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Forest fire burn areas in Western Canada modeled as self-similar criticality

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Abstract

Forest fire burn areas in the western Canadian provinces of Alberta and British Columbia have cumulative frequency-area distributions that are well described by a power law or an upper-truncated power law. The power law scaling extends over as many as five orders of magnitude and is observed for different geographical regions and for time intervals ranging from 1 to 40 years. The observed scaling exponent varies both geographically within and between provinces and temporally between annual records. The temporal variability decreases at the decadal scale, suggesting that decadal distributions may be useful for long term fire control planning within a geographical region. For example, for all of Alberta, based on the scaling parameters that describe the 1961–2000 record, we expect approximately 24 fires per year of 100 ha or larger. Unlike the original self-organized criticality (SOC) forest fire model that produces a single scaling exponent, the self-similar criticality (SSC) model replicates the range of scaling exponents observed for cumulative frequency-area distributions of natural forest fires.

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Keywords: Wildfire; Self-organized criticality; Self-similar criticality; Power laws; Scaling

1. Introduction

Frequency-area distributions of forest fires in various parts of the world have power law scaling over several orders of magnitude (e.g. [1–3]). The first objective of this work is to determine the scaling relationships that describe forest fire cumulative frequency-area distributions for different geographic regions and for various time intervals in western Canada. The second objective is to identify or develop a model that replicates the observed scaling relationships and thereby gain insight into the natural processes that cause forest fires.

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The forest fire self-organized criticality (SOC) cellular automata model [4–6] has been used to model forest fire frequency-area distributions [1–3]. The SOC model was introduced by Bak et al. [7] and later developed into a forest fire model [4]. The original forest fire model of Bak et al. [4] is referred to in this paper as the traditional SOC forest fire model. It has been shown that the SOC forest fire models are not critical when the probability of tree growth is small and the grids are large [8–10]. This is not necessarily a problem when modeling natural forest fires, since one can consider model parameters that are within the range that produces power law scaling.

The forest fire model represents the forest as a two-dimensional lattice. In the traditional SOC model [4], trees are added to randomly selected grid cells in successive steps. A match is periodically dropped in a randomly selected grid cell and, if a tree is present, that tree and all trees in non-diagonal adjacent cells burn and are removed from the grid. The traditional SOC model [4] generates forest fire non-cumulative frequency-area distributions with a power law scaling exponent equal to one.

Variations of the forest fire SOC model have been proposed. Drossel and Schwabl [11] and Albano [12,13] introduced tree immunity, where a probability is set for whether a tree will burn if an adjacent tree is burning. Sinha-Ray et al. [14] replaced periodic ignition with an auto-ignition threshold based on the age of trees. Hergarten [15] explored many variations in the forest fire model such as including random and preferred wind directions and allowing diagonal neighboring cells to burn.

2. Scaling relationships

2.1. Power law

Cumulative frequency-size distributions associated with many natural systems exhibit power law scaling. Examples include earthquakes (e.g. [16]), floods [17,18], landslide areas [19], hotspot seamount volumes [20,21], and forest fire areas [1–3]. A power law applied to a cumulative distribution has the form

$$\dot{N}(r) = Cr^{-\alpha} \tag{1}$$

where $\dot{N}(r)$ is the cumulative number of events per unit time with size greater than or equal to r, α is the scaling exponent, and C is the activity level, a constant equal to the number of events per unit time with size $r \geq 1$.

2.2. Tapered power law

The tapered power law is the product of a power law and an exponential function (e.g. [22–24]). The power law term dominates for smaller sizes and the exponential function causes a decrease from the power law at the larger event sizes. A tapered power law, $\dot{N}_{\mathrm{Ta}}(r)$, has the form

$$\dot{N}_{\mathrm{Ta}}(r) = Cr^{-\alpha} \exp\left(\frac{-r}{\theta}\right) \tag{2}$$

where $\dot{N}_{\mathrm{Ta}}(r)$ is the cumulative number of events per unit time with size greater than or equal to r, $Cr^{-\alpha}$ is the power law (Eq. (1)), $\exp(-r/\theta)$ is the exponential decrease from the power law, and θ is the event size where the cumulative number has fallen to 37% ($1/e$) of the power law value ($Cr^{-\alpha}$).

2.3. Upper-truncated power law

An upper-truncated power law [25] has been found to describe cumulative distributions associated with several natural systems including earthquake magnitudes [26], tsunami run-up heights [27], longshore erosion and accretions cells [28], hot spot seamount volumes [21], petroleum field sizes [29], and fault lengths and offsets [29]. An upper-truncated power law, $\dot{N}_T(r)$, has the form

$$\dot{N}_T(r) = C(r^{-\alpha} - r_T^{-\alpha}) \tag{3}$$

where $\dot{N}_T(r)$ is the cumulative number of events per unit time with size greater than or equal to r, α is the scaling exponent, and there are no events of size r_T or larger. Since each value in a cumulative distribution includes all larger events, upper truncation of the distribution decreases the cumulative number associated with each event size. In Eq. (3), the second term, $Cr_T^{-\alpha}$, represents this decrease, or "fall-off", from the power law, $Cr^{-\alpha}$.

2.4. Pareto functions

The Pareto, tapered Pareto, and truncated Pareto functions have been used to describe frequency-size distributions of forest fire burn areas (e.g. [24,30]). The various Pareto functions describe cumulative probability distributions, which yield the probability of occurrence of an event of a given size or smaller. The Pareto functions can be rewritten as survivor functions, which yield the probability of occurrence of an event of a given size or larger. The Pareto functions and related survivor functions are normalized by the number of events in the record, not by the time duration of the record. The power law, Eq. (1), tapered power law, Eq. (2), and upper-truncated power law, Eq. (3), are normalized by the time duration of the record, which yields the number of events occurring per unit time. To analyze the number of fires occurring per year of a given size and larger, we use Eqs. (1)–(3).

