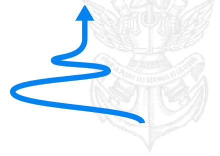
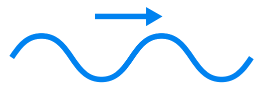
NEW DIMENSIONLESS NUMBERS

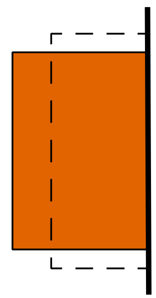
Reynolds $R_E = \dfrac{\rho U_0 L}{\mu}$



Froude $F_R = \dfrac{U_0}{\sqrt{gL}}$



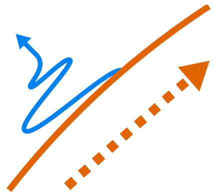
Cauchy $C_Y = \dfrac{\rho\, U_0^2}{E}$



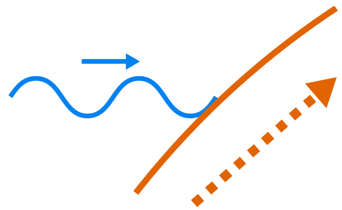
NEW DIMENSIONLESS NUMBERS

$$\boxed{U_0 \leftarrow c \quad \text{or} \quad \frac{L}{T_{Solid}}}$$

Stokes $\quad S_T = \dfrac{\rho c L}{\mu}$



Dynamic Froude $\quad F_D = \dfrac{c}{\sqrt{gL}}$



Mass $\quad M = \dfrac{\rho c^2}{E} = \dfrac{\rho}{\rho_S}$

CHOICE OF DIMENSIONLESS NUMBERS

$$R_E = \frac{\rho U_0 L}{\mu} \qquad F_R = \frac{U_0}{\sqrt{gL}} \qquad C_Y = \frac{\rho U_0^2}{E} \qquad U_R = \frac{U_0}{c}$$

$$\Downarrow$$

$$S_T = \frac{R_E}{U_R} \qquad F_D = \frac{F_R}{U_R} \qquad M = \frac{C_Y}{U_R^2} \qquad U_R = \frac{U_0}{c}$$

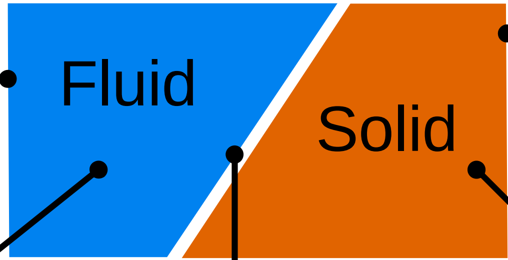
$$\Downarrow$$

$$S_T = \frac{\rho c L}{\mu} \qquad F_D = \frac{c}{\sqrt{gL}} \qquad M = \frac{\rho c^2}{E} = \frac{\rho}{\rho_S} \qquad U_R = \frac{U_0}{c}$$

$$S_T = \frac{\rho L^2}{\mu T_{Tsolid}} \qquad F_D = \frac{L}{T_{Solid}\sqrt{gL}} \qquad M = \frac{\rho c^2}{E} = \frac{\rho}{\rho_S} \qquad U_R = \frac{U_0 T_{Solid}}{L}$$

FLUID AND SOLID

Fluid boundary conditions



Solid boundary conditions

$$\mathit{div}\,\underline{\tilde{U}} = 0$$

$$\frac{1}{U_R}\frac{d\underline{\tilde{U}}}{d\bar{t}} = -\frac{1}{F_R^2}\underline{e}_Z - \underline{\nabla}\tilde{p} + \frac{1}{R_E}\Delta\underline{\tilde{U}}$$

