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Self-similar branching of aftershock sequences

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Abstract

In this paper we propose a branching aftershock sequence (BASS) model for seismicity. We suggest that the BASS model is a preferred alternative to the widely studied epidemic type aftershock sequence (ETAS) model. In the BASS model an initial, or seed, earthquake is specified. The subsequent earthquakes are obtained from the statistical distributions of magnitude, time, and location. The magnitude scaling is based on a combination of the Gutenberg–Richter scaling relation and the modified Båth's law for the scaling relation of aftershocks relative to the magnitude of the seed earthquake. Omori's law specifies the distribution of earthquake times, and a modified form of Omori's law specifies the distribution of earthquake locations. Since the BASS model is specified by the four scaling relations, it is fully self-similar. This is not the case for ETAS. We also give a deterministic version of BASS and show that it satisfies Tokunaga side-branching statistics in a similar way to diffusion-limited aggregation (DLA).

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Keywords: Aftershocks; Scaling; Branching; Epidemic-type models; Earthquakes; Hazard assessment

1. Introduction

Like many other natural processes, a fundamental property of earthquakes is the invariance of physics and dynamics across a vast range of spatial and temporal scales. Scale invariance is exemplified through well-known observational relationships such as the Gutenberg–Richter frequency–magnitude relation [1], the generalized Omori's law for aftershock decay [2], and the modified Båth's law for the magnitudes of aftershocks relative to the main shock [3]. Recent work on the temporal distribution of time between earthquakes [4] and the spatial distribution of distance between earthquakes [5] support the idea of scale invariance. The concept of self-similar branching introduced in this paper is identical to the self-similar branching studied by Gabrielov et al. [6] in a general context.

A major goal of earthquake studies is to quantify the risk of occurrence of an earthquake of a specified magnitude, in a specified area, and in a specified time window. This is done and results in hazard maps. Another goal is to specifically forecast or predict earthquakes. Many attempts have been made, but with only marginal success. The focus

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of this paper is to discuss both seismic hazard assessments and earthquake prediction in terms of our understanding of the physics and scaling of earthquakes. Two distinct approaches exist: a fault based approach and a seismicity based approach.

Fault based models consider the earthquakes that occur on recognized active faults. These models are also known as renewal models. Renewal models imply that the stress on an individual fault is "renewed" by the tectonic drive of plate tectonics. The simplest renewal model would be that of a single planar strike-slip fault subjected to a uniform rate of strain accumulation (plate motion). In this case, "characteristic" earthquakes would occur periodically. Clearly the earth's crust is much more complex with faults present at all scales and orientations. This complexity leads to chaotic behavior and statistical variability.

Renewal models have formed the basis for assessments of future earthquake probabilities in California [7]. A major problem with renewal models is that large earthquakes often occur on faults that were not previously recognized. Recent examples in California include the 1952 Kern County earthquake, the 1971 San Fernando Valley earthquake, the 1992 Landers earthquake, the 1994 Northridge earthquake, and the 1999 Hector Mine earthquake. At the time when these earthquakes occurred, the associated faults were either not mapped or were considered too small to have such large earthquakes.

An alternative approach to probabilistic seismic hazard assessment and earthquake forecasting is to use observed seismicity. The universal applicability of Gutenberg–Richter frequency–magnitude scaling allows the rate of occurrence of small earthquakes to be extrapolated to estimate the rate of occurrence and location of large earthquakes. This type of extrapolation played an important role in creating the national seismic hazard map for the United States [8].

A more formalistic application of this extrapolation methodology is known as a relative intensity (RI) forecast. This type of forecast was made on a world wide basis by Kossobokov et al. [9] and to California by Holliday et al. [10]. A related forecasting methodology is the pattern informatics (PI) method [11–16]. This method was used by Rundle et al. [11] to forecast $m = 5$ and larger earthquakes in California for the time period 2000–2010. This forecast successfully predicted the locations of 16 of the 18 large earthquakes that have subsequently occurred.

Keilis-Borok [17,18] and colleagues utilized patterns of seismicity to make formal intermediate term earthquake predictions. The most widely used algorithm, M8, has been moderately successful in the prediction of time and location of large earthquakes. More recently, this group has used chains of premonitory earthquakes to make intermediate term predictions [19,20]. Again, moderate success was achieved.

