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A Review of Earthquake Statistics: Fault and Seismicity-Based Models, ETAS and BASS

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Abstract—There are two fundamentally different approaches to assessing the probabilistic risk of earthquake occurrence. The first is fault based. The statistical occurrence of earthquakes is determined for mapped faults. The applicable models are renewal models in that a tectonic loading of faults is included. The second approach is seismicity based. The risk of future earthquakes is based on the past seismicity in the region. These are also known as cluster models. An example of a cluster model is the epidemic type aftershock sequence (ETAS) model. In this paper we discuss an alternative branching aftershock sequence (BASS) model. In the BASS model an initial, or seed, earthquake is specified. The subsequent earthquakes are obtained from statistical distributions of magnitude, time, and location. The magnitude scaling is based on a combination of the Gutenberg-Richter scaling relation and the modified Båth's law for the scaling relation of aftershock magnitudes relative to the magnitude of the main earthquake. Omori's law specifies the distribution of earthquake times, and a modified form of Omori's law specifies the distribution of earthquake locations. Unlike the ETAS model, the BASS model is fully self-similar, and is not sensitive to the low magnitude cutoff.

1. *Introduction*

Deformation of the Earth's crust is responsible for the generation of earthquakes over a wide range of scales. In terms of the resulting seismicity, the Earth's crust is clearly a self-organizing complex system (MAIN, 1996; RUNDLE *et al*., 2003). Despite this complexity, seismicity satisfies a number of universal scaling laws. These scaling laws have important implications for probabilistic seismic hazard analysis and earthquake forecasting. They also form the basis for a variety of models and simulations of earthquake activity. Examples include the epidemic type aftershock sequence (ETAS) model and the branching aftershock sequence (BASS) model. A comparison of these models will be a major focus of this paper.

Earthquakes constitute a major hazard on a worldwide basis. Although the locations of large earthquakes are concentrated near plate boundaries, they can occur within plate interiors. A specific example is the three large (magnitude ~ 7.7) earthquakes that occurred near New Madrid, Missouri in 1810 and 1811. A number of very large cities are

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located very close to plate boundaries. Examples include Tokyo, Los Angeles, San Francisco, Seattle, Lima, Jakarta, and Santiago. Much of China is a diffuse plate boundary, and major earthquakes have caused large losses of life throughout this region. A recent example was in the 1976 Tangshan earthquake with some 500 000 deaths.

A major goal of earthquake research is to quantify the risk of occurrence of an earthquake of a specified magnitude, in a specified area, and in a specified time window. This is done and results in hazard maps. Historic and paleoseismicity are major constraints on seismic hazard assessments. Slip rates and recurrence intervals of earthquakes on recognized faults are specified in so far as data are available. Examples are the sequence of studies carried out by the working groups on California earthquake probabilities (FIELD, 2007b). These reports have formed the basis for establishing rates of earthquake insurance in California.

A second major goal of earthquake research is to specifically forecast or predict earthquakes. Many attempts have been made, but with only marginal success. A number of published forecasting algorithms involve the use of past seismicity. The occurrence of recent smaller earthquakes is extrapolated to forecast the occurrence of future larger earthquakes. We first consider the relative roles of fault-based models and seismicity-based models. These alternatives will be discussed in Sections 2 and 3.

A specific type of seismicity-based forecast models is the ETAS model. This model is discussed in Section 4. In Section 5 we introduce the BASS model, and in Section 6 we compare BASS to ETAS. We conclude that BASS is preferable because it is fully scale-invariant and satisfies the major accepted scaling laws of seismicity. In Section 7 we introduce a deterministic version of the BASS model and show that it exhibits Tokunaga scale-invariant, side-branching statistics. These statistics are also satisfied by river networks, diffusion-limited aggregation (DLA) clusters, branching in biology, and cluster growth in site percolation. In Section 8 we present a numerical simulation of an aftershock sequence using the BASS model. Finally, we state our conclusions in Section 9 and discuss the future of seismic hazard assessment.

2. *Fault-Based Models*

Fault-based models consider the earthquakes that occur on recognized active faults. These models are also known as renewal models. Renewal models require that the stress on an individual fault is "renewed" by the tectonic drive of plate tectonics. The simplest renewal model would be that of a single planar strike-slip fault subjected to a uniform rate of strain accumulation (plate motion). In this case, "characteristic" earthquakes would occur periodically. Clearly the Earth's crust is much more complex with faults present at all scales and orientations. This complexity leads to chaotic behavior and statistical variability.

An important question is whether the concept of quasi-periodic characteristic earthquakes is applicable to tectonically active areas. There is extensive evidence that

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"characteristic" earthquakes do occur quasi-periodically on major faults. Many studies have been carried out to quantify the recurrence time statistics of these characteristic earthquakes (UTSU, 1984; OGATA, 1999; RIKITAKE, 1982). Recurrence time statistics can be characterized by a mean value, μ, and a coefficient of variation, C_v. The coefficient of variation is the ratio of the standard deviation to the mean. We have $C_v = 0$ for periodic characteristic earthquakes and $C_v = 1$ for a random distribution of recurrence times. ELLSWORTH *et al.* (1999) reviewed many examples of recurrence time statistics and concluded that $C_v \approx 0.5$ for characteristic earthquakes. Many probability distribution functions have been proposed for recurrence times, including the Weibull, lognormal, Brownian passage time, and gamma distributions.

Two major renewal simulation models have been developed. The first is "Virtual California" (RUNDLE *et al.*, 2004, 2005, 2006). This is a geometrically realistic numerical simulation of earthquake occurring on the San Andreas fault system and includes all major strike-slip faults in California. The second model is the "Standard Physical Earth Model" (SPEM) developed by WARD (1992) and applied to characteristic earthquakes associated with subduction at the Middle American trench. This model was further developed and applied to the entire San Andreas fault system by GOES and WARD (1994), to the San Andreas system in southern California by WARD (1996), and to the San Andreas system in northern California by WARD (2000).

