Interface Depinning, Self-Organized Criticality, and the Barkhausen Effect

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We report measurements of magnetic avalanches in a Fe-Ni-Co alloy. The distribution of avalanche sizes is given by $P(A) \propto A^{-\alpha} \exp(-A/A_0)$, with $\alpha = 1.33 \pm 0.10$. The role of demagnetizing effects in the avalanche statistics is demonstrated. We analyze a simple model of interface growth and find a power law distribution of avalanche sizes over a wide range of parameters when an infinite-range demagnetization field is included. The exponents agree with those found at the critical field in the absence of demagnetization effects.

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The dynamics of interfaces in disordered systems describe the behavior of a wide variety of physical systems, including domain wall motion in ferromagnets, fluid invasion in porous media, critical currents in superconductors, and many examples of nonequilibrium growth. Models of these phenomena usually include an interface elastic energy, random quenched disorder, and an external driving force [1-9]. The effects of thermal fluctuations can often be neglected when describing the long length scale physics. At low values of the applied force, the interface is pinned by the impurities. As the driving force is increased, there is a critical value above which the interface begins to move. The value of the critical force depends on the degree of disorder in the system, but the dynamics near the depinning threshold exhibit scaling behavior with universal exponents [8,9].

Experiments on ferromagnets, by contrast, show that dynamic activity with a broad distribution of scales is found generically without any careful tuning of parameters [10]. The primary method of study is a statistical analysis of the Barkhausen effect, the magnetic noise generated as a domain wall jumps erratically from one pinned configuration to the next. It has been suggested that this is an example of "self-organized criticality" (SOC) [11]. Given that interface dynamics in a random medium is well understood theoretically and only exhibits critical behavior at a particular value of the driving force, the experimental results on ferromagnets seem perplexing.

Several explanations have been suggested. The simplest, and the one with the most direct experimental support, is that there is no self-organization involved. If the domain wall is effectively a rigid interface, any scaling observed in the signal is a consequence of correlations in the pinning potential [12]. A recent investigation of the statistics of Barkhausen noise found no evidence for dynamic self-organization [13]. Alternatively, the critical region for the interface depinning transition is quite large. As a consequence, significant power law scaling may be observed even for fields well below the depinning field. Similar behavior is observed in the random-field Ising model (RFIM), where there is both a critical field and a critical level of disorder [14].

The purpose of this Letter is to demonstrate that the experimentally observed Barkhausen signal is strongly affected by the presence of long-range demagnetization fields, and that the inclusion of this interaction into a simple model of interface depinning dramatically changes the behavior of the model. In particular, the addition of an "infinite-range" demagnetization field results in self-organized criticality, while a local demagnetization field does not.

In magnetic systems the force locally applied to a domain wall includes not only the externally applied field, but the magnetic fields from the rest of the sample as well. Since the susceptibility of most ferromagnets is much greater than unity, the actual internal field is much smaller than the applied field (i.e., the "demagnetizing field" is opposite and almost equal to the applied field) except in special geometries, such as a torus or a long thin rod. After an increasing applied field depins an interface, the magnetic field from the spins behind the advancing domain wall decreases the net field at the interface. The demagnetizing field grows as the interface advances, until the total field is weak enough for the interface to once again become pinned. In this fashion the long-range demagnetizing fields, combined with the interface dynamics, maintain the total internal field near the critical depinning value. It has been known for some time that demagnetization fields have a clear effect on the observed power spectra of Barkhausen noise [15]. Recently it was shown that by including demagnetizing fields in a Langevin model of domain wall motion, the scaling of the power spectra with permeability and the rate of change of the applied field could be accurately described [12]. The implication for models of interface depinning has not, to our knowledge, been explored.

We have found that the effect of the long-range magnetostatic field can be observed directly in the autocorrelation of the avalanche distribution sequence, where a significant negative correlation is observed at short times. This peak repulsion is a consequence of the demagnetizing fields: After a section of a domain wall moves in response to an increase in the applied field, the effective field on the domain walls is reduced. The probability of

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a subsequent event is therefore reduced until the applied field is increased enough to bring the total field back to the critical value.

