THE FLUID-CONVEYING PIPE INSTABILITY

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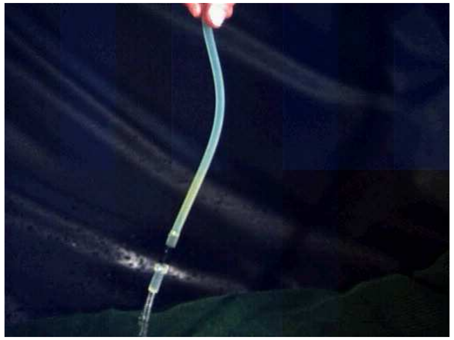
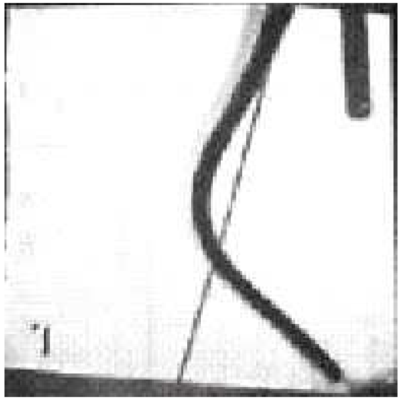
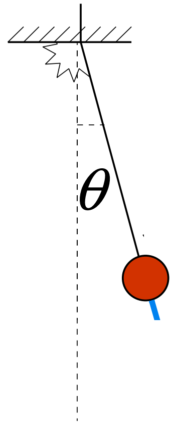




THE FLUID-CONVEYING BI-PENDULUM

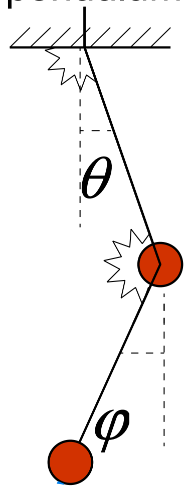
Single pendulum model



Without fluid

$$\ddot{\theta} + \theta = 0$$

Bi-pendulum model



Without fluid

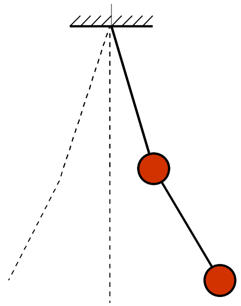
$$2\ddot{\theta} + \ddot{\varphi} + 2\theta - \varphi = 0$$

$$\ddot{\theta} + \ddot{\varphi} - \theta + \varphi = 0$$

THE FLUID-CONVEYING BI-PENDULUM

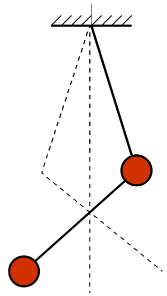
Mode 1

$$\omega_1 = \sqrt{2} - 1$$



Mode 2

$$\omega_2 = \sqrt{2} + 1$$

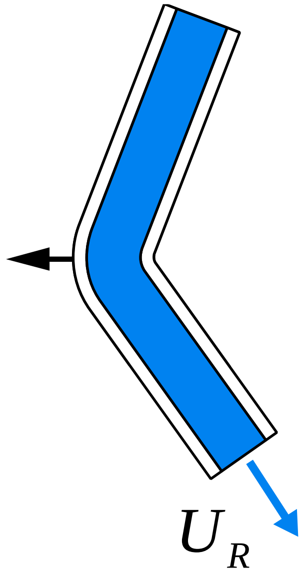


$$\begin{bmatrix} \theta(t) \\ \varphi(t) \end{bmatrix} = q_1(t) \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} + q_2(t) \begin{bmatrix} 1 \\ -\sqrt{2} \end{bmatrix}$$

$$\ddot{q}_1 + \omega_1^2 q_1 = 0$$

$$\ddot{q}_2 + \omega_2^2 q_2 = 0$$

THE FLUID-CONVEYING BI- PENDULUM AT VERY LARGE REDUCED VELOCITY



$$\begin{bmatrix} \theta \\ \varphi \end{bmatrix}$$ Frozen in time

Dimensional elbow force.

$$F = \rho S U^2 (\theta - \varphi)$$

Cauchy number

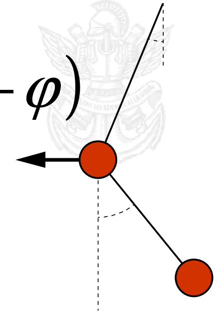
$$C_Y = \frac{\rho S U^2 L}{C}$$

Dimensionless elbow force.