It has also been proposed that there is an increase in the number of intermediate sized earthquakes prior to a large earthquake. This proposal has been quantified in terms of an accelerated moment release (AMR) prior to a large earthquake. This approach has shown considerable success retrospectively [21–23] but has not evolved into a successful prediction algorithm as of yet.

Seismicity based models are often referred to as clustering models. That is, clusters of small earthquakes indicate the future occurrence of larger earthquakes. The RI, PI, and AMR models clearly belong to this class. Another approach in this class is the epidemic type aftershock sequence (ETAS) model. This approach was first formulated by Kagan and Knopoff [24]. It is a statistical model based on applicable scaling laws. This model was further developed by Ogata and colleagues [25–40]. A modified version of ETAS was introduced by Helmstetter, Sornette, and colleagues [41–62]. Related models have been developed by Felzer and colleagues [63–65], by Console and colleagues [66–68], by Gerstenberger and colleagues [69,70], and by Lombardi and Marzocchi [71].

The ETAS model recognizes that each earthquake has an associated sequence of aftershocks. A random number generator is used to determine the magnitude of each earthquake in an aftershock sequence. In some cases, the subsequent earthquake is larger than the first earthquake. In this case the first earthquake is a foreshock and the subsequent earthquake is the main shock. Each primary aftershock will generate second-order aftershocks, the second-order aftershocks will generate third-order aftershocks, and so forth. This branching process is controlled by a prescribed branching ratio. The ratio is chosen to approximate Båth's law [72,73]: on average the largest aftershock of a main shock has a magnitude 1.2 units less than the magnitude of the main shock. In this model, each event is capable of producing secondary aftershock sequences and can be considered simultaneously as a foreshock, main shock, or aftershock. The resulting aftershock sequence is a combined effect of many aftershock sequences produced by each aftershock.

Three scaling laws are associated with the statistics of aftershocks [74]. These are:

(1) The Gutenberg–Richter (GR) frequency–magnitude scaling [1]:

$$\log_{10}[N_{\rm as}(\geq m)] = a - bm, \tag{1}$$

where m is the aftershock magnitude, $N_{as}(\geq m)$ is the number of aftershocks with magnitude equal to or greater than m, and a and b are constants. The b-value is universally in the range of $0.8 < b < 1.2$.

(2) The modified form of Omori's Law [2]:

$$R(t) = \frac{dN_{as}}{dt} = \frac{1}{\tau(1 + t/c)^p}, \quad (2)$$

where $R(t)$ is the rate of aftershock occurrence as a function of the time t since the main shock, and τ, c, and p are parameters. The power-law parameter p is generally in the range of $1.0 < p < 1.2$.

(3) The Båth's law for the magnitude of large aftershocks [72]. This law states that it is a good approximation to assume that the difference in magnitude between the main shock and its largest aftershock is a constant independent of the magnitude of the main shock. That is,

$$\Delta m = m_{ms} - m_{as}^{max}, \quad (3)$$

with m_{ms} the main shock magnitude, m_{as}^{max} the magnitude of the largest aftershock, and Δm approximately a constant taken to be $\Delta m \approx 1.2$. A number of extensive studies of the statistical variability of Δm have been carried out [50,63,64,73,75–77]. For a sequence of California earthquakes Felzer et al. [63] give $\Delta m = 1.28 \pm 0.19$. A modified form of Båth's law based on the statistics of the entire aftershock sequence has been introduced by Shcherbakov and Turcotte [3]. This form will be introduced and discussed below.

An alternative to the ETAS model is the branching aftershock sequence (BASS) model [78]. As in the ETAS model, the BASS model recognizes that each earthquake has associated aftershocks. Each main shock produces a sequence of primary aftershocks. Each of these aftershocks, in turn, produces second-order aftershocks. Each second-order aftershock can produce third-order aftershocks, and so forth. Statistically, a primary aftershock can be larger than the initial main shock. In this case the initial main shock becomes a foreshock, and the larger primary aftershock becomes the main shock of the sequence. The primary difference between BASS and ETAS is that BASS utilizes the modified form of Båth's law as an additional constraint. This uniquely constrains the aftershock sequence without any further assumptions. We first consider a deterministic version of the BASS model to illustrate its behavior. We then give statistical simulations of the BASS model and discuss their relationship to ETAS.