The importance of controlled experiments, in particular the need to restrict data to a limited field range where the macroscopic parameters can be considered approximately constant, was emphasized by Alessandro *et al.,* where the behavior of the Barkhausen signal at finite driving rates was investigated [12,16]. We are interested, at least initially, in studying the behavior of the Barkhausen signal in the low-frequency limit, where the time scale for domain wall motion is much smaller than the time between jumps. Samples of Perminvar were fabricated by melting a mixture of 30% Fe, 45% Ni, and 25% Co in an arc furnace [17]. A small piece was cut from the resulting alloy and rolled into a thin foil (100 μm), which was then cut into ellipses approximately 2 mm $\times$ 5 mm. The samples were annealed at 1000 °C for 1 h and at 450 °C for 24 h. Perminvar prepared in this fashion has a lower susceptibility than most soft magnetic materials (about 250) as a result of strong pinning of the domain walls during the annealing process [18]. The advantage of this for our study is that the Barkhausen events are well separated for moderately slow changes in the applied field (Fig. 1).

The Barkhausen signal was acquired from a 50 turn coil wrapped around the sample. The sample and coil were placed in a solenoid inside a magnetically shielded enclosure. The amplified voltage was recorded by a high speed (1 MHz, 12 bit) data acquisition board with 1 Megabyte of on-board memory. The high speed is necessitated by the short duration of the avalanches (see Fig. 1). In order to minimize the amount of data stored, peak detection was performed in software, and only data surrounding pulses above some threshold were written to disk. For the data in this paper, the threshold was about 0.5% of the maximum peak height.

We have acquired 66 000 avalanches from a small field increment over 440 cycles. (The total field cycle was a triangle wave from -5 to 5 Oe with a period of 75 sec. Data were collected from 4 to 4.5 Oe. The statistics were found to be independent of driving rate at these frequencies.) It has been observed that the distribution of voltages in the Barkhausen signal is given by a power law with an exponential cutoff [16]. A more natural quantity to compare with our model of intermittent dynamics, however, is the area A under the peaks, since this corresponds directly to the total number of magnetic moments that reverse direction during the events. The probability distribution function (PDF) for avalanche size is shown in Fig. 2. The solid line is a fit by $P(A) \propto A^{-\alpha} \exp(-A/A_0)$, with $\alpha = 1.33$ and $A_0 = 1.4 \times 10^4$. The exponent is somewhat lower than those reported in Ref. [10] (1.7–1.9), but the uncertainty in that result was quite large, and the inability to include the exponential cutoff due to limited statistics undoubtedly biased the exponent toward larger values. Ji and Robbins [8] investigated the behavior of critical interface depinning in the RFIM. They found that for moderate disorder the interface is self-affine, and at the critical field the distribution of incremental growths is described by a power law with exponent 1.28 ± 0.05, in agreement with our experimental results. Higher levels of disorder produce a fractal interface and a larger exponent. Images of domain walls in materials similar to ours indicate that the interfaces are not self-similar [19]. The essential difference between the RFIM and the model considered here is that the RFIM allows for overhangs and inclusions in the advancing interface.

The effect of the demagnetizing fields on the Barkhausen signal $B(t)$ can be seen most directly from the autocorrelation function $S(\tau) = \int dt\, B(t)B(t + \tau)$ [20]. The result is shown in the inset of Fig. 2, and clearly exhibits a dip at the origin. This represents a significant tendency for peaks to repel each other. The time scale of the correlation is about 200 msec, which is much larger than the average spacing between peaks (13 msec). The existence of these correlations in the avalanche sequence has been observed as a peak in the power spectrum at low frequencies [15]. Alessandro *et al.* showed that the

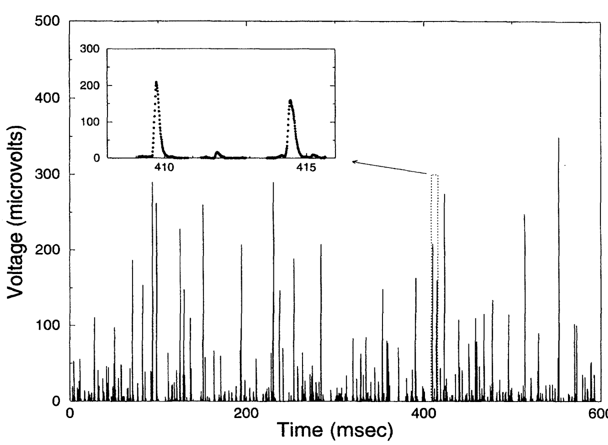

FIG. 1. Experimental Barkhausen signal (voltage produced from a pickup coil around a ferromagnet subjected to a slowly varying applied field).

