CS 391L HW2: Independent Component Analysis

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Abstract

This assignment is aimed at learning Independent Component Analysis (ICA) for Blind Signal separation. Let's say we have m signals of length t, which are actually linear combinations of n (n<=m) independent signals. Now we want to construct back the original n signals. It can be done using Gradient Descent. Here, for a particular set of 5 independent signals, their various linear combinations are considered and their reconstructions are analyzed.

1. Introduction

Informally, blind signal separation is the separation of the source signals from a set of mix signal with out using any additional information. Variety of approaches exist to perform such tasks. Major algorithms include SVD, PCA, Non negative matrix factorization. Independent Component Analysis (ICA) is one such algorithm.

ICA mainly makes two assumptions about the source signals: 1) they are non-Gaussian signals and 2) they are statistically independent. One of the main applications of it is in Cocktail party problem, which is mainly listening to one person's speech in noisy room. In this assignment also, we mix different sound signals and try to reconstruct them. Different ways of mixing and their results are analyzed here.

2. Dataset

There are mainly 5 sound signals each of length around length 4 sec, and they are sampled 11025 samples per sec. Total length in terms of samples is 44000. 2 of the signals (first and fourth) are similar in the sense that they have small dense chunks, while other 3 signals have are more or less uniformly distributed across the time. We will use these information while analyzing reconstruction of mixture of different subsets of these 5 signals.

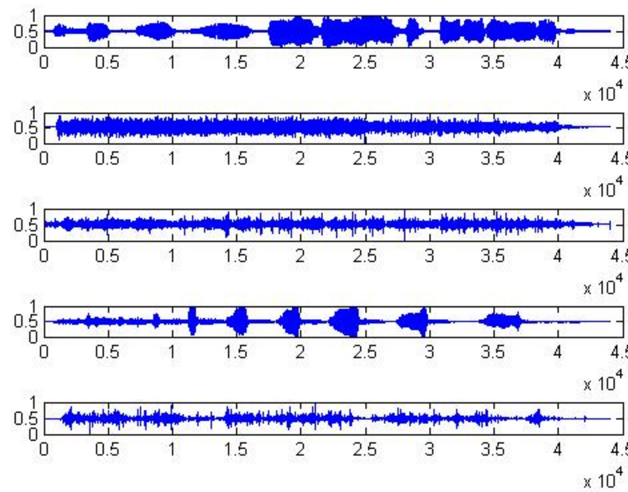

Figure 1. Original Source Signals

3. Method

First of all, we mix the source signals to generate linear combinations of them, which will serve as input to our algorithm.

Remember that number of mixed signals (m) need to be at least the number of source signals (n). To do so, we generate a random mxn matrix A, which has values ranging from 0 to 0.1.

Once we have matrix A, we can use it to generate mix signals from the original ones. Let's say we have matrix U of size nxt, where each row is a source signal of length t. Now by multiplying A with U, we get a new mxt matrix X, which represents m mixed signals, each of length t.

Now, our goal is to reconstruct the original signals from the mixed ones. To do so, we need a nxm matrix W that, we can multiply with X in order to get a reconstruction of nxt matrix Y. The only information we have is the X matrix (mixed signals) and the number of source signals n. From that we need W matrix and eventually Y matrix. We can use gradient descent algorithm for that purpose in the following manner.

First of all, initialize the matrix W with random initial values. Calculate $Y = W$ x X. It is our first estimate of source signals. Now, we need to iteratively update W and Y. For that, we need some direction along, which we can perform gradient descent. Now, as our initial signals were statistically independent, we can use maximum information separation as a direction along which, gradient descent can be performed. To do so, we take element wise sigmoid of each of the entries in Y and store them in another matrix of same dimensions Z. Now we update the W according to following equation.

$\Delta W = \eta(I + (1 - 2Z)Y')W$

$W = W + \Delta W$

Now, we again start from calculating $Y = W$ x X, and repeat this procedure until convergence or maximum iterations reached. This way, the last calculated Y will be the final reconstruction of our source signals. Reconstructed signals may not be in the same order as the source signals.

In the next section, we analyze the results of this approach on given source signals.

4. Experiments and Results

All the experiments presented in this section mainly involves variation of 3 parameters:

1) learning rate η. (0.1,0.01,0.001)

2) number of maximum iterations. (0.1m, 0.3m, 0.5m)

3) Various subsets of 5 source signals that are mixed.

Each of the following subsections correspond to different signals mixed (parameter 3) and analyzes their reconstructions with respect to different learning rates η (parameter 1) and number of iterations (parameter 2).

