1 $\mathcal{NP}$-completeness

To prove $\mathcal{NP}$-completeness, we look at the problem PartitionByPairs (PBP).

1.1 Transformation

Given an instance of PBP X, we do the following transformation. We calculate the value $B = \frac{\sum_{i=1}^{2n} s_i}{2}$. For each pair in S, (s_{2i-1}, s_{2i}) where $i \in \{1, \ldots, n\}$, we construct a graph as show in Fig. 1, where the labels are the weights of the corresponding edge.

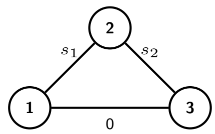

Figure 1: Transformation of a single pair

We order the edges so the mirror of the edge with weight s_{2i-1} is s_{2i} and the mirror of an edge with weight 0 is an edge with weight 0. For multiple pairs, we chain multiple pairs together as shown in Fig. 2. For example, the edge weight distribution for $S = 1, 2, 3, 4, 5, 6$, the weights will be distributed as $w(e_i) = i$ for $i \in \{1, 3, 5, 0, 0, 0, B, B, B, 0, 0, 0, 6, 4, 2\}$.

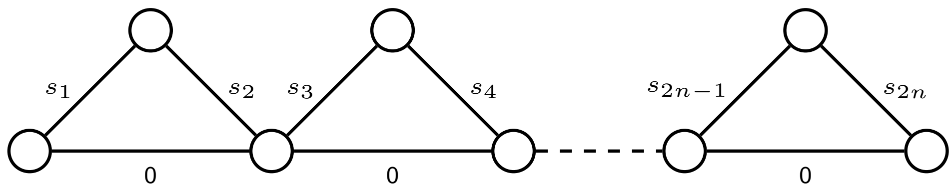

Figure 2: Graph of several pairs

Using this graph and the calculated value B we can query MFMST.

1.2 Proof

We do our transformation in polynomial time. The calculation of B is done in $O(n)$. For each pair, we construct a constant number of nodes and edges in constant time, so the graph can be created in $O(n)$, this means our transformation can be done in $O(n)$.

If the answer to the original problem instance X is YES, it means that a partition where we pick one from each pair equals B. It is possible to pick a spanning tree where we pick one from each pair as shown in Fig 3

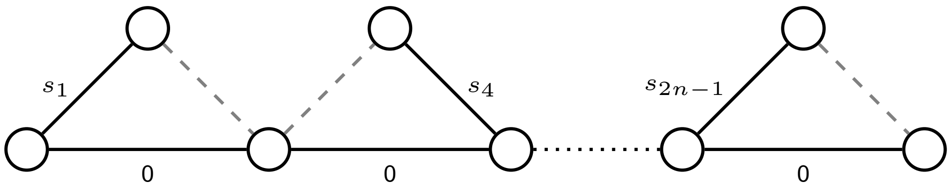

Figure 3: A spanning tree