BASS, an alternative to ETAS

D. L. Turcotte,[1] J. R. Holliday,[2,3] and J. B. Rundle[2,3]

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[1] The epidemic type aftershock sequence (ETAS) model has been widely used to model the statistics of seismicity. An essential feature of the ETAS model is the concept of generations of aftershocks. A mainshock has primary aftershocks, the primary aftershocks have secondary aftershocks, and so forth. In this paper, we introduce the branching aftershock sequence (BASS) model as an alternative to ETAS. The BASS model is fully self-similar whereas the ETAS model is not. Furthermore, the ETAS model requires the specification of branching (parent-daughter) ratios, while the BASS model instead utilizes Båth's law. We also show that the branching statistics in the BASS model are identical to the self-similar Tokunaga statistics of drainage networks. **Citation:** Turcotte, D. L., J. R. Holliday, and J. B. Rundle (2007), BASS, an alternative to ETAS, *Geophys. Res. Lett.*, *34*, L12303, doi:10.1029/2007GL029696.

1. Introduction

[2] In this paper, we propose a branching aftershock sequence (BASS) model for seismicity. This model is based on four scaling relations: (1) Gutenberg-Richter (GR) frequency-magnitude scaling [*Gutenberg and Richter*, 1954], (2) the modified form of Båth's law that specifies the *a*-value in the GR scaling [*Shcherbakov and Turcotte*, 2004], (3) Omori's law for the temporal decay of aftershocks [*Shcherbakov et al.*, 2004], and (4) a spatial form of Omori's law for the spatial distribution of aftershocks [*Helmstetter and Sornette*, 2003a]. These scaling relations completely specify the distribution of aftershocks.

[3] We first illustrate the BASS formulation using a deterministic branching structure. A basic building block of our model is the concept of primary aftershocks, second-order aftershocks which are the aftershocks of the primary aftershocks, and higher-order aftershocks. We draw a direct analogy between steam order [*Horton*, 1945; *Strahler*, 1957; *Tokunaga*, 1978] and aftershock order. We show that our branching aftershock sequence satisfies the same self-similar scaling as river networks [*Tokunaga*, 1978].

[4] There are strong similarities, but also important differences, between the BASS formulation given here and the ETAS formulation [*Ogata*, 1988, 2004; *Helmstetter and Sornette*, 2002a, 2002b, 2003a, 2003b, 2003c]. Both formulations utilize the concept of primary, second-order, and higher-order aftershocks. Both formulations utilize GR, temporal Omori, and spatial Omori scaling. The ETAS model also introduces a magnitude dependent branching (parent-daughter) ratio. We constrain this ratio with the modified form of Båth's law [*Shcherbakov et al.*, 2004]. This law directly relates the *a*-value of the GR scaling of aftershocks to the amplitude of the parent earthquake. Because of this replacement, BATH scaling is fully self-similar whereas ETAS scaling is not.

2. Illustration of the BASS Model

[5] We first illustrate the principals of the BASS model using a deterministic branching structure. This is in direct analogy to a deterministic illustration of a drainage network [*Turcotte and Newman*, 1996]. Our construction is illustrated in Figure 1. A sequence of primary aftershocks is illustrated in Figure 1a. A $m = 5$ mainshock has one $m = 4$ aftershock (Båth's law with $\Delta m = 1$), 2 $m = 3$ aftershocks, 4 $m = 2$ aftershocks, and 8 $m = 1$ aftershocks. This construction is extended to include 2nd and higher order aftershocks in Figure 1b. A first-order aftershock with magnitude 4 is denoted "45", a first-order aftershock with magnitude 3 is denoted "35", etc. The 2nd-order aftershocks of the primary aftershocks are denoted "34", "24", "14" for the $m = 4$ primary aftershock, the 2nd order aftershocks of the $m = 3$ primary aftershocks are denoted "23" and "13", and so forth. Thus we have

$$N_i = \sum_{i+1}^{n} N_{ij}, \tag{1}$$

where n is the magnitude of the mainshock (5 in this case), N_i is the total number of aftershocks of magnitude i, and N_{ij} is the number of aftershocks of magnitude i that have a magnitude j parent.

