Inflation with primordial broken power law spectrum as an alternative to the concordance cosmological model

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We consider cosmological models with a non–scale-invariant spectrum of primordial perturbations and assess whether they represent a viable alternative to the concordance ΛCDM model. We find that in the framework of a model selection analysis, the WMAP and 2dF data do not provide any conclusive evidence in favor of one or the other kind of model. However, when a marginalization over the entire space of nuisance parameters is performed, models with a modified primordial spectrum and $\Omega_\Lambda = 0$ are strongly disfavored.

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I. INTRODUCTION

In the past few years, a cosmological "concordance model" has emerged from precision measurements of the cosmological observables. According to this Λ cold dark matter (ΛCDM) model, the Universe has flat spatially geometry and its overall energy density is contributed by baryons, dark matter, and by an unclustered "dark energy" component. The structures we observe today have been formed through gravitational instability, from initial, nearly scale-invariant adiabatic Gaussian fluctuations. The observational basis for this concordance model mainly lies in the measurements of the anisotropies of the cosmic microwave background (CMB) radiation [1,2], of the large-scale structure (LSS) of the Universe [3–5], and of the Hubble diagram of distant type Ia supernovae [6–8].

However, even if the model can satisfactorily explain the majority of cosmological observations, it still remains puzzling from a theoretical point of view. In particular, while there are many theoretically motivated (from the point of view of particle physics) candidates for the role of dark matter, on the contrary a satisfactory explanation concerning the nature of dark energy still does not exist, although many candidates have been proposed [8–11]. In fact, a strong amount of fine-tuning is required in order to explain the smallness of the dark energy density with respect to any significant high-energy scale, and the fact that dark energy and matter presently give the same contribution, within a factor of 3, to the total density of the Universe.

The above difficulties have led some authors to assess the robustness of the evidence for the existence of dark energy (in the simplest form of a cosmological constant), and, in particular, its dependence on the underlying assumptions related to the choice of a particular cosmological model. Recently, it has been shown that the CMB data can be well fitted by an Einstein–de Sitter model with zero cosmological constant, by relaxing the assumption that the primordial power spectrum is a simple power law [12]. Interestingly enough, the authors of Ref. [12] find that the best-fit model with broken spectrum and no cosmological constant has a *better* χ^2 relative to the CMB power spectrum than the best concordance model. Models with zero Λ can also fit the spectrum of matter fluctuations provided that part of the matter content is given by a nonclustering component, for example, neutrinos with eV mass. The price to pay is that these models do require a very low value of the Hubble constant, $H_0 \simeq 46\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, that is inconsistent with the HST Key Project measurements of the Hubble constant: $H_0 = 72 \pm 9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [13]. A possible explanation could be that we live in an underdense region, that is thus expanding faster than the average: this would imply that the locally measured "Hubble constant" probed by the HST would be larger than the actual, cosmological Hubble constant, probed by CMB observations. In any case, the analysis in Ref. [12] would seemingly indicate that, once the usual assumptions on the form of the primordial power spectrum are dropped, the Hubble diagram of distant supernovae Ia is the only direct evidence for the presence of a cosmological constant.

Many generalization of the simple power-law shape of the primordial power spectrum have been proposed, both on physical and observational grounds. Here we will only consider models with a broken primordial power spectrum, since these were shown in Ref. [12] to provide a very good fit to the CMB data also with a zero cosmological constant. For a Bayesian analysis of other alternatives to the single power-law spectrum, see, for example, Refs. [14–16]. The aim of this work is to constrain the parameters of models with a broken power-law primordial spectrum, and to assess whether these models (and, in particular, models

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with a broken power spectrum *and* $\Omega_\Lambda = 0$) really represent a viable alternative to the concordance model. In fact, the χ^2 goodness of fit is not the proper criterium to select which model, within a set, does a better job in explaining the experimental data. A very reliable way to assess the performance of a model is given by Bayesian model selection. This method automatically encodes Occam's razor principle, since models with a larger number of parameters are naturally penalized.

