SLOSHING

FLUID SLOSHING COUPLED WITH THE SOLID DYNAMICS

Fluid sloshing ⟵ Solid motion

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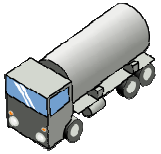


A FLUID WITH A FREE SURFACE

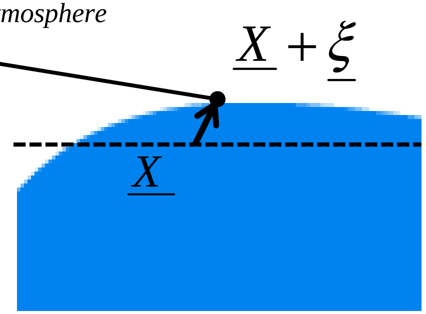
$$P = P_{Atmosphere}$$

$$div\ \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

A FLUID WITH A FREE SURFACE

$$P(\underline{X}+\underline{\xi}) = P_{Atmosphere}$$



Order 0

$$P_0 = P_{Atmosphere}$$

Order 1

$$p\left(\underline{X}\right)+\underline{\nabla}P_0\left(\underline{X}\right).\underline{\xi}$$

$$P(\underline{X}+\underline{\xi}) = P_{Atmosphere}$$

$$\Downarrow$$

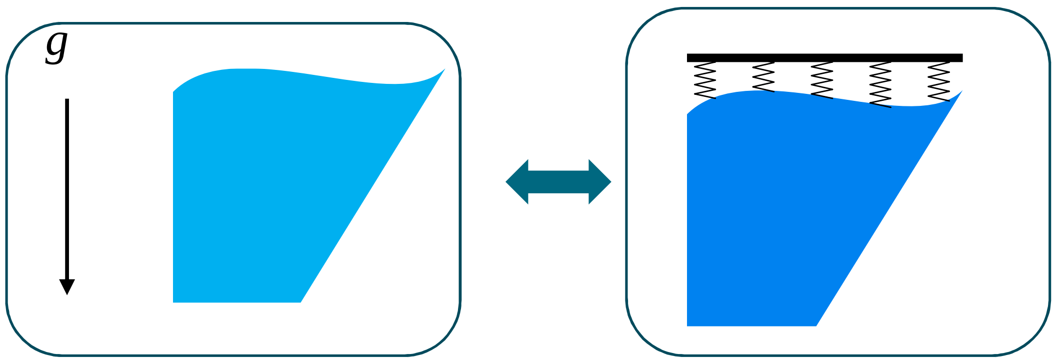
$$p+\underline{\nabla}P_0.\underline{\xi} = 0$$

A FLUID WITH A FREE SURFACE : STIFFNESS CONDITION

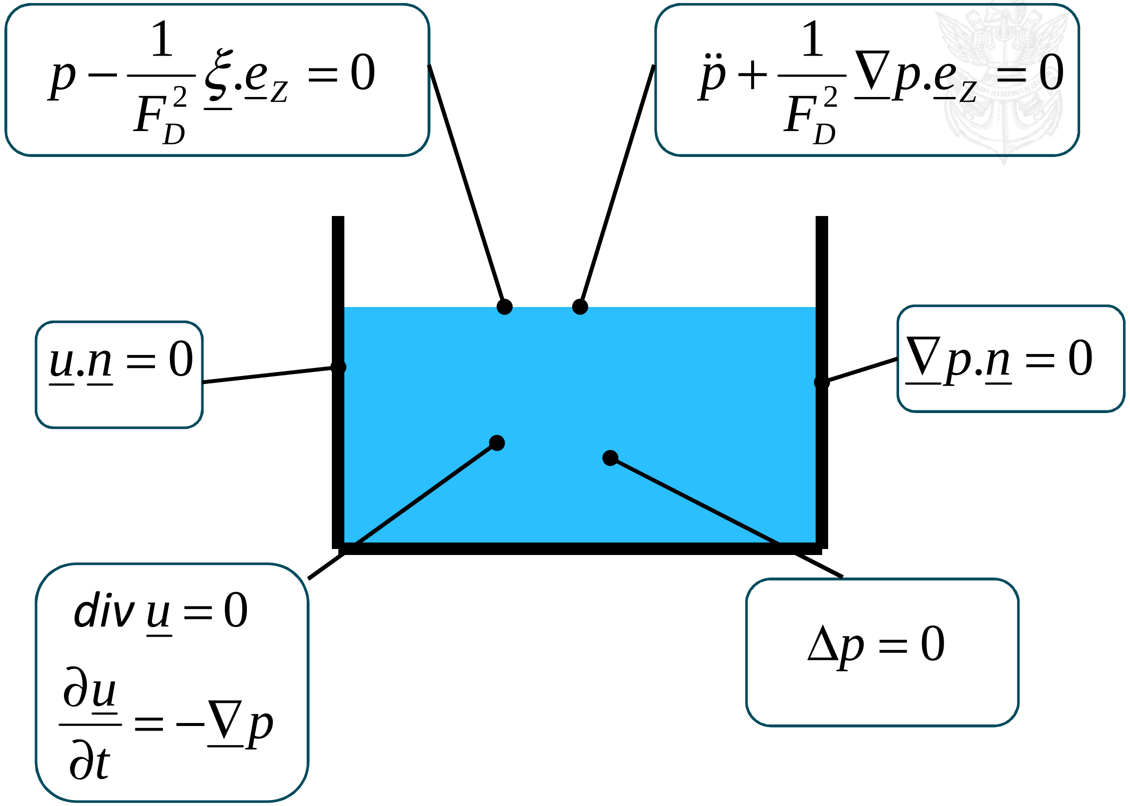
$$p + \underline{\nabla} P_0 . \underline{\xi} = 0$$

