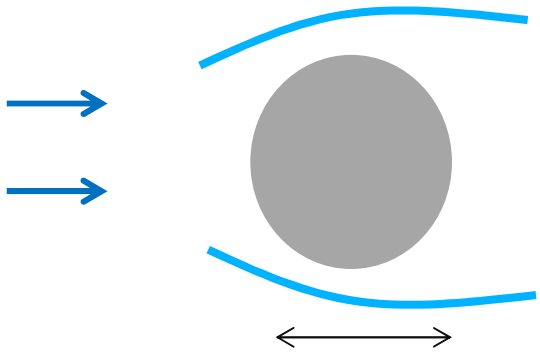
FLOW AROUND A SPHERE : DIMENSIONAL QUANTITIES



Velocity U

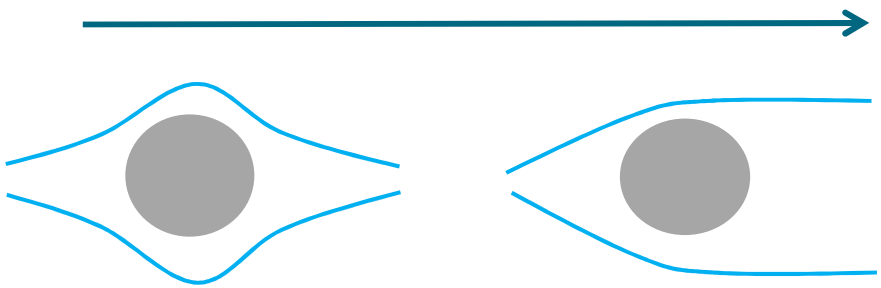
Diameter L

Density ρ

Viscosity μ

FLOW AROUND A SPHERE : THE DIMENSIONLESS REYNOLDS NUMBER

$$R_E = \frac{\rho U L}{\mu}$$



Creeping flow

Detached flow

PHYSICAL LAWS

$$f(x_1, x_2, \ldots, x_N) = 0$$

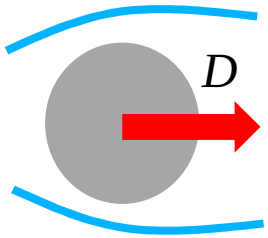
Dimensional quantities

A physical law must relate only **dimensionless** quantities

$$F(X_1, X_2, \ldots, X_P) = 0$$

Dimensionless quantities

EXAMPLE : DRAG ON A SPHERE

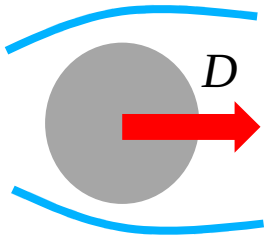


$$f(D, U, \rho, \mu, L) = 0$$

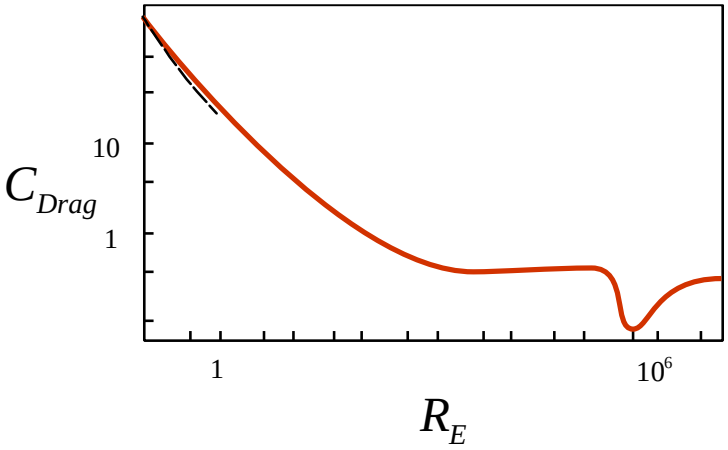
$$F\left(\frac{D}{\rho U^2 L^2}, \frac{\rho U L}{\mu}\right) = 0$$

