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Mean-field threshold systems and phase dynamics: An application to earthquake fault systems

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Abstract. – Driven mean-field threshold systems demonstrate complex observable space-time patterns of behavior that are difficult to understand or predict without knowledge of the underlying dynamics, which are typically unobservable. Here we describe a new method based on phase dynamics techniques to analyze and forecast the space-time patterns of activity in these systems. Application to earthquake data from a typical, seismically active region shows that the method holds considerable promise for forecasting the temporal occurrence of the largest future events. We demonstrate the power of our technique via an application to the difficult problem of earthquake forecasting in southern California.

Earthquake dynamics [1] are considered to be one example of a strongly correlated, high-dimensional driven threshold system, other examples of which include both neural networks [2] and magnetized domains in ferromagnets [3]. Such systems, which undergo sudden unstable transitions following application of slow but persistent forcing, are often modeled as interacting, nonlinear oscillators [4, 5]. Extensive use of simulations to understand the dynamics of strongly correlated threshold systems is critical to revealing important clues to the fundamental physics. For example, many previous studies of the earthquake fault system focused on scaling, critical phenomena, and nucleation arising from the analysis of simulation data [6, 7]. A more difficult problem is associated with using the observable space-time patterns of activity to forecast future large events when the underlying dynamics of the system are not observable [8, 9]. However, hypothesis testing must be conducted with real data if the simulation studies are to have meaning. It is from this perspective that we now consider the implications of recent simulations [9] on forecasting large earthquakes on one specific, seismically active region, southern California.

Methods to forecast large earthquakes proposed previously have generally relied upon using either an excess, or a lack, of prior activity (anomalous activation or quiescence) [10–41]. The success of these various methods has generally been mixed, and the solid earth geophysics community has failed to come to a consensus as to whether it is possible to forecast large events [42–47].

While these prior attempts at earthquake forecasting primarily use the same types of data as our method, they have not been particularly successful for several reasons. First, many of them depend on arbitrary definitions of the anomalous behavior in very local regions, which can never be uniquely identified until after the main shock has occurred [15, 24, 28]. Second, a number of them require either windowing or fitting of other important parameters, such as time, region, and magnitude [22, 26, 29]. Third, all of these methods look for only one type of precursor, either activation or quiescence, not both simultaneously [14, 17, 21, 22, 35, 37, 47]. Fourth, in the case of precursory quiescence, it is necessary to decluster the catalog [35, 37, 41]. Fifth, in the case where the parameter chosen is not seismicity but strain or energy release, questions remain as to both its appropriate physical interpretation and applicability [44, 48]. And finally, for those cases in which changes in stress are used to model the likelihood of large events, differences in data availability and interpretations can result in significant differences in the results [15]. A thorough review of the subject can be found at the website `http://helix.nature.com/debates/earthquakes`. By contrast, our method has the advantages that the criteria for using the method are objective; no arbitrary definition of the precursory activity is needed, so that both anomalous activation and quiescence are treated equally; there is no fitting, smoothing, windowing, or declustering performed on the data; and no *a priori* knowledge of the location or extent of the activation/quiescence area is required. Our method, a coarse-grained measure of the spatio-temporal variations in seismicity performed on the regional historic earthquake catalog, quantifies the change through time of a unit vector over the entire space. As such, it identifies the characteristic patterns associated with the shifting of small earthquakes from one location to another through time, prior to the occurrence of large earthquakes.

Simulations indicate [9, 49, 50] that the long-range, but weak nature of the predominant elastic interactions on earthquake faults leads to mean-field behavior [51] characterized by small fluctuations about an average frequency of events. For a large enough spatial domain of volume V and long enough time intervals $T = t - t_b$, the Gutenberg-Richter (GR) magnitude-frequency relation [10, 11, 52] implies that the frequency of earthquakes, $r(m_\mathrm{c}, t_b, t) = \overline{r(m_\mathrm{c})}$ is constant for magnitudes $m \leq m_\mathrm{c}$. m_c is a cutoff magnitude, above which all events in V are detected by seismographs. Although the GR relation describes the overall rate, various subdomains in V will be more or less active at different times, so the relative intensity of earthquake activity is observed to shift from one location to another [10,11,52]. These facts [9, 50] suggest that earthquake activity at $\boldsymbol{x}$ over the time interval $T = t - t_b$ can be represented by a function $S(\boldsymbol{x}, t_b, t)$ whose spatial mean is the rate $r(m_\mathrm{c}, t_b, t)$:

$$\frac{1}{V}\int_V S(\boldsymbol{x}, t_b, t)\,\mathrm{d}\boldsymbol{x} = r(m_\mathrm{c}, t_b, t) = \overline{r(m_\mathrm{c})} \quad \text{as } T \to \infty. \tag{1}$$