[6] The branch numbers N_{ij} constitute a square upper-triangular matrix. The matrix corresponding to the aftershock family given in Figure 1b is given in Figure 2a. It can be shown that this branching also satisfies Tokunaga side branching statistics. To show this, we introduce branching ratios T_{ij} defined by

$$T_{ij} = \frac{N_{ij}}{N_j}, \tag{2}$$

where T_{ij} is the number of aftershocks with magnitudes i for each parent with magnitude j. The branching ratios T_{ij} also constitute a square upper-triangular matrix. The matrix corresponding to the aftershock family given in Figure 1b is given in Figure 2b. Self-similar networks are defined to satisfy the condition $T_{i,j+k} = T_k$, where T_k is a branching ratio

[1]Geology Department, University of California, Davis, California, USA.
[2]Center for Computational Science and Engineering, University of California, Davis, California, USA.
[3]Department of Physics, University of California, Davis, California, USA.

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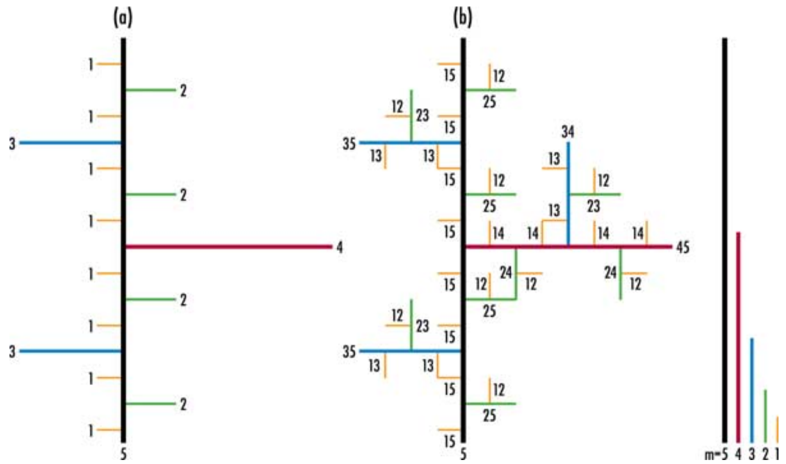

Figure 1. Illustration of our branching model using a discrete set. (a) The primary family of aftershocks is a generator for the fractal construction. (b) The full Tokunaga structure of side-branching aftershocks.

that depends on k but not on i. *Tokunaga* [1978] introduced a more restricted class of self-similar networks by requiring

$$T_k = a \cdot c^{k-1}. \tag{3}$$

For the aftershock family we have considered, we have $T_1 = 1$, $T_2 = 2$, $T_3 = 4$, $T_4 = 8$, $a = 1$ and $c = 2$. The construction given above is binary, but it can be extended to higher orders to give a range of b-values. It can also be modified to accommodate an arbitrary value of Δm.

3. Probabilistic BASS Model

[7] We now turn to a probabilistic version of the BASS model. We require that the frequency-magnitude distribution for each sequence of aftershocks satisfies a Gutenberg-Richter (GR) frequency-magnitude relation:

$$\log_{10}[N_d(\geq m_d)] = a_d - b_d m_d, \tag{4}$$

where m_d is the magnitude of a daughter earthquake, $N_d(\geq m_d)$ is the number of daughter earthquakes with magnitudes greater than or equal to m_d, and a_d and b_d are the a- and b-values of the distribution, respectively. We further require that a modified form of Båth's law [*Shcherbakov and Turcotte*, 2004] is applicable for each generation of aftershocks. The magnitude of the largest aftershock inferred from the GR relation is a fixed value Δm^* less than the magnitude of the parent earthquake, m_p:

$$N_d\left(\geq \left(m_p - \Delta m^*\right)\right) = 1. \tag{5}$$

It must be emphasized that this Δm^* is not the magnitude difference between the main shock and the largest aftershock, $\Delta m = m_{ms} - m_{as\max}$. Substitution of equation (5) into equation (4) gives

$$a_d = b_d\left(m_p - \Delta m^*\right). \tag{6}$$

Using this result, the GR relation for aftershocks is given by

$$\log_{10}[N_d(\geq m_d)] = b_d\left(m_p - \Delta m^* - m_d\right). \tag{7}$$

[8] In order to terminate the sequence of aftershocks, it is necessary to specify a minimum magnitude earthquake $m_{\min}$ in the sequence. From equation (7), the total number of daughter earthquakes N_{dT} is given by

$$N_{dT} = N(\geq (m_{\min}) = 10^{b_d\left(m_p - \Delta m^* - m_{\min}\right)}. \tag{8}$$

