

SMALL REDUCED VELOCITY, SMALL MOTION



$$U_R \ll D$$

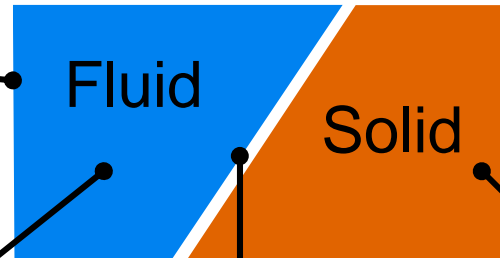
$$D \ll 1$$

FLUID AND SOLID

Fluid boundary conditions

$$\operatorname{div} \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T} \Delta \underline{u}$$



Solid boundary conditions

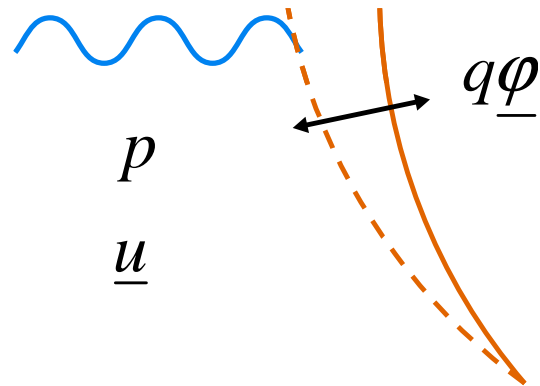
$$\ddot{q} + q = f$$

$$\underline{u} = \dot{q} \underline{\varphi}$$

$$M \int_{Interface} \underline{\varphi} \cdot \left[-p \underline{I} + \frac{1}{S_T} (\underline{\nabla} \underline{u} + \underline{\nabla}^t \underline{u}) \right] \cdot \underline{n} dS - Mq \int_{Interface} (\underline{\nabla} P_0 \cdot \underline{\varphi}) (\underline{\varphi} \cdot \underline{n}) dS = f$$

FLUID LOADING : MOTION INDUCED STRESS

$$M \int_{Interface} \underline{\varphi} \cdot \left[-p \underline{I} + \frac{1}{S_T} (\nabla \underline{u} + \nabla^t \underline{u}) \right] \cdot \underline{n} dS - Mq \int_{Interface} (\underline{\nabla} P_0 \cdot \underline{\varphi}) (\underline{\varphi} \cdot \underline{n}) dS = f$$



FLUID LOADING : MOTION INDUCED STRESS



$$\text{div } \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T} \Delta \underline{u}$$

$$\underline{u} = \dot{q} \underline{\varphi}$$

$$M \int_{\text{Interface}} \underline{\varphi} \cdot \left[-p \underline{I} + \frac{1}{S_T} (\underline{\nabla} \underline{u} + \underline{\nabla}^t \underline{u}) \right] \cdot \underline{n} dS = f$$

STOKES NUMBER

$$S_T = \frac{\rho c L}{\mu} = \frac{\rho L^2}{\mu T_{solid}}$$



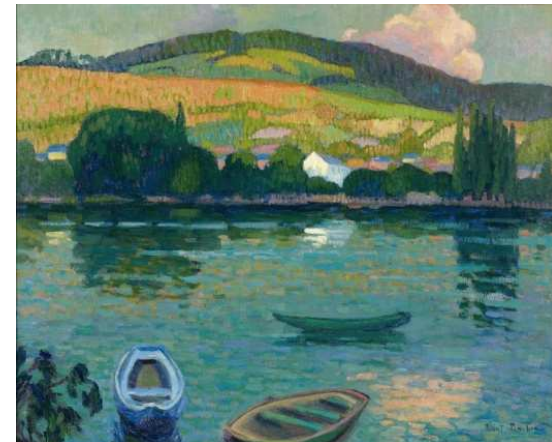
$$L = 1 \text{ m}$$

$$\mu / \rho = 10^{-6} \text{ m}^2 \text{s}^{-1}$$

$$T_{solid} = 1 \text{ s}$$



$$S_T = 10^6$$



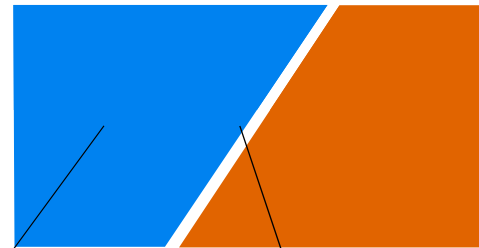
$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T} \Delta \underline{u} \quad \rightarrow \quad \frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