Our previous work with numerical simulations [9] strongly suggests that, while the overall rate of earthquake activity in V remains essentially constant, physical meaning can be attached to the shift in relative intensity of earthquake activity in V from one location $\boldsymbol{x}_i$ to another location $\boldsymbol{x}_j$ through time. We define a spatial coarse-graining of V using a d-dimensional lattice of boxes at scale size L, with the i-th small box having a volume L^d centered at $\boldsymbol{x}_i$. The idea is to use information on small events having spatial scale $\lambda < L$ to forecast the occurrence of large events having scale $\lambda > L$. As we have demonstrated in the past, if N of the boxes have at least one earthquake in them, a set of N physically meaningful, complete, orthonormal basis vectors $\phi_n(\boldsymbol{x}_i)$ can be constructed from the correlation operator $K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ constructed by cross-correlating the mean-removed time series, $z(\boldsymbol{x}_i, t)$ [9]. Diagonalizing $K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ produces the eigenfunctions $\phi_n(\boldsymbol{x}_i)$ and eigenfrequencies ω_n. Physically, the $\phi_n(\boldsymbol{x}_i)$ represent N-dimensional spatial patterns of earthquake activity that are eigenvectors (eigenpatterns) of this $N \times N$ Karhunen-Loeve correlation operator $K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ [9, 53–55]. The construction of this correlation operator and its eigenvectors quantifies the correlated activity between the various sites, the space-time patterns of seismic activity that directly reflect the space-time correlations in the underlying medium [34].

$S(\boldsymbol{x}, t_b, t)$ is the coarse-grained realization of a random process with a constant mean, $r(m_c, t_b, t)$, and a constant variance, $\|\tilde{S}(\boldsymbol{x}, t_b, t)\|^2$ [56]. In particular, a shift in locations and numbers of small events reflect the space-time correlations and encompass both precursory quiescence and precursory activation. Physically, we define an N-dimensional vector, $\tilde{S}(\boldsymbol{x}, t_b, t) \equiv S(\boldsymbol{x}, t_b, t) - r(m_c, t_b, t)$ for which the overall rate of activity remains constant, but whose phase angle changes through time. If the seismicity patterns are undergoing a systematic change due to the growth of precursory modes, then the vector is no longer experiencing a random walk about some mean. Depending on the changes in the relative rate of seismic activity at each location, its phase angle undergoes a persistent rotation away from that mean that is quantifiable. The value of $\tilde{S}(\boldsymbol{x}, t_b, t)$ describes which sites are anomalously active relative to the mean rate $r(m_c, t_b, t)$, where $\tilde{S}(\boldsymbol{x}, t_b, t) > 0$, and which are anomalously quiescent, where $\tilde{S}(\boldsymbol{x}, t_b, t) < 0$. Changes in phase angle of $\tilde{S}(\boldsymbol{x}, t_b, t)$ are then the physically meaningful quantities, rather than changes in the norm, or variance, $\|\tilde{S}(\boldsymbol{x}, t_b, t)\|$. Systems which can be analyzed in this way are often called phase dynamical systems because the physical content is carried by the phase angle of the normalized state vector rather than its amplitude [49]. We define this *normalized* unit vector as

$$\hat{S}(\boldsymbol{x}, t_b, t) \equiv \frac{\tilde{S}(\boldsymbol{x}, t_b, t)}{\|\tilde{S}(\boldsymbol{x}, t_b, t)\|}, \tag{2}$$

where

$$\|\tilde{S}(\boldsymbol{x}, t_b, t)\| = \sqrt{\int_V \left|\tilde{S}(\boldsymbol{x}, t_b, t)\right|^2 \mathrm{d}\boldsymbol{x}}. \tag{3}$$

Now consider earthquake activity in a typical, seismically active region, southern California between 32° and 37° north latitude, and 238° to 245° east longitude. We use the standard online data set available through the web site maintained by the Southern California Earthquake Center (`http://www.scecdc.scec.org`), which consists of a record of all instrumentally recorded earthquakes beginning in January 1932 and extending to the present. For this region, $m_c = 3$ is typically used to ensure catalog completeness since 1932. We tile the surface area with $(d = 2)$-dimensional boxes of horizontal scale size $L = 0.1° \sim 11\,\text{km}$. Boxes of this size correspond roughly to the linear scale size of a magnitude $m \sim 6$ earthquake.