2. Illustration of the BASS model

We first illustrate the principles of the BASS model using a deterministic branching formulation. This is in direct analogy to a deterministic illustration of a drainage network [79]. We consider the distribution of aftershocks associated with a main shock of a prescribed magnitude. The application of Båth's law introduces a characteristic earthquake magnitude Δm, the magnitude difference between the main shock and the largest aftershock. For convenience, we take $\Delta m = 1$ and consider the number of earthquakes with unit magnitudes, $m = 1, 2, 3, \ldots$. Our basic formulation introduces a branching ratio B that gives the number N_{ij} of daughter earthquakes (aftershocks) of magnitude i generated by a parent earthquake (main shock or aftershock) of magnitude j:

$$N_{ij} = B^{j-i-1}. \quad (4)$$

This basic branching is illustrated in Fig. 1 for binary branching ($B = 2$). A sequence of primary aftershocks is illustrated in Fig. 1(a). A $m = 5$ main shock has one $m = 4$ aftershock (Båth's law with $\Delta m = 1$), two $m = 3$ aftershocks, four $m = 2$ aftershocks, and eight $m = 1$ aftershocks. That is, $N_{45} = 1$, $N_{35} = B = 2$, $N_{25} = B^2 = 4$, $N_{15} = B^3 = 8$.

We extend the basic branching relation given in (4) to families of aftershocks. That is, we consider the aftershocks of aftershocks. The generalized branching relation is

$$N_{ijn} = N_{jn}N_{ij} = N_{jn}B^{j-i-1}, \quad (5)$$

where N_{ijn} is the number of magnitude i aftershocks generated by all the aftershocks of magnitude j that in turn were generated by a magnitude n main shock. The total number of aftershocks of magnitude i generated by a main shock of magnitude n, N_{in}, is given by

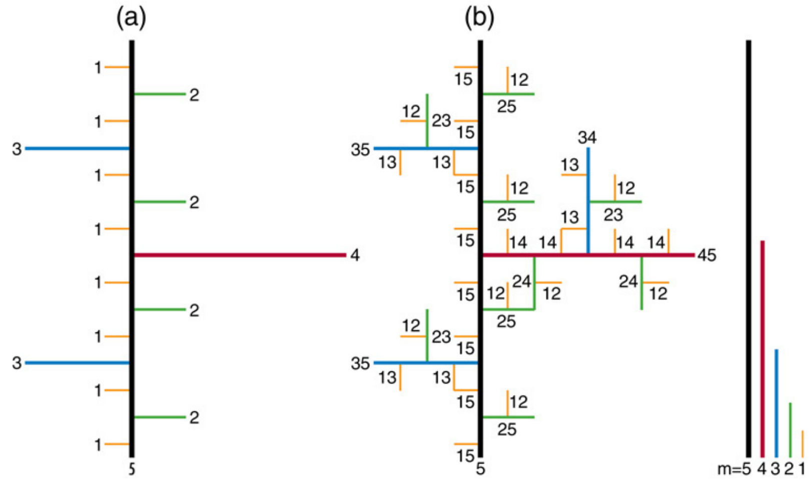

Fig. 1. Illustration of our branching model using a discrete set with binary ($B = 2$) branching. (a) The primary family of aftershocks is a generator for the fractal construction; (b) the full Tokunaga structure of side-branching aftershocks.

$$N_{in} = \sum_{j=i+1}^{n} N_{ijn} = (B+1)^{n-i-1}. \tag{6}$$

The side-branching structure of aftershocks of aftershocks is best illustrated by an example.