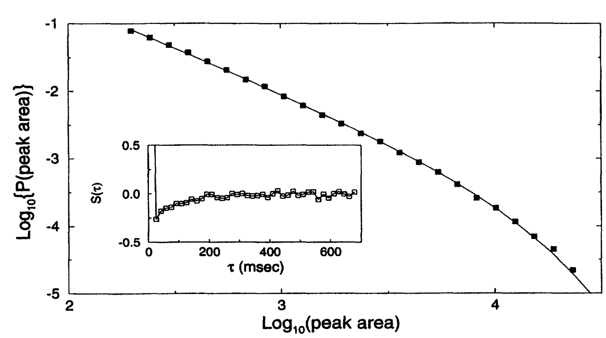

FIG. 2. Experimental avalanche size probability distribution function. The solid line is a fit to $P(A) \propto A^{-\alpha} \exp(-A/A_0)$, with $\alpha = 1.33$. Inset: normalized autocorrelation function for the Barkhausen signal.

peak could be accounted for by including a magnetostatic field in the manner described above [12]. The connection between the avalanche statistics and the demagnetization effect in our data can be easily demonstrated. According to the argument above, the average value of the internal field, $H - \eta M$, remains constant, so the average rate of magnetization change, $\langle dM/dH \rangle$, is equal to $1/\eta$. After an avalanche of size δM, the avalanche probability will be reduced until the applied field is increased an amount $\delta H \approx \eta(\delta M) \approx \delta M / \langle dM/dH \rangle$. The longest correlations in the avalanche sequence, which are a consequence of the largest events observed, should be given by $\delta t \approx A_0/[\langle dM/dH \rangle^* (dH/dt)]$, where A_0 is the cutoff of the avalanche size distribution. For the data used to generate plots in Fig. 2, this quantity is 150 msec, in good agreement with the observed dip in the autocorrelation function. We have also tested this reasoning for smaller events using conditional probability distributions with similar results. Note that this result is inconsistent with demagnetization effects whose range is much less than the size of the advancing interface.

In order to demonstrate the implications of the long-range magnetostatic fields on motion of domain walls, we have investigated a simple model of domain wall motion. The dynamical model that we have considered describes an interface by its height $h(\mathbf{x}, t)$, where $\mathbf{x}$ is a $(d-1)$-dimensional vector and includes only forces along the growth direction. We assume that $h(\mathbf{x}, t)$ is single valued so that there are no overhangs on the interface. Each element of the interface is coupled elastically to its nearest neighbors (with periodic boundary conditions) and experiences a random on-site force that is time independent. In two dimensions, the force on each element of the string is given by

$$f_i = u(h_i, i) + k(h_{i+1} + h_{i-1} - 2h_i) + H_i \,.$$

$u(h_i, i)$ is chosen randomly for each site, with a Gaussian distribution of mean zero. H_i is the driving force resulting from the applied field. Choosing H_i equal to the applied field H (AF model) yields critical depinning behavior described in the opening paragraph. We have modeled the internal field (IF) in the presence of demagnetizing effects with a global restoring force, $H_i = H - \eta M$, where $M = (1/L)\sum_{i=1}^{L} h_i$, for a lattice of width L. We have also considered a local restoring force, $H_i = H - h_i$ (local IF model). The profound dynamical effect of the demagnetization term can be understood by considering the limit of no disorder: In the AF model the interface proceeds forever, while in the IF model, there exists an equilibrium domain wall position where the applied force on the interface is zero, as is observed in very clean ferromagnets. While in the AF model, M diverges at the critical field; a plot of H vs M for the IF model is linear after an initial transient period. In this regime the average properties of the interface, such as the width and the jump size distribution, are field independent.

The simulation proceeds as follows: The wall is initially straight, with zero applied field. Wall elements with a positive net force are allowed to move forward one step, with a new random force assigned to the element with each step, until the force on each element is negative. At this point, the field is increased until the most weakly bound element goes positive. That element is moved forward one site. If the motion of that element changes the sign of the force on the neighbors, they are advanced as well. The process continues, moving forward each element with a positive force, until the force is again negative everywhere. The magnetization is updated after every individual step. The total number of sites passed between metastable configurations is defined to be the avalanche size.

We expect this model to apply when the incremental growths induced by the magnetic field are large compared to the spacing between impurities but small compared to the characteristic domain sizes. (It does not apply, for example, to very clean magnetic bubble materials, where avalanches occur as the result of topological rearrangements of the domain configuration [21]. It has been shown that this system is self-organized into a subcritical state [22].)