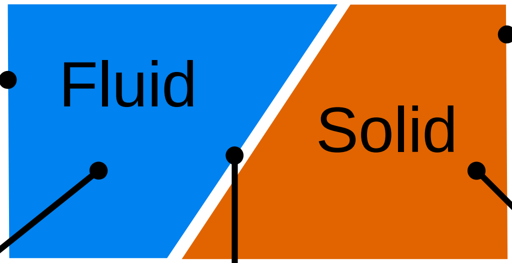
$$\frac{d^2\bar{q}}{d\bar{t}^2} + \bar{q} = \bar{f}$$

$$U_R\,\underline{\tilde{U}} = D\frac{d\bar{q}}{d\bar{t}}\,\underline{\varphi}(\underline{x})$$

$$\int_{Interface}\left\{C_Y\left[-\tilde{p}\,\underline{\underline{I}} + \frac{1}{R_E}\left(\nabla\underline{\tilde{U}} + \nabla^t\underline{\tilde{U}}\right)\right].\underline{n}\right\}.\underline{\varphi}\,dS = D\,\bar{f}$$

FLUID AND SOLID

Fluid boundary conditions



Solid boundary conditions

$$div\, \underline{\tilde U} = 0$$

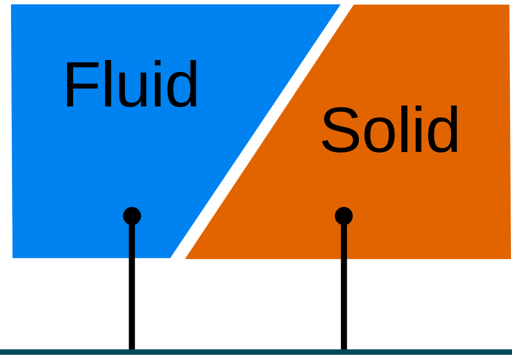
$$\frac{1}{U_R}\frac{d\underline{\tilde U}}{d\bar t} = -\frac{1}{F_R^2}\underline{e}_Z - \underline{\nabla}\tilde p + \frac{1}{R_E}\Delta\underline{\tilde U}$$

$$\frac{d^2\bar q}{d\bar t^2} + \bar q = \bar f$$

$$U_R\, \underline{\tilde U} = D\frac{\partial \bar q}{\partial \bar t}\underline{\varphi}(\underline{x})$$

$$\int_{Interface} \left\{ C_Y\left[-\tilde p\, \underline{\underline{I}} + \frac{1}{R_E}\left(\underline{\nabla}\underline{\tilde U} + \underline{\nabla}^t\underline{\tilde U}\right)\right].\underline{n}\right\}.\underline{\varphi}\, dS = D\bar f$$

DIMENSIONLESS VARIABLES



$$\underline{\tilde{x}} = \frac{\underline{x}}{L}$$

$$\underline{\tilde{U}} = \frac{\underline{U}}{U_0}$$

$$\tilde{p} = \frac{p}{\rho U_0^2}$$

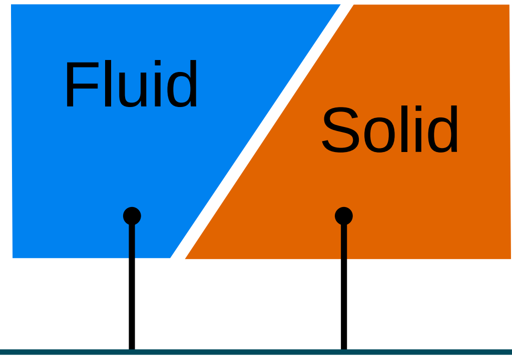
$$\bar{t} = \frac{t}{T_{\text{Solid}}}$$

$$\underline{\bar{x}} = \frac{\underline{x}}{L}$$

$$\bar{q} = \frac{q}{\xi_0}$$

$$\bar{f} = \frac{f}{k\xi_0}$$

NEW DIMENSIONLESS VARIABLES



$$\bar{x} = \frac{x}{L}$$

$$\bar{U} = \frac{U}{c}$$

$$\bar{p} = \frac{p}{\rho c^2}$$

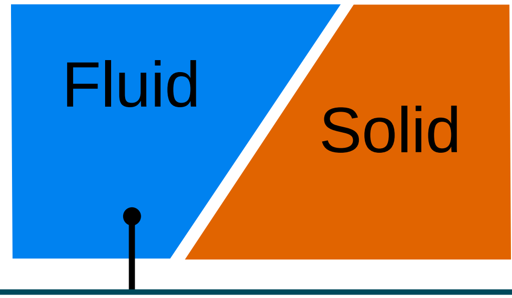
$$\bar{t} = \frac{t}{T_{\mathrm{Solid}}}$$