The full binary ($B = 2$) side-branching structure of aftershocks for a magnitude 5 main shock is given in Fig. 1(b). The second-order aftershocks of the primary aftershocks are denoted "34", "24", "14" for the $m = 4$ primary aftershock, the second-order aftershocks of the $m = 3$ primary aftershocks are denoted "23" and "13", and so forth. The branch numbers N_{ij} constitute a square upper-triangular matrix. The matrix corresponding to the aftershock family given in Fig. 1(b) is given in Table 1(a). It can be shown that this branching also satisfies Tokunaga side branching statistics. To show this, we introduce branching ratios T_{ij} defined by

$$T_{ij} = \frac{N_{ij}}{N_j}, \tag{7}$$

where T_{ij} is the number of aftershocks with magnitude i for each parent with magnitude j. The branching ratios T_{ij} also constitute a square upper-triangular matrix. The matrix corresponding to the aftershock family given in Fig. 1(b) is given in Table 1(b). Self-similar networks are defined to satisfy the condition

$$T_{ij} = T_k, \tag{8}$$

where $k = j - i$. Tokunaga [80] introduced a more restricted class of self-similar networks by requiring

$$T_k = a \cdot c^{k-1}. \tag{9}$$

Substitution of (5) into (7) gives

$$T_{ij} = T_k = B^{k-1}. \tag{10}$$

Thus, our aftershock sequence satisfies the definition of Tokunaga self-similar networks given in (9) with $a = 1$ and $c = B$. The construction given above is binary, but it can be extended to higher orders to give a range of b-values. It can also be modified to accommodate an arbitrary value of Δm. The result is a self-similar branching structure that satisfies the same Tokunaga [80] statistics as drainage networks [81]. This self-similar structure is also applicable to diffusion limited aggregation (DLA) [82] and to models that exhibit self-organized criticality [6].

3. Generalized BASS model

In the BASS model, every earthquake (the parent earthquake) generates a sequence of aftershocks (daughter earthquakes). We generate the magnitude m_d, the time delay after the occurrence of the parent earthquake t_d,

Table 1
Tabulation of the branching properties of the idealized aftershock sequences illustrated in Fig. 1(b)

(a)	$N_{12} = 9$	$N_{13} = 6$ $N_{23} = 3$	$N_{14} = 4$ $N_{24} = 2$ $N_{34} = 1$	$N_{15} = 8$ $N_{25} = 4$ $N_{35} = 2$ $N_{45} = 1$	$N_1 = 27$ $N_2 = 9$ $N_3 = 3$ $N_4 = 1$
(b)		$T_{12} = 1$	$T_{13} = 2$ $T_{23} = 1$	$T_{14} = 4$ $T_{24} = 2$ $T_{34} = 1$	$T_{15} = 8$ $T_{25} = 4$ $T_{35} = 2$ $T_{45} = 1$

(a) Branch number matrix; and (b) branching ratio matrix.

and the distance from the parent earthquake r_d for every daughter earthquake using four basic scaling laws: a Gutenberg–Richter frequency–magnitude relation, a modified form of Båth's law, a generalized Omori's law, and a spatial scaling law analogous to the temporal Omori's law.

We require that the frequency–magnitude distribution for each sequence of aftershocks satisfy the Gutenberg–Richter (GR) frequency–magnitude relation given in (1):

$$\log_{10}[N_d(\geq m_d)] = a_d - b_d m_d, \tag{11}$$

where m_d is the magnitude of a daughter earthquake, $N_d(\geq m_d)$ is the number of daughter earthquakes with magnitudes greater than or equal to m_d, and a_d and b_d are the a- and b-values of the distribution, respectively. Note that the b-value b_d for each sequence of aftershocks is not necessarily equal to the b-value for all aftershocks. This is due to the superposition of many generations of aftershock sequences for each parent earthquake.

In order to fully specify the frequency–magnitude distribution of a family of aftershocks, we apply the modified form of Båth's law [3]. As shown by Shcherbakov et al. [74], this formulation is closely related to that given by Reasenberg and Jones [83], Yamanaka and Shimazaki [84], Felzer et al. [63].