The basic activity rate function $S(\boldsymbol{x}, t_b, t)$ can be calculated from the data, where $n(\boldsymbol{x}_i, t)$ is the number of events at location $\boldsymbol{x}_i$ and time t:

$$S(\boldsymbol{x}, t_b, t) \equiv \frac{1}{(t - t_b)} \int_{t_b}^{t} n(\boldsymbol{x}_i, t)\,\mathrm{d}t. \tag{4}$$

The shift of earthquake activity from one location to another means that we are interested in time-dependent changes described by $\Delta\hat{S}(\boldsymbol{x}, t_b, t_1, t_2) \equiv \hat{S}(\boldsymbol{x}, t_b, t_2) - \hat{S}(\boldsymbol{x}, t_b, t_1)$, which can be interpreted as an angular drift of $\hat{S}(\boldsymbol{x}, t_b, t)$ in the N-dimensional space defined by the box locations. In particular, we are interested in the *mean angular drift* $\langle\Delta\hat{S}(\boldsymbol{x}, t_b, t_1, t_2)\rangle \equiv \Delta s(\boldsymbol{x}_i, t_1, t_2)$ over the years (t_1, t_2), where $\langle\rangle$ denotes the expectation over all realizations parameterized by t_b [56]. The physical picture is that $\Delta s(\boldsymbol{x}_i, t_1, t_2)$ is proportional to a mean drift angle (or vector difference) over the time period t_1 to t_2, that points in the direction of future patterns of activity occurring after t_2. This picture is qualitatively similar to that for a scalar function $f(t)$, in which $\Delta f = \frac{\mathrm{d}f(t)}{\mathrm{d}t}\Delta t$ is used to project future changes in f during a time interval Δt. The direction in which $\Delta s(\boldsymbol{x}_i, t_1, t_2)$ points has physical meaning because the $\phi_n(\boldsymbol{x}_i)$ have physical meaning, in terms of the spatial and temporal correlations in the earthquake activity, as described above [9, 53]. Thus $\Delta s(\boldsymbol{x}_i, t_1, t_2)$ is the precursor that we seek. Since we expect that one particular realization of the Brownian stochastic process is $\Delta\hat{S}(\boldsymbol{x}, t_b, t_1, t_1 + c)$, over the time interval c, we predict that the mean of $\Delta\hat{S}(\boldsymbol{x}, t_b, t_1, t_1 + c)$ with respect to the time increment c should be zero, and that the mean amplitude of excursions should be equal to the standard deviation.

Since we are interested in the increase of probability above the time-dependent background probability $\mu_{\mathrm{B}}(t_1, t_2)$, we compute $\Delta P(\boldsymbol{x}_i, t_1, t_2)$, the change in probability of an event:

$$\Delta P(\boldsymbol{x}_i, t_1, t_2) \equiv [\Delta s(\boldsymbol{x}_i, t_1, t_2)]^2 - \mu_{\mathrm{B}}(t_1, t_2), \tag{5}$$

where

$$\mu_{\mathrm{B}}(t_1, t_2) \equiv \frac{1}{V}\int_V \{\Delta s(\boldsymbol{x}_i, t_1, t_2)\}^2 \,\mathrm{d}\boldsymbol{x}. \tag{6}$$

We again note that there are *no free model parameters* in the computation of $\Delta P(\boldsymbol{x}_i, t_1, t_2)$ to be determined by fitting the data.

