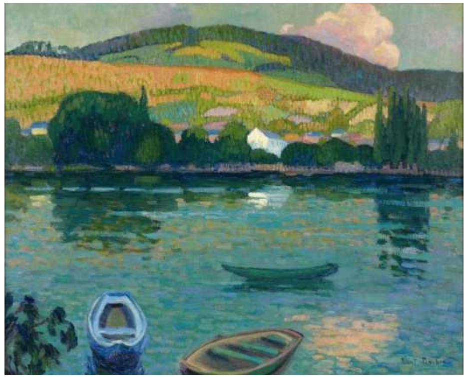
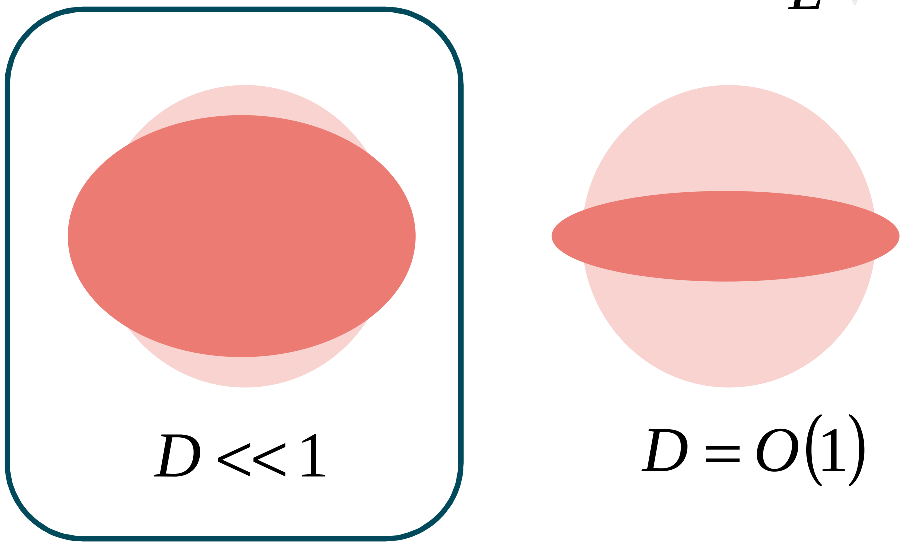
SMALL REDUCED VELOCITY, SMALL MOTION



SMALL MOTION

Displacement number $D = \dfrac{\xi_0}{L}$

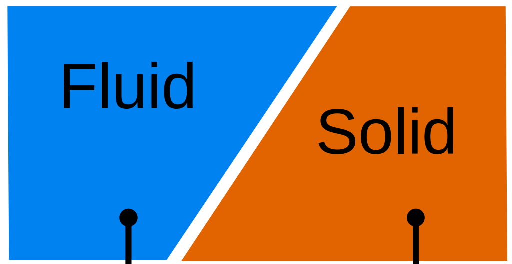


$D << 1$ $\qquad$ $D = O(1)$

$\left(\bar{\ }\right)$ noted $(\ \ )$

EXPANSION OF ALL VARIABLES

$$D << 1$$

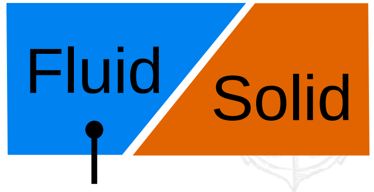


$$P = P_0 + D\, p + D^2 \ldots$$

$$\underline{U} = 0 + D\, \underline{u} + D^2 \ldots$$

$$\underline{\xi} = 0 + D\, q\, \underline{\varphi}$$

EQUATIONS AT THE ORDER ZERO IN D



$$\mathit{div}\,\underline{U} = 0$$

$$\frac{d\underline{U}}{dt} = -\frac{1}{F_D^2}\underline{e}_Z - \underline{\nabla}P + \frac{1}{S_T}\Delta\underline{U}$$

$$P = P_0 + D\ p + \dots$$

$$\underline{U} = 0 + D\ \underline{u} + \dots$$

$$\boxed{\begin{aligned} &\mathrm{div}\ 0 = 0 \\ &0 = -\frac{1}{F_D^2}\underline{e}_Z - \underline{\nabla}P_0 + 0 \end{aligned}}$$

Hydrostatics !