Shcherbakov and Turcotte [3] introduced a new way of defining this difference and obtained a value Δm^* based on the entire distribution of aftershocks, not just the largest aftershock. It is required that the magnitude of the largest aftershock inferred from the GR relation is a fixed value Δm^* less than the magnitude of the parent earthquake, m_p:

$$N_d(\geq(m_p - \Delta m^*)) = 1. \tag{12}$$

With this condition we require (using (11)) that $a_d = b_d(m_p - \Delta m^*)$ so that

$$\log_{10}[N_d(\geq m_d)] = b_d(m_p - \Delta m^* - m_d). \tag{13}$$

Shcherbakov and Turcotte [3] considered the applicability of Båth's law and the modified form of Båth's law to ten large, well documented earthquakes that occurred in California between 1987 and 2003. For Båth's law defined in (3), the mean value was $\Delta m = 1.16$ with a standard deviation $\sigma_{\Delta m} = 0.46$. For the modified form of Båth's law, it was found that $\Delta m^* = 1.11$ with a standard deviation $\sigma_{\Delta m^*} = 0.29$. The variability of Δm^* associated with the modified form of Båth's law was found to be significantly less than the variability of Δm associated with the original Båth's law. Similar results for earthquakes in Japan have been given by Nanjo et al. [85].

In order to terminate the sequence of aftershocks (daughter earthquakes), it is necessary to specify a minimum magnitude earthquake $m_{\min}$ in the sequence. From (13), the total number of daughter earthquakes $N_{d\mathrm{T}}$ is given by

$$N_{d\mathrm{T}} = N(\geq m_{\min}) = 10^{b_d(m_p - \Delta m - m_{\min})}. \tag{14}$$

From (13) and (14) we obtain the cumulative distribution function $P_{\mathrm{C}m}$ for the magnitudes of the daughter earthquakes:

$$P_{\mathrm{C}m} = \frac{N_d(\geq m_d)}{N_{d\mathrm{T}}} = 10^{-b_d(m_d - m_{\min})}. \tag{15}$$

For each daughter earthquake a random value $0 < P_{\mathrm{C}m} < 1$ is generated, and the magnitude of the earthquake is determined from (15). Note that there is a finite probability that a daughter earthquake can be bigger than the parent earthquake.

We require that the time delay t_d until each daughter earthquake after the parent earthquake satisfies the generalized form of Omori's law given in (2):

$$R(t_d) = \frac{\mathrm{d}N_d}{\mathrm{d}t} = \frac{1}{\tau(1+\frac{t_d}{c})^p}, \tag{16}$$

where $R(t_d)$ is the rate of aftershock occurrence and τ, c, and p are parameters. The number of daughter aftershocks that occur after a time t_d is then given by

$$N_d(\geq t_d) = \int_{t_d}^{\infty} \frac{\mathrm{d}N_d}{\mathrm{d}t} = \frac{c}{\tau(p-1)(1+\frac{t_d}{c})^{p-1}}. \tag{17}$$

The total number of daughter earthquakes $N_{d\mathrm{T}}$ is obtained by setting $t_d = 0$ in (17) with the result

$$N_{d\mathrm{T}} = \int_0^{\infty} \frac{\mathrm{d}N_d}{\mathrm{d}t} = \frac{c}{\tau(p-1)}. \tag{18}$$

From (17) and (18) we obtain the cumulative distribution function $P_{\mathrm{C}t}$ for the time of occurrence of the daughter earthquakes:

$$P_{\mathrm{C}t} = \frac{N_d(\geq t_d)}{N_{d\mathrm{T}}} = \frac{1}{(1+t_d/c)^{p-1}}. \tag{19}$$

For each daughter earthquake a random value $0 < P_{\mathrm{C}t} < 1$ is generated, and the time of occurrence of the earthquake is determined from (19).

The distribution of time is dependent only on the fitting parameters c and p. In our model these parameters for each generation of aftershocks are assumed to be equal. The values for the superposition of many generations of aftershocks, however, may be different.

We utilize a spatial form of Omori's law to specify the location of each daughter earthquake. The cumulative distribution function $P_{\mathrm{C}r}$ for the radial distance r_d of each daughter earthquake from the parent earthquake is given by

$$P_{\mathrm{C}r} = \frac{N_d(\geq r_d)}{N_{d\mathrm{T}}} = \frac{1}{(1+r_d/(d\cdot 10^{0.5m_p}))^{q-1}}. \tag{20}$$

The dependence on the magnitude m_p of the parent earthquake introduces a mean radial position of aftershocks that scales with the rupture length of the parent earthquake. The direction of each daughter earthquake is chosen randomly within the uniform range $0 < \theta_d < 2\pi$.