From equations (7) and (8) we obtain the cumulative distribution function P_{Cm} for the magnitudes of the daughter earthquakes:

$$P_{Cm} = \frac{N_d(\geq m_d)}{N_{dT}} = 10^{-b_d(m_d - m_{\min})}. \tag{9}$$

For each daughter earthquake a random value $0 < P_{Cm} < 1$ is generated, and the magnitude of the earthquake is determined from equation (9).

[9] We require that the time delay t_d until each daughter earthquake after the parent earthquake satisfies a generalized form of Omori's law [*Shcherbakov et al.*, 2004]:

$$R(t_d) = \frac{\mathrm{d}N_d}{\mathrm{d}t} = \frac{1}{\tau\left(1 + \frac{t_d}{c}\right)^p}, \tag{10}$$

$$\begin{array}{ccccc}
N_{12}=9 & N_{13}=6 & N_{14}=4 & N_{15}=8 & N_1=27 \\
 & N_{23}=3 & N_{24}=2 & N_{25}=4 & N_2=9 \\
 & & N_{34}=1 & N_{35}=2 & N_3=3 \\
 & & & N_{45}=1 & N_4=1
\end{array}$$

(a)

$$\begin{array}{cccc}
T_{12}=1 & T_{13}=2 & T_{14}=4 & T_{15}=8 \\
 & T_{23}=1 & T_{24}=2 & T_{25}=4 \\
 & & T_{34}=1 & T_{35}=2 \\
 & & & T_{45}=1
\end{array}$$

(b)

Figure 2. Illustration of the branching properties of the idealized aftershock sequences illustrated in Figure 1b. (a) Branch number matrix. (b) Branching ratio matrix.

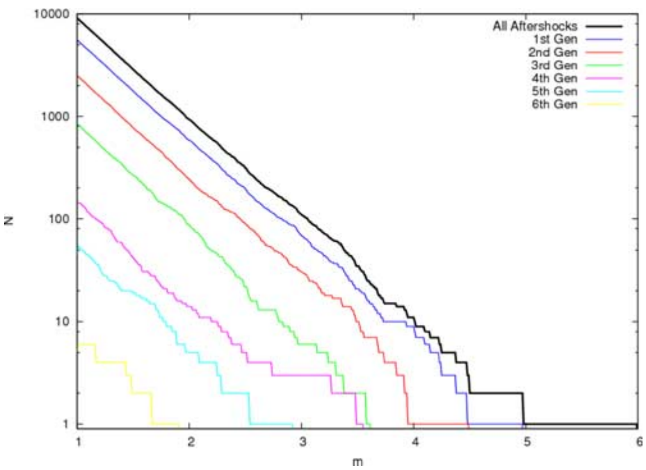

Figure 3. Cumulative number N of aftershocks with magnitudes greater than m. All aftershocks as well as various generations of aftershocks are shown.

where $R(t_d)$ is the rate of aftershock occurrence and τ, c, and p are parameters. The number of daughter aftershocks that occur after a time t_d is then given by

$$N_d(\geq t_d) = \int_{t_d}^{\infty} \frac{dN_d}{dt} dt' = \frac{c}{\tau(p-1)\left(1+\frac{t_d}{c}\right)^{p-1}}. \quad (11)$$

The total number of daughter earthquakes N_{dT} is obtained by setting $t_d = 0$ in equation (11) with the result

$$N_{dT} = \int_{0}^{\infty} \frac{dN_d}{dt} dt' = \frac{c}{\tau(p-1)}. \quad (12)$$

From equations (11) and (12) we obtain the cumulative distribution function P_{Ct} for the times of occurrence of the daughter earthquakes:

$$P_{Ct} = \frac{N_d(\geq t_d)}{N_{dT}} = \frac{1}{(1+t_d/c)^{p-1}}. \quad (13)$$

For each daughter earthquake a random value $0 < P_{Ct} < 1$ is generated, and the time of occurrence of the earthquake is determined from equation (13).