EQUATIONS AT THE ORDER ONE IN D

$$div\,\underline{U} = 0$$

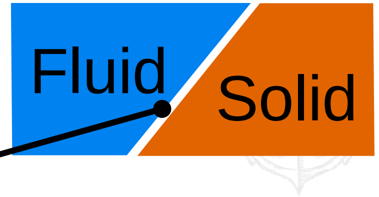
$$\frac{d\underline{U}}{dt} = -\frac{1}{F_D^2}\underline{e}_Z - \underline{\nabla}P + \frac{1}{S_T}\Delta\underline{U}$$

$$P = P_0 + D\,p + \ldots$$

$$\underline{U} = 0 + D\,\underline{u} + \ldots$$

$$\boxed{\begin{gathered} div\,\underline{u} = 0 \\ \frac{\partial\underline{u}}{\partial t} = -\underline{\nabla}p + \frac{1}{S_T}\Delta\underline{u} \end{gathered}}$$

EQUATIONS AT THE INTERFACE



Kinematic condition

$$\underline{U} = D\frac{dq}{dt}\underline{\varphi}$$

$$\underline{U} = 0 + D\,\underline{u} + \dots$$

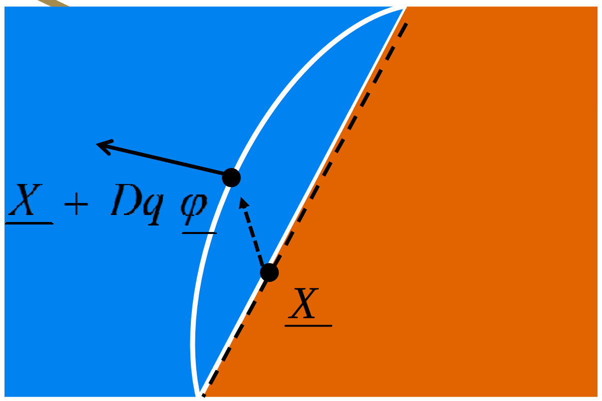
$$\boxed{\underline{u} = \frac{dq}{dt}\underline{\varphi}}$$

Dynamic condition

$$\int_{Interface}\left\{M\left[-P\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla\underline{U} + \nabla^t\underline{U}\right)\right].\underline{n}\right\}.\underline{\varphi}\,dS = Df$$

EQUATIONS AT THE INTERFACE

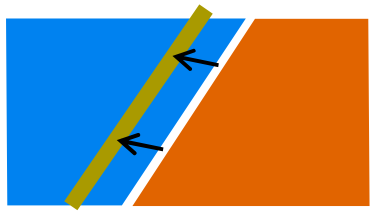
$$\int_{Interface} \left\{ M \left[-P\,\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla\underline{U} + \nabla^t\underline{U}\right)\right].\underline{n}\right\}.\underline{\varphi}\, dS = Df$$



EQUATIONS AT THE INTERFACE

$$\int_{Interface} \left\{ M\left[-P\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla\underline{U}+\nabla^t\underline{U}\right)\right].\underline{n}\right\}.\underline{\varphi}\, dS = Df$$

Pure translation



$$M\underline{\varphi}.\int_{Interface}\left[-P\underline{\underline{I}}+\frac{1}{S_T}\left(\nabla\underline{U}+\nabla^t\underline{U}\right)\right].\underline{n}\,dS = Df$$

