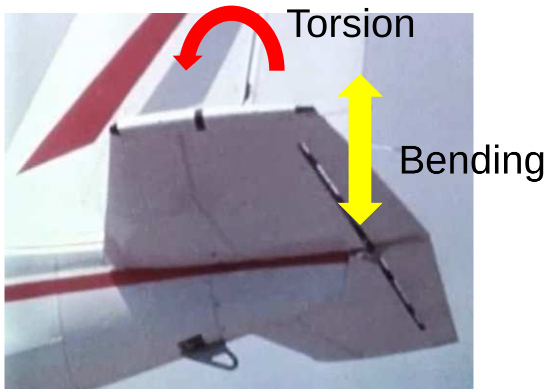
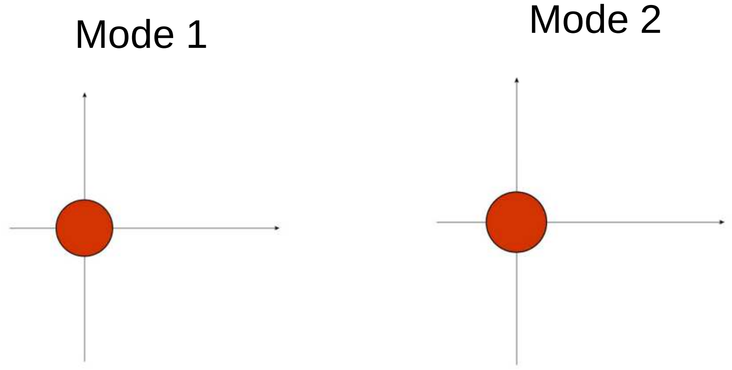
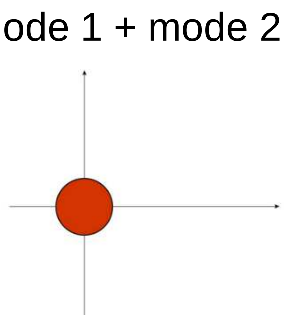
FLOW-INDUCED OSCILLATION OF A PLANE EMPENNAGE



TWO-MODES APPROXIMATION

$$\underline{\xi}\left(\underline{x},t\right)=Dq_1\left(t\right)\underline{\varphi}_1\left(\underline{x}\right)+Dq_2\left(t\right)\underline{\varphi}_2\left(\underline{x}\right)$$





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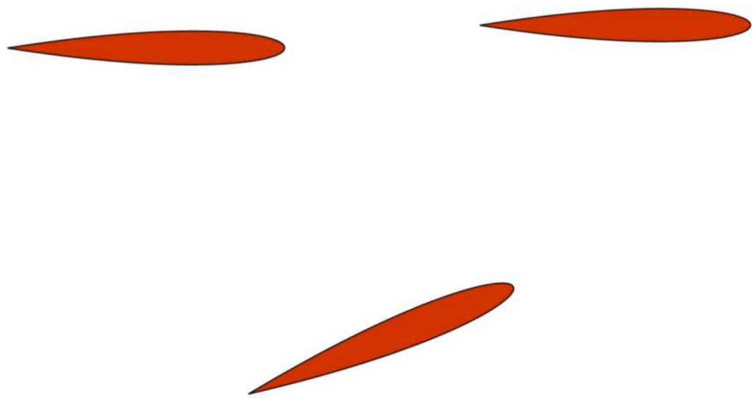

TWO-MODES APROXIMATION

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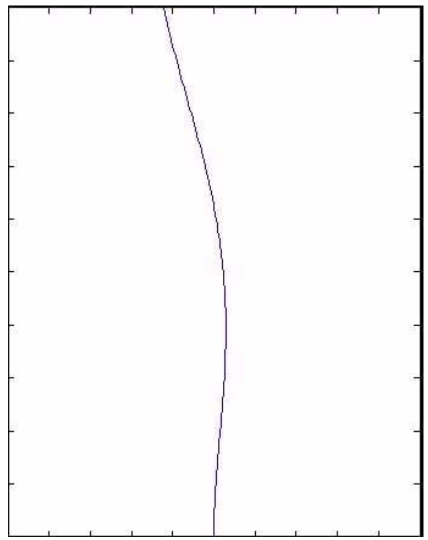