3. Data

We analyze forest fire burn areas cataloged for the Canadian provinces of Alberta from 1961 to 2002 and British Columbia from 1998 to 2002. The Alberta forest fire catalog, obtained from the website of the Forest Protection Division of the Government of Alberta, reports some fires as small as 0.01 ha. We analyze fires greater than one hectare for Alberta. The Protection Branch of the Ministry of Forests of British Columbia reports fires of five hectares and larger. We analyze fires of 5 ha and larger for British Columbia. The Alberta and British Columbia data are subdivided into fire regions and districts, which we use for regional analysis. For Alberta, fires designated "S" and "P" in the catalog all occur within the same southeast slopes region and we combine these for regional analysis.

4. Analysis

The cumulative frequency-area distributions for recorded forest fires in both Alberta and British Columbia exhibit strong power law scaling over as many as five orders of magnitude (Fig. 1). For the largest fires there is a "fall-off" from the power law. The distributions, including the "fall-off" from the power law, are well-described by the tapered power law (Eq. (2)) and the upper-truncated power law (Eq. (3)).

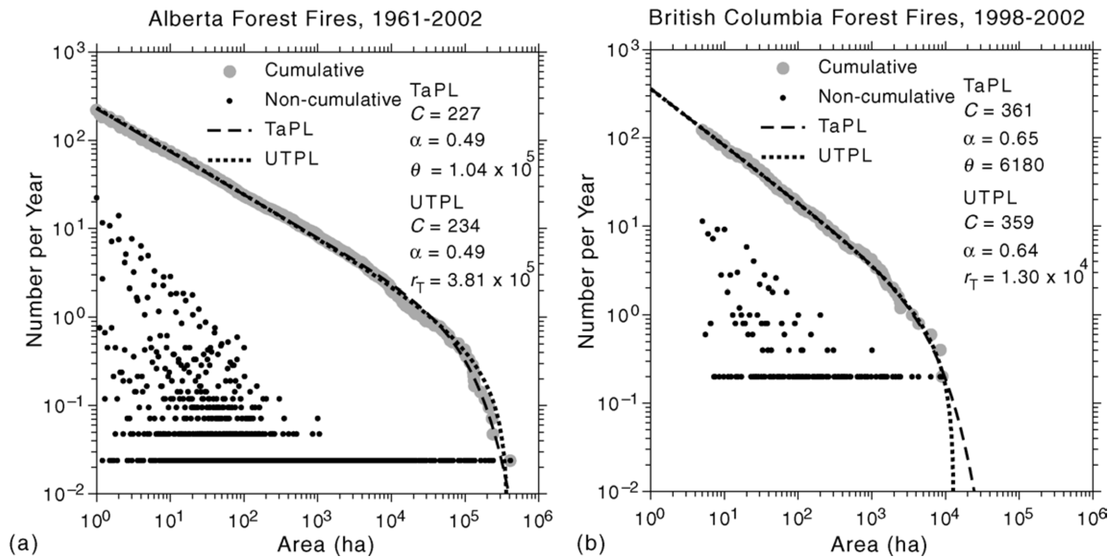

Fig. 1. Cumulative (gray circles) and non-cumulative (black dots) frequency-area distributions for recorded forest fires in the Canadian provinces of (a) Alberta and (b) British Columbia plotted at the same scale. The vertical axis is number of fires per year and the horizontal axis is fire area in hectares (ha). The distributions are well-described by both the tapered power law (TaPL, dashed line) and the upper-truncated power law (UTPL, dotted line).

Applying Eqs. (2) and (3) to forest fire data, $\dot{N}_{\mathrm{Ta}}(r)$ and $\dot{N}_T(r)$ are the observed cumulative frequency (number of fires per year) and r is forest fire area. The parameters for each function are found by using the Levenberg-Marquardt algorithm to minimize chi-squared [31]. The tapered power law and upper-truncated power law yield nearly identical values for the scaling exponent and activity level of each distribution (Fig. 1).

Differences between the tapered power law and upper-truncated power law only become evident at fire sizes larger than those recorded in the time intervals analyzed (Fig. 1b). For the tapered power law, there is no upper limit to the largest fire size as the function continues to decrease exponentially. The upper-truncated power law has an upper limit, as does the data set being analyzed. The value of r_T is determined from the observed "fall-off" in the data and is the event size where the upper-truncated power law that best fits that data equals zero. The term r_T should not be misinterpreted as the maximum fire size that can possibly occur.

A tapered power law (Eq. (2)) requires both a power law and an exponential function to describe the forest fire burn area distributions. An upper-truncated power law (Eq. (3)) describes the observed distribution without introducing an exponential term and is physically reasonable. Upper-truncation may be controlled by the finite size of a study area or by temporal limitations of the collected data [29]. For forest fires, maximum possible burn area is limited by the size of the contiguous forest. Also, the limited duration of the analyzed record may not include the largest fires that can occur in these regions. Due to the physical and temporal limitations of the forest fire data, we use the upper-truncated power law for the rest of our analysis.

4.1. Temporal variability

Annual records of forest fires in both Alberta and British Columbia can be analyzed for temporal variability. For Alberta, we compare the total distribution from 1998 to 2002 (Fig. 2a) to the distributions for each individual year (Fig. 2b–f). All the distributions exhibit power law scaling over several orders of magnitude. The dotted lines in Fig. 2b–f represent the best fit of the upper-truncated power law to the combined record in Fig. 2a. In 2001 and 2002 there was no "fall-off" from a power law. The parameters of the best-fit functions vary from year to year. The annual distributions fall above the dotted line in more active years, such as 1998 (Fig. 2b), and below the dotted line in less active years, such as 2000 and 2001 (Fig. 2d–e). There is also an annual variation in the largest recorded fires. The largest fire in 2002 was two orders of magnitude larger than the largest fire that occurred in 2000. The scaling exponent, α, relates the number of small fires to the number of large fires. A larger scaling exponent indicates a larger ratio of small fires to large fires. The scaling exponent for Alberta varies temporally, with a minimum of 0.25 in 1999 to a maximum of 0.46 in 2000.