The intensity of seismic activity over the years 1932–1991 is shown in fig. 1a. Figure 1b is a color contour plot of $\mathrm{Log}_{10}\,\Delta P(\boldsymbol{x}_i, t_1, t_2)$, $\Delta P(\boldsymbol{x}_i, t_1, t_2) > 0$, where $t_1 = 01/01/1978$, and $t_2 = 12/31/1991$. In computing the contours in fig. 1b, the average over t_b in eq. (4) was taken over the time interval 1/1/1932–12/31/1991. Here we plot only the *increase* in probability, normalized to the maximum which is, as expected, one of the events which occurred during the calculation time period, in this case the 1983 Coalinga earthquake. The shaded anomalies are associated with large (*i.e.* $m \geq 5.0$) events for both the current (inverted triangles, $t_1 < t < t_2$) and future (circles, $t_2 < t$) time periods. The color scale is logarithmic, as noted above. While those events that occur during the calculation time period have the highest probability of occurrence, as one would anticipate should be the case, many of the earthquakes which occur after that time period are also highlighted. *No data were used in the colored anomalies of fig. 1b from the time after December 31, 1991, 6 months prior to the June 27, 1992, (moment magnitude)* $m \sim 7.3$ *Landers* [13] *earthquake (*$34°\ 13'$ *N Lat.,* $116°\ 26'$ *W Long.).*

Visual inspection of fig. 1b shows that the method has forecast skill, but statistical testing is needed. We used two types of null hypotheses to test the forecast in fig. 1b. First, we constructed thousands of random earthquake catalogs from the observed catalog by using the same total number of events, but assigning occurrence times from a uniform probability distribution over the years 1932–1991, and distributing them with uniform probability over the original event locations. Thus all events in the randomized catalogs remain at seismically active locations. Our procedure produces a Poisson distribution of events in space with an exponential distribution of interevent times. Randomizing the catalog in this way destroys whatever coherent space-time structure may have existed in the data. These random catalogs are used to construct a set of null hypotheses, since any forecast method using such a catalog cannot, by definition, produce useful information. For the second null hypothesis, we used the actual seismic intensity data in fig. 1a directly as a probability density at $\boldsymbol{x}_i$.

The Maximum Likelihood (ML) test is currently accepted as the standard method of testing earthquake forecasts [13, 57]. We therefore carried out ML tests to evaluate the accuracy with which our probability measure $\Delta P(\boldsymbol{x}_i, t_1, t_2)$ can forecast future ($t > t_2$) large