TWO-MODES APROXIMATION

$$\underline{\xi}(\underline{x},t) = Dq_1(t)\underline{\varphi}_1(\underline{x}) + Dq_2(t)\underline{\varphi}_2(\underline{x})$$

$$m_1\ddot{q}_1 + k_1 q_1 = f_{FS}^1$$

$$m_2\ddot{q}_2 + k_2 q_2 = f_{FS}^2$$

HIGH REDUCED VELOCITIES : ONE MODE

Steady-state
fluid dynamics

$$\tilde{p}(Dq)$$

$$\underline{\tilde{U}}(Dq)$$

Single mode approximation

$$\underline{\xi}(\underline{x},t) = D\,q(t)\underline{\varphi}(\underline{x})$$

$$C_Y \int_{\partial\Omega_{FS}} \left\{ \left[-\tilde{p}\underline{\underline{I}} + \frac{1}{R_E}\left(\nabla\underline{\tilde{U}} + \nabla^t\underline{\tilde{U}}\right) \right] . \underline{n} \right\} . \underline{\varphi}\, dS = F_{FS}(R_E, Dq, \ldots)$$

Stiffness force $$f_{FS} = C_Y \left(\frac{\partial F}{\partial q} \right)^0 q = -k_F q$$

HIGH REDUCED VELOCITIES : TWO MODES

Two modes approximation

$$\underline{\xi}(\underline{x},t) = Dq_1(t)\underline{\varphi}_1(\underline{x}) + Dq_2(t)\underline{\varphi}_2(\underline{x})$$

Steady-state fluid dynamics

$$\tilde{p}(Dq_1, Dq_2)$$

$$\underline{\tilde{U}}(Dq_1, Dq_2)$$

Projection on $\underline{\varphi}_1$

$$F_{FS}{}^{1} = C_Y F_1(R_E, Dq_1, Dq_2, \ldots)$$

Projection on $\underline{\varphi}_2$

$$F_{FS}{}^{2} = C_Y F_2(R_E, Dq_1, Dq_2, \ldots)$$

HIGH REDUCED VELOCITIES : TWO MODES

$\boxed{i = 1;\, 2}$

$$F_{FS}{}^{i} = C_Y F_i^0 + DC_Y \left(\frac{\partial F_i}{\partial q_1}\right)^0 q_1 + DC_Y \left(\frac{\partial F_i}{\partial q_2}\right)^0 q_2 + \ldots$$

$$\begin{aligned} m_1 \ddot{q}_1 + k_1 q_1 &= C_Y K_{11} q_1 + C_Y K_{12} q_2 \\ m_2 \ddot{q}_2 + k_2 q_2 &= C_Y K_{21} q_1 + C_Y K_{22} q_2 \end{aligned}$$

$$K_{ij} = \left(\frac{\partial F_i}{\partial q_j}\right)^0$$

Coupled flow-induced stiffness forces
between the two modes

TWO MODES COUPLED THROUGH FLOW-INDUCED STIFFNESS FORCES

$$m_1\ddot{q}_1 + k_1 q_1 = C_Y K_{11} q_1 + C_Y K_{12} q_2$$

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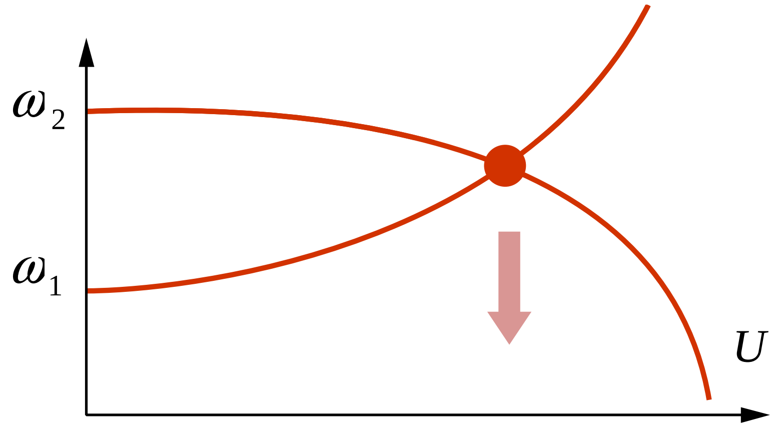