The paper is thus organized as follows. In Sec. II we describe our data analysis method used, and we recall the basics of Bayesian model comparison. In Sec. III we show our results, and finally in Sec. IV we draw our conclusions.

II. LIKELIHOOD ANALYSIS

We consider three different cosmological models: the standard ΛCDM model, a CDM model (i.e. a model with $\Omega_m = 1$) with a modified spectrum of primordial fluctuations, and a ΛCDM model also with a modified spectrum. In particular, we consider the case of a broken power-law spectrum $P(k)$, i.e.:

$$P(k) = \begin{cases} A_1(\frac{k}{k_p})^{n_1} & \text{for } k < k_*, \\ A_2(\frac{k}{k_p})^{n_2} & \text{for } k \geq k_*, \end{cases} \tag{1}$$

with the continuity condition $A_1 k_*^{n_1} = A_2 k_*^{n_2}$. Then, if the usual, single power-law spectrum is defined in terms of two parameters, namely, its amplitude A_s (as measured at the pivot wave number $k_p = 0.002\ \mathrm{Mpc}^{-1}$) and spectral index n_s, the broken power-law spectrum will be defined in terms of four out of the five parameters $\{A_1, A_2, n_1, n_2, k_*\}$. We choose the four independent parameters to be n_1, n_2, k_*, and the amplitude A_s at k_p. This coincides with A_1 or A_2 depending if $k_* > k_p$ or $k_* < k_p$.

The parameters of the ΛCDM model are, as usual, the physical baryon density ω_b, the cold dark matter density ω_c, the ratio θ between the sound horizon and the angular diameter distance to the last scattering surface, the neutrino fraction f_ν, the reionization optical depth τ, the amplitude A_s of the primordial spectrum at k_p, and the scalar spetral index n_s. The CDM model with a broken spectrum, that we will call for simplicity "modified CDM" (MCDM), is described by the following eigth parameters: ω_b, θ, f_ν, τ, A_s, n_1, n_2, k_*. The value of ω_c is derived by the request that $\Omega_m = 1$. Finally, the ΛCDM model with a broken spectrum ("modified ΛCDM", or MΛCDM) is described by nine parameters, namely ω_b, ω_c, θ, f_ν, τ, A_s, n_1, n_2, k_*. In all three models, we impose spatial flatness and purely adiabatic initial conditions, and we consider three neutrino families with equal mass. We take implicit flat priors on all the parameters. In particular, we take $0.8 \leq n_1 \leq 2$, $0.2 \leq n_2 \leq 1$ and $0.001\ \mathrm{Mpc}^{-1} \leq k_* \leq 0.02\ \mathrm{Mpc}^{-1}$. It is clear that the M$\Lambda$CDM model encompasses the other two for particular values of the parameters: it reduces to the ΛCDM when the two spectral indices are equal, $n_1 = n_2$, and thus the spectrum becomes a single power law; and it reduces to the MCDM when $\Omega_m = 1$.

We have modified the CAMB code in order to compute the spectrum of CMB anisotropies for models with a broken power-law spectrum like the one in Eq. (1). First, we constrain the variation of the parameters in the more general MΛCDM model by means of a Markov Chain Monte Carlo (MCMC) analysis of recent CMB data. The analysis method we adopt is based on the publicly available Markov Chain Monte Carlo package COSMOMC with a convergence diagnostics done through the Gelman and Rubin statistics. Since we are interested, among other things, in assessing whether modifying the assumptions on the shape of the primordial power spectrum reopens the possibility for a purely matter-dominated Universe, we want to make sure that our chains sufficiently explore the low-probability tails of the posterior distribution. For this reason, we have performed additional MCMC runs at different temperatures.

Our basic data set is the five-year WMAP data (temperature and polarization) with the routine for computing the likelihood supplied by the WMAP team. We marginalize over a possible contamination from Sunyaev-Zeldovich component, rescaling the WMAP template at the corresponding experimental frequencies. Then we also consider data on the matter power spectrum from the 2dF galaxy survey [4].