$$F\left(C_{Drag}, R_E\right) = 0$$

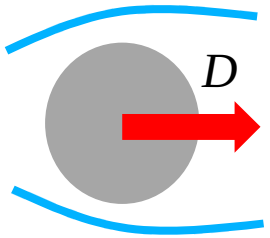
EXAMPLE : DRAG ON A SPHERE



$$F\left(C_{Drag}, R_E\right) = 0$$



EXAMPLE : DRAG ON A SPHERE



$$f(D, U, \rho, \mu, L) = 0$$

Dimensional

$$F\left(C_{Drag}, R_E\right) = 0$$

Dimensionless

DIMENSIONS OF A VARIABLE

$$[x] = L^{\alpha} M^{\beta} T^{\gamma}$$

length mass time

Example : gravity

$$g = 9.81 \text{ m/s}^2$$

$$[g] = L^1 M^0 T^{-2}$$

DIMENSIONS OF A VARIABLE

$$f(x_1, x_2, \ldots, x_N) = 0$$

Dimensional quantities

$$F(X_1, X_2, \ldots, X_P) = 0$$

Dimensionless quantities

$$[X_i] = L^0 M^0 T^0$$

VASCHY-BUCKINGHAM (OR PI) THEOREM

$$f(x_1, x_2, ..., x_N) = 0$$

Dimensional quantities

$$[x_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

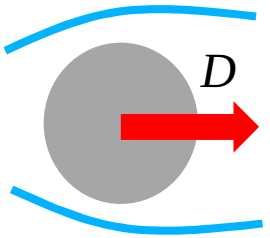
$$R = \text{rank} \begin{bmatrix} \alpha_1 & ... & \alpha_N \\ \beta_1 & ... & \beta_N \\ \gamma_1 & ... & \gamma_N \end{bmatrix}$$

$$F(X_1, X_2, ..., X_P) = 0$$

Dimensionless quantities

$$P = N - R$$

EXAMPLE : DRAG ON A SPHERE

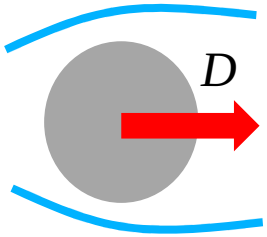


$$\begin{bmatrix} D & U & \rho & \mu & L \end{bmatrix}$$

$$R = \text{rank}\begin{pmatrix} \text{L} \\ \text{M} \\ \text{T} \end{pmatrix}\begin{bmatrix} 1 & 1 & -3 & -1 & 1 \\ -1 & 0 & 1 & 1 & 0 \\ -2 & -1 & 0 & -1 & 0 \end{bmatrix} = 3$$

$$P = 5 - 3 = 2$$

EXAMPLE : DRAG ON A SPHERE



$$f\left(D,U,\rho,\mu,L\right)=0$$

$$P=5-3=2$$

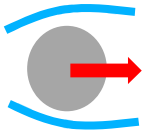
$$F\left(\frac{D}{\rho U^2L^2},\frac{\rho UL}{\mu}\right)=0$$

MEANING OF DIMENSIONLESS PARAMETERS

$$[\,X\,] = L^0 M^0 T^0$$

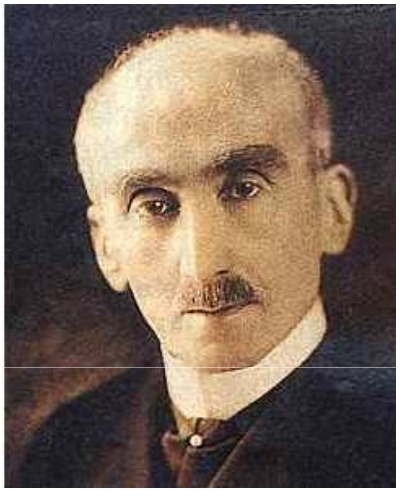
$$X = \frac{time}{time} = \frac{length}{length} = \frac{force}{force} = ...$$

Drag on a sphere



$$C_{Drag} = \frac{D}{\rho U^2 L^2} = \frac{\text{Drag force}}{\text{Sum of dynamic pressure}}$$

PHILOSOPHY.. OR PHYSICS ?



H. Bergson (1859-1941)

« If all motions were to happen twice as fast nothing would have to be changed in our formulas»