GENERAL FORM OF FLUID LOADING

Steady-state
fluid dynamics

$$\tilde{p}(Dq)$$

$$\underline{\tilde{U}}(Dq)$$

Single mode
approximation

$$\underline{\xi}(\underline{x},t) = D\,q(t)\underline{\varphi}(\underline{x})$$

$$C_Y \int_{\partial\Omega_{FS}} \left\{ \left[-\tilde{p}\,\underline{\underline{I}} + \frac{1}{R_E}\left(\nabla\underline{\tilde{U}} + \nabla^t\underline{\tilde{U}}\right) \right].\underline{n} \right\}.\underline{\varphi}\,dS = F_{FS}(R_E, Dq, \ldots)$$

Position of the interface

EXPANSION OF THE FLUID LOADING

$$F_{FS} = C_Y \int_{\partial\Omega_{FS}} \left\{ \left[-\tilde{p}\,\underline{\underline{I}} + \frac{1}{R_E}\left(\nabla \underline{\tilde{U}} + \nabla^t \underline{\tilde{U}}\right) \right] . \underline{n} \right\} . \underline{\varphi}\, dS$$

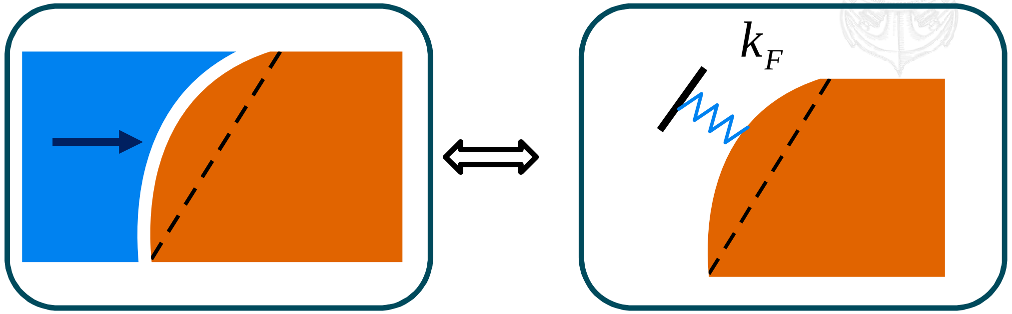
$$F_{FS} = C_Y F\left(R_E, Dq, \ldots\right)$$

$$F_{FS} = C_Y F^0 + DC_Y \left(\frac{\partial F}{\partial q}\right)^0 q + ..$$

$$\left[\frac{\partial F}{\partial q} \text{ notation for } \frac{\partial F}{\partial (Dq)}\right]$$

$$\boxed{f_{FS} = C_Y \left(\frac{\partial F}{\partial q}\right)^0 q}$$

FLOW-INDUCED STIFFNESS



$$f_{FS} = C_Y \left(\frac{\partial F}{\partial q}\right)^0 q = -k_F q$$

$$k_F = -C_Y \left(\frac{\partial F}{\partial q}\right)^0$$

Depends on the flow velocity and the geometry
Positive…. or NEGATIVE

FLOW-INDUCED STATIC INSTABILITY

$$U_R^2 \frac{\partial^2 q}{\partial \tilde{t}^2} + q = f_{FS}$$

$$\ddot{q} + q = f_{FS}$$

Using $\bar{t} = t / t_{\text{solid}}$

$$f_{FS} = C_Y \left(\frac{\partial F}{\partial q} \right)^0 q$$

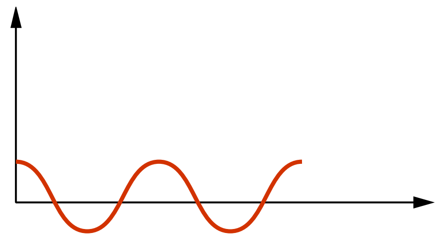
$$\ddot{q} + \left[1 - C_Y \left(\frac{\partial F}{\partial q} \right)^0 \right] q = 0$$

