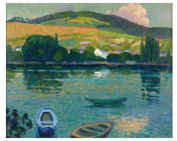
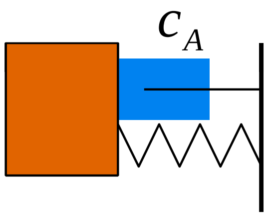
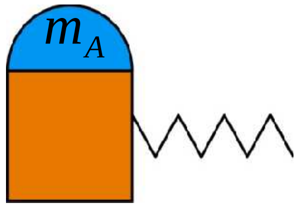
INTERMEDIATE STOKES NUMBERS

$$S_T = \frac{\rho L^2}{\mu T_{solid}}$$

?







SINGLE MODE APPROXIMATIONS FOR THE FLUID

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T} \Delta \underline{u}$$

$S_T << 1$

$$0 = -\underline{\nabla} p + \Delta \underline{u}$$

$$\underline{u} = \dot{q}\, \underline{\varphi}_u$$

$$0 = -\underline{\nabla} \varphi_p + \Delta \underline{\varphi}_u$$

$$f = -c_A \dot{q}$$

$S_T >> 1$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

$$\underline{u} = \dot{q}\, \underline{\varphi}_u$$

$$\underline{\varphi}_u = -\underline{\nabla} \varphi_p$$

$$f = -m_A \ddot{q}$$

SINGLE MODE APPROXIMATIONS FOR THE FLUID

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T}\Delta \underline{u}$$

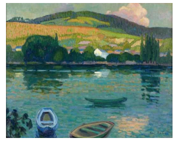
$$\underline{u} = A(t)B(x)$$

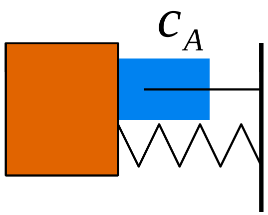
$$\frac{\partial Y}{\partial t} + \frac{1}{\tau}Y = 0 \quad \Rightarrow \quad Y = e^{-t/\tau}$$

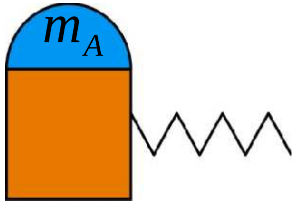
INTERMEDIATE STOKES NUMBERS

Internal time scale

$$S_T = \frac{\rho L^2}{\mu T_{solid}}$$







Instantaneous viscous diffusion

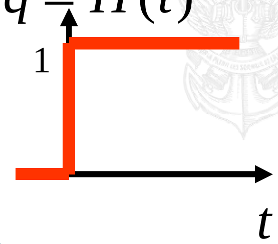
Instantaneous inertial response

IMPULSE RESPONSE

$$div\,\underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T}\Delta \underline{u}$$

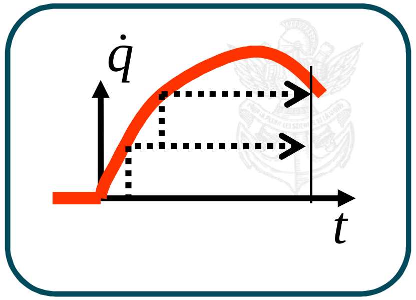
$$\dot{q} = H(t)$$



$$\underline{u}^I(\underline{x},t) \quad p^I(\underline{x},t)$$

$$f^I(t) = M \int_{Interface} \underline{\varphi}.\left[-p^I \underline{\underline{I}} + \frac{1}{S_T}\left(\nabla \underline{u}^I + \nabla^t \underline{u}^I\right)\right].\underline{n}\,dS$$