In order to generate a full aftershock sequence, we perform the following steps:

(1) An initial (seed) earthquake with magnitude m_s is specified. This "main shock" defines time $t = 0$ and position $r = 0$.
(2) The total number of primary aftershocks is obtained from (14) after a minimum aftershock magnitude $m_{\min}$ has been specified.
(3) For each of the primary aftershocks, three random numbers in the range $P_{\mathrm{C}} \in (0, 1)$ are selected. Using these random cumulative probabilities the magnitude m_d, the time of occurrence t_d, and the radial distance r_d of each aftershock is determined from (15), (19) and (20). A random number in the range $\theta_d \in (0, 2\pi)$ gives the direction of each aftershock relative to the seed earthquake.
(4) Each of the primary aftershocks are then treated as a parent earthquake and the entire procedure is repeated for each parent to generate an ensemble of second-order aftershock sequences. The procedure is further extended to third- and higher-order aftershock sequences until no more aftershocks are generated.

In order to constrain a BASS simulation it is necessary to specify six parameters: b_d, Δm, c, p, d, and q. For the GR–Båth parameters, we take $b = 1$ and $\Delta m = 1.2$ [3]. For the temporal Omori parameters, we take $c = 0.1$ days and $p = 1.25$ [2]. For the spatial Omori parameters, we take $d = 4.0$ m and $q = 1.35$ [86].

For our sample simulation, we consider an $m_s = 7$ earthquake and take $m_{\min} = 2.0$. From (14) we have $N_{d\mathrm{T}} = 6310$ primary aftershocks. Magnitude as a function of event occurrence time for the $m = 7.0$ seed sequence is presented in Fig. 2. From this plot it is easy to see that large aftershocks generate their own aftershock sequences using

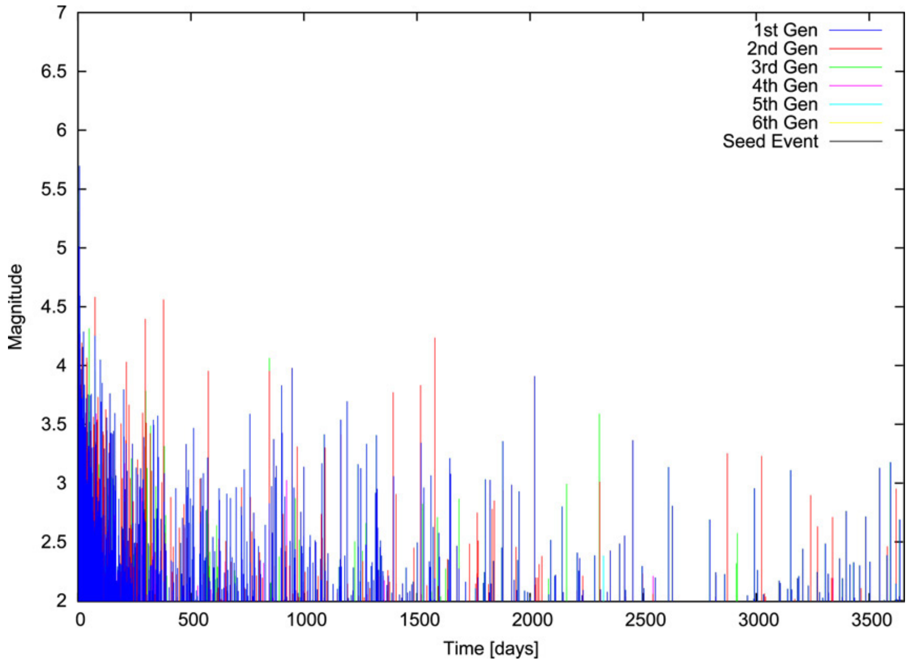

Fig. 2. Plot of magnitude as a function of time (in days) over the first year for the first four generations of an aftershock sequence based on an $m = 7.0$ seed event at time $t = 0$. Note that large aftershocks generate their own aftershock sequences.

this model. A similar plot of aftershock position as a function of time is given in Fig. 3. It is seen that the aftershocks for each generation are clustered about their respective parent earthquakes. Gutenberg–Richter frequency–magnitude relations for the aftershock sequences are presented in Fig. 4.

