Analysis of a Complex Kind

Week 1

Lecture 1: History of Complex Numbers

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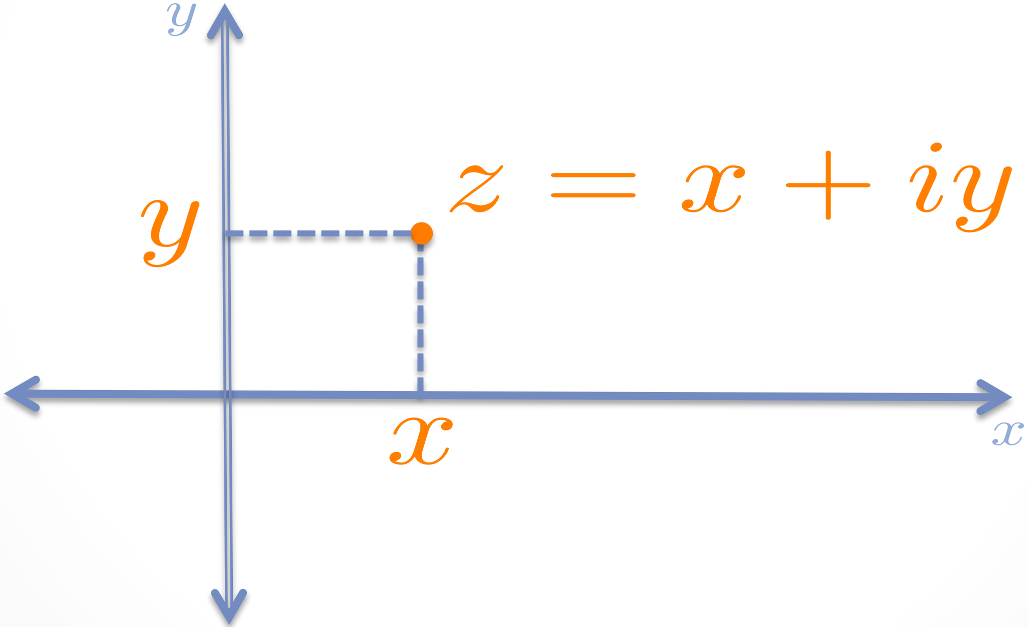
Welcome

About me:

- Petra Bonfert-Taylor
- Born, raised and educated in Germany (Berlin).
- Ph.D. 1996, Technical University of Berlin.
- Postdoc at University of Michigan.
- Professor at Wesleyan University since 1999.

About this Course

- Complex numbers, their geometry and algebra.