Finally, we perform a model comparison between the three models (for an introduction to Bayesian model comparison, see, for example, Ref. [17]). The relevant quantity assessing a model's performance in explaining the data is the Bayesian evidence, namely, the probability of the data given the model: $p(d|\mathcal{M})$. This can be related to the quantity we are really interested in, the posterior probability of the model (i.e., the probability of the data given the model), by applying Bayes' Theorem, so that:

$$p(\mathcal{M}|d) \propto p(d|\mathcal{M})p(\mathcal{M}), \tag{2}$$

where we have dropped an irrelevant normalization constant $p(d)$ that depends only on the data. The quantity $p(\mathcal{M})$ is the prior probability assigned to the model.

When comparing two models $\mathcal{M}_0$ and $\mathcal{M}_1$, one is interested in the ratio between the posterior probabilities:

$$\frac{p(\mathcal{M}_0|d)}{p(\mathcal{M}_1|d)} = \frac{p(d|\mathcal{M}_0)p(\mathcal{M}_0)}{p(d|\mathcal{M}_1)p(\mathcal{M}_1)}. \tag{3}$$

In absence of significant preference for any of the models, one can take the prior to be equal, $p(\mathcal{M}_0) = p(\mathcal{M}_1)$, and the ratio of the posteriors is thus given by the ratio of the evidences, the so-called Bayes factor B_{01}:

$$\frac{p(\mathcal{M}_0|d)}{p(\mathcal{M}_1|d)} = \frac{p(d|\mathcal{M}_0)}{p(d|\mathcal{M}_1)} \equiv B_{01}. \tag{4}$$

A value of B_{01} larger than unity means that the current data favor model 0 with respect to model 1, and vice versa.

The Bayesian evidence for each model can be in principle evaluated by writing

$$p(d|\mathcal{M}) = \int p(d|\theta, \mathcal{M}) p(\theta|\mathcal{M}) d\theta \tag{5}$$

i.e. as the integral of the product of the likelihood times the prior over the whole parameter space of the model. Unfortunately, evaluation of this integral is in general not easy, since it involves integration over a highly-dimensional parameter space. Also, it requires that the tail of the posterior distribution for the parameter have been adequately explored. However, this calculation can be simplified when the two models that are compared are nested one into the other, i.e., when one of the models reduces to the other for particular values of the parameters, as it is the case for the models considered here.

III. RESULTS

A. Parameter estimation

We show the constraints for the parameters of the MΛCDM model, obtained using the WMAP and WMAP + 2dF data sets, in Table I. For the sake of comparison, we also show in Table II the constraints on some of the parameters of the MΛCDM together with the constraints obtained in the ΛCDM model, using in both cases the WMAP-only data set. First of all, we notice that the value of $\Omega_\Lambda = 0.62 \pm 0.10$ for the WMAP data set is lower than the ΛCDM value $\Omega_\Lambda = 0.67 \pm 0.07$. The uncertainty in the determination of Ω_Λ is also larger, probably as an effect of the introduction of the additional parameters describing the modified power spectrum. Adding the 2dF data shifts the dark energy to larger values and reduces the error, $\Omega_\Lambda = 0.71^{+0.04}_{-0.03}$. The value of the Hubble constant is also lower in the MΛCDM model when using only the WMAP data, and is shifted to larger values when the 2dF data are included. In the top panels of Fig. 1 we show the posterior distribution for Ω_Λ and H_0 for the

TABLE I. MΛCDM model parameters and 68% credible intervals (first and third column) and best-fit values (second and fourth column), obtained using the WMAP and WMAP + 2dF data sets.