GENERAL CASE



$$\dot{q}(t) = \int_0^t \ddot{q}(\tau) H(t-\tau)\, d\tau$$

Linear equations

$$f(t) = \int_0^t \ddot{q}(\tau) f^I(t-\tau)\, d\tau$$

Force acting on the solid

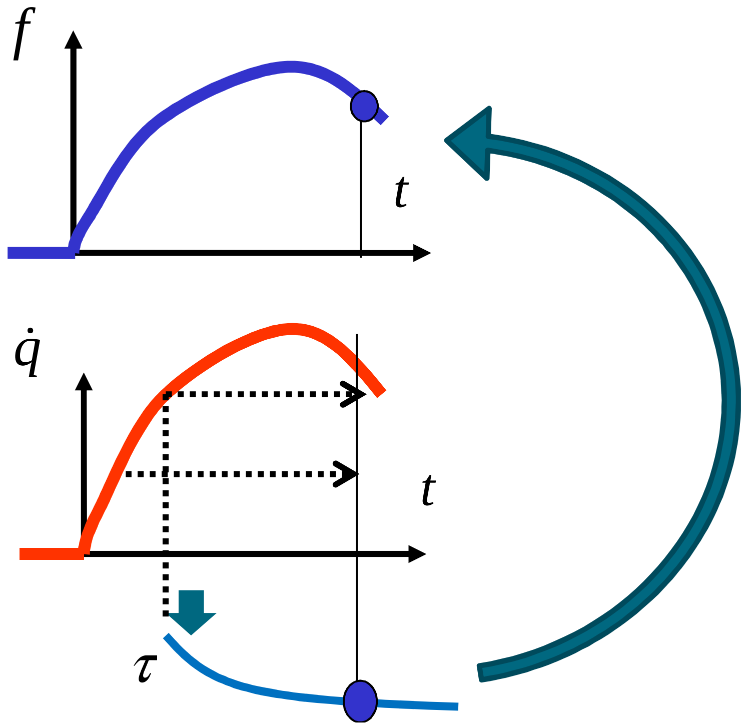
Acceleration of the solid

Impulse force

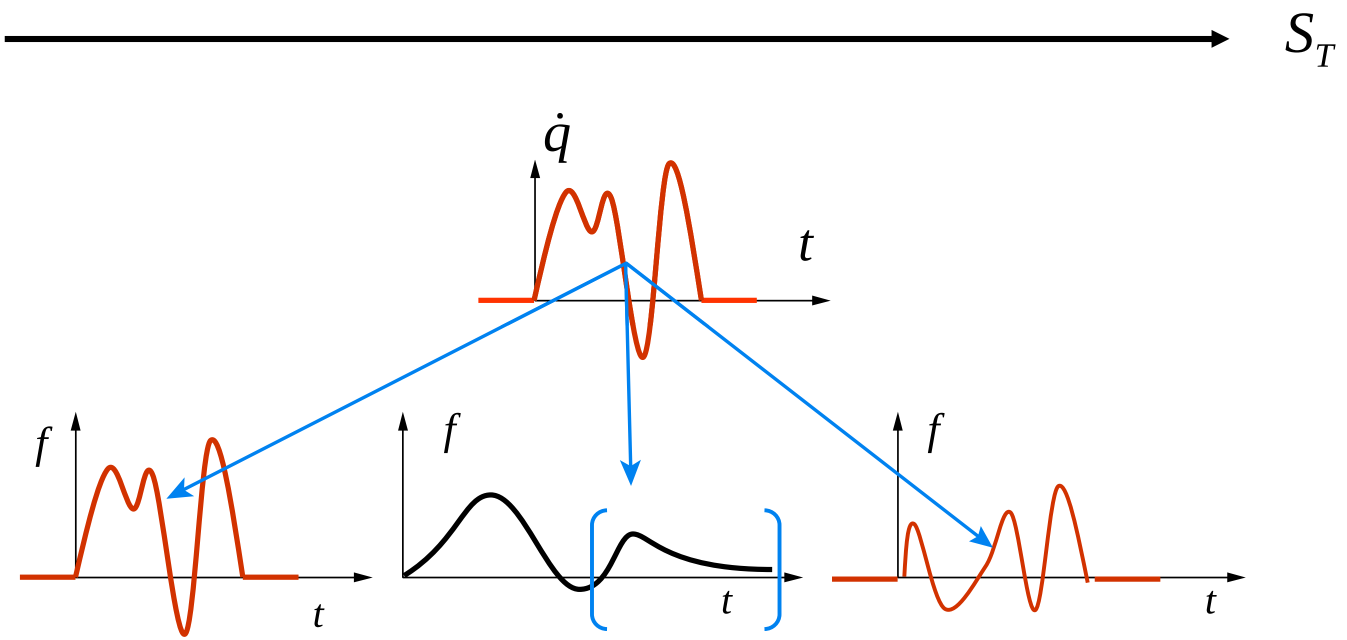
Convolution product

MEMORY EFFECT

$$f(t) = \int_0^t \ddot{q}(\tau) f^{I}(t-\tau) d\tau$$



EFFECT OF THE STOKES NUMBER

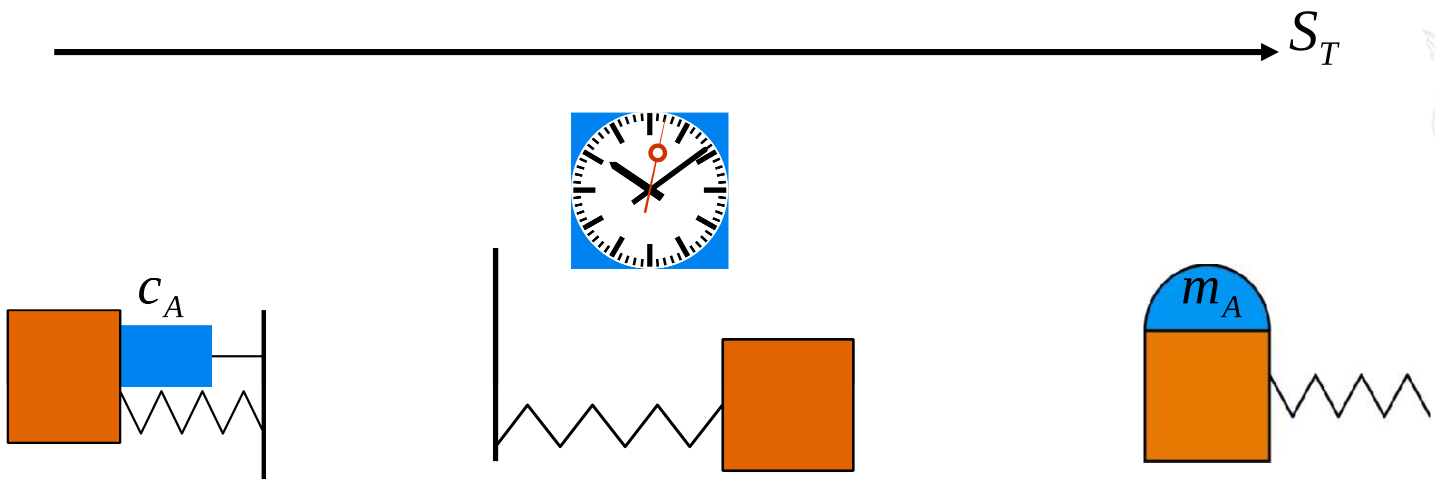