The forest fire record for British Columbia may be analyzed for the same years that we analyzed for Alberta. We compare the forest fire area distribution from 1998 to 2002 (Fig. 3a) to the distributions for each individual year (Fig. 3b–f). As seen for Alberta, all the distributions exhibit power law scaling over several orders of magnitude and there are annual variations in the scaling parameters (Fig. 3).

Comparison of the annual distributions for Alberta and British Columbia reveals several years in which the two provinces experienced similar deviations from the 5-year record (Figs. 2 and 3). In 1998, both provinces had a more active fire year than the 5-year record with the distributions lying above the dotted line. In 1999 and 2001 the largest fires recorded in both provinces were roughly an order of magnitude smaller than the largest fires in each 5-year record. For both provinces, the distributions exhibit no "fall-off" from a power law in 2001 and 2002. While there are many similarities between the two provinces, there are also differences. In 2000, Alberta experienced its least active fire year (Fig. 2d) while fire activity in British Columbia was consistent with the 5-year record (Fig. 3d).

Alberta has a continuous forest fire record extending back to 1961, so decadal variations can be analyzed. The cumulative frequency-area distribution for 1961–2000 (Fig. 4a) is compared to the distributions for each decade (Fig. 4b–e). All the decadal distributions are well-described by an upper-truncated power law. The scaling parameters that describe each decade are consistent with the 40-year record (Fig. 4), with far less variability than observed for the annual records (Figs. 2 and 3). The consistency in the observed distributions from one decade to the next in Alberta suggests that the scaling parameters are representative of the region and may be used for forecasting.

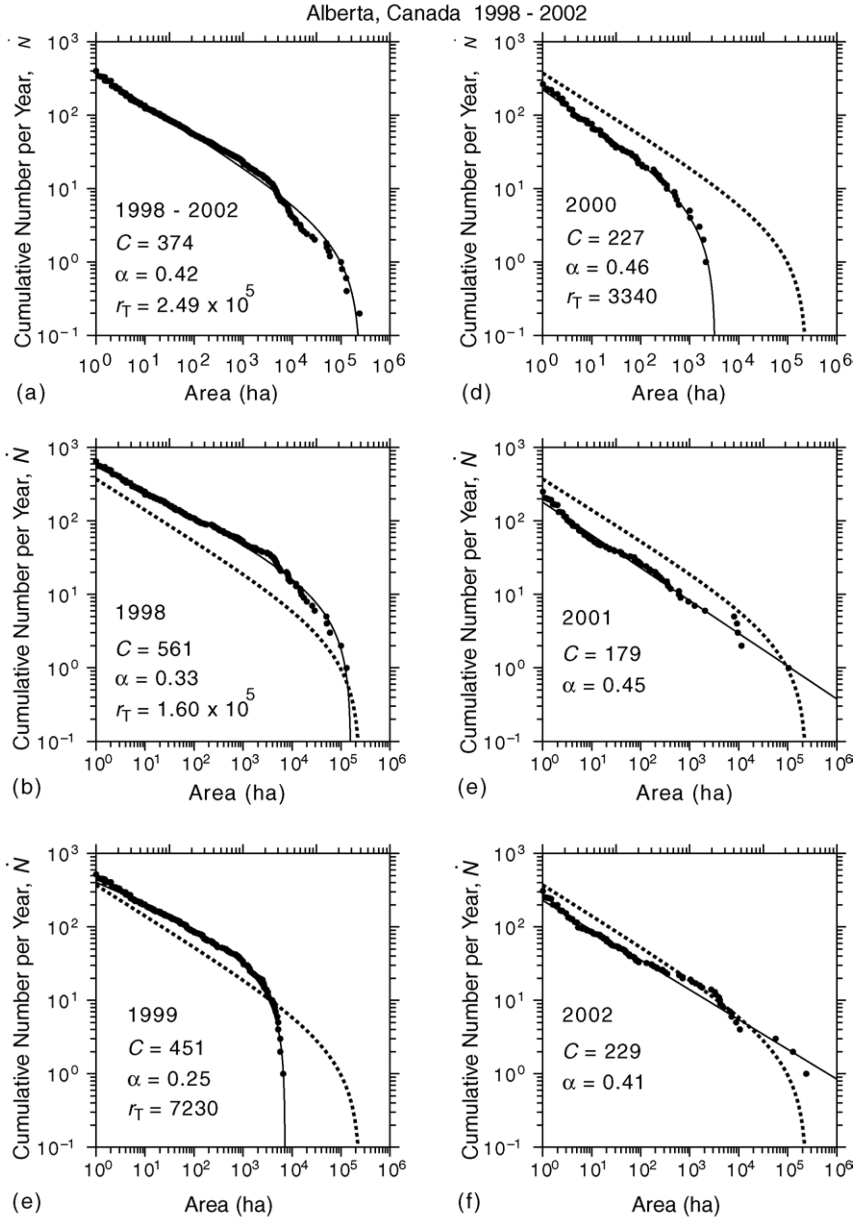

Fig. 2. Cumulative frequency-area distributions for forest fires in Alberta for the time interval (a) 1998–2002, and for the individual years (b) 1998, (c) 1999, (d) 2000, (e) 2001, and (f) 2002. Parameters are given for the power law or upper-truncated power law (thin solid line) that describes each distribution. The upper-truncated power law that describes the entire time interval (a) is shown as a dotted line in subsequent plots (b–f).