4.1. Mixing all the 5 signals

We mix all the 5 source signals and create new 5 signals. Then we use the mentioned approach with different learning rates and iterations mentioned. Reconstruction results are shown in 2 to 10. It is easy to see that their quality increases with number of iterations and learning rate of 0.1 seems to be working best. Maybe 0.01 and 0.001 will take too long to reach the minimum error. In either case (small steps and large iterations or large steps and small iterations), we are not moving enough down the gradient. So, in those cases, we can see that reconstructed signals still have information from other signals and they are not fully separated in terms of maximum information separation. The best configuration here is highest no. of iterations 0.5m and largest learning rate of 0.1. Here for illustration purpose, all the 9 figures are presented. From the next subsections, the best and the worst performing figures will be presented.

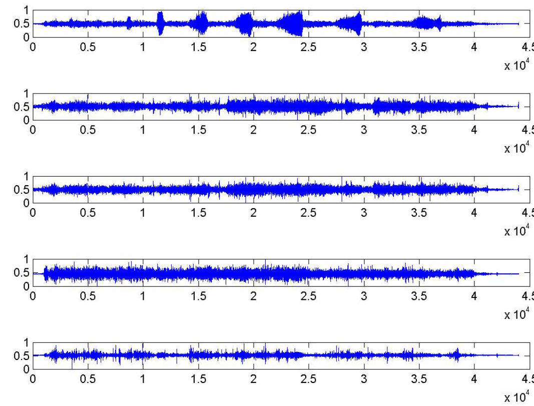

Figure 2. 5x5, LR=0.001, itr=0.1m

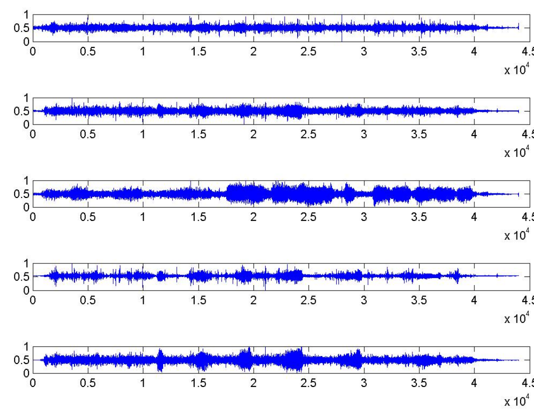

Figure 3. 5x5, LR=0.01, itr=0.1m

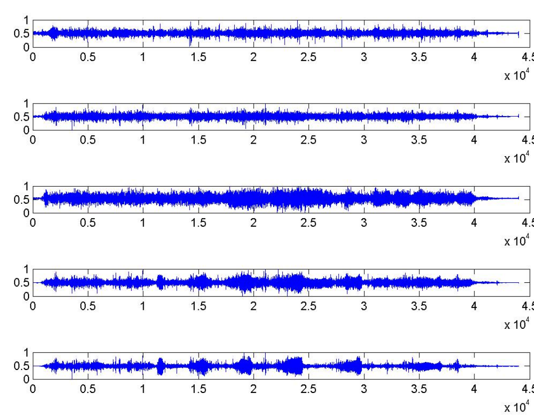

Figure 4. 5x5, LR=0.1, itr=0.1m