Both simulation models utilize backslip, with the accumulation of a slip deficit on each fault segment prescribed using available data. Ideally the tectonic drive would be applied directly to the edges of a region, say 200 km on each side of the San Andreas fault system. But the long-term evolution of this approach requires that faults become longer as slip accumulates. The resulting geometrical incompatibility leads to serious numerical problems. In the backslip models, continuous displacements are applied to all faults until the frictional constraints result in backslip (an earthquake). Both models "tune" the prescribed static friction to give recurrence times that are consistent with available data. In both models fault segments are treated as dislocations where characteristic earthquakes occur, and all fault segments interact with each other elastically utilizing dislocation theory. These chaotic interactions result in statistical distributions of recurrence times on each fault. The resulting coefficients of variation are measures of this interaction.

YAKOVLEV *et al.* (2006) utilized the Virtual California model to test alternative distributions of recurrence times. They concluded that the Weibull distribution is preferable and based its use on its scale invariance. The hazard rate is the probability distribution function (pdf) that a characteristic earthquake will occur at a time t_0 after the last characteristic earthquake. The Weibull distribution is the only distribution that has a power-law (scale-invariant) hazard function. YAKOVLEV *et al.* (2006) found that the coefficient of variation of the recurrence times of 4606 simulated great earthquakes on the northern San Andreas fault is $C_v = 0.528$. GOES and WARD (1994) using the SPEM simulator found that $C_v = 0.50 - 0.55$ on this fault. The two simulations are quite different, so the statistical variability appears to be a robust feature of characteristic

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earthquakes. A similar simulation model for New Zealand has been given by ROBINSON and BENITES (1995, 1996.)

Renewal models have also formed the basis for three formal assessments of future earthquake probabilities in California. These assessments were carried out by the United States Geological Survey (WORKING GROUP ON CALIFORNIA EARTHQUAKE PROBABILITIES, 1988, 1990, 1995, 2003). A major problem with renewal models is that large earthquakes often occur on faults that were not previously recognized. Recent examples in California include the 1952 Kern County earthquake, the 1971 San Fernando Valley earthquake, the 1992 Landers earthquake, the 1994 Northridge earthquake, and the 1999 Hector Mine earthquake. At the times when these earthquakes occurred, the associated faults were either not mapped or were considered too small to have such large earthquakes. To compensate for this problem, renewal models often include a random level of background seismicity unrelated to recognized faults.

3. Seismicity-Based Models

An alternative approach to probabilistic seismic hazard assessment and earthquake forecasting is to use observed seismicity. The universal applicability of Gutenberg-Richter frequency-magnitude scaling allows the rate of occurrence of small earthquakes to be extrapolated to estimate the rate of occurrence and location of large earthquakes. This type of extrapolation played an important role in creating the national seismic hazard map for the United States (FRANKEL *et al.*, 1996).

A more formalistic application of this extrapolation methodology is known as a relative intensity (RI) forecast. This type of forecast was made on a worldwide basis by KOSSOBOKOV *et al.* (2000) and for California by HOLLIDAY *et al.* (2005). A related forecasting methodology is the pattern informatics (PI) method (RUNDLE *et al.*, 2002; TIAMPO *et al.*, 2002a, b; HOLLIDAY *et al.*, 2006a, b, 2007). This method was used by RUNDLE *et al.* (2002) to forecast $m = 5$ and larger earthquakes in California for the time period 2000–2010. This forecast successfully predicted the locations of 16 of the 18 large earthquakes that have subsequently occurred.

KEILIS-BOROK (1990, 2002) and colleagues utilized patterns of seismicity to make formal intermediate term earthquake predictions. Two notable successes were the 1988 Armenian earthquake and the 1989 Loma Prieta, California, earthquake. However, a number of large earthquakes were not predicted and the approach remains controversial. More recently, this group has used chains of premonitory earthquakes to make intermediate term predictions (SHEBALIN *et al.*, 2004; KEILIS-BOROK *et al.*, 2004). Again, moderate success was achieved.

It has also been proposed that there is an increase in the number of intermediate-sized earthquakes prior to a large earthquake. This proposal has been quantified in terms of an accelerated moment release (AMR) prior to a large earthquake. This approach has shown considerable success, retrospectively (BUFE and VARNES, 1993; BOWMAN *et al.*, 1998;

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Sammis *et al.*, 2004) but has not evolved into a successful prediction algorithm as of yet (Gross and Rundle, 1998; Main, 1999).

Seismicity-based models are often referred to as clustering models. That is, clusters of small earthquakes indicate the future occurrence of larger earthquakes. The RI, PI, and AMR models clearly belong to this class. A rational for the application of clustering models is that the clustering is related to families of foreshocks, main shocks, and aftershocks. This rational forms the basis for the use of both the ETAS and the BASS models. It should be emphasized that neither model introduces renewal, the tectonic drive responsible for the energy input dissipated in earthquakes. One way to overcome this problem has been to introduce a random background seismicity to excite aftershocks. An alternative approach is to couple a clustering model, such as ETAS or BASS, to a fault-based model, such as Virtual California or SPEM.

4. ETAS

A clustering model that has been widely studied is the ETAS model. This approach was first formulated by Kagan and Knopoff (1981). It is a statistical model based on applicable scaling laws. This model was further developed by Ogata and colleagues (Ogata, 1988, 1989, 1992, 1998, 1999, 2001a, b, 2004; Ogata *et al.*, 1993, 2003; Guo and Ogata, 1997; Ogata and Zhuang, 2006; Zhuang and Ogata, 2006; Zhuang *et al.*, 2002, 2004; Vere-Jones, 2005). Modified versions of ETAS were introduced by Helmstetter, Sornette, and colleagues (Helmstetter, 2003; Helmstetter *et al.*, 2003a, b, 2004, 2006; Helmstetter and Sornette, 2002a, b, 2003a, b, c, d; Saichev *et al.*, 2005; Saichev and Sornette, 2004, 2005a, b, 2006a, b, c, 2007a, b; Sornette and Helmstetter, 2002; Sornette and Werner, 2005a, b) and by Lepiello *et al.* (2007). Related models have been developed by Felzer and colleagues (Felzer *et al.*, 2002, 2003, 2004), by Console and colleagues (Console and Murru, 2001; Console *et al.*, 2003, 2006), and by Gerstenberger and colleagues (Gersterberger *et al.*, 2004, 2005). Before discussing the details of the ETAS model, we will first introduce the BASS model.