Coincidence of frequencies

A MODEL SYSTEM

$$\boxed{\begin{aligned} m_1\ddot{q}_1 + k_1 q_1 &= C_Y K_{11} q_1 + C_Y K_{12} q_2 \\ m_2\ddot{q}_2 + k_2 q_2 &= C_Y K_{21} q_1 + C_Y K_{22} q_2 \end{aligned}}$$

$$\boxed{\begin{aligned} m_1\ddot{q}_1 + \left(k_1 - C_Y K_{11}\right) q_1 &= C_Y K_{12} q_2 \\ m_2\ddot{q}_2 + \left(k_2 - C_Y K_{22}\right) q_2 &= C_Y K_{21} q_1 \end{aligned}}$$

Symmetric

$$\begin{aligned} \ddot{q}_1 + q_1 &= \varepsilon\, q_2 \\ \ddot{q}_2 + q_2 &= \varepsilon\, q_1 \end{aligned}$$

$$\varepsilon << 1$$

Antisymmetric

$$\begin{aligned} \ddot{q}_1 + q_1 &= \varepsilon\, q_2 \\ \ddot{q}_2 + q_2 &= -\varepsilon\, q_1 \end{aligned}$$

MODEL SYSTEM : SYMMETRIC COUPLING

Modes of the coupled system

$$\ddot{q}_1 + q_1 = \varepsilon\, q_2$$
$$\ddot{q}_2 + q_2 = \varepsilon\, q_1$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_0 e^{i\omega t}\right]$$

$$\begin{bmatrix} 1-\omega^2 & -\varepsilon \\ -\varepsilon & 1-\omega^2 \end{bmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_0 e^{i\omega t} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 1-\omega^2 & -\varepsilon \\ -\varepsilon & 1-\omega^2 \end{vmatrix} = 0$$

$$\varepsilon << 1$$

$$\boxed{\omega \approx 1 \pm \frac{\varepsilon}{2}}$$

TWO MODES WITH REAL FREQUENCIES AND REAL EIGENVECTORS

$$\ddot{q}_1 + q_1 = \varepsilon\, q_2$$
$$\ddot{q}_2 + q_2 = \varepsilon\, q_1$$

$$\omega_A = 1 + \frac{\varepsilon}{2} \qquad \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\omega_B = 1 - \frac{\varepsilon}{2} \qquad \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Two modes with real frequencies and real eigenvectors

MODEL SYSTEM : ANTISYMMETRIC COUPLING

Modes of the coupled system

$$\begin{aligned} \ddot{q}_1 + q_1 &= \varepsilon\, q_2 \\ \ddot{q}_2 + q_2 &= -\varepsilon\, q_1 \end{aligned} \qquad \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_0 e^{i\omega t} \right]$$

$$\begin{bmatrix} 1-\omega^2 & -\varepsilon \\ +\varepsilon & 1-\omega^2 \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_0 e^{i\omega t} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 1-\omega^2 & -\varepsilon \\ +\varepsilon & 1-\omega^2 \end{vmatrix} = 0$$

$$(1-\omega^2)^2 + \varepsilon^2 = 0$$

$$\varepsilon << 1 \qquad \boxed{\omega \approx 1 \pm i\frac{\varepsilon}{2}}$$

TWO MODES WITH COMPLEX FREQUENCIES AND EIGENVECTORS

$$\ddot{q}_1 + q_1 = \varepsilon\, q_2$$

$$\ddot{q}_2 + q_2 = -\varepsilon\, q_1$$

$$\omega_A = 1 + i\,\frac{\varepsilon}{2}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_A = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\omega_B = 1 - i\,\frac{\varepsilon}{2}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_B = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