Static instability
(buckling, divergence) when

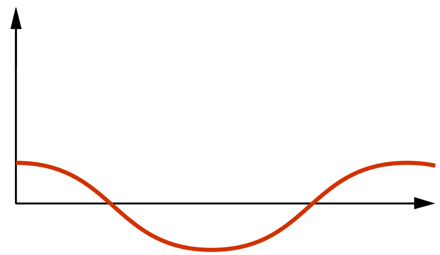
$$C_Y \left(\frac{\partial F}{\partial q} \right)^0 = 1$$

FLOW-INDUCED STATIC INSTABILITY

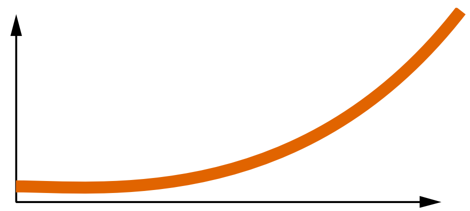
$U = 0$



$U < U_C$

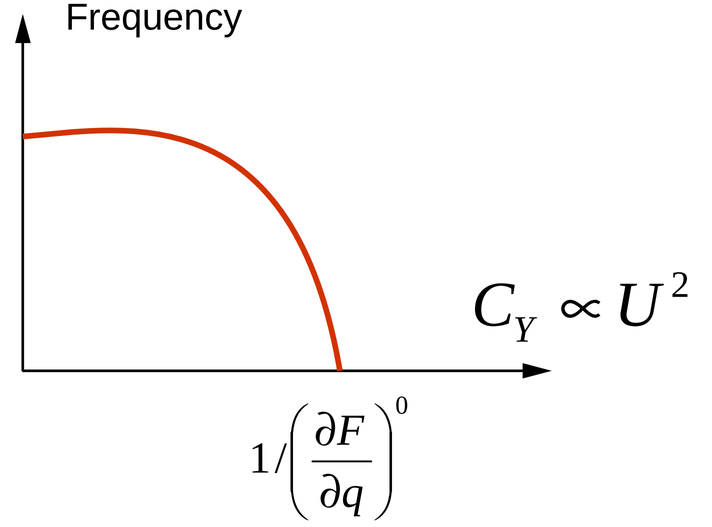


$\boxed{U > U_C}$

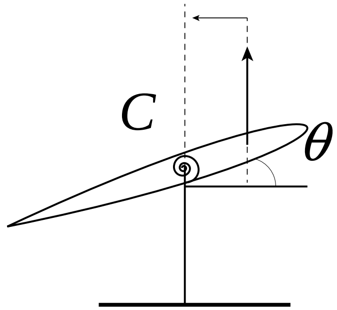


FLOW-INDUCED STATIC INSTABILITY

$$\ddot{q} + \left[1 - C_Y \left(\frac{\partial F}{\partial q} \right)^0 \right] q = 0$$