About this Course

- Historical explorations

connectedness

Riemann

e

Cauchy

Weierstrass

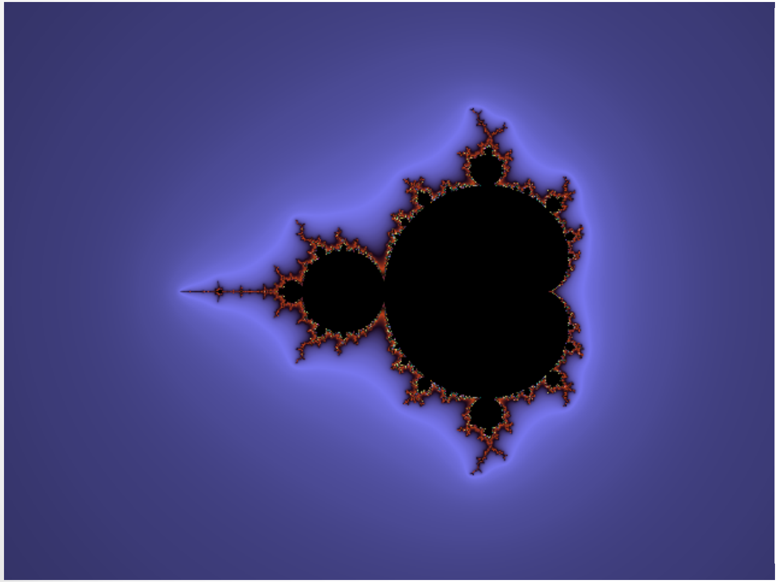
lim

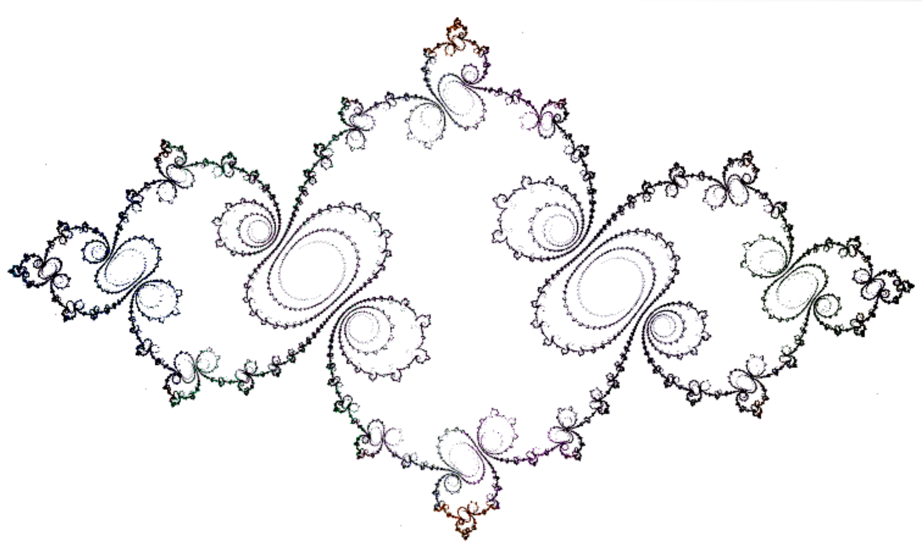
i

open set

About this Course

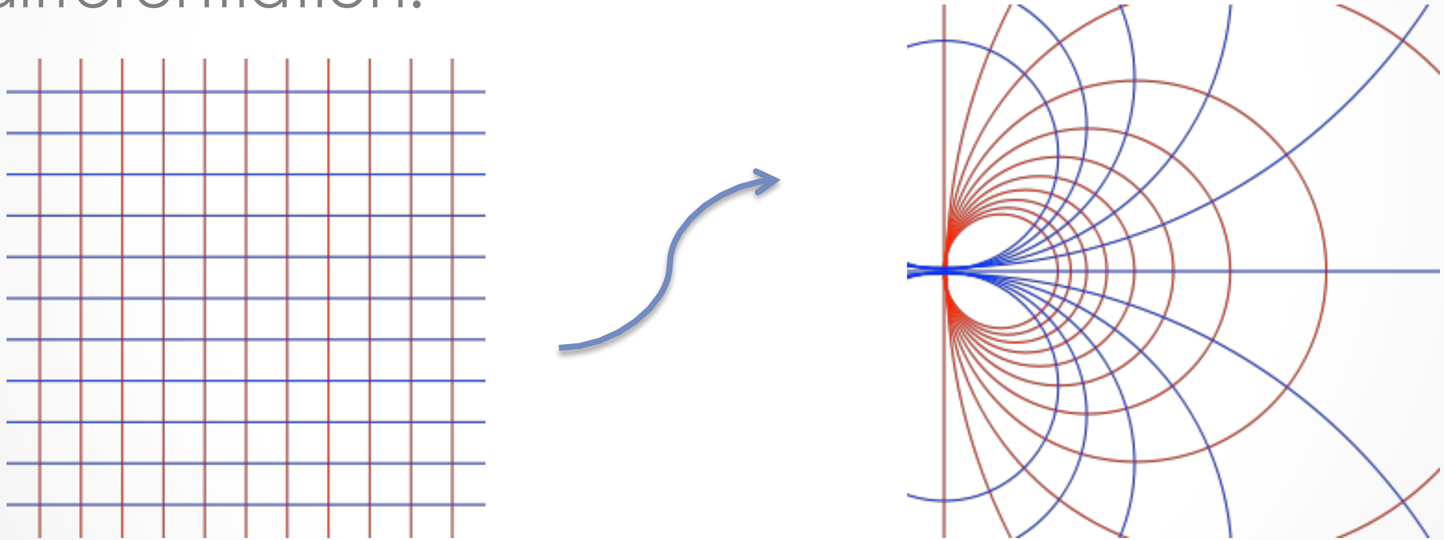
- Complex dynamics: Mandelbrot set, Julia sets.