HIGH STOKES NUMBER

$$S_T \gg 1$$



Negligible viscous effects



$$\begin{aligned}\operatorname{div} \underline{u} &= 0 \\ \frac{\partial \underline{u}}{\partial t} &= -\nabla p\end{aligned}$$

$$\begin{aligned}\underline{u} \cdot \underline{n} &= \dot{q} \underline{\varphi} \cdot \underline{n} \\ M \int_{\text{Interface}} (-p \underline{n}) \cdot \underline{\varphi} dS &= f\end{aligned}$$

HIGH STOKES NUMBER : A SINGLE MODE SOLUTION

$$\underline{u}(\underline{x}, t) \cdot \underline{n} = \dot{q}(t) \underline{\varphi}(\underline{x}) \cdot \underline{n}$$



$$\underline{u}(\underline{x}, t) = \dot{q}(t) \underline{\varphi}_u(\underline{x})$$

$$p(\underline{x}, t) = \ddot{q}(t) \underline{\varphi}_p(\underline{x})$$

$$\begin{aligned} \operatorname{div} \underline{u} &= 0 \\ \frac{\partial \underline{u}}{\partial t} &= -\nabla p \end{aligned}$$

$$\begin{aligned} \operatorname{div} \underline{\varphi}_u &= 0 \\ \underline{\varphi}_u &= -\nabla \underline{\varphi}_p \end{aligned}$$

$$\begin{aligned} \underline{u}(\underline{x}, t) \cdot \underline{n} &= \dot{q}(t) \underline{\varphi}(\underline{x}) \cdot \underline{n} \\ f &= M \int_{\text{Interface}} (-p \underline{n}) \cdot \underline{\varphi} dS \end{aligned}$$

$$\begin{aligned} \underline{\varphi}_u \cdot \underline{n} &= \underline{\varphi} \cdot \underline{n} \\ f &= -\ddot{q} \left[M \int_{\text{Interface}} \underline{\varphi}_p \cdot \underline{n} \cdot \underline{\varphi} dS \right] \end{aligned}$$

ADDED MASS

$$f = -\ddot{q} \left[M \int_{Interface} \varphi_p \underline{n} \cdot \underline{\varphi} dS \right]$$



The fluid force is an inertia force

$$f = -m_A \ddot{q}$$

↑
Added mass

The fluid response is instantaneous

$$f(t) = -m_A \ddot{q}(t)$$

ADDED MASS

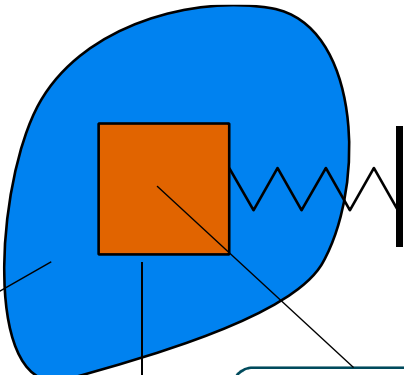


Diagram illustrating a rigid body (orange square) submerged in a fluid (blue region). The body is connected to a fixed wall by a spring. The fluid is represented by a blue region surrounding the body.

Equations governing the fluid flow:

$$\text{div } \underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p$$

Equation governing the body motion:

$$\ddot{q} + q = f$$

Boundary conditions at the interface:

$$\underline{u}(x, t) \cdot \underline{n} = \dot{q}(t) \underline{\varphi}(x) \cdot \underline{n}$$

$$f = M \int_{\text{Interface}} (-p \underline{n}) \cdot \underline{\varphi} dS$$

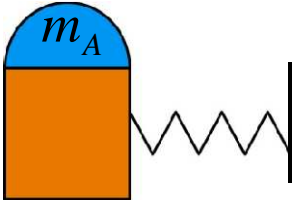



Diagram illustrating a simplified mass-spring system. The mass is represented by an orange rectangle with a blue semi-circular top labeled m_A . The mass is connected to a fixed wall by a spring.

Equation governing the simplified system:

$$(1 + m_A) \ddot{q} + q = 0$$


DIMENSIONLESS NUMBERS AND STEPS OF APPROXIMATION

Initial choice	D	F_R	R_E	C_Y	U_R
New choice	D	F_D	S_T	M	U_R
Small reduced velocity	D	F_D	S_T	M	
Small amplitude		F_D	S_T	M	
Small viscosity		F_D		M	

$$f = - \left[-\frac{M}{F_D^2} \int_{\text{Interface}} (\underline{\varphi} \cdot \underline{e}_Z)(\underline{\varphi} \cdot \underline{n}) dS \right] q - \left[M \int_{\text{Interface}} \varphi_p \underline{n} \cdot \underline{\varphi} dS \right] \ddot{q}$$

$$f = -k_F q - m_A \ddot{q}$$