5. BASS

An alternative to the ETAS model is the BASS model (Turcotte *et al.*, 2007). As in the ETAS model, the BASS model recognizes that each earthquake has an associated sequence of aftershocks. Each main shock produces a sequence of primary aftershocks. Each of these aftershocks, in turn, produces second-order aftershocks. Each second-order aftershock can produce third-order aftershocks, and so forth. Statistically, a primary aftershock can be larger than the initial main shock. In this case the initial main shock becomes a foreshock, and the larger primary aftershock becomes the main shock of the

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sequence. In principal, a higher-order aftershock could be the main shock. The probability of this occurring, however, is extremely small.

It has been demonstrated by many authors that the frequency-magnitude distribution of aftershocks satisfy the Gutenberg-Richter (GR) relation to a good approximation (GUTENBERG and RICHTER, 1954; SHCHERBAKOV *et al.*, 2005). In the BASS formulation, we require that the frequency-magnitude distribution of each order of aftershocks satisfies the GR relation in the form

$$\log_{10}[N_d(\geq m_d)] = a_d - b_d m_d, \tag{1}$$

where m_d is the magnitude of a daughter earthquake, $N_d(\geq m_d)$ is the number of daughter earthquakes with magnitudes greater than or equal to m_d, and a_d and b_d are the *a*- and *b*-values of the distribution, respectively. Note that the *b*-value b_d for each sequence of aftershocks is not necessarily equal to the *b*-value for all aftershocks. This is due to the superposition of many generations of aftershock sequences for each parent earthquake and will be discussed in detail later in the paper.

In order to fully specify the frequency-magnitude distribution of a family of aftershocks, we apply the modified form of Båth's law (SHCHERBAKOV and TURCOTTE, 2004). As shown by SHCHERBAKOV *et al.* (2005), this formulation is closely related to that given by REASENBERG and JONES (1989), YAMANAKA and SHIMAZAKI (1990), and FELZER *et al.* (2002).

In its original form, Båth's law (BÅTH, 1965; VERE-JONES, 1969) states that the magnitude difference between a main shock and its largest aftershock Δm is nearly constant with a value near 1.2. SHCHERBAKOV and TURCOTTE (2004) introduced a new way of defining this difference and obtained a value Δm^* based on the entire distribution of aftershocks, not just the largest aftershock. It is required that the magnitude of the largest aftershock inferred from the GR relation is a fixed value Δm^* less than the magnitude of the parent earthquake, m_p:

$$N_d(\geq (m_p - \Delta m^*)) = 1. \tag{2}$$

With this condition we require (using Eq. (1) that $a_d = b_d\,(m_p - \Delta m^*)$ so that

$$\log_{10}[N_d(\geq m_d)] = b_d(m_p - \Delta m^* - m_d). \tag{3}$$

This relation fully specifies the frequency-magnitude distribution of each family of aftershocks (daughter earthquakes). However, this distribution implies an infinite number of small earthquakes. To eliminate this singularity, it is necessary to prescribe a minimum magnitude earthquake $m_{\min}$ that is to be considered.

We obtain this total number of aftershocks in a family by setting $m_d = m_{\min}$ in Eq. (3):

$$N_{dT} = N(\geq m_{\min}) = 10^{b_d(m_p - \Delta m^* - m_{\min})}. \tag{4}$$

This relation is the essential feature of the BASS model. Utilizing Eqs. (3) and (4), we obtain the cumulative distribution function P_{Cm} for the magnitudes of the daughter earthquakes:

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$$P_{Cm} = \frac{N_d(\geq m_d)}{N_{dT}} = 10^{-b_d(m_d - m_{\min})}. \tag{5}$$

The magnitude m_d of each daughter earthquake is selected from this distribution. For each daughter earthquake a random value for P_{Cm} in the range $0 < P_{Cm} < 1$ is generated, and the magnitude of the earthquake is determined from Eq. (5). Note that there is a finite probability that a daughter earthquake can be larger than the parent earthquake. The probability that this occurs is obtained by substituting Eq. (4) into Eq. (5) and setting $m_d = m_p$ with the result

$$P_{Cm}(m_d > m_p) = 10^{-b_d \Delta m^*}. \tag{6}$$

Taking $b_d = 1$ and $\Delta m^* = 1$, we have $P_{Cm}(m_d > m_p) = 10\%$. With these values, a well defined foreshock would be expected about 10% of the time. This is in reasonable agreement with observed values of $13 \pm 5\%$ (REASENBERG, 1999). It should be emphasized that this value is independent of the choice for $m_{\min}$. The selection of a $m_{\min}$ is necessary, but the choice does not significantly influence the distribution of magnitudes above this cutoff.

Having specified the magnitude m_d of each daughter earthquake by selecting a random number P_{Cm} in the range $0 < P_{Cm} < 1$ and using Eq. (5) in the form

$$m_d = -\frac{1}{b_d}\log P_{Cm} + m_{\min}, \tag{7}$$

we next specify the time of occurrence of the daughter earthquakes.

We require that the time delay t_d until each daughter earthquake after the parent earthquake satisfies a general form of Omori's law (SHCHERBAKOV *et al.*, 2004):

$$R(t_d) = \frac{\mathrm{dN}_d}{\mathrm{d}t} = \frac{1}{\tau(1 + t_d/c)^p}, \tag{8}$$

where $R(t_d)$ is the rate of aftershock occurrence and τ, c, and p are parameters. The number of daughter aftershocks that occur after a time t_d is then given by

$$N_d(\geq t_d) = \int_{t_d}^{\infty} \frac{\mathrm{dN}_d}{\mathrm{d}t} = \frac{c}{\tau(p-1)(1 + t_d/c)^{p-1}}. \tag{9}$$

The total number of daughter earthquakes is obtained by setting $t_d = 0$ in Eq. (9) with the result

$$N_{dT} = \frac{c}{\tau(p-1)}. \tag{10}$$

It should be emphasized that this result is only valid for $p > 1$. If $p \leq 1$, the integral is not convergent and a maximum time must be specified. From Eqs. (9) and (10) we obtain the cumulative distribution function P_{Ct} for the times of occurrence of the daughter earthquakes:

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$$P_{Ct} = \frac{N_d(\geq t_d)}{N_{dT}} = \frac{1}{(1 + t_d/c)^{p-1}}. \tag{11}$$

The time of occurrence t_d of each daughter earthquake is selected from this distribution. For each daughter earthquake a random value for P_{Ct} in the range $0 < P_{Ct} < 1$ is generated, and the time of occurrence of the earthquake is determined using Eq. (11) in the form

$$t_d = c(P_{Ct}^{-1/(p-1)} - 1). \tag{12}$$

The distribution of times is dependent only on the fitting parameters c and p. In this paper, these parameters for each generation of aftershocks are assumed to be equal. The values for the superposition of many generations of aftershocks, however, may be different. In general, the parameter p should be in the range $1.1 \leq p \leq 1.3$.