[10] We utilize a spatial form of Omori's law to specify the location of each daughter earthquake. This follows the scaling used in ETAS [*Helmstetter and Sornette*, 2003a] and is consistent with observations [*Felzer and Brodsky*, 2006]. The cumulative distribution function P_{Cr} for the radial distance r_d of each daughter earthquake from the parent earthquake is given by

$$P_{Cr} = \frac{N_d(\geq r_d)}{N_{dT}} = \frac{1}{(1+r_d/(d \cdot 10^{0.5m_p}))^{q-1}}. \quad (14)$$

The dependence on the magnitude m_p of the parent earthquake introduces a mean radial position of aftershocks that scales with the rupture length of the parent earthquake. The direction of each daughter earthquake is chosen randomly within the uniform range $0 < \theta_d < 2\pi$. This isotropic distribution was also used in ETAS. An elliptical distribution about the rupture plane could easily be obtained using a different azimuthal dependence.

[11] In order to constrain a BASS simulation it is necessary to specify six parameters: b_d, Δm, c, p, d, and q. For the GR-Båth parameters, we take $b = 1$ and $\Delta m = 1.25$ [*Shcherbakov and Turcotte*, 2004]. For the temporal Omori parameters, we take $c = 0.1$ days and $p = 1.25$ [*Shcherbakov et al.*, 2004]. For the spatial Omori parameters, we take $d = 4.0$ meters and $q = 1.35$ [*Felzer and Brodsky*, 2006]. For our sample simulation, we consider a $m_s = 6$ initial (seed) earthquake and take $m_{\min} = 1.0$. We first determine the magnitude, time of occurrence, and location of the primary aftershocks. For our parameter values, we find from equation (8) that there are $N_{dT} = 5{,}623$ primary aftershocks. We generate three random numbers in the range 0 to 1 for each primary aftershock and determine its magnitude, time of occurrence, and radial position from equations (9), (13), and (14).

[12] As a specific example, consider the random numbers 0.28208, 0.65993, and 0.48812. From equations (9), (13), and (14) we have $m_d = 1.55$, $t_d = 0.427$ days, and $r_d = 27$ km. If the random number P_{Cm} is in the range $0 < P_{Cm} < 10^{-5}$, the primary aftershock will be bigger than the seed earthquake. Since there are 5,623 primary aftershocks, there is a 5.6% chance that the seed earthquake will be a foreshock and a primary aftershock will become the mainshock. A random number in the range 0 to 2π gives the direction of each aftershock relative to the seed earthquake. Each of the $N_{dT}(\geq m_{\min})$ primary aftershocks is then treated as a parent earthquake and the entire procedure is repeated for each parent to generate an ensemble of second-order aftershock sequences. The procedure is further extended to third- and higher-order aftershock sequences until no more aftershocks are generated.

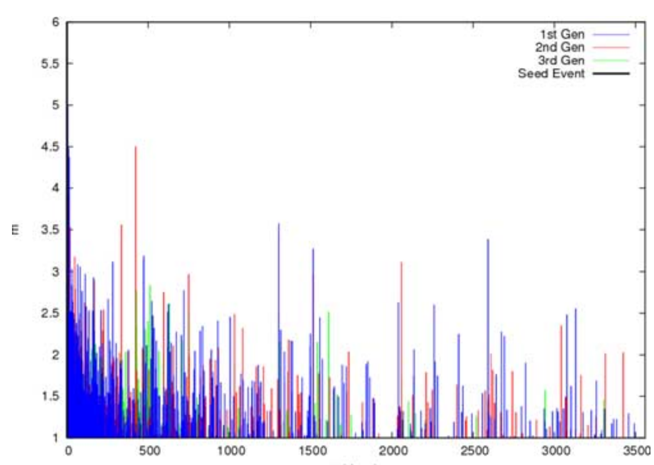

Figure 4. Magnitudes m of aftershocks as a function of the time at which they occur.