Parameter	Mean WMAP	Max Like WMAP	Mean WMAP + 2dF	Max Like WMAP + 2dF
$100\Omega_b h^2$	$2.14^{+0.10}_{-0.11}$	2.07	$2.206^{+0.080}_{-0.078}$	2.211
$\Omega_c h^2$	0.1221 ± 0.011	0.1214	$0.1118^{+0.0054}_{-0.0058}$	0.1097
Ω_Λ	$0.621^{+0.095}_{-0.094}$	0.648	$0.708^{+0.039}_{-0.034}$	0.728
Ω_m	$0.379^{+0.094}_{-0.095}$	0.352	$0.292^{+0.034}_{-0.039}$	0.271
H_0 [km sec^{-1} Mpc^{-1}]	62.8 ± 5.9	63.5	$68.2^{+3.3}_{-3.1}$	69.6
f_ν	≤ 0.064	0.014	≤ 0.042	0.0057
τ	$0.0882^{+0.0075}_{-0.0090}$	0.0810	$0.0914^{+0.0078}_{-0.0092}$	0.0838
z_r	11.1 ± 1.6	10.7	10.9 ± 1.5	10.2
t_0 [Gyrs]	$14.23^{+0.33}_{-0.32}$	14.18	13.98 ± 0.21	13.92
n_1	$1.017^{+0.061}_{-0.063}$	1.02	$1.006^{+0.068}_{-0.067}$	0.979
n_2	$0.921^{+0.037}_{-0.039}$	0.897	$0.948^{+0.025}_{-0.023}$	0.944
k_* [Mpc^{-1}]	≤ 0.012	0.012	≤ 0.011	0.009
$\log[10^{10} A_s]$	$3.204^{+0.055}_{-0.056}$	3.165	$3.182^{+0.051}_{-0.054}$	3.160
$n_1 - n_2$	$0.095^{+0.087}_{-0.089}$	0.12	$0.057^{+0.085}_{-0.088}$	0.035

TABLE II. Comparison between the MΛCDM and ΛCDM 68% credible intervals for selected parameters, obtained using the WMAP dataset.

Parameter	MΛCDM (WMAP)	ΛCDM (WMAP)
Ω_Λ	$0.621^{+0.095}_{-0.094}$	0.669 ± 0.066
Ω_m	$0.379^{+0.094}_{-0.095}$	0.331 ± 0.066
H_0 [km sec^{-1} Mpc^{-1}]	62.8 ± 5.9	65.7 ± 4.9
n_1	$1.017^{+0.061}_{-0.063}$	0.950 ± 0.017
n_2	$0.921^{+0.037}_{-0.039}$	0.950 ± 0.017
k_* [Mpc^{-1}]	≤ 0.012	$\cdots$
$\log[10^{10} A_s]$	$3.204^{+0.055}_{-0.056}$	$3.211^{+0.051}_{-0.058}$
$n_1 - n_2$	$0.095^{+0.087}_{-0.089}$	0