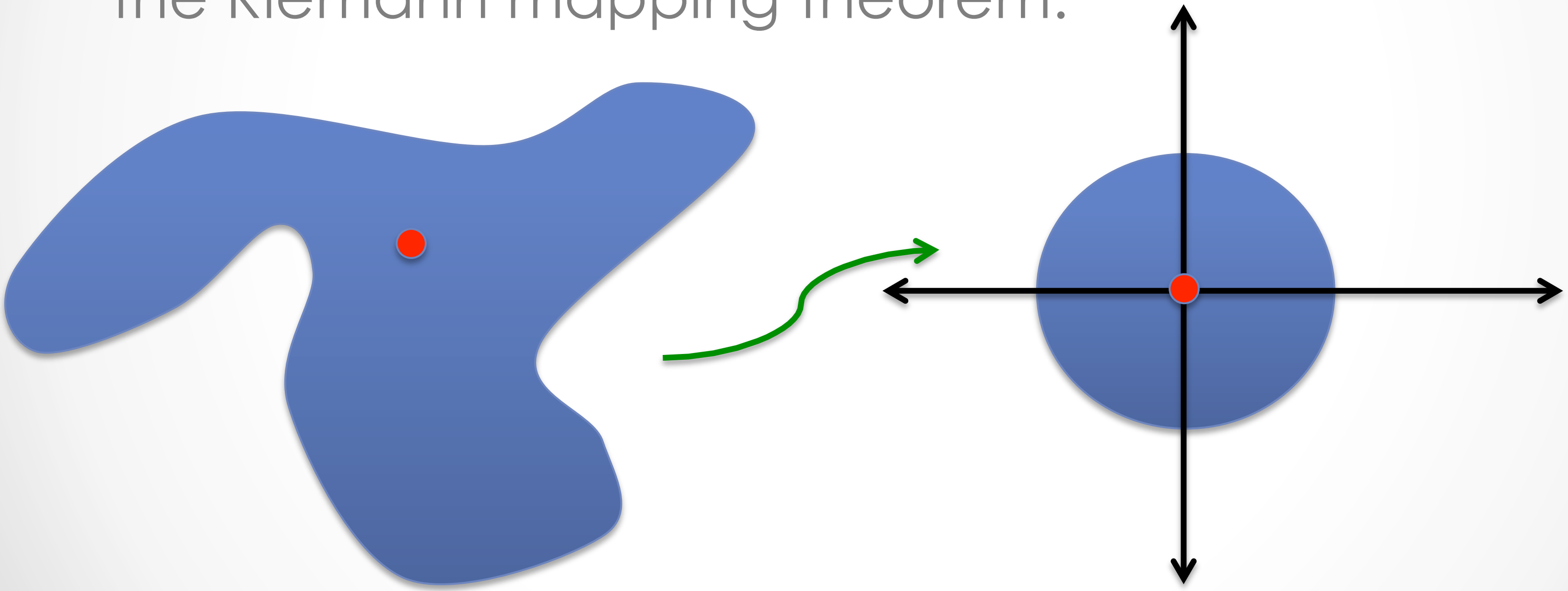
About this Course

- Complex functions, continuity, complex differentiation.



About this Course

- Conformal mappings, Möbius transformations and the Riemann mapping theorem.



About this Course

- Complex integration, Cauchy theory and consequences.

$$f(a) = \frac{1}{2\pi i}\oint_\gamma \frac{f(z)}{z-a}dz$$

Fundamental Theorem of Algebra:

$$a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0$$

$$= \quad a_n(z-z_1)(z-z_2)\cdots(z-z_n)$$

About this Course

- Power series representation of analytic functions, Riemann hypothesis.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

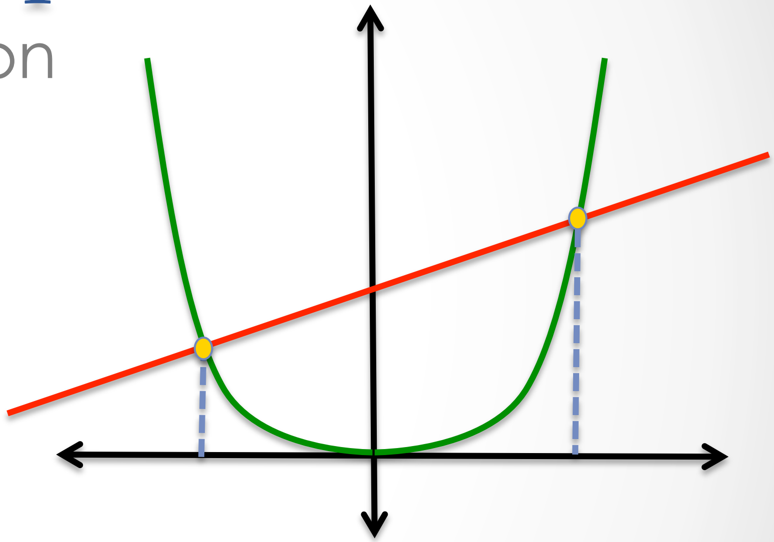
prime numbers?