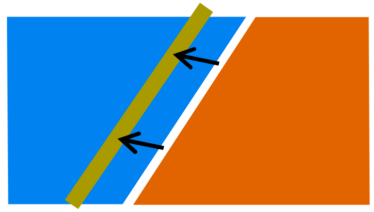
$$P\left(\underline{X}+Dq\underline{\varphi}\right) = P_0\left(\underline{X}+Dq\underline{\varphi}\right) + D\ p\left(\underline{X}+Dq\underline{\varphi}\right) + \ldots$$

$$= P_0\left(\underline{X}\right) + Dq\underline{\varphi}.\underline{\nabla P_0} + D\ p\left(\underline{X}\right) + \ldots$$

$$\underline{U}\left(\underline{X}+Dq\underline{\varphi}\right) = 0 + D\ \underline{u}\left(\underline{X}\right) + \ldots$$

EQUATIONS AT THE INTERFACE

Pure translation



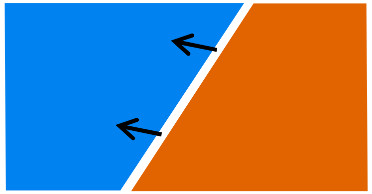
$$M\,\underline{\varphi}.\int_{Interface}\left[-P\underline{\underline{I}}+\frac{1}{S_T}\left(\nabla\underline{U}+\nabla^t\underline{U}\right)\right].\underline{n}\,dS = Df$$

$$P = P_0\left(\underline{X}\right)+Dq\underline{\varphi}.\underline{\nabla}P_0+D\;p\left(\underline{X}\right)+\ldots$$

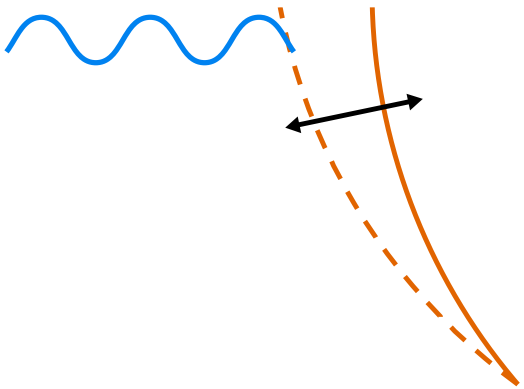
$$\underline{U} = 0 + D\;\underline{u}\left(\underline{X}\right)+\ldots$$

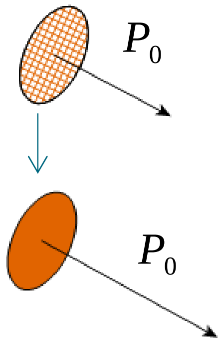
$$DM\,\underline{\varphi}.\int_{Interface}\ldots\ldots\underline{n}\,dS = Df$$