4.2. Regional variability

The Alberta and British Columbia forest fire catalogs are subdivided spatially into fire regions and centers. To examine regional variability, we analyze the cumulative frequency-area distributions for four Alberta regions (Fig. 5) and for five British Columbia fire centers (Fig. 6). All the cumulative frequency-area

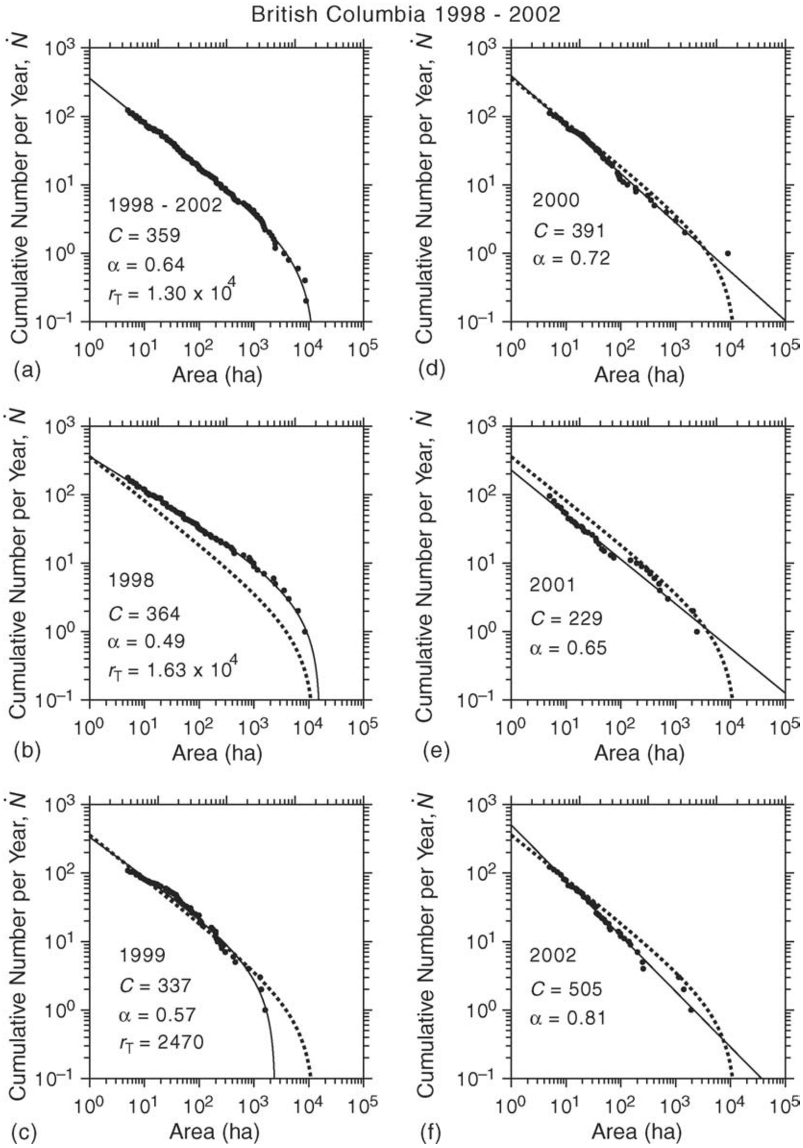

Fig. 3. Cumulative frequency-area distributions for forest fires in British Columbia for the time interval (a) 1998–2002, and for individual years (b) 1998, (c) 1999, (d) 2000, (e) 2001, and (f) 2002. See Fig. 2 for further details.

distributions are well-described by a power law or an upper-truncated power law (Figs. 5 and 6). There is considerable variability in fire activity level, scaling exponent, and maximum fire size between the regions within each province (Figs. 5 and 6). The observed variability is likely caused by differences in region sizes and local conditions. For example, the Coastal center in British Columbia has lower fire activity and the largest fires are an order of magnitude smaller than the largest fires reported at any other center. These lower values may be due to the relatively wet environment along the coast.

5. Modeling

Many natural phenomena that exhibit power-law frequency-size distributions are modeled as self-

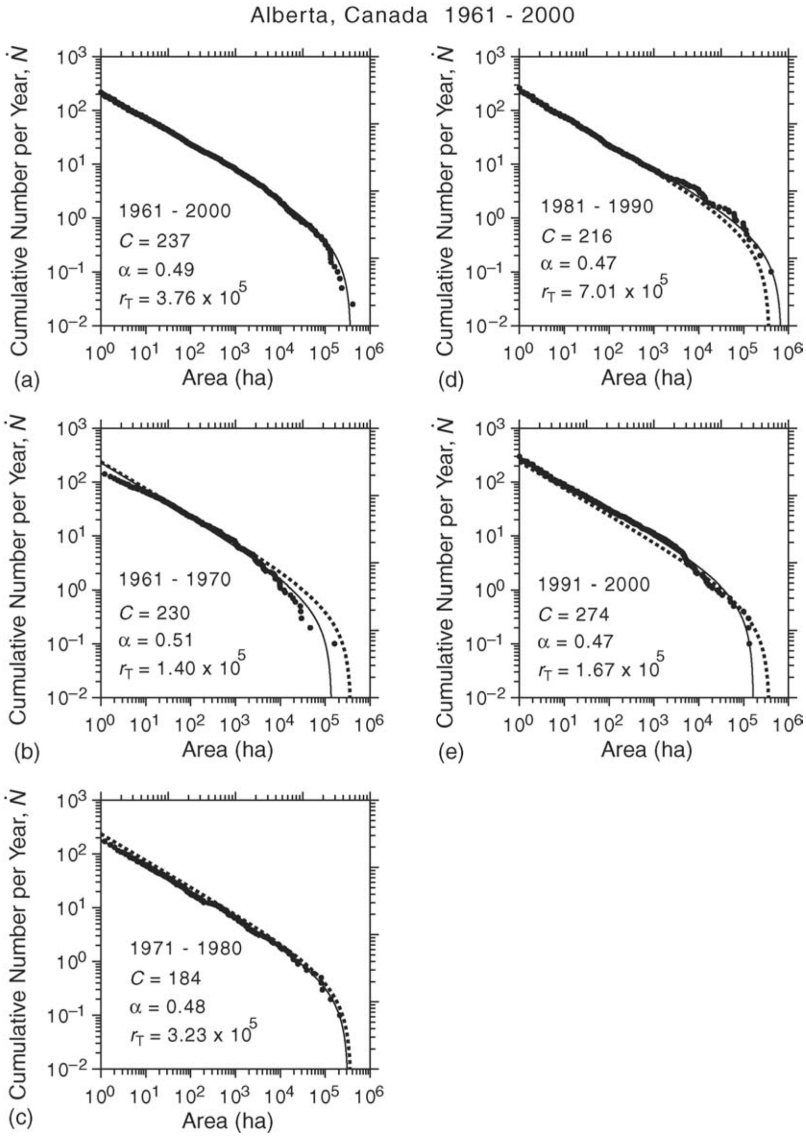

Fig. 4. Cumulative frequency-area distributions for forest fires in Alberta for the time interval (a) 1961–2000, and for the decades (b) 1961–1970, (c) 1971–1980, (d) 1981–1990, and (e) 1991–2000. Parameters are given for the upper-truncated power law (thin solid line) that describes each distribution. The upper-truncated power law that describes the entire time interval (a) is shown as a dotted line in subsequent plots (b–e).

