_2$
$\exists l_1, l_2: \quad a \in B_{l_1}, b \in B_{l_2}, l_1 \neq l_2$

успех

$$R = \frac{R_{11} + R_{00}}{R_{11} + R_{00} + R_{01} + R_{10}} = \frac{R_{11} + R_{00}}{C_N^2}$$

3) R_{10} 4) R_{01}

$A: \; A_1 \ldots A_n$

$B: \; B_1 \ldots B_s$

$J_{ij} = A_i \cap B_j \qquad |J_{ij}| = n_{ij}$

$$R_{00} = |(x, y): x, y \in \mathcal{O}^*, \; \exists i, j : x, y \in J_{ij}| =$$

$$= \sum_{ij} |(\exists i: x, y \in A_i) \cap (\exists j: x, y \in B_j)| =$$

$$= \sum_{ij} C_{n_{ij}}^2 = \sum_{ij} \frac{n_{ij}(n_{ij}-1)}{2} = \ldots = \frac{1}{2} \sum_{ij} n_{ij}^2 - \frac{1}{2} N$$

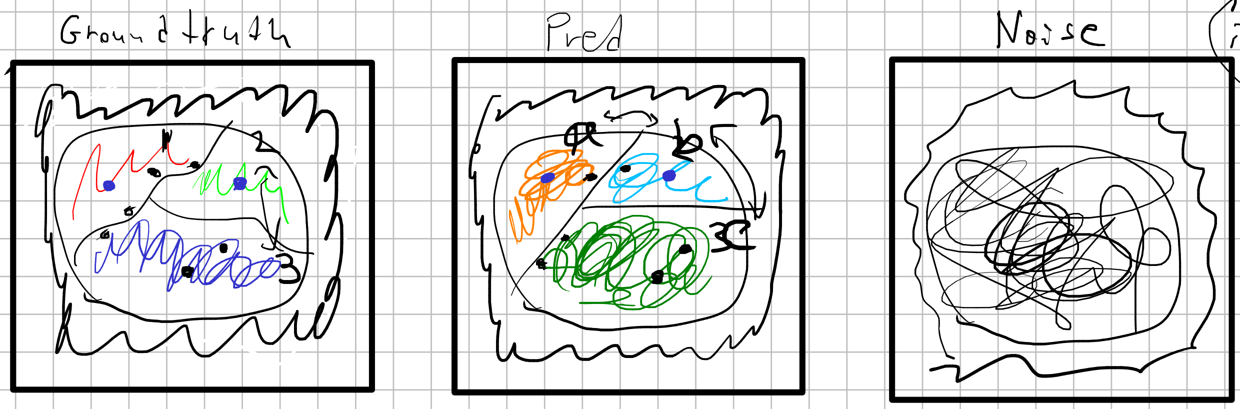
$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

$$K = (K_1 \quad K_2 \quad K_3)^T$$

$$R_{11} = C_N^2 - R_{00} - R_{01} - R_{10}$$

$$R_{01} = J \cdot K - \sum_{ij} n_{ij}^2$$

$$R_{10} = J^T \cdot S - \sum_{ij} n_{ij}^2$$



Ігри про ізоморфізм графів

Звʼязність (графів)

$[0, 1]$ $\quad R = \dfrac{R_{00} + R_{11}}{C_N^2}$ $\quad \underline{R_{00}} \quad \underline{R_{11}}$

$ER \sim 0.5, \ 0.7, \ 0.95, \ 0.99$

AR Adjusted for a chance

$S \quad k$

$n_{\cdot i} \quad n_{j \cdot}$

$$AR = \frac{R - ER}{\max R - ER}$$

$A : A_1 - - A_S$

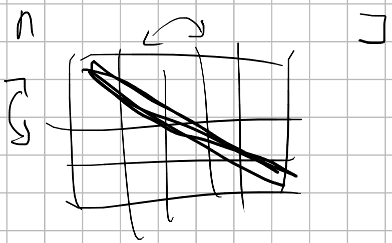
$B : B_1 - - B'_S$

$A \leftrightarrow B$

JJ^T

$J^T J$

$tr(JJ^T) = \sum_{ij} n_{ij}^2$



$A : A_1 - - A_S$

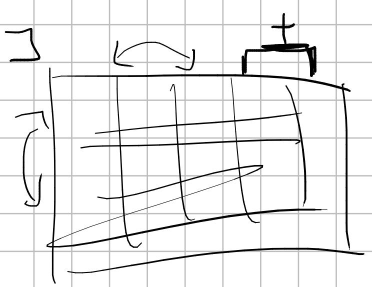
$B : B_1 - - B_K$

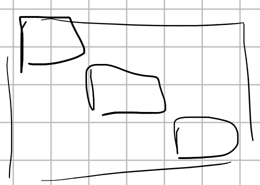
$k > s$

$S \neq k$

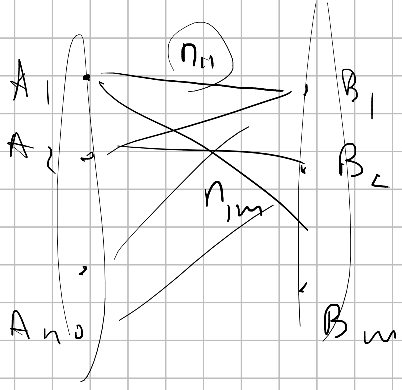
$s, k < 10$

$R \to \max$





SVD



n_{11}
A_1
A_2
A_n
B_1
B_2
B_m
n_{1m}