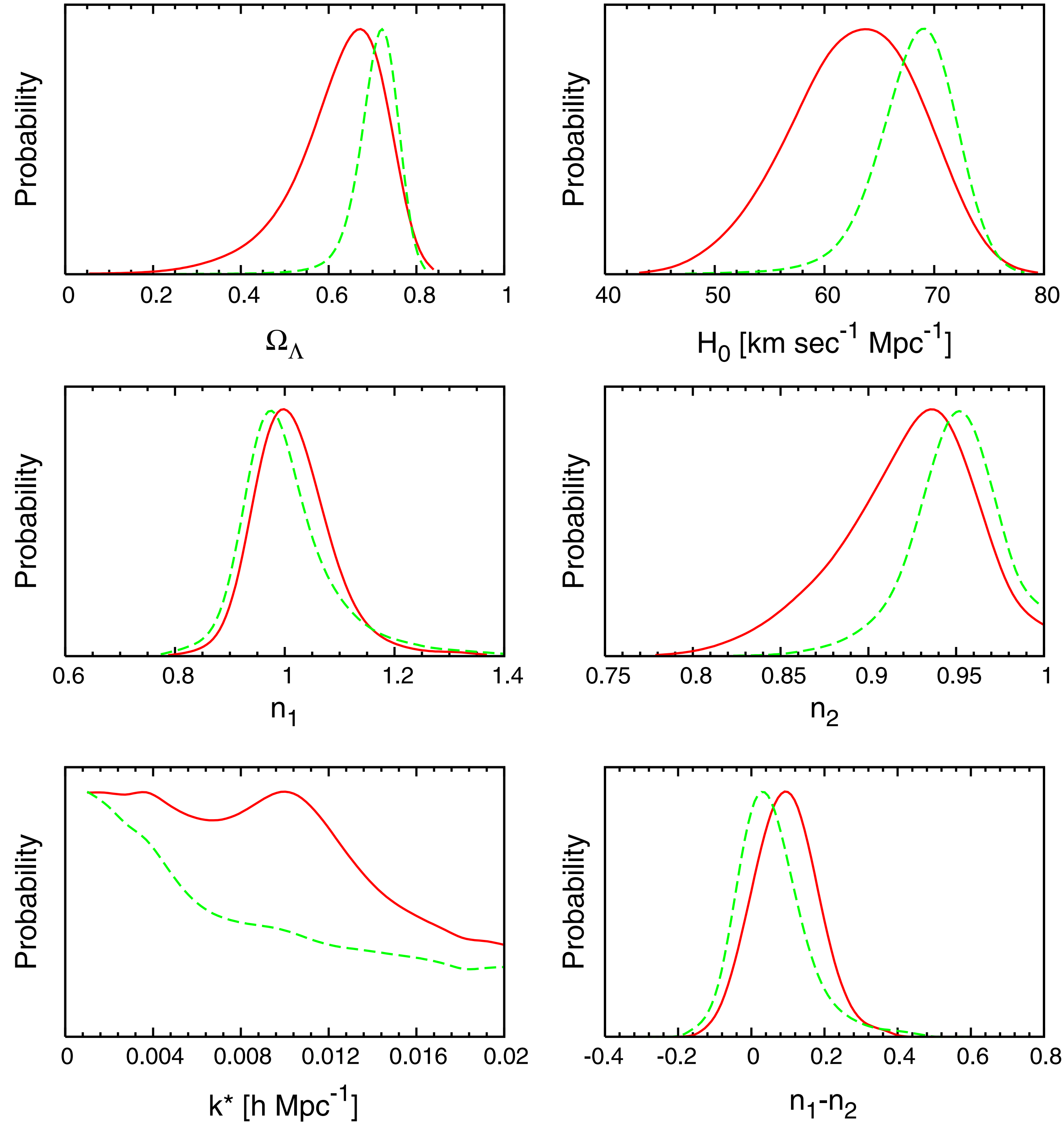

FIG. 1 (color online). Marginalized one-dimensional posteriors for Ω_Λ, H_0, n_1, n_2, k_* and Δ_n using the WMAP data (solid [red] curves) and the WMAP + 2dF data set (green [dashed] curves).

MΛCDM model, using the WMAP data only and the WMAP + 2dF data set.

For what concerns the parameters describing the shape of the primordial power spectrum, using the WMAP-only data set, we find that the low wave number spectral index $n_1 = 1.02 \pm 0.06$, so that the tilt of the spectrum could be either blue or red, while the high wave number spectral index is constrained to be red, $n_2 = 0.92 \pm 0.04$. For comparison, the overall spectral index n_s in the ΛCDM model lies somewhere in the middle, $n_s = 0.95 \pm 0.02$. It is interesting to note that the 68% credible intervals for n_1 and n_2 do not overlap. This is made more clear if one looks a the marginalized posterior for $\Delta n \equiv n_1 - n_2$. We have that $\Delta n = 0.095^{+0.087}_{-0.089}$ and thus $\Delta n = 0$ lies slightly outside the 68% credible interval, indicating a weak preference for models with $n_1 \neq n_2$. The wave number at the break of the spectrum, k_* is poorly constrained by the data and could lie nearly anywhere in its prior range, although values smaller than $\simeq 0.012\ \mathrm{Mpc}^{-1}$ are preferred. The main effect of adding the 2dF data is that the mean value of n_2 is shifted to larger values, $n_2 = 0.95 \pm 0.01$, and then there is a small overlap of the 68% credible intervals for n_1 and n_2. We show the posterior distributions for n_1, n_2, k_* and Δn in the three lower panels of Fig. 1.