There are 8491 total aftershocks in the simulation, and therefore 2181 are second- and higher-order aftershocks. The magnitude of the largest aftershock in this realization is $m = 5.7$ so that $\Delta m = 1.3$. The frequency–magnitude distribution for all aftershocks is well approximated by the scaling relation (13) taking $b = 1$ and $\Delta m^* = 1.0$. With these values, a well defined foreshock would be expected 10% of the time. This is in reasonable agreement with observed values of $13 \pm 5\%$ [87].

4. BASS versus ETAS

There are many similarities between the BASS model and the ETAS model, but there are also fundamental differences. Both utilize the concept of multiple orders of aftershocks. The main shock generates a sequence of primary aftershocks, these in turn generate families of secondary aftershocks, and so forth. In considering the ETAS model we will utilize the formulation given by Helmstetter and Sornette [48]. In our notation, their Eq. (2) becomes

$$N_{dT} = K 10^{\alpha m_p}. \tag{21}$$

Their other basic ETAS equations (3)–(5) are essentially identical to our BASS equations (16), (20) and (15). If (18) and (21) are identical then ETAS is essentially identical to BASS. This is the case if

$$\alpha = b_d \tag{22}$$

and

$$K = 10^{-b_d \Delta m^* + m_{\min}}. \tag{23}$$

Felzer et al. [63] argue that (22) is in fact satisfied. Proponents of ETAS, however, require that $\alpha < b_d$. A discussion of this relation and its application to Båth's law has been given by Helmstetter and Sornette [50] who favor $\alpha \approx 0.8$.

5. Discussion

Scale invariance is a fundamental aspect of seismology. The Gutenberg–Richter frequency–magnitude relation (1) is the key to this behavior. This fractal relation cannot be defined as a probability distribution function (pdf) because

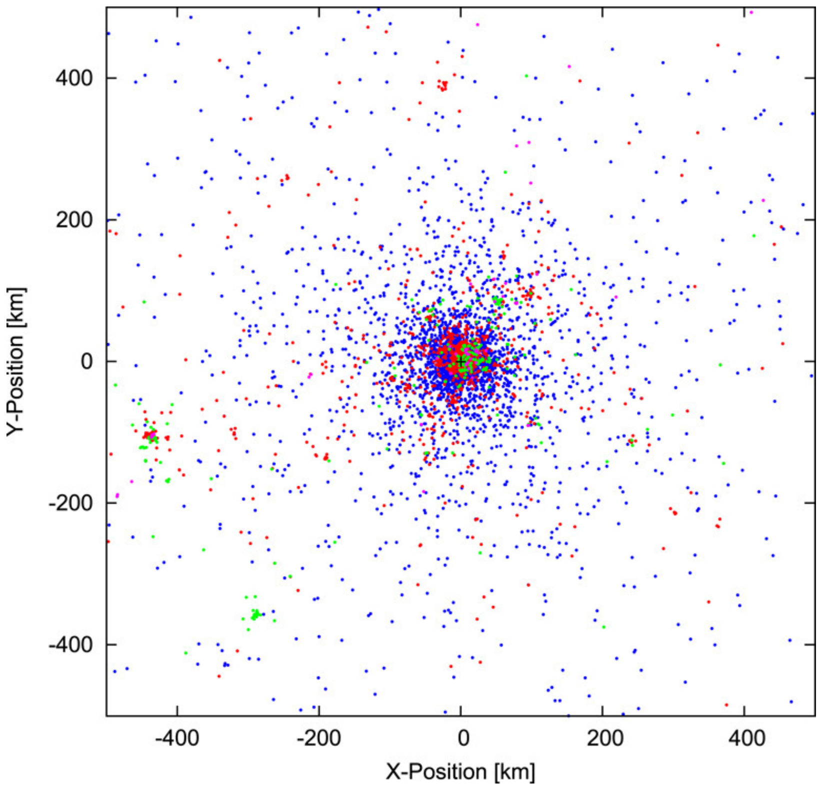

Fig. 3. Plot of aftershock position for the first four generations of an aftershock sequence based on an $m = 7.0$ seed event at location $r = 0$. Note that each generation's aftershocks (color coded) are clustered about their respective main shocks. Only plotted are aftershocks that fall within a 500 km box surrounding the main shock. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