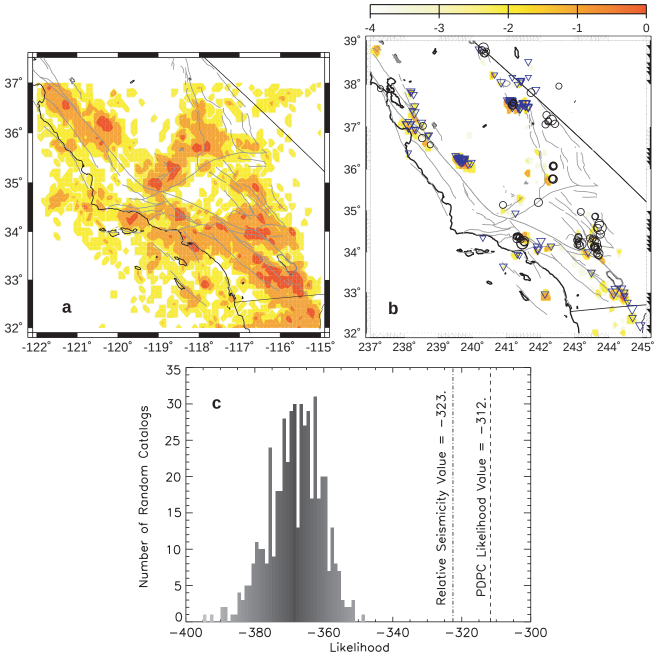

Fig. 1

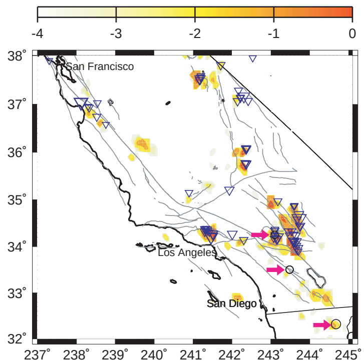

Fig. 2

Fig. 1 – (a) Relative seismic intensity in southern California for the period 1932–December 31, 1991, plotted on a logarithmic color scale. (b) Color contour plot of $\mathrm{Log}_{10}\,\Delta P(\boldsymbol{x}_i, t_1, t_2)$, for locations at which $\Delta P(\boldsymbol{x}_i, t_1, t_2) > 0$. $t_1 =$ January 1, 1978, and $t_2 =$ December 31, 1991. Values are scaled by the maximum. Inverted triangles are events that occurred from 1978–1991 for events of $5 < m$ (triangle size scales with magnitude). Circles are events that occurred from 1992–present, again for $5 < m$ (circle size also scales with magnitude). (c) $\mathrm{Log}_{10}[\boldsymbol{L}]$ plots for 500 random catalogs (histogram); for the seismic intensity map of (a) (dash-dotted line); and for the forecast in (b) (dashed line).

Fig. 2 – Plot of $\mathrm{Log}_{10}\,\Delta P(\boldsymbol{x}_i, t_1, t_2)$, for $\Delta P(\boldsymbol{x}_i, t_1, t_2) > 0$, $t_1 = 1/1/1989$, and $t_2 = 12/31/1999$. Inverted triangles are events, $m \geq 5.0$, during 1989–1999. Events of $m \geq 5.0$ that occurred between 1/1/2000 and 3/31/2002, are plotted with circles. The color scale indicates the logarithmic exponent.

($m \geq 5.0$) events, relative to forecasts from the null hypotheses. Define $P[\boldsymbol{x}]$ to be the union of a set of N Gaussian density functions $p_{\mathrm{G}}(|\boldsymbol{x} - \boldsymbol{x}_i|)$ [57] centered at each location $\boldsymbol{x}_i$. Each elementary Gaussian density $p_{\mathrm{G}}(|\boldsymbol{x} - \boldsymbol{x}_i|)$ has a peak value $\Delta P(\boldsymbol{x}_i, t_1, t_2) + \mu_{\mathrm{B}}(t_1, t_2)$, the probability change including the background, if $\Delta P(\boldsymbol{x}_i, t_1, t_2) > 0$; a peak value $\mu_{\mathrm{B}}(t_1, t_2)$ if $\Delta P(\boldsymbol{x}_i, t_1, t_2) \geq 0$; as well as a standard deviation $\sigma = L \sim 11\,\mathrm{km}$. $P[\boldsymbol{x}(e_j)]$ is then a probability measure that a future large event e_j occurs at location $\boldsymbol{x}(e_j)$. If there are J future large events, the likelihood $\boldsymbol{L}$ that all J events are forecast is

$$\boldsymbol{L} \equiv \Pi_j \left\{ \frac{P[\boldsymbol{x}(e_j)]}{\sum_i P[\boldsymbol{x}_i]} \right\}. \tag{7}$$

In fig. 1c, we show computations of i) $\mathrm{Log}_{10}[\boldsymbol{L}]$ for 500 random catalogs of the first type (histogram); ii) $\mathrm{Log}_{10}[\boldsymbol{L}]$ for the seismic intensity map in fig. 1a (vertical dash-dotted line); and iii) $\mathrm{Log}_{10}[\boldsymbol{L}]$ for our forecast of fig. 1b (dashed line). Since larger values of $\mathrm{Log}_{10}[\boldsymbol{L}]$ indicate a more successful hypothesis, we conclude that our method has some forecast skill.

The diffusive, mean field nature of the dynamics [6–9, 13], leads to several important hypotheses: 1) Forecasts such as fig. 1b should convey information for times t approximately in the range: $t_2 + (t_2 - t_1) > t > t_2$; and 2) anomalies of elevated probability having area A should persist for a characteristic time (*i.e.*, alarm time), $\tau \propto A^{\eta}$, where $\eta \sim 1$ [7, 14]. Using the hypothesis in (1), above, we provide the forecast shown in fig. 2 for future large events. The most unbiased test possible, this forecast is for the decade beginning in 2000, based on changes during the years 1989–1999 [58]. Superimposed are those earthquakes that have occurred since January 1, 2000 of $m \geq 5.0$, current as of March 2002. Note that the occurrence of each of these new events corresponds to a colored anomaly on the original map, substantiating the potential forecast capability of this method.

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