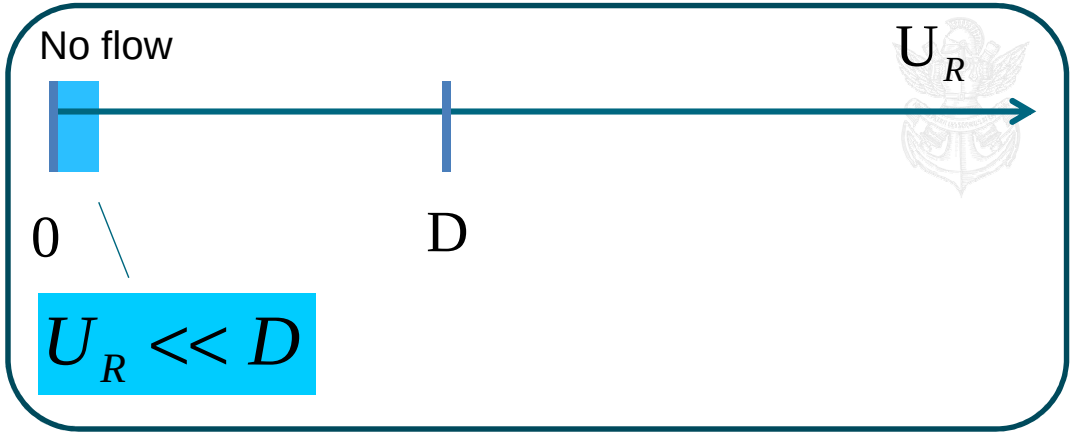
# SMALL REDUCED VELOCITY, SMALL MOTION




$$D << 1$$

$$M \int_{Interface} \underline{\varphi}.[....].\underline{n}\, dS - Mq \int_{Interface} \left(\underline{\nabla} P_0.\underline{\varphi}\right)\left(\underline{\varphi}.\underline{n}\right) dS = f$$

# FLUID-INDUCED STIFFNESS

$$M \int_{Interface} \underline{\varphi}.[....].\underline{n}\, dS - Mq \int_{Interface} (\underline{\nabla} P_0.\underline{\varphi})(\underline{\varphi}.\underline{n}) dS = f$$

Modal displacement | Mass number | Hydrostatic pressure | Modal shape | Geometry

$$0 = -\frac{1}{F_D^2}\underline{e}_Z - \underline{\nabla} P_0$$

$$f = q\frac{M}{F_D^2}\int_{Interface} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS$$

# FLUID-INDUCED STIFFNESS

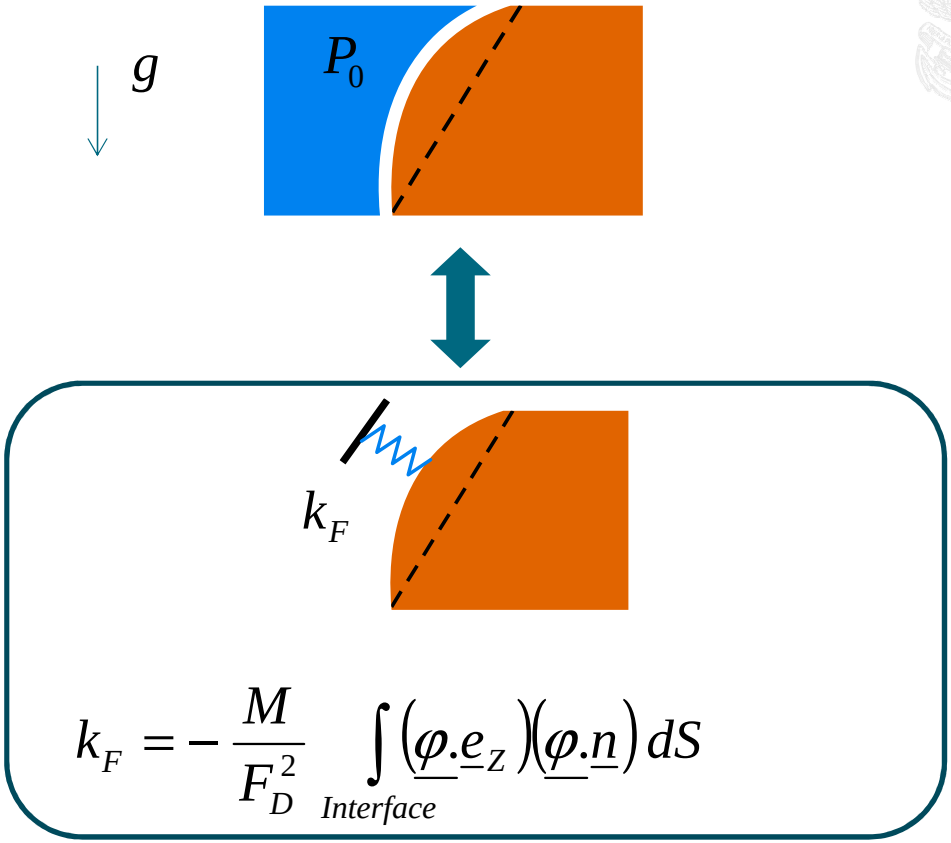
$$f = q \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS$$

