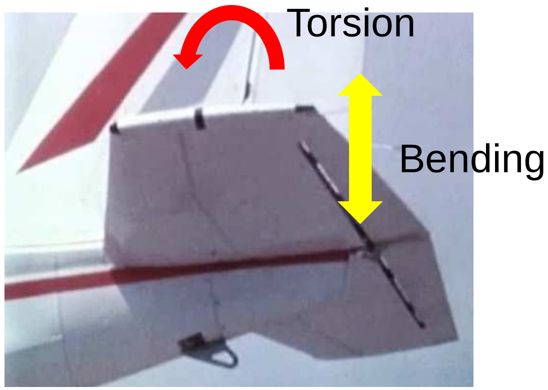
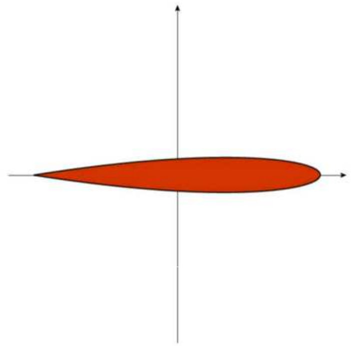
COUPLED-MODE FLUTTER OF A WING

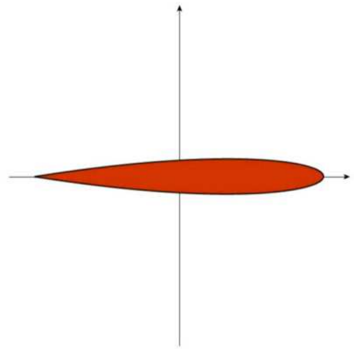


COUPLED-MODE FLUTTER OF A WING

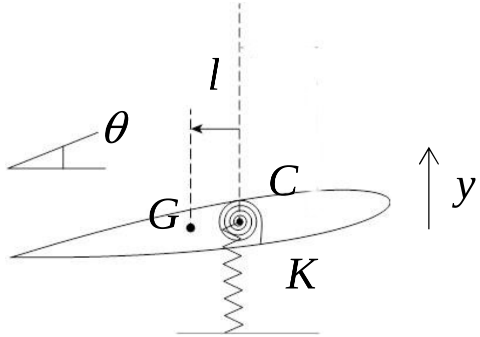
Mode 1 : plunge



Mode 2 : torsion



TWO-MODES APPROXIMATION



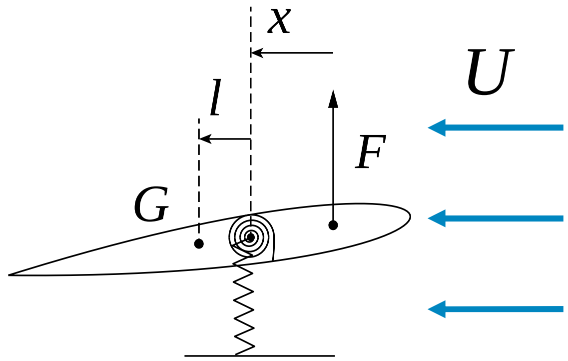
Kinetic energy $K_C = \frac{1}{2} M \left(\dot{y} - l\dot{\theta}\right)^2 + \frac{1}{2} J\dot{\theta}^2$

Elastic energy $W = \frac{1}{2} Ky^2 + \frac{1}{2} C\theta^2$

Using Lagrange equation

$$\boxed{\begin{aligned} &M\ddot{y} - Ml\ddot{\theta} + Ky = 0 \\ &J\ddot{\theta} + C\theta = -Kyl \end{aligned}}$$

FLUID FORCES



Quasi-static aeroelasticity approximation

$$F = \frac{1}{2}\rho U^2 L C_L(\theta)$$

$$M\ddot{y} - Ml\ddot{\theta} + Ky = \boxed{\frac{1}{2}\rho U^2 L C_L(\theta)}$$

$$J\ddot{\theta} + C\theta = -Kyl + \boxed{\frac{1}{2}\rho U^2 L(x+l) C_L(\theta)}$$

TWO MODES WITH THE FLUID FORCE

Expansion of the force at the first order

$$F = \frac{1}{2}\rho U^2 L C_L(\theta) \approx \frac{1}{2}\rho U^2 L \left(\frac{\partial C_L}{\partial \theta}\right)_0 \theta$$

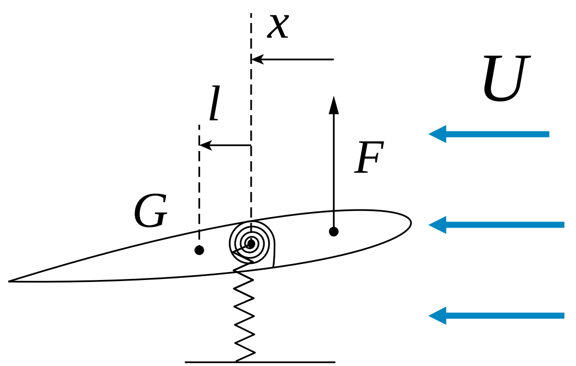
$$\frac{\partial C_L}{\partial \theta} = 2\pi$$