$$= \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

Brief History of Complex Numbers

- Consider a quadratic equation

$$x^2 = mx + b$$



- Solutions are

$$x = \frac{m}{2} \pm \sqrt{\frac{m^2}{4} + b}$$

and represent intersection of $\boxed{y = x^2}$ and $\boxed{y = mx + b}$.

Solutions: $x = \frac{m}{2} \pm \sqrt{\frac{m^2}{4} + b}$.

- What if $\frac{m^2}{4} + b < 0$?

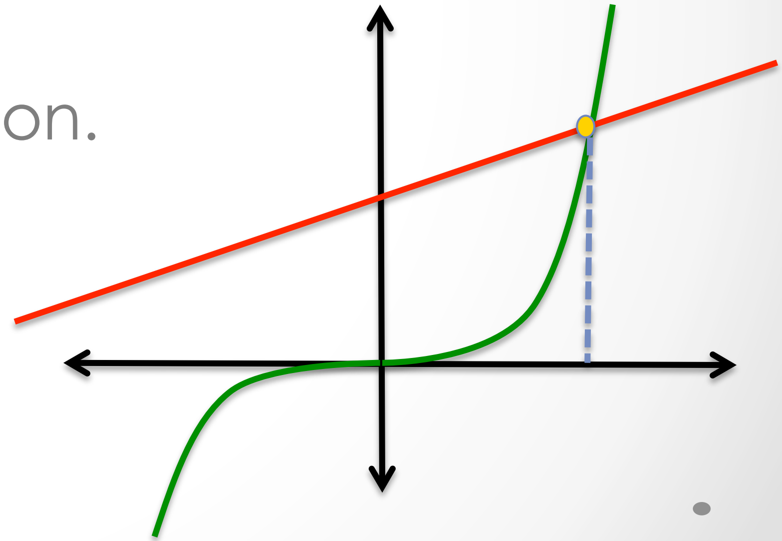
- In particular, $x^2 = -1$ has no real solutions.
- It is often argued that this led to $i = \sqrt{-1}$.
- But... Historically, no interest in non-real solutions since the graphs of $y = x^2$ and $y = mx + b$ simply don't intersect in that case.

History

- Cubic equations were the real reason. Consider

$$x^3 = px + q$$

- Represents intersection of $y = x^3$ and $y = px + q$.
- There always must be a solution.



Solution to Cubic

- Del Ferro (1465-1526) and Tartaglia (1499-1577), followed by Cardano (1501-1576), showed that
$$x^3 = px + q$$
has a solution given by
$$x = \sqrt[3]{\sqrt{\frac{q^2}{4} - \frac{p^3}{27}} + \frac{q}{2}} - \sqrt[3]{\sqrt{\frac{q^2}{4} - \frac{p^3}{27}} - \frac{q}{2}}$$

- Try it out for $x^3 = -6x + 20$!

Bombelli’s Problem

- About 30 years after the discovery of this formula, Bombelli (1526-1572) considered the equation

$$x^3 = 15x + 4$$

- Plugging $p = 15$ and $q = 4$ into the formula yields

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

- Bombelli had a “wild thought”…

Bombelli's Idea

- Bombelli discovered that

$$\sqrt[3]{2+\sqrt{-121}} = 2+\sqrt{-1} \quad \text{and} \quad \sqrt[3]{2-\sqrt{-121}} = 2-\sqrt{-1}$$

- These clearly add up to 4, the desired solution.
- Check it out:

$$(2+\sqrt{-1})^3 = 2+\sqrt{-121} \quad \text{and} \quad (2-\sqrt{-1})^3 = 2-\sqrt{-121}$$

Check it out...

$$(2+\sqrt{-1})^3 =$$

The Birth of Complex Analysis

- Bombelli's discovery is considered the "Birth of Complex Analysis".
- It showed that perfectly real problems require complex arithmetic for their solution.
- Note: Need to be able to manipulate complex numbers according to the same rules we are used to from real numbers (distributive law, etc).
- We'll study this next.