Random intercept model, integrated mean:

$$\begin{aligned}\text{judgment}_i &\sim N\left(\alpha_{j[i]} + \beta_1(\text{issue}_{\text{non-at-}}), \sigma^2\right)\\ \alpha_j &\sim N\left(\mu_{\alpha_j}, \sigma^2_{\alpha_j}\right), \text{ for id j} = 1, \ldots, \text{J}\end{aligned}$$

Random intercept model, separate mean, treatment coding:

$$\begin{aligned}\text{judgment}_i &\sim N\left(\mu, \sigma^2\right)\\ \mu &= \alpha_{j[i]} + \beta_1(\text{issue}_{\text{non-at-}})\\ \alpha_j &\sim N\left(\mu_{\alpha_j}, \sigma^2_{\alpha_j}\right), \text{ for id j} = 1, \ldots, \text{J}\end{aligned}$$

Random intercept model, separate mean, sum coding:

$$\begin{aligned}\text{judgment}_i &\sim N\left(\mu, \sigma^2\right)\\ \mu &= \alpha_{j[i]} + \beta_1(\text{issue}_1)\\ \alpha_j &\sim N\left(\mu_{\alpha_j}, \sigma^2_{\alpha_j}\right), \text{ for id j} = 1, \ldots, \text{J}\end{aligned}$$

By-item and by-participant random intercepts:

$$\begin{aligned}\text{judgment}_i &\sim N\left(\mu, \sigma^2\right)\\ \mu &= \alpha_{j[i],k[i]}\\ \alpha_j &\sim N\left(\gamma_0^\alpha + \gamma_1^\alpha(\text{issue}_1), \sigma^2_{\alpha_j}\right), \text{ for item j} = 1, \ldots, \text{J}\\ \alpha_k &\sim N\left(\mu_{\alpha_k}, \sigma^2_{\alpha_k}\right), \text{ for id k} = 1, \ldots, \text{K}\end{aligned}$$

By-participants random intercepts and random slopes (at-issueness):

$$\begin{aligned}\text{judgment}_i &\sim N\left(\mu, \sigma^2\right)\\ \mu &= \alpha_{j[i]} + \beta_{1j[i]}(\text{issue}_1)\\ \begin{pmatrix}\alpha_j\\ \beta_{1j}\end{pmatrix} &\sim N\left(\begin{pmatrix}\mu_{\alpha_j}\\ \mu_{\beta_{1j}}\end{pmatrix}, \begin{pmatrix}\sigma^2_{\alpha_j} & \rho_{\alpha_j\beta_{1j}}\\ \rho_{\beta_{1j}\alpha_j} & \sigma^2_{\beta_{1j}}\end{pmatrix}\right), \text{ for id j} = 1, \ldots, \text{J}\end{aligned}$$

By-participant random intercepts and issue-slopes + by-item random intercepts and issue-slopes:

$$\begin{aligned}\text{judgment}_i &\sim N\left(\mu, \sigma^2\right)\\ \mu &= \alpha_{j[i],k[i]}\\ \begin{pmatrix}\alpha_j\\ \gamma_{1j}\end{pmatrix} &\sim N\left(\begin{pmatrix}\gamma_0^\alpha + \gamma_{1k[i]}^\alpha(\text{issue}_1)\\ \mu_{\gamma_{1j}}\end{pmatrix}, \begin{pmatrix}\sigma^2_{\alpha_j} & \rho_{\alpha_j\gamma_{1j}}\\ \rho_{\gamma_{1j}\alpha_j} & \sigma^2_{\gamma_{1j}}\end{pmatrix}\right), \text{ for item j} = 1, \ldots, \text{J}\\ \begin{pmatrix}\alpha_k\\ \gamma_{1k}\end{pmatrix} &\sim N\left(\begin{pmatrix}\mu_{\alpha_k}\\ \mu_{\gamma_{1k}}\end{pmatrix}, \begin{pmatrix}\sigma^2_{\alpha_k} & \rho_{\alpha_k\gamma_{1k}}\\ \rho_{\gamma_{1k}\alpha_k} & \sigma^2_{\gamma_{1k}}\end{pmatrix}\right), \text{ for id k} = 1, \ldots, \text{K}\end{aligned}$$

From fixed effects models to random effects models to mixed effects ones:

Data:
Assume we measure to travel time on rails, 6 in total, three times per rail.

Here's the intercept-only model:

mean travel time

$$\boxed{Y_{ij}} = \boxed{\beta} + \boxed{\epsilon_{ij}}, \quad \boxed{i = 1, \dots, M}, \quad \boxed{j = 1, \dots, n_i}$$

observed travel time for observation j on rail i

independent $N(0, \sigma^2)$ errors

rails

number of observations per rail (3 per)

equivalent to: lm(travel ~ 1)

now lets add a fixed effect: the rails

effect of each rail

$$Y_{ij} = \boxed{\beta_i} + \epsilon_{ij}, \quad i = 1, \dots, M, \quad j = 1, \dots, n_i$$

equivalent to: lm(travel ~ rail $\boxed{-1}$) removes the intercept

Now, a random effects model (~ treating the rails as samples from a population of rails)

$$Y_{ij} = \boxed{\bar{\beta}} + (\beta_i - \bar{\beta}) + \epsilon_{ij}$$

average travel time for the rails in the experiment

Now with the by-rails classification:

mean travel time across the sampled rails

$$Y_{ij} = \boxed{\beta} + \boxed{b_i} + \epsilon_{ij}$$

random variable representing the deviation from the population mean of the mean travel time for the ith rail

In addition, we need the distribution of the random variables b_i + ϵ_{ij}:

$$b_i \sim N(0, \boxed{\sigma_b^2}), \quad \epsilon_{ij} \sim N(0, \boxed{\sigma^2})$$

between-rail variability

within-rail variability

et+k
edition text + kritik
im RICHARD BOORBERG VERLAG
Levelingstraße 6a
81673 München
Tel. (089) 43 60 00 12
Fax (089) 43 60 00 19
www.etk-muenchen.de
info@etk-muenchen.de