Dimensionless quantities

$$q_1 = \frac{y}{DL};\ q_2 = \frac{\theta}{D}$$

$$\Omega = \sqrt{\frac{KJ}{CM}};\ \kappa = \frac{KL^2}{C};\ \varepsilon = \frac{l}{L};\ \chi = \frac{x}{L}$$

$$C_Y = \frac{\rho U^2 L^2}{2C} \qquad \bar{t} = \sqrt{\frac{C}{J}}t$$



Coupled equations in flow

$$\begin{aligned} \ddot{q}_1 - \varepsilon\,\ddot{q}_2 + \Omega^2 q_1 &= C_Y 2\pi \frac{\Omega^2}{\kappa} q_2 \\ \ddot{q}_2 + \left(1 - C_Y 2\pi(\varepsilon + \chi)\right) q_2 &= -\kappa\varepsilon\, q_1 \end{aligned}$$

COUPLED EQUATIONS IN FLOW

$$\ddot{q}_1 - \varepsilon\,\ddot{q}_2 + \Omega^2 q_1 = C_Y\, 2\pi \frac{\Omega^2}{\kappa} q_2$$

$$\ddot{q}_2 + \left(1 - C_Y\, 2\pi(\varepsilon + \chi)\right) q_2 = -\kappa\varepsilon\; q_1$$

Coincidence

+

Non-symmetric
stiffness coupling

= Dynamic instability !

COUPLED EQUATIONS IN FLOW

$$\ddot{q}_1 - \varepsilon\, \ddot{q}_2 + \Omega^2 q_1 = \boxed{C_Y\, 2\pi \frac{\Omega^2}{\kappa} q_2}$$

$$\ddot{q}_2 + \left(1 - \boxed{C_Y\, 2\pi(\varepsilon + \chi)}\right) q_2 = -\kappa\varepsilon\; q_1$$

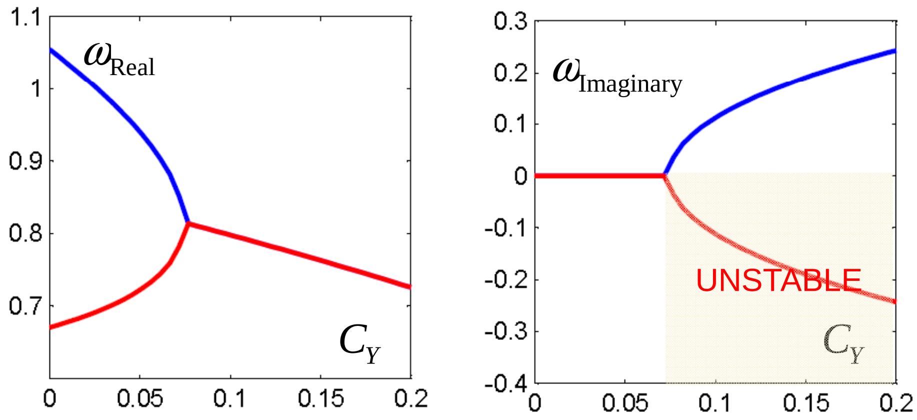
Modes for a given set of parameters

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_0 e^{i\omega t}\right] \Rightarrow \begin{array}{ll} \omega_1 & \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_1 \\ \omega_2 & \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_2 \end{array}$$

$$\boxed{\omega(C_Y) = \omega_{\text{real}}(C_Y) + i\,\omega_{\text{imag}}(C_Y)}$$

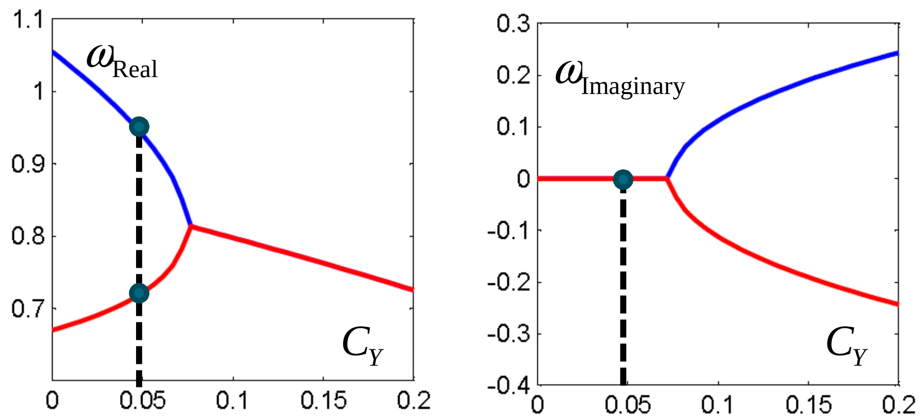
EFFECT OF THE CAUCHY NUMBER ON FREQUENCIES