Finally, we specify the location of each daughter earthquake relative to its parent. There are a wide variety of distributions that we could choose from, but in this paper we assume that a daughter earthquake occurs at a randomly chosen radial distance from the parent earthquake in a randomly chosen direction. Based on results given by FELZER and BRODSKY (2006), we assume a power-law dependence of the radial position in direct analogy to Omori's law. The cumulative distribution function P_{Cr} for the radial distance r_d of each daughter earthquake from the parent earthquake is given by

$$P_{Cr} = \frac{N_d(\geq r_d)}{N_{dT}} = \frac{1}{(1 + r_d/(d \cdot 10^{0.5m_p}))^{q-1}}. \tag{13}$$

The dependence on the magnitude m_p of the parent earthquake introduces a mean radial position of aftershocks that scales with the rupture length of the parent earthquake. The radial position r_d of the daughter earthquake relative to the parent earthquake is selected from this distribution. For each daughter earthquake a random value for P_{Cr} in the range $0 < P_{Cr} < 1$ is generated, and the radial distribution is determined using Eq. (13) in the form

$$r_d = d \cdot 10^{0.5m_p}({P_{Cr}}^{-1/(q-1)} - 1). \tag{14}$$

In order to completely specify the location of the daughter earthquake, its direction relative to the parent earthquake θ_d must be specified. The direction is therefore chosen randomly from the uniform range $0 < \theta_d < 2\pi$.

6. *BASS versus ETAS*

There are many similarities between the BASS model and the ETAS model. More importantly, however, there are also fundamental differences. In considering the ETAS model, we will utilize the formulation given by HELMSTETTER and SORNETTE (2003a). Both

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models utilize the concept of multiple orders of aftershocks. The main shock generates a sequence of primary aftershocks; these in turn generate families of secondary aftershocks, and so forth.

The primary difference between the two models is the way in which the number of daughter earthquakes is specified. In the ETAS model, the number of daughter earthquakes produced by a mother earthquake N_{dT} takes the form (SORNETTE and WERNER, 2005b, eq. 3)

$$N_{dT} = k \cdot 10^{\alpha(m_p - m_{\min})}, \tag{15}$$

and the two constants k and α must be specified. In some formulations of ETAS, the mean number n of direct aftershocks per earthquake, averaged over all magnitudes, is used to specify k (SORNETTE and WERNER, 2005b, eq. 4). If Eqs. (4) and (15) are identical, then ETAS is essentially identical to BASS. This is the case if

$$\alpha = b_d \tag{16}$$

and

$$k = 10^{-b_d \Delta m^*}. \tag{17}$$

FELZER *et al.* (2002) argue that Eq. (16) is in fact satisfied. Proponents of ETAS, however, require $\alpha < b_d$ so that n is less than one. The BASS formulation utilizes the modified form of Båth's law as given in Eq. (3) to constrain the number of daughter earthquakes N_{dT} (productivity).

The general ETAS formulation does not satisfy Båth's law. The association of Båth's law to ETAS has been discussed in some detail by HELMSTETTER and SORNETTE (2003c). As shown in their Figure 1, Δm has strong magnitude dependence. In fact, the values of Δm become negative in the vicinity of the minimum magnitude earthquakes $m_{\min}$. In this vicinity, the average largest aftershock is greater than the main shock. This clearly violates scale-invariance and makes results very sensitive to the choice of $m_{\min}$. In the BASS model, the distribution of daughter earthquake magnitudes is fully scale-invariant and is insensitive to the choice of the minimum size earthquake considered.

In terms of the times and positions of daughter earthquakes, BASS and ETAS use identical formulations. Both use Omori's law for times and a modified form of Omori's law for radial positions.

7. *Illustration of the BASS Model*

We first illustrate the principals of the BASS model using a deterministic branching formulation. We begin by considering the distribution of aftershocks associated with a main shock of a prescribed magnitude. The application of Båth's law introduces a characteristic earthquake magnitude Δm, the magnitude difference between the main

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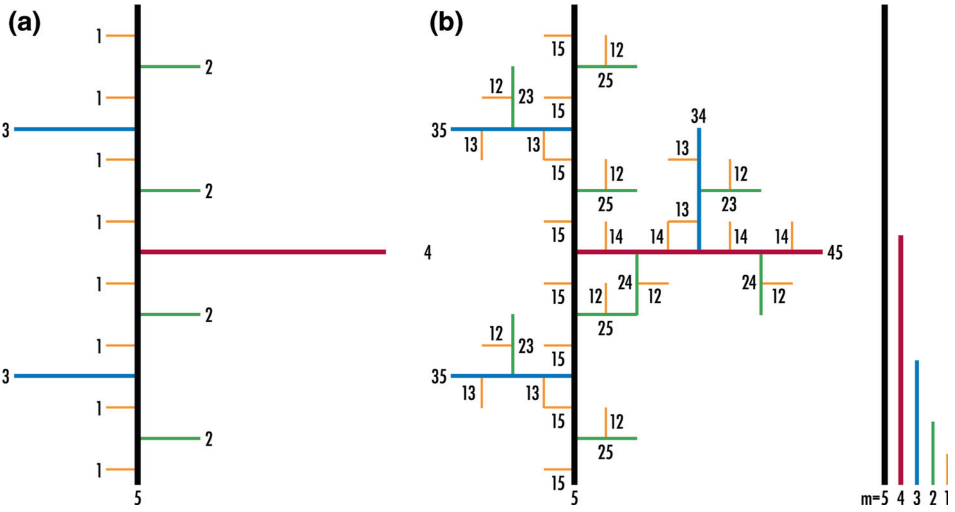

Figure 1
Illustration of our branching model using a discrete set. (a) The primary family of aftershocks is a generator for the fractal construction; (b) the full Tokunaga structure of side-branching aftershocks.

shock and the largest aftershock. For convenience, we take $\Delta m = 1$ and consider the number of earthquakes with unit magnitudes, $m = 1, 2, 3, \ldots$.