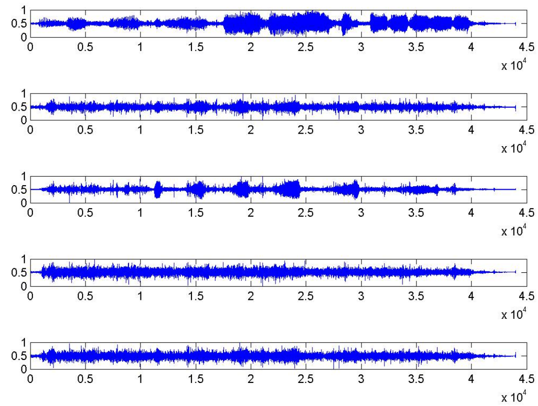

Figure 5. 5x5, LR=0.001, itr=0.3m

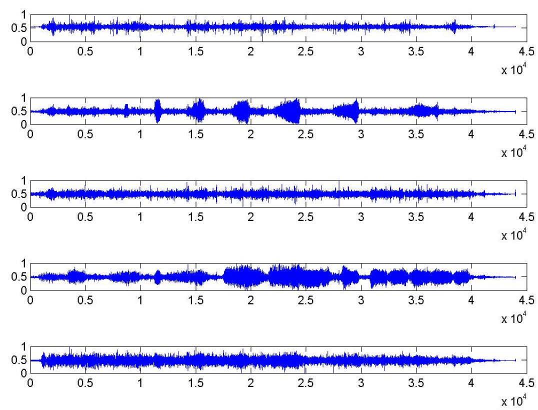

Figure 6. 5x5, LR=0.01, itr=0.3m

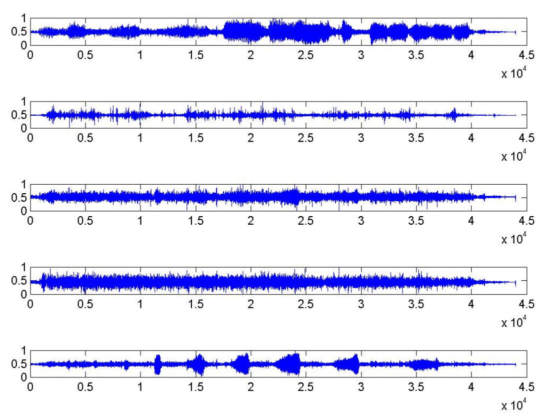

Figure 7. 5x5, LR=0.1, itr=0.3m

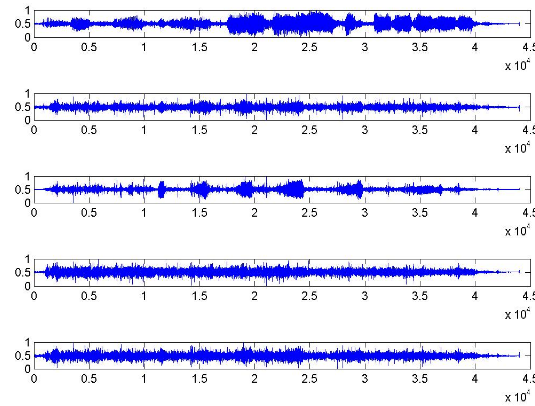

Figure 8. 5x5, LR=0.001, itr=0.5m

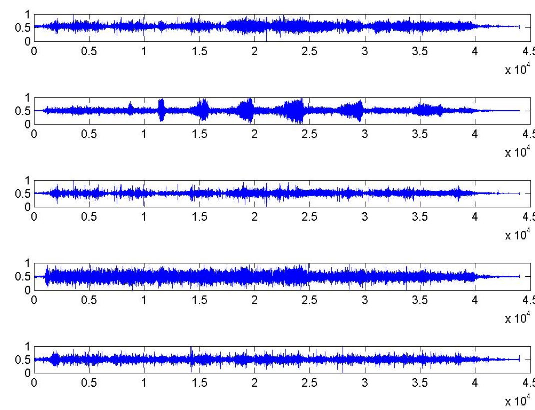

Figure 9. 5x5, LR=0.01, itr=0.5m

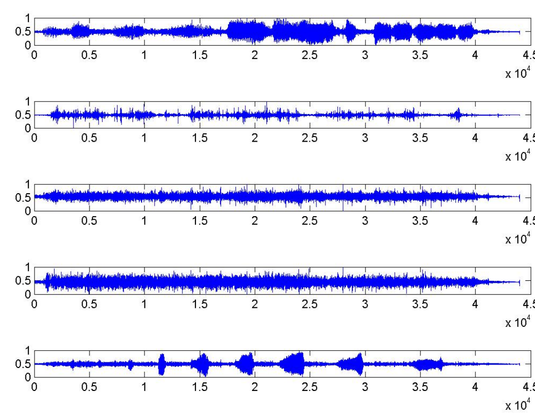

Figure 10. 5x5, LR=0.1, itr=0.5m

4.2. Mixing signals 1 and 4(Containing dense chunks)

Now, we try to see whether mixing just these 2 signals ends in better reconstruction than mixing all of them or not. 2 new mixed signals are created. For these signals also, experiments were run on 9 configurations and similar trends were observed, i.e., more iterations and higher learning rate helped improve upon the quality. Worst and best results are shown in 11 and 12 respectively. We can see that first one has much more noise than the second one, and dense chunks are easily separable in the later one. Slightly observable improvement was found in the quality with respect to the previous ones. Main reason can be that, this time not continuous disturbance was present in the mixing due to other 3 signals.