EQUATIONS AT THE INTERFACE

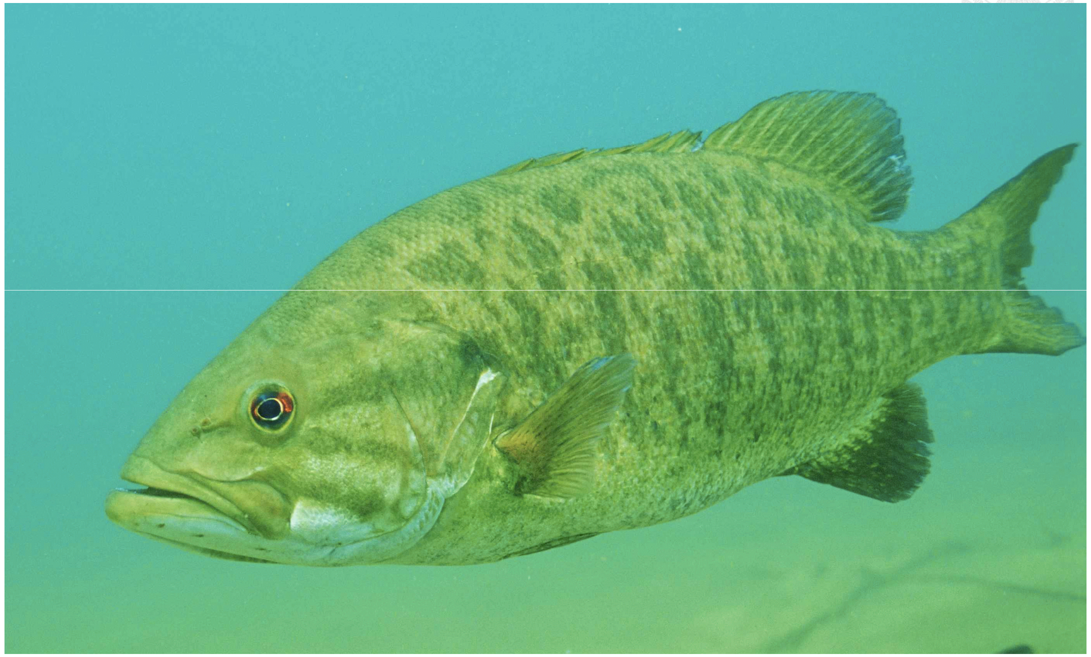


$$M\,\underline{\varphi}.\int_{Interface}\left[-p\underline{\underline{I}}+\frac{1}{S_T}\left(\nabla\underline{u}+\nabla^t\underline{u}\right)\right].\underline{n}\,dS-Mq\underline{\varphi}.\int_{Interface}\left(\underline{\nabla P_0}.\underline{\varphi}\right).\underline{n}\,dS=f$$



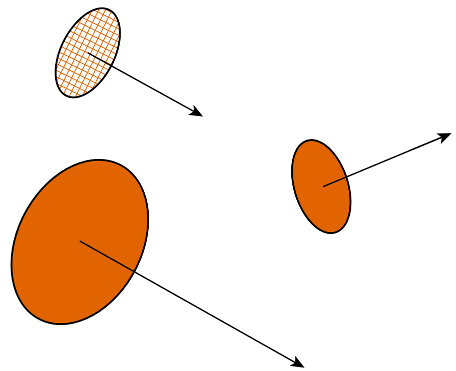


TWO KINDS OF FORCES



TWO KINDS OF FORCES

$$Mq\underline{\varphi}.\int_{Interface}\left(\underline{\nabla}P_0.\underline{\varphi}\right).\underline{n}\,dS$$

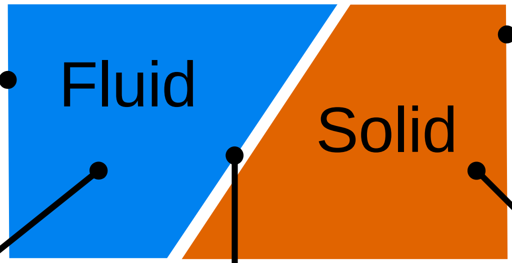


$$-Mq\int_{Interface}\left(\underline{\nabla}P_0.\underline{\varphi}\right)\left(\underline{\varphi}.\underline{n}\right)dS$$

FLUID AND SOLID

Fluid boundary conditions

Solid boundary conditions



$$div\,\underline{u} = 0$$

$$\frac{\partial \underline{u}}{\partial t} = -\underline{\nabla} p + \frac{1}{S_T}\Delta \underline{u}$$

$$\frac{d^2 q}{dt^2} + q = f$$

$$\underline{u} = \frac{dq}{dt}\underline{\varphi}$$

$$M \int_{Interface} \underline{\varphi}.\left[-p\underline{\underline{I}} + \frac{1}{S_T}\left(\nabla \underline{u} + \nabla^t \underline{u}\right)\right].\underline{n}\,dS - Mq \int_{Interface} \left(\underline{\nabla} P_0.\underline{\varphi}\right)\left(\underline{\varphi}.\underline{n}\right)dS = f$$

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