Our basic formulation introduces a branching ratio B into the prescription of the number of smaller earthquakes of magnitude i generated by a larger earthquake of magnitude j. The number of daughter earthquakes of magnitude i generated by a parent earthquake of magnitude j, N_{ij}, can be written

$$N_{ij} = B^{j-i-1}. \tag{18}$$

This basic branching is illustrated in Figure 1a for binary branching ($B = 2$). We take the "parent" to have a magnitude $j = 5$. From Eq. (18) we have one daughter earthquake with magnitude $i = 4$, two daughter earthquakes with magnitude $i = 3$, four daughter earthquakes with magnitude $i = 2$, and eight daughter earthquakes with magnitude $i = 1$. That is $N_{45} = 1$, $N_{35} = B = 2$, $N_{25} = B^2 = 4$, and $N_{15} = B^3 = 8$.

Tokunaga (1978) introduced the basic branching ratio concept given in Eq. (16) for river network branching. Extensive examples have been given by Peckham (1995) and Pelletier (1999). This branching was also found to be applicable to the structure of diffusion limited aggregation (DLA) clusters (Ossadnik, 1992), to examples in biology (Turcotte *et al.*, 1998), and to clustering (Gabrielov *et al.*, 1999). Deterministic examples have been given by Turcotte and Newman (1996) and by Newman *et al.* (1997). In this paper, we show that this same branching structure is applicable to seismicity.

We extend the basic branching relation given in Eq. (18) to families of aftershocks. That is, we consider the aftershocks of aftershocks. The number of daughter earthquakes

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of magnitude i generated by a parent earthquake of magnitude j which was a daughter of a parent earthquake of magnitude k, N_{ijk}, ks given by

$$N_{ijk} = N_{jk}N_{ij}, \tag{19}$$

where N_{ij} is the number of magnitude j aftershocks generated by a parent of magnitude i. Substituting N_{ij} from Eq. (18) into Eq. (19). gives

$$N_{ijk} = N_{jk}B^{j-i-1}. \tag{20}$$

The total number of aftershocks of magnitude i generated by a main shock of magnitude k, N_{ik}, is given by

$$N_{ik} = \sum_{j=i+1}^{k} N_{ijk} = (B+1)^{k-i-1}. \tag{21}$$

The validity of this result can be verified by noting that it gives

$$\begin{aligned} N_{jk} &= 1 \quad \text{if} \quad j = k \\ N_{jk} &= (B+1)^{j-k-1} \quad \text{if} \quad j < k \end{aligned}. \tag{22}$$

Substitution of Eq. (22) into Eq. (21) and carrying out the sum verifies the validity of Eq. (21). The side-branching structure of aftershocks of aftershocks is best illustrated by an example.

The full binary ($B = 2$) side-branching structure of aftershocks for a magnitude 5 main shock is given in Figure 1b. The corresponding numbers N_{ij5} and N_{i5} are given in Table 1. From Eq. (21), we predict $N_{i5} = 3^{4\text{-}i}$, which is what we find. A discrete form of the Gutenberg-Richter frequency-magnitude scaling can be written:

$$\log N_{ik} = a - bi. \tag{23}$$

For the sequence given in Table 1, we have $a = \log 81$ and $b = \log 3 = 0.477$, which is low relative to actual aftershock sequences.

Table 1

Illustration of the deterministic BASS model for binary branching $B = 2$, main shock magnitude $m_k = 5$, modified Båth's law $\Delta m = 1$, and minimum magnitude $m_{min} = 1$. The numbers of aftershocks N_{ij5} with magnitude i generated by a parent earthquake of magnitude j are given. The total numbers of aftershocks N_{i5} of magnitude i are also given

Aftershock Magnitude	Parent Earthquake Magnitude				Total N_{ij}
	$j = 5$	$j = 4$	$j = 3$	$j = 2$	
$i = 4$	$N_{455} = 1$				$N_{45} = 1$
$i = 3$	$N_{355} = 2$	$N_{345} = 1$			$N_{35} = 3$
$i = 2$	$N_{255} = 4$	$N_{245} = 2$	$N_{235} = 3$		$N_{25} = 9$
$i = 1$	$N_{155} = 8$	$N_{145} = 4$	$N_{135} = 6$	$N_{125} = 9$	$N_{15} = 27$

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Table 2

Illustration of the deterministic BASS model for a branching ratio $B = 9$, main shock magnitude $m_k = 8$, modified Båth's law $\Delta m^* = 1$, and minimum magnitude $m_{min} = 1$. The numbers of aftershocks N_{ij8} with magnitude i generated by a parent earthquake of magnitude j are given. The total number of aftershocks N_{i8} of magnitude i are also given. Note that the total numbers satisfy Gutenberg-Richter frequency-magnitude scaling with $b = 1$

Aftershock Magnitude	Parent Earthquake Magnitude							Total N_{ij}
	$j = 8$	$j = 7$	$j = 6$	$j = 5$	$j = 4$	$j = 3$	$j = 2$	
$i = 7$	1							1
$i = 6$	9	1						10
$i = 5$	81	9	10					100
$i = 4$	729	81	90	100				1000
$i = 3$	6561	729	810	900	1000			10000
$i = 2$	59049	6561	7290	8100	9000	10000		100000
$i = 1$	53144	59049	65610	72900	81000	90000	100000	1000000

As a more realistic example, we take $B = 9$ and a main shock magnitude $k = 8$. The corresponding numbers N_{ij8} and N_{i8} are given in Table 2. We again assume that $\Delta m = 1$ and that $m_{\min} = 1$. The numbers of aftershocks N_{ij8} with magnitude i generated by a parent earthquake of magnitude j are given. The total numbers of aftershocks N_{i8} of magnitude i are also given.