$$f = -q\, k_F$$

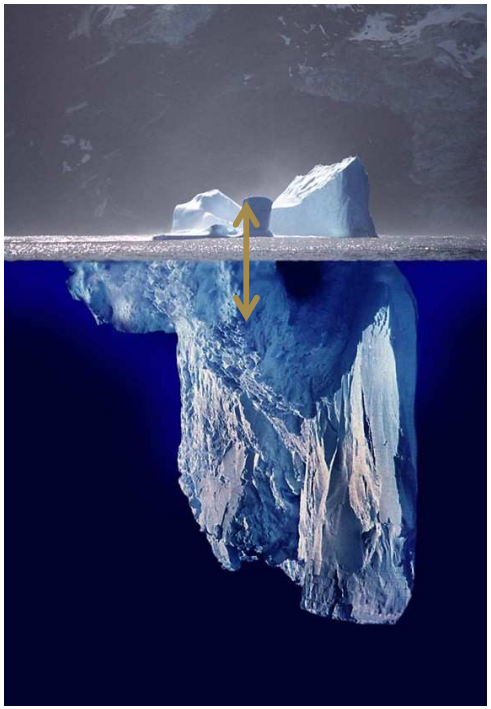
$$k_F = -\frac{M}{F_D^2} \int_{Interface} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS$$

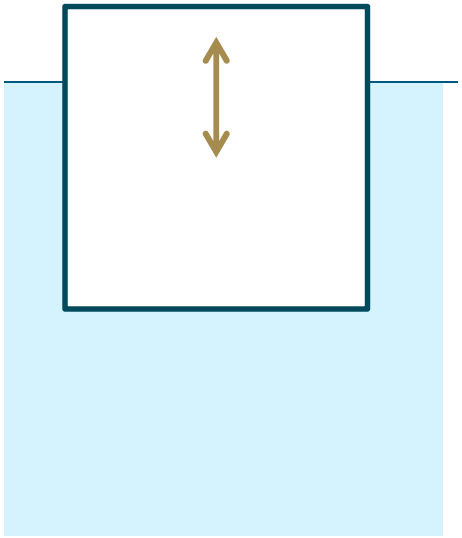
$$M = \frac{\rho}{\rho_S} \qquad F_D{}^2 = \frac{L}{T_{Solid}^2\, g}$$