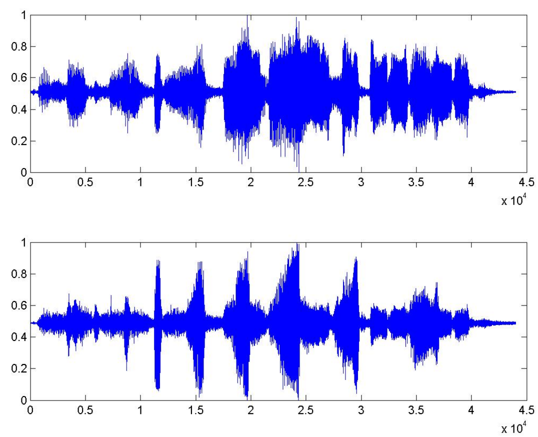

Figure 11. 2x2, LR=0.001, itr=0.1m

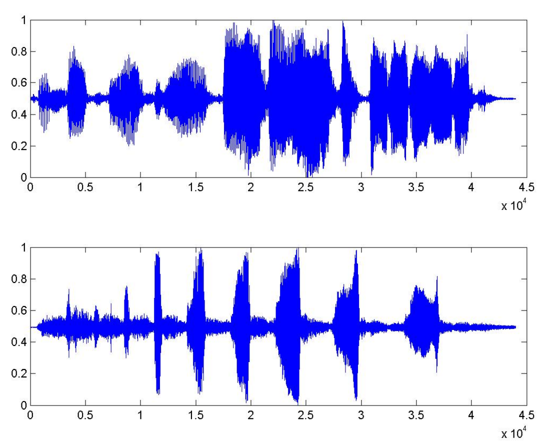

Figure 12. 2x2, LR=0.1, itr=0.5m

4.3. Mixing signals 2,3 and 5(Containing uniform denseness throughout the time)

Again, in these part we try mixing remaining 3 signals, which more or less have uniform density through out the time. 3 new mixed signals are created. Here, for no. of iterations trends were same, but learning rates 0.01 and 0.001 found to be performing equally bed. Only learning rate 0.1 could perform better. From the analysis, it looks like these 3 signals are much harder to separate. Also, as expected, the quality of best performing case, was slightly poorer than when all the 5 signals were mixed; which is exactly opposite to previous section. Reconstructions are shown in 13 (bad quality) and 14 (good quality).

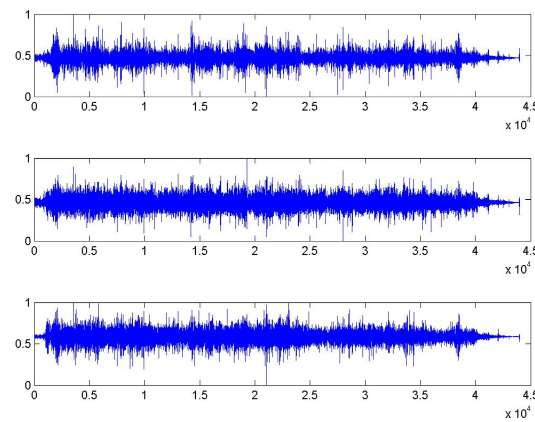

Figure 13. 3x3, LR=0.01, itr=0.1m

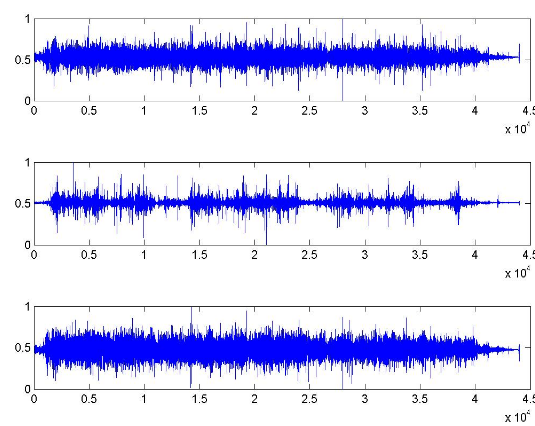

Figure 14. 3x3, LR=0.1, itr=0.5m

4.4. Mixing signals 1, 3, 4 and 5

Now, we take 2 signals from both the previous types and produce new 4 mixed signals. Apart from normal trends;

here, an interesting observation came into notice. For configurations with learning rate 0.01, 2 of the 4output signals were almost same, while for the learning rate 0.1 it was well separated. It seems like at the learning rate of 0.01, both these signals have same information and afterwards by increasing the learning rate they move in the opposite direction in terms of gaining maximum information separation. This can be an example that the algorithm descents on the gradient indicating maximum information separation. Reconstructions are shown in 15 and 16. We can easily see that in 15, row 2 and row 3 are almost same.