From Eq. (21), we predict $N_{i8} = 10^{7\text{-}i}$, which is what we find. From Eq. (23), we find $a = 7$ and $b = 1$ which is a reasonable value for an aftershock sequence. The results given in Table 2 are directly analogous to Table 1. The total families of aftershocks are given. In Table 2 we have $B = 9$ and $m_{ms} = 8$. In Table 1 we have $B = 2$ and $m_{ms} = 5$.

We next determine the entire inventory of earthquakes in a region using our deterministic model. In order to do this, we must specify the magnitude of the largest earthquake in the region, $m_{\max}$. For our example, we take $m_{\max} = 8$. We first specify the numbers of main shocks of magnitude i, N_{im}, using Gutenberg-Richter scaling in the form

$$N_{im} = B^{8-i}. \tag{24}$$

Taking $B = 9$ and $m_{\min} = 1$, the numbers of main shocks, N_{im}, are given in Table 3. Utilizing the results given in Table 2, the numbers of aftershocks N_{ik} with aftershock magnitude i generated by a main shock with magnitude k are also given in Table 3. The total numbers of aftershocks of each magnitude N_{ia} and the total numbers of earthquakes of each magnitude N_{iT} are also given. It is seen that the total number of earthquakes, all main shocks and their aftershocks, satisfy the scaling relation

$$N_{iT} = (B + 1)^{8-i}. \tag{25}$$

The fraction of all earthquakes that are aftershocks increases systematically from 10% at magnitude seven to 52% at magnitude one.

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Table 3

Entire inventory of earthquakes in a region given by our deterministic model. We specify the largest earthquake to have $m_{max} = 8$. The numbers of main shocks of magnitude i, N_{im} as obtained from Eq. (24) with $B = 9$ are given. The total numbers of aftershocks of magnitude i generated by a main shock of magnitude k, N_{ik}, are also given as well as the numbers of aftershocks and total numbers of earthquakes of magnitude k

Earthquake Magnitude	Number of Main shocks	Parent Earthquake Magnitude							Total Aftershocks	Total Earthquakes
		$j = 8$	$j = 7$	$j = 6$	$j = 5$	$j = 4$	$j = 3$	$j = 2$		
$i = 8$	1									1
$i = 7$	9	1							1	10
$i = 6$	81	10	9						19	100
$i = 5$	729	100	90	81					271	1000
$i = 4$	6561	1000	900	810	729				3439	10000
$i = 3$	59049	10000	9000	8100	7290	6561			40451	100000
$i = 2$	531441	100000	90000	81000	72900	65610	59049		468559	1000000
$i = 1$	4782969	1000000	900000	810000	729000	656100	590490	531441	5217031	10000000

8. *Probabilistic BASS Simulation*

We now give a specific probabilistic simulation using the BASS model. A complete stochastic aftershock sequence will be generated. In order to start the simulation, it is necessary to choose a main shock amplitude, m_k. In this example we take $m_k = 7$.

A. We first determine the distribution of primary aftershocks generated by the main shock by taking the main shock to be the parent earthquake.
   1. The first step is to determine the total number of primary aftershocks from Eq. (4). In addition to the parent earthquake magnitude $m_p = m_k = 7$ it is necessary to specify a b-value for the daughter earthquakes, we take $b_d = 1$ throughout the simulation, and the modified Båth's law magnitude difference Δm^*, we take $\Delta m^* = 1.0$ for simplicity. It is also necessary to specify the minimum magnitude considered. For this example, we take $m_{\min} = 2$. With these values, we find from Eq. (4) that the total number of primary aftershocks is $N_{dT} = 10^{4.0} = 10000$.
   2. We generate $N_{dT} = 10000$ random numbers for the P_{Cm} in the range $0 < P_{Cm} < 1$, and the magnitudes of the $N_{dT} = 10000$ primary aftershocks are determined using Eq. (7) and the parameter values given above.
   3. We next utilize the generalized form of Omori's law given in Eq. (8) to obtain the time of occurrence of each aftershock. In order to specify the cumulative distribution function P_{Ct} given in Eq. (11), we require the two parameters c and p. Based on the results given by YAMANAKA and SHIMAZAKI (1990), FELZER *et al.* (2003), and SHCHERBAKOV *et al.* (2004), we take $c = 0.1$ days and $p = 1.25$. We again generate $N_{dT} = 10000$ random numbers for P_{Ct} in the range $0 < P_{Ct} < 1$, and the times of occurrence of the $N_{dT} = 10000$ primary aftershocks are determined using Eq. (12) and the parameter values given above. Note that the

times of occurrence of the aftershocks t_d are not correlated with their magnitudes m_d.

4. Finally, we utilize the cumulative distribution function P_{Cr} given in Eq. (13) to specify the radial distance of the daughter earthquakes from the parent earthquake. For primary aftershocks, these distances are from the main shock. To fully specify P_{Cr}, we require two parameters d and q. Based on the results given by FELZER and BRODSKY (2006), we take $d = 4$ m and $q = 1.35$. We generate another $N_{dT} = 10000$ random numbers for P_{Cr} in the range $0 < P_{Cr} < 1$, and the radial distances of the $N_{dT} = 10000$ primary aftershocks are determined using Eq. (14) and the parameter values given above. Again, these radial positions of these aftershocks are not correlated with either their magnitudes or their times of occurrence. A final set of $N_{dT} = 10000$ random numbers are generated in the range $0 < \theta < 2\pi$. The value of θ for each aftershock is taken as the angle of the aftershock relative to some reference direction.