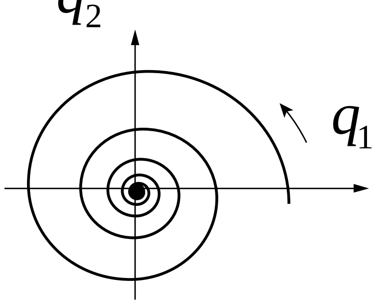
Two modes with complex frequencies and complex eigenvectors

A DAMPED MODE

$$\boxed{\begin{gathered}\omega_A = 1 + i\frac{\varepsilon}{2} \\ \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_A = \begin{pmatrix} 1 \\ -i \end{pmatrix}\end{gathered}}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_A e^{i\omega_A t}\right] = \mathrm{Re}\left[\begin{pmatrix} 1 \\ -i \end{pmatrix}_A e^{i(1+i\varepsilon/2)t}\right]$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^{-\varepsilon t/2}$$

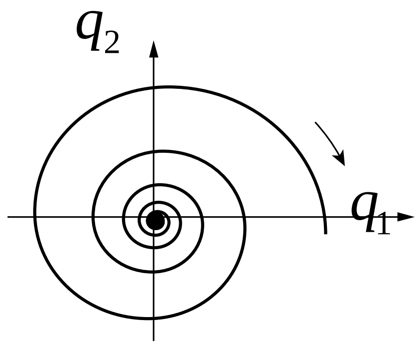


AN UNSTABLE MODE

$$\boxed{\begin{aligned} \omega_B &= 1 - i\frac{\varepsilon}{2} \\ \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_B &= \begin{pmatrix} 1 \\ i \end{pmatrix} \end{aligned}}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_B e^{i\omega_B t} \right] = \mathrm{Re}\left[\begin{pmatrix} 1 \\ i \end{pmatrix}_B e^{i(1 - i\varepsilon/2)t} \right]$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{\varepsilon t/2}$$



NEUTRAL, DAMPED AND UNSTABLE MODES

Symmetric

$$\ddot{q}_1 + q_1 = \varepsilon\, q_2$$
$$\ddot{q}_2 + q_2 = \varepsilon\, q_1$$

Two neutral modes

Antisymmetric

$$\ddot{q}_1 + q_1 = \varepsilon\, q_2$$
$$\ddot{q}_2 + q_2 = -\varepsilon\, q_1$$

One damped mode , one unstable mode

AN UNSTABLE MODE

Symmetric

Potential

$$\Phi = \varepsilon q_1 q_2$$

$$\ddot{q}_1 + q_1 = \varepsilon q_2 = \frac{\partial \Phi}{\partial q_1}$$

$$\ddot{q}_2 + q_2 = \varepsilon q_1 = \frac{\partial \Phi}{\partial q_2}$$

Conservative

Two neutral modes

Antisymmetric

No Potential

Non conservative

One damped mode, one unstable mode

BACK TO THE GENERAL CASE

$$\boxed{\begin{aligned} m_1\ddot{q}_1 + (k_1 - C_Y K_{11})q_1 &= C_Y K_{12} q_2 \\ m_2\ddot{q}_2 + (k_2 - C_Y K_{22})q_2 &= C_Y K_{21} q_1 \end{aligned}}$$

$$K_{ij} = \left(\frac{\partial F_i}{\partial q_j}\right)^0$$

$$F_{FS}{}^i = C_Y F_i(R_E, Dq_1, Dq_2, \ldots)$$

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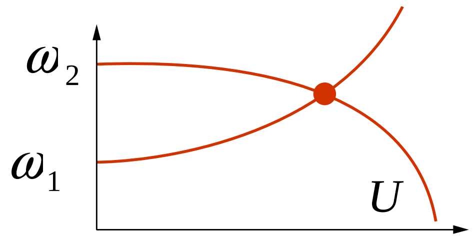
FREQUENCY COINCIDENCE AND NON-SYMMETRIC COUPLING

Two modes that coincide



+ Non symmetric coupling

Dynamic instability

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