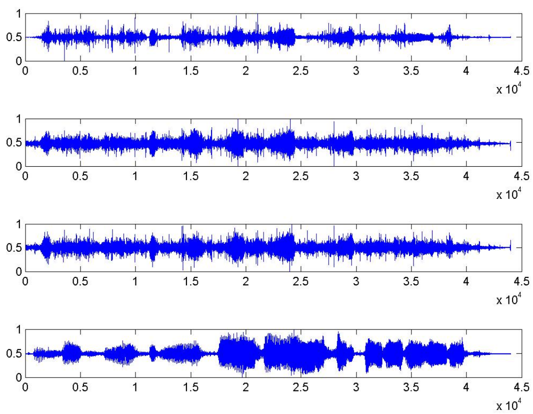

Figure 15. 4x4, LR=0.01, itr=0.5m

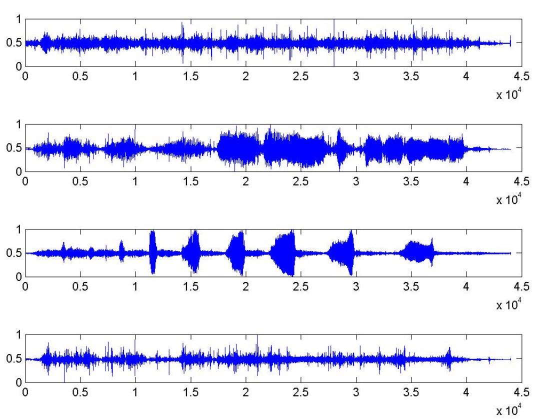

Figure 16. 4x4, LR=0.1, itr=0.5m

4.5. Mixing signals 1 and 5

This is almost same as previous section. But instead of taking 2 signals of each type, we just take 1 signal of each type and create 2 new signals. Here, a bit different trends were observed. Performance did not change at all for different no. of iterations and learning rate of 0.01 turned out to be the best. It might be the case as there are only 2 signals, and they are of different type, so it is some how much easier to separate them. So, convergence may have occurred much before 0.1m iterations. And more iterations might not make any sense. Also for learning rate the best explanation is, 0.001 might be to small to converge while with 0.1 it might start diverging. So, 0.01 maybe the best fit. Reconstructions are shown in 17 and 18.

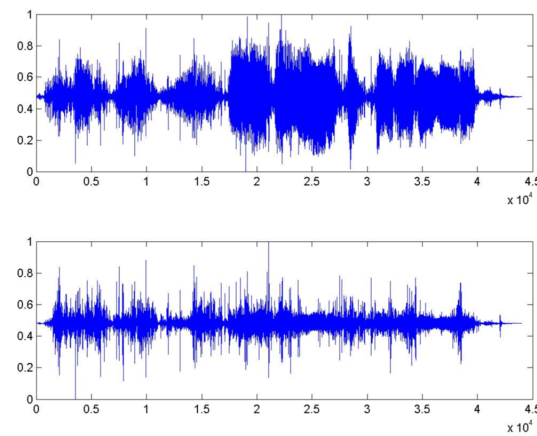

Figure 17. 2x2, LR=0.01, itr=0.5m

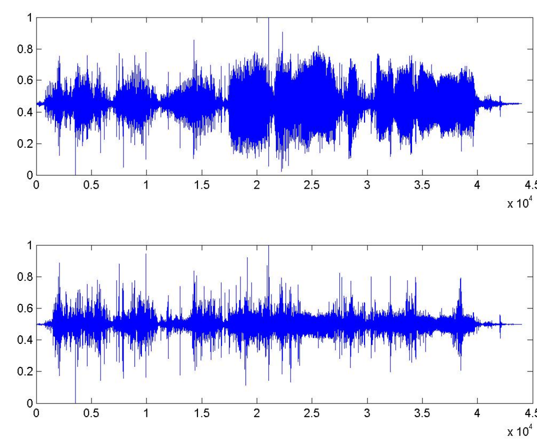

Figure 18. 2x2, LR=0.1, itr=0.5m

So, in this section we analyzed the reconstructed signals for various combinations of source signals and with different parameter configurations for learning rate η and no. of iterations.

5. Conclusion

In this paper, we implemented Independent Component Analysis (ICA) for the task of blind source separation. We had 5 source signals, which were statistically independent. We created signals which were linear combinations of various subsets of these 5 signals. We analyzed how good original signals in that subset were reconstructed from the mix signals, with different parameter configurations. It was found that in general learning rate η and no. of iterations are main parameters, but type and no. of signals being mixed can also affect the final performance and can have impact on the values of those parameters. Finally from the reconstruction results, we can say that Independent Component Analysis (ICA) is indeed a good method for blind source separation and though recovery is not perfect, it is extremely close to the original source signals.