B. Each of the primary aftershocks are next treated as a parent earthquake, and steps A1 to A4 are repeated.
   1. For each primary aftershock, the number of secondary aftershocks is obtained using Eq. (4). This number has a strong dependence on the magnitude of the primary aftershock under consideration. The magnitude of each secondary aftershock is then determined using random numbers and the distribution given in Eq. (7). Note that the magnitudes to be determined do not depend on the magnitude of the parent earthquake (the primary aftershock).
   2. The time of occurrence of each secondary aftershock is then determined using random numbers and the distribution given in Eq. (12). Note that the time of occurrence of each secondary aftershock is the time since the occurrence of the parent earthquake (the primary aftershock).
   3. The radial position of each secondary aftershock is determined using random numbers and the distribution given in Eq. (14). Note that the radial position is relative to the position of the parent earthquake (the primary aftershock). The direction relative to the parent earthquake is also randomly selected. Note also that the parent magnitude in Eq. (14) is the magnitude of the parent primary aftershock.

C. Each secondary aftershock is taken to be a parent earthquake, and a family of daughter second-order aftershocks is generated using the procedure outlined in B1 through B3. The procedure is further repeated to higher orders until no more aftershocks are generated.

The magnitudes of the aftershocks as a function of times of occurrence since the main shock are given in Figure 2. There are 101015 aftershocks in the simulation spanning twenty two generations. Since, as was pointed out, there are 10000 primary aftershocks, this simulation generated 91015 second- and higher-order aftershocks. The magnitude of the largest aftershock in this simulation is $m = 6.6$, thus $\Delta m = 0.4$. Note that the Δm in

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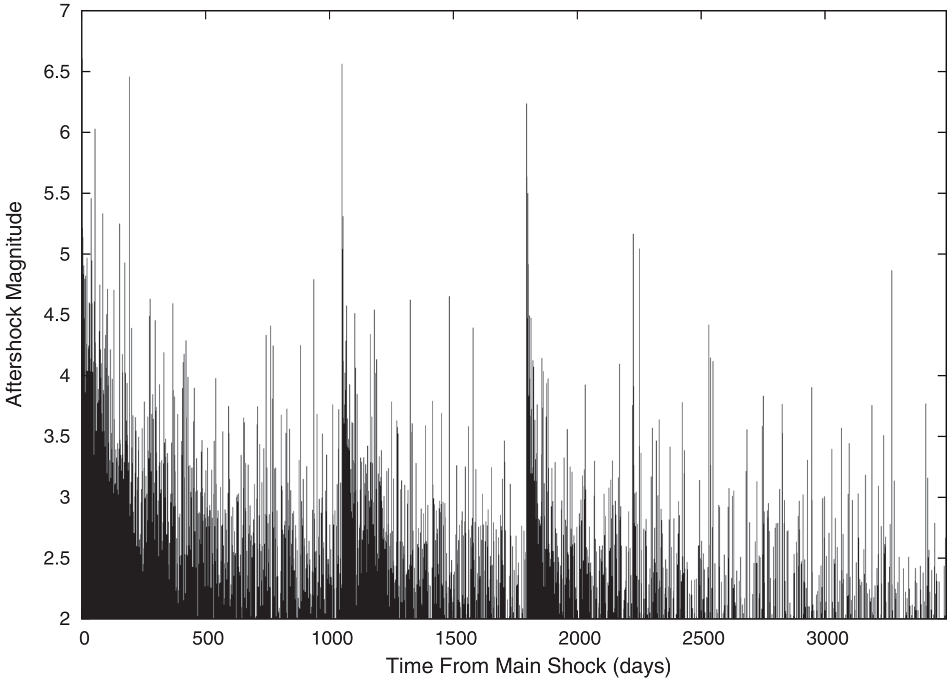

Figure 2
Plot of magnitudes as a function of time (in days) over the first year for the first four generations of an aftershock sequence based on an initial $m = 7.0$ event at time $t = 0$. Note that large aftershocks generate their own aftershocks sequences.

each simulation is different although we take $\Delta m^{*} = 1$. It is clearly seen in Figure 1 that large primary aftershocks generate their own sequences of higher-order aftershocks.

The positions of the aftershocks relative to the main shock are given in Figure 3. Again, it is clearly seen that clusters of higher-order aftershocks surround the large primary aftershocks. The cumulative Gutenberg-Richter frequency-magnitude statistics of the aftershocks are given in Figure 4. The frequency-magnitude distribution for all aftershocks is well approximated by the Gutenberg-Richter relation (Eq. (1) taking $b = 1$ and $\Delta m^{*} = 1$.

9. Discussion

Probabilistic seismic hazard assessments play many roles. These include: 1) alerting the public to the level of risk, 2) influencing seismic building codes and seismic retrofitting, 3) setting earthquake insurance premiums, and 4) motivating earthquake hazard preparations. We have discussed two distinct approaches to seismic hazard

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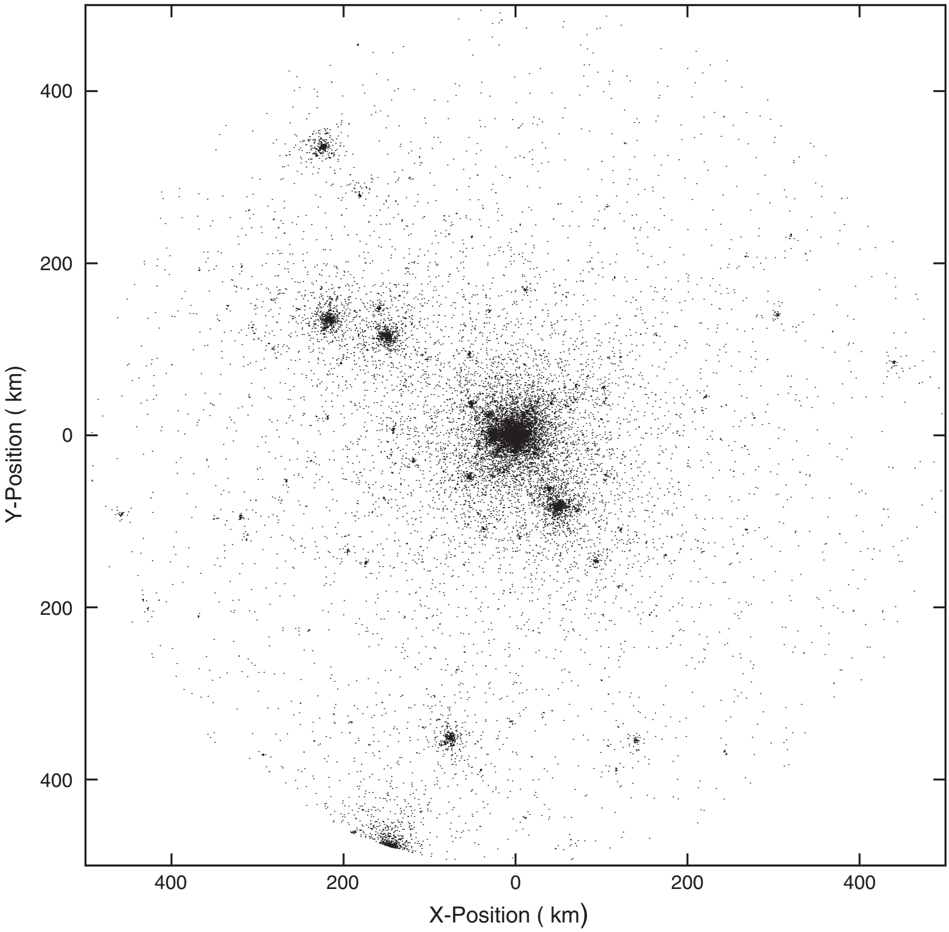