$$\Omega^2 = 1/2;\ \kappa = 1;\ \varepsilon = 1/4;\ \chi = 1/4$$

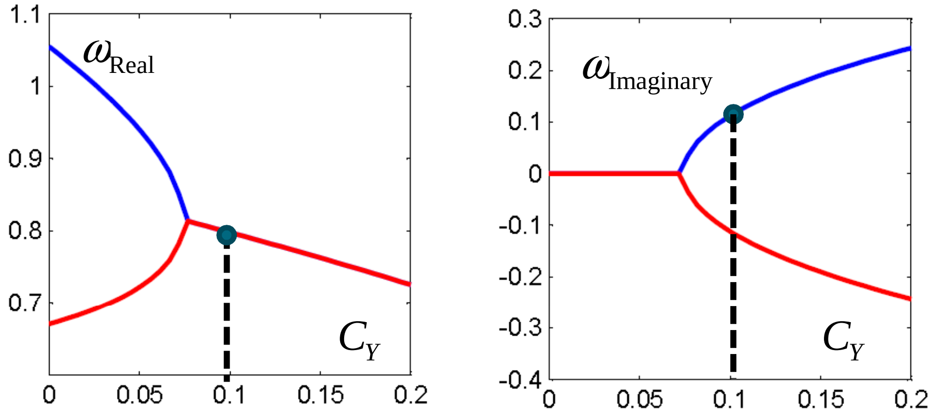


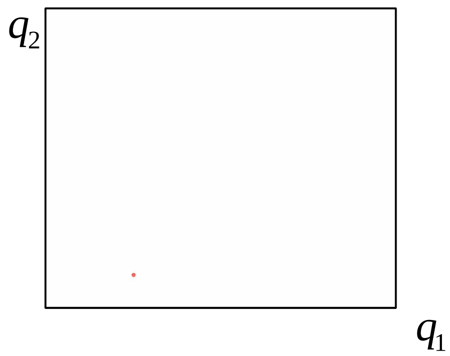
Coincidence

BEFORE COINCIDENCE

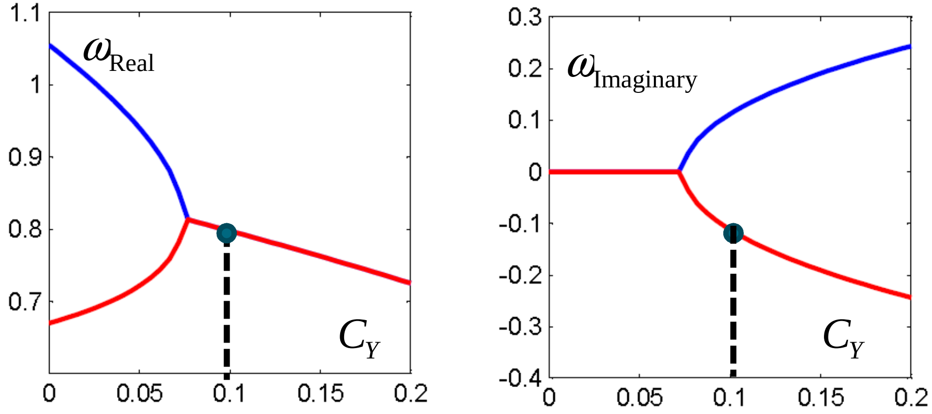


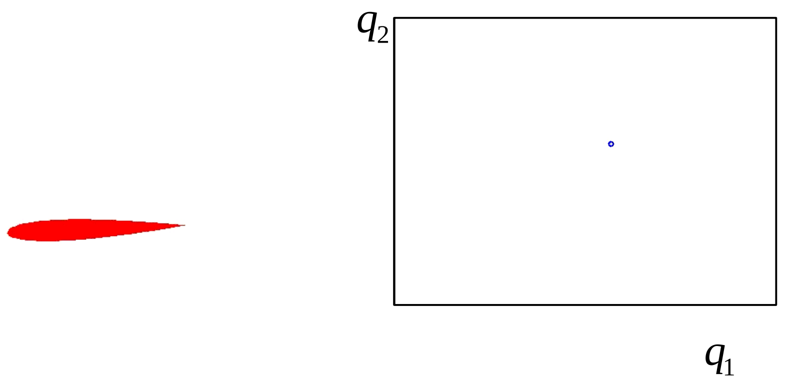
AFTER COINCIDENCE : DAMPED MODE



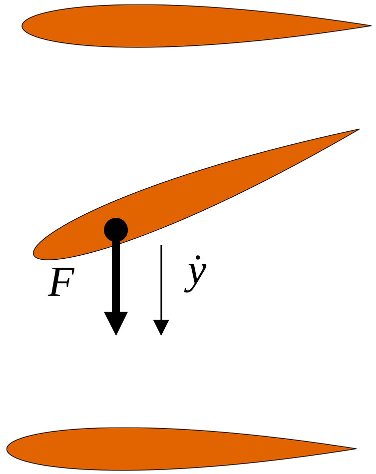


AFTER COINCIDENCE : UNSTABLE MODE





UNSTABLE MODE : BALANCE OF ENERGY



$$W_{FS} = \int_T F\dot{y}\, dt > 0$$

COUPLED MODE FLUTTER OF A WING

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