organized critical (SOC) systems (e.g. [7]). Malamud and Turcotte [1] applied the forest fire SOC model to observed forest fire areas. The traditional forest fire SOC model [4] generates non-cumulative frequency-size distributions that follow a power law with a scaling exponent equal to one. This implies that a logarithmic function, rather than a power law, describes the cumulative distribution [1,29]. The scaling exponent of the traditional SOC model [4] is inconsistent with actual forest fire frequency-area distributions. For the Canadian provinces examined in this work, the scaling exponent, α, ranges from 0.15 to 0.92. Non-cumulative forest fire frequency-area distributions for regions in the United States and Australia fol-

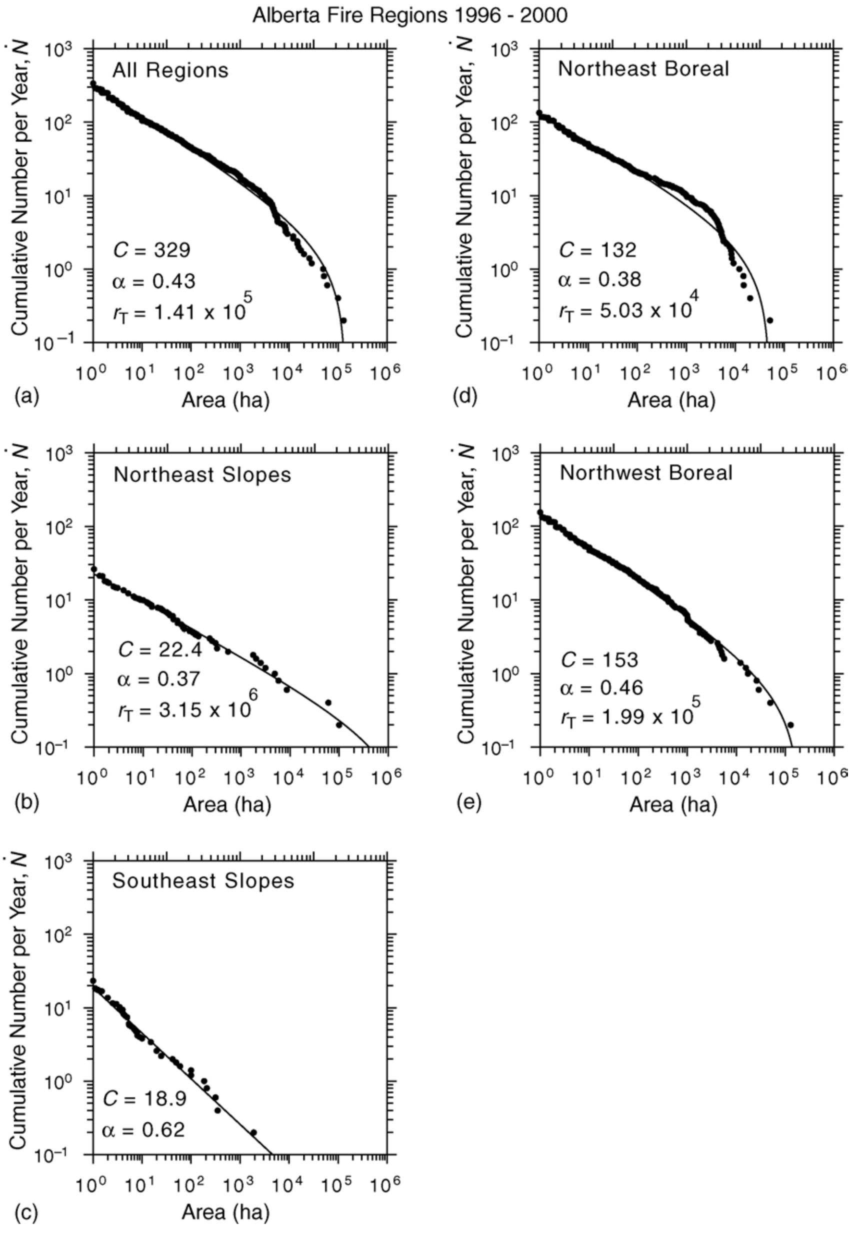

Fig. 5. Cumulative frequency-area distributions for forest fires in Alberta from 1996 to 2000 for (a) the entire province and for the regions (b) Northeast slopes, (c) Southeast slopes, (d) Northeast Boreal, and (e) Northwest Boreal. Parameters are given for the power law or upper-truncated power law (thin solid line) that describes each distribution.

low a power law with scaling exponents ranging from 1.31 to 1.49 [1]. These scaling exponents correspond to cumulative distributions with scaling exponents, α, ranging from 0.31 to 0.49 [1,29]. Observed forest fire cumulative frequency-area distributions exhibit a range of scaling exponents that are all less than one, inconsistent with the traditional SOC model [4].