Figure 3
Plot of aftershock positions for the first four generations of an aftershock sequence based on an initial $m = 7.0$ event at location $r = 0$. Note that each generation's aftershocks are clustered about their respective main shocks. Only plotted are aftershocks that fall within a 500 km radius surrounding the main shock.

assessment. The first uses fault-based models. The risk of an earthquake on mapped faults is assessed. This can be done in several ways. The statistics of occurrence of a characteristic earthquake on each fault is prescribed. This requires the magnitude, mean recurrence time, coefficient of variation, and a distribution function for recurrence times. Evidence favors the applicability of the Weibull distribution (YAKOVLEV *et al.*, 2006).This distribution includes Poisson (random) and periodic limits. Fault-based models can be constrained using simulations. Two examples we have discussed are Virtual California and SPEM. These models include a specified tectonic drive and interactions between fault segments.

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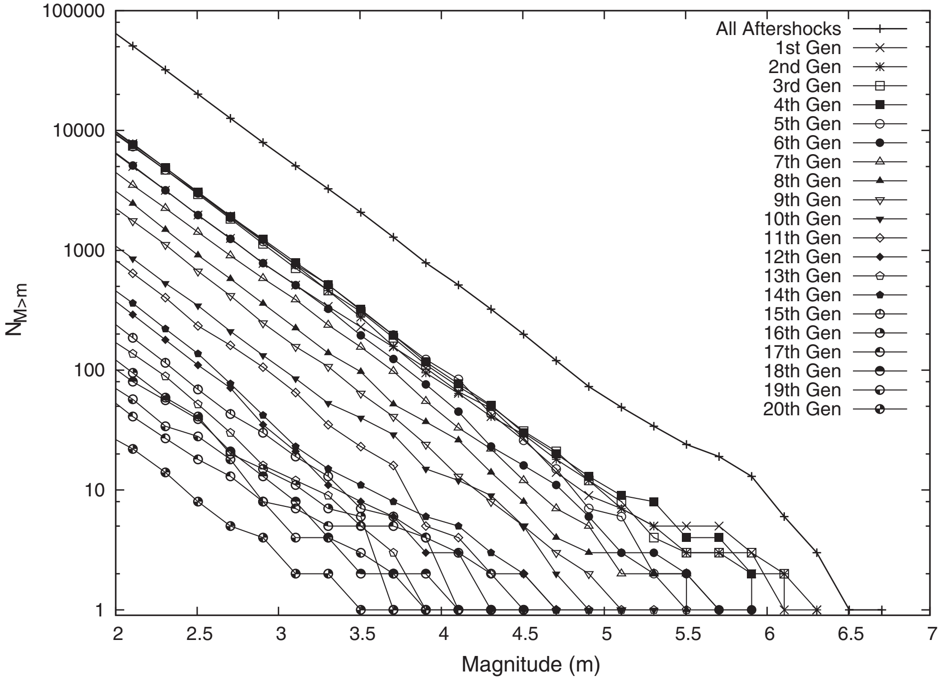

Figure 4
Plot of the Gutenberg-Richter frequency-magnitude relation for an aftershock sequence based on an initial $m = 7.0$ event. Note that this particular realization consisted of twenty two generations (only the first twenty are shown).

A major difficulty with fault-based models is that many large earthquakes occur on faults that have not been mapped. This difficulty can be alleviated by introducing a random background of seismicity. This background, however, should be correlated the regional seismicity.

The second approach to seismic hazard assessment is to use seismicity-based models. Future earthquakes are associated with past earthquakes. The simplest approach is relative intensity (RI) models. The rates of occurrence of small earthquakes (say, $m = 2$) in gridded cells (say, $0.1^\circ \times 0.1^\circ$) are extrapolated to larger magnitudes using Gutenberg-Richter frequency-magnitude scaling.

It is also possible to extrapolate past seismicity forward in time using aftershock models. The ETAS model has been used extensively for this purpose. Each past earthquake can be used as a parent earthquake, and future aftershocks can be determined. In this paper we present the BASS model as an alternative to ETAS. The BASS model is fully self-similar, satisfies all relevant scaling laws, and is simple to implement.

It is clear that there are many alternative approaches to probabilistic seismic hazard assessment. In order to test alternative models for California earthquakes, a competition for Regional Earthquake Likelihood Models (RELM) was sponsored by the Southern

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California Earthquake Center (SCEC). Forecasts for $m > 5$ earthquakes during the period 1 January, 2006 to 31 December, 2011 on a $0.1° \times 0.1°$ grid of cells were solicited. Probabilities of occurrence were required for each grid. Nine competing forecasts were submitted and have been summarized by FIELD (2007a). The contrasts between the smeared fault based forecasts and the highly gridded seismicity-based forecasts stands out. Several of the seismicity-based forecasts utilized ETAS models. At the end of the five-year period, the forecasts will each be scored and a winner declared. A direction for future work would be to drive a BASS simulation with a Virtual California simulation. This would combine the fault based and seismicity based approaches.

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