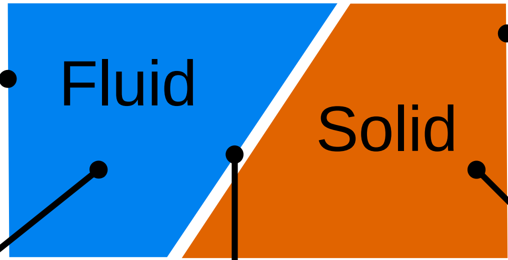
SMALL REDUCED VELOCITY, SMALL MOTION

$U_R << D$

$\mathrm{D} << 1$

FLUID AND SOLID

Fluid boundary conditions



Solid boundary conditions

$$div\,\underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T} \Delta \underline{u}$$

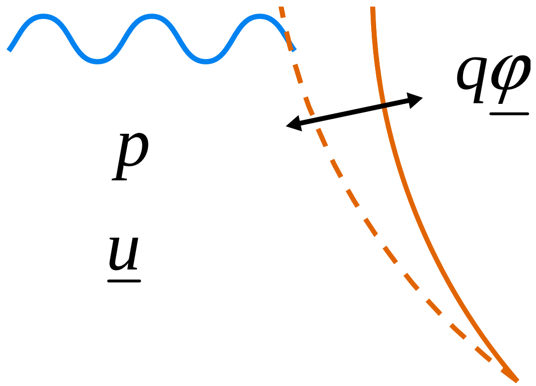
$$\ddot{q} + q = f$$

$$\underline{u} = \dot{q}\underline{\varphi}$$

$$M \int_{Interface} \underline{\varphi}.\left[-p\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla \underline{u} + \nabla^t \underline{u}\right)\right].\underline{n}\, dS - Mq \int_{Interface} \left(\underline{\nabla} P_0.\underline{\varphi}\right)\left(\underline{\varphi}.\underline{n}\right) dS = f$$

FLUID LOADING : MOTION INDUCED STRESS

$$M\int_{Interface}\underline{\varphi}.\left[-p\underline{\underline{I}}+\frac{1}{S_T}\left(\nabla\underline{u}+\nabla^t\underline{u}\right)\right].\underline{n}\,dS-Mq\int_{Interface}\left(\underline{\nabla P_0}.\underline{\varphi}\right)\left(\underline{\varphi}.\underline{n}\right)dS=f$$



FLUID LOADING : MOTION INDUCED STRESS

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STOKES NUMBER

$$S_T = \frac{\rho c L}{\mu} = \frac{\rho L^2}{\mu T_{solid}}$$

$$L = 1\ \mathrm{m}$$

$$\mu / \rho = 10^{-6}\ \mathrm{m^2 s^{-1}}$$

$$T_{solid} = 1\ \mathrm{s}$$

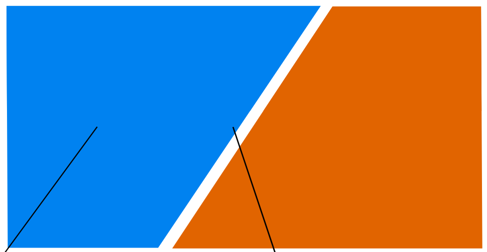
$$S_T = 10^6$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \boxed{\frac{1}{S_T}} \Delta \underline{u} \quad \Rightarrow \quad \frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

HIGH STOKES NUMBER

$$S_T >> 1$$

Negligible viscous effects



$$\mathrm{div}\, \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

$$\underline{u}.\underline{n} = \dot{q}\, \underline{\varphi}.\underline{n}$$

$$M \quad \int_{Interface} \left(-p\underline{n}\right).\underline{\varphi}\, dS = f$$

HIGH STOKES NUMBER : A SINGLE MODE SOLUTION

$$\underline{u}(x,t).\underline{n} = \dot{q}(t)\,\underline{\varphi}(x).\underline{n}$$

$$\underline{u}(\underline{x},t) = \dot{q}(t)\,\underline{\varphi}_u(\underline{x})$$

$$p(\underline{x},t) = \ddot{q}(t)\varphi_p(\underline{x})$$

$$div\,\underline{u} = 0 \qquad \frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

$$div\,\underline{\varphi}_u = 0 \qquad \underline{\varphi}_u = -\underline{\nabla}\varphi_p$$

$$\underline{u}(x,t).\underline{n} = \dot{q}(t)\,\underline{\varphi}(x).\underline{n} \qquad f = M \int_{Interface} (-p\underline{n}).\underline{\varphi}\,dS$$

$$\underline{\varphi}_u.\underline{n} = \underline{\varphi}.\underline{n} \qquad f = -\ddot{q}\left[M \int_{Interface} \varphi_p \underline{n}.\underline{\varphi}\,dS \right]$$

ADDED MASS

$$f = -\ddot{q}\left[M \int_{Interface} \varphi_p \underline{n}.\underline{\varphi}\, dS \right]$$

The fluid force is an inertia force

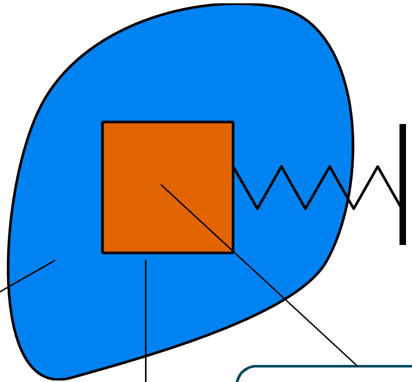
$$f = -m_A \ddot{q}$$

Added mass

The fluid response is instantaneous

$$f(t) = -m_A \ddot{q}(t)$$

ADDED MASS



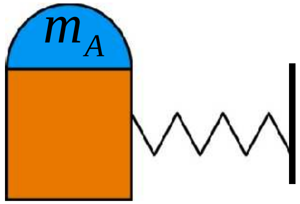
$$div\ \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

$$\ddot{q} + q = f$$

$$\underline{u}(x,t).\underline{n} = \dot{q}(t)\,\underline{\varphi}(x).\underline{n}$$

$$f = M \int_{Interface} (-p\underline{n}).\underline{\varphi}\, dS$$



$$(1 + m_A)\ddot{q} + q = 0$$

DIMENSIONLESS NUMBERS AND STEPS OF APPROXIMATION

Initial choice	D	F_R	R_E	C_Y	U_R
New choice	D	F_D	S_T	M	U_R
Small reduced velocity	D	F_D	S_T	M	
Small amplitude		F_D	S_T	M	
Small viscosity		F_D		M	

$$f = -\left[-\frac{M}{F_D^2} \int_{Interface} \left(\underline{\varphi}.\underline{e}_Z\right)\left(\underline{\varphi}.\underline{n}\right) dS \right] q - \left[M \int_{Interface} \varphi_p \underline{n}.\underline{\varphi}\, dS \right] \ddot{q}$$

$$f = -k_F q - m_A \ddot{q}$$