# FLUID-INDUCED STIFFNESS



# EXAMPLE : ICEBERGS AND ICE CUBES





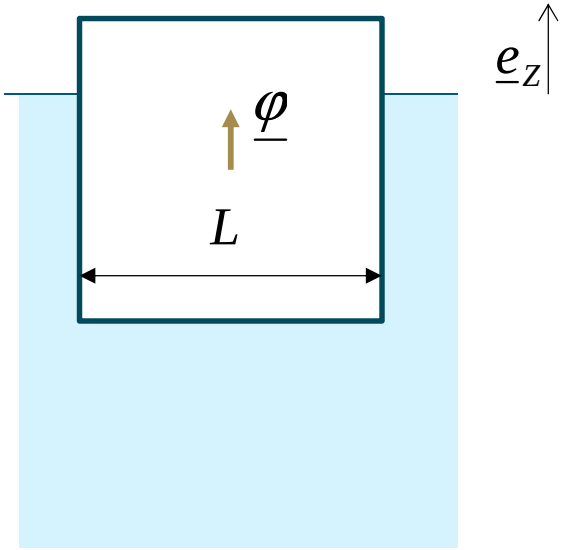
# FROM ICE CUBES TO ICEBERGS

$$f = q \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS$$

$$M = \frac{\rho_{water}}{\rho_{ice}}$$

$$F_D = \frac{L}{T_{solid}\sqrt{gL}}$$

$$\underline{\varphi} = \underline{e}_Z$$

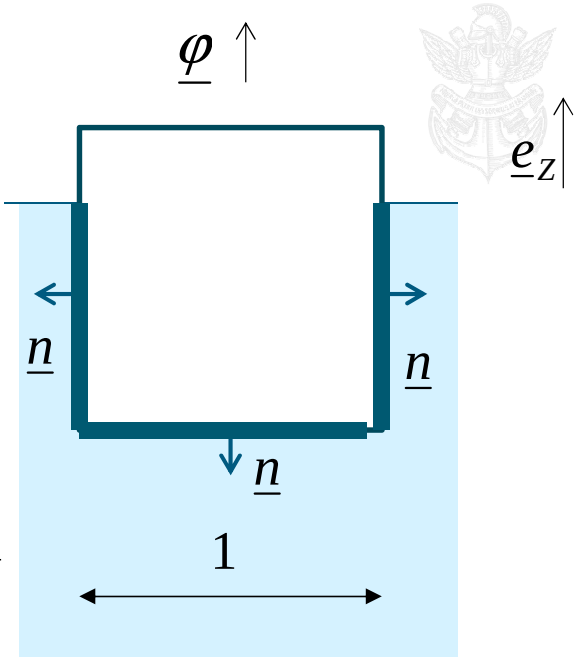


# FROM ICE CUBES TO ICEBERGS

$$\int_{Interface} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS$$

$$\int_{Sides} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS = 0$$

$$\int_{Bottom} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS = -1$$

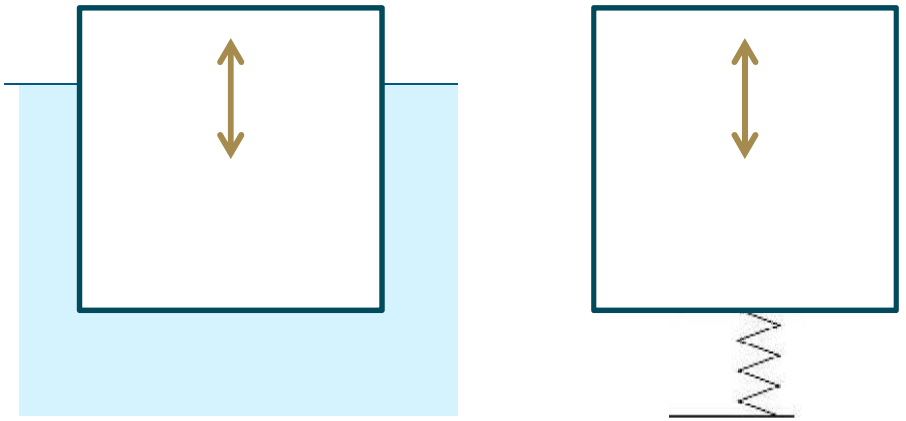



$$\int_{Interface} (\underline{\varphi}.\underline{e}_Z)(\underline{\varphi}.\underline{n})\, dS = -1$$

# FROM ICE CUBES TO ICEBERGS

$$M = \frac{\rho_{water}}{\rho_{ice}} \qquad F_D = \frac{1}{T_{solid}}\sqrt{\frac{L}{g}} \qquad \int_{Interface} \left(\underline{\varphi}.\underline{e}_Z\right)\left(\underline{\varphi}.\underline{n}\right) dS = -1$$

$$f = -\left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{LT_{solid}^2}\right] q$$