Added damping

Memory effect

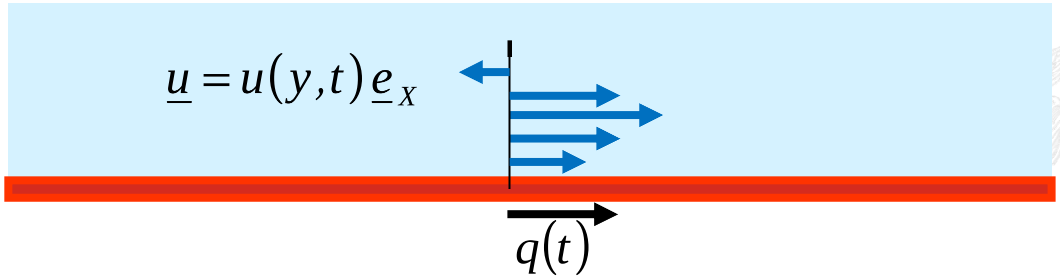
Added mass

EFFECT OF THE STOKES NUMBER



Added damping | Memory effect | Added mass

EXAMPLE : AN INFINITE PLATE BOUNDED BY A FLUID



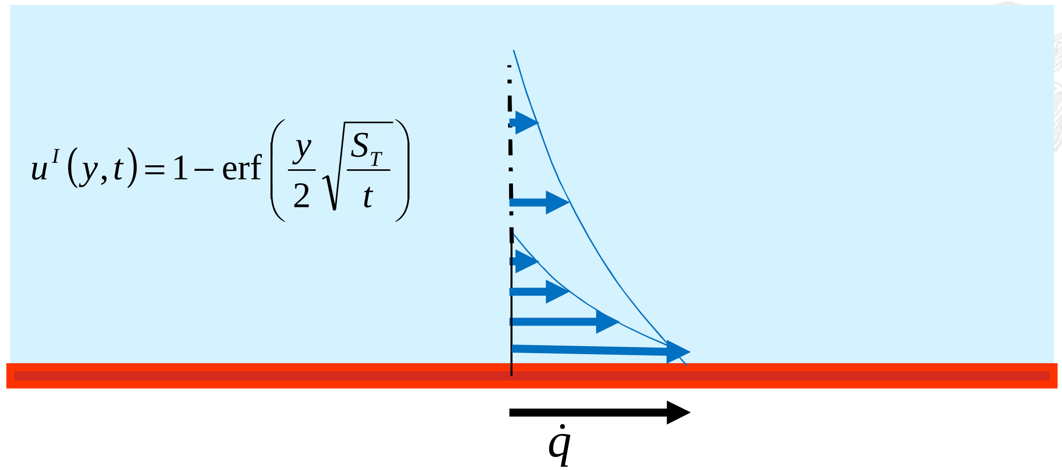
$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T}\Delta \underline{u} \quad \Rightarrow \quad \frac{\partial u}{\partial t} = \frac{1}{S_T}\frac{\partial^2 u}{\partial y^2}$$