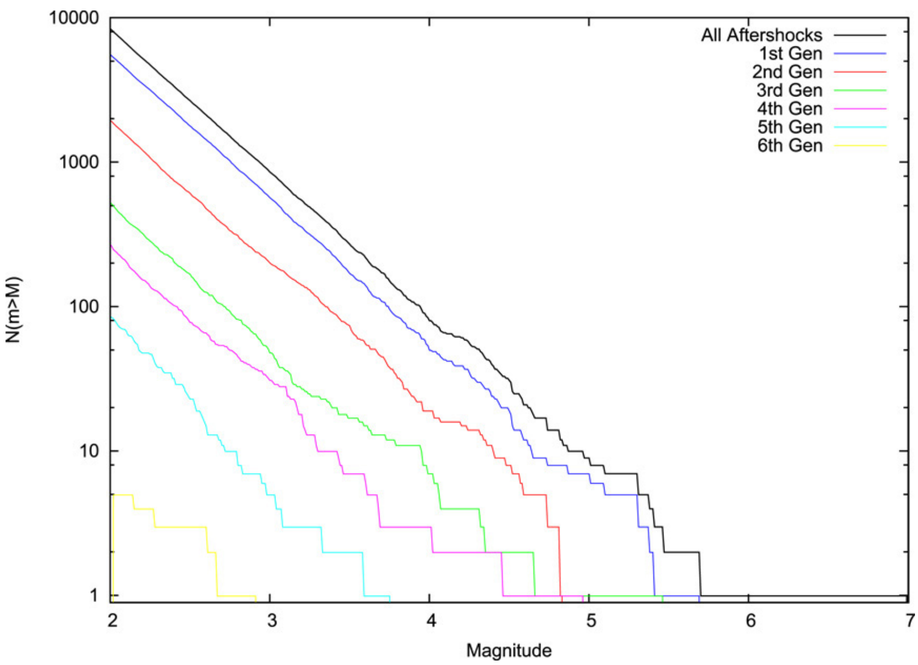

Fig. 4. Plot of the Gutenberg–Richter frequency–magnitude relation for an aftershock sequence based on an $m = 7.0$ seed event (this particular realization consisted of 6 generations).

the total number of earthquakes cannot be defined. Although there must certainly be a smallest earthquake, practically there are so many earthquakes below the detection limit that the lower threshold is meaningless. The Båth's law is a second consequence of scale invariance in seismicity. The relation of aftershock statistics to the main shock is independent of the main shock magnitude.

Our BASS model is also scale invariant. The aftershock sequence is independent of the main shock magnitude. It is only necessary to specify the difference $m_{\mathrm{p}} - m_{\min}$. This is required in the same way as the instrumental limitation restricts the applicability of GR statistics. The BASS model is fractal in the same sense that the Gutenberg–Richter relation is fractal. The fractality of the BASS model was illustrated in Fig. 1. The ETAS model was based on branching considerations. The key parameter was the average number n of daughter earthquakes created per mother [47]. In order to keep n finite it was necessary to take $\alpha < b_d$. This precludes the self-similar BASS model and the applicability of Båth's law.

We have presented a single BASS simulation in this paper. Clearly, many simulations must be made to understand the implications of our new model. These implications include: (1) the statistics of foreshock occurrence in terms of time, space, and magnitude; (2) whether the c-value for total sequences have a magnitude dependence; (3) the relationship of aftershock occurrence to background seismicity; and (4) the correlation statistics of all earthquakes, both in space and time. We believe our self-similar branching model is likely to find applicability in other fields, specifically biology.

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