The reason for the fact that Ω_Λ tends to be smaller in the enlarged MΛCDM model with respect to the concordance ΛCDM model is that it exists a partial degeneracy between n_2 and Ω_Λ (or, equivalently, between n_2 and Ω_m). A mildly red spectrum at the large wave numbers can partially compensate for the increased matter content and thus mimic of the effect a cosmological constant. This is clear in the first panel of Fig. 2, where we show the two-dimensional 68% and 95% credible intervals in the $n_2 - \Omega_\Lambda$ plane. In the same figure, we also show the credible intervals in the $n_2 - H_0$ and $n_2 - n_1$ planes. On the other hand, we do not find any appreciable correlation between the value of n_1 and that of Ω_Λ. One could naively expect that the late integrated Sachs-Wolfe effect would make the effect of Ω_Λ at the low multipoles partially degenerate with a change of the slope of the primordial spectrum in the same region. However this effect is probably masked by the large cosmic variance. The slightly blue best-fit value that we find for n_1 is probably due to the effect of the low quadrupole in the WMAP data.

B. Model comparison

In order to assess the performance of the models we consider here in explaining the WMAP and 2dF data, we

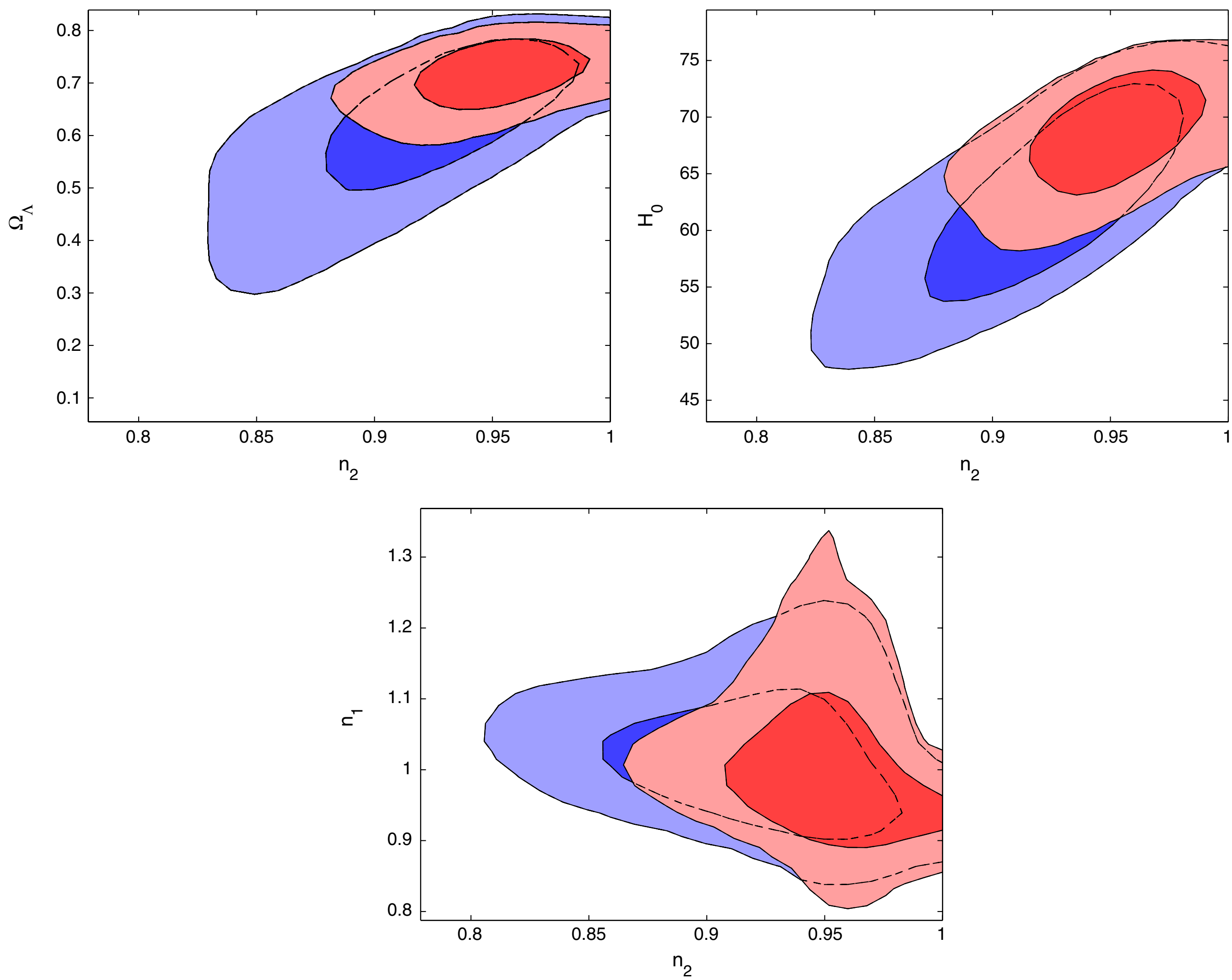

FIG. 2 (color online). Two-dimensional 68% (darker regions) and 95% (lighter regions) credible regions for (n_2, Ω_Λ) [upper left panel], (n_2, H_0) [upper right panel], (n_1, n_2) [lower panel], using the WMAP-only [blue] and WMAP + 2dF [red] data sets.

compute the Bayes factor between pairs of models. We assume equal priors for all the models. We start by considering the MΛCDM model and the MCDM model, namely, the model studied in Ref. [12]. The MCDM model is equivalent to the more general MΛCDM when $\Omega_\Lambda = 0$. Assuming that the prior is separable, the Bayes factor can be shown to be equal to the Savage-Dickey density ratio, i.e. the ratio between the one-dimensional posterior and the prior, evaluated at the point where the more complex model reduces to the simpler one. Then we can write for the Bayes factor $B_{\rm MCDM}$:

$$B_{\rm MCDM} \equiv \frac{p(d|{\rm MCDM})}{p(d|{\rm M\Lambda CDM})} = \frac{p(\Omega_\Lambda = 0|d, {\rm M\Lambda CDM})}{p(\Omega_\Lambda = 0|{\rm M\Lambda CDM})}. \quad (6)$$

Since Ω_Λ is a derived parameter, we do not know the exact form of the prior $p(\Omega_\Lambda = 0|{\rm M\Lambda CDM})$. However, it is reasonable to assume that it is nearly flat between 0 and 1, so that we can take $p(\Omega_\Lambda = 0|{\rm M\Lambda CDM}) = {\rm const} = 1$. Another possible issue is due to the fact that the calculation of $B_{\rm MCDM}$ involves the value of the one-dimensional posterior for Ω_Λ in $\Omega_\Lambda = 0$, i.e. at the very extreme of the parameter space and very far from the region of maximum probability. Usually the tails of the posterior are poorly sampled by the Monte Carlo methods used for parameter estimation, like the Metropolis-Hastings algorithm. To ensure that the value of $p(\Omega_\Lambda = 0)$ that we get is reliable, we have performed different MC runs at different temperatures, meaning that samples are drawn from $p(d|\theta, \mathcal{M})^{1/T} p(\theta|\mathcal{M})$ instead than from $p(d|\theta, \mathcal{M}) p(\theta|\mathcal{M})$ (i.e., the χ^2 for each model is divided by T). This lowers the height of the peak of the likelihood relatively to the tails and allows for better exploration of the latter. Under these assumptions, we find, using the WMAP data, that $\log(B_{\rm MCDM}) \simeq -4.3$, indicating a quite strong preference for the more complex MΛCDM model. In particular, the odds are $\sim$77:1 in favor of this model. Adding the 2dF data makes the evidence in favor of the MΛCDM model even stronger: $\log(B_{\rm MCDM}) \simeq -4.9$, corresponding to odds of $\sim$133:1.