In order to understand the effect of the demagnetizing field, it is instructive to review the behavior of the model without it. In this case, as the applied field is increased, the probability of a given configuration being strong enough to pin the interface drops, resulting in larger and larger displacements. The resulting magnetization diverges at a critical field. Just below the critical field, we find that the distribution of avalanche sizes is described by a power law with no detectable cutoff (the simulation is unbounded in the growth direction). We have simulated a lattice width of 250, $k = 100$, $\eta = 1$, and a distribution of random forces with a standard deviation of 1000. These numbers are chosen so that $H - \eta M \gg 1$, and so that the interface will be smooth on length scales of the lattice size. After an initial transient period, the growth of the domain is linear in field with a wide range of event sizes, including those that span the length of the interface. For the growth within 4% of the critical field, we find $P(A) \propto A^{-\alpha}$ over 5 orders of magnitude, with $\alpha = 1.00 \pm 0.01$. A simulation of a two-dimensional interface of width 40×40 yields an exponent of 1.17 ± 0.05.

We have investigated the behavior of the IF model using the same parameters. The PDF of avalanche sizes for a one-dimensional interface of width 500 shown in Fig. 3 does exhibit power law scaling, cut off only by finite size effects. This has been confirmed with lattice widths up to 5000. The slope of the logarithmic plot of the avalanche distribution is 1.00 ± 0.01. We have varied the value of k from 20 to 1000 with no detectable change in slope. A simulation of the IF model for a two-dimensional interface of width 40×40 yields an exponent of 1.13 ± 0.02. Within the statistical uncertainty, the IF model produces the same exponents for one- and two-dimensional interfaces as the AF model when the latter is tuned to criticality. Also, we find that

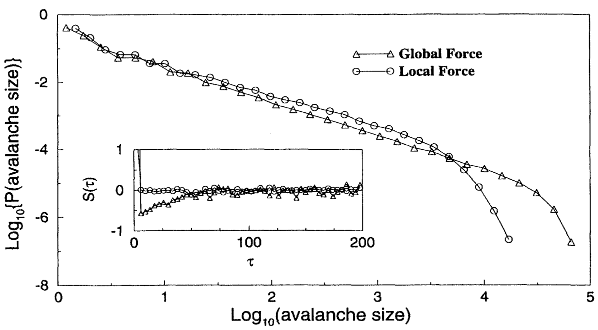

FIG. 3. Avalanche size probability distribution functions for the local and global IF models. Inset: normalized autocorrelation function for each model. The units of the horizontal axis are increments of the applied field.

the net internal field, $H - \eta M$, fluctuates around a value slightly below the value of H_c determined from the AF model with the same parameters.

Given that the AF model exhibits critical behavior, it is clear that a global demagnetizing field will cause an infinite system to sit at the critical point, and the fluctuations in the internal field observed in our simulations will diminish as the system size is increased. An analogous behavior is predicted for ^{4}He near the superfluid transition temperature if a finite heat current is applied [23]. In this case, the long-range interaction is provided by the divergent thermal conductivity of the superfluid, which acts to keep the system at the critical state without tuning of the temperature.

The behavior of the local IF model is qualitatively different. The PDF for the avalanche sizes is shown in Fig. 3, using the parameter values above and a lattice width of 500, including about 9000 avalanches. For the local model, the distribution follows a power law only approximately with a slope slightly less than 1. Increasing the lattice width does not improve the quality of the scaling. The autocorrelation for the avalanche sequence produced by the simulations is shown in the inset of Fig. 3. The local model shows no detectable correlation. This is a somewhat surprising result, since avalanches do release tension along some portion of the interface. The autocorrelation function for the global force model, by contrast, is qualitatively similar to the experimental result. This is a graphic demonstration of the importance of the long-range interactions. Similarly, elastic forces in mechanical systems and pressure forces in fluid problems are typically long range. Our results suggest that purely local forces cannot adequately account for the experimental results.

Our results clearly demonstrate the importance of long-range magnetic interactions, both in the experimental measurements of the Barkhausen effect and in simulations of interface motion. The question of whether Barkhausen noise is a consequence of self-organized criticality, "plain old criticality," or correlated disorder without any cooperative effects is unlikely to have a unique answer for all ferromagnets. Detailed statistical analyses similar to Ref. [13] are necessary, with different materials parameters and different sample geometries. (The annular geometry employed in that study eliminates the demagnetization effect.) A very rigid domain wall, with or without demagnetization effects, will be well described by the model of Alessandro *et al.* With increasing disorder or decreasing elastic energy, the interface will become spatially complex [8], and the long-range interactions will play a critical role. Finally, for small magnetic domains or large changes in magnetization, models with more complex topology are required [14,22].

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