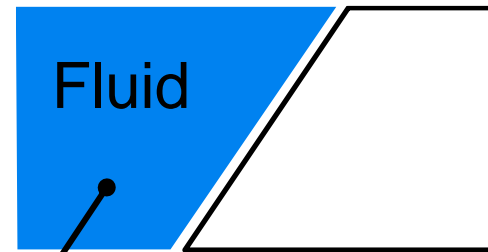


EQUATIONS IN THE FLUID DOMAIN



Mass Balance

$$\text{div } \underline{U} = 0$$

Momentum Balance

$$\rho \frac{d\underline{U}}{dt} = -\rho g \underline{e}_z - \underline{\nabla} p + \mu \Delta \underline{U}$$

EQUATIONS IN THE SOLID DOMAIN



Continuum mechanics

Modal approximation

Single mode approximation



SINGLE MODE APPROXIMATION IN THE SOLID DOMAIN

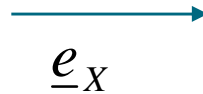


$$\underline{\xi}(\underline{x}, t) = q(t) \underline{\varphi}(\underline{x})$$

Modal
displacement

Modal
shape

SINGLE MODE APPROXIMATION IN THE SOLID DOMAIN



$$\underline{\xi}(\underline{x}, t) = q(t) \underline{\varphi}(\underline{x})$$

Modal
displacement

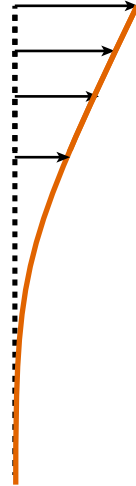
Modal
shape

$$\underline{\varphi}(\underline{x}) = \underline{e}_x$$

SINGLE MODE APPROXIMATION IN THE SOLID DOMAIN

$$\underline{\xi}(\underline{x}, t) = q(t) \underline{\varphi}(\underline{x})$$

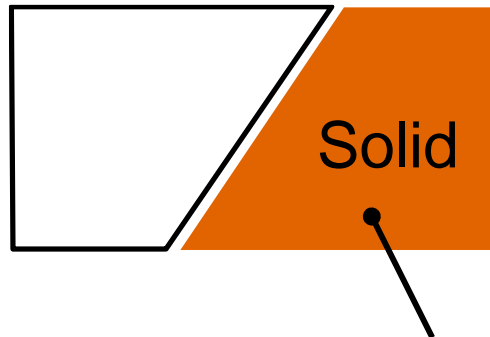
Modal shape



The diagram illustrates a modal shape, which is a spatial distribution of displacement. It features a vertical dashed line on the left, representing the undeformed state. To the right of this line is a solid orange curve that represents the deformed state. Four horizontal arrows of increasing length point from the dashed line to the orange curve at different heights, indicating the magnitude of displacement at those points. A blue arrow points from the term $\underline{\varphi}(\underline{x})$ in the equation above to the orange curve.



MODAL EQUATIONS IN THE SOLID DOMAIN



$$\underline{\xi}(\underline{x}, t) = q(t) \underline{\varphi}(\underline{x})$$

Modal
displacement
(unknown)

$$m \frac{d^2 q}{dt^2} + kq = f$$

Modal
shape
(known)

MODAL EQUATIONS IN THE SOLID DOMAIN

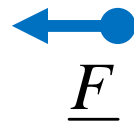


$$m \frac{d^2 q}{dt^2} + kq = f$$

Modal
mass
(known)

Modal
stiffness
(known)

Modal
load
(known)



$$f = \underline{F} \cdot \underline{e}_x$$

MODAL EQUATIONS IN THE SOLID DOMAIN



$$m \frac{d^2 q}{dt^2} + kq = f$$

Modal
mass
(known)

Modal
stiffness
(known)

Modal
load
(known)

A diagram showing a blue point mass on a curved orange line. A blue arrow labeled \underline{F} points to the right from the mass. To the left of the mass, a vertical dashed line is labeled $\underline{\varphi}$. Below the diagram, the equation $f = \int \underline{F} \cdot \underline{\varphi} dx$ is written.

$$f = \int \underline{F} \cdot \underline{\varphi} dx$$

FLUID AND SOLID

Fluid

Solid



Mass Balance

$$\operatorname{div} \underline{U} = 0$$

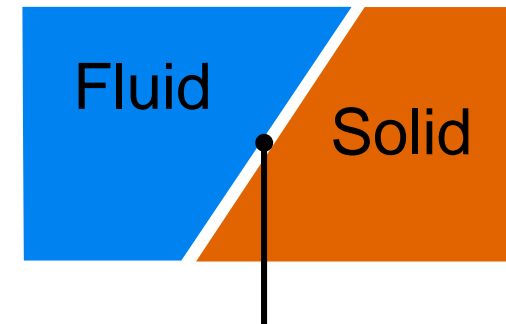
Momentum Balance

$$\rho \frac{d\underline{U}}{dt} = -\rho g \underline{e}_z - \nabla p + \mu \Delta \underline{U}$$

$$\underline{\xi}(\underline{x}, t) = q(t) \underline{\varphi}(\underline{x})$$

$$m \frac{d^2 q}{dt^2} + kq = f$$

AT THE INTERFACE

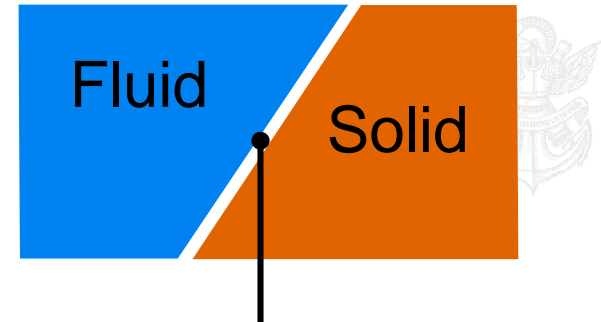


Kinematic condition

$$\underline{U} = \frac{\partial \underline{\xi}}{\partial t}$$

$$\underline{U}(\underline{x}, t) = \frac{dq}{dt}(t) \underline{\varphi}(\underline{x})$$

AT THE INTERFACE



Dynamic condition

$$\left[-p \underline{\underline{I}} + \mu (\nabla \underline{U} + \nabla^t \underline{U}) \right] . \underline{n}$$

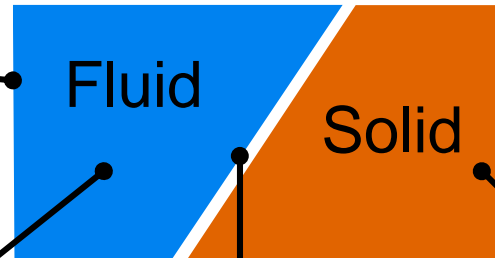


$$\int_{Interface} \left\{ \left[-p \underline{\underline{I}} + \mu (\nabla \underline{U} + \nabla^t \underline{U}) \right] . \underline{n} \right\} . \underline{\varphi} dS = f$$

FLUID AND SOLID

Fluid boundary
conditions

Solid boundary
conditions



$$\operatorname{div} \underline{U} = 0$$

$$\rho \frac{d\underline{U}}{dt} = -\rho g \underline{e}_z - \nabla p + \mu \Delta \underline{U}$$

$$m \frac{d^2 q}{dt^2} + kq = f$$

$$\underline{U}(\underline{x}, t) = \frac{dq}{dt}(t) \underline{\varphi}(\underline{x})$$

$$\int_{Interface} \{ [-p \underline{I} + \mu(\nabla \underline{U} + \nabla^t \underline{U})] \cdot \underline{n} \} \cdot \underline{\varphi} dS = f$$