$$\bar{x} = \frac{x}{L}$$

$$\bar{q} = \frac{q}{\xi_0}$$

$$\bar{f} = \frac{f}{k\xi_0}$$

NEW DIMENSIONLESS EQUATIONS



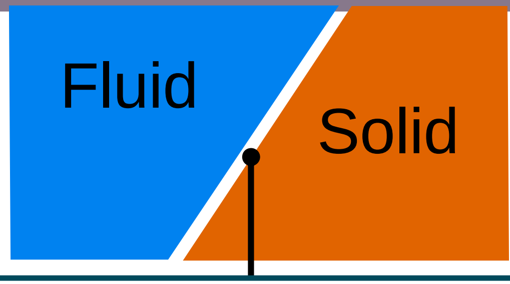
$$div\, \underline{\tilde{U}} = 0$$

$$\frac{1}{U_R}\frac{d\underline{\tilde{U}}}{d\tilde{t}} = -\frac{1}{F_R^2}\underline{e}_Z - \underline{\nabla}\tilde{p} + \frac{1}{R_E}\Delta\underline{\tilde{U}}$$

$$\mathrm{div}\, \underline{\overline{U}} = 0$$

$$\frac{d\underline{\overline{U}}}{d\overline{t}} = -\frac{1}{F_D^2}\underline{e}_Z - \underline{\nabla}\overline{p} + \frac{1}{S_T}\Delta\underline{\overline{U}}$$

NEW DIMENSIONLESS EQUATIONS



$$U_R \underline{\tilde{U}} = D \frac{\partial \bar{q}}{\partial t} \underline{\varphi}(\underline{x})$$

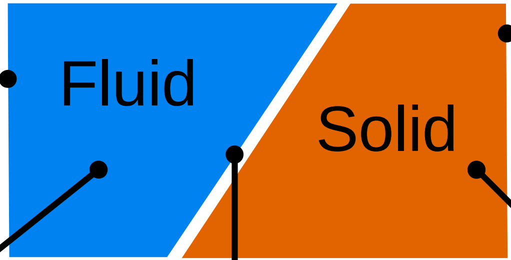
$$\int_{Interface} \left\{ C_Y \left[-\tilde{p}\,\underline{\underline{I}} + \frac{1}{R_E}\left(\nabla \underline{\tilde{U}} + \nabla^t \underline{\tilde{U}}\right)\right].\underline{n} \right\}.\underline{\varphi}\, dS = D \bar{f}$$

$$\underline{\bar{U}} = D \frac{d\bar{q}}{dt} \underline{\varphi}(\underline{x})$$

$$\int_{Interface} \left\{ \boxed{M} \left[-\bar{p}\,\underline{\underline{I}} + \boxed{\frac{1}{S_T}}\left(\nabla \underline{\bar{U}} + \nabla^t \underline{\bar{U}}\right)\right].\underline{n} \right\}.\underline{\varphi}\, dS = D \bar{f}$$

FLUID AND SOLID

Fluid boundary conditions



Solid boundary conditions

$$\mathrm{div}\,\underline{\overline{U}} = 0$$

$$\frac{d\underline{\overline{U}}}{d\overline{t}} = -\frac{1}{F_D^2}\underline{e}_Z - \underline{\nabla}\overline{p} + \frac{1}{S_T}\Delta\underline{\overline{U}}$$

$$\frac{d^2\overline{q}}{d\overline{t}^2} + \overline{q} = \overline{f}$$

$$\underline{\overline{U}} = D\frac{d\overline{q}}{d\overline{t}}\underline{\varphi}(\underline{x})$$

$$\int_{Interface}\left\{M\left[-\overline{p}\,\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla\underline{\overline{U}} + \nabla^t\underline{\overline{U}}\right)\right].\underline{n}\right\}.\underline{\varphi}\,dS = D\overline{f}$$

SMALL REDUCED VELOCITY