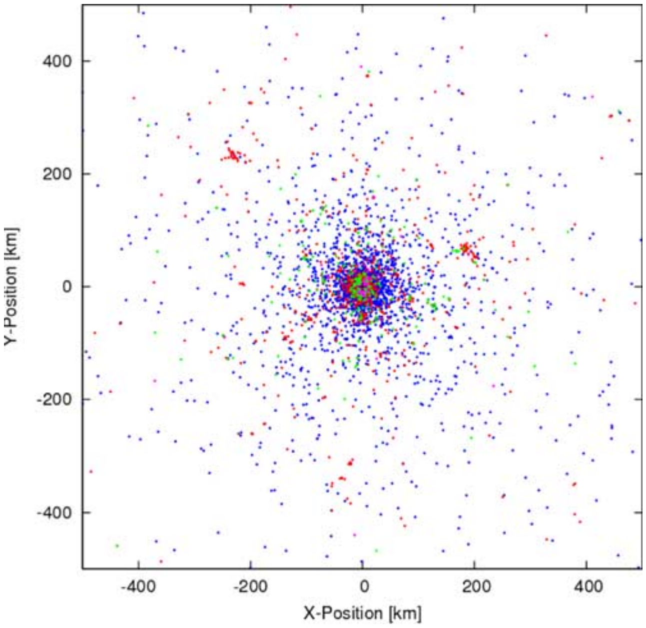

Figure 5. Positions of aftershocks relative to the position $X = Y = 0$ of the initial mainshock.

[13] The cumulative frequency-magnitude statistics for an aftershock sequence generated by a $m_s = 6$ seed earthquake are given in Figure 3. The results for each generation of aftershocks are given as well as the distribution for all aftershocks. There are 9,221 total aftershocks in the simulation, thus there are 3,598 second- and higher-order aftershocks. The b-value for all aftershocks is 0.96, $\Delta m^* = 0.91$ for all aftershocks, and the largest aftershock has a magnitude $m = 4.98$, yielding a value $\Delta m = 1.02$. The value of Δm will be different for each realization of the model. In some realizations the largest aftershock will be larger than the parent earthquake. In this case the parent earthquake is a foreshock and the largest aftershock is the main shock.

[14] Magnitude as a function of event occurrence time for the $m = 6.0$ seed sequence is presented in Figure 4. From this plot it's easy to see that large aftershocks generate their own aftershock sequences using this model. A similar plot of aftershock positions as a function of time is given in Figure 5. It is seen that the aftershocks for each generation are clustered about their respective parent earthquakes.

4. Conclusions

[15] In this paper, we have introduced a new approach to the quantification of aftershock sequences. Our BASS model is fully self-similar. It can be extended to an arbitrary range of earthquake magnitudes without changing the structure of the solution. The result is a self-similar branching structure that satisfies the same *Tokunaga* [1978] statistics as drainage networks [*Pelletier*, 1999]. This self-similar structure is also applicable to diffusion limited aggregation (DLA) [*Ossadnik*, 1992] and to models that exhibit self-organized criticality [*Gabrielov et al.*, 1999].

[16] We have presented a single BASS simulation in this paper. Clearly, many simulations must be made to better understand the implications of our new model. These implications include: (1) the statistics of foreshock occurrence in terms of time, space, and magnitude; (2) whether the c-values for total sequences have a magnitude dependence; (3) the relationship of aftershock occurrence to background seismicity; and (4) the correlation statistics of all earthquakes, both in space and in time.

[17] There are certainly strong similarities between BASS and ETAS simulations, but there are also significant differences. The BASS model contains six parameters whereas the ETAS model contains seven. Five of the parameters–b_d, c, p, d, and q–are basically the same. In the BASS model the sixth parameter is the Δm^* associated with the application of the modified form of Båth's law. In the ETAS model two parameter, K and α, are introduced to define the branching ratio [*Helmstetter and Sornette*, 2003a]. The additional parameter leads to a breakdown in self-similarity. In an ETAS simulation Δm^* is not constant; it is a function of m_p. It is not necessary to specify a branching ratio in the BASS model. The aftershock sequence is fully self-similar to an arbitrarily specified smallest aftershock. This specification does not influence the distribution of larger aftershocks.

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J. R. Holliday and J. B. Rundle, Center for Computational Science and Engineering, University of California, Davis, One Shields Avenue, Davis, CA 95616-8677, USA. (holliday@cse.ucdavis.edu; jbrundle@ucdavis.edu)

D. L. Turcotte, Geology Department, University of California, Davis, One Shields Avenue, Davis, CA 95616, USA. (turcotte@geology.ucdavis.edu)