$$\dot{q} = H(t)$$

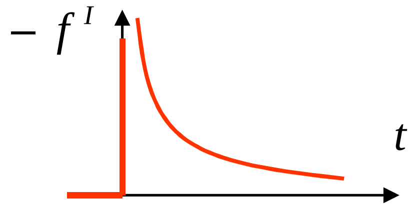
$$\boxed{u^I(y,t) = 1 - \operatorname{erf}\left(\frac{y}{2}\sqrt{\frac{S_T}{t}}\right)}$$

$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}}\int_0^u e^{-v^2}\,dv$$

EXAMPLE : AN INFINITE PLATE BOUNDED BY A FLUID

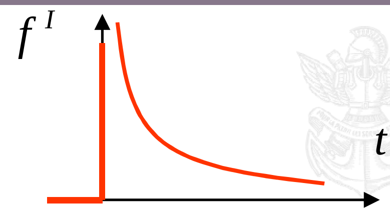


$$f^I(t) = \frac{M}{S_T}\frac{\partial u^I}{\partial y}(0,t) = -\frac{M}{\sqrt{\pi S_T t}}$$



EXAMPLE : AN INFINITE PLATE BOUNDED BY A FLUID

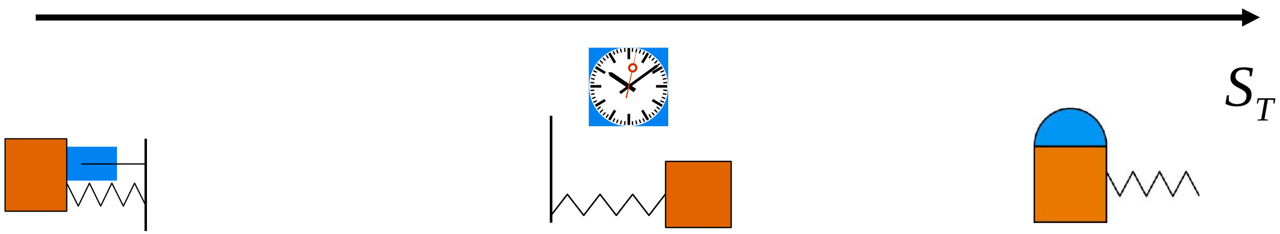
$$f^{I}(t) = -\frac{M}{\sqrt{\pi S_T t}}$$



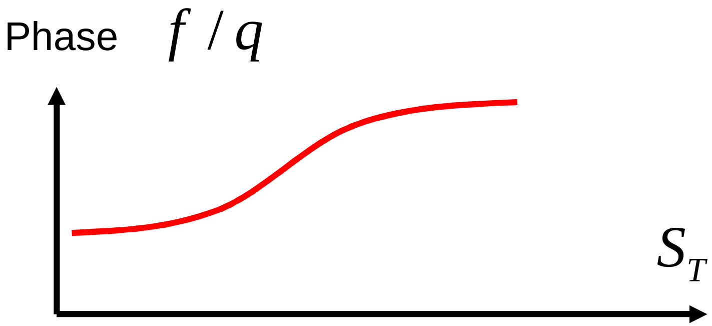
$$f(t) = \int_0^t \ddot{q}(\tau) f^{I}(t-\tau)\, d\tau$$

$$\boxed{f(t) = -\int_0^t \frac{M\ddot{q}(\tau)}{\sqrt{\pi S_T (t-\tau)}}\, d\tau}$$

ALL STOKES NUMBERS



MEASURING VISCOSITY USING VIBRATIONS



Food engineering