$$\boxed{p - \frac{1}{F_D^2} \underline{\xi} . \underline{e}_Z = 0}$$

Pressure Displacement



SLOSHING IN A RECTANGULAR TANK

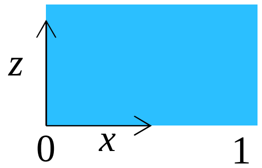


FIRST SLOSHING MODE

$$\boxed{\Delta p = 0}$$

$$\boxed{\ddot{p} + \frac{1}{F_D^2}\underline{\nabla} p . \underline{e}_Z = 0}$$

$$\boxed{\underline{\nabla} p . \underline{n} = 0}$$



$$p(x, z, t) = e^{i\omega t}\phi_P(x, z)$$

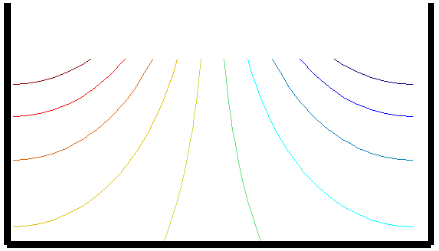
$$\phi_P(x, z) = F(x)G(z)$$

$$\boxed{\frac{F''}{F} = \frac{-G''}{G} = cst}$$

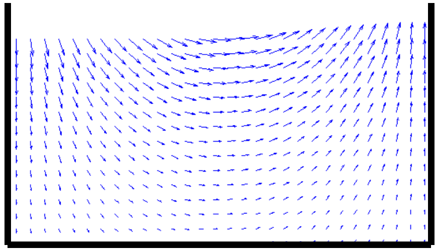
$$\boxed{\begin{array}{l} -\omega^2 G(1) + \dfrac{1}{F_D^2} F(x)G'(1) = 0 \\ F'(0) = F'(1) = 0 \\ G'(0) = 0 \end{array}}$$

FIRST SLOSHING MODE

$$\phi_P(x,z) = \cos \pi x \frac{\cosh \pi z}{\cosh \pi / 2}$$

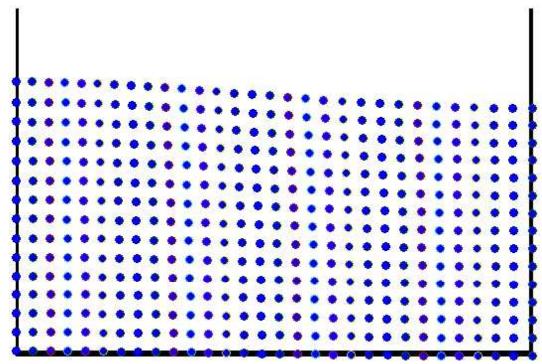


$$\underline{\phi}_u = -\underline{\nabla} \phi_p$$



FIRST SLOSHING MODE

$$\underline{u} = \underline{\phi}_u e^{i\omega t}$$



$$\omega = \frac{\sqrt{\pi \tanh\left(\pi / 2\right)}}{F_D}$$

SLOSHING MODAL MASS AND STIFFNESS

$\phi_p \quad \underline{\phi}_u \quad \omega$ $\qquad p(\underline{x},t) = e^{i\omega t}\phi_P(\underline{x})$

Local $\qquad \ddot{p} + \frac{1}{F_D^2}\underline{\nabla} p.\underline{e}_Z = 0$

Projected $\qquad \int_{\text{Free Surface}} \left[\ddot{p} + \frac{1}{F_D^2}\underline{\nabla} p.\underline{e}_Z\right]\phi_P dS = 0$

$$-\omega^2\left[\int_{\text{Free Surface}} \phi_p^2 \, dS\right] + \left[\frac{1}{F_D^2}\int_{\text{Free Surface}} \left(-\underline{\phi}_u.\underline{e}_Z\right)\phi_p dS\right] = 0$$

$$-\omega^2 M_F + K_F = 0$$

Mass $\qquad$ Stiffness

A SINGLE MODE APPROXIMATION FOR THE FLUID DYNAMICS

$\phi_p \quad \underline{\phi}_u \quad \omega \quad M_F \quad K_F$

$$\underline{u}(\underline{x},t) = \dot{Q}(t)\underline{\phi}_u(\underline{x})$$

$$p(\underline{x},t) = \ddot{Q}(t)\phi_p(\underline{x})$$

Free motion $$M_F\ddot{Q} + K_F Q = 0$$

Forced motion $$M_F\ddot{Q} + K_F Q = F$$

RESONANCE OF A SLOSHING MODE