APPLICATION : TORSIONAL DIVERGENCE OF AN AIRFOIL



Without flow

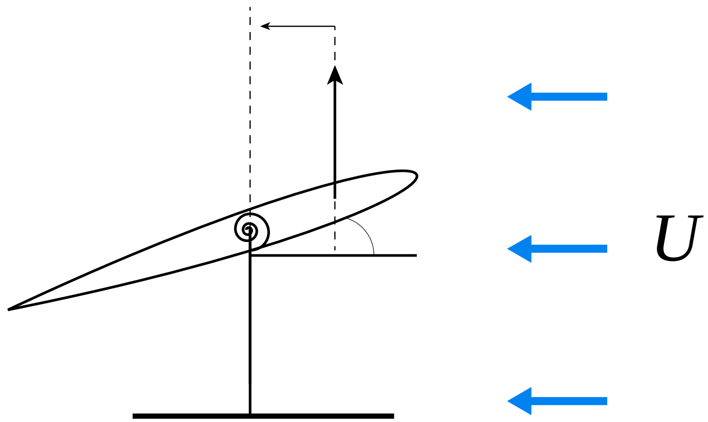
$$J\ddot{\theta} + C\theta = 0$$

$$T_{\text{solid}} = \sqrt{J / C}$$

$$\bar{t} = t / T_{\text{solid}}$$

$$\boxed{\ddot{\theta} + \theta = 0}$$

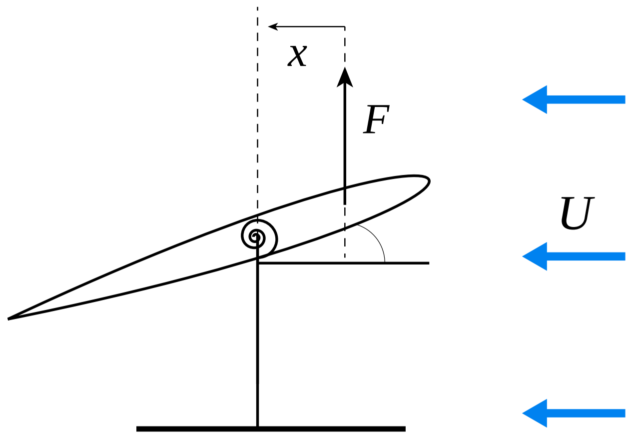
APPLICATION : TORSIONAL DIVERGENCE OF AN AIRFOIL



$$U_R = \frac{T_{\text{SOLID}}}{T_{\text{FLUID}}} = \frac{\sqrt{J/C}}{L/U} \qquad U_R \approx 100$$

$$D = \frac{\theta_0 L}{L} = \theta_0 \qquad D \approx \pi/100$$

APPLICATION : TORSIONAL DIVERGENCE OF AN AIRFOIL



$$U_R >> D \qquad F = \frac{1}{2}\rho U^2 L C_L(\theta)$$

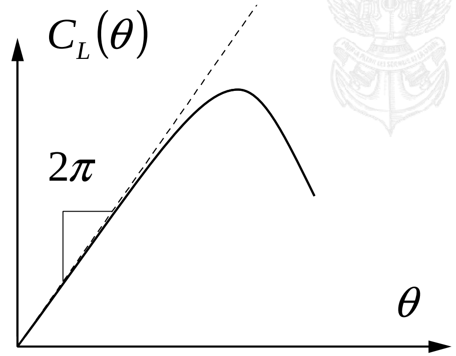
$$C_Y = \frac{\rho U^2 L^2}{2C}$$

$$\ddot{\theta} + \theta = C_Y C_L(\theta)\frac{x}{L}$$

$$\ddot{\theta} + \theta = C_Y \frac{x}{L}\left(\frac{\partial C_L}{\partial \theta}\right)^0 \theta$$

APPLICATION : TORSIONAL DIVERGENCE OF AN AIRFOIL

Lift coefficient

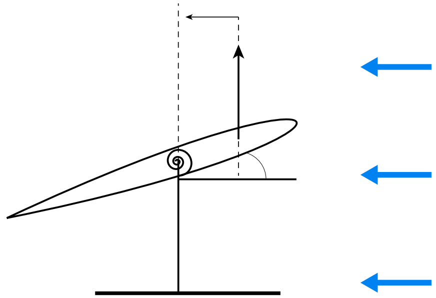


$$\ddot{\theta} + \theta = C_Y \frac{x}{L} \left(\frac{\partial C_L}{\partial \theta} \right)^0 \theta$$

$$\ddot{\theta} + \theta = C_Y \frac{x}{L} 2\pi\theta$$

$$\boxed{\ddot{\theta} + \left(1 - C_Y \frac{x}{L} 2\pi \right) \theta = 0}$$

APPLICATION : TORSIONAL DIVERGENCE OF AN AIRFOIL



$$\ddot{\theta}+\left(1-C_Y\frac{x}{L}2\pi\right)\theta=0$$

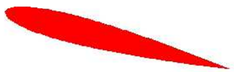
$$C_Y^{Critical}=\frac{L}{2\pi x}$$

$$C_Y=\frac{\rho U^2 L^2}{2C}$$

$$\boxed{U^{Critical}=\sqrt{\frac{C}{\rho\pi L x}}}$$

APPLICATION : TORSIONAL DIVERGENCE OF AN AIRFOIL

$$\boxed{U < U^{Critical}}$$



$$\boxed{U > U^{Critical}}$$

FLOW-INDUCED STATIC INSTABILITY