

Numerical Methods (ENUME 2019) – Project
Assignment C: Solving ordinary differential equations

1. Develop a program for solving the following differential equation:

$$9y'' + 6y' + 10y = 0 \text{ for } t \in [0, 10], \quad y(0) = 0 \text{ and } y'(0) = 2$$

by means of the implicit Gauss-Legendre order 5 method defined by the following Butcher table:

$\frac{1}{2} - \frac{\sqrt{15}}{10}$	$\frac{5}{36}$	$\frac{2}{9} - \frac{\sqrt{15}}{15}$	$\frac{5}{36} - \frac{\sqrt{15}}{30}$
$\frac{1}{2}$	$\frac{5}{36} + \frac{\sqrt{15}}{24}$	$\frac{2}{9}$	$\frac{5}{36} - \frac{\sqrt{15}}{24}$
$\frac{1}{2} + \frac{\sqrt{15}}{10}$	$\frac{5}{36} + \frac{\sqrt{15}}{30}$	$\frac{2}{9} + \frac{\sqrt{15}}{15}$	$\frac{5}{36}$
	$-\frac{5}{6}$	$\frac{8}{3}$	$-\frac{5}{6}$

Compare the solution, obtained by means of this program for the constant integration step $h = 0.01$, with the solution, obtained by means of the MATLAB operator *ode45*. Optimise the parameters *RelTol* and *AbsTol* of the latter in such a way as to get the most accurate solution $\hat{\mathbf{y}}(t; h)$ (to be used as a reference in the following sections).

2. Carry out a systematic investigation of the dependence of the accuracy of the solution $\hat{\mathbf{y}}(t; h)$ on the integration step h . Use the following accuracy indicators for this purpose:

$$\delta_2(h) = \frac{\|\hat{\mathbf{y}}(t; h) - \dot{\mathbf{y}}(t, h)\|_2}{\|\dot{\mathbf{y}}(t, h)\|_2} \quad (\text{the root-mean-square error})$$

$$\delta_\infty(h) = \frac{\|\hat{\mathbf{y}}(t; h) - \dot{\mathbf{y}}(t, h)\|_\infty}{\|\dot{\mathbf{y}}(t, h)\|_\infty} \quad (\text{the maximum error})$$

Make the graphs $\delta_2(h)$ and $\delta_\infty(h)$.

3. Repeat the systematic investigation, defined in Section 3, for the implicit Euler method. Add the curves representative of $\delta_2(h)$ and $\delta_\infty(h)$ to the graph obtained in Section 2.