A goal of SOC models is scale invariance, with the model producing distributions described by a single

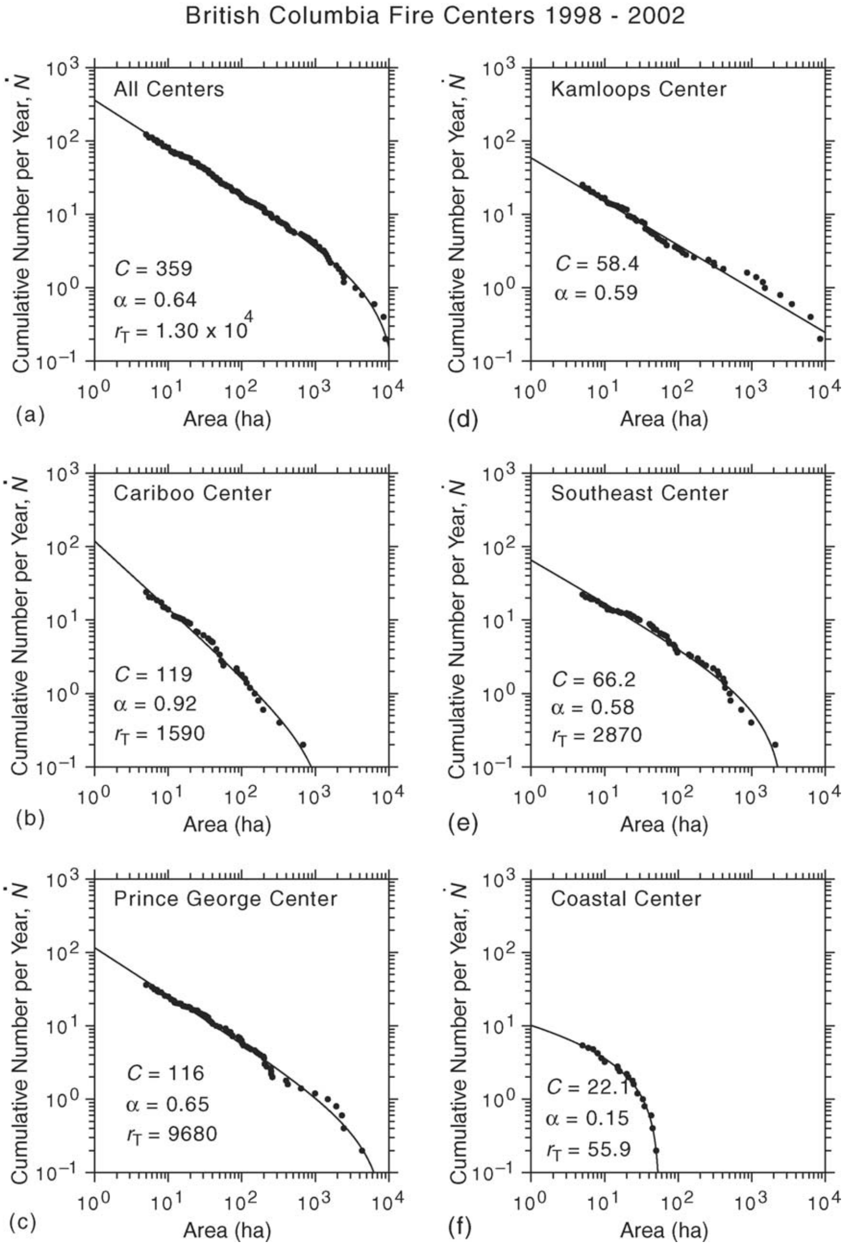

Fig. 6. Cumulative frequency-area distributions for forest fires in British Columbia from 1998 to 2002 for (a) the entire province and for the fire centers (b) Cariboo, (c) Prince George, (d) Kamloops, (e) Southeast, and (f) Coastal. Parameters are given for the power law or upper-truncated power law (thin solid line) that describes each distribution.

scaling exponent. In contrast to the traditional SOC model [4], varying the parameters of the self-similar criticality (SSC) model [20,32] produces power law cumulative distributions with different scaling exponents. The SSC model has been applied to the geophysical process of hotspot seamount formation [21] and may provide insight into natural systems that exhibit a range of scaling exponents.

The SSC model is discussed in detail by Burroughs [20], Hergarten [15], and Tebbens and Burroughs [32] and is summarized here. A square grid contains a stochastic fractal pattern of critical cells. Cells selected as critical do not change and remain in the critical state throughout each model run. The model consists of randomly selecting cells in successive steps. At each step, each cell is in one of three possible states: empty, occupied, or critical. As applied to forest fires, there are three possible outcomes at each step. If an empty cell is selected, a tree is added. If a cell is selected that already contains a tree, nothing happens. If a critical cell is selected, all trees in surrounding non-diagonal adjacent cells are cleared from the grid, simulating a forest fire. The critical cells represent regions where forest fires originate. An event is the occurrence of a fire. The number of trees cleared from the grid is the event size, representing forest fire area.

Applying the SSC model to forest fires, the critical cells represent locations of cloud-to-ground lightning strikes that ignite fires. Approximately 85% of the area burned in Canada annually is a result of lightning-induced fires [33]. Ignition site locations differ for each run of the model, as do the locations of fires caused by lightning in different years [34]. The modeled forest fire size distributions depend on the fractal dimension of the critical cells, not on the particular critical cell locations in each model run.

For a given fractal distribution, the density of critical cells in some regions of the grid will be greater than in other regions of the grid. Regions with fewer critical cells have fewer fire initiation sites and tend to have larger fires. Increasing the fractal dimension of the critical cells results in more small fires and fewer large fires, increasing the activity level and the scaling exponent.

We create stochastic fractal patterns following the approach of Mandelbrot [35]. We start with a square grid generator of size n_{r} rows by n_{r} columns. The generator contains a fixed number, n_{c}, of randomly located critical cells. The second order stochastic fractal is obtained by replacing each critical cell of the first order fractal with the generator containing n_{c} critical cells at reselected random locations. The process is repeated to create higher order fractal patterns. The spatial fractal dimension of the critical cells, D_{c}, is

$$D_{\mathrm{c}} = \frac{\ln n_{\mathrm{c}}}{\ln n_{\mathrm{r}}} \tag{4}$$

We create third-order stochastic fractal patterns with n_{r} equal to 7, producing grids of 343 by 343 cells. By selecting n_{c} to be 6 and 10, we create stochastic fractals with D_{c} equal to 0.92 and 1.18, respectively. We run the SSC model on these grids for 2.03 million iterations to produce event size distributions. For each dimension, the model is run five times using an independently generated pattern of critical cells each time. Varying the fractal dimension of the critical cells in the SSC model produces cumulative frequency-size distributions with a range of scaling exponents [32], similar to the range observed for natural forest fires.