$$F = C_Y (\theta - \varphi)$$

THE FLUID-CONVEYING BI- PENDULUM
AT VERY LARGE REDUCED VELOCITY

$$f = C_Y\left(\theta - \varphi\right)$$



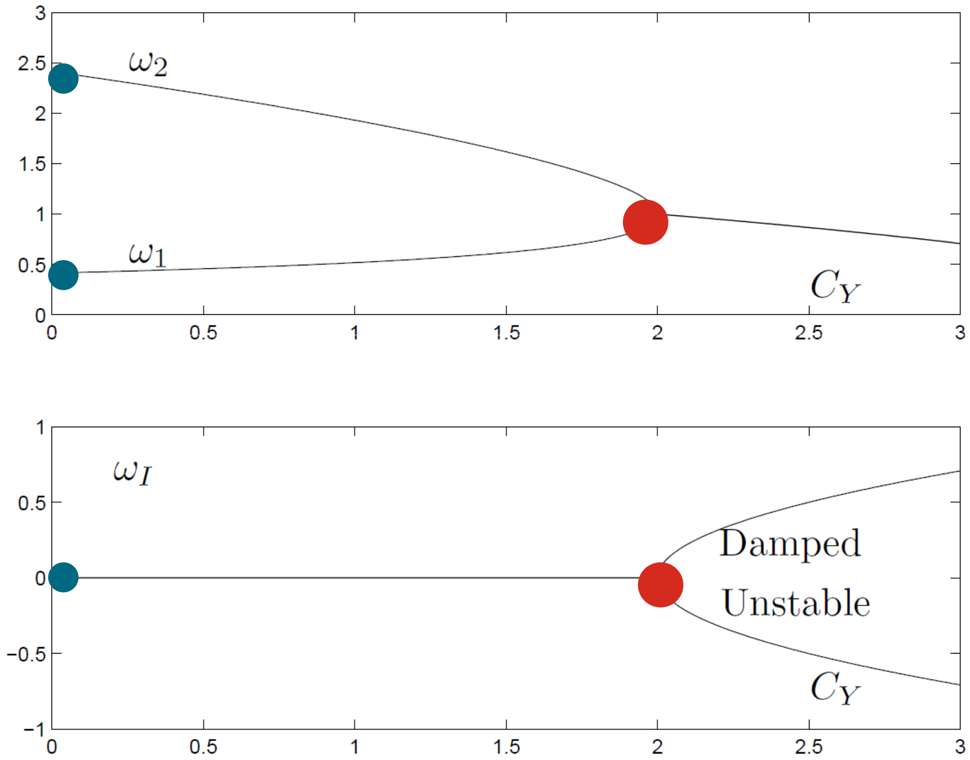
$$\ddot{q}_1 + \omega_1^2 q_1 = -C_Y \frac{\sqrt{2}-1}{4+2\sqrt{2}}\, q_1 + C_Y \frac{\sqrt{2}+1}{4+2\sqrt{2}}\, q_2$$

$$\ddot{q}_2 + \omega_2^2 q_2 = -C_Y \frac{\sqrt{2}-1}{4-2\sqrt{2}}\, q_1 + C_Y \frac{\sqrt{2}+1}{4-2\sqrt{2}}\, q_2$$

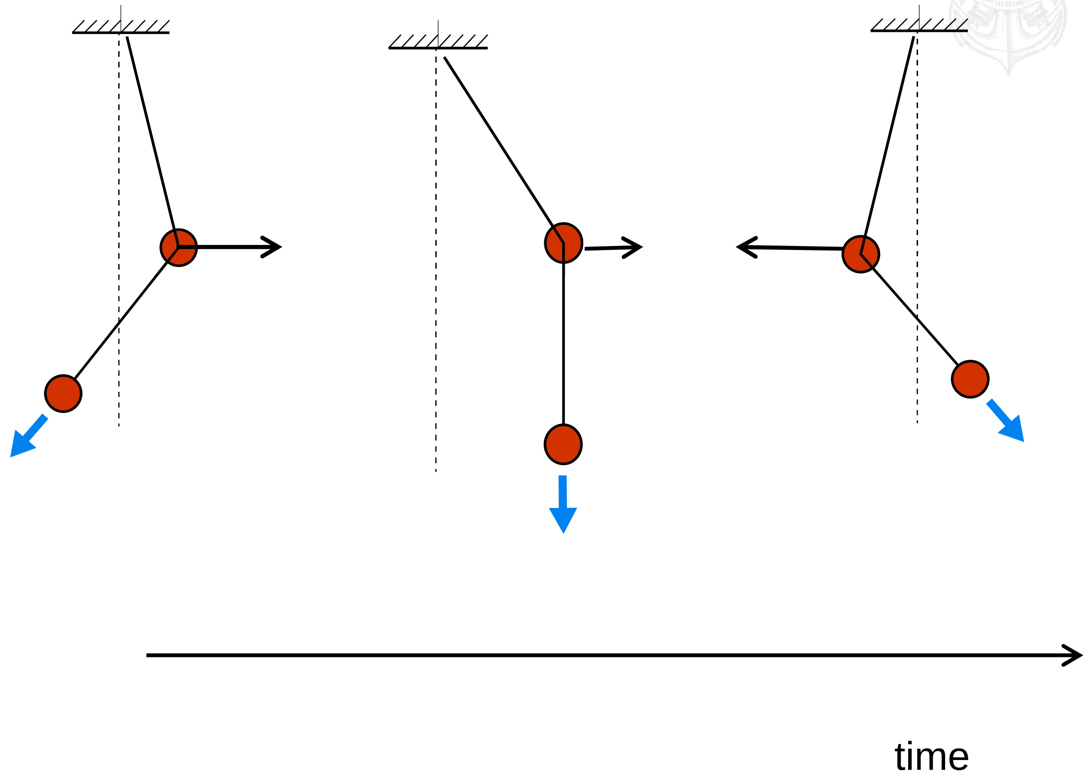
$$\ddot{q}_1 + \left[\omega_1^2 + C_Y \frac{\sqrt{2}-1}{4+2\sqrt{2}}\right] q_1 = C_Y \frac{\sqrt{2}+1}{4+2\sqrt{2}}\, q_2$$

$$\ddot{q}_2 + \left[\omega_2^2 - C_Y \frac{\sqrt{2}+1}{4-2\sqrt{2}}\right] q_2 = -C_Y \frac{\sqrt{2}-1}{4-2\sqrt{2}}\, q_1$$

THE FLUID-CONVEYING BI- PENDULUM
AT VERY LARGE REDUCED VELOCITY



THE FLUID-CONVEYING BI- PENDULUM AT VERY LARGE REDUCED VELOCITY



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