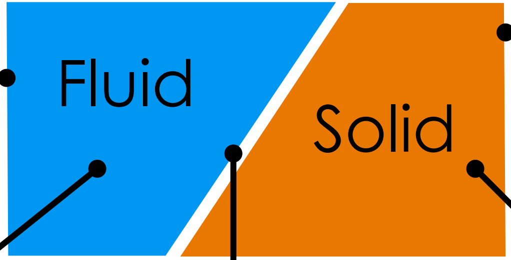
FLUID AND SOLID

Fluid boundary conditions



Solid boundary conditions

$$\mathit{div}\ \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T} \Delta \underline{u}$$

$$\ddot{q} + q = f$$

$$\underline{u} = \dot{q} \underline{\varphi}$$

$$M \int_{Interface} \underline{\varphi}.\left[-p\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla \underline{u} + \nabla^t \underline{u} \right) \right].\underline{n}\, dS - Mq \int_{Interface} \left(\underline{\nabla} P_0 . \underline{\varphi} \right)\left(\underline{\varphi} . \underline{n} \right) dS = f$$

STOKES NUMBER

1

$$S_T = \frac{\rho L^2}{\mu T_{solid}}$$

$L = 10^{-5}$ m

$\mu / \rho = 10^{-3}$ m^{2}s^{-1}

$T_{solid} = 0.1$ s

$$S_T = 10^{-6}$$

Added mass effect

$$f = -m_A \ddot{q}$$

LOW STOKES NUMBER

$S_T \ll 1$ Dominant viscous effects

$$\cancel{\frac{\partial \underline{u}}{\partial t}} = -\underline{\nabla} p + \frac{1}{S_T}\Delta \underline{u}$$

$$p \leftarrow p S_T$$

$$0 = -\underline{\nabla} p + \Delta \underline{u}$$

$$\text{div}\ \underline{u} = 0$$

$$\underline{u} = \dot{q}\ \underline{\varphi}$$

$$f = \frac{M}{S_T} \int_{Interface} \underline{\varphi}.\left[-p\underline{\underline{I}} + \left(\nabla \underline{u} + \nabla^t \underline{u}\right)\right]\underline{n}\, dS$$

LOW STOKES NUMBER : A SINGLE MODE SOLUTION

$$\underline{u} = \dot{q}\,\underline{\varphi}$$

$$\underline{u}(\underline{x},t) = \dot{q}(t)\,\underline{\varphi}_u(\underline{x})$$

$$p(\underline{x},t) = \dot{q}(t)\varphi_p(\underline{x})$$

$\text{div}\,\underline{u} = 0$ $0 = -\underline{\nabla}p + \Delta\underline{u}$	$\Rightarrow$	$\text{div}\,\underline{\varphi}_u = 0$ $0 = -\underline{\nabla}\varphi_p + \Delta\underline{\varphi}_u$

interface

$\underline{u} = \dot{q}\,\underline{\varphi}$	$\Rightarrow$	$\underline{\varphi}_u = \underline{\varphi}$

ADDED DAMPING

$$f = \frac{M}{S_T} \int_{Interface} \underline{\varphi}.\left[-p\underline{\underline{I}} + \left(\nabla \underline{u} + \nabla^t \underline{u}\right)\right]\underline{n}\, dS$$

$$\underline{u} = \dot{q}\, \underline{\varphi}_u \quad p = \dot{q}\varphi_p$$

$$f = -\left[-\frac{M}{S_T} \int_{Interface} \underline{\varphi}.\left[-\varphi_p \underline{\underline{I}} + \left(\nabla \underline{\varphi}_u + \nabla^t \underline{\varphi}_u\right)\right]\underline{n}\, dS\right]\dot{q}$$

The fluid force is a damping force

$$f = -c_A \dot{q}$$

Added damping

The fluid response is instantaneous

$$f(t) = -c_A \dot{q}(t)$$

ADDED DAMPING AND ADDED MASS

1 $\qquad S_T$

Added damping effect

$$f(t) = -c_A \dot{q}(t)$$

$$\boxed{\begin{aligned} \mathit{div}\,\underline{\varphi}_u &= 0 \\ 0 &= -\underline{\nabla}\varphi_p + \Delta\underline{\varphi}_u \end{aligned}}$$

$$c_A = \frac{M}{S_T} \int_{Interface} \ldots dS$$

Added mass effect

$$f(t) = -m_A \ddot{q}(t)$$

$$\boxed{\Delta\varphi_p = 0}$$

$$m_A = M \int_{Interface} \ldots dS$$

ADDED DAMPING AND ADDED MASS

1 S_T

Added damping effect

$$f(t) = -c_A \dot{q}(t)$$

$$c_A = -\frac{M}{S_T} \int_{Interface} \ldots dS$$

$$C_A \propto \mu$$

Dimensional

Added mass effect

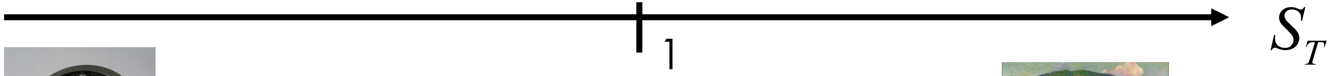
$$f(t) = -m_A \ddot{q}(t)$$

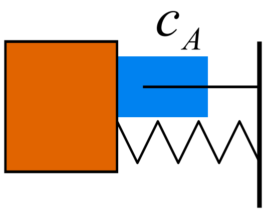
$$m_A = M \int_{Interface} \ldots dS$$

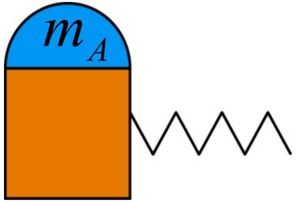
$$M_A \propto \rho$$

Dimensional

ADDED DAMPING AND ADDED MASS

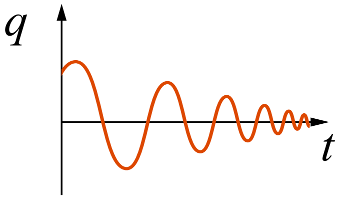


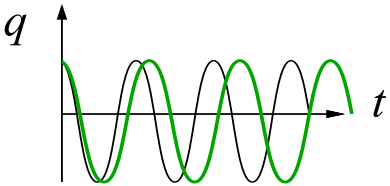




$$\ddot{q} + c_A \dot{q} + q = 0$$

$$(1 + m_A)\ddot{q} + q = 0$$





ADDED DAMPING

Fluid bearing

Spiruline

Food processing

INTERMEDIATE STOKES NUMBERS

$$S_T = \frac{\rho L^2}{\mu T_{solid}}$$

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