The resulting cumulative frequency-size distributions follow Eq. (3) with scaling exponents, α, of approximately 0.4 for $D_{\mathrm{c}} = 0.92$ and 0.6 for $D_{\mathrm{c}} = 1.18$ (Fig. 7). For the range of values of D_{c} appropriate for modeling forest fires, α is approximately equal to $D_{\mathrm{c}}/2$ [32]. This $D_{\mathrm{c}}/2$ relationship is consistent with the scaling exponent of void size distributions derived for a Euclidean embedding space and applied to void sizes between galaxies [36]. The forest fires are contained within the space between the critical cells and α represents the scaling exponent of the forest fire size distribution. For SSC model values of D_{c} greater than approximately 1.3, the relationship between D_{c} and α is nonlinear [32]. Possible causes of this nonlinearity, such as finite size effects, are discussed in [32] and [36]. Fig. 8 shows examples of three fires generated by the SSC model using the critical cell pattern illustrated in Fig. 7 with D_{c} equal to 0.92. For a given event, the edge of the burn area is uneven and there tend to be internal pockets of unburned trees, similar to the pattern produced by natural forest fires.

6. Discussion

6.1. Forecasting

If a particular scaling function is found to describe a system, that function may be used for probabilistic forecasting. For instance, Gutenberg and Richter [16] demonstrated that earthquake frequency size distributions in southern California follow a power law. The scaling exponent of the power law, called the *b*-value, has been determined by numerous researchers for different regions worldwide and is the basis of probabilistic earthquake forecasting. We adopt a sim-

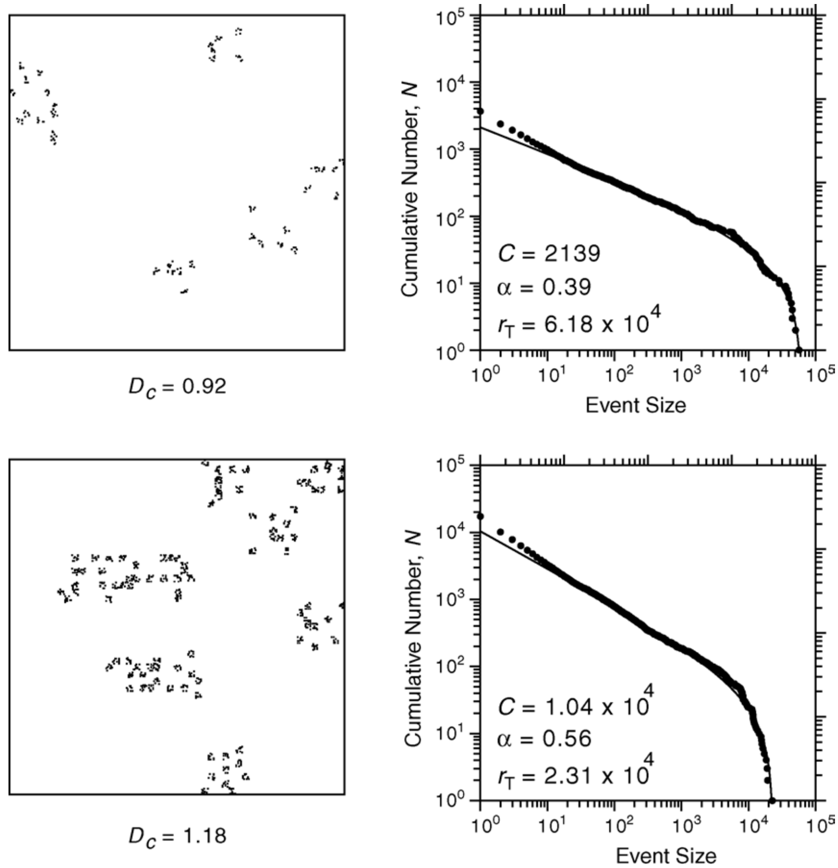

Fig. 7. Sample of SSC model output for 2.03 million iterations on a 343 by 343 grid with critical cell dimensions, D_c, 0.92 and 1.18. The cumulative distributions are well-described by an upper-truncated power law (Eq. (3)) with scaling exponents 0.39 and 0.56. The scaling exponent of the distribution depends on the fractal dimension of the critical cells. The SSC model replicates the cumulative frequency-area distributions observed for natural forest fires.

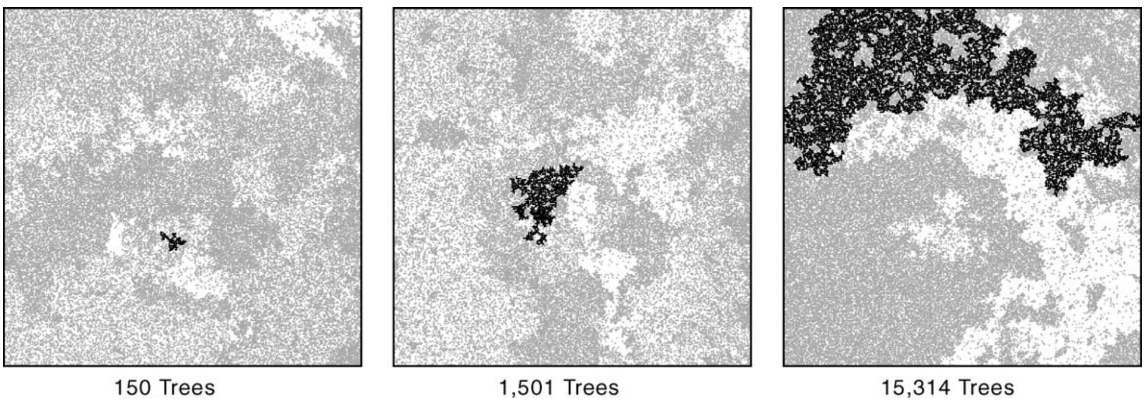

Fig. 8. Individual fire events generated by the SSC model using the critical cell pattern in Fig. 7 with D_c equal to 0.92. Light gray represents cells occupied by trees. Black represents cells that burn in a single event of the size indicated. White represents unoccupied cells and often reveals the outline of past events.

ilar approach to the probabilistic forecasting of forest fires.