# FROM ICE CUBES TO ICEBERGS

$$f = -\left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{LT_{solid}^2}\right]q$$

$$\frac{d^2q}{dt^2} = -\left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{LT_{solid}^2}\right]q$$

$$\frac{d^2q}{dt^2} + \left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{LT_{solid}^2}\right]q = 0$$

$$\frac{d^2q}{dt^{-2}} + \left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{LT_{solid}^2}\right]q = 0$$

# FROM ICE CUBES TO ICEBERGS

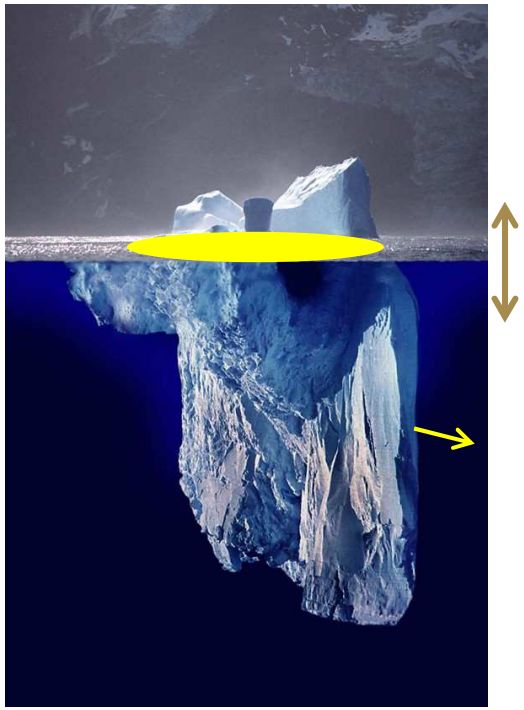
$$\frac{d^2q}{d\bar{t}^{-2}} + \left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{LT_{solid}^2}\right]q = 0 \qquad \bar{t} = \frac{t}{T_{solid}}$$

$$\frac{d^2q}{dt^2} + \left[\frac{\rho_{water}}{\rho_{ice}}\frac{g}{L}\right]q = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{\rho_{water}}{\rho_{ice}}\frac{g}{L}}$$

$$T = 2\pi / \sqrt{\frac{\rho_{water}}{\rho_{ice}}\frac{g}{L}}$$

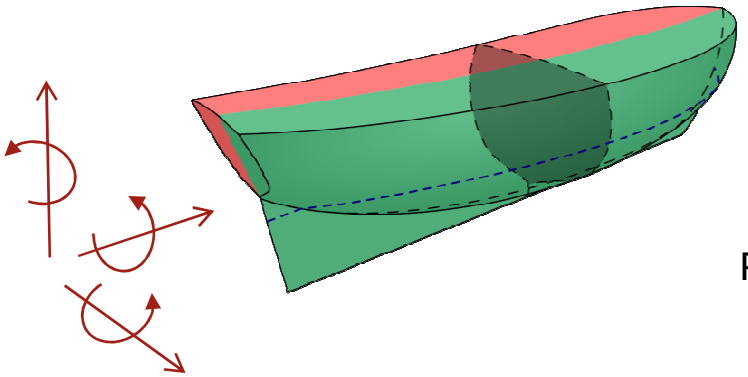
# FROM ICE CUBES TO ICEBERGS

$$f = q\,\frac{M}{F_D^2}\int_{Interface}\left(\underline{\varphi}.\underline{e}_Z\right)\left(\underline{\varphi}.\underline{n}\right)dS$$



$$\int_{Interface}\left(\underline{e}_Z.\underline{n}\right)dS = S_0$$

# MORE GENERAL CASE



$\underline{\varphi}$

Pitch
Roll
Yaw

$$k_F = -\frac{M}{F_D^2} \int_{Interface} \left(\underline{\varphi}.\underline{e}_Z\right)\left(\underline{\varphi}.\underline{n}\right) dS$$

Stiffness in each type of motion

$$f = -q\, k_F$$

# FLUID INDUCED FORCE

$$M \int_{Interface} \underline{\varphi}.[....].\underline{n}\, dS - Mq \int_{Interface} \left(\underline{\nabla} P_0.\underline{\varphi}\right)\left(\underline{\varphi}.\underline{n}\right) dS = f$$

$$\left[ -p\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla \underline{u} + \nabla^t \underline{u}\right) \right]$$

Linearized fluid dynamics

Hydrostatics

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