Then we turn to the comparison of the MΛCDM with the concordance ΛCDM model. In this case the former model reduces to the latter for $n_1 = n_2$, whatever the value of k_*. In this case, always assuming the separability of the prior, the following generalization of the Savage-Dickey density ratio holds:

$$B_{\Lambda{\rm CDM}} \equiv \frac{p(d|\Lambda{\rm CDM})}{p(d|{\rm M\Lambda CDM})} = \int \frac{p(n_1, n_2|d, {\rm M\Lambda CDM})}{p(n_2|{\rm M\Lambda CDM})} \bigg|_{n_2 = n_1} dn_1. \quad (7)$$

We find that $\log(B_{\Lambda{\rm CDM}}) \simeq 0.77$ when using only the WMAP data, meaning that the evidence in favor of one or the other model is inconclusive. Adding the 2dF data yields $\log(B_{\Lambda{\rm CDM}}) \simeq 1.3$, corresponding to odds 7:2, thus indicating a weak preference for the simpler ΛCDM model.

Finally, we can combine the two results for $B_{\rm MCDM}$ and $B_{\Lambda{\rm CDM}}$ to compute the evidence of the ΛCDM model relative to the MCDM model, simply given by $B_{\Lambda{\rm CDM}}/B_{\rm MCDM}$. We get that $\log(B_{\Lambda{\rm CDM}}/B_{\rm MCDM}) \simeq 5.1$ or 6.2 using the WMAP-only or WMAP + 2dF data sets, respectively, indicating a strong preference for the concordance model with respect to broken power law spectrum, no-Λ models.

IV. CONCLUSIONS

In this paper we have investigated the constraints on cosmological models with a broken power-law spectrum of primordial fluctuations. We have found that, using the WMAP data, it exists a weak preference for models with a redder spectrum after the break. In fact we find the constraint $\Delta n \equiv n_1 - n_2 = 0.095^{+0.087}_{-0.089}$ at 68% C.L. This preference tends to disappear when the 2dF data are taken into account. We also find that the limits on Ω_Λ are slightly relaxed with respect to the concordance ΛCDM model, allowing for smaller values of Ω_Λ. However, models with $\Omega_\Lambda = 0$ are still incompatible with the observations. The constraints obtained in this paper could be improved by using the small-scale data in the galaxy power spectrum on one side, and the Supernovae Ia and Hubble Space Telescope data on the other. The information from the smallest scales in the galaxy power spectrum would allow a more precise determination of n_2, although it would crucially rely on a proper treatment of the nonlinear effects. Using a prior on H would indirectly allow to reduce the constraints on n_2 by virtue of the correlation between the two parameters, as it is clear from the second panel of Fig. 2. We do not expect instead that any of these additional data will improve the constrants on n_1, since it mainly affects the largest scales and is poorly degenerate with other parameters. Overall, using the nonlinear scales and the Supernovae Ia or Hubble data will lead to a better determination of Δn.

We have also performed a Bayesian model comparison analysis in order to assess whether models with a modified primordial spectrum provide a better interpretation of the WMAP and 2dF data. We find that the WMAP data alone are not yet able to discriminate between the models with a modified primordial spectrum considered here, and the concordance model. Considering also the 2dF data, we find that the concordance model is slightly favored. On the other hand, models with $\Omega_\Lambda = 0$ and a broken power-law primordial spectrum like those considered in Ref. [12] are strongly disfavored with respect to the concordance ΛCDM model.

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