In Alberta, similarities in frequency-area distributions from one decade to the next (Fig. 4) suggest that the scaling parameters may be used to forecast forest fire burn areas. Based on the 1961 through 2000 record, for all Alberta we expect approximately 24 fires per year of 100 ha or larger, 8 fires per year of 1000 ha or larger, 2 fires per year of 10,000 ha or larger, and 1 fire every 2.5 years of 100,000 ha or larger (Fig. 4a). Forecasts are appropriate within the observed range of the data, but may be inaccurate if extrapolated to larger fire sizes.

The variability in scaling relationships observed for different centers and regions within each province (Figs. 5 and 6) indicates that localized forecasts may be possible. While a 5-year record is probably insufficient to accurately characterize the long term behavior of each region, variations between regions are apparent. Scaling relationships determined for one region cannot be applied to other geographic regions. Within Alberta more fires are expected each year in the boreal forests (Fig. 5d and e) than in the northeast and southeast slopes (Fig. 5b and c). Based on the available record, roughly one fire per year of 100 ha or larger is expected in the Southeast slopes region while the Northeast boreal region is expected to have roughly 21 fires per year of this size (Fig. 5). Regional variability is also observed within British Columbia. For instance, fires of 10 ha or larger are expected to be reported three to four times per year in the Coastal center and fifteen times per year in the Kamloops center (Fig. 6). Longer records for these regions are needed to refine or confirm these forecasts.

6.2. Implications for forecasting large events

The tapered power law and upper-truncated power law both describe the observed “fall-off” in the forest fire distributions. The upper-truncated power law quickly falls to zero beyond the largest observed fire sizes and therefore cannot be used to forecast fires larger than those in the record. For some natural hazards, such as tsunamis and earthquakes, the power law portion of the upper-truncated power law (Eq. (3)) may be used to forecast events larger than those observed in a short-term record [26,27]. For forest fires in British Columbia, extrapolation of the power law portion of Eq. (3) predicts approximately eleven fires of 90,000 ha or larger every 50 years. Only four fires of this size or larger have been observed since 1950, indicating that this approach is not appropriate for forecasting large fires. An ancillary implication of this result is that the observed upper-truncation of forest fire area distributions is not due to the short duration of the data set, as found for earthquakes [26] and tsunamis [27], but to another cause such as the finite size of the contiguous forest. The tapered power law cannot be extrapolated to larger fire sizes because the function predicts too few large events. The tapered power law under-predicts the number of large fires, predicting only one fire of 90,000 ha or larger every 9.7 million years. Four fires of 90,000 ha or larger have been reported since 1950. Thus, the power law, the tapered power law, and the upper-truncated power law cannot be extrapolated to make meaningful forecasts of the probability of fires larger than those in the observed data set.

6.3. SSC model and lightning

The ability of the SSC model to replicate the observed forest fire burn area distributions suggests that locations of cloud-to-ground lightning strikes that ignite fires may be fractally distributed. Spatial clustering is a property of a fractal distribution. Spatial clustering has been observed for lightning strike density in Ontario, with the cluster pattern varying from year to year [34]. Future studies could examine whether or not the patterns of fire-igniting lightning strikes are fractal with dimensions similar to those predicted by the SSC model.

6.4. Forest fire size classes

Several forest fire size classes have been used over the past decades without consistency between various agencies. For example, a report on forest fires by the Corpo Forestale dello Stato lists four size classes: less than 1, 1–5, 5–100 ha, and greater than 100 ha [37]. A fire season report created by the Montana Forestry Division defines seven size classes: 0.01–0.25, 0.26–9.9, 10.0–99.9, 100.0–299.9, 300.0–999.9, 1000.0–4999.9, and 5000 acres and larger, designated size class A–G, respectively [38]. In 1986, a logarithmic scale for fire size class was adopted by the Canadian Interagency Forest Fire Center (CIFFC) [39] (Table 1). This clas-

Table 1
Forest fire size classes

Number	Letter	Area (ha)
1	A	Up to 0.1
2	B	0.11–1.0
3	C	1.1–10
4	D	10.1–100
5	E	100.1–1,000
6	F	1000.1–10,000
7	G	10,000.1–100,000
8	H	Over 100,000

After CIFFC [39].

sification scheme is similar to the logarithmic binning of the Gutenberg-Richter scale for earthquake magnitudes [16]. Observed forest fire area distributions in Canada (e.g. Figs. 1–6), Italy [2], China [3], the western United States [1], and Australia [1] are all well described by a power law or an upper-truncated power law over several orders of magnitude. Logarithmic divisions are used in many fields to represent a quantity if the range of values extends over many orders of magnitude. Examples include sound (decibels), earthquakes (Gutenberg-Richter magnitude scale), and acidity (pH scale). Logarithmic binning is particularly appropriate for quantities that exhibit power law scaling since one unit on a logarithmic scale represents one order of magnitude. Since forest fire area distributions exhibit power law scaling, the use of logarithmic bins to define fire size classes is appropriate (e.g. [39]) (Table 1).

7. Conclusions

Forest fire cumulative frequency-area distributions for the Canadian provinces of Alberta and British Columbia are found to be well-described by a power law or an upper-truncated power law with scaling exponents ranging from 0.15 to 0.92. A classification scheme with logarithmic bins is therefore appropriate for describing forest fires (e.g. [39]). Regional variability in the scaling parameters is observed both within and between provinces. Temporal variability is found for annual records, but the variability decreases at the decadal scale suggesting that decadal distributions may be used for long term fire control planning within a geographic region. The SSC model as applied to forest fires may provide a link between fractal geometry in nature and observed power law frequency-size distributions of forest fire burn areas.

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